Evaluation of fluid trajectory in time-resolved PIV

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ABSTRACT

A new time-resolved PIV (TR-PIV) method is introduced by integrating both two concepts, the fluid parcel tracking [1] and the ensemble averaged correlation [2, 3]. The dynamic range is extended by applying the polynomial trajectory as the fluid motion [1]. The tracking procedure is not performed by using individual locations of the fluid parcel along the time, but the ensemble averaged cross-correlation value of the trajectory. The random errors caused by the cross-correlation maps could be then reduced. Each polynomial coefficient is considered individually for economizing the computational cost by introducing the V-cycle search [4]. To taking into account the curved fluid motion, a new iterative corrector based on the shear rate of deformed image according to the trajectory is proposed for stabilizing the process. Not only the present method, but also the state-of-the-art TR-PIV methods, such as the trajectory correlation method [1] and the multi-frame pyramid correlation method [3] are carried out for the comparison. The quantitative evaluations are performed by analyzing synthetic image sequences of translation and rotation motions. An experimental image sequence of the flow around an airfoil is obtained by the High Rate-PIV measurement system, and used to evaluate the performances of the methods. The material acceleration from each methods is also calculated from the trajectory profile or the spatial velocity derivatives with respect to the measurement grid.

1. INTRODUCTION

In recent several years, the time-resolved particle image velocimetry (TR-PIV) has been emerged from the improvement in the experimental equipment such as high-speed camera and highly-repeatable laser system. The particle images acquired from the time-resolved system generally shows low image quality due to low sensitivity of camera sensor and weak illumination, and corresponding velocity fields from the conventional 2-frame PIV algorithms might be suffer from it. On the other hand, information inherent in consecutive multiple images, which are related each other chronologically, could be converted into temporal relative physical properties [5], which is also available in revealing instantaneous pressure distribution. Therefore the development of methods to analyze the time-resolved particle images become of high interest in in experimental fluidics fields [1, 3, 6].

Ways of using successive particle images in PIV analysis for acquiring the velocity field and its temporal derivatives could be stated as two concepts by perspectives on a material acceleration term of the Navier-Stokes equation [7]. For the N-S equation under incompressible flow conditions

\[ \nabla p = -\rho \frac{Du}{Dt} + \mu \nabla^2 u \]  

(1)

, the material derivative term \( \frac{Du}{Dt} \) can be expressed by Eulerian and Lagrangian perspectives respectively.

\[ \frac{Du}{Dt} = \begin{cases} \frac{\partial u}{\partial t} + (u \cdot \nabla)u & \text{(Eulerian perspective)} \\ \frac{du(x_p(t), t)}{dt} & \text{(Lagrangian perspective)} \end{cases} \]  

(2)

, where \( u_p \) and \( x_p \) are the velocity and the position of the fluid particle at time \( t \) respectively. In the Eulerian perspective, the material acceleration could be evaluated indirectly by calculating a local and a convective acceleration respect a stationary reference frame. The algorithms in the Eulerian perspective set an objective at improving robustness and accuracy of velocity field corresponding to the reference frame, i.e. each interrogation grid point. The improvements of robustness and accuracy could be achieved by increasing the number of averaged correlation maps [2, 3] and by locally
optimizing the time separation [6] respectively. The former complemented the cross-correlation map by using others from neighboring time steps, which are based on different particle image patterns. The latter improved the measurement precision by enhancing the dynamic velocity range locally. Recently, [3] introduced the multi-frame pyramid correlation method, which combined both two concepts into the balanced one with providing practical guidelines for selecting algorithmic parameters. The only limitation in the pyramid correlation method is that the optimal time separation is strongly limited by both temporal and spatial derivatives of velocity.

Considering fluid trajectories could be regarded as the Lagrangian perspective [1, 8, 9]. Here, it is assumed that a fluid parcel follows a modeled trajectory, and corresponding material acceleration could be easily obtained by taking derivative on the model function. The most improved algorithm for tracking the fluid trajectory was proposed by [1]. They assumed the fluid motion as a polynomial model, and implemented a pattern tracking concept into the multiple frame PIV by means of a cross-correlation. Because of considering the material acceleration directly, the limitation of dynamic range could be enhanced significantly in comparing with the PIV algorithms which consider a velocity field only. There might be the limitation due an image deformation caused by flow shear, but the flow trajectory correlation (FTC) method showed the robust result under extremely large deformation condition (1.5 pixels/pixel) also.

