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The numerical model WAVEWATCH: a third generation model for hindcasting of wind waves on tides in shelf seas

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The numerical model WAVEWATCH:
A third generation model for
hindcasting of wind waves on
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Abstract

In this report the analytical and numerical formulations, testing and calibration of the third generation wave hindcast model WAVEWATCH are presented. WAVEWATCH is specially designed to operate in a combined wave-current model. The main attention is focussed on numerical aspects of the model, including academic test cases and comparison with other third generation wave models (which do not include current effects). Furthermore, preliminary results of calculations for a south westerly storm in the southern North Sea are presented to illustrate the effects of wave-current interactions in the southern North Sea in such conditions.
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1 INTRODUCTION

In this report the analytical and numerical formulations, testing and calibration of a wave hindcast model, designed to operate in a combined wave-current model, are presented. The model is called WAVewatch (WAVE height, WAterdepth and Current Hindcasting). It is developed as a part of a study of the significance of wave-current interactions at large scales, such as those of the North Sea. As the main aim of the study is the research into the significance of different types of interactions, an not investigation of new numerical methods, the state of the art (physical and numerical) is used as much as possible. This report mainly deals with the numerical model. The selection of the different physical formulations used in the model will be discussed elsewhere.

Up to the present (to the knowledge of this author), wind waves and currents (surges) at sea have always been treated separately, and hindcasting (forecasting) took place with separate, unrelated models. This separate treatment is in contrast with the close correlation between current and wave occurrences, which exists at sea (e.g. the close correlation between storm surges and extreme wave conditions and e.g. the existence of tidal races near many headlands). Whereas inhabitants of coastal areas have shared this knowledge for centuries, scientists seem to have neglected this until recently. Only the last decades the interaction between waves and currents has been addressed by various authors. Textbooks (e.g. Whitham, 1974, Phillips, 1977, Mei, 1983), review papers (e.g. Peregrine, 1976) and reports (e.g. Peregrine and Jonsson, 1982) show this growing interest. Although large progress has been made in the theoretical field, the actual magnitude of wave current interactions for large scale areas such as the North sea has never been assessed.

To do so, one has to solve the equations for tides (storm surges) and for wind waves, including their interactions. In conventional models, mean water depths and current velocities are usually determined by solving momentum and mass conservation equations, whereas wind wave spectra are calculated by solving energy or action balance
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equations. For realistic situations both types of equations have to be solved numerically. To solve the equations for wind waves and surges, including their interactions, three different strategies can be recognised:

- Simultaneous solution of all equations.
- Solution of equations for wind waves and equations for surges with independent numerical modules, used by a single wave-current model. Information needed to calculate interactions in one module is obtained from the other module at the previous time step. In literature this method is known as parallel running (e.g. Abbott, 1979).
- Solutions of equations for wind waves and surges with independent models. Calculations are performed with one of the models for the entire period considered, using the previously obtained results of the other model as input. Iterative calculations with both models are needed to obtain the final solution. This method was first suggested by Skovgaard and Jonsson (1976), who called it a two level approach.

On the simultaneous solutions of wind wave and surge equations no literature is known to the author. As the wave and current equations are of completely different nature (as sketched above), efforts to solve the equations simultaneously are expected to encounter various (unexpected) problems. As an investigation into new numerical methods is out of the scope of this study, simultaneous solution of all equations is not considered further.

The parallel running approach and the two level approach, are (from a numerical point of view) very similar, as both approaches use two or more separate models for the calculation of wind waves and surges. In both cases the wind wave parameters are assumed to be constant during a single calculational step of the surge model and vice versa. The only difference occurs in the data exchange between the wind wave and surge modules / models, which is needed to calculate the magnitude of interaction terms in both models / modules.

The two level approach has been used successfully in the model of Skovgaard and Jonsson (1976) (later extended by Christoffersen, 1982)
and by Dingemans et al. (1986). These models are however not suitable for this study, as they considered stationary (depth and current) conditions, either due to the model design (Dingemans et al., 1986) or due to numerical problems (data transfer and calculational efforts, Christoffersen, 1982).

On parallel running wind wave and surge models no literature is known to the author. However, in view of the similarity with the two level approach, parallel running models seem as feasible as two level models when wave-current models are considered. Furthermore parallel running models have been used successfully in other fields of computational hydraulics where equations of completely different nature are to be solved simultaneously (e.g. in pollution transport models, e.g. Abbott, 1979).

As for the investigation of wave-current interactions on large scales such as the North Sea no models are available, either a parallel running or two level model has to be developed. Before a selection between those two types of models is made, attention should be focussed on the availability of the two basic models.

Existing surge models only need minor adaptations to incorporate wind wave influences. First the wind forcing should be changed so that all momentum fluxes between the atmosphere, waves and surges are accounted for correctly. Secondly the bottom friction should be changed to allow for an influence of wind waves on the bottom boundary layer. These adaptations do not change the nature of the equations, so that conventional solution techniques for depth integrated surge equations can be expected to render good results.

Existing wind wave models need more adaptations. First none of the models known to the author include the shift in absolute frequency of waves due to the instationarity of depth and current (Barber, 1949). Secondly the numerical treatment of refraction and convection is usually badly balanced, in particular in grid models (which will be discussed in section 4.2.2).

In view of the above the logical step in the development of a (research) model for the combined hindcasting of wind waves and surges on large scales is the development of a wind wave hindcast model for instationary depths and currents. Such a model (WAVEWATCH) is
presented in this report. For such a model, a Eulerian approach (grid model) is more appropriate than a Lagrangian approach (ray model), as the wave model has to produce data at fixed locations in view of the exchange of data with the surge model. Furthermore a full discrete spectral approach is used, where source functions are treated in a so-called third generation approach (WAMDI group, 1988), to minimize the use of empirical information. Although calculations with such a third generation wave model are expensive, this is not necessarily intolerable for a research model. The calculation and integration of source functions is performed using techniques from the third generation WAM model (WAMDI group, 1988), which could be adapted to WAVEWATCH with minor changes. The propagation module of the WAVEWATCH is entirely new and incorporates the mayor part of current influences on wind waves. In this module the action propagation equation for action density spectra defined on absolute frequency and direction is solved.

In chapters 2 through 4 the governing equations, the numerical model and the testing and calibration of the numerical model are discussed respectively. In chapter 5 the results of an example hindcast are given and chapter 6 consists of conclusions and recommendations.
2 GOVERNING EQUATIONS

Random wind waves are generally described with a spectrum giving energy or variance density as a function of wavenumber $k$, frequency $f$ and direction $\theta$. This makes the spectra essentially three-dimensional, e.g. the variance density spectrum $F(k,f,\theta)$, or six-dimensional if variations in space and time are also considered $F(x,t,k,f,\theta)$. By applying the linear wave theory to components of a spectrum, its wavenumber and frequency are interrelated by the dispersion relation:

$$\omega = \sigma + k \cdot U$$  \hspace{1cm} (2-1)

with

$$\sigma^2 = gk \tanh(kd)$$  \hspace{1cm} (2-2)

where

- $\omega$: absolute (radian) frequency, as observed in a frame of reference fixed to the bottom, ($\omega = 2\pi f_a$),
- $\sigma$: intrinsic or relative frequency, as observed in a frame of reference which moves with the average current velocity $U$ ($\sigma = 2\pi f_r$),
- $k$: wavenumber vector with absolute value $k$ and direction $\theta$,
- $g$: acceleration of gravity,
- $d$: water depth, local average over wave field.

Due to the above relation between $\omega$ and $k$, the variance and energy density spectra become two-dimensional. One is free to choose the two parameters to describe the components of the spectra, e.g. $(\omega, \theta)$, $(k)$ etc. In the model described here the two-dimensional variance density spectrum $F(\omega, \theta)$ will be used, which is defined so that:

$$\int_0^{2\pi} \int_0^{\infty} F(\omega, \theta) \, d\omega \, d\theta = \langle \eta^2 \rangle$$  \hspace{1cm} (2-3)
Governing Equations

where \(<..>\) denotes the ensemble average and \(\eta\) is instantaneous surface elevation due to the wind waves (\(\eta = 0\) for mean water level). The variance density spectrum is closely related to the energy density spectrum \(E(\omega, \theta)\)

\[
E(\omega, \theta) = \rho g F(\omega, \theta)
\]  

(2-4)

During propagation over inhomogeneous and unsteady depth and current fields the energy or variance of the wave field is not conserved, whereas the wave action \(N = F/\sigma\) is (e.g. Whitham, 1965, 1974, Bretherthon and Garrett, 1968). The action conservation equation is therefore a more suitable equation to describe the propagation of the wave field than the energy conservation equation. For the two-dimensional action density \(N(\omega, \theta)\), dropping independent variables, the propagation equation including source and sink functions reads:

\[
\frac{\partial N}{\partial t} + \nabla \left[ (c_g + U)N \right] + \frac{\partial}{\partial \theta} \left[ c_g N \right] + \frac{\partial}{\partial \omega} \left[ c_\omega N \right] = \frac{S}{\sigma}
\]  

(2-5)

where

\(\nabla\) differential operator in \(x_1-x_2\) space,
\(c_g\) energy/action propagation velocity in \(x_1-x_2\) space (frame of reference moving with velocity \(U\))
\(c_\theta\) propagation velocity in the directions space,
\(c_\omega\) propagation velocity in the frequency space,
\(S\) source and sink functions for wave variance.

In the left hand side of equation (2-5) the local rate of change of the action density \(\partial N/\partial t\) and the effects of propagation are gathered, while the right hand side represents source and sink functions such as wind input. The different terms of this equation are discussed one by one in the following.

The second term on the left hand side of equation (2-5) represents the convection of wave action in the \(x_1-x_2\) space due to both the mean current \(U\) and the energy propagation velocity \(c_g\) (defined in a frame
of reference moving with the current velocity \( \mathbf{U} \). The direction of \( \mathbf{c}_g \) is given by \( \theta \) and its absolute value is

\[
\mathbf{c}_g = \frac{\partial \sigma}{\partial \mathbf{k}} = n \frac{\sigma}{k}
\]  

(2-6)

with

\[
n = \frac{1}{2} + \frac{kd}{\sinh(2kd)}
\]  

(2-7)

This term includes depth and current shoaling due to spatial variations of \( (\mathbf{U} + \mathbf{c}_g) \) in the propagation direction of the wave energy.

The third term on the left hand side of equation (2-5) represents refraction. The change of direction as would occur for monochromatic waves is represented here by transport of action in the \( \theta \)-space. The propagation velocity is determined by differentiating equations (2-1) and (2-2), using the equation for the conservation of wavenumber density (e.g. Whitham, 1974, his page 11)

\[
\frac{\partial k}{\partial t} + \mathbf{v}_\omega = 0
\]  

(2-8)

The propagation velocity in the \( \theta \)-space then becomes (e.g. Mei, 1982):

\[
\mathbf{c}_g = \frac{\partial \theta}{\partial t} = -\frac{1}{k} \left[ \frac{\partial \sigma}{\partial d} \frac{\partial d}{\partial m} + k \cdot \frac{\partial \mathbf{U}}{\partial m} \right]
\]  

(2-9)

where

\[m\] coordinate perpendicular to the propagation direction \((\theta)\) of the waves

The fourth term on the left hand side of equation (2-5) represents the change of absolute frequency due to the instationarity of depth and current (e.g. Barber, 1949). The change of frequency as would occur for monochromatic waves is represented here by transport of action in the \( \omega \)-space. The derivation of the propagation velocity in the \( \omega \)-space is similar to the above derivation for the propagation velocity in the \( \theta \)-space (e.g. Mei, 1982).
Governing Equations

\[ c_\omega = \frac{d\omega}{dt} = \frac{\partial \sigma}{\partial d} \frac{d}{dt} + k \cdot \frac{\partial U}{\partial t} \quad (2-10) \]

The source functions at the right hand side of equation (2-5) basically consist of a wind input term \( S_{in} \), a term representing nonlinear wave-wave interactions \( S_{nl} \) and a dissipation term \( S_{ds} \). The latter term can be divided in dissipation due to wave breaking at the surface (whitecapping, \( S_{ds,s} \)) and dissipation in the bottom boundary layer (\( S_{ds,b} \)).

\[ S = S_{in} + S_{nl} + S_{ds,s} + S_{ds,b} \quad (2-11) \]

Unlike propagation, for which theories with closed analytical formulations are available, the source functions are poorly understood and the available formulations are mainly fitted to measurements. In the model presented here, theory is followed as much as possible. Concepts of the WAM model (WAMDI group, 1988, hence denoted as WAM) have been used, as this model uses the physically most sound approaches to source function available at the moment. The source functions as used in the WAM model are easily adapted for the average current considering that they are basically applicable in a frame of reference which moves with the local current velocity \( \bar{U} \), as long as changes in the propagation velocity \( \bar{U} \) occur on a time scale which is much larger than the time scale of variations of a single wave. The source functions of WAM in the moving frame of reference \( S(\sigma, \theta) \) are transformed to source functions in the fixed frame of reference \( S(\omega, \theta) \) by applying the following Jacobian transformation:

\[ S(\omega, \theta) = J_{\omega \sigma} S(\sigma, \theta), \quad J_{\omega \sigma} = (1 + \frac{U/}{c g})^{-1} \quad (2-12) \]

where

- \( J_{\omega \sigma} \) Jacobian for transformation of moving frame to fixed frame,
- \( U/ \) current velocity in the propagation direction of the waves \( k \cdot \bar{U}/k \).
Wind input is described using the formulation of Snyder et al. (1981), rescaled with the wind friction velocity $U^*$ (WAM):

$$S_{in}(\sigma, \theta) = \max \left[ 0, \left[ 0.25 \frac{\rho_r}{\rho_w} \frac{28 U^*}{k/\sigma} \cos(\theta - \theta_w) - 1 \right] \right] \sigma F(\sigma, \theta) \quad (2-13)$$

with

$$U^* = U_{10} \left[ (0.8 + 0.065 U_{10}) \times 10^{-3} \right]^{1/2} \quad (2-14)$$

where

- $\rho_r$ relative density, i.e. density of air divided by density of water $\rho_a/\rho_w$,
- $\theta_w$ wind direction,
- $U_{10}$ wind speed at ten meters above the sea surface.

