Numerical modeling of the flow in extended stinger completions

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Abstract

Pressure drop over horizontal wells causes unequal draw down along the well, thus reducing the effectiveness of the toe of the well, and increasing the tendency for water and gas coning at the heel. Completions have been proposed to combat this effect using a stinger that extends from the heel into the horizontal section. This reduces the magnitude of the pressure drop over the well bore, since the well is effectively split in two segments of reduced length. If the stinger also has an inflow opening at the heel of the well, a further reduction in the well bore pressure drop can be obtained.

This report presents a semi-analytical model to compute the flow in extended stinger completions with or without inflow at the heel. The model is an extension of the classic paper of Dikken (JPT, Nov. 1990, 1426-1433) which gives a coupled analysis of steady-state well bore and reservoir flow for a conventional horizontal well in terms of two first order differential equations. Here we arrive at two systems of first order equations describing pressure and flow in the annulus around the stinger and in the part of the well without stinger. The model can be used to obtain a rough indication of the optimum configuration of a stinger completion, especially for quick evaluation as a precursor for more detailed analysis with a reservoir simulator. The report presents a worked example illustrating the drastic reduction in pressure drop along the well bore with an extended stinger completion as compared to a conventional completion.
1 Introduction

1.1 Well bore pressure drop

Pressure drop over horizontal oil wells has been identified as a potential problem in various publications; see e.g. References [1] to [4]. The immediate consequence of well bore pressure drop is an unequal draw down distribution along the well, as indicated in Figure 1. Here, the draw down $\Delta p$ is defined as the difference between a reference reservoir pressure $p_R$ at a large distance from the well and the well bore pressure $p$:

$$\Delta p(x) = p_R - p(x),$$

where $x$ is a co-ordinate along the well bore running from the heel to the toe of the well. The reduced draw down near the toe of the well reduces the effectiveness of the well, and results in a reduced benefit of increasing the well length. Another effect of the uneven draw down profile is that the well has a much higher tendency for water and gas coning at the heel than at the toe. This may lead to early water or gas breakthrough at the heel, which in turn may drastically reduce oil production and, in the worst case, render the well useless.

![Figure 1: Horizontal well with draw down $\Delta p$ decreasing from the heel to the toe.](image)

1.2 Extended stinger completion

Figure 2 depicts a horizontal well with length $L_w$ completed with a stinger which extends from the packer at the heel of the well into the horizontal section over a length $L_{st}$. The purpose of such an “extended stinger” is to move the point of highest draw down from the heel of the well towards the toe [5, 6, 7]. This reduces the magnitude of the pressure drop over the well bore, since the well is effectively split in two segments of reduced length. If the stinger is also connected to the annulus at the heel of the well, a further reduction in the well bore pressure drop can be obtained, see Figure 2. In practice the intermediate inflow opening
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can be created through the use of a sliding sleeve which can be opened or closed through well intervention. For the purpose of modeling, we will consider the opening as a flow restriction with similar characteristics as a surface choke.

\[ L_w \]
\[ L_{st} \]
\[ \Delta p \]
\[ q_{heel} \]
\[ q_{st} \]
\[ q_{an} \]
\[ q_{s,R} \]
\[ q_w \]

**Figure 2:** Horizontal well with an extended stinger allowing for inflow from the annulus to the stinger at two points: at the end of the stinger, and at an intermediate inflow point near the heel of the well. The draw down \( \Delta p \) varies along the length of the well, but the difference between maximum and minimum values is much lower than in case of the conventional completion depicted in Figure 1.
2 Governing equations

2.1 Well bore pressure drop

We restrict the analysis to single phase (liquid) flow through a homogeneous reservoir, drained by a fully penetrating horizontal well bore. The pressure drop per unit length over the well bore is characterized with a standard expression for single-phase flow in pipes with a circular cross section [8]:

\[
\frac{dp}{dx} = \frac{\rho}{2d} f(v) * v^2 \text{sgn}(v) = -\frac{8 \rho}{\pi^2 d^5} f(q) * q^2 \text{sgn}(q) .
\]

(2)

Here \( \rho \) is liquid density, \( d \) is the inside diameter of the reservoir conduit (e.g. perforated casing, slotted liner or pre-packed screen), \( f \) is the Moody-Weissbach friction factor, \( v \) is the flow velocity and \( q \) is the flow rate. The abbreviation \( \text{sgn}(x) = x/|x| \) represents the sign of variable \( x \). The velocity \( v \) and the flow rate \( q \) are positive for injection wells and negative for production wells. They are functions of \( x \), as is the friction factor \( f \). The friction factor \( f \) is also a function of \( q \), through its dependence on the Reynolds number \( N_{Re} \) which is defined as

\[
N_{Re} = \frac{\rho d |v|}{\mu} = \frac{4 \rho |q|}{\pi \mu d} ,
\]

(3)

where \( \mu \) is the dynamic viscosity, and of the dimensionless pipe roughness \( \varepsilon \), defined as

\[
\varepsilon = \frac{e}{d} ,
\]

(4)

where \( e \) is the pipe roughness with dimension length. For Reynolds numbers lower than 2000 the flow is laminar, and \( f \) is given explicitly by

\[
f = \frac{64}{N_{Re}} ,
\]

(5)

while for Reynolds numbers larger than 3000 the flow is turbulent and \( f \) is given implicitly by the Colebrook-White equation:

\[
\frac{1}{\sqrt{f}} = -2 \log_{10} \left( \frac{\varepsilon}{3.71} + \frac{2.51}{N_{Re} \sqrt{f}} \right) .
\]

(6)

For flow in the intermediate regime, characterized by Reynolds numbers between 2000 and 3000, we use a linear interpolation between equations (5) and (6).