The present method aimed at integrating both the fluid trajectory correlation method [1] and the pyramid correlation method [3]. We have tried to pursue the advantages of both algorithms for evaluating a fluid trajectory with enhanced accuracy and robustness. Therefore two objectives of the present method could be defined as following; “Enhancement of the dynamic range by introducing the fluid trajectory model” and “Evaluation of the fluid trajectory from the ensemble averaged cross-correlations”.

2. ENSEMBLE AVERAGED CROSS-CORRELATION MAP

In the present method, a fluid motion is assumed as a polynomial model as following,

\[
x(t) = g + \Gamma(t) = g + \sum_{p=1}^{P} a_p t^p
\]

, here \(x(t)\) denotes a position of fluid parcel at the time \(t\), \(g\) denotes an interrogating grid point, \(\Gamma(t)\) is a relative fluid trajectory from \(g\), \(P\) is a polynomial order, and \(a_p\) is polynomial coefficients. The fluid trajectory correlation method [1] obtains the polynomial trajectory by means of a least square fitting as following expression.

\[
\begin{bmatrix}
\Gamma(t_1) \\
\Gamma(t_2) \\
\vdots \\
\Gamma(t_N)
\end{bmatrix} = \begin{bmatrix} t_1^0 & t_1^1 & \cdots & t_1^p \\
\vdots & \vdots & \ddots & \vdots \\
t_N^0 & t_N^1 & \cdots & t_N^p
\end{bmatrix} \begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_P
\end{bmatrix} \Rightarrow A = \left(T^T \Gamma \right)^{-1} T^T \Gamma \tag{4}
\]

Here \(t_n = n\Delta t\) and \(N\) is the maximum sequence length from the central time \(t_0 = 0\), and the total number of images used in the process \(M\) is equal to \(2N+1\) including the central image. In the procedure of the least square fitting, the discrete fluid trajectory \(\Gamma(t_0)\) is equivalent with a peak position of a cross-correlation map:

\[
\begin{align*}
R_x(\Delta x) &= \left[I_0 \star I_x \right](\Delta x) \\
\Gamma(t_n) &= I_n \otimes I_n = \Delta x \max \left[R_x(\Delta x) \right]
\end{align*}
\]

It means that random errors due to qualities of each cross-correlation maps could affect the final result of the fitting.

To enhance the fitting quality, the present method adopted an ensemble averaged cross-correlation method based on the homothetic transformation introduced in the pyramid correlation method (Sciacchitano et al. 2012) and reformed into a trajectory-adaptive method. The ensemble averaged cross-correlation value along a certain trajectory \(\Gamma(t)\), \(C(\Gamma(t))\) is introduced as following
\[
C(\Gamma(t)) = \frac{1}{2N} \sum_{n=1}^{N} R_n(\Gamma(t)) = \frac{1}{2N} \sum_{n=1}^{N} R_n\left( \sum_{p=1}^{P} a_{p,n} t^p \right)
\]

and it could be assumed that \(C(\Gamma(t))\) has the maximum value when the polynomial coefficients \(a_n\) is most appropriate. Here, \(R_0\) is regarded as 0 and \(2N\) is used instead of \(M\). In contrast to the peak finding process in the pyramid method which is able to obtain one velocity vector from one ensemble averaged cross-correlation map directly, the present method has to obtain all the polynomial coefficients simultaneously from multiple averaged correlation maps:

\[
R_{\text{ens}}(\Gamma_{\text{prediction}}(t), \Delta a_n) = \frac{1}{2N} \sum_{n=1}^{N} R_n\left( \sum_{p=1}^{P} a_{\text{prediction},n} t^p + \Delta a_n t^p \right).
\]

By comparing the Eqns. 5 and 7, dimensions of the peak finding problems could be stated simply as \(S^D\) and \(S^{PD}\) respectively, where \(S\) is a final size of the ensemble averaged cross-correlation, which is equivalent to the number of steps of \(\Delta x\) or \(\Delta a_q\) along one axis, and \(D\) is a dimension of map. These values could be also regarded as computational costs. In taking into account an orthogonal relation between odd- and even- order polynomial functions, the computational cost of the present method could be reduced as

\[
\text{Computational cost} = \begin{cases} 
S^D, & \text{when } P = 1 \\
2S^{PD}, & \text{when } P > 1 \text{ and } P \text{ is even} \\
S^{(P-1)D/2} + S^{(P+1)D/2}, & \text{otherwise}
\end{cases}
\]

