Cross-Polarized Radar Reflections from Wet Snow and Ice Droplets at Weather Radar Wavelengths

DAVID A. DE WOLF, FELLOW, IEEE, H. W. J. RUSSCHENBERG, AND L. P. LIGTHART

Abstract—The co-polar radar reflection cross sections Q_{HH} and Q_{VV} from a collection of spheroid scatterers in the melting layer formed by ice precipitation prior to becoming rain have been analyzed in recent work. The model is extended to calculate the cross-polarized cross section Q_{VH} due to reception at an electric-field polarization orthogonal to that of transmission.

I. INTRODUCTION

IN RECENT WORK [1], radar reflections from the melting layer at 8 cm wavelength were analyzed in terms of an incoherent sum of reflected flux from composite-permittivity wet-snow particles. The melting layer is formed by ice precipitation that melts as it reaches lower altitudes; the melting hydrometeors are irregular mixtures of dry snow (= ice + air) and water which coalesce and give rise to strong reflections before they drop at higher velocities and lower densities as fully melted rain hydrometeors. The effective permittivity of wet snow was modeled as if it consisted of dry-snow spheroids embedded in water. Such a model is at best approximate [2], [4] but the conclusions drawn from it do not appear to depend so critically upon the details of shape that they are useless. We shall need only that each wet-snow droplet can be characterized by an effective real permittivity somewhere between $\epsilon' \approx 3$ (ice) and $\epsilon' \approx 80$ (water), and that the shape is a spheroid. The imaginary part of the permittivity ϵ'' is not of importance here, and statistical averaging over shape and orientation of wet snow makes the assumption of spheroids reasonable for individual composite particles. It appears that the permittivity of wet snow is a universal function of the fractional volume of dry snow [1] because the dry-snow particles that make up a composite wet-snow particle are presumably randomly shaped and oriented. The purpose of this work is to extend the calculations of the co-polar cross sections for reflection, Q_{HH} and Q_{VV} , to the cross-polarized cross section for reflection, Q_{VH} , due to reception of an electric field polarized linearly but at right angles to the transmitted field. We are specifically interested in the cross-polarized differential reflectivity $L_{dr} = 10 \log_{10} (Q_{VH}/Q_{HH})$.

Manuscript received August 16, 1989; revised June 8, 1990.

D. A. de Wolf is with the Laboratory for Telecommunications and Remote Sensing Technology, Department of Electrical Engineering, Technical University of Delft, the Netherlands, on leave from the Bradley Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0111.

H. W. J. Russchenberg and L. P. Ligthart are with the Laboratory for Telecommunications and Remote Sensing Technology, Department of Electrical Engineering, Technical University of Delft, the Netherlands.

IEEE Log Number 9038603.

II. BACKSCATTER MODEL

The geometry to be considered is as shown in [1, figs. 7 and 8]. The pertinent unit vectors forming a right-handed coordinate system are $\hat{e}_V$, $\hat{e}_H$, and $\hat{k}_0$ with,

$$\begin{aligned}\hat{e}_V &= (\cos \phi \cos \theta_0, \quad \sin \phi, \quad \cos \phi \sin \theta_0), \\ \hat{e}_H &= (-\sin \phi \cos \theta_0, \quad \cos \phi, \quad -\sin \phi \sin \theta_0), \\ \hat{k}_0 &= (-\sin \theta_0, \quad 0, \quad \cos \theta_0).\end{aligned}\quad (1)$$

The radar is pointed along $\hat{k}_0$ which makes an angle of θ_0 with the vertical axis. $\hat{z}$. The two $\hat{e}$ unit vectors represent mutually orthogonal linear polarizations of the electric field vector $\vec{E}_0$ emitted by the radar. When $\phi = 0$, the vector $\hat{e}_H$ is horizontal along the y axis and $\hat{e}_V$ lies in the $y = 0$ plane. Increase in ϕ implies simultaneous counterclockwise rotation of the polarization vectors when looking in the positive $\hat{x}$ direction.