The source function $S_{nl}$ describes the resonant exchange of energy between wave components due to non-linear interactions (e.g. Hasselmann (1961, 1962, 1963 a and b), Phillips (1966, 1977)). For deep water the smallest number of components to come to resonance is four. For these components the resonance conditions are given as:

$$k_1 + k_2 - k_3 = k_4 \quad (2-15)$$

$$\sigma_1 + \sigma_2 - \sigma_3 = \sigma_4 \quad (2-16)$$

where the components 1, 2 and 3 exchange energy with component 4.

Expressions for the exchange of energy due to this mechanism are given by e.g. Hasselmann (1968), Sell and Hasselmann (1972) and WAM.

The dissipation of wave energy due to whitecapping is described by (WAM, Komen et al., 1984, Hasselmann, 1974):

$$S_{ds,s}(\sigma, \theta) = -3.4 \bar{\sigma} k/k_0 \left[ E k^2 \right]^2 F(\sigma, \theta) \quad (2-17)$$

The mean frequency $\bar{\sigma}$ and mean wavenumber $k$ are determined as:
Governing Equations

\[ \bar{\sigma} = 1 / \langle 1/\sigma > \]  
\[ \bar{k} = < 1 / \sqrt{k} >^2 \]  

where \(<..>\) denotes the average over the spectrum, e.g.

\[ < 1/\sigma > = \frac{\sum (1/\sigma) F(\sigma, \theta) \Delta \sigma \Delta \theta}{\sum F(\sigma, \theta) \Delta \sigma \Delta \theta} \]  

The WAM model uses \(\bar{\sigma}\) and \(\bar{k}\) rather than \(<\sigma>\) and \(<k>\) as the latter two are much more sensitive to variations in the high frequency tail of the spectrum than the first two. \(\bar{\sigma}\) and \(\bar{k}\) therefore result in a more stable behaviour of the model, allowing for larger time steps in the numerical integration of the source functions (WAM).

The dissipation of wave energy due to bottom friction is described using the formulation of Madsen et al. (1988):

\[ S_{ds, b}(\sigma, \theta) = - \frac{8}{3\pi} f_w \frac{1}{\lambda} \left[ n - \frac{1}{2} \right] u_{b,r} F(\sigma, \theta) \]  

where:

\(f_w\) friction factor.
\(u_{b,r}\) representative (maximum) near bottom current velocity in a frame of reference moving with \(U\).

This formulation includes some more physics than the JONSWAP formulation (which is used in the WAM model) and allows for a mean current influence by means of a dynamically adapted friction factor \(f_w\). The orbital velocity at the bottom \(u_{b,r}\) is estimated as

\[ u_{b,r} = \left[ 2 \int \int \frac{\sigma^2}{\sinh^2(kd)} F(\sigma, \theta) \, d\sigma \, d\theta \right]^{0.5} \]  

which essentially is the root mean square value of the extreme near bottom current velocities. The friction factor \(f_w\) is calculated using the formulation of Jonsson (1966)

\[ \frac{1}{\log_{10}\left[ \frac{a}{\sqrt{f_w}} \right]} = m_f \quad + \quad \log_{10}\left[ \frac{u_{b,r}}{k_N} \right] \]  

- 10 -
where

\[ m_f \] is constant, \( m_f = -0.08 \) as determined experimentally by Jonsson and Carlsson (1976),
\[ a_{b,r} \] is representative near bottom excursion amplitude of the spectrum,
\[ k_N \] is equivalent roughness scale of bottom.

The representative near bottom excursion amplitude \( a_{b,r} \) is calculated as the rms value of the extreme near bottom excursions (similar to \( u_{b,r} \) of equation (2-22)):

\[
a_{b,r} = \left[ 2 \int \int \frac{1}{\sinh^2(kd)} F(\sigma, \theta) \, d\sigma \, d\theta \right]^{0.5}
\]  

(2-24)

The basic problem in estimating the wave friction factor of equation (2-23) is the estimation of the bottom roughness \( k_N \). Specially if sandy bottoms are considered, such as that of the North Sea, ripple forming can change the bottom roughness dramatically. For the southern North Sea this implies that the bottom roughness can range from sand grain roughness (with values as low as \( k_N = 200 \) \( \mu m \)) to ripple roughness (with values as large as \( k_N = 20 \) cm, e.g. Weber et al, 1988). Models have been proposed to estimate ripple roughness from sand grain roughness and wave data (e.g. Grant and Madsen, 1982), but application of these models leads to extreme variations of roughness in both space and time. Numerical experiments considering homogeneous conditions have shown that small changes in selected grain diameter can lead to either no differences or large differences in the calculated wave height. In view of the uncertainties in such grain roughness models, they will not be used. Instead the roughness length \( k_N \) as used in equation (2-23) will be calibrated for the model to fit available measurements (chapter 4).
3 NUMERICAL MODEL

3.1 Introduction

The action balance equation (2-5) is essentially a five-dimensional \((x, t, \omega, \theta)\) hyperbolic equation including source functions, which can be dominant. Such an equation is difficult to solve in a single (numerical) step. All grid models known to the author solve the basic energy or action balance equation using a fractional step method (Yanenko, 1971), which allows for a step by step solution of the equation. In general this implies that propagation and source functions are treated separately (e.g. WAM), whereas some models also use several steps in the numerical integration of propagation and/or source functions (e.g. Golding, 1983). In the model presented here a fractional step method is used in which all aspects of propagation (convection, refraction and change of absolute frequency) are treated in the first step and all source functions are treated in the second step, as shown in figure 1. The propagation module is described in section 3.2 and the source function module is described in section 3.3. If propagation and source functions are coupled some precautions are needed to preserve stability, as is discussed in section 3.4.

Fig. 1 Flow chart for the wave model
3.2 Propagation

3.2.1 Basic concepts

In view of the scope of the study, for which the numerical model has been developed, existing numerical schemes have been considered for the propagation module. As a discrete spectral model is considered, the action conservation equation has to be solved for a large number of spectral bins (i.e. discrete combinations of frequency $\omega$ and direction $\theta$ in the spectrum, typically 500 bins in practical situations) for every spatial grid point and time step. To solve these equations economically, an explicit or single sweep implicit numerical method is preferable. This excludes the use of full implicit and ADI methods. To minimize the computer storage needed, the numerical scheme should additionally include only two time levels. Therefore leap frog and equivalent schemes have not been considered. Several explicit two layer schemes for short wave propagation in full discrete spectral grid models have been reviewed:

- First order upstream (e.g. WAM).
- Lax-Wendroff scheme modified by Gadd (1978) (denoted as LWG), (e.g. Golding, 1983).
- Iterative Crank-Nicholson (denoted as ICN) (e.g. Booij and Holthuijsen, 1987).
- Spatial interpolation techniques (e.g. Greenwood et al., 1985).

Of the above schemes the first order upstream scheme has not been used as it is a first order scheme and therefore by far the least accurate of the four (groups of) schemes. Its prime advantage of being cheap is not relevant here, as the main calculational effort is expected to go to the calculation of propagation velocities and not to the actual numerical solution of the equations. Potentially the most accurate are the spatial interpolation techniques. These techniques, however, require data at many grid points in the numerical description of terms of the equation considered, which makes boundary management difficult. To avoid time consuming experiments with boundary management, this approach has not been further considered.
The choice between the LWG scheme and the ICN scheme is more difficult. Theoretically the ICN scheme is of a higher order of accuracy than the LWG scheme (second order in space and time versus second order in space and first order in time). However, the ICN scheme is not stable when taken centrally in time. Known solutions to this problem (as will be discussed later) bring down the order of accuracy. Of the LWG scheme the main disadvantage is the use of staggered grid points at half time steps. This makes an expansion to more dimensions difficult and can lead to different propagation properties in different directions. As the ICN scheme is easier to apply in a multidimensional situation and does not show different propagation behaviour in different directions, it has been used in the model presented here, in spite of its stability problems.

Using the Crank-Nicholson scheme centrally in both space and time, and using a predictor which is forward in time and central in space, the following ICN scheme is constructed (for simplicity of presentation propagation in \( x_1 \) space only):

**Predictor** \( N_i^{n'} \):

\[
N_i^{n'} = N_i^n + \frac{\Delta t}{2\Delta x_1} \left[ (c_{x_1} N)_i^{n-1} - (c_{x_1} N)_i^{n+1} \right]
\]  (3-1)

**Half time value** \( N_i^{n''} \):

\[
N_i^{n''} = 0.5 \left( N_i^n + N_i^{n'} \right)
\]  (3-2)

**Corrector** \( N_i^{n+1} \):

\[
N_i^{n+1} = N_i^n + \frac{\Delta t}{2\Delta x_1} \left[ (c_{x_1} N)_i^{n''} - (c_{x_1} N)_i^{n'''} \right]
\]  (3-3)

Where:

- \( n \) discrete time counter
- \( i \) discrete space counter
- \( c_{x_1} \) component in \( x_1 \) direction of \( \mathbf{c}_{g+u} \).
Which is the numerical representation of the following partial differential equation:

\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial x_1} \left[ c_{x_1} N \right] = 0
\]  

(3-4)

Two problems occur when this scheme is used centrally in time. These are the occurrence of spurious solutions and the occurrence of negative action. Both problems are discussed below.

**Spurious solutions**

The ICN scheme (equations (3-1) through (3-3)) calculates changes in time of the action density for a grid point (counter i) in some space (e.g. \( x_1 \) space) using the action density at the surrounding points in that space only (counters i+1 and i-1). However, the action density at the grid point itself (counter i) is not used. Therefore the following spurious solution satisfies equations (3-1) through (3-3):

\[
(c_{x_1} N)_i = (-1)^i F_0
\]

(3-5)

where \( F_0 \) is some constant action flux \( c_{x_1} N \). Numerical exercises have shown that these spurious solutions occur mainly for propagation in the two-dimensional \( x_1-x_2 \) space, but not for propagation in the direction or frequency space. This might be caused by the strong variation in the propagation velocities in the latter two spaces, but no satisfactory explanation for this behaviour has been found.

The spurious solution of equation (3-5) cannot exists when the changes at some grid point not only depend on the action density at the surrounding grid points, but also on the action density at the point itself. This can be achieved by applying diffusion in space. The following methods of introducing diffusion have been considered:

- Introduction of a diffusion term in the action conservation equation (2-5).
- Spatial smoothing in the \( x_1-x_2 \) space after each time step, e.g. by application of the following symmetric three point smoothing algorithm after each time step (Abbott, 1979):
\[ N_i := (1-2\beta) N_i + \beta \left( N_{i-1} + N_{i+1} \right), \quad 0 < \beta < 0.5 \] (3-6)

- Making both predictor and corrector of equations (3-1) through (3-3) partially central and partially first order upstream (equations given in following section).

Since spatial smoothing is very similar to introducing a diffusion term, but much easier to implement in a numerical model, the first option is not further considered. Numerical experiments with the remaining two options have shown that both remedies are able to suppress the spurious solutions effectively. As the partially upstream scheme can be incorporated in the basic ICN scheme without adding an extra calculational loop for every time step, it has been selected (for propagation in \( x_1-x_2 \) space only).

**Negative action**

In the basic ICN scheme (equations (3-1) through (3-3)) negative action occurs at the "upstream" flank of the spatial distribution of action density propagating in space. For one-dimensional propagation this is shown in figure 2. Numerical experiments have shown that such negative action occurs mainly when the spatial distribution of the action density is narrow compared to the mesh size of the grid (distribution described by approximately 10 grid points or less). Such negative action tends to grow and ultimately leads to instability. There are several methods available to eliminate negative action:

- Introduction of diffusion.
- Limited flux approach, i.e. the action flux between grid points is limited such that the flux in one time step is always smaller than the total action of the "upstream" grid point. From a numerical point of view the limited flux approach represents an upstream Courant condition.
- Conservative elimination, i.e. for every spectral bin at each location and time all negative action is reset to zero. Subsequently each spectrum is rescaled such that no net action is generated (method discussed in following section).
For propagation in the $x_1$-$x_2$ space the introduction of diffusion serves a dual purpose, as it suppresses the occurrence of both negative action and spurious solutions. This approach is therefore chosen for propagation in the $x_1$-$x_2$ space (see following section).