Figure 1 Schematic diagrams for obtaining polynomial coefficients along the modeled trajectories. (a), (b) For the case of \(P = 1\). (c), (d) and (e) for the case of \(P = 2\)
The computational cost is not an issue under $P < 3$, however as reported by [1, 10] about the higher polynomial order cases, the polynomial order 2 could not prevent a divergence of measurement bias. In the case of applying the present method about the higher order case, i.e. $P > 2$, the computational cost is exponential to the polynomial order $P$. Fig. 1 shows schematic diagrams of the trajectories and their peak finding processes for the simplest cases; $P = 1$ and 2. For the case of $P = 2$, as shown in Fig. 1(d) and Fig. 1(e), the processes of evaluating $R_{ens, 1}$ and $R_{ens, 2}$ are not related each other due to its orthogonal property.

Figure 2  (a) Schematic diagrams for obtaining polynomial coefficients for the case of $P = 3$, (b) Exact solution at $t \rightarrow x/t$ plane, (c) Least square fitting by using correlation peaks, (d) Procedure of the present method for the low-high cycle ($a_1 \rightarrow a_2 \rightarrow a_3$), and (e) the completed V-cycle ($a_1 \rightarrow a_2 \rightarrow a_3 \rightarrow a_2 \rightarrow a_1$).

As the first step of economization, the averaged correlation maps in Eqn. 7 are calculated about neighboring $(2D + 1)$ points only including $C(x_{predictor}(t))$, and the corresponding polynomial coefficient is corrected individually by means of 3-points Gaussian fitting. Then, the computational cost is reduced to $P (2D + 1)^3$, which is linear to the $P$. The correction performed by this simplified process is not enough due to that the odd- or even- order coefficients are related to themselves. To compensate this, each coefficient is sequentially considered by following a low-high-low order, i.e. $R_{ens, 1} \rightarrow R_{ens, 2} \rightarrow \ldots \rightarrow R_{ens, P} \rightarrow R_{ens, P-1} \rightarrow \ldots \rightarrow R_{ens, 1}$. This cycle is called as V-cycle, generally used in multi-grid scheme in computational researches [4]. It contrast to the least square fitting, a result of this method converges into some appropriate solution. The difference is graphically illustrated in Fig. 2. As shown in Fig. 2(d) and Fig. 2(e), the convergence could not be achieved by one V-cycle due to its accuracy is limited by the value of $\Delta a_q$. To match a scale of $\Delta a_q$ with one pixel size of the $RN$, $\Delta a_q$ is initially set as $1/N^q$ pixels, and scaled by $\gamma$, $0 < \gamma < 1$, when one cycle is over:

$$\Delta a_q^c = \frac{\gamma^c}{N^q} \text{ pixels}$$

(9)

, here $c$ and $\gamma$ indicate the number and the scale factor of the cycle, respectively. In the present method, $\gamma = 0.5$ is used in all the case.
3. ITERATIVE CORRECTOR

In the iterative PIV process based on the image deformation method [11, 12, 13], an equation of the iterative correction in the simplest form could be expressed as:

\[
V^{k+1} = V^k + I_i \left( x - V^k / 2 \right) \otimes I_a \left( x + V^k / 2 \right)
\]  

(10)

Here \( V^k \) is the velocity field at the \( k \)-th iteration and the correction term bases on the case when a centered deformation is employed. Similarly the position of fluid parcel \( x(t) \) could be updated and be used in deforming particle image:

\[
x^{k+1}(t_n) = x^k(t_n) + I_a(x) \otimes I_a \left( x + x^k \left( t_n \right) \right)
\]  

(11)

Due to both the curved trajectory and corresponding cross-correlation between the central and inconsecutive time step, implications from the geometrical transformation of the cross-correlation map should be considered carefully [1, 14]. For enhancing the stability of the process, [15] proposed predictor and corrector filtering, and [1] introduced the rotated corrector, which is suitable for the trajectory based approach:

\[
x^{k+1}(t_n) = x^k(t_n) + R_s \left[ I_0 \left( x \right) \otimes I_a \left( x + x^k \left( t_n \right) \right) \right]
\]  

where \( R_s = \begin{bmatrix} r_s^T \\ r_r^T \end{bmatrix} = \begin{bmatrix} \cos \Delta \theta & -\sin \Delta \theta \\ \sin \Delta \theta & \cos \Delta \theta \end{bmatrix} \) at 2D case

(12)

where \( \Delta \theta \) is the estimated fluid parcel rotation, and then the stability of the iterative process was significantly improved.