Referring to the analysis and notation of [1], it follows that the co- and cross-polar backscatter fields involve

$$\begin{aligned}E_{HH} &\leftrightarrow \hat{e}_H \cdot \vec{\tau} \cdot \hat{e}_H \\ &= (\Lambda_1 \cos^2 \theta_0 + \Lambda_3 \sin^2 \theta_0) \sin^2 \phi + \Lambda_2 \cos^2 \phi, \\ E_{VV} &\leftrightarrow \hat{e}_V \cdot \vec{\tau} \cdot \hat{e}_V \\ &= (\Lambda_1 \cos^2 \theta_0 + \Lambda_3 \sin^2 \theta_0) \cos^2 \phi + \Lambda_2 \sin^2 \phi, \\ E_{VH} &\leftrightarrow \hat{e}_V \cdot \vec{\tau} \cdot \hat{e}_H \\ &= -(\Lambda_1 \cos^2 \theta_0 - \Lambda_2 + \Lambda_3 \sin^2 \theta_0) \sin \phi \cos \phi,\end{aligned}\quad (2)$$

and we may assume that $\Lambda_2 = \Lambda_1$ for spheroids. The dyadic $\vec{\tau} = \sum_{i=1}^3 \Lambda_i \hat{u}_i \hat{u}_i$ expresses the effect of the particle shape upon the dipole moment in terms of $\Lambda_i = 1/[1 + \lambda_i(\epsilon_r - 1)]$, where λ_i are the depolarization parameters [4] for ellipsoids. Incoherent addition of backscattered fluxes inside a range cell then yields

$$\begin{aligned}Q_{HH} &= \frac{k_0^4}{4\pi} N \langle v^2 \rangle \langle |\epsilon_r - 1|^2 \rangle q_{HH}, \\ Q_{VV} &= \frac{k_0^4}{4\pi} N \langle v^2 \rangle \langle |\epsilon_r - 1|^2 \rangle q_{VV}, \\ Q_{VH} &= \frac{k_0^4}{4\pi} N \langle v^2 \rangle \langle |\epsilon_r - 1|^2 \rangle q_{VH},\end{aligned}\quad (3)$$

where the Q represent Rayleigh-regime backscatter cross sections after averaging over size and effect permittivities of

the composite particles. Here, N is the number of particles in a range cell and $\langle v^2 \rangle$ is the mean-square particle volume. The q contain particulars concerning the shape and orientation of the particles. It is easily corroborated that

$$\begin{aligned} q_{HH} &= \Lambda_1^2(1 - \sin^2 \phi \sin^2 \theta_0) + \Lambda_3^2 \sin^4 \phi \sin^4 \theta_0 \\ &\quad + 2\Lambda_1\Lambda_3 \sin^2 \phi \sin^2 \theta_0(1 - \sin^2 \phi \sin^2 \theta_0), \\ q_{VV} &= \Lambda_1^2(1 - \cos^2 \phi \sin^2 \theta_0)^2 + \Lambda_3^2 \cos^4 \theta_0 \\ &\quad + 2\Lambda_1\Lambda_3 \cos^2 \phi \sin^2 \theta_0(1 - \cos^2 \phi \sin^2 \theta_0), \\ q_{VH} &= (\Lambda_1^2 + \Lambda_3^2 - 2\Lambda_1\Lambda_3) \sin^4 \theta_0 \sin^2 \phi \cos^2 \phi. \quad (4) \end{aligned}$$

Should averaging over shape be desired, then simply replace all quadratic Λ factors in (4) by averages. These are given in [1, eqs. (21)] for a uniform distribution of λ_3 between $0 < a < \lambda_3 < b < 1$. Choices of the bounds a and b of this uniform distribution are discussed in [3].