For the propagation in the $\omega$ and $\theta$ space diffusion is not needed to suppress spurious solutions. The other two options are then preferable as they mainly correct the ICN scheme when and where needed (the upstream flank of the distribution and bins in which negative action occurs). Numerical exercises show excellent results for both options. For the propagation in the $\theta$ space the conservative elimination approach is selected, as this method showed slightly better results than the limited flux approach. For the propagation in the $\omega$ space the limited flux approach was selected for numerical convenience (boundary management and implementation of variable frequency increments, equation (3-16)).

3.2.2 The propagation module

The basic outline of the propagation module, as discussed in the previous section, is shown in figure 3. In the propagation module the action conservation equation, i.e. the action balance equation (2-5) without source functions, is solved. Replacing $\nabla$ by $(\partial/\partial x_1, \partial/\partial x_2)$ and $c_\theta + \tilde{U}$ by $(c_{x_1}, c_{x_2})$, the action conservation equation becomes:

$$\frac{\partial N}{\partial t} = - \frac{\partial}{\partial x_1} \left[ c_{x_1} N \right] - \frac{\partial}{\partial x_2} \left[ c_{x_2} N \right] - \frac{\partial}{\partial \theta} \left[ c_\theta N \right] - \frac{\partial}{\partial \omega} \left[ c_\omega N \right]$$

(3-7)
For the numerical representation of the first and second term on the right hand side of this equation (propagation in $x_1-x_2$ space), a partially ICN and partially upstream scheme is used. For the third term (propagation in $\theta$ space) the ICN scheme together with a conservative elimination approach is used. For the last term (propagation in $\omega$ space) the ICN scheme together with a limited flux approach is used. Using the ICN scheme in the predictor-corrector structure of equations (3-1) through (3-3), the structure of the predictor is identical to the structure of the corrector. It is therefore sufficient to discuss the predictor only. The numerical treatment of boundary points differs from those of internal grid points and is therefore discussed separately in the following. Finally the conservative elimination algorithm is discussed.
Internal grid points

For internal grid points the predictor of the numerical representation of equation (3.7) is given as:

\[
\left[ \frac{N_{n'} - N^n}{\Delta t} \right]_{i_1, i_2, i_3, i_4} = L_{x_1} + L_{x_2} + L_\omega + L_\theta =
\]

\[
\left[ \frac{(1+\alpha) \left[ c_{x_1} N \right]_{i_1, i_2, i_3, i_4} - 2\alpha \left[ c_{x_1} N \right]_{i_1, i_2, i_3, i_4} - (1-\alpha) \left[ c_{x_1} N \right]_{i_2, i_3, i_4}}{2\Delta x_1} \right]^n +
\]

\[
\left[ \frac{(1+\alpha) \left[ c_{x_2} N \right]_{i_1, i_2, i_3, i_4} - 2\alpha \left[ c_{x_2} N \right]_{i_2, i_3, i_4} - (1-\alpha) \left[ c_{x_2} N \right]_{i_2, i_3, i_4}}{2\Delta x_2} \right]^n +
\]

\[
\left[ \frac{F_{i_3, i_2, i_3, i_4} - F_{i_3, i_3, i_4} + 1}{\Delta \omega} \right]_{i_1, i_2, i_3, i_4} =
\]

\[
\left[ \frac{[c_{\theta} N]_{i_1, i_2, i_3, i_4} - [c_{\theta} N]_{i_1, i_2, i_3, i_4} + 1}{2\Delta \theta} \right]_{i_1, i_2, i_3, i_4} \tag{3.8}
\]

with

\[
F'_{i_3, i_3, i_4} + 1 = 0.5 \left[ \left[ c_{\omega} N \right]_{i_3, i_3, i_4} + \left[ c_{\omega} N \right]_{i_3, i_3, i_4} + 1 \right]_{i_1, i_2, i_4} \right]
\]

\[
F_{i_3, i_3, i_4} + 1 = \max \left[ \left( N\Delta \omega \right)_u / \Delta t , F'_{i_3, i_3, i_4} + 1 \right] \right]
\]

where:

\[ \alpha \] upstream fraction \hspace{1cm} 0 \leq \alpha \leq 1 \quad \text{for} \quad c_{x_1} > 0

\[ -1 \leq \alpha \leq 0 \quad \text{for} \quad c_{x_1} < 0 \]

\[ n \] discrete time counter
\[ i_1-i_4 \] grid counters in \( x_1, x_2, \omega \) and \( \theta \) space respectively
\[ u \] suffix indicating "upstream" bin in \( \omega \)-space
Boundary points

For the boundary points in the $x_1-x_2$ space, i.e., sea points for which at least one of the surrounding eight points is a land point, an angle derivative upstream scheme is used (propagation in $x_1-x_2$ space only). Using this scheme, the first and second term in the right hand side of equation (3-8) are replaced by:

$$
\left( \frac{[c_{xs}N]_{int} - [c_{xs}N]_{i_1,i_2}}{\Delta s} \right)_{i_3,i_4}^{n}
$$

(3-10)

Where:

- $\Delta s$ distance between spatial grid point and interpolation point,
- $c_{xs}$ convection velocity in $s$ direction, $c_{xs} = (c_g + U_e \cdot e_\theta \cdot e_s)$, $e_\theta$ is the unit vector in direction $\theta$,
- int suffix denoting interpolation point.

Fig. 4 Interpolation point in the angle derivative upstream scheme for boundary points in the $x_1-x_2$ space. Waves propagate from point interpolation point 5 to prediction point 1. Flux in point 5 calculated using linear interpolation between points 2 and 4.
This scheme introduces an interpolation point as shown in figure 4 (point 5). The change of action density in the point 1 is calculated from the action fluxes $c_{xs} N$ in points 1 and 5. Point 5 is located at the first upstream intersection of the wave propagation line and a grid line in either $x_1$ or $x_2$ direction. In determining the location of point 5 influences of current are neglected. The flux in point 5 is determined by means of linear interpolation from the fluxes at points 2 and 4. Points 2, 3, and 4 can either be land or sea points. In land points the fluxes are assumed to be zero.

In the frequency space the boundaries should preferably be closed, as the total action should be conserved during propagation. Therefore the boundaries should be located at frequencies where the propagation velocity $c_\omega$ (equation (2-10)) or the action $N$ is approximately zero. The propagation velocity is composed of a depth dependent part $c_{\omega d}$ and a current dependent part $c_{\omega U}$:

$$c_\omega = \frac{k \sigma}{\sinh(2kd)} \frac{\partial d}{\partial t} + k \cdot \frac{\partial u}{\partial t} = c_{\omega d} + c_{\omega U}$$  \hspace{1cm} (3-11)

The behaviour of both parts of the propagation velocity is illustrated in figure 5. This figure shows that both terms of equation (3-11) become zero at low frequencies. Thus a closed boundary at low frequencies is in agreement with the local flux $c_{\omega} N$, as long as the lowest frequency is sufficiently small. At high frequencies $c_{\omega d}$ goes to zero but the (current induced part of the) propagation velocity $c_{\omega U}$ goes to infinity in which cases the closed boundary does not seem to be acceptable. However, applying the linear wave theory and assuming deep water conditions and small current velocities, the wavenumber $k$ and the propagation velocity $c_{\omega U}$ are proportional to $\omega^2$. The action density in the high frequency range can be expected to be proportional to $\omega^{-5}$ or $\omega^{-6}$ (see e.g. Phillips, 1977). The action flux $c_\omega N$ (proportional to $\omega^{-3}$ or $\omega^{-4}$) consequently becomes zero at sufficient high frequencies and a closed boundary can be applied.

As the direction space is a closed (circular) space, no boundary points exist and no boundary management is needed.
Conservative elimination

The conservative elimination is used to stabilize the propagation in the \( \theta \)-space only. To assure that \( N(\omega) \) is conserved, the conservative elimination is applied for every location and frequency separately. First all negative action for the \( x_1-x_2 \) grid point and frequency considered is removed, after which the action density distribution over the directions is multiplied with the factor:

\[
\frac{\sum N(x,t,\omega,\theta) \Delta \theta}{\sum \max\left[ 0, N(x,t,\omega,\theta) \right] \Delta \theta}
\]

(3-12)

3.2.3 Numerical stability

As the propagation schemes used are explicit, the stability is governed by Courant criteria. For the propagation in the frequency space this criterion is satisfied automatically due to the limited flux approach. For the propagation in \( x \) and \( \theta \) spaces the Courant criteria are not satisfied automatically. Normally increments \( \Delta x_1 \), \( \Delta x_2 \), \( \Delta \theta \) and \( \Delta t \) are chosen, such that Courant criteria are satisfied. For the propagation in the \( x_1-x_2 \) space the time step is limited by the grid increment and the maximum propagation velocity, i.e. the propagation velocity of the spectral component with the lowest frequency at the grid point with the largest depth (or \( kd = 1.2 \)). Using such a time step and directional increments \( \Delta \theta = 15^\circ \), the Courant criterion for the propagation in the directions space is usually satisfied. However,
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for extremely shallow water, the propagation velocity $c_y$ of equation (2-9) can become extreme so that the integration time step should be much smaller to satisfy the Courant condition. To avoid that refraction at some extremely shallow points results in time steps, which are much smaller than needed for refraction at all other grid points and for all other parts of the model, the absolute value of the propagation velocity in the directions space is actively restricted to satisfy the Courant criterion. The above limitation influences mainly the refraction of low frequency components in coastal areas. In such areas low frequency components usually carry significant energy when propagating towards the coast only. Errors due to the limitation of propagation velocities then propagate out of the model without spreading over the model.

Naturally one should ensure that the automatic Courant limitations in the frequency and directions space are used for a limited number of bins, i.e. the frequency and directions steps $\Delta \omega$ and $\Delta \theta$ have to match the time step $\Delta t$ and propagation velocities $c_\omega$ and $c_y$ for the bulk of the bins. If this is not the case, the Courant limitations will systematically influence the propagation velocities $c_\omega$ and $c_y$, thus introducing systematic errors.

3.3 Source functions

3.3.1 Basic concepts

In the source function module the following reduced version of the action balance equation (2-5) is solved

$$\frac{\partial N}{\partial t} = \frac{S}{\sigma}$$  \hspace{1cm} (3-13)

Only two third generation models, from which concepts can be used in the calculation and integration of source functions, are available. These models are EXACT-NL (Hasselmann and Hasselmann, 1985a) and WAM (WAMDI group, 1988). In EXACT-NL the nonlinear transfer integral is calculated exactly, whereas in WAM only discrete interactions for a single mirror symmetrical pair of quadruplets are considered.
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Therefore EXACT-NL seems the physical most sound model and therefore concepts of this model should be used. However, due to the calculational effort needed to calculate the interaction integral exactly, this model is too expensive for use in an operational environment. Thus WAM is the only third generation model from which concepts can be used.

In the WAM model an implicit second order centered scheme is used for the integration of the source functions in time. For most bins in a spectrum, however, this implicit scheme degenerates to an explicit first order scheme (van Vledder, 1989). Computations of standard growth curves (SWAMP case II) using a one-dimensional version of the WAM model (homogeneous sea), in which either a second order implicit or a first order explicit integration scheme can be selected, show that the choice between those two integration schemes has a negligible effect on calculated spectra. Only instantaneous values of nonlinear interactions are strongly influenced by the choice of the integration scheme. As the time needed to calculate source functions decreases by a factor of approximately 2 when the second order scheme is replaced by a first order scheme (for calculations on an IBM 3083-JX1, on which the model has been implemented), and as the model is not designed to investigate nonlinear interactions, the first order explicit integration scheme is used:

\[
\frac{N^{n+1} - N^n}{\Delta t} = \frac{S}{\sigma}
\]

(3-14)

In the WAM model variance density spectra \( F(f_r, \theta) \) are calculated. To utilize algorithms and numerical techniques of the WAM model directly, the action density \( N(\omega, \theta) \) is converted to variance density \( F(f_r, \theta) \) before the source functions are calculated and integrated.

3.3.2 The source function module

In the source functions module first the action density \( N(\omega, \theta) \) is converted to variance density \( F(f_r, \theta) \), secondly source functions are calculated and integrated, thirdly the change in variance density is converted to the change in action density and finally the new action density is calculated. A flow chart for the source function module is
shown in figure 6. The different parts of the module are discussed below.

\[ N(\lambda, \omega, \theta) \text{ at } t = t \]

- conversion \( N(\omega, \theta) \rightarrow F(\lambda, \theta) \)

- calculate and integrate source functions: \( \Delta F(\lambda, \theta) = \int S \, dt \)

- conversion \( \Delta F(\lambda, \theta) \rightarrow \Delta N(\omega, \theta) \)

- calculate new action density spectrum

\[ N(\lambda, \omega, \theta) \text{ at } t = t + \Delta t \]

**Fig. 6 Flow chart for source function module**

**Conversions**

The calculation of \( F(\lambda, \theta) \) from \( N(\omega, \theta) \) is performed using a Jacobean transformation and the definition of wave action \( (N(\omega, \theta) = F(\omega, \theta)/\sigma) \):

\[
F(\lambda, \theta) = \sigma \, 2\pi \, J_{\sigma\omega} \, N(\omega, \theta)
\]  

(3-15)

where \( 2\pi \, J_{\sigma\omega} \) (equation (2-12)) is the Jacobean for a transformation from \( N(\omega) \) to \( N(\lambda) \). As the propagation module uses a the same \( \omega \) grid for every spatial grid point and time and as \( \mathcal{U} \) varies in space and time, the variance spectrum \( F(\lambda, \theta) \) thus obtained is defined on an \( \lambda \) grid, which varies in space and time. To allow for economic calculation of the non-linear interaction \( S_{nl} \), however, the WAM model uses an
exponential distribution of discrete frequencies, which does not vary in space or time (WAM : $\xi = 1.10$):

$$f_{r,m+1} = \xi f_{r,m}, \quad \xi > 1$$  \hspace{1cm} (3-16)

To adopt the algorithm of WAM for the present model, the variance spectrum is interpolated on such a fixed exponential grid from the variance spectrum at the variable grid by linear interpolation.

