Improved simulation of main flow and secondary flow in a curved open channel

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Report no. 1 - 84

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Department of Civil Engineering
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Summary

For the computation of depth averaged flows in tidal channels and rivers a recently developed fully implicit finite difference method of the ADI-type proved vastly superior to the partly explicit variants, commonly used. The fully implicit variant allowed a relatively large time step in combination with a realistic lateral diffusion coefficient.

The fully implicit method, used, requires a square grid for the time being. The irregular numerical representation of the sidewalls of a curved channel or flume gives rise to disturbances in the computed flow field. In this report a correction scheme to keep the disturbances small is considered. In this scheme a correction is realized by a modification of the depth near the sidewalls. The correction scheme is actually a combination of two different corrections. The first correction is aimed at forcing the velocity at corner points of the computational grid in the direction of the physical boundary. The second correction compensates for the local narrowing and widening of the flow by the irregular numerical representation of the sidewalls.

The influences of both corrections appear to be small when applied separately, but the combination of the two corrections has, however, important consequences. The application of the combined corrections on the simulation of steady flow in a curved flume with a rectangular cross-section annihilates nearly the important disturbances caused by the use of a rectangular grid. For a curved flume with the geometrical proportions of a river, in which case the disturbances proved much less important, a less important amelioration is obtained. For flow in tidal channels even smaller disturbances and less improvement are expected. The effect of the corrections is very sensitive to the exact waterdepth at the sidewalls. In time dependent flows it is therefore difficult to apply the right corrections. Further improvement will have to await the possibility to use a curvilinear grid in the fully implicit finite difference method.
This research is aimed at the computation of secondary flow in tidal channels based on the depth averaged velocity field. The improved computation of the depth averaged velocity field brings about an improved computation of the secondary flow. Knowledge of the secondary flow in a tidal channel is essential for predictions about the morphology of the bottom.
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1. **Introduction**

A thorough knowledge of the secondary flow in tidal channels with alluvial bottoms is required for the predictions of their morphology, because this secondary flow gives rise to bottom slopes transverse to the main flow. This research, which is financially supported by the directorate of the Deltadienst of Rijkswaterstaat, concerns the determination of the secondary flow in tidal channels of estuaries like the Eastern Scheldt, based on a known depth averaged velocity field. The depth averaged velocities must be computed with a high accuracy in order to make possible a reasonable determination of the secondary flow.

For the computation of the depth averaged velocities generally an implicit finite difference method of the ADI-type is used. In such a method the depth averaged equations of motion and the depth averaged continuity equation, together called the shallow water equations, are solved by means of an Alternating Direction Implicit computation using a staggered spatial grid.

Although the velocity and waterlevel parameters are treated implicitly, in general the convective and diffusion terms are, however, treated explicitly in the difference equations. In this partly explicit representation a large diffusion coefficient is required in order to suppress a possible instability lest an uneconomically small time step has to be used (Vreugdenhil and Wijbenga, 1982). Such a large diffusion coefficient, compared to the physical eddy viscosity, severely hampers the representation of the velocity distributions in the considered steady or quasi-steady flow (Pennekamp and Booij, 1983 and Booij, 1983).

Recently the Dienst Informatieverwerking of Rijkswaterstaat developed a fully implicit finite difference method of the ADI-type. This method is usually referred to as Miniwaqua. In this fully implicit method no diffusion coefficient is required for stability, so a realistic diffusion coefficient can be introduced (Stelling, 1983). To investigate the reproduction of the depth averaged flow in circumstances comparable to bends in tidal channels, using this fully implicit method, computations were executed for steady flow in a curved flume of the Delft Hydraulics Laboratory (Booij and Pennekamp, 1983). The computations concerned two different bottom topographies, for both of which extensive measurements were available: a rectangular cross-section (de Vriend and Köch, 1977) and an uneven bottom
topography as found in river bends (de Vriend and Koch, 1978).