Inflow from the reservoir into production wells, or outflow to the reservoir from injection wells has an influence on the flow profile in the well bore and thus on the friction forces. Furthermore, the fluid velocity over the length of the well bore is not constant, and the equations used above neglect the corresponding acceleration forces. Reference [9] gives a detailed account of inflow and acceleration effects. In many practical circumstances the effects are relatively small, and we will therefor neglect them in our analysis.
2.2 Annular flow

To model pressure drop in an annulus, we use the concept of a hydraulic radius \( r \) defined as the ratio of the cross-sectional area \( A \) and the wetted perimeter \( P \) \[8\]. For pipe flow the hydraulic radius becomes:

\[
r = \frac{A}{P} = \frac{\pi d^2}{4 \pi d} = \frac{1}{4} d,
\]

while for flow in an annulus with inside diameter \( d_i \) and outside diameter \( d_o \) we find

\[
r_a = \frac{A_{an}}{P_{an}} = \frac{\pi (d_o^2 - d_i^2)}{2 \pi (d_o + d_i)} = \frac{1}{4} (d_o - d_i).
\]

Consequently we can rewrite equations (2) to (4) for the annular case as:

\[
\frac{dp_{an}}{dx} = -\frac{\rho}{2(d_o - d_i)} f_{an}(v_{an}) v_{an}^2 \text{ sgn}(v_{an}),
\]

\[
N_{Re,an} = \frac{\rho v_{an} (d_o - d_i)}{\mu}, \quad \text{and}
\]

\[
e_{an} = \frac{\sqrt{e_o + e_i}}{(d_o - d_i)}.
\]

2.3 Reservoir inflow

Following Dikken [1], flow from the reservoir into the well bore is described with a specific Productivity Index \( J_s \) according to:

\[
q_{s,R}(x) = J_s \Delta p(x) = J_s [p_R - p(x)],
\]

where \( q_{s,R} \) is specific flow from the reservoir, i.e. flow from the reservoir per unit length. The specific Productivity Index \( J_s \) is a function of reservoir dimensions, and fluid and formation properties as illustrated in more detail in Appendix B. Note that we follow a two-dimensional approach, i.e. we assume that all reservoir flow occurs perpendicular to the well bore only. The inflow from the reservoir per unit length should now equal the increase in well bore flow:

\[
\frac{dq}{dx} = q_{s,R}.
\]

Substitution of equation (12) in equation (13), differentiation with respect to \( x \), and subsequent substitution of equation (2) results in:

\[
\frac{d^2 q}{dx^2} - \frac{8J_s \rho}{\pi^2 d^3} f(q) * q^2 \text{ sgn}(q) = 0.
\]
Expression (14) is the governing equation for steady-state single-phase flow in a well bore conduit with constant reservoir fluid inflow. It is a nonlinear second order differential equation. Two boundary conditions are required to fully specify the problem.
3 System model

3.1 System of equations

Figure 3 depicts an extended stinger completion as a network of nodes and elements. This representation is similar to those used in the analysis of electrical or mechanical systems. We distinguish a total of 24 unknowns: 10 internal element variables as indicated in the Figure, 4 nodal pressures, 8 branch flow rates (upstream and downstream of each element), and 2 flow rates at the toe and the heel of the system. To solve for these unknowns we need 24 equations: 10 element equations, 4 nodal equations, 8 branch equations, and 2 prescribed system boundary conditions. Flow rates in the positive $x$ direction are defined positive. Therefore, we use superscripts - to indicate upstream flow rates and pressures, and superscripts + for the downstream variables.

The element equations for the well bore follow from (2), (12) and (13) as:

$$\frac{dp_w}{dx} = -\frac{8\rho}{\pi^2 d_{ci}^2} \int_0^x \left(q_{R}(x) - q_{W}(x)\right) \mathrm{sgn}(q_{W}) \ dx,$$

$$q_{s,R}(x) = J_x \left[p_{R} - p_{w}(x)\right],$$

$$\frac{dq_w}{dx} = q_{s,R},$$

where $d_{ci}$ is the inside diameter of the reservoir conduit. Similarly, for the annulus we find from (9), (12) and (13):

$$\frac{dp_{an}}{dx} = -\frac{8\rho}{\pi^2 (d_{ci} - d_{st,0})^2} \int_0^x \left(q_{an}(x) - q_{an}^2 \right) \mathrm{sgn}(q_{an}) \ dx,$$

$$q_{s,R}(x) = J_x \left[p_{R} - p_{an}(x)\right],$$

$$\frac{dq_{an}}{dx} = q_{s,R},$$

where $d_{st,0}$ is the outside diameter of the stinger. The first element equation for the stinger follows from equation (9), with slightly adapted variables, while the second is a trivial equation expressing that $q_{st}$ is a constant. Denoting the inside diameter of the stinger with $d_{st,i}$, we can write:
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\[
\frac{dp_{st}}{dx} = -\frac{8\rho}{\pi^2 d_{st,i}^2} f_{st}(q_{st}) \cdot q_{st}^2 \cdot \text{sgn}(q_{st}) ,
\]
(21)

\[
\frac{dq_{st}}{dx} = 0 .
\]
(22)

The pressure drop over the choke (i.e. the intermediate inflow point), \( \Delta p_{ch} \), as a function of the flow through the choke, \( q_{ch} \), is defined with a standard expression for sub-critical flow of liquids through a restriction [8]. The continuity in flow rate results in a trivial equation again:

\[
\Delta p_{ch} = -\frac{\rho}{2C_{ch} A_{ch}} q_{ch}^2 \cdot \text{sgn}(q_{ch}) ,
\]
(23)

\[
\Delta q_{ch} = 0 .
\]
(24)

