Integrated Flood Risk Analysis and Management Methodologies

Sea Dikes Breaching Initiated by Breaking Wave Impact

DETAILED COMPUTATIONAL MODEL

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SUMMARY

Sea dikes are of crucial importance in the defence systems of low-lying coastal areas in Germany as well as in other countries such as The Netherlands and Denmark. Breaches of sea dikes due to storm surges are regarded as the main cause of flood disasters, so that a reliable prediction of both breach initiation, breach formation and breach development is urgently needed. One may distinguish several causes of dike breaching, depending on the structure of the dike as well as on the morphologic, geotechnical and on hydraulic boundary conditions. A breach may be initiated either from the landside by wave overtopping and overflow or from the seaside by repeated breaking wave impacts on a dike slope. For a dike breach initiated by wave overtopping, a PhD research has been completed (D’Eliso, 2007). Because the processes associated with the dike breach initiation from the seaside as well as the breach growth itself are completely different, there is an urgent need to investigate the dike breaching initiated from the seaside. The knowledge of the process of breach initiation and development is crucial for the prediction of the initial conditions at the defence line needed to model the flood wave propagation. In order to satisfy those needs, a tiered and modular strategy similar to that adopted by D’Eliso (2007) is applied. Similarly, the complete model consists of a simplified preliminary model and a more process-oriented detailed model. In this report the assumptions of the preliminary model, its implementation, example application and discussion on the limitations and capabilities is presented.
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1. Introduction

1.1 Motivations and problem formulation

Low-lying coastal areas in Germany as well as in other countries such as The Netherlands and Denmark are protected from flooding by sea dikes. A reliable prediction of both breach initiation and breach development are needed, since for sea dikes breaching is regarded as the main cause of flood disasters. Depending on the structure of the dike and on the hydraulic conditions, a breach may be initiated either from the landside by wave overtopping and overflow or from the seaside by repeated breaking wave impact acting on a dike slope.

For a dike breach initiated by wave overtopping, a PhD research work has been completed (D’Eliso, 2007), but for the dike breaching initiated from the seaside research is still needed. In fact, differences would be expected with the development of the breach as well as in the final breach width, in the breach time and maximum discharge through the breach.

The knowledge of the process of breach initiation and development is crucial for the prediction of the initial conditions at the defence line needed to model the flood wave propagation. In order to satisfy those needs, a model system has been developed (Stanczak et al, 2006a for instance). The complete model consists of a simplified, preliminary model and a more process-oriented, detailed model - see Fig. 1.

![Figure 1: Overall approach](image).

The modules, assumptions and implementation of the preliminary model was comprehensively described by Stanczak et al. (2007g). The application of the model for the calculation of the dike failure probability were discussed by Stanczak et al. (2007e). The analysis of the results delivered by the preliminary model revealed that the duration of the breach initiation phase has the most significant influence on the warning time. Therefore, in order to improve the understanding of the processes that initiated the breach, a number of laboratory experiments were performed. The surface erosion of the grass cover and clay layers of different quality as well as the shear failure of water-filled cracks subject to impact pressures were investigated (see also Stanczak et al, 2007d and Stanczak et al, 2007f).
This report addresses the development of the detailed model and the implementation of the surface erosion models based on the results of the experimental tests. The strategy which has been adopted for the detailed computational model is outlined in Fig.2.

### INPUTS

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<th>Sea state data</th>
<th>Dike geometrical parameters</th>
</tr>
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<tbody>
<tr>
<td>Material characteristics</td>
<td>Discretisation</td>
</tr>
</tbody>
</table>

### DETAILED HYDRODYNAMIC MODULE

- Definition of storm surge history
- Impact pressures (Liu and Lin, 1997)
- Run-up and run-down (Liu and Lin, 1997)
- Energy dissipation (Larson and Kraus, 1990)
- Overtopping and overflow (D’Eliso, 2007)
- Simplified models (D’Eliso, 2007)
- Wave breaking
- Flow through breach channel
- Water infiltration into the dike

### DETAILED MORPHODYNAMIC MODULE

- Grass roots properties (LWI - experiments)
- Grass erosion coefficients (LWI - experiments)
- Clay erosion coefficients (LWI - experiments)
- Clay undermining and collapse (LWI - experiments)
- Equilibrium profile model (Larson and Kraus, 1990)
- Shear failure in cracks (LWI - experiments)
- Sand erosion (Larson et al., 2004)
- Same as in the preliminary model
- Transition phase between clay and sand erosion
- Sand core erosion
- Sand core wash-out

### OUTPUTS

- Breach evolution in time and space (numerical and graphical data)
- Outflow hydrograph (numerical and graphical data)

**Figure 2: Strategy for the detailed computational model**

First, the detailed hydrodynamic module is described, including wave breaking impact, wave run up and run down on the dike slope as well as the flow through the breach channel. The submodule that calculates the loading on the outer slope using a numerical model is completely new when compared to the preliminary model which was based on the empirical equations. The same concerns
the simulation of the flow during core erosion. Although the wave overtopping is also accounted for, the flow through the breach channel is essentially calculated using the same method as in the preliminary model. Additionally to the preliminary model, the infiltration is also simulated.

Second, the detailed morphodynamic module is described, including the application of the results gained during laboratory experiments performed at the LWI for the simulation of grass and clay erosion. The transition phase between clay and sand erosion, mass stability of the clay and front-face core erosion are simulated according to the results of small-scale tests in a wave flume.

Finally, the model implementation and results of the simulations are provided. The sensitivity analysis is performed as well.

2. Hydrodynamic module

The main task of the hydrodynamic module is to provide the information on (i) the forces that act on the dike and (ii) on the water infiltration into the dike. This chapter presents the basis of the numerical models that have been selected for the application in the detailed hydrodynamic module. The following is included in this chapter:

1. brief description of the formulation and application of the numerical model COBRAS which is used to simulate the impact pressures and flow of wave run-up and run-down during grass and clay erosion and during the transition phase between clay and sand erosion;
2. description and application of the SBeach model, which is applied in order to calculate the flow parameters during beach profile formation;
3. summary of the empirical equations used for the simulation of the wave overtopping and overflow;
4. information on the simplified infiltration models that are selected for the implementation in order to provide information on the saturated and infiltration water fronts which is essential for the calculation of water content in the soil and consequently the soil erodibility coefficients.

2.1 Simulation of the breaking wave impact on the slope and flow fields of the wave run-up

A large amount of information on several aspects of wave impact on slopes is available in the literature. For the purposes of the dike breaching model, the following parameters are particularly needed:

- Maximal values of maximal impact pressures on the dike slope
- Location of the impact point on the dike slope
- Shape of impact area
- Wave run-up and run-down levels
- Velocity field on the dike slope
- Thickness of water layer on the dike slope

The detailed description of the available formulae for the calculation of the above mentioned parameters can be found in Stanczak et al (2007h). However, they are based on the results of laboratory tests and are therefore valid only in a given range of input data corresponding more or less to the testing conditions.

These formulae were found to be sufficient in the case of preliminary model, however for the detailed model a numerical model based on the Reynolds Averaged Navier Stokes 2DV equations will be needed. For this purpose, the RANS-VOF model COBRAS (Cornell Breaking Wave and Structures) has been adopted. This model is based on the Reynolds Averaged Navier Stokes 2DV equations, with a nonlinear, three-dimensional \( k-\varepsilon \) turbulence model. The model was originally developed by the NASA (Kothe et al, 1991). After modifications at Cornell University the model is able to cope with breaking waves and flow within porous media. The main features of the model may be summarised as follows:
the equations of motion are solved using two finite difference methods, which significantly improves the accuracy of results
• a Volume-of-Fluid (VOF) method is used to track the free surface
• complete information of pressure, kinetic energy, horizontal and vertical velocities, free surface configuration, etc are provided (see Section 2.1.2)

2.1.1 Mathematical formulation

The motion of an incompressible fluid can be described using the Reynolds Averaged Navier-Stokes equations:

Continuity:

\[ \frac{\partial <u_i>}{\partial x_i} = 0 \]  \hspace{1cm} (1)