If averaging over canting angle is desired, then the calculations will become somewhat more complicated. To do it, we orient each spheroid by angles θ_m , ϕ_m with respect to the axes such that the main spheroid axis makes an angle θ_m with the $\hat{z}$ vector and an angle ϕ_m with the $-\hat{x}$ vector; i.e., $\hat{u}_3 = -\hat{x} \sin \theta_m \cos \phi_m + \hat{y} \sin \theta_m \sin \phi_m + \hat{z} \cos \theta_m$. Thus, $\theta_m = 0$ would coincide with the above-treated vertical orientation of spheroids. With the unit vectors from (1) we obtain

$$\begin{aligned} \hat{e}_H \cdot \hat{u}_3 &= t_1 \sin \theta_m - t_H \cos \theta_m, \\ \hat{e}_V \cdot \hat{u}_3 &= -t_3 \sin \theta_m + t_V \cos \theta_m, \\ t_1 &\equiv \cos \theta_0 \sin \phi \cos \phi_m + \cos \phi \sin \phi_m, \quad t_H \equiv \sin \theta_0 \sin \phi, \\ t_3 &\equiv \cos \theta_0 \cos \phi \cos \phi_m - \sin \phi \sin \phi_m, \quad t_V \equiv \sin \theta_0 \cos \phi. \end{aligned} \quad (5)$$

The combinations t_H and t_V play an important role, hence are indicated by these labels. Referring to definitions above, we now make use of $\Lambda_2 = \Lambda_1$, and specifically rewrite

$$\begin{aligned} q_{HH} &= (\hat{e}_H \cdot \vec{\tau} \cdot \hat{e}_H)^2 = [\Lambda_1 + (\Lambda_3 - \Lambda_1)(\hat{e}_H \cdot \hat{u}_3)^2]^2, \\ q_{VV} &= (\hat{e}_V \cdot \vec{\tau} \cdot \hat{e}_V)^2 = [\Lambda_1 + (\Lambda_3 - \Lambda_1)(\hat{e}_V \cdot \hat{u}_3)^2]^2, \\ q_{VH} &= (\hat{e}_V \cdot \vec{\tau} \cdot \hat{e}_H)^2 = [(\Lambda_3 - \Lambda_1)(\hat{e}_H \cdot \hat{u}_3)(\hat{e}_V \cdot \hat{u}_3)]^2. \end{aligned} \quad (6)$$

Remember that we must carry out the squaring above before averaging over the Λ factors. Repeating the analysis for a totally random distribution of canting angle [1], we then obtain,

$$\begin{aligned} \langle (\hat{e}_H \cdot \hat{u}_3)^2 \rangle &= \langle (\hat{e}_V \cdot \hat{u}_3)^2 \rangle = \frac{1}{3}, \\ \langle (\hat{e}_H \cdot \hat{u}_3)^4 \rangle &= \langle (\hat{e}_V \cdot \hat{u}_3)^4 \rangle = \frac{1}{5}. \end{aligned} \quad (7a)$$

Without repeating all the steps explicitly, we obtain similarly

for the cross-polarized quantity,

$$\begin{aligned} \langle (\hat{e}_H \cdot \hat{u}_3)^2 (\hat{e}_V \cdot \hat{u}_3)^2 \rangle &= \frac{1}{15} [8\langle t_1^2 t_3^2 \rangle + 3t_H^2 t_V^2 \\ &\quad + 2(t_V^2 \langle t_1^2 \rangle + t_H^2 \langle t_3^2 \rangle)] + \frac{8}{15} t_H t_V \langle t_1 t_3 \rangle, \\ \langle t_1 t_3 \rangle &= -\frac{1}{2} t_H t_V, \\ \langle t_1^2 t_3^2 \rangle &= \frac{1}{8} \left[1 - (t_H^2 + t_V^2) + \frac{3}{8} t_H^2 t_V^2 \right], \\ \rightarrow \langle (\hat{e}_H \cdot \hat{u}_3)^2 (\hat{e}_V \cdot \hat{u}_3)^2 \rangle &= \frac{1}{15}. \end{aligned} \quad (7b)$$