A conversion similar to that of equation (3-15) is performed to calculate the change in action density $\Delta N(\omega, \theta)$ from the change in variance density $\Delta F(f, \theta)$:

$$\Delta N(\omega, \theta) = \left[ \sigma \cdot 2\pi \cdot J_{\sigma \omega} \right]^{-1} \Delta F(f, \theta)$$  \hspace{1cm} (3-17)

Such a conversion again incorporates linear interpolation between fixed $f$ and $\omega$ grids. Notice that such interpolation is basically non-conservative. As the interpolation is performed on the change of action density due to source functions (rather than the action density itself), the non-conservative character of the interpolation is limited to the source functions only (i.e. the propagation module remains conservative).

**Calculation and integration of source functions**

As far as input and dissipation are considered, the calculation of the source functions is straightforward algebra. The calculation of nonlinear interactions, however, is more complicated and involves integration over a five-dimensional continuum of resonant quadruplets. In the WAM model this is reduced to a two-dimensional continuum by considering a mirror symmetrical pair of quadruplets of the following form only (WAM : $\lambda = 0.25$):

$$\begin{align*}
\sigma_2 &= \sigma_1 \\
\sigma_3 &= (1+\lambda) \sigma_1 \\
\sigma_4 &= (1-\lambda) \sigma_1
\end{align*}$$  \hspace{1cm} (3-18)
An economic integration method for such mirror symmetrical pairs of quadruplets was developed by Hasselmann and Hasselmann (1985b) (Expressions not given here). For application in shallow water the expression for $S_{n1}$ is simply scaled with the following factor (WAM) (where $\kappa$ is given by equation (2.19)):

$$R = 1 + \frac{5.5}{x} \left[ 1 - \frac{5}{6} x \right] e^{-1.25 x}, \quad x = 0.75 \frac{\kappa d}{k_p d} \quad (3-19)$$

Integrating the source functions using equation (3-14), the change of the variance spectrum simply becomes:

$$\Delta F(f_r', \theta) = S \Delta t \quad (3-20)$$

However, if the integration of the source functions is performed without any precautions, complication occur at the high frequency range of the spectrum. In particular the action density at high frequencies reacts fast to changes in wind conditions. To assure numerical stability for reasonable time steps (e.g. $\Delta t = 20$ min, as used in WAM), the high frequency tail of the spectrum is parameterized for frequencies higher than $f_{r_1}$ (WAM):

$$f_{r_1} = \max (4 f_{r, PM}, 2.5 \bar{f}_r) \quad (3-21)$$

$$f_{r, PM} = 5.6 \times 10^{-3} \ U^* \cdot 1 \quad (3-22)$$

The following shape of the tail is applied to frequencies higher than $f_{r_1}$ (WAM, suffix 1 indicates values belonging to $f_r = f_{r_1}$):

$$F(f_r, \theta) = \frac{c}{c_g} \left[ \frac{k_1}{k} \right] 2.5 \ F(f_{r_1}, \theta) \quad , \quad f_r > f_{r_1} \quad (3-23)$$

which corresponds to a $f^{-4}$ tail in deep water. This tail is applied before the source functions are calculated.

Finally the stability at high frequencies and/or for fast changes in wind conditions is assured by limiting the change of the action contents of every single bin during each time step. Following the WAM model, the maximum change of each spectral component in a single time
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step is limited to 10% of a rough estimate of the highest spectral level to be reached (PM equilibrium range)

\[ |\Delta F(f_r, \theta)| \leq 0.1 \cdot \frac{0.081 \pi}{c} k^{-3} \]  \hspace{1cm} (3-24)

If the absolute value of the variation of the variance density of a bin is larger than this limiting value, it becomes the limiting value (either positive or negative). Note that only the maximum variation in a single time step is related to some equilibrium range, but not the shape of the frequency spectrum itself.

**New action density**

The new action density after one time step is simply calculated by adding the change of action density in one time step to the old action density:

\[ N(\omega, \theta)^{n+1} = N(\omega, \theta)^n + \Delta N(\omega, \theta) \]  \hspace{1cm} (3-25)

Negative action as might occur due to the source functions is simply eliminated (action density of such bins becomes zero).

3.4 Coupling of propagation and source functions

When the propagation and source function modules of the previous sections are used simultaneously without further adaptions, unstable behaviour occurs in fetch limited conditions as will be illustrated in the following chapter. Although a satisfactory explanation for this behaviour has not been found, it seems to be related to the unstable behaviour of the "upstream flank" of some spatial action density distribution (section 3.1), which may be enhanced by the introduction of source functions (i.e. the instability is typical for the propagation scheme selected here). Following this hypothesis, a logical solution is to increase the upstream fraction \( \alpha \) in equation (3-8). To avoid a generally high value of \( \alpha \), which would bring down the accuracy of the propagation module, \( \alpha \) varies dynamically as a function of
frequency $\omega$, direction $\theta$, place $x$ and time $t$. The selection of an expression for such a dynamically varying $\alpha$ is discussed in appendix A. The study as described in this appendix resulted in the following expression for $\alpha$.

$$ |\alpha| = \max \left[ \alpha_c, \alpha_{\min} \right] \quad (3-26) $$

$$ \alpha_c = \min \left[ 1, 0.10 + 0.75 \gamma \frac{B\Delta x}{c} \right] \quad (3-27) $$

where

$$ B = S_{in}(\sigma, \theta) / F(\sigma, \theta) \quad (3-28) $$

with

$$ \gamma \quad \text{calibration constant, } \gamma = 5 \text{ (see appendix A).} $$

(S_{in} given by equation (2-13)) The sign of $\alpha$ is determined by the propagation direction of the waves (equation (3-8)). Separate values for $\alpha$ are determined for propagation in $x_1$- and $x_2$-space. In these equations $\alpha_{\min}$ is determined so that spurious solutions of the propagation scheme in the $x_1$-$x_2$ space are suppressed. $B$ is the exponential growth rate due to wind input (equation (2-13)), which is used here as an estimate for the time scale of response of a wave field to some wind speed. Finally $B\Delta x/c \gamma$ is a non-dimensional parameter which indicates the relative importance of wave generation compared to wave propagation.
4 TESTING AND CALIBRATION

4.1 Introduction

In this chapter results of tests and calibration of the wave model will be discussed. A distinction is made between tests for the propagation module (section 4.2), test for the source function module (section 4.3) and test for the combined propagation and source function modules (section 4.4). The main attention is focussed on tests for the propagation module as the propagation module essentially describes the wave-current interactions (changes of wavenumber and frequencies and exchange of energy with mean current).

The propagation module is tested for propagation in $x_1$-$x_2$ space, $\omega$ space or $\theta$ space separately and for combined propagation in $x_1$-$x_2$ and $\theta$ space (refraction) or in $x_1$-$x_2$ and $\omega$ space (changes in absolute frequency). For cases including refraction and change of absolute frequency, no practical test cases (for which e.g. analytical solutions would be available) were found. Examples of calculations for such conditions, however, are presented by Tolman (1988). Apart from establishing the effect of the minimum upstream factor $\alpha_{\text{min}}$, no calibration is needed for the propagation module.

The source function module is tested to compare time limited growth characteristics of WAVEWATCH with those of the WAM model (deep water), in particular in situations with mean currents. Situation with shallow water are considered to calibrate the bottom roughness $k_N$ as used in the source term for energy dissipation due to bottom friction $S_{ds,b}$ (equations (2-21) and (2-23)).

Finally tests with the combined propagation and source function modules are performed to compare the fetch limited growth characteristics of WAVEWATCH with those of other models.
4.2 Propagation

4.2.1 Propagation in $x_1$-$x_2$ space

The selected schemes for propagation in the $x_1$-$x_2$ space (equations (3-8) and (3-10)) have been tested in various situations to check the propagation properties of the model and the ability of the schemes to represent stationary situations. Four groups of test conditions have been considered. In the first group of tests one-dimensional deep water propagation is considered to check the general propagation properties of the numerical schemes and to establish the influence of the upstream fraction $\alpha$. (In all tests of the propagation module $\alpha_c = 0$ so that $\alpha$ of equation (3-27) equals $\alpha_{\text{min}}$). In the second group of tests two-dimensional propagation is considered to check propagation properties for propagation under an angle with the $x_1$ and $x_2$ axis (deep water). The third group of tests consists of a single deep water propagation test in which waves propagate along a coastline under 45° with the $x_1$ and $x_2$ axis to check the behaviour of the scheme for the boundary points. In the last group of tests inhomogeneous but stationary depth and current conditions are considered to check the representation of depth and current shoaling by the numerical model. Notice that propagation tests in shallow water with homogeneous and stationary depth or tests with homogeneous and stationary currents are from a numerical point of view identical to deep water propagation, as the propagation velocity $c + \bar{u}$ will be different but remains constant in space (i.e. only the Courant number will be different).

One-dimensional deep water propagation

In the tests of one-dimensional deep water propagation (no currents, propagation along a grid line), a single spectral component (bin) with an initial Gaussian action density distribution in space is propagated. Both a situation with a relatively small and a relatively large initial spread (equation (4-2)) compared to the mesh size $\Delta x$ are considered ($2.0 \Delta x$ and $7.5 \Delta x$ respectively). Results of calculations with relatively high Courant number (0.94) are presented, as the IGN scheme shows largest negative action for such conditions. Results for various upstream fractions $\alpha$ ($= \alpha_{\text{min}}$) are shown in figure 7. The
analytical solution to this problem consists of convection with velocity \( v \) without changing the spatial shape of the distribution. Notice that from a numerical point of view only the Courant number, resolution (initial spread) and number of time steps, but not the velocity, grid step and time step are of importance.

\[
\begin{array}{c}
t/\Delta t = 0 & t/\Delta t = 60 \\
\alpha = 0 & \alpha = 0.1 & \alpha = 0.25 & \alpha = 0.5 & \alpha = 1
\end{array}
\]

Fig. 7 One-dimensional deep water propagation without currents. Results of calculations with various upstream fractions \( \alpha \) (Courant number 0.94). a) large initial spread (spr\(_{x} = 7.5\Delta x\)) b) small initial spread (spr\(_{x} = 2.0\Delta x\))

<table>
<thead>
<tr>
<th>Table 1 Results of one-dimensional deep water propagation tests without currents. Courant number 0.94, 60 time steps.</th>
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<tr>
<td>0.00</td>
</tr>
<tr>
<td>0.10</td>
</tr>
<tr>
<td>0.25</td>
</tr>
<tr>
<td>0.50</td>
</tr>
<tr>
<td>1.00</td>
</tr>
<tr>
<td>anal.</td>
</tr>
</tbody>
</table>

' Initially 21.00
" Small part of action distribution has passed through the downstream model boundary

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The total action of the waves is perfectly conserved (changes less than 0.1 %), as long as no action crosses the physical boundaries of the numerical model. In table 1 test results for integrated parameters other than the total action are presented. Presented are mean offset $x_m$ and spread $spr_x$, both relative to the mesh size $\Delta x$:

$$x_m = \frac{m_1}{m_0} \quad (4-1)$$

$$spr_x = \frac{m_2}{m_0} - x_m^2 \quad (4-2)$$

$$m_n = \sum x^n N(x) \Delta x \quad (4-3)$$

Where $m_n$ is n-th spatial moment of the action distribution (for a given bin of the spectrum). Figure 7 a) and the first columns of table 1 show the ability of the ICN scheme without adoptions ($\alpha = 0$) to propagate an action distribution well, as long as the mesh size in the model is small enough to represent the spatial action distribution well. Errors in $x_m$ and $spr_x$ are small and only moderate tail forming occurs. A small upstream fraction ($\alpha = 0.10$) is sufficient to remove the tail while $spr_x$ shows a somewhat larger error compared to the case with $\alpha = 0$. The results of the ICN scheme with a small upstream fraction shows much better results than the upstream scheme (e.g. $\alpha = 1$). For smaller Courant numbers the ICN scheme shows comparable errors in $x_m$ and $spr_x$, however, less tail forming occurs (test results not presented here). In a situation with small initial spread (figure 7 b), right hand side of table 1) the ICN scheme without adoptions shows unstable behaviour (case $\alpha = 0$ not shown in figure). A small upstream fraction ($\alpha = 0.05$ to 0.10) is sufficient to stabilize the scheme, but will not remove the tail and negative action. In the case considered an upstream fraction $\alpha = 0.25$ was needed to remove all negative action, introducing significant diffusion (error in spread) but no significant error in the mean convection velocity ($offset x_m$).