Momentum:

\[ \frac{\partial <u_i>}{\partial t} + <u_j> \frac{\partial <u_i>}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p_i}{\partial x_j} + g_i + \frac{1}{\rho} \frac{\partial <\tau_{ij}>}{\partial x_j} \]  \hspace{1cm} (2)

with:

• \( i, j \) - 1,2,3 - for each of the fluid three dimensions
• \( u_i \) - \( i \) th component of velocity vector
• \( \rho \) - density
• \( p_i \) - pressure
• \( g_i \) - \( i \) th component of the gravity acceleration
• \( \tau_{ij} \) - viscous stress
• \( <> \) - symbol representing time-averaging

Kinematic boundary condition:

\[ \frac{\partial <\rho>}{\partial t} + <u_i> \frac{\partial <\rho>}{\partial x_i} = 0 \]  \hspace{1cm} (3)

\( k-\varepsilon \) turbulence transport model

\[ \frac{\partial k}{\partial t} + <u_j> \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\nu_t}{\sigma_k} + \nu \right) \frac{\partial k}{\partial x_j} \right] - <u_i u_j> \frac{\partial <u_i>}{\partial x_j} - \varepsilon \]  \hspace{1cm} (4)

\[ \frac{\partial \varepsilon}{\partial t} + <u_j> \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\nu_t}{\sigma_\varepsilon} + \nu \right) \frac{\partial \varepsilon}{\partial x_j} \right] + 2C_{1\varepsilon} \frac{\varepsilon}{k} \nu_S \frac{\partial <u_i>}{\partial x_j} - C_{2\varepsilon} \frac{\varepsilon^2}{k} \]  \hspace{1cm} (5)

with:

• \( k = \frac{1}{2} <u_i u_i> \) - turbulent kinetic energy
• \( \varepsilon = \nu <\left( \frac{\partial u_i}{\partial x_k} \right)^2> \) - turbulent dissipation rate
• \( \nu_t = C_d \frac{k^2}{\varepsilon} \) - eddy viscosity

where:

• \( \sigma_k = 1.0 \)
• \( \sigma_\varepsilon = 1.3 \)
• \( C_{1\varepsilon} = 1.44 \)
• \( C_{2\varepsilon} = 1.92 \)
• \( C_a = \frac{2}{3} \left( \frac{1}{7.4 + S_{\text{max}}} \right) \)
• \( S_{\text{max}} = \frac{k}{\varepsilon} \max (I \frac{\partial < u_i >}{\partial x_i}) \)

**Linear closure model** for Reynolds stresses - isotropic eddy viscosity

\[
< u_i u_j > = -2\nu_s \delta_{ij} + \frac{2}{3} k \delta_{ij}
\]  

(6)

**VOF function**

\[
\frac{\partial F}{\partial t} + \frac{\partial (u F)}{\partial x} + \frac{\partial (v F)}{\partial y} = 0
\]

(7)

where:
• \( \rho(x, y, t) = F(x, y, t) \rho_f \)
• \( F(x, y, t) \rho_f \) is the cell density

**Partial cells treatment - obstacle boundaries**

\[
\frac{\partial (\theta u_i)}{\partial x_i} = 0
\]

\[
\frac{\partial (\theta u_i)}{\partial t} + \theta u_j \frac{\partial (\theta u_i)}{\partial x_j} = \frac{\theta}{\rho} \frac{\partial p}{\partial x_i} + \theta \frac{\partial}{\partial x_j} \tau_{ij}
\]

where \( \theta \) - is the relation between the cell surface open to flow and the total cell surface.

A more detailed description of the COBRAS model can be found in the original publication (Liu and Lin, 1997).

### 2.1.2 Results provided by the COBRAS model

The COBRAS model provides the complete information on the temporal and spatial distribution of all the essential parameters that are necessary for the simulation of sea dikes breaching, including (i) impact pressure, (ii) flow velocity and (iii) layer thickness.

The model was used to simulate wave breaking on the slope. The main input parameters for the calculation of loading on a prototype dike are listed in Tab. 1

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<td>Water depth ( h ) [m]</td>
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<tr>
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<tr>
<td>( t_{\text{max}} ) [s]</td>
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</table>

**Table 1: Main inputs for the calculation of the wave loading on the prototype dike**
Figure 3 shows a series of free surface elevations during the wave breaking on the dike slope calculated by the model using input data summarized in Tab. 1.

![Wave breaking simulated by COBRAS - free surface elevation](image)

In comparison to the information provided by the simplified, preliminary hydrodynamic model, the data given by RANS-VOF model is considered to be more reliable, as:

- the process-oriented model solving the equations of fluid dynamics is more reliable as the empirical equations used in the preliminary model
- the interaction between waves is included - in Figures 4d - 4f the variations of the parameters for particular waves can be observed

The available version of the model (built 250605) has still a number of limitations:

- significant computational effort - the simulation of 1 second of flow requires about 1 hour of CPU time, when a PC computer with two processors running at 2GHz is used
- only regular waves can be simulated
• only constant still water level can be assumed
• as the assumed water level has to be equal on the both sides of a dike (see also Figure 3), the simulation of the overtopping and overflow parameters on the inner slope during the breach widening and deepening phase has to be performed with the means of other available models

2.1.3 Flow model for the calculation of the beach profile development

A number of small-scale tests on the dike breaching initiated by breaking wave impact (e.g. Husrin, 2007) have shown that the progressing erosion of a dike sand core is similar to the erosion of a beach profile (Figure 6). Therefore, the application of an existing beach profile model for this study is recommended.

![Figure 6: Erosion of a sand core of a dike during breaching process](image)

In order to select the most appropriate beach profile model to be implemented in the detailed dike breaching model, a review of available beach profile models was made (Tab.2). The following selection requirements were adopted to identify the most appropriate beach profile model:

• it should simulate the erosion and sediment transport induced by breaking waves
• it should properly reproduce the morphological changes, including erosional and accretional processes as well as bar formation as observed during small-scale tests on the dike breaching (Figure 6)
• it should include the effects of wave run-up and run-down
• it should be affordable, easy to implement and cost-effective in terms of computational effort

The SBEACH model by Larson and Kraus (1990) was found to best fulfill the above listed requirements.
<table>
<thead>
<tr>
<th>Model</th>
<th>Sediment transport modelling approach</th>
<th>Theoretical basis</th>
<th>Pros</th>
<th>Cons</th>
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<td>Energetics; efficiency of the water to transport sediment</td>
<td>Reasonably sound theoretical basis</td>
<td>The model has been improved by Stive (1988), Nairn (1990) and Roelvink (1993)</td>
<td>Constant efficiency factors and drag coefficient; only valid for a planar bed</td>
<td>The data cover a large range of conditions</td>
<td>Bad correlation between predicted and measured profiles in small-scale tests</td>
<td>available</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The effect of the bottom slope is included</td>
<td>Turbulence due to breaking waves is ignored;</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Both erosion and accretion are simulated</td>
<td>Sheet flow is assumed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Watanabe (1998)</td>
<td>Wave power (excess shear stress multiplied with the mean velocity)</td>
<td>Largey based on theory, but some aspects are empirical</td>
<td>The model is complicated and requires extensive calibration</td>
<td></td>
<td></td>
<td>Due to poorly simulated breaking wave induced turbulences inappropriate for dike breaching model</td>
<td>unknown</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Is capable of simulating sand bars</td>
<td>The effects of turbulence due to wave breaking on the bottom shear stress is poorly simulated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Water level variations are taken into account</td>
<td>Extensive computing facilities required</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Both erosion and accretion are simulated</td>
<td>The effects of grain size distribution on the sediment transport are not considered</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kriebel and Dean (1964)</td>
<td>Equilibrium beach profile, wave energy dissipation</td>
<td>Is a simple model</td>
<td>Wave characteristics, water level variations and storm surges are taken into account</td>
<td>The model is incapable of predicting sand bars</td>
<td>Extensive calibration has been carried out, including prototype data</td>
<td>No incipient motion criterion is applied</td>
<td>available</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Includes the effects of wave run-up</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Larson and Kraus (1990) SBEACH</td>
<td>Equilibrium beach profile, wave energy dissipation</td>
<td>The dynamics of macroscale profile change including growth and movement of berms and break point bars is simulated</td>
<td>The model is very stable and reproduces the correct rate of profile change</td>
<td></td>
<td></td>
<td>Extensive calibration has been carried out, including prototype data</td>
<td>available</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Breaking waves are the sole driving force causing sediment transport</td>
<td>Although the beach profiles are adequately predicted, actual cross-shore transports need to be verified</td>
<td></td>
<td></td>
<td>An incipient motion criterion is employed</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Avalanching is simulated</td>
<td>No overwash is simulated</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steetzel (1993)</td>
<td>Suspended sediment flux</td>
<td>Although the model is simple, it simulates the most important coastal processes</td>
<td>Fair agreement between measured and computed profiles has been found</td>
<td></td>
<td></td>
<td>Extensive calibration has been carried out, including prototype data</td>
<td>No incipient motion criterion is applied</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sand bars can be handled and predicted in an erosive setting, they cannot be however predicted under accretive conditions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.1.4 **General description of the SBEACH model**