**Figure 3**  Deformed grid schemes along the trajectory and corresponding correction vectors (a) rotating motion and (b) shear motion.

In the present method, another corrector has been newly adopted based on the shear rate of deformed image with respect to the previous studies [16, 17, 18, 19]. Fig. 3 illustrates the simple situations about rotating and shear flow respectively. The cross-correlation between a non-deformed image \( I_0(x) \) and a deformed image \( I_a \left( x + x^k \left( t_n \right) \right) \) is evaluated with respect to the deformed frame. It means that the cross-correlation map is physically defined with respect to the deformed frame also. Then the shear corrector \( S_n \) could be expressed as following by introducing vectors \( s_i \) which is unit vector of the deformed axis.

\[
S_n = \begin{cases} 
\{ s_s^T, s_s^T \}^T & \text{at 2D case} \\
\{ s_s^T, s_s^T, s_s^T \}^T & \text{at 3D case}
\end{cases}
\]  

(13)
, where the vector $s_i = \partial x / \partial x_i$. By this definition, the shear corrector is exactly same as the transpose matrix of Jacobian matrix, i.e. $S = J^T$. In the case of 2D where $x = <x, y>$, the shear corrector at each discrete time step could be evaluated by:

$$S_n = \left[ J_n \left( x(t_n) \right) \right]^T = \left[ \begin{array}{cc}
\frac{\partial x(t_n)}{\partial x} & \frac{\partial y(t_n)}{\partial x} \\
\frac{\partial x(t_n)}{\partial y} & \frac{\partial y(t_n)}{\partial y}
\end{array} \right]$$

(14)

This could be estimated by referring the neighboring grid points in the central difference scheme [20] as following:

$$S_n = \frac{x(t_{n+1}) - x(t_{n-1})}{2\Delta x} \quad \frac{y(t_{n+1}) - y(t_{n-1})}{2\Delta y}$$

$$\frac{x(t_{n+1}) - x(t_{n-1})}{2\Delta x} \quad \frac{y(t_{n+1}) - y(t_{n-1})}{2\Delta y}$$

(15)

, here $\Delta x_i$ denotes the grid spacing. This formulation is exactly equivalent to the rotated corrector $R$ in the rotating flow as shown in Fig. 3(a). However in contrast to $r_i$ of $R$, $s_i$ are not orthogonal to each other when flow shear exists and stretched due to the shear.

### 3. HOMOTHETIC TRANSFORMATION OF CROSS-CORRELATION MAP

As the rotated or the shear correctors are introduced, the disagreement between the physical domain $\Delta x_{phys}$ and the computational domain of cross-correlation $\Delta x_{com}$ is inevitable as:

$$\Delta x_{phys} = R \Delta x_{com} \text{ or } S \Delta x_{com}$$

(16)

In the case of considering one peak from one cross-correlation map such as the fluid trajectory correlation method [1], the disagreement could be corrected by applying the iterative corrector to the displacement vector directly as the equation (x). As the ensemble averaged concept is integrated [2], each polynomial coefficient is able to be iteratively corrected by:

$$a_{q+1} = a_q + \Delta a_q \max \left[ R_{n, q} \left( \Gamma^t (t), \Delta a_q \right) \right]$$

, where $R_{n, q} \left( \Gamma^t (t), \Delta a_q \right) = \frac{1}{2N} \sum_{n=1}^{N} \sum_{q=1}^{N} a_{p, q} \Gamma^t (t) + \Delta a_q t^q$.