Here, \( C_{ch} \) is a dimensionless choke coefficient and \( A_{ch} \) is the surface area of the choke opening. The choke coefficient depends on the choke geometry and needs to be determined from experiments. The nodal equations are obtained from the requirement that the sum of the flow rates in each of the nodes vanishes:

\[
\text{node 1: } q_{\text{heel}}^- - q_{\text{w}}^- - q_{\text{t}}^- = 0 \\
\text{node 2: } q_{\text{ch}}^- - q_{\text{an}}^- = 0 \\
\text{node 3: } q_{\text{an}}^- + q_{\text{st}}^- - q_{\text{w}}^- = 0 \\
\text{node 4: } q_{\text{w}}^- - q_{\text{hoe}}^- = 0
\]
(25)

while the branch equations express the that elements connected to the same node see the same pressure:

\[
p_{\text{ch}}^- = p_{\text{w}}^- = p_1 , 
\quad p_{\text{ch}}^+ = p_{\text{an}}^- = p_2 , 
\quad p_{\text{an}}^+ = p_{\text{st}}^- = p_{\text{w}}^- = p_3 , 
\quad p_{\text{w}}^+ = p_4 .
\]
(26)

The remaining 2 equations follow from the boundary conditions specifying either pressure or flow rate at each end of the system. Here, we choose:

\[
p_1 = \hat{p}_{\text{heel}} , 
\quad q_{\text{hoe}} = 0 ,
\]
(27)

where the hat on \( \hat{p}_{\text{heel}} \) indicates that it’s value has been prescribed. The second boundary condition, which states that the well bore flow vanishes at the toe of the well, is in line with our two-dimensional approach, i.e. with the assumption that all reservoir flow occurs perpendicular to the well bore.

### 3.2 Fully penetrating stinger

First we consider the case were \( L_{st} = L_w \). In that case the total number of unknowns reduces from 24 to 18, and we do not need the element equations for \( q_w^- , q_{\text{s,r}}^- \) and \( p_w^- \), and the branch equations for \( q_w^- \) and \( q_w^+ \), while the nodal equations for nodes 3 and 4 can be combined to:

\[
q_{\text{an}}^+ + q_{\text{st}}^- - q_{\text{hoe}}^- = 0 .
\]
(28)
Because \( q_{st} \) in equation (21) is independent of \( x \) we can integrate the element equations for the stinger directly to obtain

\[
p_{st}^+ = p_{st}^- - \frac{8 \rho L_{st}}{\pi^2 d_{st,j}^5} f_{st}(q_{st}) \cdot q_{st}^2 \cdot \text{sgn}(q_{st}) \quad \text{and} \quad q_{st}^+ = q_{st}^- = q_{st}^-.
\] (29)

Similarly we can derive from the element equations for the choke that

\[
p_{ch}^+ = p_{ch}^- - \frac{\rho}{2 C_{ch} A_{ch}} q_{ch}^2 \cdot \text{sgn}(q_{ch}) \quad \text{and} \quad q_{ch}^+ = q_{ch}^- = q_{ch}^-.
\] (30)

We can now reduce the problem to a system of two first order differential equations for the annulus: one for the pressure, given by expression (18), and one for the flow rate which follows from combining equations (19) and (20) leading to

\[
\frac{dq_{an}}{dx} = J_s \left[ p_R - p_{an}(x) \right].
\] (31)

The boundary conditions can be derived through manipulation of the various system equations specified above, leading to:

\[
p_{an}^- = \hat{p}_{heel} - \frac{\rho}{2 C_{ch} A_{ch}} \left( q_{an}^- \right)^2 \cdot \text{sgn}(q_{an}^-)
\]

\[
p_{an}^+ = \hat{p}_{heel} + \frac{8 \rho L_{an}}{\pi^2 d_{st,j}^5} f_{st}(q_{an}^+)^2 \cdot \text{sgn}(q_{an}^+)
\] (32)

where we have used the shorthand notation

\[
p_{an}^- = p_{an} \big|_{x=0}, \quad p_{an}^+ = p_{an} \big|_{x=L_a}, \quad q_{an}^- = q_{an} \big|_{x=0} \quad \text{and} \quad q_{an}^+ = q_{an} \big|_{x=L_a}.
\] (33)

We can simplify equations (31), (32) and (33) by making use of the following dimensionless variables:

\[
x_D = \frac{x}{L_w}, \quad \lambda = \frac{L_{st}}{L_w}, \quad p_{an,D} = \frac{\Delta p_{an}}{p_R - \hat{p}_{heel}}, \quad q_{an,D} = \frac{q_{an}}{J_s \Delta p_{heel} L_w},
\]

\[
\xi_{ch} = \frac{J_s \rho L_{an}^2 \Delta p_{heel}}{2 C_{ch} A_{ch}^2}, \quad \xi_{an} = \frac{8 J_s^2 \rho L_{an} \Delta p_{heel}}{\pi^2 (d_{c,i}^2 - d_{st,o}^2)(d_{c,j}^2 - d_{st,o}^2)^3}, \quad \xi_{st} = \frac{8 J_s^2 \rho L_{an} \Delta p_{heel}}{\pi^2 d_{st,j}^5}.
\] (34)

