IMPROVED PARTIAL DIFFERENTIAL EQUATION-BASED METHOD TO REMOVE NOISE IN IMAGE ENHANCEMENT

H.Y. Tian1, H.M. Cai1, X. Xu2, J.H. Lai1

1 The Sun Yat-Sen University, Department of Automation, Guangzhou, China
2 Brigham and Women’s Hospital, Department of Radiology, Boston, MA, USA

ABSTRACT

Removing noise effectively without destroying important fine structure is a challenging problem in image processing. Among many techniques, diffusion method is shown to be attractive due to its rigorous mathematical background and efficiency. In this paper we propose a well-balanced diffusion method by redefining the diffusion and regularization function. Results obtained from processing synthetic and real images demonstrate that our new method can obtain better performance in terms of removing noises without destroying detailed features of images.

Index Terms— Partial diffusion equation, Adaptive diffusion coefficient, Medical image

1. INTRODUCTION

Nonlinear diffusion methods have proven to be very useful in many applications of image processing, from enhancing image quality [1, 2] to improving image analyses [3–5]. The main idea of a nonlinear diffusion method is to use the observed noisy image as the initial condition of a diffusion equation and allow it to evolve in both spatial and temporal domain by adding a dummy variable of time. Ideally, this diffusive flow should on one hand eliminate the noises and on the other hand keep the important features, such as edges and boundaries, unchanged.

The smoothing effect can be controlled by a diffusion coefficient, which is a monotonically decreasing function of local spatial gradient. When the diffusion coefficient is small in an area with a large local spatial gradient, diffusion process is discouraged but still has effect. In a homogeneous region with a small local spatial gradient, diffusion coefficient is large and diffusion process is encouraged to remove the noise.

Nonlinear diffusion methods, though effective in removing noise, typically are prone to create blocky effects [6]. These artifacts, in fact, are due to the trade-off between noise removal and edge preservation embedded in the diffusion process. Other methods have been developed to circumvent this problem. Rudin-Osher-Fatemi [7] developed another nonlinear diffusion method by minimizing the total variation (TV) of the images under certain conditions. This approach introduces nonlinear diffusion filters and has been used as a regularization method for many other applications where one seeks to identify discontinuous functions [8, 9]. However, it remains an open problem in removing the noisy pixels located near image edges while preserving the edge information. Wang [10] considered the noise and low frequency information as two class classification problems, thus introducing a kernel mapping to magnify their difference before diffusion scheme is employed.

In this paper, we propose a new nonlinear diffusion method that has better performance in the sense that it can reduce noise and blocking effects and preserve the edges and boundaries by an ‘selective’ diffusion coefficient. The remainder of this paper is organized as follows. Section 2 presents classical and widely used diffusion models. Section 3 describes the proposed adaptive diffusion model. Experimental results are shown in Section 4 to demonstrate the performance of the new method. Finally, we conclude the paper in Section 5.

2. NONLINEAR DIFFUSION FILTERING

The basic idea behind these diffusion methods originated from a well known physical heat transfer process which equilibrates concentration differences without creating or destroying mass. This process can be modeled by partial differential equations, and their solutions describe the heat transfer at any particular time. Let the image domain be an open rectangle \( \Omega = (0, a_1) \times (0, a_2) \), \( \Gamma \equiv \partial \Omega \) be its boundary and the observed image \( u_0(x) \) be represented by a bounded function \( u : \Omega \rightarrow \mathbb{R} \). The classical Perona and Malik (PM) diffusion equation [11] is

\[
\begin{align*}
\frac{d}{dt} u(x, y, t) &= \text{div} \left( g(|\nabla u|) \nabla u \right), \\
\frac{d}{dt} u(x, y, 0) &= u_0.
\end{align*}
\]

Here the diffusion coefficient \( g(\cdot) \) (also called flux term) is a nonnegative function of the magnitude of the image gradient...

---

This work was supported by NSFC under award number 60902076, and NSF of Guangdong Province, China under award number 9451027501002551
$|\nabla u| = \sqrt{u_x^2 + u_y^2}$ and set to be $g(x,y,t) = \frac{1}{1+\left(\frac{t-t_0}{st} \right)^2}$, where $k$ is a trade-off term that can be set depending on the estimation of the noise level.