The reproduction of the depth averaged velocity field was satisfactory in the uneven bed case. The influence of the secondary flow on the main flow cannot be reproduced by Miniwaqua. This influence is not very large because of the gentle curvature of the flume. The influence of the secondary flow is very small in the case of the rectangular cross-section. Here disturbances connected with the numerical representation, which also appear in the uneven bed configuration, but are not very important there, are however very strong, because of the large sidewalls in this plane bed configuration.

The reproduction of the secondary flow is reasonable. Only in regions in which the depth averaged velocity field is strongly influenced by the sidewall disturbances, a defective reproduction occurs.

In this report a correction procedure to suppress the disturbances connected with the numerical representation is investigated.
2. **Mathematical description**

2.1 **Depth averaged flow**

The computation of the depth averaged flow is based on shallow water equations of the form (Booij and Pennekamp, 1983).

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial \zeta}{\partial x} + \frac{g}{C^2} \frac{u \sqrt{u^2 + v^2}}{h} - \frac{\tau_{wx}}{\rho h} - \Omega v
\]

\[
- \frac{\partial}{\partial x} \left( \epsilon \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left( \epsilon \frac{\partial u}{\partial y} \right) = 0
\]

(1)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial \zeta}{\partial y} + \frac{g}{C^2} \frac{v \sqrt{u^2 + v^2}}{h} - \frac{\tau_{wy}}{\rho h} + \Omega u
\]

\[
- \frac{\partial}{\partial x} \left( \epsilon \frac{\partial v}{\partial x} \right) - \frac{\partial}{\partial y} \left( \epsilon \frac{\partial v}{\partial y} \right) = 0
\]

(2)

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0
\]

(3)

In these equations the following notation is used (see also definition sketch, fig. 1):  

x, y  horizontal coordinated, z is the vertical coordinate;  
  t  time;  
  u, v  depth-averaged velocity-component in x-, y-direction;  
  \zeta  waterlevel above reference level;  
  g  acceleration due to gravity;  
  h  waterdepth;  
  \rho  mass density;  
  \tau_{wx}, \tau_{wy}  components of surface shear stress;  
  \Omega  Coriolis parameter: = 2\omega \sin \phi , where \phi is the geographic latitude and \omega is the angular velocity of the rotation of the earth;  
  C  Chézy coefficient;  
  \epsilon  diffusion coefficient.
Shallow water equations can be obtained by integrating the Reynolds' equations for turbulent flow over the depth, assuming a hydrostatic pressure distribution along each vertical. Some additional assumptions about the shear stresses are reflected in the form of the shallow water equations used (eqs. 1, 2 and 3).

The bottom shear stresses are assumed to act opposite to the direction of the mean velocity vector and to vary with the mean velocity squared. The effective stresses in vertical planes are replaced by diffusion terms, with an isotropic diffusion coefficient, $\varepsilon$, which is constant in time and throughout the flow. A good choice for $\varepsilon$ is an average value of the lateral eddy viscosity ($\text{Booij and Pennekamp}, 1983$)

$$\varepsilon = 0.01 \frac{h v^2 + v^2}{u^2}$$

(4)

The overbar in expression (4) indicates averaging over the flow field.

The shallow water equations are solved numerically with Miniwaqua.

2.2 Secondary flow

The flow pattern in river and channel bends is quite complex. A main flow can be defined by the horizontal velocity component, $u_s(z)$ in the direction, $s$, of the depth averaged velocity. In addition to this main flow a secondary flow, defined by the horizontal velocity component, $u_n(z)$, in the normal direction, $n$, can be important. The main flow velocity can be described properly by its depth averaged value, $u_s$, but the depth averaged value of the secondary flow velocity is zero. The secondary flow can be described by its intensity, i.e. half the averaged absolute value ($\text{de Vriend}, 1981$),

$$I_n = \frac{1}{2h} \int_{\text{depth}} |u_n(z)| \, dz$$

(5)

In tidal channels two contributions to the secondary flow can be distinguished, curvature of the main flow and the Coriolis acceleration. In the curved flume considered in this report the only important source of secondary flow is the curvature of the main flow ($\text{Booij and Kalkwijk}, 1982$). In gently curved shallow
flows the secondary flow can be assumed fully developed everywhere. The intensity of the fully developed secondary flow caused by a curvature of the main flow with a radius of curvature \( R \) is (Booij and Pennekamp, 1983)

\[
I_n = c \left| \frac{u_n h}{\kappa^2 R} \right|
\]