The tests of one-dimensional deep water propagation show the ability of the chosen scheme to represent action propagation along a grid line very well as long as the resolution of the model is relatively high and only small upstream fractions $\alpha$ (e.g. $\alpha < 0.1$) are needed to
Testing and Calibration

prevent the occurrence of negative action. In cases with low resolution an upstream fraction \( \alpha = 0.10 \sim 0.25 \) is needed to prevent the generation of negative action. For the IGN scheme with such upstream fractions the results are much better than that of a pure upstream scheme, however, significant diffusion occurs.

Two-dimensional deep water propagation

In the test of two-dimensional deep water propagation (no currents) a single spectral component with an initial two-dimensional Gaussian distribution in space is propagated under an angle of 30° with the \( x_1 \) axis. Mesh sizes in \( x_1 \) and \( x_2 \) spaces are equal (\( \Delta x \)). Only one case with large initial spread (spr \( x \) = 5.33 \( \Delta x \) in \( x_1 \) and \( x_2 \) direction) and with an upstream fraction \( \alpha = 0 \) is considered. Propagation takes place over about 10 meshes in \( x_1 \) direction and 6 meshes in \( x_2 \) direction with a Courant number 0.79. The analytical solution to this problem again is convection without changes in shape of the spatial distribution. The results of the numerical model are shown in figure 8.

Fig. 8 Two-dimensional deep water propagation without currents. Propagation under 30° with the horizontal axis. Two-dimensional Courant number 0.79, a) initial situation b) situation after 24 time steps. Contours at 0, 0.125, 0.25 and 0.5 times the initial maximum.

In this test the maximum value of the spatial action density distribution changes less than 0.2%, mean errors in the propagation
velocity of the top are less than 5%. Negative action occurs in large areas, but in absolute value it is always smaller than 0.5% of the maximum positive action density.

The two-dimensional propagation test shows the ability of the selected schemes to propagate action density under an angle with the grid axes. Results of an absorption test are not presented here, but no significant reflection of wave action occurs for waves traveling out of the model. With upstream factors \( \alpha > 0 \), deformations of the spatial distribution similar to those of the one-dimensional test can be expected to occur.

**Propagation along a 45° shore**

In test of two-dimensional deep water propagation along a coast under 45° with the \( x_1 \) axis, the spatial discretization and initial action density distribution are identical to those in the previous test except for the propagation direction (now 45°) and the presence of the coast. The results of this test are shown in figure 9.

![Figure 9](image)

**Fig. 9** Two-dimensional deep water propagation along a coast. Propagation under 45° with \( x_1 \) axis. Courant number 0.79 (two-dimensional), a) initial distribution b) situation after 24 time steps.

The results with respect to (errors in) maximum value, mean convection velocity and negative action are comparable to those of the
previous test. Near the coast, however, some diffusion can be recog-
nized, as expected when using an upstream scheme for boundary points. 
The general propagation properties are good and the angle derivative 
upstream scheme proves to be suitable for boundary management in the 
\(x_1-x_2\) space.

Depth and current shoaling

The fourth set of tests have been carried out to test the behaviour 
of the numerical model for stationary depths and currents, which are 
inhomogeneous along the propagation path of wave energy only, i.e. in 
cases of depth and current shoaling without refraction. Stationary 
wave fields are obtained by applying stationary boundary conditions 
(for a single bin, i.e. monochromatic unidirectional waves) at the 
"upwind" boundary of the model and by calculating forward in time for 
a sufficiently long period. As the poorest results can be expected for 
cases with relatively large upstream fractions, all test results 
presented have been calculated with \(\alpha = 0.4\) (Courant numbers of ap-
proximately 0.5).

First conventional depth shoaling without currents is considered. In 
these tests a constant bottom slope is used with deep water at the 
input (i.e. upwind) model boundary and shallow water at the absorbing 
boundary. From the input boundary waves with a period of 10 s 
propagate perpendicularly across straight parallel depth contours. 
Several tests have been performed, both with propagation along grid 
lines and with propagation under an angle with the grid lines (but 
still perpendicularly to bottom contours) and furthermore with dif-
ferent bottom slopes. The test conditions of the here presented 
results are collected in table 2.

The analytical solution for the short wave energy \(E\) (or variance \(F\)) 
in stationary conditions without currents is given as (e.g. Phillips, 
1977):

\[
E = E_0 \cdot \frac{\overline{E_0}}{c} \cdot \frac{c}{g} \quad (4.4)
\]
where the suffix o indicates the deep water conditions at the input boundary. The test results are collected in figure 10. This figure shows good agreement between the numerical and analytical solutions, even for extremely shallow water (case I), for propagation under an angle with the grid lines (case III) and in situations with large differences of depth in neighbouring points (cases II and III).

### Table 2 Depth shoaling cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Propagation along $x_1$ axis only, depth steps of 2 m, smallest depth 2 m, $\Delta x = 15$ km, $\Delta t = 30$ min. Maximum Courant number 0.56.</td>
</tr>
<tr>
<td>II</td>
<td>Like case I with depth steps of 5 m and smallest depth 2.5 m.</td>
</tr>
<tr>
<td>III</td>
<td>Propagation under $45^\circ$ with $x_1$ and $x_2$ axis, depth steps in $x_1$ and $x_2$ direction 10 m, $\Delta x_1 = \Delta x_2 = 25$ km, $\Delta t = 30$ min, maximum Courant number 0.47.</td>
</tr>
</tbody>
</table>

Fig. 10 Depth shoaling in several test cases without currents (cases described in table 2). --- : analytical solution, symbols : numerical model

From a numerical point of view current shoaling is identical to depth shoaling (when action conservation is considered instead of
energy conservation). Current shoaling tests can therefore be expected to show similar results as depth shoaling tests. Here only results of a one-dimensional test are shown to show the capability of the numerical model to calculate current shoaling effects. In the test the (absolute) period of the waves is 10 s and the propagation direction parallel to the $x_1$ axis. The waves propagate in the $x_1$ direction with or against a current (deep water). The current velocity has a constant gradient in the $x_1$ direction. At the input boundary the current velocity is zero. Cases with both a following and an opposing current with velocities up to 3.25 m/s have been considered. The analytical solution for wave energy in such a case is given as (e.g. Longuet-Higgins and Stewart, 1960, Phillips, 1977):

\[
\frac{E}{E_0} = \frac{c^2}{c (c + 2U)} \quad ; \quad \frac{c}{c_0} = \frac{1}{2} + \frac{1}{2} \left[ 1 + \frac{4U}{c_0} \right]^{1/2} \quad (4-5)
\]

in which $U$ indicates the current velocity and the index $0$ refers to a situation where $U = 0$. The test results are gathered in figure 11. The behaviour of the numerical model in the current shoaling case is comparable to the behaviour in the depth shoaling cases.

Fig. 11 Current shoaling in one-dimensional current shoaling test cases (deep water). — analytical solution, □ numerical model
The depth and current shoaling tests show the ability of the numerical schemes to handle spatial variations of depth and current in the propagation direction of the waves.

4.2.2 Refraction

The selected schemes for the refraction (equation (3-8), terms \( L_{x_1} \), \( L_{x_2} \) and \( L_\theta \)) have been tested in two ways. First the scheme for propagation in the \( \theta \) space (\( L_\theta \)) is isolated from the schemes for propagation in the \( x_1\times x_2 \) space (\( L_{x_1} \) and \( L_{x_2} \)) and tested separately to check the propagation properties of this part of the propagation model. Secondly a plane beach test is considered to show the ability of the model to represent depth refraction. As from a numerical point of view current refraction is identical to depth refraction, current refraction tests are not considered. In both tests cases the water level is constant and horizontal, so that all depth variations correspond to bottom level variations only.

Propagation in \( \theta \) space only

In the first set of tests the following reduced version of the action conservation equation (2-5) is solved:

\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial \theta} \left[ c_\theta N \right] = 0
\]  

(4-6)

This equation describes the change of the mean direction of a wave field to some equilibrium direction, without propagation in the \( x_1\times x_2 \) space (depth and depth gradients are constant). Considered is a situation without currents and with a bottom slope in the \( x_1 \) direction only. In such a case the propagation velocity \( c_\theta \) becomes (using the linear wave theory)

\[
c_\theta = - \frac{c(n-0.5)}{d} \frac{\partial d}{\partial x} \sin(\theta) = C_1 \sin(\theta) = \frac{D\theta}{Dt}
\]  

(4-7)

where \( c \) is the phase velocity of the waves (\( c = \sigma/k \)). If \( \partial d/\partial x_1 < 0 \) the equilibrium direction is \( \theta = 0^\circ \). In this simple case the following analytical solution for the direction of a component of the wave field can be found by integrating equation (4-7) with respect to time:
\[ \theta(t) = C_0 \arctan \left( e^{C_1 t + C_2 t} \right) \]  

(4-8)

where

- \( C_0 = -2 \) for \(-180^\circ < \theta < 0\)
- \( 2 \) for \( 0^\circ < \theta < 180^\circ \)
- \( C_2 = \ln (|\tan(0.5\theta_0)|) \)
- \( \theta_0 \) is the angle at \( t = 0 \) for the component considered.

With this analytical solution for the direction, a semi-analytical solution for the directional distribution of the action density as a function of time can be obtained. When the distribution at \( t = 0 \) is known at intervals \( \Delta \theta' \) (\( \Delta \theta' \ll \Delta \theta \), grid counter \( j \)), the analytical solution can be estimated as:

\[ N(\theta(t)_j, t) = N(\theta(0)_j, 0) \frac{2\Delta \theta'}{\theta(t)_{j+1} - \theta(t)_{j-1}} \]  

(4-9)

where \( \theta(t) \) is given by equation (4-8). The mean direction \( \theta_m \) and directional spread \( \text{spr}_{\theta} \) for the spectral component with absolute frequency \( \omega_0 \) are calculated as (Kuik et al., 1988):

\[ \theta_m = \arctan \left( \frac{b}{a} \right) \]  

(4-10)

\[ \text{spr}_{\theta} = \left[ 2 \left( 1 - \sqrt{a^2 + b^2} \right) \right]^{0.5} \]  

(4-11)

with:

\[ a = \sum \cos(\theta) D(\theta) \Delta \theta \]  

(4-12)

\[ b = \sum \sin(\theta) D(\theta) \Delta \theta \]  

(4-13)

\[ D(\theta) = \frac{N(\omega_0, \theta)}{\sum N(\omega_0, \theta)} \Delta \theta \]  

(4-14)

Where \( a \) and \( b \) are the first Fourier components of the directional distribution \( D(\theta) \).
In figures 12 and 13 the results of such a case with a bottom slope in x₁ direction only are presented. The initial mean direction in the calculations is 60° and the initial distribution is of the type \( \cos^2(\theta - \theta_m) \). The maximum Courant number \((C_1 \Delta t/\Delta \theta)\) is 0.80 and the directional increment \(\Delta \theta = 15°\) (apart from resolution, only these two parameters are relevant from a numerical point of view). The semi-analytical solution was obtained by using equations (4-8) and (4-9) with \(\Delta \theta' = 1°\).

![Fig. 12 Mean angle θ and angular spread spr_θ for case with propagation in directions space only. --- semi-analytical solution (equations (4-8) and (4-9)), • numerical model.](image)

![Fig. 13 Directional distributions for cases with propagation in directions space only. --- initial distribution, semi-analytical solution, x numerical model. a) time t_a in fig. 12, spr_x > Δx, b) time t_b in fig. 12, spr_x < Δx.](image)
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Figure 12 shows a good agreement between results of the numerical model and semi-analytical solutions for the mean direction $\theta_m$ and the spread $\delta_\theta$, in spite of the large change in direction and the small spread compared to the directional increment $\Delta \theta = 15^\circ$. Figure 13 shows similar good results for the directional distribution. Considering the small directional spread, specially at time $t_B$, the results of the numerical model are excellent.

These tests shows the ability of the term $L_\theta$ (equation (3-8)) combined with the conservative elimination to describe the change of direction of a spectrum. Notice that all components of the refraction velocity $c_\theta$ (equation (2-9)) are dependent on a sine or cosine of the direction and are therefore of the same nature as the depth refraction velocity. It is therefore not necessary to present results of tests with e.g. shearing currents.