The SBEACH (Storm-induced BEAch CHanges) developed by Larson and Kraus (1990) is a deterministic numerical model that calculates the morphological changes of a beach profile assuming that these changes are essentially induced by the action of breaking waves. Therefore, the cross-shore transport rate is determined from the local properties of wave, water level and beach profile. The beach profile change in time is calculated by solving the mass conservation equation. The model consists of three modules:

- wave module which calculates the wave height across the shore
- sediment transport module that provides the information on the net cross-shore transport rate
- morphological module which revises the bathymetry for each time step

**Wave model**

The wave characteristics across-shore from a specified water depth offshore (dike outer toe) to the break point are determined using a linear wave theory. Shoreward of the break point, a generalized form of the wave decay by Dally, Dean and Dalrymple (1984, 1985) is used for the calculation of wave height distribution across-shore. Assuming the wave conditions to be uniform alongshore and the bottom contours to be straight and parallel, the equation for conservation of energy flux incorporating energy dissipation associated with wave breaking may be written as:

\[
\frac{d}{dx}(F \cdot \cos \theta_w) = \frac{\kappa}{h}(F - F_s)
\]  

(8)

where:

- \(x\) - cross-shore coordinate, positive directed seaward, [m]
- \(F\) - actual incident wave energy flux \([Nm/m\cdot s]\)
- \(F_s\) - stable wave energy flux \([Nm/m\cdot s]\)
- \(\theta_w\) - wave angle with respect to the bottom contours
- \(\kappa\) - empirical wave decay coefficient, recommended value \(\kappa = 0.15\)
- \(h\) - water depth [m]

Energy dissipation that is used for the sediment transport calculation is assumed proportional to the excess energy flux beyond a stable energy flux below which a wave will not decay. The wave energy flux is given by:

\[
F = E \cdot C_g
\]  

(9)

where:

- \(E\) - wave energy density, \([Nm/m^2]\)
- \(C_g\) - wave group velocity [m/s]

The wave energy density is written using linear wave theory as:

\[
E = \frac{1}{8} \rho \cdot g \cdot H^2
\]  

(10)

Wave group speed is a function of wave celerity \(C\) and a factor \(n_w\)

\[
C_g = n_w \cdot C
\]  

(11)

where:

- \(n_w\) - parameter which depends on the relative water depth \(h/L\):

\[
n = \frac{1}{2} \left[ 1 + \frac{2\pi h}{L} \sinh \left( \frac{2\pi h}{L} \right) \right]
\]  

(12)
The wave celerity $C$ is determined through the dispersion relationship:

$$C = C_0 \cdot \tanh\left(\frac{2\pi h}{L}\right)$$

(13)

where:

- $C_0$ - wave celerity in deep water [m/s]:

$$C_0 = \frac{gT}{2\pi}$$

(14)

The stable energy flux determines the amount of energy dissipation necessary for stable conditions to occur once breaking is initiated. The stable energy flux is expressed as:

$$F_s = E_s \cdot C_g$$

(15)

where:

- $E_s$ - stable wave energy density [J/m$^2$];

The stable wave energy flux corresponds to a stable wave height $H_s$ that is a function of water depth:

$$H_s = \Gamma \cdot d$$

(16)

where:

- $\Gamma$ - stable wave height coefficient [-], recommended value $\Gamma = 0.4$

Wave setup or setdown is determined from the cross-shore variation of radiation stress:

$$\frac{dS_{ss}}{dx} = -\rho g h \frac{d\mu}{dx}$$

(17)

where:

- $S_{ss} = \frac{1}{8} \rho g H^2 [n - \frac{1}{2}]$

(18)

The setdown at the first calculation cell is determined analytically as:

$$\eta = -\frac{\pi h^2}{4L \sinh\left(\frac{4\pi h}{L}\right)}$$

(19)

The point of incipient wave breaking is determined from an empirical breaking criterion expressed in terms of surf similarity parameter $\xi$:

$$\frac{H_b}{h_b} = 1.14 \cdot \xi^{0.21}$$

(20)
Shoreward of the break point $\kappa$ is set to zero, and no energy dissipation takes place since bottom friction is neglected. Once the breaking is initiated, the wave energy dissipation per unit water volume is obtained as:

$$D = \frac{\kappa}{d^2} (F - F_s) \quad (21)$$

**Sediment transport model**

The identification of regions with different wave characteristics requires an analysis of the net transport rates in the following zones (see also Figure 7):

- **prebreaking zone** (zone I) - from the seaward depth of effective sand transport to the breaking zone
- **breaker transition zone** (zone II) - from the break point to the plunge point
- **broken wave zone** (zone III) - from the plunge point to the swash zone
- **swash zone** (zone IV) - from the shoreward boundary of the surf zone to the shoreward limit of runup

![Figure 7: Principal zones of cross-shore transport](image)

The applied transport rate relationships can be summarized as:

- **Zone I** ($x_b < x$):
  $$q = q_b \cdot e^{-\lambda(x-x_b)} \quad (22)$$

- **Zone II** ($x_p < x \leq x_b$)
  $$q = q_p \cdot e^{-\lambda_2(x-x_p)} \quad (23)$$

- **Zone III** ($x_z < x \leq x_p$)
  $$q = \begin{cases} 
  K(D - D_{eq} + \frac{\varepsilon}{K} \frac{dh}{dx}) & D > (D_{eq} - \frac{\varepsilon}{K} \frac{dh}{dx}) \\
  0 & D \leq (D_{eq} - \frac{\varepsilon}{K} \frac{dh}{dx}) 
  \end{cases} \quad (24)$$

- **Zone IV** ($x_r < x \leq x_z$)
\[ q = q_r \frac{x - x_r}{x_z - x_r} \]  \hspace{1cm} (25)

where:

- \( q \) - net cross-shore transport rate \([m^3/m \cdot s]\)
- \( \lambda_1 \) and \( \lambda_2 \) - spatial decay coefficients \([m/m]\) - see Equations
- \( K \) - transport rate coefficient \([m^3/N]\), assumed to be equal \( K = 3 \cdot 10^{-6} \)
- \( \varepsilon \) - slope-related transport rate coefficient \([m^2/s]\), equal \( \varepsilon = 6 \cdot 10^{-4} \)
- \( h \) - still-water depth \([m]\)
- \( D_{eq} \) - equilibrium energy dissipation, calculated as:

\[
D_{eq} = \frac{5}{24} \rho \cdot g \cdot \left( \frac{H_b}{h_b} \right)^{1.5} \cdot A^{1.5} \ [Nm/m^3/sec] \]  \hspace{1cm} (26)

where \( A \) is the shape parameter calculated as: \( A = 2.25 \cdot (w_s/\sqrt{g})^{2/5} \) with \( w_s \) being the sediment fall velocity.

The subscripts b, p, z and r stand for the quantities evaluated at the break point, plunge point, end of the surf zone and runup limit, respectively. The spatial decay coefficients used in zones I and II describe the decrease in transport rate with distance and are calculated as:

\[
\lambda_1 = 0.4 \left( \frac{D_{50}}{H_b} \right)^{0.47} \]  \hspace{1cm} (27)

and

\[
\lambda_2 = 0.2 \cdot \lambda_1 \]  \hspace{1cm} (28)

where:

- \( D_{50} \) - median grain size \([mm]\)

The locations of the break point, plunge point, end of the surf zone and runup height are calculated applying the following formulæ:

- the break point is calculated directly from the wave height calculations

\[
H_B = 0.53 \cdot H_0 \left( \frac{H_0}{L_0} \right)^{0.24} \]  \hspace{1cm} (29)

- the plunge point is located \( 3 H_B \) shorewards from the impact point
- the end of the surf zone is defined arbitrarily as the point where water depth \( h < 0.4 \) m
- the runup limit is calculated as a function of surf similarity parameter \( \xi_D \) (c.f. Eq.)