(6)

where \( \kappa \) is von Karman's constant and \( c \) is a function of the Chézy coefficient only. The value of \( c \) derived in most theoretical examinations is slightly lower than the value suggested by measurements in flumes (de Vriend, 1981). For \( C = 50 \text{ m}^1/\text{s} \) the theoretical value is about 0.25 but the value obtained from measurements is about \( 4^{1/3} \) times as large.

The computation of the intensity of the secondary flow in this report is based on equation 6. The radius of curvature of the main flow, \( R \), is calculated using

\[
\frac{1}{R} = \frac{1}{u_s} \frac{\partial u_n}{\partial s}
\]

(7)

where \( u_n \) is the depth averaged velocity component normal to the direction of the flow at the point in which \( R \) is calculated. The intensity of the secondary flow as given by equation (6) does only depend on the depth averaged flow. An incorrect reproduction of the depth averaged flow is reflected in a corresponding incorrect reproduction of the secondary flow. Disturbances of the depth averaged flow near the sidewalls of the flume can deteriorate the computation of the secondary flow as the radius of curvature of the main flow depends on a gradient of the main flow velocity (see eq. 7). Its correct computation is hence severely impaired by a scatter of the depth averaged velocities in neighbouring grid points, caused by the irregular numerical representation of the sidewalls.
3. The computation of depth averaged flow

3.1 The flow configurations

To investigate the accuracy of the computations of the depth averaged flow by Miniwauqua a comparison with measurements was executed (Booij and Pennekamp, 1983). No measurements of flow in tidal channels, with enough precision and detail to make an investigation of this accuracy feasible, are known. Only flows in Laboratory flumes are investigated thoroughly enough.

The calculations are executed for a flume in the Delft Hydraulics Laboratory, called the DHL-flume in this report. In this large flume, with a rather gentle bend \((B/R_f = 0.12\), with \(B\) the width and \(R_f\) the radius of curvature of the channel axis) of almost \(90^\circ\), extensive measurements were executed for two different bed configurations. In the first series of experiments the bed of the flume was plane and the cross-section rectangular (see fig. 2) (de Vriend and Koch, 1977). In the other series of experiments the flume was provided with a fixed uneven bottom of more or less the same shape as in a natural river bend (see fig. 3) (de Vriend and Koch, 1978). The flume with the uneven bed is then also a fair model of a bend in a tidal channel (Pennekamp and Booij, 1983). The cross-sections in which the measurements were executed are indicated in fig. 2 and fig. 3. The measurements were limited to steady flow. Measurements of time dependent flow in a curved flume in the Laboratory of Fluid Mechanics of the Delft University of Technology are being elaborated.

The flow is mainly controlled by the bottom friction. The distributions of the depth averaged velocity in the cross-sections reflect therefore mainly the depth distributions. Deviations from the measurements are somewhat easier to analyse in the plane bed configuration than they are in the uneven bed configuration. Besides, the effects of the irregular numerical representation of the sidewalls by the square grid, used, are much larger in the plane bed configuration. The plane bed configuration is properly speaking too strong a test for the reproduction of flow in a tidal channel. It gives however an indication of the results to be expected from the numerical reproduction of the flow, measured at the Delft University of Technology. The cross-section of the flume in which these measurements were executed is also rectangular.
The distance between neighbouring grid points in the considered computation was $\Delta = 0.40$ m (see fig. 4), and the time step used was $\Delta t = 1.5$ s. These values were chosen because of available memory space, accuracy, stability and efficiency.

The computations were executed with a free-slip boundary condition

$$\frac{\partial u_s}{\partial n} |_{\text{wall}} = 0 \quad (8)$$

The direction of $n$ is perpendicular to the wall. The wall shear stresses, when using the physically more attractive no-slip boundary condition, are much too large because of the relatively large grid spacing (Booij and Pennekamp, 1983).