Plane beach refraction

In the plane beach test the combined propagation in $x_1$, $x_2$ and $\theta$ space (including shoaling) is tested. For the simple geometry of a plane beach semi-analytical solutions, similar to those of the previous tests, are available. Considering a situation with a bottom slope in $x_1$ direction only, the angle as a function of depth can be calculated from

$$\frac{\sin(\theta)}{c} = \text{constant} \quad (4-15)$$

where the constant is determined from the deep water boundary conditions. Using this analytical solution for the directions, a semi-analytical solution for the directional distribution as a function of depth is found, similar to equation (4-9):

$$N(\theta_i) = N(\theta_{i-1}) \frac{2 \Delta \theta'}{\theta_{i+1} - \theta_{i-1}} \left[ \frac{c_{g,0} \cos(\theta_{i-1})}{c_g \cos(\theta_i)} \right] \quad (4-16)$$

where the suffix $o$ indicates known parameter values at the deep water boundary.
The beach considered in the tests has a slope $\partial d/\partial x_1 = -10^{-3}$ and is described in the model with increment $\Delta x_1 = 5$ km and a depth range of 50-5 meters. The directional step $\Delta \theta = 15^\circ$, time step $\Delta t = 5$ min and upstream fraction $\alpha = 0.4$ Two cases, both with a wave period of 10 s, have been considered. In case I a situation is considered with a relatively large angle $\theta_m = 45^\circ$ and small spread ($\cos^{12}(\theta - \theta_m)$) at the deep water boundary. In case II a situation is considered with a relatively small angle $\theta_m = 15^\circ$ and large spread ($\cos^2(\theta - \theta_m)$) at the deep water boundary. Results for both tests are shown in figures 14 and 15.

Fig. 14 Depth refraction on plane beach, integrated parameters, a) mean direction $\theta_m$, b) directional spread $\operatorname{spr}_\theta$, c) total action $m_o$, normalized with total action at boundary $m_{ob}$ ($m_o = \Sigma N(\omega, \theta) \Delta \omega \Delta \theta$).
In case I (small initial spread) the mean direction $\theta_{m}$ and total action $m_{o}$ (equation (4.12)) as obtained from the numerical model show good agreement with the semi-analytical solutions (again obtained with $\Delta \theta' = 1^\circ$). The directional spread $s_{t}$, however, is represented badly. This can be explained by the extremely small directional spread compared to the directional increment $\Delta \theta$. Case II, in which much larger spreads occur, shows much better results.

In view of the poor resolution in the directions space, refraction is represented well in the numerical model. Again separate tests for current refraction will not give more information about the model behaviour.

It should be noticed that in published models the propagation in the directions space is usually described using a first order upstream scheme (e.g. Golding, 1983, Sakai et al., 1983, Gao, 1986). Test results presented in these papers show a good representation of refraction by such models. However, the test results presented do not give complete insight in the behaviour of the numerical models, as the only parameter assessed quantitatively is the mean direction for a plane beach refraction case. This mean direction is expected to be well presented when using a first order upstream scheme. The weak point of the upstream scheme, however, is the introduction of numerical diffusion resulting in a smoothing of the directional
distribution. As in general the resolution in the directions space is poor (see e.g. figure 15), the diffusion is expected to have a significant influence on the spread. As the above papers provide no information on the directional spread, the influence of the numerical diffusion cannot be established from them. It is easy to imagine that a local diffusion in the directions space will cause errors in the \( x_1 \)-\( x_2 \) space at large distances, specially on irregular bottoms. As the plain beach test is basically two-dimensional (\( x_1, \theta \)) the above three-dimensional effect will not occur and the error in directional distribution will not influence the mean direction. Consequently one should consider the directional spread as a parameter to be tested quantitatively in refraction models or one should considered true three-dimensional test conditions with quantitative assessment of integral parameters such as the directional spread.

4.2.3 Change of absolute frequency

The selected schemes for the change of absolute frequency due to unsteady depths and currents (equation (3-8), terms \( L_{x_1}', L_{x_2} \) and \( L_{\omega} \)) have been tested in two ways, similar to the tests for refraction. First the propagation of action in the \( \omega \) space (\( L_{\omega} \)) has been isolated and tested separately to show the propagation properties of this part of the propagation model. Secondly a situation with propagation in \( x_1 \) and \( \omega \) space is considered to show the ability of the model to represent changes of absolute frequency in a more realistic situation.

Propagatio in \( \omega \) space only

In the first set of tests the following reduced version of the action conservation equation (2.5) is solved:

\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial \omega} \left[ \omega N \right] = 0
\]  

This equation describes the transport of action in the \( \omega \) space, without convection in the \( x_1 \)-\( x_2 \) space. As explained above, problems can be expected in the treatment of the high frequency boundary if unsteady currents are considered. A situation is therefore considered with current variation only. To simplify the situation even more, a

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purely academic case is considered where $\partial U/\partial t$ is positive but where the current velocity remains zero. The propagation velocities $c_{\omega}(\omega)$ thus calculated are constant in time. Furthermore only one direction is considered. The numerical solution can be compared to the results of an approach similar to that for the case of propagation in the $\theta$ space only (equations (4-8) and (4-9)). The frequencies $\omega(t)$ are calculated numerically using equation (2-10), as no analytical solution for $\omega(t)$ was found.

Three different discretizations of the frequency space are considered. In case A the frequencies range from 0.12 to 0.36 Hz with increments of 0.01 Hz. In case B an exponential frequency distribution is considered (equation (3-16)), with $\xi = 1.1$ and frequencies ranging from 0.04 to 0.394 Hz (as in WAM). In case C an such a distribution is considered with $\xi = 1.14$ and frequencies range from 0.02 to 0.464 Hz. In all three cases 25 frequencies are considered. Time step $\Delta t$, and the time derivative of the current velocity are chosen to give a maximum Courant number of 0.8 for discretization A ($\Delta t$ and $\partial U/\partial t$ identical for all three cases). The initial action density spectrum is a Pierson-Moskowitz spectrum with a mean frequency of 0.2 Hz. The results for the different discretizations are presented in figure 16.

\[ \frac{N}{N_{\text{max}, t=0}} \]

\[ f_a \text{ (Hz)} \]

\[ \text{model boundaries} \]

\[ \bullet \text{ A} \bullet \text{ B} \times \text{ C} \]

\[ \text{A} \]

\[ \text{B} \]

\[ \text{C} \]

Fig. 16 Spectral transformations due to an instationary current without propagation in other spaces.
For all three discretizations the results of the numerical model show good agreement with the above described semi-exact results for most of the discrete frequencies. Case A shows large differences between the results of the numerical model and the semi-exact solutions for discrete frequencies close to the high frequency boundary. If the high frequency boundary is located at higher frequencies than in case A, and \( \Delta \omega \) at high frequencies is larger (cases B and C) the results are much better. The frequency distribution C seems suitable if closed boundaries are assumed. Notice that the problems with the high frequency flank can be eliminated by the numerical treatment of the source functions if the parametric high frequency tail (section 3.3) is always applied to the highest discrete frequency of the numerical model (making discretization B, i.e. the discretization as used in WAM, suitable).

**propagation in \( x_1 \) and \( \omega \) space**

In this test a situation is considered, where waves travel in water of constant depth (both in space and time), over a homogeneous current which varies in time only. Furthermore unidirectional waves in extremely shallow water are considered, which become non-dispersive, since the propagation velocity of all spectral components becomes equal to \( \sqrt{gd} \). Therefore changes in spectral shape only occur due to the instationarity, and not due to dispersion in the \( x_1(-x_2) \) space. In such homogeneous current conditions with homogeneous and stationary depth both energy and action of the waves is conserved and both wavenumber and intrinsic frequency remain constant (e.g. Tolman, 1988).

A situation is considered with a depth of 1 m and a current velocity which is zero at \( t = 0 \) and goes to 1 m/s in 30 h. The change in time of the current velocity is described by the function \( 1 - \cos(t/T) \), where \( T = 60 \) h. After 30 h the current velocity remains constant. Initially the action density has a Gaussian distribution in the frequency space with a mean frequency of 0.1 Hz and a spread of 0.07 Hz. The spatial increments are 25 km and the time step is 1.5 h.
Fig. 17 Spatial distribution of normalized total energy in test with homogeneous instationary current — analytical solution, numerical model with $c_\omega \neq 0$, numerical model with $c_\omega = 0$.

Since the energy is conserved during propagation and the waves are non-dispersive, the total energy (integrated over the spectrum) is convected in the $x_1$ space without changes in the shape of the distribution in this space. The ability of the numerical model to do so is illustrated in figure 17, which shows the spatial distribution of the action density at $t = 0$ and at $t = 30$ h ($t/\Delta t = 20$). The agreement between the exact solution and the results of the numerical model is excellent. Also shown in this figure are the results as obtained with the numerical model when $c_\omega = 0$, i.e. when changes of absolute frequency due to the unsteady current are neglected. The differences between the thus obtained results and the results as obtained with the full numerical model ($c_\omega \neq 0$) show the importance of the change in absolute frequency in the situation considered here.

Finally figure 18 shows normalized frequency spectra as a function of either absolute frequency $f_a$ or intrinsic frequency $f_\pi$, both for the initial situation at $x = 300$ km and after 30 h at $x = 700$ km.

Figure 18 a) shows a significant change of the shape of the absolute frequency spectra, whereas figure 18 b) shows only small variations in the shape of the intrinsic frequency spectrum. Considering the poor resolution in the frequency space in the test situation considered,
the latter figure shows a good agreement with the analytical solution (i.e. no change in the shape of the intrinsic frequency spectrum).

Fig. 18 Normalized frequency spectra in the test with homogeneous unsteady current, $\circ: t = 0$, $x = 300$ km, $+$: $t = 30$ h, $x = 700$ km. a) absolute frequency spectrum, b) relative frequency spectrum.

4.3 Source functions

4.3.1 Introduction

Results of test calculations of duration limited growth curves are presented, for which the source function module in WAVEWATCH is used only (i.e. homogeneous conditions, no propagation). These calculations have been performed for three reasons. First the wave growth characteristics of the model presented here (WAVEWATCH) are compared with those of other models presented in literature (e.g. SWAMP group, 1985), in particular with the WAM model. Secondly the influence of the interpolation needed to obtain $F(f_r, \theta)$ from $N(\omega, \theta)$ in case of the presence of currents is established. Such interpolation introduces diffusion in the frequency space, which might influence growth characteristics of the model. Finally the bottom roughness scale as used in the source function for dissipation in the bottom boundary layer is calibrated. The first two sets of tests are performed simultaneously, by calculating non-dimensional growth curves and spectral shapes for cases with and without currents, with identical "relative" wind speeds.
\( \bar{U}_{10} - U \) (section 4.3.2). The bottom roughness length scale \( k_N \) is calibrated considering cases without currents only (section 4.3.3).

In all tests of the source functions the following discretization of the spectrum is used: 24 discrete angles over 360°, resulting in an angular increment of \( \Delta \theta = 15^\circ \) (WAM: \( \Delta \theta = 30^\circ \)) and 26 frequencies, ranging from 0.041 through 0.453 with \( \xi = 1.10 \) (equation (3-16), identical to WAM). Empirical constants in the description of dissipation due to whitecapping and non-linear interactions are identical to those of the WAM model (values as presented in chapter 2 and section 3.3).

4.3.2 Duration limited growth in deep water

To compare the results of WAVEWATCH with the results of other models (in particular WAM), deep water duration limited growth curves have been computed, following test case II of the SWAMP report (SWAMP group, 1985). The non-dimensional variance \( \bar{F} \) (integrated over the spectrum, in the SWAMP report called energy) and the non-dimensional peak frequency \( \bar{\nu}_p \) are considered as a function of the non-dimensional time \( \bar{t} \)

\[
\bar{F} = g^2 \int_0^{2\pi} \int_0^\infty F(\omega, \theta) \, d\omega \, d\theta \, / \, U_*^4
\]  
(4-18)

\[
\bar{\nu}_p = \frac{U_* \bar{\nu}_p}{g}
\]  
(4-19)

\[
\bar{t} = gt / U_*
\]  
(4-20)

where \( U_* \) is the wind friction velocity \( U_* = \sqrt{C_d} U_{10} \), comparable to equation (2-14). In the SWAMP study \( C_d \) is assumed to be a constant \( (C_d = 1.83 \times 10^{-3}) \), whereas \( C_d \) in equation (2-14) is a function of the wind speed \( U_{10} \). In the calculations with the numerical model \( C_d \) of equation (2-14) is used, whereas \( C_d = 1.83 \times 10^{-3} \) (SWAMP) is used in the normalization of the non-dimensional parameters.

Growth curves for the variance \( \bar{F} \) and the peak frequency \( \bar{\nu}_p \), calculated with WAVEWATCH for current velocities of 1, 0 and -1 m/s

- 50 -
respectively (in the wind direction) and wind speeds of 20 m/s in a frame of reference moving with the current are presented in figure 19. In addition growth curves for the WAM model are presented, as calculated with the one-dimensional WAM model, mentioned in section 3.3 \((U_{10} = 20 \text{ m/s})\). For these calculations the one-dimensional WAM model was provided with a discretization of the spectrum, identical to that of WAVEWATCH. Finally figure 19 shows the envelope of the results from the SWAMP study.

\[\text{Fig. 19} \quad \text{a) Non-dimensional variance } \bar{F} \text{ and b) peak frequency } \bar{t}_p \text{ as a function of time } \bar{t} \text{ for } U_{10} = 20 \text{ m/s in a frame of reference moving with the current velocity.}\]

The results of WAVEWATCH in the case without currents \((U = 0)\) show excellent agreement with the results of the one-dimensional WAM model (figure 19). Furthermore the results of WAVEWATCH for the cases with currents show good agreement with the results of WAVEWATCH for the cases without currents, which indicates that the diffusion as introduced by the interpolation in the source function model has no significant influence on the growth characteristics of WAVEWATCH.