The transport rate is first calculated at the plunge point \((x=x_p)\) and at the end of the surf zone \((x=x_z)\), after that the Equations (Fehler! Verweisquelle konnte nicht gefunden werden.), (Fehler! Verweisquelle konnte nicht gefunden werden.), (Fehler! Verweisquelle konnte nicht gefunden werden.), (Fehler! Verweisquelle konnte nicht gefunden werden.) are applied to completely define the transport rate distribution. In the model also the concept of avalanching is implemented, i.e. if the local
slope exceeds the angle of initial yield $\psi_i$, soil is redistributed along the slope and a new stable slope with the angle $\psi_f$ is formed. In the model, $\psi_i$ is set to $28^0$ while $\psi_f$ takes the value of $18^0$.

2.2 Simulation of overtopping and overflow

Together with the onshore directed progress of erosion, the dike crest becomes lower and overtopping occurs, resulting in the erosion of the landside slope of the dike. As the available version of the RANS-VOF model applied for the calculation of the flow parameters on the outer slope cannot provide the required information for the inner slope, a selection of the model for the simulation of the flow on the inner slope was made. This recently developed model for dike breaching initiated by wave overtopping (D’Eliso, 2007) contains the formulae that enable one the calculation of the relevant parameters of wave overtopping, overflow as well as combined wave overtopping and wave overflow.

2.2.1 Wave overtopping

In order to calculate the flow parameters that are necessary for the assessment of shear stress on the landside slope i.e. flow velocity and layer thickness, the approach proposed by Schüttrumpf and Oumeraci (2005) is applied. The principle sketch and definitions for the calculations is shown in Figure 8.

The following steps are needed for the calculation of the maximal velocity and layer thickness of wave run-up for regular waves:

1. the calculation of wave run-up:
   \[ z_r = 2.25 \cdot \tanh(0.5 \cdot \xi_d); \quad \xi_d = \frac{\tan \alpha}{\sqrt{H/L}}; \quad L = \frac{gT^2}{\pi} \]  
   \hspace{1cm} (30)

2. the calculation of the layer thickness and velocity of the wave run-up on the outer slope, i.e. $z_r \leq R_c$:
   \[ h_s(x_s) = 0.057 \cdot (x_{cr} - x_s) \]  
   \hspace{1cm} (31)
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Contract No: GOCE-CT-2004-505420

Preliminary model 30 05 2008

3. the calculation of the flow on the crest; \(x_c < \left(\frac{R}{\tan \alpha} + B_d\right)\):

\[
h_c(x_c) = \frac{h_c(H_d)}{h_c(H_d)} = \exp(-0.58 \cdot \frac{c_s}{B_d})
\]

\[
v_c(x_c) = v_r(H_d) \cdot \exp(-\frac{f \cdot x_c}{2 \cdot h_c(x_c)}); \quad f = 0.02;
\]

4. the calculation of the landward flow; \(s \leq H_d \cdot \sin \beta:\)

\[
v_l(s) = \frac{v_c(B_d) + k_l(s) \cdot h_l(s) \cdot \tanh\left(\frac{k_l(s) \cdot t}{2}\right)}{1 + \frac{f \cdot v_c(B_d)}{h_l(s) \cdot k_l(s) \cdot \tanh\left(\frac{k_l(s) \cdot t}{2}\right)}}
\]

\[
h_l(s) = \frac{v_c(B_d) \cdot h_c(B_d)}{v_l(s)}
\]

\[
t = \frac{v_c(B_d)}{g \sin \beta} + \sqrt{\frac{v_l(s)^2}{g^2 \sin^2 \beta} + \frac{2 \cdot s}{g \sin \beta}}; \quad k_l = \sqrt{\frac{2 \cdot f \cdot g \cdot \sin \beta}{h_l(s)}}
\]

The flow discharge per unit width:

\[
q = h_l \cdot v_l
\]

The flow parameters obtained with Eqs. (Fehler! Verweisquelle konnte nicht gefunden werden.) - (Fehler! Verweisquelle konnte nicht gefunden werden.) represent the peak flow within a wave period. In order to take also the flow variation into consideration, each event is divided into five time steps, with the following assumptions imposed (D’Eliso, 2007):

- the flow discharge variation in time are assumed to have a triangular shape (Fig. 9a)
- the Froude number defined at each point along the dike profile \(Fr = \frac{v_i}{\sqrt{g \cdot h_i}}\) is constant with time (Fig. 9b)
- the flow velocity and depth are calculated as (Figs. 9c and 9d):

\[
q = h_i \cdot v_i; \quad Fr = \frac{v_i}{\sqrt{g \cdot h_i}} \Rightarrow h = \left(\frac{q}{Fr \cdot \sqrt{g}}\right)^{2/3}; \quad v_i = \frac{q}{h_i}
\]
2.3 Combined wave overtopping and overflow

Combined wave overtopping and overflow is defined as the sum of the wave overtopping and overflow due to high water level. The combined flow region consisting of two flow transitions and two subregions is shown in Figure 10. In the model an approach developed by D’Eliso (2007) for the dike breaching initiated by wave overtopping is selected for the implementation.

The flow discharge in the first region, i.e. between wave overtopping and combined flow ($h_{over} < H/2$) is calculated as:

$$q_{comb} = q_{comb} - \frac{(H/2 - h_{over})}{H/2} \cdot (q_{comb} - q)_{h_{over}=0}$$  \hspace{1cm} (40)
where:

\[ h_{over} = MWL - H_d \]  \hspace{1cm} (41)

\[ q_{comb} = \frac{1}{T} \int_0^T \left( \frac{2}{3} \cdot \mu_{comb} \cdot \sqrt{2 \cdot g \cdot (h_{over} + \eta(t))^{1.5}} \right) dt; \quad \mu_{comb} = 0.4728 \cdot \xi_d \]  \hspace{1cm} (42)

In the second region, i.e. \( h_{over} > H/2 \) the flow discharge is calculated as:

\[ q_{comb} = \frac{2}{3} \cdot \sqrt{2 \cdot g \cdot (\mu_{comb} \cdot h_{comb}^{1.5} + \frac{h_{over}-H/2}{h_{comb}-H/2} \cdot (\mu_{over} \cdot h_{over}^{1.5} - \mu_{comb} \cdot h_{comb}^{1.5}))} \]  \hspace{1cm} (43)

### 2.4 Flow through the breach channel

The discharge through the breach channel is calculated with the weir formula of Poleni (D’Eliso, 2007):

\[ q = \frac{2}{3} \cdot \mu_{over} \cdot \sqrt{2 \cdot g \cdot h_{over}^{1.5}} \]  \hspace{1cm} (44)

and

\[ Q = \frac{2}{3} \cdot \mu_{over} \cdot \sqrt{2 \cdot g \cdot h_{over}^{1.5}} \cdot B \]  \hspace{1cm} (45)

where:

- \( \mu_{over} \) is the coefficient taking the value \( \approx 0.58 \) (Visser, 1998)
- \( B \) is the width of the breach
- \( h_{over} \) - overflow head

The discharge through the breach is calculated as

\[ Q_{cor} = Q \cdot S_Q \]  \hspace{1cm} (46)

where \( S_Q \) is the backwater level coefficient:

\[ S_Q = (1 - \left( \frac{h_p-Z_b}{h-Z_b} \right)^{1.5})^{0.385} \quad h_p > Z_b \]  \hspace{1cm} (47)

\[ S_Q = 1 \quad h_p < Z_b \]

where:

- \( h_p \) - backwater level calculated as in the preliminary model, i.e. as the function of discharge through the breach and the polder area
- \( Z_b \) - elevation of the breach bottom at the breach entrance