3.2 Reproduction of the velocity field

The reproduction by Miniwaqua of the depth averaged velocity fields of the DHL-flume for both bottom configurations was satisfactory (Booij and Pennekamp, 1983). The comparison of the computations and the measurements are given in fig. 5 and fig. 6. Fig. 5 shows the results for the rectangular cross-section and fig. 6 for the uneven bed configuration. In both figures the reproduction of the shifting of the maximum velocity to the inner side of the bend at the beginning of the bend can be appreciated. This effect shows the flow in this region to behave like a potential flow. The overall distribution of the depth averaged velocities are satisfactory. The importance of the bottom friction is evident and is reproduced in the computations. This is an important improvement compared to the finite difference schemes of the ADI-type hitherto, in which horizontal diffusion of momentum appeared to be too important.

The influence of secondary flow on the main flow is not reproduced by Miniwaqua. This influence is very small in the plane bed configuration, but somewhat stronger in the uneven bed configuration. Consequently, the gradual shifting of the main flow to the outer side of the bend is slightly too small in the computation, as only the shift caused by the bottom topography is accounted for. This effect is to be expected in the computation of flow in tidal channels too, but it will be somewhat smaller.
there, because the flow is relatively shallower.

The most important failure in the reproduction of the depth averaged flow is connected with the irregular numerical representation of the sidewalls. Two different kinds of disturbances can be distinguished (Booij and Pennekamp, 1983). One disturbance is a scatter in the values of the depth averaged velocity in neighbouring grid points, caused by the irregular boundary. This effect is especially obvious at the outer side in the first half of the bend and at the inner side in the second half, where the irregularities in the representation of the sidewalls appear to act as obstructions to the flow (see fig. 7).

A more important disturbance connected with the numerical representation of the sidewalls is found at the inner side in the first part of the bend and at the outer side in the second part of the bend. There the flow does not follow the local widenings of the flow, presented by the irregularities in the representation of the sidewalls (see fig. 8).

In Chapter 4 correction schemes to suppress these disturbances will be considered.
4. Correction schemes for the computation of depth averaged flow

4.1 Velocity direction correction

Correction schemes to improve the computation of the depth averaged velocity near the sidewall boundaries in the curved part of the flume are related to the exact numerical representation of the flow there. In order to discuss possible correction schemes a short description of the staggered grid, used, can be helpful.

In Miniwawa the surface level, the bottom level and the two components of the velocity are defined at different places. The locations in the horizontal plane of the various places, making up a staggered grid, are shown below.

```
+ - + - +
| o | o |
+ - + - +
```

\[ y\text{-direction} \]
\[ x\text{-direction} \]

Four different neighbouring elements make up a computational molecule in which these different elements bear the same indices although their locations in the horizontal plane are not the same.

```
+ - + - +
| o | o | o |
+ - + - +
```

\[ \text{molecule } i,j \]
Whenever an impermeable boundary occurs in the space-staggered grid, the existence of this boundary is simulated in the numerical model by setting the velocity of the nearest velocity element permanently to zero, where the physical boundary crosses a line between two neighbouring surface level points.

As a consequence in flow regions where the sidewall is not in a coordinate direction some surface level points are surrounded by only two non-zero velocity elements. In a stationary flow the continuity of the flow around such surface level points requires that the discharges through these two velocity elements are equal, $q_v = q_u$. When the local bottom level is horizontal, this means that the velocities in the two velocity elements have to be equal, $u = v$. The velocity at the considered surface level points is the mean velocity of the four surrounding velocity elements. Consequently, the velocity at the surface level points will always be oriented at an angle of 45 degrees with respect to the grid orientation.

This means an important difference between the flow in a flume, river or tidal channel and its numerical reproduction by Miniwaqua or another ADI simulating system. In the prototype a velocity direction near the sidewall parallel to this wall is expected and measured. In the numerical simulation the velocity direction in the considered points does not depend on the direction of the sidewall in the prototype, but it has a
specified angle with the grid direction. For a horizontal local
bottom level this angle is $45^\circ$; for sloping bottoms other angles
will appear. This ill-fitting of the velocity direction near the
boundary is passed on to a more extensive region by the computation.