The results of (7) allow us to substitute in (6) to obtain for a completely random distribution of spheroid orientation (and averaging over shape factor),

$$\begin{aligned} \langle q_{HH} \rangle &= \frac{1}{15} [8\langle \Lambda_1^2 \rangle + 4\langle \Lambda_1 \Lambda_3 \rangle + 3\langle \Lambda_3^2 \rangle], \\ \langle q_{VV} \rangle &= \frac{1}{15} [8\langle \Lambda_1^2 \rangle + 4\langle \Lambda_1 \Lambda_3 \rangle + 3\langle \Lambda_3^2 \rangle], \\ \langle q_{VH} \rangle &= \frac{1}{15} [\langle \Lambda_1^2 \rangle - 2\langle \Lambda_1 \Lambda_3 \rangle + 3\langle \Lambda_3^2 \rangle]. \end{aligned} \quad (8)$$

Thus, clearly $\langle Z_{dr} \rangle = 0$, and $\langle L_{dr} \rangle$ is a constant that is independent of the radar and polarization angles, as expected.

Averaging over a Gaussian distribution is more complicated in detail, but follows the same procedure. We retain the model of [1] in which ϕ_m is still randomly distributed but $\xi \equiv \sin \theta_m$ has a Gaussian distribution around zero:

$$P(\sin g q_m) = \frac{2}{\sqrt{\pi} \xi_0} e^{-\xi^2 / \xi_0^2}, \quad \xi_0^2 \ll 1. \quad (9)$$

This distribution yields [1, eqs. (27)] and in addition to those

$$\begin{aligned} \langle (\hat{e}_H \cdot \hat{u}_3)^2 (\hat{e}_V \cdot \hat{u}_3)^2 \rangle &= \left(1 - \frac{3}{2} \xi_0 + \frac{57}{32} \xi_0^4 \right) t_H^2 t_V^2 \\ &\quad + \frac{1}{16} \xi_0^2 \left(1 - \frac{3}{2} \xi_0^2 \right) (t_H^2 + t_V^2). \end{aligned} \quad (10)$$

These equations can be inserted into (6) to obtain the $\langle q_{HH} \rangle$, etc., factors for a Gaussian distribution of polar angles of spheroid main axes around the vertical. There is no point in presenting the ensuing lengthy expressions, but it is obvious that we obtain (2) in the limit $\xi_0 \rightarrow 0$.

III. THE RESULTS FOR CROSS-TO-CO-POLAR CROSS-SECTION RATIOS (L_{dr})

The analyses of the preceding sections serve as the basis for graphic results we will now describe. The quantity of interest is labeled L_{dr} , and it is defined as

$$L_{dr} \equiv 10 \log_{10} (Q_{VH}/Q_{HH}) = 10 \log_{10} (q_{VH}/q_{VV}). \quad (11)$$

The second equality derives from (3). It is plotted in Fig. 1 as a function of the polarization parameter λ_3 for relative permittivity $\epsilon_r = 3$ (ice), 10, 25, 50 (wet snow values), and for

Fig. 1. Graphs of $L_{dr} = 10 \log_{10}(q_{VH}/q_{HH})$ vs λ_3 based upon (7), under the assumption of vertical alignment of spheroids: for $\phi = 15^\circ$ (X), 30° (Δ), and 45° ($\square$): (a) $\epsilon_r = 3$, $\theta_e = 67.5^\circ$. (b) $\epsilon_r = 3$, $\theta_e = 22.5^\circ$. (c) $\epsilon_r = 10$, $\theta_e = 67.5^\circ$. (d) $\epsilon_r = 10$, $\theta_e = 22.5^\circ$. (e) $\epsilon_r = 25$, $\theta_e = 67.5^\circ$. (f) $\epsilon_r = 25$, $\theta_e = 22.5^\circ$. (g) $\epsilon_r = 50$, $\theta_e = 67.5^\circ$. (h) $\epsilon_r = 50$, $\theta_e = 22.5^\circ$.

various choices of the elevation angle $\theta_e = 90^\circ - \theta_0$ and ϕ . Each figure shows a dip at $\lambda_3 = 1/3$ (spheres) which value $q_{VH} = 0$. The figures show little dependence upon ϵ_r beyond $\epsilon_r \approx 10$. This is significant in that it tells us that we need not be too precise about the effective value of the permittivity of wet-snow particles in the melting layer whenever $\epsilon_r \geq 10$

(which we may fairly safely assume to be the case for wet snow). We also note a decrease in L_{dr} with increase in elevation angle θ_e (i.e., with a decrease in θ_0). Not shown but easily checked is the limit $L_{dr} \rightarrow -\infty$ when θ_0 tends to zero (radar is pointed up vertically).