The flow velocity and depth are then calculated with an explicit forward calculation - see Figure 11:

\[ \frac{H_{i+1} - H_i}{dx_{i+1}} = -J_{i+1} \Rightarrow z_{i+1} + h_{i+1} + \frac{v_{i+1}^2}{2g} = z_i + h_i + (1 - \alpha_c) \cdot \frac{v_i^2}{2g} - J_{i+1} \cdot dx_{i+1} \]  \hspace{1cm} (48)

where:

- energy slope:

\[ J_{i+1} = \frac{J_i + J_{i+1}}{2} \]  \hspace{1cm} (49)

and
\[
J = \frac{n^2 \cdot v^2}{h^{4/3}}; \quad n \text{ - Manning coefficient}
\]  

\[\text{• breach section contraction-expansion coefficient} \]

\[
\alpha_c = \left(1 - \frac{\Omega_{i+1}}{\Omega_i}\right) \cdot \frac{v_{i+1}^2}{2 \cdot g} \quad \text{for} \quad \Omega_{i+1} \leq \Omega_i
\]  

\[
\alpha_c = \left(1 - \frac{\Omega_{i+1}}{\Omega_i}\right) \cdot \frac{v_{i+1}^2}{2 \cdot g} \quad \text{for} \quad \Omega_{i+1} > \Omega_i
\]

Figure 11: Computational scheme (D’Eliso, 2007)

### 2.5 Infiltration

The erosion resistance of the cohesive soils used for the construction of the dike revetment strongly depends on the water content. Therefore a reliable model for the calculation of water infiltration into the soil is needed. As already discussed by D’Eliso et al (2007) the application of the numerical solver of the Richard equation (Richards, 1931) is unfeasible for the purpose of a dike breaching module. Instead, the implementation of one of the available simplified solutions is suggested. Therefore only the application of a simplified model will be here addressed.

#### 2.5.1 Wang Z model

The Wang Z model (Wang, 2000) is a simplified infiltration model that calculates the saturated water front using the Darcy’s equations as:

\[
z_{\text{inf}}(t) = \sqrt{2 \cdot \alpha_i \cdot k_s \int_0^t h(t) dt}
\]

with:

- \(z_{\text{inf}}\) - vertical coordinate of the saturated water front \([m]\)
- \(\alpha_i\) - coefficient, \(\alpha = 4.5\) \([-\]\]
- \(k_s\) - saturated hydraulic conductivity \([m/s]\) - see Tab.3
- \(h\) - thickness of the water layer \([m]\)

The following values of \(k_s\) are suggested:
Table 3: Typical values of the hydraulic conductivity $k_s$

The infiltration water front is calculated as:

$$z_w(t) = \sqrt{2 \beta_i k_i \int_0^t h(t) dt}$$

where: $\beta_i -$ coefficient, $\beta_i = 15 \cdot 15 \cdot 67.5 \cdot [\cdot]

The volumetric water content $\theta$ can be calculated either using the model of Wang Q or assuming the linear distribution along the vertical infiltration path as:

$$\theta(z_s) = \theta_s$$

$$\theta(z_w) = \theta_i$$

where:

- $\theta_s$ - saturated volumetric water content $[m^3/m^3]$ - see Tab.4
- $\theta_i$ - initial volumetric water content $[m^3/m^3]$ - see Tab.5

Table 4: Typical values of the saturated volumetric water content $\theta_s$ (modified from D’Eliso, 2007)

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$\theta_s$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.35 – 0.43</td>
<td>Brankensiek et al., 1981; Weissmann, 2003</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.41 – 0.44</td>
<td>Weissmann, 2003; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.39 – 0.48</td>
<td>Weissmann, 2003; Brankensiek et al., 1981</td>
</tr>
<tr>
<td>Clay</td>
<td>0.36 – 0.48</td>
<td>Weissmann, 2003; Brankensiek et al., 1981</td>
</tr>
</tbody>
</table>

Table 5: Typical values of the initial volumetric water content $\theta_i$ (D’Eliso 2007)

<table>
<thead>
<tr>
<th>Soil condition</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>$\theta_i \div \theta_s + 0.3(\theta_s \theta_i)$</td>
</tr>
<tr>
<td>Medium</td>
<td>$\theta_i + 0.3(\theta_i \theta_i \div \theta_s + 0.6(\theta_s \theta_i))$</td>
</tr>
<tr>
<td>Wet</td>
<td>$\theta_i + 0.6(\theta_s \theta_i) \div \theta_s$</td>
</tr>
</tbody>
</table>

Table 6: Typical values of the residual volumetric water content $\theta_r$ (modified from D’Eliso, 2007)

where $\theta_r$ is the residual volumetric water content (Tab.6)

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$\theta_r$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.020 – 0.054</td>
<td>Brankensiek et al., 1981; Weissmann, 2003</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.045 – 0.118</td>
<td>Weissmann, 2003; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.095 – 0.185</td>
<td>Weissmann, 2003; Brankensiek et al., 1981</td>
</tr>
<tr>
<td>Clay</td>
<td>0.068 – 0.226</td>
<td>Weissmann, 2003; Brankensiek et al., 1981</td>
</tr>
</tbody>
</table>
The following assumptions are imposed in the model:

- the hydraulic gradient is a function of the mean water depth on the surface of the dike, including wave run-up and run-down
- the calculation domain is limited only to the saturated soil, the influence of the unsaturated soil on the process of infiltration is limited to the empirical coefficient $\alpha$

### 2.5.2 Wang Q model

The Wang Q model (Wang et al, 2003) is based on the following assumptions:

- 1D vertical infiltration is simulated
- the water-soil characteristic curve by Brooks and Corey (1964) are used
- the Richard equation is solved analytically using a Taylor series method

The infiltration water front is calculated solving Eq. **(57)**, while the volumetric water content is calculated using Eq. **(58)**.

$$t = \frac{\theta_s - \theta_i}{(1 + \alpha) k_i} \left( z_w - \frac{\ln(\beta z_w + 1)}{\beta} \right)$$  \hspace{1cm} (57)

$$\theta(z) = \theta_i + \left( 1 - \frac{z}{z_w} \right)^\alpha (\theta_s - \theta_i)$$  \hspace{1cm} (58)

where:

- $\alpha_i = N/M$ ; $M = 2 + 3N$

with $N$ - pore size distribution index [\text{-}]

- $\beta_i = Mh_b$ with $h_b$ - air entry value [cm]

The typical values of the pore size distribution index $N$ and air entry value $h_b$ are given in Tab. 7 and Tab. 8, respectively.

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$N$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>0.4 – 1.68</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>0.40 – 0.89</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay loam</td>
<td>0.28 – 0.40</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay</td>
<td>0.09 – 0.41</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
</tbody>
</table>

*Table 7: Typical values of the pore size distribution index $N$ (modified from D'Eliso, 2007)*

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>$h_b$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>3.58 – 35.30</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>9.09 – 29.21</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay loam</td>
<td>31.25 – 69.55</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
<tr>
<td>Clay</td>
<td>10.0 – 125.00</td>
<td>Brankensiek et al., 1981; Carsel and Parrish, 1988</td>
</tr>
</tbody>
</table>

*Table 8: Typical values of the air entry value $h_b$ (modified from D'Eliso, 2007)*
2.5.3 Comparison of the infiltration models

In order to compare the results provided by the simplified infiltration model, an idealized simulation has been performed. The following data were used:

- dike height \( H_d = 10m \)
- mean water level at the toe of the dike \( h = 6m \)
- irregular waves, \( H_s = 1.47m \) \( T_p = 4.7s \)
- thickness of the clay layer \( d_c = 1.5m \)
- time of simulation \( t_{sim} = 120min \)
- soil parameters for clay and sand as given in Tabs. 3-8

In Fig.12 the comparison of the results provided by the Wang Q and Wang Z models is shown. Although the same input data set has been employed, the results differ significantly. The source of this difference is unknown. In the detailed model both the methods for the calculation of the saturated front are implemented and the one to be used can be freely chosen by the user.