The importance of this boundary direction effect depends on
the relative flow depth at the boundaries. For a flume with a
rectangular cross-section a large disturbance is found. In the
DHL-flume with an uneven bed the effect is smaller because of the
smaller depth at the sidewalls and the consequently lesser
importance of the flow direction at the walls. In tidal flow an
even less important effect can be expected.

To correct this ill-fitting velocity direction, local
bottom slopes can be assumed. In this way the velocity direction
can be made to correspond with the sidewall direction. These
local bottom slopes are introduced by a change of the bottom
elevations around the considered surface level point. The
equality $q_u = q_v$ still applies but the flow direction is changed.
This can be understood by the following reasoning. The velocity
in an element can be computed by the division of the local discharge
($q_u$ or $q_v$) by the area of the flow section. The area of the
flow section in the numerical simulation is the width of the grid
spacing multiplied by the mean differences between surface level and
bottom level in two neighbouring grid points.

The bottom levels of the
computational molecules of which
the surface level point is
just outside the sidewall
boundary, are changed to
give the good velocity direction,
without violating continuity in the
considered corner surface level
point. To impose an angle $\alpha$ with
respect to the grid direction in
correspondence with the angle of
the sidewall, the bed levels have to satisfy

$$\tan \alpha = \frac{v}{u} = \frac{q_v}{(\zeta - \frac{1}{2}(b_B + b_C)) \Delta} / \frac{q_u}{(\zeta - \frac{1}{2}(b_A + b_C)) \Delta}$$

(9)
where $b_A, b_B, b_C$ are the bottom levels in points $A, B$ and $C$ respectively. Substituting $q_v = q_u$, the values of $b_A$ and $b_B$ have to satisfy

$$\tan \alpha = \frac{\xi - \frac{1}{4}(b_A + b_C)}{\xi - \frac{1}{4}(b_B + b_C)} \quad (10)$$

The bottom level $b_C$ in point $C$ may not be altered because this bottom level directly influences other velocities. Consequently the value of one of the remaining bottom levels $b_A$ or $b_B$ determines the other.

Very unrealistic bottom levels have to be imposed in regions with small angles between sidewall and grid direction. Other regions where very unrealistic values can develop are regions where the angle is about $45^\circ$. In these regions the choice of one bottom level determines a whole chain of bottom levels along the boundary.

4.2 Flow area correction

The physical boundaries are translated into the numerical configuration by imposing the value of the velocity in the nearest velocity elements at zero permanently. The exact position of the physical boundary within the grid spacing is lost.

Especially when the physical boundary has a small angle with respect to the grid orientation this coarse representation of the
boundary impairs the flow simulation. The relation between the local discharge $q_v$ and the velocity $v$ in the numerical representation depends only on the depth $h$. In the prototype the exact place of the physical boundary with respect to the coordinates (and so to the grid) is important, however, for the area of the flow section.

To correct for this deviation in the area of the flow section in the numerical simulation, the bottom level of the computational molecule of which the surface level point is just outside the flow is to be changed in order to obtain the correct area of the flow section of the molecule at the sidewall.

Both the corrections are dependent on the waterlevel. This makes both corrections less suitable to use in time-varying flow.
5. **Improved reproduction of the depth averaged flow**

Application of the correction schemes modifies the bottom configuration near the sidewalls. This modified bottom configuration is used in the computation of the depth averaged flow by Minwaqua. The correction on the bottom configuration depends on the water depth at the sidewalls, hence the correction should be determined at every time step, especially in time dependent flow. This procedure would consume much computation time. In this investigation the corrections were obtained from the water depths computed in the computation without a correction for the numerical representation of the sidewalls.

Computations based on the combination of both correction schemes as well as computations based on each correction scheme separately were executed. The resulting depth averaged velocities are compared to those resulting from the uncorrected computation and to the depth averaged values of the measured velocities. The improvement of the reproduction of the depth averaged flow by the various correction schemes can be appreciated in this way. Most comparisons are carried out for the rectangular cross-section, where the effect of the use of the correction schemes is largest, because of the serious disturbances in the uncorrected flow in connection with the high vertical sidewalls.