Substitution of expressions (34) results in:

\[
\begin{align*}
\frac{dq_{an,D}}{dx_D} &= p_{an,D} \cdot \\
\frac{dp_{an,D}}{dx_D} &= \xi_{an} f_{an}(q_{an,D}) \cdot q_{an,D}^2 \cdot \text{sgn}(q_{an,D}) \cdot \\
p_{an,D}^- = 1 + \xi_{ch} \cdot (q_{an,D}^-)^2 \cdot \text{sgn}(q_{an,D}^-) \quad \text{and} \quad p_{an,D}^+ = 1 - \lambda \xi_{st} f_{st}(q_{an,D}^+) \cdot (q_{an,D}^+)^2 \cdot \text{sgn}(q_{an,D}^+),
\end{align*}
\] (36)

where
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\[ q_{an,D}^+ = q_{an,D}^{|x_0=0}, \quad q_{an,D}^- = q_{an,D}^{|x_{n-1}=1}, \quad p_{an,D}^+ = p_{an,D}^{|x_0=0}, \quad \text{and} \quad p_{an,D}^- = p_{an,D}^{|x_{n-1}=1}, \]

with \( \lambda = 1 \) for the case of the fully penetrating stinger. The friction factors \( f_{an} \) and \( f_{st} \) are functions of the annulus and stinger Reynolds numbers and wall roughnesses:

\[ N_{Re,an} = \frac{4 \rho q_{an} (d_{c,i}^2 - d_{st,o}^2)}{\pi \mu}, \quad N_{Re,st} = \frac{4 \rho q_{st}}{\pi \mu d_{st,i}}, \quad \varepsilon_{an} = \frac{1}{2} \frac{e_{c,i} + e_{st,o}}{d_{c,i}^2 - d_{st,o}^2}, \quad \varepsilon_{st} = \frac{e_{st,j}}{d_{st,j}^2}. \]

Equations (35) can be solved numerically, e.g. using the shooting method [10]. This involves numerical integration from the heel to the toe, starting with an arbitrary choice for \( q_{an,D}^+ \) and varying it’s value until both the boundary conditions are satisfied.

### 3.3 Partially penetrating stinger

In case \( \lambda < 1 \), i.e. for \( L_s < L_w \), the problem can be represented by two systems of first order equations: one for the annulus around the stinger, as presented in expression (35) above, and one for the well bore to the right of the stinger. Introducing the additional dimensionless variables

\[ p_{w,D} = \frac{\Delta p_{w}}{\Delta p_{heel}}, \quad q_{w,D} = \frac{q_{w}}{J_s \Delta p_{heel} L_w}, \quad \xi = \frac{8 \rho L_w^3}{\pi^2 d_{c,i}^2}, \]

we can write the system of equations for the well bore as:

\[
\begin{aligned}
\frac{dq_{w,D}}{dx} &= p_{w,D}, \\
\frac{dp_{w,D}}{dx} &= \xi f_w (q_{w,D})^{*} q_{w,D}^{2} \text{sgn}(q_{w,D}),
\end{aligned}
\]

with a friction factor \( f_w \) which is a function of \( N_{Re,w} \) and \( e_w \) in the usual fashion. The boundary conditions are specified at \( x_D = 0 \) for annulus and at \( x_D = 1 \) for the well bore as

\[ p_{an,D}^- = 1 + \xi_{ch} \cdot (q_{an,D})^2 \text{sgn}(q_{an,D}) \quad \text{and} \quad q_{w,D}^+ = 0, \]

while continuity of flow and pressure at \( x_D = \lambda \) requires that

\[ p_{an,D}^+ = p_{w,D}^- = 1 - \lambda \xi f_{st} (q_{an,D}^+ - q_{w,D}^-) \cdot (q_{an,D}^+ - q_{w,D}^-)^2 \text{sgn}(q_{an,D}^+ - q_{w,D}^-). \]

The two systems of equations (35) and (40) can be solved through “shooting to a fitting point” [10]. This involves simultaneous numerical integration of each system from the outside towards the fitting point at \( x_s = \lambda \) with known boundary conditions \( p_{an,D}^- \) and \( q_{w,D}^+ \) as specified in (41) and with arbitrarily chosen values for \( q_{an,D}^- \) and \( p_{an,D}^+ \). Using Newton Raphson iteration, the integration is repeated with improved estimates for \( q_{an,D}^- \) and \( p_{w,D}^- \) until the equations for \( p_{an,D}^+ \) and \( p_{w,D}^- \) at the fitting point, as specified in (42), are satisfied.
3.4 Additional inflow points

A further improvement of the tilted bucket concept could be obtained by moving the position of the choke towards the heel of the well to create annular flow towards the stinger inflow point both from the left and from the right. A next step would be the introduction of multiple inflow points (chokes) along the stinger. The corresponding equations for the various system elements are identical to those presented above, but their interrelationships becomes quite complicated. Such systems are best analyzed using a systematic matrix-vector formalism, which is outside the scope of this report. In addition, rather than using direct numerical integration of the governing equations, it may be more advantageous to discretize them using finite difference or finite element techniques. Commercial piping network analysis packages based on these techniques are available.
4 Example