One-dimensional spectra \(F(\omega)\), as calculated with WAVEWATCH for current velocities \(U = -1, 0 \text{ and } 1 \text{ m/s and relative wind speeds } U_{10} - U = 20 \text{ m/s}\), are shown in figure 20.
The spectra as calculated for the three different current velocities show a similar general shape of the spectrum (within the resolution of the model), but different peak frequencies and high frequency saturation levels. As explained below, such differences were to be expected. For all three current velocities identical source functions $S(\sigma, \theta)$ (formulation and magnitude) are applied in a frame of reference moving with the average current velocity, as the wind speed in this frame is identical for all three cases (see chapter 2). Therefore the relative frequency spectra $F(f_r, \theta)$ or $F(\sigma, \theta)$ are identical for all three cases at any time (including identical relative peak frequencies and identical high frequency saturation levels).

The absolute peak frequency $f_{a,p}$ is estimated using equation (2-1), the relative peak frequency $f_{r,p}$ and the current velocity $U$ (for simplicity ignoring directional spread). As the relative peak frequencies are equal and as the current velocities are different for the three cases considered, the absolute peak frequency for the three cases will be different. Considering equation (2-1), the absolute peak frequency for $U < 0$ (or $U > 0$) is smaller (or larger) than that of the case without currents, (cf. figure 20).
The saturation level of the absolute frequency spectrum is calculated from the saturation level of the relative frequency spectrum using a Jacobean transformation. To determine the saturation level of the absolute frequency spectra in the cases with currents relative to the saturation level in a case without currents, consider the following shape of the high frequency tail of the relative frequency spectrum:

\[ F(f_r) = C f_r^{-m} \]  \hspace{1cm} (4-22)

Where \( C \) is a constant with dimension \( L^2_t \) and \( m \) is a non-dimensional constant, equal to 4 or 5. Applying a Jacobean transformation to this spectral shape (equations (2-1) and (2-12) and for simplicity assuming that the waves are unidirectional), the absolute frequency spectrum at high frequencies becomes:

\[ F(f_a) = C f_a^{-m} \frac{(1 - k \cdot \frac{U}{\omega})^{1-m}}{1 + k \cdot \frac{U}{\omega}} \]  \hspace{1cm} (4-23)

Consequently the ratio between the high frequency range of absolute frequency spectra calculated from identical relative frequency spectra with or without currents becomes:

\[ \frac{F(f_a)_{U=0}}{F(f_a)_{U=0}} = \left[ \frac{1 + k \cdot \frac{U}{\omega}}{1 - k \cdot \frac{U}{\omega}} \right]^{m-1} \]  \hspace{1cm} (4-24)

For \( k \cdot U < 0 \) (or \( k \cdot U > 0 \)) the spectral level at high frequencies thus is lower (or higher) than for \( U = 0 \) (cf. figure 20).

The test cases for source function in deep water (no currents) show the ability of the model to reproduce results of the WAM model for homogeneous deep water conditions. Calculations with homogeneous currents show that the diffusion in the frequency space as introduced by the interpolation between the fixed and moving frame of reference has no significant influence on the results.
4.3.3 Duration limited growth in shallow water

To calibrate the bottom roughness length scale $k_N$ in the bottom dissipation source term a homogeneous situation is considered with wind speed $U_{10} = 20$ m/s and without currents. For several non-dimensional depths $\bar{d}$ and roughness scales $k_N$ the non-dimensional equilibrium wave height $\bar{H}_\infty$ and wave period $\bar{T}_\infty$ are plotted in figure 21 (the suffix $\infty$ indicating an "infinite" integration time).

$$\bar{d} = g d / U_{10}^2$$  \hspace{1cm} (4-25)

$$\bar{H}_\infty = g H_\infty / U_{10}^2 = 4 g / U_{10}^2 \int_0^{2\pi} \int_0^\infty F_\infty(\omega, \theta) \, d\omega \, d\theta$$  \hspace{1cm} (4-26)

$$\bar{T}_\infty = g T_\infty / U_{10} = g / (f_{p\infty} U_{10})$$  \hspace{1cm} (4-27)

![Fig. 21 Non-dimensional wave height $\bar{H}_\infty$ and wave period $\bar{T}_\infty$ as a function of non-dimensional depth $\bar{d}$ for different roughness scales. Partially adapted from Holthuijsen (1980).](image)

Also shown in this figure is the envelope of observations reviewed by Holthuijsen (1980). A comparison with Holthuijsen (1980) shows that results for values of $k_N$ between 0.02 m and 0.05 m show excellent agreement with several recent analytical expressions for $\bar{H}_\infty$ and $\bar{T}_\infty$ as a function of $\bar{d}$ (e.g. Bretschneider, 1973, Krylov, 1976, Groen en

The test cases with homogeneous shallow water conditions show that the bottom dissipation model with a constant bottom roughness results in a good agreement between numerical results and measurements (or analytical expressions) for $\bar{h}_\infty$ and $\bar{T}_\infty$ if $k_N$ ranges from approximately 0.02 m to 0.05 m. Such roughness would correspond to relatively small ripples (e.g. Weber et al., 1988, see chapter 2).

4.4 Propagation and source functions

4.4.1 Introduction

Results of test calculations of fetch limited growth curves are presented, for which both the propagation module and the source function module in WAVEWATCH are used. These calculations have been performed to compare the fetch limited growth characteristics of WAVEWATCH with those of other models presented in literature (e.g. SWAMP group, 1985), in particular with the WAM model and with the numerical model EXACT-NL (see e.g. chapter 2). Following SWAMP test II, deep water without currents is considered.

The discrete representation of the spectra is identical to the discretization as described in section 4.3.1.

4.4.2 Fetch limited growth in deep water

To compare the fetch limited growth characteristics of WAVEWATCH with those of other models, in particular WAM, calculations have been performed for a wind ($U_{10} = 20$ m/s) blowing perpendicular across a straight coastline over deep water (SWAMP test II). The time step $\Delta t = 15$ min., the spatial increment $\Delta x = 25$ km and $a_{\text{min}} = 0.25$. Presented are non-dimensional variance $\tilde{F}$ and peak frequency $\tilde{f}_p$ as a function of non-dimensional fetch $\tilde{x}$ and frequency spectra $F(\omega)$.

$$\tilde{x} = \frac{g x}{U^2}$$  \hspace{1cm} (4-28)
Non-dimensional fetch limited growth curves for $\tilde{F}$ and $\tilde{f}_p$ are shown in figure 22. The results of WAM, SWAMP and JONSWAP as presented in this figure are taken from the WAMDI group (1988), whereas the results of EXACT-NL are taken from the SWAMP group (1985).

![Graphs showing non-dimensional variance $\tilde{F}$ and peak frequency $\tilde{f}_p$ as a function of fetch $\tilde{x}$ (U$_{10}$ = 20 m/s).](image)

Fig. 22 a) Non-dimensional variance $\tilde{F}$ and b) peak frequency $\tilde{f}_p$ as a function of fetch $\tilde{x}$ (U$_{10}$ = 20 m/s).

The agreement between the non-dimensional variance $\tilde{F}$ of WAM and WAVEWATCH for small fetches ($\tilde{x} = 10^5$) is poor, but the agreement for larger fetches ($\tilde{x} = 10^7$) is good. The agreement with respect to peak frequency $\tilde{f}_p$ is good for small fetches, but poor for larger fetches. Although some differences between WAM and WAVEWATCH could be expected in view of the different propagation schemes used, differences as large as those presented in figure 22 are not likely caused by the differences in the propagation scheme only. Further investigations into the differences between WAM and WAVEWATCH are out of the scope of this study, but nevertheless interesting.

Growth curves for EXACT-NL are also presented in figure 22. Compared with EXACT-NL, WAVEWATCH also shows large differences, but the results of WAVEWATCH are in general closer to those of EXACT-NL then the results of WAM. In view of the differences between WAM and EXACT-NL, the results of WAVEWATCH for fetch limited conditions are acceptable.
Spectra for several non-dimensional fetches are shown in figure 23. In these spectra an overshoot is present. The overshoot is somewhat larger than that of WAM (WAM, their figure 5), but less pronounced than that of EXACT-NL (SWAMP group, 1985, their figure 7.3d).

![Figure 23](image)

**Fig. 23** Spectra in fetch limited equilibrium situation at different fetches (WAVEWATCH).

The tests in fetch limited situations show the capability of WAVEWATCH to produce acceptable fetch limited growth curves, for which spectra show a clear overshoot. However, the differences between the results of WAM, WAVEWATCH and EXACT-NL suggest that further investigations are needed into the behaviour of third generation models in fetch limited situations.
5.1 Introduction

In this chapter some preliminary results for wind waves on the (southern) North Sea due to south-westerly winds are presented to give a first impression of the importance of wave-current interactions at the (southern) North Sea for such conditions. As the results presented here are meant only to illustrate the importance of wave-current interactions, the results will not be discussed extensively. Presented are the spatial distribution of parameters such as significant wave height, mean absolute or relative period and mean wave length. Furthermore time series of integral wave parameters are presented for the location Euro-0 (see figure 24, water depth 26 m). Finally the influence of depth and current variations due to the tide is illustrated by comparing results of calculation incorporating wave-current interactions with results of calculations in which current and depth variations are neglected.

A situation is considered with a moderate south-westerly storm on the southern North Sea (Jan. 1st and 2nd 1988). The wind conditions will be discussed in the following section. For the calculation of the depth and current fields an 8 km x 8 km plane grid is used, which is described by Voogt (1985) (Mercator projection, figure 24). For the wave model a 24 km x 24 km grid was extracted from the original grid, by considering every third grid point in x₁ and x₂ direction only. The spectra are described using 24 directions and 26 frequencies, identical to the description used in the test calculations as presented in sections 4.3 and 4.4 (see section 4.3.1). All values of constants are taken from the the WAM model, as discussed in chapter 2 and section 3.3. The minimum upstream fraction is set to \( \alpha_{\text{min}} = 0.25 \) and the bottom roughness length scale is set to \( k_{\text{N}} = 4 \) cm.
5.2 Synoptical description

In the period from December 27th 1987 to January 5th 1988 a series of depressions passes over Scotland in north-easterly directions to Scandinavia, resulting in south-westerly winds over the southern North sea for a period of several days. A weather map for January 2 is shown in figure 25. Wind fields for this period where given on a grid with increments of 1° in longitude and latitude. Wind speeds for grid points of the plane grids used in the calculation of currents and waves where obtained from the spherical grid by bilinear interpolation. The time increment of the wind fields is three hours. In both current and wave calculations the wind speed and direction is kept...
constant for a period of three hours, centered around the time of the wind field. Wind speeds and wind directions from these fields for location Euro-0 are presented in figure 27. (Directions are defined as directions from which the wind blows).

Fig. 25 Weather map jan. 2nd 1988, 1200 h GMT (source: dagelijkse weerberichten ISSN 0168-9371)

5.3 Depth and current fields

Depth and current fields for the period considered where calculated using the numerical model DUCHESS (see e.g. Wang, 1989), which solves the depth integrated shallow water long wave equations. At the open boundaries of the depth and current model (figure 24) astronomical boundary conditions where applied, consisting of O1, K1, N2, M2, S2 and K2 components for both the northern and southern boundary and M4 and MS4 components for the southern boundary only (Voogt, 1985). The wind friction \( r_w \) was calculated from the wind friction velocity \( U^* \) as given by equation (2-14) \( U^* = \sqrt{r_w / \rho_a} \). Calculations of depth and current started at december 29, 0 h GMT to allow for a two day start-up period for the depth and current model and a one day start-up.
Example Hindcast period for the wave model. An example current and water level field for this period is shown in figure 26. Water level, current velocity and current direction (defined similar to the wind direction) for location Euro-0 are presented in figure 27.
Fig. 27 Wind speed $U_{10}$, wind direction, water level, current velocity and current direction at location Euro-0
5.4 Results of the wave model

Calculations with WAVEWATCH started at dec. 31th, 1987, 0 h GMT to allow for a start-up period of one day. All results presented here have been calculated with WAVEWATCH including all wave-current interactions, unless specified otherwise.

The spatial distribution of significant wave height $H_s$, mean absolute period $T_a$, mean relative period $T_r$ and mean wavelength $L$ for jan. 1st 1988, 1200 h GMT and jan. 2nd 1988, 1200 h GMT are shown in figures 28 and 29. These parameters are defined as:

$$H_s = 4 \sum \sum F(\omega, \theta) \Delta \omega \Delta \theta$$

(5-1)

$$T_a = \langle 2\pi/\omega \rangle$$

(5-2)

$$T_r = \langle 2\pi/\sigma \rangle$$

(5-3)

$$L = \langle 2\pi/k \rangle$$

(5-4)

where $\langle . . \rangle$ denotes the average over the spectrum variance density spectrum $F(\omega, \theta)$ (equation (2-20)).