(17)

Note that each cross-correlation $R_q$ is defined with respect to the non-deformed grid, and its elements are computed on the deformed grid, and the relation between both is:

$$R \left( \Gamma^t (t) + \Delta x \right) = R_{com} \left( \Gamma^t (t) + S \Delta x_{com} \right)$$

(19)

It means that the cross-correlation value, which is defined on the physical domain, should be interpolated from the computational domain [3]. For the interpolation, the position on the computational domain $\Delta x_{com}$ is essential:

$$\Delta x_{com} = S^{-1} \Delta x$$

(20)

Then the equation (x) could be rewritten into calculable form as following.

$$R_{n, q} \left( \Gamma^t (t), \Delta a_q \right) = \frac{1}{2N} \sum_{n=1}^{N} \sum_{p=1}^{N} R_{n, com} \left( \sum_{q=1}^{P} a_{p, q} \Gamma^t + S^{-1} \Delta a_q t^q \right)$$

(21)

By the temporal order term $t^q$, the ensemble averaging method follows the hypothetic transformation introduced [3] in taking into account each polynomial coefficient.
4. PRACTICAL PROCEDURE

As mentioned in the section 2, the taking into account the curved trajectory should be paid attention on the stability. Therefore the present method follows carefully designed procedure. The present study consisted of three processes; expanding trajectory, iterative deformation process, and convergence of polynomial coefficient by V-cycle. As reported by [6], the velocity field from two consecutive images is robust but suffers from random noise, and it could be enough to estimate the fluid trajectory. The expansion of the trajectory based on the displacement fields at the both ending tips of the current trajectory is called as the pseudo-tracking [21, 22]:

\[
\Gamma_{z=n} = \Gamma_{z(n-1)} + I_{z(n-1)} \odot I_{z(n-1)} \left( \Gamma_{z(n-1)} - \Gamma_{z(n-1)} \right)
\]  

In the present method the pseudo-tracking is performed at every step when the trajectory expands and the polynomial order is also limited to avoid singularity:

\[
P_n \leq \text{min}(2n, P) \tag{23}
\]

At the starting, \( n = 1 \), the trajectory should be estimated from two displacement field:

\[
\Gamma_1 = \sum_{p=1}^{\frac{1}{2}} \left( I_0 \odot I_1 + I_0 \odot I_0 \right) I_p \tag{24}
\]

At the consecutive expansions, \( n > 1 \), the discrete trajectory points from \( \Gamma_{z(n-1)} \) to \( \Gamma_{z(n-1)} \) could be obtained from the existed coefficients. Then the discrete trajectory is delivered into the iterative image deformation and coefficient correction processes.

After the particle images are deformed, the polynomial coefficients are correctly by following the V-cycle as decreasing the scale of \( \Delta a_i \). The correction process would be finished if the scale of \( \Delta a_i \) is below than the input criterion \( \Delta a_{\text{crit}} \). Until that the expansion is completed, the accuracy of the trajectory is not a matter. Therefore the number of iterations at the expansion stage could be reduced 1 or 2, and the input criterion \( \Delta a_{\text{crit}} \) could be set as a large value such as 0.1 pixels. At the iteration stage of full trajectory, the iteration number and the input criterion would be newly set, i.e. 5~10 and \( \Delta a_{\text{crit}} < 10^{-4} \). Note that the final accuracy on the iterative process of each polynomial coefficient is \( 1/Nq \Delta a_{\text{crit}} \).

5. PROGRAMMING FEATURES

Every TR-PIV method mentioned in the present text is coded by C++ based on the SLIP library of the Institut PPRIME [23] and the FFTW. The codes are parallelized via the OpenMP for enhancing the executing performance. Gaussian weighted interrogation window [24] is applied for enhancing the measurement accuracy. For interpolating the intensity at inter-pixel region, 8 by 8 pixels kernel was sampled for applying the sinc image interpolation scheme [13]. The sinc function, which might occupy considerable processing time, is not calculated when it is recalled, but stored as discretized form with \( \Delta s = 5 \times 10^{-7} \) increment.

5. RESULTS – SYNTHETIC IMAGES

An image sequence with uniform displacement from 0 to 2 pixels has been generated synthetically and used to compare the present algorithm with others, e.g. iterative-multigrid image deformation [13], sliding averaging method [2], optimal time sequence method [6], flow trajectory correlation [1] and pyramid correlation method [2]. The particles are randomly distributed over an image plane of 500 by 500 pixels with a uniform seeding density 0.1 particles/pixel. The particle diameter was uniformly set as 1.0 pixel. The final interrogation windows (IW) were set as 15 by 15 pixels with 75 % overlapping (4 by 4 pixels spacing). The material accelerations of the image deformation method, the pyramid correlation method, and the cases of \( P = 1 \) in the trajectory based methods were computed by using neighboring velocity vectors as respect to Eqn. 2 in temporally backward scheme.