4.1 Pressure drop number

The equations developed above were programmed in a small C++ program to enable assessment of the effect of various types of stinger completions. As an example, consider a fully penetrating horizontal well in a box-shaped reservoir. The reservoir has quite a good permeability: $k_h = 0.493 \times 10^{-12} \text{ m}^2$ (500 mD), $k_v = 0.025 \times 10^{-12} \text{ m}^2$ (25 mD). The resulting good inflow performance, apparent as a high specific PI, has been derived in Appendix B. The well is 2000 m long and has a large inside diameter (ID) of 0.163 m. Other input parameters are given in Tables 1 and B-1. Table 1 also displays the results of the numerical simulation for the conventional completion without a stinger. To characterize the effect of well bore pressure drop compared to the draw down at the heel, we introduce the pressure drop number $N_{\Delta p}$ defined as the maximum pressure difference in the well bore divided by the maximum draw down:

$$N_{\Delta p} = \frac{P_{\text{max}} - P_{\text{min}}}{P_R - P_{\text{min}}} = \frac{P_{\text{toe}} - P_{\text{heel}}}{P_R - P_{\text{heel}}}, \quad 0 < N_{\Delta p} < 1,$$

where the second equality is valid for conventional wells only. A pressure drop number close to one indicates that the well bore pressure drop is of the same order of magnitude as the draw down. Mainly as a result of the high PI and the large well ID, the well in our example has quite a high pressure drop number, $N_{\Delta p} = 0.71$, as illustrated in Figure 4.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Value</th>
<th>SI units</th>
<th>Value</th>
<th>Field units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_w$</td>
<td>Well bore length</td>
<td>2.000 * 10^3 m</td>
<td>6.562 * 10^3 ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{c,j}$</td>
<td>Casing inside diameter</td>
<td>1.626 * 10^-1 m</td>
<td>6.400 * 10^-0 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{c,j}$</td>
<td>Casing inside roughness</td>
<td>5.867 * 10^-4 m</td>
<td>2.310 * 10^-2 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_R$</td>
<td>Reservoir pressure</td>
<td>2.365 * 10^-7 Pa</td>
<td>3.430 * 10^-3 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{\text{heel}}$</td>
<td>Pressure at heel</td>
<td>2.350 * 10^-7 Pa</td>
<td>3.409 * 10^-3 psi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>8.498 * 10^-2 kg m^-3</td>
<td>3.500 * 10^-1 deg. API</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>5.000 * 10^-4 Pa s</td>
<td>5.000 * 10^-1 cP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J_s$</td>
<td>Specific PI</td>
<td>2.803 * 10^-10 m^-2 Pa^-1 s^-1</td>
<td>3.200 * 10^-1 bpd/(psi*ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{Re,\text{heel}}$</td>
<td>Reynolds number at the heel</td>
<td>4.897 * 10^5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
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| $\xi_w$ | Well bore parameter | 5.656 * 10^2 | - | - | - |
| $\varepsilon_w$ | Dimensionless well bore roughness | 3.609 * 10^{-3} | - | - | - |

$\xi_w$ and $\varepsilon_w$ are well bore parameters.

Output variables

| $q_{D,\text{heel}}$ | Dimensionless flow rate at the heel | -4.43 * 10^{-1} | - | - | - |
| $q_{\text{heel}}$ | Flow rate at the heel | -3.68 * 10^{-2} m^3 s^{-1} | -2.00 * 10^4 bpd |
| $p_{D,\text{toe}}$ | Dimensionless pressure at the toe | 2.95 * 10^{-1} | - | - | - |
| $p_{\text{toe}}$ | Pressure at the toe | 2.361 * 10^7 Pa | 3.424 * 10^3 psi |
| $N_{\Delta p}$ | Pressure drop number | 7.05 * 10^{-1} | - | - | - |

Figure 4: Reservoir pressure $p_r$ and well bore pressure $p_w$ as a function of along-hole distance $x$ for a conventional completion with parameters given in Table 1.

4.2 Stinger completion

We will examine the effect of a partially penetrating stinger completion with intermediate inflow at the heel. We require that the flow rate at the heel remains equal to the 3.68 * 10^{-2} m^3 s^{-1} (20,000 bbl/day) that was obtained with the conventional completion. This will require a different bottom hole pressure, i.e. a different value of the stinger pressure at the heel of the well. The relevant completion details are given in Table 2.

In the following we will use the letter $p$ to indicate the pressure in the annulus and the pressure in the well bore:

\[ 0 \leq x \leq L_{st}: p = p_{an}, \]
\[ L_{st} < x \leq L_w: p = p_w. \] (44)

The profile of $p$ along the well has two minima: one at the choke, i.e. at the heel of the well, and one at the end of the stinger. It also has two maxima: one somewhere in between the heel
of the well and the end of the stinger, and one at the toe of the well. The first equality in
definition (43) states that \( N_{\Delta p} \) depends on the difference between the maximum and the
minimum value of \( p \). Therefore, to minimize \( N_{\Delta p} \), the two maxima in \( p \) should be equal and the
two minima should be equal. For a given flow rate and given values of the stinger’s inside
and outside diameters and roughnesses we have two variables left to influence the profile of \( p \)
along the well, namely the stinger length \( L_{st} \) and the choke flow area \( A_{ch} \).

For a stinger with a 0.114 m (4.5 in) outside diameter (OD) and a 0.102 m (4 in) ID, we can
adjust \( L_{st} \) and \( A_{ch} \) by trial and error to arrive at the profile depicted in Figure 5. The
corresponding pressure drop number is \( N_{\Delta p} = 0.29 \), which represents a considerable reduction
compared to the conventional case \( (N_{\Delta p} = 0.71) \).