PM diffusion process can effectively preserve the edge having strong contrast gradient. But in the inferior contrast gradient situation, the edge feature can be smoothed out. To levigate this problem, Total Variation (TV) was introduced to preserve level image while removing noises simultaneous, $E_L(u) = \int_{\Omega} |\nabla u| dx$. (2)

In order to obtain a stable solution, Rudin, Osher and Fatemi propose to add a regularization term (RTV) [7]. The corresponding Euler-Lagrange equation gives

$u_t = \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - \lambda (u - u_0)$ (3)

In [12], the authors proposed a speedy version of the diffusion (ITV, in abbreviated),

$u_t = |\nabla u| \nabla \cdot \left( \frac{\nabla u}{|\nabla u|} \right) - \lambda (u - u_0)$ (4)

where $\lambda$ is a trade-off parameter and defined empirically. Eq. (4) approximately satisfies morphology theory, so stair case effect can be largely removed. Wang et al. [10] suggested to use a kernelized gradient operator instead to refine edge detection and better control of the diffusion. The diffusion equation has similar form to the classical ones

$u_t = \nabla \cdot (g(|\nabla \Phi(u)|) \nabla u)$ (5)

The new edge detector, $|\nabla \Phi(u)|$, is the gradient magnitude in a feature space defined by a mapping function $\Phi$,

$|\nabla \Phi(u)|_p = \left\{ \frac{1}{|\xi_p|} \sum_{q \in \xi_p} (K(u_p, u_q) + K(u_q, u_p) - 2K(u_p, u_q)) \right\}^{0.5}$ (6)

where $\xi_p$ represents the spatial neighborhood of pixel $p$ (eight neighboring pixels around $p$), and

$K(x, y) = \exp \left( -\frac{|x-y|^2}{2 \sigma^2} \right)$ (7)

This kernelized adaptive diffusion (KAD) method is proved to be effective in edge detection for images with low signal-to-noise ratios (SNR). However, it still suffers from the blocky effects, leading to small clusters located near to edges.

3. AN ADAPTIVE DIFFUSION COEFFICIENT SCHEME

In this section, we describe an improved adaptive diffusion method to remove noises and enhance image simultaneously.

Our model takes the following form:

$$u_t = \text{div} (f \cdot s(k - |\nabla u|) \cdot \nabla u) - f \cdot (1 - s(k - |\nabla u|))(u - u_0)$$ (8)

where $\text{div}$ is the divergence operator and $s(t)$ is a unit step function defined as:

$$s(t) = \begin{cases} 
1 & \text{if } t > 0 \\
0 & \text{otherwise}
\end{cases}$$

The parameter $k$ is a trade-off term depending on image gradient. The power of diffusion is controlled by function $f$ and set to be

$$f = \frac{(||\nabla u|| - k)^2}{1 + (||\nabla u|| - k)^2}.$$ (9)

This new diffusion scheme achieves two objectives: (1) when it is in a relatively homogeneous region, inequality $||\nabla u|| < k$ is satisfied, thus leading to $f \cdot s = f$, which implies that high level of diffusion is allowed and regularization is discouraged. (2) when it is near an edge, usually we have $||\nabla u|| \geq k$, or $f \cdot s = 0$, thus resulting in slight diffusion and larger regularization. In this case, the second term of Eq. (8) plays a more dominant role when the diffusion process is approaching edges and thus adaptively preserves the edges.

Fig. 1. Illustration of adaptive diffusion coefficient. solid line and dashed line denote $f \cdot s(k - ||\nabla u||)$ and $f \cdot (1 - s(k - ||\nabla u||))$ (solid line) are drawn to illustrate their behavior. In homogeneous region where $||\nabla u|| < k$, the diffusion level could be raised higher in order to quickly remove noises, meanwhile preservation of edge information is unnecessary. Thus, we choose a large value of flux term $f \cdot s(k - ||\nabla u||)$ to direct diffusion, and a small value $(1 - f \cdot s(k - ||\nabla u||))$ to direct preservation of fine details. On the contrary, in area nearing edges with high gradient altitude, low level diffusion is desired in preventing over-smoothing and regularized term should be employed to penalize the diffusion process. The design of the diffusion and regularization terms in Eq. (8) satisfy these two requirements.