The results of the sliding averaging method [2] and the optimal time sequence method [6] are shown in Figs. 4(left) and 4(right), respectively. Here \( \Delta t \) indicates the time separation between each image, \( N_{\text{cell}} \) is the total number of cross-correlation map for one displacement field, and \( M \) is the number of images used in each method. The dynamic ranges are identical in the cases of Fig. 4(left) [2], thus the biases shows similar profiles. The random errors decrease
proportional to $1/N_{\text{col}}$, but there might be the limitation barrier. For the cases of those only 2 images with the optimal time separation, in this case the maximum time separation, the results of different $\Delta t$ show homothetic results (Fig.4 (right)).

**Figure 4**  Bias and random errors of displacement and corresponding random error of material acceleration (left) sliding averaging method [2], (right) displacement from 2 images with the time separation [6]

Figure 5(left) shows the performance of the pyramid correlation method [2]. In the notation $N_{\text{col}} = A(B)$, $A$ is a number of averaged cross-correlation map from the different time separation, and $B$ is a total number of calculated cross-correlation maps. If the uniform flow whose displacement does not exceed $2IW$, $B$ could be regarded as $A(A+1)/2$ [3]. The dynamic range is closely related to the maximum time separation $\Delta t_{\text{max}}$ of the cross-correlation. In the case of the pyramid correlation, $\Delta t_{\text{max}}$ are equal to $M-1$. Therefore each profile follows a similar trend with the result with same $\Delta t$
shown in Fig. 4(right). However the signifi
cantly improved results are shown because of its feature that the pyramid method integrates the concept of [2] and [6].

The comparison between the fluid trajectory correlation (FTC) method [1] and the pyramid correlation method [3] is shown in Fig. 5(right). The bias and random errors of displacement $u$ of $P = 1$ and 2, and of $P = 3$ and 4 show similar profiles respectively due to the orthogonality of the polynomial function. As the same reason, the random error of material acceleration of $P = 2$ and 3 also shows identical profile. In the viewpoint of the dynamic range, $\Delta t_{\text{max}}$ is equal to
Furthermore the polynomial order should also be considered. In comparing Fig. 4(right) and Fig. 5(right), the profiles of [$FTC_{P=2}$] and [$FTC_{P=3}$] are located between $[\Delta t = 2, M=3]$ and $[Pyramid M = 3 (\Delta t_{max} = 2)]$. Therefore the effective maximum time separation $\Delta t_{FTC, effective}$ might be assumed as:

$$\Delta t_{FTC, effective} = \frac{\Delta t_{max}}{Number\ of\ odd-order\ polynomial\ coefficients} = \frac{(M-1)/2}{\text{round}(P/2)}$$

#### Figure 6
Comparison between the fluid trajectory correlation (FTC) method and the present method (left) about the direction $u$ and (right) the vertical direction $v$ of flow motion


, where the round ( ) is a rounding function. Similar to the previous, the effective number of cross-correlation map $N_{\text{FTC, col, effective}}$ could be also assumed as:

$$N_{\text{FTC, col, effective}} = \left\{ \begin{array}{ll}
\frac{N_{\text{FTC, col}}}{\text{round}(P/2)}, & \text{displacement} \\
\frac{N_{\text{FTC, col}}}{\text{round}((P+1)/2)}, & \text{acceleration.}
\end{array} \right. \quad (26)$$

By following above assumption, the random errors of Fig. 5(right) could be explainable.

Fig.6 shows the comparison between the fluid trajectory correlation method and the present method. The capable information from image, such as the cross-correlation maps, used in the present method is identical to the fluid trajectory correlation (FTC) method. Therefore the bias errors $\beta (u)$ and $\beta (v)$, which depend on the dynamic range, shows almost same values. The random errors $\sigma (u)$ and $\sigma (v)$ of the present method shows improved results. It shows 22–33 % reduced value at the local maxima. This is obvious because the polynomial coefficients are calculated from the ensemble averaged cross-correlation maps. The random errors of material accelerations, $\sigma (Du/Dt)$ and $\sigma (Dv/Dt)$, also show 21–31% reduced results.

