Influence of Controlled Quantum-Mechanical Charge and Phase Fluctuations on Josephson Tunneling

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We study the interplay of controlled quantum-mechanical charge and phase fluctuations by measuring their influence on the supercurrent through a small superconducting grain. The grain is connected to leads by two small Josephson tunnel junctions. The phase fluctuations are controlled by the magnetic flux threading a SQUID that connects the grain to a large superconductor. The charge fluctuations are modulated by a voltage on a gate capacitor. As a function of gate voltage we observe a broadening of the Coulomb resonance of the supercurrent for decreasing phase fluctuations, in agreement with theoretical predictions.

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The supercurrent through a small superconducting grain depends on the charging energy E_c of the grain and the Josephson coupling energy E_J of the tunnel junctions that connect the grain to macroscopic leads [1,2]. For large coupling energy ($E_J \gg E_c$) the phase of the superconducting order parameter of the grain is a classical, nonfluctuating variable and the quantum-mechanical fluctuations in the grain charge are very large. As a result, the critical current through the junctions is close to the classical value that is determined by the resistance of the junction and the energy gap of the superconductor [3]. In the other limit ($E_c \gg E_J$) the free motion of Cooper pairs through the grain is blocked by the charging energy. Consequently, the charge fluctuations and the critical current are suppressed and the phase fluctuations are very large. In the intermediate regime ($E_J \sim E_c$) both fluctuations are comparable, and it is interesting to study how their interplay influences the Josephson tunneling through the grain.

In this Letter we report on an experiment in which, for the first time, both the quantum-mechanical charge and phase fluctuations of the grain can be modulated by two independent control parameters. In a specially designed sample it is possible to control the phase fluctuations directly by varying the Josephson coupling between the grain and a large superconducting reservoir [4]. The reservoir has a large capacitance to ground and its phase is a classical variable. Consequently, the energy cost of fluctuations in the grain phase increases when the coupling energy becomes larger. The tunable Josephson coupling between the grain and the reservoir is realized by a SQUID. The SQUID can be viewed as a single junction with an effective Josephson coupling energy that depends on the magnetic flux in the SQUID loop. The supercurrent through the grain depends periodically on the flux in the SQUID. The maximum critical current is reached at zero flux, when the coupling between grain and reservoir is largest. The minimum critical current is observed at one-half flux quantum, corresponding to smallest coupling.

The charge and the phase of the grain are conjugate variables, and their fluctuations are connected by the Heisenberg uncertainty relation $\Delta Q \Delta \phi \geq e$ [5]. Consequently, suppressing the phase fluctuations results in larger charge fluctuations. However, the energy required to change the number of Cooper pairs on the grain by one can be altered by changing the voltage on a gate electrode that is capacitively coupled to the grain. Several groups have shown that in this manner the charge fluctuations can be directly changed, which results in a modulation of the supercurrent with gate voltage [6–9]. The modulation is $2e$ periodic in the charge on the gate capacitor when the charging energy is lower than the odd-even free energy difference and no quasiparticles are tunneling to the grain [7,8]. The maximum supercurrent is observed when resonant tunneling of Cooper pairs takes place at gate charge $Q_g = (2n + 1)e$, where n is an integer.

The layout of the device is shown in Fig. 1. Without the SQUID, this system is known as the superconducting double junction. The grain consists of a small aluminum strip. We have explicitly put the SQUID outside the bias-current loop. Unlike in the experiment described in Ref. [10], the value of the supercurrent through the double junction is not changed by direct quantum interference of this current. The reservoir is formed by a macroscopic measurement lead that has a large capacitance to ground and is otherwise unconnected. Consequently, no dc current can flow through the SQUID. The Al-Al₂O₃-Al junctions are fabricated using electron beam lithography and the standard shadow evaporation technique. The resistance of the junctions is determined from the high-voltage asymptote of the I - V characteristics of the sample. Since the sample is a three-terminal device, the resistance of the two single junctions connected to the grain can be determined independently. Of the junctions in the SQUID the resistance of the two junctions in parallel is known. The resistance of the single junctions is 25 k Ω per junction, giving a Josephson coupling energy $E_J =$

FIG. 1. Schematic of the modified superconducting double junction. The capacitance connecting the superconducting reservoir to ground is much larger than the junction and gate capacitances. The phase of the reservoir θ_{res} is a classical variable, like the phase of the left ($\theta/2$) and right ($-\theta/2$) leads. The phase ϕ and charge Q of the grain form a conjugate pair and fluctuate quantum mechanically. Although the sample is current biased, the frequency-dependent impedance of the leads makes the current source nonideal.