The depth averaged velocities computed for an originally plane bottom configuration, modified by the combination of the two correction schemes are plotted in fig. 9. (The velocities at all the grid points within a strip with a width of 1.5 m around each cross-section are used to compose the plot, in order to provide a better velocity distribution by the inclusion of more grid points. The use of this relatively broad strip also gives an estimate of the scatter in the velocity distribution.)

The improvement obtained by the application of this correction is obvious. From the large disturbances at the inner side in the first part of the bend and at the outer side in the second part (see fig. 5) only traces remain. This means that after this correction the flow is capable to follow the flume wall at the local widenings in the irregular numerical representation of the sidewalls. The cross-section at 13.8 degrees shows a small remainder of the originally very important disturbances of this kind.
The correction of the bottom levels of the grid points near the wall in these cross-sections is extremely large because of the small angle of the flow with respect to the grid direction. Hence exactly at these cross-sections a deviation of the measured value will show up easily.

It is remarkable that both corrections, flow area correction and velocity direction correction, separately barely improve the depth averaged velocity field (see figs. 10 and 11), whereas the combined correction is so effective. The correct reproduction appears to be quite sensitive to the exact correction applied.

The scatter in the values of the depth averaged velocity, mainly caused by the local obstructions of the flow in the irregular numerical representation of the sidewalls, is also diminished. The combined correction gives the best results for this kind of disturbance too.

The corrections have less effect on the reproduction of the depth averaged flow in the uneven bed configuration (see fig. 12). The reproduction by the uncorrected computation was already quite good, because of the low sidewalls in this configuration. The same low sidewalls make an important improvement difficult. Comparison of the results of the uncorrected computation (see fig. 6) and of the corrected computation show an improvement with respect to both kinds of disturbances that is small but relatively not unimportant. Only at 13.8 degrees and at 82.5 degrees considerable disturbances remain because of the small angles with respect to the grid directions.

The influence of the secondary flow on the main flow is appreciable in this configuration. This influence is not accounted for in Miniwaqua, so an important deviation of the computed depth averaged velocities in the last cross-sections is to be expected. The velocity field obtained by the computation appears to be mainly determined by the bottom friction as can be concluded from fig. 13. In fig. 13 the depth averaged velocities calculated from the surface slope and a local Chézy coefficient are plotted. The local Chézy coefficient is assumed to be only dependent on the water depth. These velocities that are determined by the bottom friction and the computed velocities based on the combined corrections are more or less similar except for the first part of the bend, where the flow behaves like a potential
flow, and the region around 82.5 degrees, where the flow is pushed slightly to the inner side of the bend by the remaining disturbance at the outer sidewall.

It may be concluded that the improvement of the reproduction of the depth averaged flow is impressive for the plane bed configuration. The uncorrected reproduction of the uneven bed flow was already better and the improvement is smaller. A still less deviant reproduction can be expected for tidal channels, but the effect of the corrections will probably be much smaller too. In all cases the combined corrections for velocity direction and flow area are to be recommended.
6. Reproduction of secondary flow

The reproduction of the secondary flow in this research is based on the computed depth averaged flow. Deviations in the reproduction of the depth averaged flow will, hence, have direct consequences for the reproduction of the secondary flow. The reproduction of the secondary flow in the plane bed configuration was severely hampered by the disturbances of the depth averaged flow computed without the bottom level corrections at the sidewalls (see fig. 14). Especially the scatter by local obstructions has a devastating effect. The use of the depth averaged velocity field computed with the corrections applied increases greatly the part of the flume where reasonable values of the secondary flow appear (see fig. 15).