Image sequence with rigid rotation from 0 to 90 degrees has been generated with every 1 degree step. The image size is 300 by 300 pixels, and other parameters are same as the uniform motion. Because the optimal time separation is strongly limited, only the fluid trajectory correlation method and the present method are tested. Following the algorithm performance described above, the bias errors show almost identical results. Therefore only the random errors of tangential velocity $u_\theta$ and its material acceleration are compared as following tables.

<table>
<thead>
<tr>
<th>$\Delta \theta$</th>
<th>$M = 5$</th>
<th>$M = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = 2$</td>
<td>$P = 3$</td>
</tr>
<tr>
<td>$\sigma (u_\theta)$</td>
<td>FT C</td>
<td>0.012589</td>
</tr>
<tr>
<td></td>
<td>Present</td>
<td>0.009428</td>
</tr>
<tr>
<td>$r$</td>
<td>25.1%</td>
<td>29.3%</td>
</tr>
</tbody>
</table>

| $\sigma (D u_\theta/D t)$ | FT C | 0.022356 | 0.022347 | 0.099556 | 0.010223 | 0.010209 | 0.036836 |
|                          | Present | 0.016852 | 0.016856 | 0.054481 | 0.007730 | 0.007719 | 0.023123 |
| $r$                      | 24.6%  | 24.6%  | 45.3%  | 24.4%  | 24.4%  | 37.2%  |

<table>
<thead>
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<th>$\Delta \theta$</th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>$P = 2$</td>
<td>$P = 3$</td>
</tr>
<tr>
<td>$\sigma (u_\theta)$</td>
<td>FT C</td>
<td>0.014970</td>
</tr>
<tr>
<td></td>
<td>Present</td>
<td>0.011291</td>
</tr>
<tr>
<td>$r$</td>
<td>24.6%</td>
<td>29.7%</td>
</tr>
</tbody>
</table>

| $\sigma (D u_\theta/D t)$ | FT C | 0.021437 | 0.021345 | 0.105250 | 0.010701 | 0.009548 | 0.036256 |
|                          | Present | 0.016343 | 0.016284 | 0.056521 | 0.008670 | 0.007327 | 0.022662 |
| $r$                      | 23.8%  | 23.7%  | 46.3%  | 19.0%  | 23.3%  | 37.5%  |

Here $\Delta \theta$ indicates a rotation angle between consecutive images and $r$ denotes the decreasing ratio of each random error. Similar with the uniform displacement case, the random errors are decreased also.

6. RESULTS – EXPERIMENT

TR-PIV measurement of the flow around a NACA0015 airfoil of chord $c = 80$mm with an angle of attack $\alpha = 30^\circ$ at Re $= 10^4$ was carried out [25]. The image sequence was obtained at a rate 1500 Hz with 1024 by 1024 pixels resolution. The final interrogation windows (IW) were set as 31 by 31 pixels with 75 % overlapping (8 by 8 pixels spacing). The number of image $M = 9$ is selected based on the maximum displacement (~ 8.0 pixels) [3, 6]. The multi-grid iterative window deformation algorithm [13], the pyramid correlation method [3], the trajectory correlation method with $P = 2$ and 3, and the present method with $P = 2$ and 3 are tested as demonstrated in Figs. 7 and 8.
Figure 7  Instantaneous velocity fields (a) the multi-grid iterative window deformation algorithm, (b) pyramid correlation method, (c) and (d) the trajectory correlation method with $P = 2$ and 3, and (e) and (f) the present method with $P = 2$ and 3.
7. CONCLUSION

The new TR-PIV algorithm, which integrates the fluid trajectory tracking [1] and the ensemble averaged cross-correlation map [2, 3], has been developed. The trajectory is assumed as the polynomial form, and corresponding coefficients are obtained individually from the ensemble averaged cross-correlation maps. To achieve the convergence of the process, the iteration follows the V-cycle search. To taking into account the curved fluid motion and corresponding deformed image, a new iterative corrector according to the deformed image scheme has been introduced. The present method and other state-of-the-art TR-PIV methods have been coded in C++ and tested by using both the synthetic images and the experimental images.

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