![Graph showing comparison of saturated fronts](image)

**Figure 12: Comparison of the saturated fronts calculated using Wang Q and Wang Z models**

3. Morphodynamic module

The information on the loading and infiltration provided by the hydrodynamic module forms the input for the morphodynamic module. The main task of this module is the calculation of the soil erosion and consequently the dike profile changes resulting from the wave action. The total progress of erosion is divided into four main phases:

1. grass erosion;
2. clay erosion;
3. sand core erosion;
4. sand core wash-out.

Furthermore, two transition phases: (i) between clay and sand core erosion, and (ii) between front-face core erosion and core wash-out are included. The main improvements made in the detailed morphodynamic model concern the grass and clay erosion phases, as during the tests with the preliminary model they were found to have the most influence on the total breaching time. The new models for soil erosion and root reinforcement that were developed after the laboratory experiments are used. The core wash-out model is essentially the same as in the preliminary model.
3.1  Effect of breaking wave impact on the stability of the dike slope

The breaking wave impact on a dike slope may result in a very short (0.01 to 0.1 s) and very high impact pressures (in the range up to 150 kPa) concentrated on relatively small areas. This impact load occurs intermittently in time intervals of at least one wave period (usually longer with predominant impact in the water pad, that remains after the preceding wave), so that the actual loading duration (0.1 to 0.01 s) is small in comparison with the time period between the loads (5- 12s). In fact, the damage of the dike revetment is caused by a variety of mechanisms that are related to:

1. the impact pressures acting directly on the slope that are induced by breaking waves (represented as force A in Fig.13b) and may lead to surface erosion
2. the washing-out of soil particles and aggregates due to pressures acting from within the dike (force B in Fig.13b);
3. the effect of the impact pressures acting on water-filled cracks (force E in Fig.13c);
4. the water movement over the dike slope following the expansion of the water jet hitting the slope (Forces C and D in Figs. 13b and 13c).

Surface erosion of the dike revetment resulting from the impact pressures and from the flow of wave run-up and run-down will be here treated independently.

![Figure 13: Impact forces on and within a dike slope](image)

3.2  Simulation of the grass erosion

In order to improve the knowledge on the processes that lead to the erosion of the grass cover number of experimental investigations were performed at the LWI (see Stanczak et al, 2007d and Stanczak et al, 2007f for more details). Based on the obtained results, the models for the erosion of clay cover as well as the model for the reinforcement of the clay by grass roots have been developed.
3.2.1 Geotechnical properties of the grass cover

As shown in Fig. 14 a typical clay revetment with a grass cover used for the reinforcement of the sea dikes is composed of a clay layer that forms a substrate and vegetation which is represented by a grass rooted in soil.

![Figure 14: Structure of a grass cover - definition sketch and a illustrative photo](image)

The soil directly under the slope surface consists of small and larger aggregates, between which pores and roots are found. The very fine hair roots and symbiotic fungal threads due to connections with the finest particles keep the aggregate together. The coarser plant roots keep large and smaller particles together in a sort of network. This network of fine and coarse roots is the main reason, why the grass cover of a dike is a strong, springy and flexible layer that can deform without tearing. The parameters of this root network, especially the ratio of the roots in the soil (so-called Root Volume Ratio - RVR) describe the influence of the grass on the geotechnical parameters of the clay layer. The information on the distribution of the RVR as a function of the depth is provided by Sprangers (1999) and Stanczak et al (2007f). Both proposed the same function:

\[
RVR = A_{rvr} \cdot D_{rvr}^{(d-d_{cor})} \%
\]

where \(A_{rvr}\), \(D_{rvr}\), and \(d_{cor}\) are the empirical coefficients that depend on the quality of grass cover while \(d\) is the depth under the surface of the soil given in centimeters. The coefficients \(A_{rvr}\) and \(D_{rvr}\) are supposed to decrease with the increase of clay quality, as stronger clay prevents the growth of a dense root network. In Tab. 9 the suggested coefficients are summarized, while in Fig. 15 the graphical representation of Eq. (59) is shown.

<table>
<thead>
<tr>
<th>(A_{rvr} [-])</th>
<th>(D_{rvr} [-])</th>
<th>(d_{cor} [cm])</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.67</td>
<td>0.8</td>
<td>1.5</td>
<td>Sprangers (1999)</td>
</tr>
<tr>
<td>1.58</td>
<td>0.75</td>
<td>2.0</td>
<td>Stanczak et al (2007f)</td>
</tr>
</tbody>
</table>

![Table 9: Coefficients describing the grass roots distribution](image)

![Figure 15: Root volume ratio as the function of the depth under the surface](image)
3.2.2 Surface erosion resulting from impact pressures

The erosion of the grass cover due to a single impact pressure can be described as (Stanczak et al, 2007f):

\[ R_d = k_{dp} \cdot p_{max} \cdot e^{-h_s} \]  

(60)

with:

- \( R_d \) - volume of soil eroded after a single impact event \([m^3]\)
- \( k_{dp} \) - empirical detachability coefficient \([m^3/Pa]\)
- \( p_{max} \) - maximal impact pressure \([Pa]\)
- \( w \) - empirical coefficient describing the effectiveness of a water layer to damp impact pressures \([-]\). Typically the values of \( w \) are in the range \( 0.1 < w < 3 \), generally the higher values are observed in the case of more erosion-resistant soils
- \( h_s \) - thickness of the water layer on the slope \([m]\)

The following empirical equation based on the results of the laboratory experiments gives the dependency of the detachability parameter \( k_{dp} \) on the root volume ratio (Stanczak et al, 2007f):

\[ k_{dp} = \frac{k_{d,p}}{b \cdot RVR^2} \]  

(61)

where:

- \( k_{d,p} \) - clay erosion coefficient \([m^3/Pa]\) - see Section 3.1
- \( b \) is a dimensionless parameter describing the influence of the roots on the erodibility of the grass cover. During the performed laboratory experiments the value \( b = 5 \) was obtained.

The comparison of the measured values with the ones calculated using Eq.(61) is shown in Fig.16. A relatively good agreement is observed - the correlation coefficient takes the value of \( R = 0.963 \) and the coefficient of variation is equal \( CoV = 0.24 \)

![Figure 16: Measured and calculated values of the detachability parameter \( k_{dp} \) with respect to the depth under the surface](image)

Figure 16: Measured and calculated values of the detachability parameter \( k_{dp} \) with respect to the depth under the surface
At the depth where the $RVR = 0.44\%$ the reinforcing influence of the grass roots becomes negligible - the detachability parameter of the grass cover $k_{d,e,p}$ reaches the value of the detachability parameter of the clay $k_{d,p}$. This is the critical depth of erosion $d_{crit}$.

### 3.2.3 Surface erosion due to of run-up and run-down flow

In the process of wave run-up and run-down the grass cover is subject to the flow-induced shear stresses. If the effective shear stress is greater than the critical shear stress for the grass cover, the erosion occurs. The calculation of the effective shear stress taking the influence of the vegetation is performed using the following approach (Temple et al, 1987):

$$\tau_{0,e} = \tau_0(1 - C_f)(\frac{n_{soil}}{n_{tot}})^2 \quad \text{with} \quad \tau_0 = \rho \cdot g \cdot h \cdot J$$

with:

- $\tau_{0,e}$ - effective shear stress [$N/m^2$]
- $\tau_0$ - bottom shear stress [$N/m^2$]
- $C_f$ - empirical factor describing the influence of the grass cover on the effective shear stress [$-$]
- $n_{soil}$ - Manning roughness of the soil [$s \cdot m^{-1/3}$]
- $n_{tot}$ - total Manning roughness (taking the grass cover into account) [$s \cdot m^{-1/3}$]
- $J$ - energy slope [$-$]
- $h$ - flow depth [$m$]
- $\rho$ - water density [$N/m^3$]

The empirical factor $C_f$ depends on the type of the grass cover and takes values in the range $C_f = 0 - 1$. $C_f$ -values for a number of species are given by Temple et al (1987).