In Fig. 1, the diffusion term $f \cdot s(k - ||\nabla u||)$ (dashed line) and regularization term $f \cdot (1 - s(k - ||\nabla u||))$ (solid line) are drawn to illustrate their behavior. In homogeneous region where $||\nabla u|| < k$, the diffusion level could be raised higher in order to quickly remove noises, meanwhile preservation of edge information is unnecessary. Thus, we choose a large value of flux term $f \cdot s(k - ||\nabla u||)$ to direct diffusion, and a small value $(1 - f \cdot s(k - ||\nabla u||))$ to direct preservation of fine details. On the contrary, in area nearing edges with high gradient altitude, low level diffusion is desired in preventing over-smoothing and regularized term should be employed to penalize the diffusion process. The design of the diffusion and regularization terms in Eq. (8) satisfy these two requirements.
One important step involved in Eq. (8) is to estimate the value of $k$. A larger value leads to a “cleaner” image but “smoother” edges, while a smaller $k$ achieves better edge preservation but less noise removal. In this paper, the value of $k$ is estimated by the help of $p$-norm suggested in [13]. Given a 2-D image $u(x, y, t)$ at time $t$, the $p$-norm is defined as

$$||u||_p = \sum_{(i,j) \in u} (||u(x, y, t)||^p)^{\frac{1}{p}}$$

and the value $k(t)$ is estimated by

$$k(t) = \frac{\lambda \sigma \cdot ||u(t)||_p}{r \times c}$$

where $r$ and $c$ are row and column numbers of image $u$, respectively. $\sigma$ is a constant proportional to image intensity, and $\lambda$ is a trade-off parameter. In this paper, we set the value of $\lambda$ to be 0.45.

4. EXPERIMENTS

In this section, we validated the proposed diffusion methods on two kinds of different images: synthetic digital image and computer tomography (CT) images of human lymph nodes, provided by State Key Laboratory of Oncology, the Sun Yat-Sen University.

4.1. Synthetic Digital Image

We apply our method to a synthetic image, shown in Fig. 2(a), which is degraded by additive Gaussian noise with SNR value of 9.46db. The noisy image is processed by the proposed adaptive diffusion algorithm, and classical methods including RTV, KAD, and ITV algorithm. The results are shown in Fig. 2(b-e), respectively. For comparison, we apply Canny method to detect the edges in the processed images. The corresponding results are shown in Fig. 2(f-i), respectively, for visual validation. The differences between our edge enhanced diffusion and the other three are fairly clear. Our method preserves the edges while efficiently reducing the noise. In particular, the two small circles inside the cross have sharper boundaries and are closer to the true image than the images derived from the other methods. Our method also effectively removes the noises and thus has a clean edges, compared to edges after ITV method, which leading to undesired intensity clusters.

4.2. Medical Image

As a second example, we apply the proposed method on a CT image of human lymph node, Fig. 3. The raw image, Fig. 3(a) was obtained from a 32 slice CT scanner. The purpose of this medical experiment is to segment lymph node from surrounding organs, thus having accurate nodal edge information. Medical technician was asked to label interested nodes in Fig. 3(a) by green arrows. The results from the four diffusion methods are shown in Fig. 3(b-e). Their corresponding edges are shown in Fig. 3(f-i), respectively. From the figure we can conclude that the proposed adaptive diffusion method performs better in preserving fine edge than other schemes. The edges obtained after processing the raw image by the new method clearly stand out from the surrounding organs. In Fig. 3(f) the three lymph nodes are enclosed by complete contours that are separated from nearby edges and boundaries. While in Fig. 3(g-i) the boundaries of the three lymph nodes are either broken and unseparated from nearby edges.

5. CONCLUSION

We developed a well-balanced noise removal diffusion method. Our method achieves good results in removing image noise while preserving image details such as edges and contours without smoothing out important structure information and is less sensitive to noise level. By comparing our method with the iterative total variation method, KAD and the well-know regularized TV method, we found the new method demonstrate better performance on both synthetic images and real medical images. In future, we shall refine the estimation of parameter $k$ involved in Eq. (8).
Fig. 3. Comparison results among the four diffusion methods on CT image. (a) Zoomed region on a CT image. The interested lymph nodes have already been labeled with green arrows. (b-e) are diffused results after the proposed method, RTV, KAD, and ITV, respectively. (f-i) are edge images by applying Canny operator on (b-e), respectively.

6. REFERENCES