| Table 2: Example data for a partially penetrating stinger completion with intermediate
inflow. |
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
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<td>Symbol</td>
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<td>Value</td>
<td>SI units</td>
<td>Value</td>
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<tr>
<td>L_{st}</td>
<td>Stinger length</td>
<td>8.750 \times 10^2</td>
<td>m</td>
<td>2.871 \times 10^1</td>
</tr>
<tr>
<td>d_{st,o}</td>
<td>Stinger outside diameter</td>
<td>1.143 \times 10^{-1}</td>
<td>m</td>
<td>4.500 \times 10^0</td>
</tr>
<tr>
<td>d_{st,i}</td>
<td>Stinger inside diameter</td>
<td>1.016 \times 10^{-1}</td>
<td>m</td>
<td>4.000 \times 10^0</td>
</tr>
<tr>
<td>e_{st,o}</td>
<td>Stinger outside roughn.</td>
<td>5.334 \times 10^{-5}</td>
<td>m</td>
<td>2.100 \times 10^{-3}</td>
</tr>
<tr>
<td>e_{st,i}</td>
<td>Stinger inside roughness</td>
<td>5.334 \times 10^{-5}</td>
<td>m</td>
<td>2.100 \times 10^{-3}</td>
</tr>
<tr>
<td>C_{ch}</td>
<td>Choke coefficient</td>
<td>1.000 \times 10^0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A_{ch}</td>
<td>Choke flow area</td>
<td>1.858 \times 10^{-4}</td>
<td>m^2</td>
<td>2.880 \times 10^{-1}</td>
</tr>
<tr>
<td>p_{heel}</td>
<td>Stinger pressure at heel</td>
<td>2.275 \times 10^7</td>
<td>Pa</td>
<td>3.299 \times 10^3</td>
</tr>
</tbody>
</table>

| Intermediate variables |
|---|---|---|---|---|
| \bar{\xi}_w | Well bore parameter | 3.446 \times 10^3 | - | - | - |
| \bar{\xi}_{an} | Annulus parameter | 4.540 \times 10^4 | - | - | - |
| \bar{\xi}_{st} | Stinger parameter | 3.613 \times 10^4 | - | - | - |
| \bar{\xi}_{ch} | Choke parameter | 3.495 \times 10^3 | - | - | - |
| \vec{\varepsilon}_{an} | Dimensionless annulus roughness | 6.632 \times 10^{-3} | - | - | - |
| \vec{\varepsilon}_{st} | Dimensionless stinger roughness | 5.250 \times 10^{-4} | - | - | - |
| \lambda | Dimensionless stinger length | 4.419 \times 10^{-1} | - | - | - |

| Output variables |
|---|---|---|---|---|
| q_{D,heel} | Dimensionless flow rate | -7.28 \times 10^{-2} | - | - | - |
Flow in extended stinger completions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{\text{heel}}$</td>
<td>Flow rate at the heel</td>
<td>$-3.68 \times 10^{-2}$ m$^3$/s</td>
</tr>
<tr>
<td>$p_{\text{max}}$</td>
<td>Maximum annulus/wellbore pressure</td>
<td>$2.359 \times 10^1$ Pa</td>
</tr>
<tr>
<td>$p_{\text{min}}$</td>
<td>Minimum annulus/wellbore pressure</td>
<td>$2.357 \times 10^1$ Pa</td>
</tr>
<tr>
<td>$N_{\Delta p}$</td>
<td>Pressure drop number</td>
<td>$2.93 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Figure 5: Reservoir pressure $p_R$, wellbore pressure $p_w$, annulus pressure $p_a$ and stinger pressure $p_s$ as a function of along-hole distance $x$ for a stinger completion with parameters given in Table 2.

Figure 6: Combined view of the reservoir, wellbore and annulus pressures for the conventional and stinger completions of Figures 4 and 5 to illustrate the reduction in pressure drop along the wellbore.
5 Discussion

5.1 Optimization

- In the example discussed above, we minimized the pressured drop number \( N_{\Delta p} \) by varying the stinger length and the choke flow area for a given stinger diameter and wall thickness. If we also could vary the diameter of the stinger, we would have a third optimization variable available. This would allow us to not only minimize \( N_{\Delta p} \), but also the pressure drop over the choke which is, in the optimized situation, equal to the pressure drop over the stinger. The choke pressure drop does not directly influence the well inflow performance along the well, but it does effect the well’s outflow performance, i.e. it’s lifting capacity.

5.2 Modelling limitations

- The model used in our study assumes a homogeneous reservoir. In practice, reservoirs will never be homogeneous, and the optimization will be driven by the reservoir heterogeneities. In fact, there may be situations were the well bore pressure drop will be beneficial, e.g. to achieve a balanced inflow performance in a reservoir with an increasing permeability towards the toe of the well.

- Our model assumes a two-dimensional cylindrical inflow perpendicular to the well. In reality, flow towards a horizontal well will also have a component parallel to the well, in particular near the ends of the well. A crude way to take into account end effects is described in Reference [1] and involves the addition of semi-spherical inflow at the heel and the toe.

- Our model assumes single phase (liquid) flow. Obviously, once gas coning occurs and free gas is produced with the liquid this assumption breaks down and the pressure drops in the well and the stinger will change drastically. However, the intention of an extended stinger completion is to prevent the breakthrough of gas or water in the first place, and therefore the limitation to single phase flow is appropriate for the optimization of the completion.

- Our model assumes a concentric stinger. Usually, a stinger will not be centralised but will rest on the bottom of the well. The change in friction factor for the annular flow in an eccentric annulus can be taken into account with techniques discussed in Reference [8].

- The roughness of the tubulars is usually poorly known. In particular the effective roughness of the outer tubular, which will contain slots or perforations, will generally be unknown. Additional uncertainty arises from the effects of inflow or outflow on the flow regime; see also Reference [9].