The total Manning roughness is calculated using the following formula given by Temple et al (1987):

$$n_{tot} = 0.3048 \cdot \frac{1}{3} \exp\{C_f[0.01331ln^2(10.7639q) - 0.0954ln(10.7639q) + 0.297] - 4.16\}$$

where $C_f$ denotes the curve retardance factor calculated as $C_f = 2.5(L_s\sqrt{\rho_g})^{0.333}$, and:

- $L_s$ - stem length. The suggested values are in range $L_s = 0.08 - 0.2m$ depending on the grass type and maintenance method
- $\rho_g$ - number of stems per square meter. Typically $\rho_g = 500 - 5500$ stems/m$^2$, depending on the grass species. Generally, the higher $\rho_g$ values are observed in the case of more erosion-resistant grass species.

When the grass species are unknown, the curve retardance factor $C_f$ and grass cover factor $C_f$ can be assumed as given in Tab.10 (Temple and Hanson, 1994):
Grass cover quality | Grass cover factor $C_f$ | Curve retardance factor $C_i$
--- | --- | ---
Good | 0.75 | 5.6
Moderate | 0.5 | 5.0
Poor | 0.25 | 4.4

Table 10: Grass cover factor and curve retardance factor for idealized scenarios

3.2.4 Weaker spots on the grass cover

The dike breach initiation is facilitated by possible weaker spots in the protective grass vegetation. As the grass cover is a composite material that consists of subsoil that is reinforced by a grass roots network and of a sword above the soil surface (Fig. 14), the damages of the grass cover can be divided into damages of the root network and damages of the sword. Each type of the grass damage differently affects the erosion resistance of the revetment. The following, tentative classification of the grass damages is proposed:

- Damages of the sward (Fig.17) - the sward is removed or damaged, but the root network underneath is still present. This type of damage has significant influence on the surface erosion due to the flow of wave run-up and run-down, as the presence of sward significantly decreases the effective shear stress $\tau_0$ on the surface. At the locations where the sward is removed, the vegetal cover factor that describes the influence of the grass cover is equal to zero, resulting in greater shear stresses compared to the locations where the sward is still present. The location and dimensions of the patches where no stems are damaged is fully random. According to the Dutch requirements, the patches should be smaller than 2cm$^2$ (TAW, 1997), but the ones observed at existing dikes can reach up to 500cm$^2$, resulting in significant decrease of the erosion resistance.

![Figure 17: Damage of the grass stems (grass roots still present)](image-url)
• Damages of the root network. The density of grass root network plays an important role in the estimation of the erosion resistance against the breaking wave induced impact pressures. The location of the weaker patches is fully random and their size is probably smaller than the size of the patches on the surface. There are no regulations that prescribe the maximal size of the areas where no roots are present, but as even small patches are believed to have a significant influence on the erosion resistance, it is necessary to investigate this problem in more detail.

• Combined damage of roots and stems - at the locations where both the roots and stems are damaged or removed, the erosion resistance is decreased to the value for unprotected clay layer, as both parameters describing the influence of the grass, i.e. $RVR=0$ and $C_f=0$.

• Animal footprints - this type of damage does not directly affect the erosion resistance properties of the grass cover, but represents a location of flow concentration and thus of greater loading (Figure 18)

![Flow concentration in the locally damaged grass revetment](image)

**Figure 18: Flow concentration in the locally damaged grass revetment**

### 3.3 Simulation of clay erosion

The erosion of the grass revetment results on uncovering of the next layer, i.e. clay layer. This section presents the mathematical formulation and application of the erosion models derived after laboratory experiments performed at the LWI.

#### 3.3.1 Surface erosion due to impact pressures

In order to improve the knowledge on the processes that lead to the surface erosion of clay experimental investigations were performed at the LWI (Husrin, 2007 and Stanczak et al, 2007f). The following formula has been derived from the laboratory experiments (see also Eq. (60)):

$$R_d = k_{d,p} \cdot p_{\text{max}} \cdot e^{-wh_s}$$  \hspace{1cm} (64)
with:

- $k_{d,p}$ - empirical detachability coefficient for clay \([m^3/\text{Pa}]\)
- $p_{max}$ - impact pressure \([\text{N/m}^2]\)
- $w$ - empirical coefficient describing the damping effectiveness of a water layer \([-]\)

Both empirical coefficients $k_{d,p}$ and $w$ depend on the soil parameters, i.e. on the type of clay and on the water content. For the strong\(^1\) clay the measured damping effectiveness coefficient is $w = 1$ while the erodibility coefficient $k_{d,p}$ can be calculated as an empirical function of the gravimetric water content $w_c$ \([-\]):

$$k_d = 0.35 \cdot \arctan[110 - (wc - 0.43)] \cdot 10^{-12} \quad [m^3/\text{Pa}] \quad (65)$$

The comparison of the measured erodibility coefficients with Eq. (65) is shown in Fig. 19. The measured values for other types of clay can be found in Stanczak et al (2007f).

\[Figure\ 19: \ Erodibility\ of\ the\ strong\ clay\ as\ a\ function\ of\ water\ content\]

### 3.3.2 Surface erosion due to the wave run-up and run-down flow

The detachment of the soil particles during the surface erosion of the clay cover due to the flow induced by wave run-up and run-down resulting from both breaking and non-breaking waves is calculated according to the excess effective shear stress approach (Meyer, 1964) as:

$$\frac{dz}{dt} = k_d (\tau_{0,e} - \tau_{0,cr})dt \quad (66)$$

with:

$$\tau_{0,e} = \rho_w ghJ \quad (67)$$

\(^1\)erosion resistant category I according to the Dutch requirements (TAW, 1996)
where:

- $dz$ - incremental erosion depth [m]
- $k_d$ - detachability coefficients that depends on the soil properties [$m \cdot N^{-1} \cdot s^{-1}$], $k_d$ is calculated as a function of  percentage clay content in the soil (Eq. (68))
- $\tau_{0,cr}$ - critical shear stress for the given type of soil [Pa] calculated as a function of mass concentration of particles (Eq. (69))

The detachability coefficient is calculated as a function of the dry soil density $\rho_{c,d}$ and the weight percentage of the clay in the soil $c_{sg}$ as (Temple and Hanson, 1994):

$$k_d = 10^{-6} \frac{10 \rho_w}{\rho_{c,d}} \exp\left[-0.121c_{sg}^{0.406}\left(\frac{\rho_{c,d}}{\rho_w}\right)^{3.1}\right]$$  \hspace{1cm} (68)

where $\rho_{c,d}$ is the dry density of the soil.

The critical shear stress $\tau_{0,cr}$ is calculated as (D’Eliso, 2007):

$$\tau_{0,cr} = 5.43 \cdot 10^{-6} \cdot (\rho_{c,d} - \rho_{c,w})$$ \hspace{1cm} (69)

where $\rho_{c,w}$ denotes the density of water-soil mixture, equal to about 1100 $kg/m^3$.

The energy slope $J$ is calculated using the Manning formula for shallow water:

$$J = \frac{n^2 \cdot v^2}{h^{4/3}}$$ \hspace{1cm} (70)

where $v$ and $h$ denote the flow velocity and depth, respectively.

The Manning roughness $n$ is calculated as a function of the grain size $D_{75}[mm]$ (Temple and Hanson, 1994):

$$n = \frac{D_{75}^{1/6}}{14.2301}$$ \hspace{1cm} (71)

3.3.3 Shear failure in water-filled cracks

The clay cover in a sea dike is subject to changes in water content that occur due to drying and wetting of a dike. Differences in suction pressure result in changes in the water content and thus to changes of the clay volume. As a result, clay shrinks and expands leading to the formation of two types of cracks in the unsaturated zone (TAW, 1996):

- **pull-cracks**: usually occur when soil shrinks. These cracks are differently oriented according to their size - larger shrinkage cracks are almost always vertical, smaller cracks may occur in all directions;
- **shear cracks**: usually occur in shear areas that are caused by the swelling of clay. Those cracks may occur in all directions.