In the next set of three graphs (Fig. 2), we repeat the

Fig. 2. Graphs of $\langle L_{dr} \rangle = 10 \log_{10} (\langle q_{VH} \rangle / \langle q_{HH} \rangle)$ versus ϵ_r , under the assumption of vertical alignment of spheroids and with rectangular averaging over the shape factor $0.29 < \lambda_3 < 0.49$, for $\phi = 15^\circ$ (X), 30° (Δ), and 45° ($\square$). (a) $\theta_e = 67.5^\circ$. (b) $\theta_e = 45^\circ$. (c) $\theta_e = 22.5^\circ$.

calculations after averaging over the quadratic shape factors in (4), using [3, eqs. (15)]. Here, we center $P_\lambda(\lambda_3)$ around $\bar{\lambda}_3 = 0.39$ with a 0.1 spread on either side, and $\langle L_{dr} \rangle$ is plotted as a function of ϵ_r for $\theta_e = 67.5^\circ$, 45° , and 22.5° . Each of these shows graphs for $\phi = 15^\circ$, 30° , and 45° . The relative independence for values of ϵ_r is manifested clearly. The decrease in $\langle L_{dr} \rangle$ with increase in elevation angle is also obvious. The dip at $\epsilon_r = 1$ implies the also obvious lack of cross-polarized returns when there are no scatterers.

The graphs of Fig. 3 repeat the calculations of Fig. 2, but with an additional averaging over a Gaussian distribution of polar canting (and random distribution of azimuthal canting) angle, with choice $\xi_0 \equiv \langle \sin \theta \rangle = 0.3$, see (9), (10).

Finally, we have repeated the calculations of Fig. 3 in Fig. 4 with one change: the Gaussian canting distribution has been replaced by a totally random one over all angles θ_m , ϕ_m

Fig. 3. Graphs of $\langle L_{dr} \rangle = 10 \log_{10} (\langle q_{VH} \rangle / \langle q_{HH} \rangle)$ versus ϵ_r , under the assumption of Gaussian distribution of spheroid alignment around the vertical. Same shape averaging and angles as in Fig. 2.

Fig. 4. Graph of $\langle L_{dr} \rangle = 10 \log_{10} (\langle q_{VH} \rangle / \langle q_{HH} \rangle)$ versus ϵ_r , under the assumption of totally random alignment of spheroids. No dependence upon θ_0 or ϕ .

using (8). It is clear (and not unexpected) that there can be no dependence upon elevation and polarization angles. The randomizing of θ shows generally higher values of $\langle L_{dr} \rangle$ in comparison to those of Fig. 2.

REFERENCES

- [1] D. A. de Wolf, H. W. J. Russchenberg, and L. P. Ligthart, "Effective permittivity of and scattering from wet snow and ice droplets at weather radar wavelengths," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 1317-1325, Sept. 1990.
- [2] E. Schanda, "Microwave modelling of snow and soil," *J. Electromagn. Waves Appl.*, vol. 1, pp. 1-24, Jan. 1987.
- [3] D. A. de Wolf, H. W. J. Russchenberg, and L. P. Ligthart, "Modelling of particle distribution in the melting layer," *Inst. Elec. Eng. Proc.*, pt. H, to be published.
- [4] A. H. Sihvola and J. A. Kong, "Effective permittivity of dielectric mixtures," *IEEE Trans. Geosci. Remote Sensing*, vol. 26, pp. 420-429, July 1988.

David A. de Wolf (SM'65-F'88), for a photograph and biography please see page 1325 of the September 1990 issue of this TRANSACTIONS.

H. W. J. Russchenberg, for a photograph and biography please see page 1325 of the September 1990 issue of this TRANSACTIONS.

L. P. Ligthart, for a photograph and biography please see page 1325 of the September 1990 issue of this TRANSACTIONS.