- Our model assumes a perfect bonding between the casing and the formation. In reality, horizontal wells usually have a non-cemented completion such as a slotted liner or a pre-packed screen. In that case there will be a second annulus, between the well bore tubular and the formation. Usually it is unknown to what extent this external annulus is blocked by debris.
• Undulations in the well bore can cause an additional pressure drop, in particular when water or gas accumulate in the lower or higher areas of the well bore. In case of multiphase flow, there may also occur differences between the flow regimes, and the associated pressure drops, in uphill and downhill flow.
6 Conclusions

- The model presented in this report can be used to obtain a rough indication of the optimum configuration of an extended stinger completion.
- Detailed design requires more refined modelling, e.g. with the aid of a numerical reservoir simulator that is capable of handling flow in well bores containing loops.
- Reservoir heterogeneity, friction factors and the amount of communication in the external annulus remain uncertain, whatever the sophistication of the well bore model.
- The effect of pressure drop over a horizontal well bore can be quantified with the aid of a dimensionless pressure drop number $N_{dp}$, defined as the maximum pressure difference in the well bore divided by the maximum draw down.
- Extended stinger completions can drastically reduce the pressure drop over a horizontal well bore.
- A price to pay for this pressure drop reduction is the requirement for a lower well bore pressure at the heel of the well and therefore a reduced lifting capacity of the well.
- In a homogeneous reservoir, a partially penetrating stinger is more effective than a fully penetrating stinger.
- In a homogeneous reservoir, further reduction of the well bore pressure drop can be achieved through the use of one or more intermediate inflow points in the stinger.
Acknowledgments

This report was inspired by contacts with staff of Shell Gabon, in particular Duncan Green Armitage who first developed the concept of the extended stinger with intermediate inflow, and Rob Kleibergen, who brought the concept to my attention. Shell Gabon pioneered the development and application of extended stinger completions which are known within that company as “Tilted bucket completions”. This name stems from the shape of the inflow profile over the section with the stinger, which can, with some imagination, be seen to resemble a bath tub or a bucket; see Figure 2. By changing the flow opening of the sleeve between the annulus and the stinger, it is possible to influence the shape of the inflow profile, i.e. to “tilt the bucket”. I also like to thank Arjen Wagenvoort for his help in testing the C++ program and verifying the results, as part of his final thesis work [7].
References


## Appendix A - Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
<th>SI units</th>
<th>Field units</th>
<th>Conv. factor field to SI units</th>
</tr>
</thead>
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<tr>
<td>$A$</td>
<td>Area</td>
<td>$L^2$</td>
<td>m$^2$</td>
<td>in.$^2$</td>
<td>645.2*10^{-6}</td>
</tr>
<tr>
<td>$C_{ch}$</td>
<td>Choke coefficient</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter</td>
<td>$L$</td>
<td>m</td>
<td>in.</td>
<td>25.40*10^{-3}</td>
</tr>
<tr>
<td>$e$</td>
<td>Absolute wall roughness</td>
<td>$L$</td>
<td>m</td>
<td>in.</td>
<td>25.40*10^{-3}</td>
</tr>
<tr>
<td>$f$</td>
<td>Moody-Weissbach friction factor</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$h$</td>
<td>Height</td>
<td>$L$</td>
<td>m</td>
<td>ft</td>
<td>0.3048</td>
</tr>
<tr>
<td>$J$</td>
<td>Productivity Index (PI)</td>
<td>$L^4m^{-1}t$</td>
<td>$m^3/(Pa \ s)$</td>
<td>bpd/psi</td>
<td>0.27*10^{-9}</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Specific PI, i.e. PI per unit length</td>
<td>$L^3m^{-1}t$</td>
<td>$m^2/(Pa \ s)$</td>
<td>bpd/(psi ft)</td>
<td>0.88*10^{-9}</td>
</tr>
<tr>
<td>$k$</td>
<td>Equivalent isotropic permeability</td>
<td>$L^2$</td>
<td>m$^2$</td>
<td>mD</td>
<td>0.9869*10^{-15}</td>
</tr>
<tr>
<td>$L$</td>
<td>Length</td>
<td>$L$</td>
<td>m</td>
<td>ft</td>
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<td>$N_{Re}$</td>
<td>Reynolds number</td>
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<tr>
<td>$N_{Ap}$</td>
<td>Pressure drop number</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>$L^{-1}m^2t^{-2}$</td>
<td>Pa</td>
<td>psi</td>
<td>6.895*10^{3}</td>
</tr>
<tr>
<td>$P$</td>
<td>Wetted perimeter</td>
<td>$L$</td>
<td>m</td>
<td>in.</td>
<td>25.40*10^{-3}</td>
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<tr>
<td>$q_{s,R}$</td>
<td>Specific inflow from reservoir, i.e. inflow per unit length</td>
<td>$L^2t^{-1}$</td>
<td>$m^2/s$</td>
<td>bpd/ft</td>
<td>0.5608*10^{-6}</td>
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<tr>
<td>$q$</td>
<td>Well bore flow rate</td>
<td>$L^3t^{-1}$</td>
<td>$m^3/s$</td>
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<td>1.840*10^{-6}</td>
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<tr>
<td>$r$</td>
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<tr>
<td>$v$</td>
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<td>$Lt^{-1}$</td>
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<td>ft/s</td>
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<tr>
<td>$w$</td>
<td>Width</td>
<td>$L$</td>
<td>m</td>
<td>ft</td>
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<td>$x$</td>
<td>Horizontal co-ordinate along well bore</td>
<td>$L$</td>
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<td>ft</td>
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<td>$y$</td>
<td>Horizontal co-ordinate perpendicular to well bore</td>
<td>$L$</td>
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<td>ft</td>
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<tr>
<td>$z$</td>
<td>Vertical co-ordinate</td>
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<td>m</td>
<td>ft</td>
<td>0.3048</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>Pressure drop</td>
<td>$L^{-1}m^2t^{-2}$</td>
<td>Pa</td>
<td>psi</td>
<td>6.895*10^{3}</td>
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<tr>
<td>$\lambda$</td>
<td>Dimensionless stinger length</td>
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<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>$L^{-1}m^2t^{-1}$</td>
<td>Pa s</td>
<td>cP</td>
<td>1.000*10^{-3}</td>
</tr>
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</table>
Flow in extended stinger completions