The reproduction of the secondary flow in the uneven bed configuration was already satisfactory when the uncorrected computation was used (see fig. 16). A factor of about $1^{1/3}$ between the computed and the measured values, connected with the uncertainty of $c$ in equation 6, was discussed in chapter 2. The reproduction using the corrected depth averaged velocity field is slightly better (see fig. 17) because of the more correct depth averaged field, and especially the smaller scatter. The scatter at 82.5 degrees is caused by the large variation of the radius of curvature of the main flow over a short distance at the end of the flume, because of the bad reproduction of the flow there, connected with the short outflow length.
7. **Conclusions**

The reproduction of the depth averaged flow in a large curved flume of the Delft Hydraulic Laboratory with a fully implicit finite difference method of the ADI-type, Miniwaqua, is satisfactory on the whole but certain defects remain (Booij and Pennekamp, 1983). These defects can, in so far as they are connected with the numerical representation of the sidewalls in the bend, considerably be reduced by means of a correction scheme. The correction scheme combines two kinds of corrections which have only a slight effect separately. Both corrections modify the bottom level at the sidewalls to correct the velocity direction in corner points of the computational grid and to correct the area of the flow section at the sidewalls.

The reproduction of the depth averaged flow using this correction scheme is importantly improved. For a rectangular cross-section, in which configuration the uncorrected computation showed quite large disturbances caused by the representation of the sidewalls, only traces of the original disturbances remain when the correction scheme is used. The effect of the correction scheme is less in the case of a flume with the geometrical proportions of a river, for which flow the disturbances obtained by the uncorrected computation are small, however, because of the smaller sidewalls. In tidal channels smaller disturbances and less improvement is to be expected. Further improvement requires a more natural choice of the grid configuration of the sidewalls and the bed. The use of a curvilinear grid is not yet possible in Miniwaqua, but it is foreseen in the near future.

The reproduction of the secondary flow, which is based on the computed depth averaged flow, is consequently also much improved when the correction scheme for the computation of the depth averaged flow is used. The part of the flume, where reasonable values of the secondary flow are computed, increases greatly especially in the plane bed configuration.
References


Notation

- **b**: bottom level
- **b_p**: bottom level in grid point P
- **B**: width of the flume
- **c**: coefficient in the secondary flow intensity
- **C**: Chézy coefficient
- **g**: acceleration due to gravity
- **h**: depth of flow
- **i,j**: subscripts indicating a computational molecule
- **In**: secondary flow intensity
- **n**: local flow coordinate perpendicular to the direction of the depth averaged flow
- **q_u,q_v**: discharges through velocity elements
- **R**: radius of curvature of the main flow
- **R_f**: radius of curvature of the channel axis
- **s**: local flow coordinate in the direction of the depth averaged flow
- **t**: time
- **u**: depth averaged velocity in x-direction
- **u_n**: depth averaged velocity in n-direction
- **u_n(z)**: secondary flow velocity at level z
- **u_s**: depth averaged velocity
- **u_s(z)**: main flow velocity at level z
- **v**: depth averaged velocity in y-direction
- **x**: horizontal coordinate
- **y**: horizontal coordinate
- **z**: vertical coordinate
- **α**: local angle between flow direction and grid direction
- **Δ**: distance between grid points
- **Δt**: numerical time increment
- **ε**: diffusion coefficient
- **ζ**: water level with respect to a horizontal reference level
- **κ**: Von Karman's constant
- **ρ**: mass density
- **τ_wx,τ_x**: components of the surface shear stress
- **φ**: latitude
- **ω**: angular rotation of the earth
- **Ω**: Coriolis parameter
Fig. 1 Definition sketch.
Fig. 2. Geometry of the DHL-flume with the plane bed.
CONTOUR KEY

1 - 0.2000
2 - 0.2500
3 - 0.3000
4 - 0.3500
5 - 0.4000
6 - 0.4500
7 - 0.5000
8 - 0.5200

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Fig. 13 Depth averaged velocity distributions in several cross-sections (uneven bed configuration), determined by the bottom friction.
Fig. 14 Secondary flow intensity distributions in several cross-sections (plane bed configuration). No correction applied.
Fig. 15 Secondary flow intensity distributions in several cross-sections (plane bed configuration). Combined corrections.
Fig. 17 Secondary flow intensity distributions in several cross-sections (uneven bed configuration). Combined corrections.