25 μeV . For the junctions forming the SQUID this value is $E_j^{\text{SQ}} = 21 \mu\text{eV}$ per junction, assuming they are equal. The total grain capacitance C_Σ is estimated from the offset voltage at high bias voltages to be 1.3 fF, corresponding to a charging energy $E_c = 62 \mu\text{eV}$. The area of the SQUID loop is much larger than the area of the junctions so the magnetic fields of a few mT used in the experiment have negligible effects on the junctions.

The measurement leads are filtered at room temperature by rfi-feedthrough filters and by high-frequency copper powder filters at mixing chamber temperature. The sample is mounted in a microwave tight copper box and protected from high-frequency radiation by extra copper shields at 700 and 150 mK. The leads are made of normal metal connecting to the aluminum electrodes a few microns away from the junctions. The extensive filtering and electromagnetic shielding is needed to prevent photons from creating quasiparticles in the superconductors. The normal metal close to the junctions makes it possi-

ble for the quasiparticles to reach thermal equilibrium and prevents the poisoning of the Josephson tunneling by out-of-equilibrium quasiparticles [8].

The I - V characteristic of the modified double junction shows a supercurrent branch with a residual resistance that is lower than the measuring sensitivity of 10 Ω . We observe a switching current at which the voltage jumps to the superconducting gap. In Fig. 2 the switching current is shown as a function of the charge Q_g on the gate capacitor for different values of the magnetic flux in the SQUID. The magnetic flux Φ is given in units of the flux quantum $\Phi_0 = h/2e$. The ratio is indicated by $f = \Phi/\Phi_0$. The switching current is periodically modulated by a gate voltage with a maximum at gate charge $Q_g = (2n + 1)e$. We clearly observe that the Coulomb resonance broadens when f is varied from one-half to zero. The inset shows that the value of the switching current at resonance is larger at $f = 0.5$ compared to its value close to 0.5, although the width of the resonance is smallest at $f = 0.5$. We believe this unexpected behavior is caused by the noise of the reservoir as explained below. The measurement temperature $T = 30 \text{ mK}$ is low enough to keep the even-odd free energy difference [7] higher than the charging energy for the parameters of our device.

In Fig. 3 the switching current is plotted as a function of the magnetic flux in the SQUID for different values of the gate charge. We observe a clear periodic modulation with a period of one flux quantum for all values of the gate charge. For gate charge e the modulation is small (a factor of 1.1 between maximum and minimum), which is consistent with the fact that, at this value of the gate charge, the Coulomb blockade for Cooper pairs is lifted and the charge fluctuations are already large. Suppressing the phase fluctuations in this case has only a little effect on the critical current. Away from the resonance, the modulation becomes larger, changing from a factor of 2 at $Q_g = 0.75e$ to a factor of 3.4 at $Q_g = 0$. Note the anomaly of the switching current near half-integer values of f at gate charge $Q_g = e$ and $Q_g = 0.75e$, also caused by the noise of the reservoir.

The system can be described by the following Hamiltonian:

$$H = \frac{(Q - Q_g)^2}{2C_\Sigma} - E_J [\cos(\phi - \theta/2) + \cos(\phi + \theta/2)] - E_J^{\text{SQ}} [\cos(\phi - \theta_{\text{res}} - \pi f) + \cos(\phi - \theta_{\text{res}} + \pi f)], \quad (1)$$

in which Q is the net grain charge and Q_g is the charge on the gate capacitor. The conjugate variable to Q is the phase of the grain ϕ . The left and right leads are characterized by phases $\theta/2$ and $-\theta/2$, respectively, and θ_{res} is the phase of the reservoir. The Josephson coupling of the SQUID depends on the magnetic flux f . From this Hamiltonian the ground-state energy can be calculated using only a finite number of charge states [1]. The energy

is minimized with respect to the phase of the reservoir, which corresponds to zero dc current through the SQUID. The ground-state energy E_{gr} determines the Josephson current via the relation $J(\theta) = (2e/\hbar)(\partial E_{\text{gr}}/\partial \theta)$ [2]. By maximizing $J(\theta)$ the critical current is obtained. The critical current depends on the gate charge Q_g and on the flux in the SQUID.

FIG. 2. Switching current as a function of gate charge for different values of the magnetic flux in the SQUID, indicated by $f = \Phi/\Phi_0$ ($f = 0, 0.25, 0.5$, top to bottom). The inset shows the same plot on a smaller scale for $f = 0.5$ (dashed line) and $f = 0.44$ (solid line). Note that the Coulomb resonance for $f = 0.5$ is smaller and higher than the one for $f = 0.44$ most likely caused by a small, but finite noise level on the reservoir. The switching current is compared to the calculated minimum escape current I_m (solid lines in the main figure). See the text for a definition of I_m .