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>Parameter</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
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</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Density</td>
<td>( L^{-3}\text{m} )</td>
<td>( \text{kg/m}^3 )</td>
<td>( ^\circ\text{API} )</td>
<td>( 141.5 \times 10^3/(131.5+^\circ\text{API}) )</td>
</tr>
</tbody>
</table>

**Subscripts**

- \( an \) Annulus
- \( c \) Casing
- \( ch \) Choke
- \( D \) Dimensionless
- \( h \) Horizontal
- \( i \) Inside
- \( min \) Minimum
- \( o \) Outside
- \( R \) Reservoir
- \( s \) Specific
- \( st \) Stinger
- \( v \) Vertical
- \( w \) Well

**Superscripts**

- - Upstream variable (Note: positive flow is from low to high co-ordinate values)
- + Downstream variable
Appendix B – Specific PI for a box-shaped reservoir

The specific PI $J_s$ is a function of reservoir dimensions and fluid and formation properties. As an illustration, we derive the PI for a simple, box-shaped reservoir with simultaneous gas cap and aquifer drive. In line with the two-dimensional approach followed in the body of the text, we consider a cross-section of unit thickness through the reservoir perpendicular to a fully penetrating well; see Figure B-1. The well is positioned mid in between two vertical no-flow boundaries at distances $w_R/2$ and two horizontal constant pressure boundaries at distances $h_R/2$. The formation is anisotropic with horizontal and vertical permeabilities $k_h$ and $k_v$. An equivalent isotropic reservoir is defined through re-scaling the reservoir dimensions in the usual fashion [11]. The scaled dimensions become

$$
\hat{w}_R = \sqrt{\frac{k_v}{k}} \frac{w_R}{h_R}, \quad \hat{h}_R = \sqrt{\frac{k_h}{k}} \frac{h_R}{h_R},
$$

(B-1)

where

$$
k = \sqrt{k_h k_v},
$$

(B-2)

is the equivalent isotropic permeability. The relation between reservoir pressure, well pressure and well flow rate for a steady state situation is obtained from the classic expression for the flow potential a well of unit length mid between two no-flow boundaries [11]:

$$
p_R(\hat{y}, \hat{z}) - p_w = \frac{q_{s,R} \mu}{2 \pi k} \ln \left( \cosh \frac{2 \pi \hat{z}}{\hat{w}_R} - \cos \frac{2 \pi \hat{y}}{\hat{w}_R} \right).
$$

(B-3)

Here $\hat{y}$ and $\hat{z}$ are the scaled co-ordinates

$$
\hat{y} = \sqrt{\frac{k_v}{k}} y, \quad \hat{z} = \sqrt{\frac{k_h}{k}} z,
$$

(B-4)

and $y$ and $z$ are the unscaled co-ordinates in horizontal and vertical direction, with the origin in the center of the well. Use of equation (B-3) is justified if $\hat{h}_R >> \hat{w}_R$ and if the influence of gravity is small compared to the effect of draw down. If we choose the reference point for the reservoir pressure at $(\hat{y}, \hat{z}) = \left(0, \frac{h_R}{2}\right)$, the specific Productivity Index follows as:

$$
J_s = \frac{q_{s,R}}{p_R\left(0, \frac{h_R}{2}\right) - p_w} = \frac{2\pi}{\ln \left( \cosh \frac{2 \pi \hat{h}_R}{\hat{w}_R} - 1 \right)} \frac{k}{\mu}.
$$

(B-5)

A worked-out example is presented in Table B-1.
Flow in extended stinger completions

Figure B-1: Schematic representation of a box-shaped reservoir with a fully penetrating horizontal well.

Table B-1: Example calculation of specific PI for a horizontal well in a box-shaped reservoir.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Value</th>
<th>SI units</th>
<th>Value</th>
<th>Field units</th>
</tr>
</thead>
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<td></td>
<td>Input variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>$w_R$</td>
<td>Reservoir width</td>
<td>250</td>
<td>m</td>
<td>820</td>
<td>ft</td>
</tr>
<tr>
<td>$h_R$</td>
<td>Reservoir height</td>
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<td>m</td>
<td>328</td>
<td>ft</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
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<td>kg m$^{-3}$</td>
<td>35</td>
<td>deg. API</td>
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<td>$\mu$</td>
<td>Dynamic viscosity</td>
<td>5.00 * 10$^{-4}$</td>
<td>Pa s</td>
<td>0.5</td>
<td>cP</td>
</tr>
<tr>
<td>$k_h$</td>
<td>Horizontal permeability</td>
<td>4.93 * 10$^{-13}$</td>
<td>m$^2$</td>
<td>500</td>
<td>mD</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Vertical permeability</td>
<td>2.47 * 10$^{-14}$</td>
<td>m$^2$</td>
<td>25</td>
<td>mD</td>
</tr>
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<td>Output variables</td>
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</tr>
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<td>$k$</td>
<td>Equivalent permeability</td>
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<td>m$^2$</td>
<td>112</td>
<td>mD</td>
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<td>$\hat{w}_R$</td>
<td>Scaled reservoir width</td>
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<td>388</td>
<td>ft</td>
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<tr>
<td>$\hat{h}_R$</td>
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<td>m</td>
<td>694</td>
<td>ft</td>
</tr>
<tr>
<td>$J_s$</td>
<td>Specific PI</td>
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<td>m$^2$ Pa$^{-1}$ s$^{-1}$</td>
<td>0.32</td>
<td>bpd/(psi*ft)</td>
</tr>
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</table>