The significant wave height ($H_s$), the mean absolute and relative period ($T_a$, $T_r$) and the mean wavelength ($L$) for the location Euro-0 are shown in figure 30 as a function of time. In this figure results of WAVEWATCH are presented as calculated with wave-current interactions and without wave-current interactions. The latter results where obtained by ignoring currents and mean water level variations in WAVEWATCH. The figure shows a clear current influence on integral wave parameters, in particular on significant wave height and absolute frequency.

Finally significant wave heights of WAVEWATCH at a fixed location, calculated with or without wave-current interactions, are intercompared by considering the rms difference (in time) between the wave
heights of the two calculations, normalized with the average significant wave height as calculated with wave-current interactions (i.e. scatter index):

\[
SI(H_s) = \frac{\left[ \frac{1}{T} \sum_{s_1} (H_{s_1} - H_{s_2})^2 \Delta t \right]^{0.5}}{\frac{1}{T} \sum_{s_1} H_{s_1} \Delta t}
\] (5-5)

Fig. 28 Spatial distribution of significant wave height \(H_s\), mean absolute and relative period \(T_a\) and \(T_r\), and mean wave length \(L\) for Jan. 1st 1988, 1200 h\(\text{GMT}\).
where $T = \Sigma \Delta t$ is the period over which the scatter index is determined and where the suffix 1 (or 2) denotes results of WAVEWATCH including (or without) interactions. Similarly scatter indices for absolute period, relative period and wavelength ($SI(T_a)$, $SI(T_r)$ and $SI(L)$ respectively) are defined. The spatial distributions of these scatter indices, based on integration over two days, are presented in figure 31.

![Diagram showing significant wave height, mean wave length, mean absolute period, and mean relative period contours.]

Fig. 29 Like figure 28 for Jan. 2nd 1988, 1200 h GMT.
Figure 31 shows that wave-current interactions influence wave parameters in a large part of the southern North Sea. In particular, the significant wave height $H_s$ and the absolute period $T_a$ show significant changes.

Fig. 30 Significant wave height, mean and absolute and relative period and mean wave length as a function of time for location Euro-0. WAVEWATCH without interactions.
significant interactions with scatter indices of up to 15% and 10% respectively. In view of the claimed accuracy of third generation models (e.g. WAM, their figure 13), such scatter indices are not negligible.

**Fig. 31** Scatter indices for significant wave height, mean and absolute and relative period and mean wave length, comparing results of WAVEWATCH as calculated with and without interactions (Jan. 1st and 2nd 1988).
Tests for WAVEWATCH as presented in this report show good results. In the following several points will be discussed, i.e. propagation in the $x_1-x_2$ space, propagation in the directions space, the calibration of the bottom roughness length scale $k_N$, and the differences between EXACT-NL, WAM and WAVEWATCH with respect to fetch limited growth curves.

- The accuracy of the numerical model for propagation in the $x_1-x_2$ space over many meshes depends strongly on the resolution in this space and on the value of the upstream fraction (e.g. figure 7). For practical reasons (such as available computer memory and computational effort) the number of degrees of freedom in the numerical model (i.e. number of discrete spectral bins $\times$ number of spatial grid points) cannot be much larger than in the present computations for a storm in North Sea (chapter 5, using computer facilities which presently are available at the Delft University of Technology, i.e. an IBM 3083 JX1). Thus the spatial resolution can only be improved at the expense of the spectral resolution, which is relatively poor. The spatial resolution of 24 km as used in the present computations (chapter 5) is relatively poor in particular in the southern North sea. This, and the relatively large upstream fraction therefore needed (e.g. $\alpha_{\min} = 0.25$) doubtlessly results in numerical errors. However, errors as large as indicated in figure 7 are not expected to occur in the case of the present North Sea computations, as propagation only takes place over a limited number of meshes.

- The present directional resolution (chapters 4 and 5, $\Delta \theta = 15^\circ$) is relatively poor (e.g. figure 15). Even when higher order numerical schemes are used to describe the change of direction due to refraction (as in WAVEWATCH), the poor resolution can result in errors in the change of direction and in the directional spread for situations where the spectrum shows a small but otherwise fairly realistic directional spread (e.g. swell, see figure 14). Note that the directional resolution of WAVEWATCH ($\Delta \theta = 15^\circ$) is
relatively good compared to e.g. WAM (Δθ = 30°), and that all other wave models known to the author use a first order upstream scheme to describe the change of direction due to refraction, resulting in much larger errors. Published tests for refraction are insufficient to illustrate some of the effects of the poor directional resolution or of the first order schemes used in most wave models (see pages 44 and 45).

- The bottom roughness $k_N$ is calibrated using academic test cases only. The value for $k_N$ (≈ 4 cm) thus obtained is fairly realistic and applicable for a research model. For application in an operational wave model the bottom roughness length scale could be further calibrated using observed situations to obtain the best possible results.

- The differences between the fetch limited growth curves of WAM and of WAVEWATCH (and of EXACT-NL, see figure 22) are much larger than expected at the onset of this study. Since WAM and WAVEWATCH use identical approximations to the physics and show identical duration limited growth curves, these differences seem to be caused by numerical effects in the coupling of propagation and source functions. The WAMDI group (1988) assumes that the differences (in spectral shape for fetch limited conditions) between WAM and EXACT-NL are caused by the discrete interaction approximation used in WAM (and WAVEWATCH). However, the differences between WAM and WAVEWATCH presented here suggest that part of the differences between WAM and EXACT-NL are caused by numerical effects.

Preliminary results for a moderate south-westerly storm in the southern North Sea show that in such conditions wave-current interactions result in variations of wave height and absolute period of the order of 10%. Work is on progress to determine the effects of wave-current interactions for other situations (in particular north-westerly storm conditions). The results will be compared with measurements, and a sensitivity analysis will be performed for different wave-current interaction mechanisms, such as e.g. current refraction and of the unsteadiness of the current.
Acknowledgements

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APPENDICES

A Dynamic upstream fraction

The influence of a dynamic upstream fraction in the numerical propagation scheme on the unstable behaviour of the combined propagation and source function modules is investigated numerically. To consider many situations with little calculational effort, a model including growth and deep water propagation only is considered (denoted as the simple model), instead of the third generation model presented in this report. After results have been obtained with this model, their implication for application in a third generation model will be discussed. The simple model is described by the following equation:

$$\frac{\partial E}{\partial t} + c \frac{\partial E}{\partial x} = A + B E \quad (A-1)$$

with

$$A(f_x) = 2.2 \times 10^{-11} \ U_{10}^4 \quad (A-2)$$

$$B = 1.23 \left( \frac{U^*}{c} \right)^2 \exp \left[ -3.2 \times 10^{-8} \left( \frac{U^*}{c} \right)^4 \right] \quad (A-3)$$

In this equation $A$ is the linear growth rate and $B$ is an exponential growth rate (Inoue, 1966). To keep this model as simple as possible only one (fixed) frequency propagating in the wind direction is considered. Note that as this model includes no dissipation or limiting form, it can result in a stationary solution in fetch limited situations only.

Following the hypothesis of section 3.4, the magnitude of the upstream fraction $\alpha$ can be related to the relative importance of source functions compared to propagation. This relative importance is determined by considering the time scales of generation (source functions) and propagation respectively. In the model of equation (A-1) the exponential growth rate $B$ is dominant in all practical
Dynamic upstream fraction

situations, so that $1/B$ is a typical time scale of generation. Similarly a typical time scale for changes in $E$ due to propagation between two grid points is given by $\Delta x/c_g$. Thus the relative importance of growth and propagation can be based on the parameter

$$\frac{B\Delta x}{c_g}$$

(A-4)

For $B\Delta x/c_g \ll 1$ the propagation is dominant and for $B\Delta x/c_g \gg 1$ the generation is dominant. For $B\Delta x/c_g = O(1)$, propagation and generation are of roughly equal influence.

To investigate the existence of a critical upstream fraction $\alpha_c$, needed to stabilize the propagation scheme, and to investigate the relation between such a critical upstream fraction and the parameter $B\Delta x/c_g$, equation (A-1) is solved numerically using a fractional step method as described in section 3.1. The left hand side of equation (A-1) is solved using equations (3-8) and (3-10). A fetch limited situation is considered where a uniform, stationary wind blows perpendicularly off an infinite straight coast over a deep sea, which is initially at rest. For all wind speeds $U_{10}$ and wave frequencies considered, calculations are performed with different upstream fractions $\alpha$, to determine the smallest upstream fraction $\alpha$ for which the numerical model shows stable behaviour (i.e. the critical value $\alpha_c$). A stable solution is defined as a solution where the variation $\Delta E/E$ per time step averaged over all grid points becomes smaller than $\gamma_1$ (typical $\gamma_1 = 10^{-5}$). To assure that the stable solution is reached within a reasonable time, the model is considered to be stable only if the above condition is satisfied within a time period $T < \gamma_2 \frac{\alpha}{c_e}$ (typical $\gamma_2 = 5.0$), where $\alpha$ is an estimate for the time in which the (analytical) solution is reached. To estimate $\alpha$, consider the analytical solutions for equation (A-1) for both a fetch limited stationary situation ($\partial E/\partial t = 0$) and a duration limited homogeneous situation ($\partial E/\partial x = 0$). Using these analytical solutions $\alpha$ is estimated as the time needed in the duration limited situation to reach the energy level at the location with the largest fetch ($x_{\text{max}}$) in the fetch limited situation. After some algebra, $\alpha$ then simply becomes:
\[ \tau_e = \frac{X_{\text{max}}}{c_g} \]  

(A-5)

Numerical experiments with this model have shown that a critical upstream fraction \( \alpha_c \) in fact exists. This upstream fraction \( \alpha_c \) is insensitive to variations in \( \gamma_1 \) and \( \Delta t \) (as long as \( \gamma_1 < 10^{-4} \) say and as long as Courant criteria are satisfied). The magnitude of \( \alpha_c \) varies with wind speed \( U_{10} \) (i.e. A and B), propagation velocity \( c_g \) and spatial increment \( \Delta x \) through the non-dimensional parameter \( B \Delta x/c_g \), as is shown in figure 32. Considering the results as presented in figure 32, \( \alpha_c \) is safely described by the following expression for \( B \Delta x/c_g \) is not too close to 0:

\[
\alpha_c = \min \left[ 1, 0.10 + 0.75 \frac{B \Delta x}{c_g} \right]
\]  

(A-6)

For small values of \( B \Delta x/c_g \) the propagation becomes dominant and e.g. the spatial resolution of the numerical model becomes material for the upstream fraction needed, as is shown in chapter 4. For such cases a minimum upstream fraction \( \alpha_{\text{min}} \) is needed to suppress the occurrence of spurious solutions and negative action, so that the upstream fraction \( \alpha \) becomes the maximum of \( \alpha_c \) and \( \alpha_{\text{min}} \).

In a spectral third generation model, the application of a dynamic upstream fraction is more complicated than in the simple model considered in this appendix. As the linear wave theory is used in the propagation model, the bins propagate independently from each other and an upstream fraction can be considered for every bin separately (as the instability arises during propagation). In a third generation model equation (A-6) can be used for every bin, if the estimate \( 1/B \) for the time for wave generation is replaced by a similar time scale for the third generation model. Considering equation (A-1), a logical choice would be to replace the exponential growth rate \( B \) of the simple model by \( S/F \), where \( S \) is the total of all source functions for the third generation model (equation (2-11)). However, for reasons of implementation, (available memory on the IBM 3083-JXL on which the model is implemented), \( B \) is replaced by \( \gamma S_{\text{in}}/F \), where \( S_{\text{in}} \) is the wind input source function of the third generation model (equation (2-13)).
Dynamic upstream fraction

i.e. \( B \) in (A-4) is replaced by the exponential growth rate of the third generation model, multiplied with a calibration constant \( \gamma \). As second generation models (which for the situation considered roughly resemble the simple model used here) need exponential growth rates, which are larger than the exponential growth rate of a third generation model by a factor of approximately 5, to account for the growth of the low frequency range of the spectrum (the SWAMP group, 1985), \( \gamma \) is expected to have a value of approximately 5. Calculations of fetch limited growth curves using WAVEWATCH have shown that \( \gamma = 5 \) results in stable behaviour of the model, whereas \( \gamma = 2.5 \) results in slowly growing instabilities. Therefore \( \gamma = 5 \) is used in all calculations of WAVEWATCH.

![Graph](image)

Fig. 32 Critical upstream fraction \( \alpha_c \) as a function of \( B \Delta x / c_\xi \). Data of numerical calculations gathered in table 3.

**Table 3** Wave and wind data for numerical calculations of figure 32

<table>
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<tr>
<td>Spatial increment</td>
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<tr>
<td>Time step</td>
<td>( \Delta t = 15 ) min</td>
</tr>
<tr>
<td>Wind speed ( U_{10} )</td>
<td>( \text{min} : 5 ) ( \text{m/s} )</td>
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<tr>
<td></td>
<td>( \text{max} : 50 ) ( \text{m/s} )</td>
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<tr>
<td>Frequency ( f_r )</td>
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<td></td>
<td>( \text{max} : 0.357 ) ( \text{Hz} )</td>
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<tr>
<td></td>
<td>( \xi : 1.2 )</td>
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</table>

<table>
<thead>
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<tr>
<td></td>
<td>( \gamma_2 : 3, 5, 10 )</td>
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</table>

A-4