Although the theoretical model described in the previous paragraph contains all the basic physical ingredients, it is known that even small classical noise on the leads can have a considerable effect on the value of the critical current. The small noise is caused by pickup of the large leads in the dilution refrigerator and by thermal fluctuations. The switching current can be significantly lower than the calculated critical current. The relation between the two is

FIG. 3. Switching current as a function of the magnetic flux in the SQUID for different values of the gate charge $Q_g/e = 1, 0.75, 0.5$, and 0 (top to bottom). Note the anomaly near half-integer values of f , which is also responsible for the effect shown in the inset of Fig. 2. Again, the switching current is compared to the calculated minimum escape current I_m (solid lines in the figure).

complicated and depends on the electromagnetic environment of the double junction [8,11]. The theoretical model (1) qualitatively describes the effects shown in Figs. 2 and 3 very well, except for the anomaly near half-integer values of f . However, to take into account the noise and the electromagnetic environment, we compare our experimental results to a model described in Ref. [8] that is a minimal extension of (1). The electromagnetic environment is modeled by a general admittance $Y(\omega)$ shunting the double junction. The admittance influences the dynamics of the phase difference of the leads θ . The phase difference θ plays the same role as the phase difference of a single junction in the resistively and capacitively shunted junction (RCSJ) model [12]. The phase is described as a particle moving in a periodic potential $E_{gr}(\theta) - (\Phi_0/2\pi)\theta I$, where I is the bias current. In the zero-voltage state the phase particle is stable in one of the minima of the periodic potential. When the bias current exceeds a certain value, the so-called minimum escape current I_m [11], the particle can run down the potential, because above this current the energy gain due to the tilt in the potential exceeds the energy loss due to friction. The first energy barrier the particle sees will be overcome by thermal fluctuations. When the phase particle starts to move a voltage across, the double junction occurs and the system leaves the zero-voltage state. The minimum escape current in this model, in the weak friction limit, can be calculated giving [8]

$$I_m = \frac{\beta}{\Phi_0^2} \frac{Y''(0)}{|Y'(0)|^{3/2}} (E_0)^{3/2}, \quad (2)$$

where E_0 is half the bandwidth of the ground-state energy band $E_{gr}(\theta)$ and β is a dimensionless coefficient. The model assumes that the response time of the electromagnetic environment is much shorter than the characteristic time of the evolution of the external phase difference θ . Furthermore, we assume that the probability for the system to be in the ground state with an odd number of electrons is negligibly small. This is justified, considering our low temperature, small magnetic fields, and good electromagnetic shielding.

In Figs. 2 and 3, we compare our experimental results to the calculated minimum escape current. Given the fact that the scaling parameter $Y''(0)/|Y'(0)|^{3/2}$ is independent of the magnetic flux in the SQUID, we obtain a good fit using an E_J/E_c ratio of 0.28. This is in reasonable agreement with the value of 0.4 determined from the I - V characteristic. The ratio $E_J/E_c = 0.28$ is used to fit all curves. Slightly different values of the scaling parameter $Y''(0)/|Y'(0)|^{3/2}$ are needed to fit the different curves; however, these values do not deviate more than 5%. In Fig. 2, at the top of the curve taken at zero flux, there is a small discrepancy between the measured switching current and the calculated one. This can be explained by the influence of noise on the reservoir. Since the reservoir is formed by a measurement lead, the noise on the reservoir is also caused by pickup and finite temperature. A small,

but finite noise voltage on the reservoir makes its phase fluctuate. As a result, the reservoir phase deviates from its equilibrium value. At this value, the barrier height $2E_0$ for the phase particle is maximum. As a consequence, the fluctuations in the reservoir phase decrease E_0 and thus the minimum escape current. However, the influence of the fluctuating reservoir phase on E_0 depends on the effective Josephson coupling to the grain and thus on the flux in the SQUID. When f approaches half-integer values the switching current gradually returns to its value with a smaller influence of the reservoir noise, which gives rise to the observed anomalies near $f = 0.5$. This effect is not taken into account by theory, but only slightly modifies the main results. In the curves of Fig. 3 taken at $Q_g = e$ and $0.75e$, we observe the same effect. The calculated minimum escape current deviates from the measured switching current near half-integer flux quanta in the SQUID. The curves of the measured switching current in Fig. 3 taken at $Q_g = 0.5e$ and 0 have small deviations from the minimum escape current at more values of the flux in the SQUID. We do not have an explanation for this behavior yet.

In conclusion, we have clearly shown the effect of modulating the quantum-mechanical phase and charge fluctuations on the Josephson current. This was done by using two independent control parameters in one device. Figures 2 and 3 clearly show the competition between the two characteristic energies of the system E_J and E_C . The Josephson energy tries to maintain a constant phase of the grain, which involves the free motion of Cooper pairs through the system. The charging energy tends to block this motion. The competition leads to an interesting behavior of the critical current, which can be qualitatively predicted by quantum mechanics, using a

simple Hamiltonian. Our data fit well by theory when taking into account the electromagnetic environment.

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