A cross-section of a dike clay layer with significant pull-cracks is shown in Fig.20.
The water-filled cracks subject to impact pressures have often been reported as the starting points for the seaward slope damage during storm surges (Träger, 1962 or Wohlberg, 1963). After the disastrous flood event in Germany in 1962, Führböter (1966) proposed the following conceptual model to calculate the effect of impact pressures acting on water-filled cracks in clay cover: if a water-filled crack of depth $a$ and length $L_c$ (the width of the crack is not involved in the calculation) is subjected to an impact pressure $p_{\text{max}}$ then the pressure is instantly (speed of sound in water $C = 1485 \text{ m/s} = \text{speed of pressure propagation}$) transferred in the full magnitude to the two side walls of the crack. The force acting on the wall of the crack is then calculated as follows (see also Fig. 21):

$$F_{\text{crack}} = a \cdot L_c \cdot p_{\text{max}}$$

(72)

with:

- $F_{\text{crack}}$ - force acting on the wall of the crack [N]
- $a$ - depth of the crack [m]
- $L_c$ - length of the crack [m]
These forces are absorbed by the compression and the shear strength of the soil behind the walls of the crack. The weight of the soil body is considered by Führböter (1966) to be negligibly small in comparison to the possible impact forces. Using this assumption, only the shear strength provided by cohesion $c$ of the soil acts as a resistance.

The shear stress acts on a plane leaning to the surface with an angle $\alpha$ and provides the following shear forces (see Fig. 21b):

$$ S = a \cdot L_c \cdot p_{\text{max}} \cdot \cos \alpha $$

the resistance force is provided by the shear strength described by cohesion $c$ only:

$$ W = l \cdot L \cdot c $$

setting

$$ l = \frac{a}{\sin \alpha} $$

gives

$$ W = \frac{a \cdot L_c \cdot c}{\sin \alpha} $$

Solving the limit state equation $S = W$ for $\sin \alpha$ will provide the angle of shear failure $\alpha$:

$$ \sin \alpha = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \left(\frac{c}{p_{\text{max}}}\right)^2} $$

solving of Eq. (77) leads to $p_{\text{max}} = 2c$ as the critical impact pressure, i.e. the shear failure occurs for impact pressures $p_{\text{max}}$ greater or equal to the double of cohesion $c$.

The applicability of the discussed model was investigated by Stanczak et al (2007f) who have noted that the weight of the soil block and forces on the side walls of the crack that have been neglected by Führböter (1966) also play a significant role in the process of shear failure and should therefore be taken into account (see also Fig. 22).
For this purpose, the following extension of the conceptual model by Führböter (1966) was proposed (Stanczak et al, 2007f) and will be used for the implementation in the detailed model:

\[
\frac{a \cdot L_c \cdot c}{\sin \alpha} + 0.5 \cdot a^2 \cdot L_c \cdot \tan \alpha \cdot \rho_s \cdot g \cdot \sin \alpha + \frac{a^2 \cdot c}{\tan \alpha} = a \cdot L_c \cdot p_{\text{max}} \cdot \cos \alpha
\]  

(78)

### 3.4 Sand core erosion - simulation of cross-shore profile development

#### 3.4.1 Transition phase between clay erosion and sand core erosion

According to the small scale simulations of dike breaching initiated from the seaside by breaking wave impact (Husrin, 2007, Stanczak et al, 2007g) the remaining part of the clay cover after breach initiation still plays an important protective role. The assumption made in the preliminary model, stating that the entire clay cover is removed from the dike after the breach initiation is therefore not consistent. Actually, a transition phase containing both the erosion of the clay cover and the sand core was observed between the clay erosion phase and the sand core erosion phase (Figure 23). This transition phase begins immediately after the end of the clay erosion phase, i.e. when the eroded hole in the clay layer has reached at least one point of the sand core and ends when the dimensions of the scour hole have grown up to the point when the plunge point is located on the uncovered sand core.
During the transition phase the erosion of clay is calculated as in the preceding phase (Eqs.(64)-(71)), while the progress of sand core erosion is calculated as in the preliminary model, i.e. by applying the model of Larson et al. (2004) for sand dune erosion. The governing equation for this phase reads (Larson et al., 2004):

\[ Q_{\text{single}} = C_E \cdot F_{\text{impact}} \]  

(79)

where \( F_{\text{impact}} \) is the impact force for a single wave [N] (as given by the VOF-model), while \( C_E \) \([m^3/N]\) is the coefficient of erosion calculated as:

\[ C_E = \frac{2 \cdot C_s \cdot C_U \cdot \rho_s^2 \cdot (1 - p)^2}{\rho_w} \]  

(80)

\[ C_s = A_s \cdot e^{-b \cdot \frac{H}{D_{50}}} \]  

(81)
The empirical coefficients in Eqs. ((80)) and ((81)) were calibrated using a large number of the experimental results and take the values (Larson et al, 2004):

- \( A_3 = 1.34 \cdot 10^{-3}[-] \)
- \( b = 3.19 \cdot 10^{-4}[-] \)
- \( C_{ic} = 1.83[-] \)

During this transition phase the progress of sand core erosion is significantly faster than that of the clay layer resulting in undermining and consequently in the collapse of the clay layer (Fig. 24).

Figure 24: Undermining of the clay layer during the transition phase

Three failure modes for the clay cantilever can theoretically occur: bending, shearing and sliding. However, based on the observations during tests in the wave flume it can be stated, that bending is the dominant failure mode. The critical length of the undermined clay cantilever (Fig. 25) is then given as:

\[
\text{eq:82} \quad l_{crit} = \frac{c \cdot d_c}{3 \cdot \rho_c \cdot g}
\]

with:

- \( l_{crit} \) - critical cantilever length [m]

Figure 25: Bending of the undermined clay cantilever - definition sketch

3.4.2 Simultaneous erosion of front face and landside slope

During the dike breaching process, wave overtopping can occur, resulting in the erosion of the inner slope. In order to account for this phenomenon, from the begin of the sand core erosion till the begin of the overflow, also the erosion of the inner slope is accounted for. The erosion of the outer
slope is simulated by applying the complete SBEACH model (Section 2.1). If overtopping occurs, the flow conditions are calculated applying the model of D’Eliso (2007) described in Section 3.1 and the profile change is calculated using the excess shear approach as described in Section 3.1.

3.4.3 Erosion due to overflow - introduction of the third dimension

During the erosion process the dike crest level decreases and consequently wave overtopping becomes overflow. The erosion due to overflow is simulated essentially by applying the same models that have been used in the preliminary model, i.e. the weir formula of Poleni is used for the flow simulation, while the breach channel growth is calculated by applying one of the sediment transport models and the 1D Exner equation. The breach initiation and erosion of the clay cover as well as the first phase of the erosion of the sand core are simulated as 2D processes in the $x-z$ plane. The third dimension is introduced after the erosion has reached the inner slope of the dike and wave overtopping has become overflow. The shape of the scour hole in the $x-y$ plane (Fig 27) is assumed basing on the reported damages of the sea-dikes (Landesamt für Wasserwirtschaft - Schleswig-Holstein, 1962; Zitscher, 1962; Wohlenberg, 1963; Stephan, 1981). According to those reports and available pictures (examples in Fig. 26) taken after the flood events, the width of the initial breach $B_{ini}$ (the removal of the grass cover) is usually in the range $B_{ini} = r_h$ to $B_{ini} = 2r_h$ where $r_h$ is the length of the initial breach. During the progress of the erosion, the side walls of the breach are not formed parallel but the breach remains wider at the seaside. The angle between side walls and $x$-axis $\alpha_{breach}$ can be roughly estimated to be equal $\alpha_{breach} = 30^\circ$.

![Figure 26: Shape of a partial breach in a dike - examples](image1)

![Figure 27: Initial conditions for the 3D modeling - $y-z$ plane](image2)
3.4.4 Sediment transport model

For the detailed model, like for the preliminary model, two sediment transport models were selected: (i) Bagnold-Visser sediment transport formula (Visser, 1998) and (ii) Bagnold-Bailard formula (Bailard, 1981). Both formulae have already been used for the breaching of sea dikes under overflow conditions (D’Eliso, 2007). The sediment transport calculated by those models is a function of the energy spent by the flow in transporting the sediments (energetic models).

Governing equations

One of the following sediment transport models can be freely selected by the user. Generally, the models are similar, but the Bagnold-Visser model is suggested for use, as it was reported that the Bagnold-Bailard model can provide too high erosion rate (D’Eliso, 2007).

1. Bagnold-Visser sediment transport formula:

The volumetric bed load transport is calculated as:

$$ q_{sb} = \frac{\tau_0 \cdot \nu}{\rho_w \cdot g \cdot \Delta \cdot \tan \phi - \tan \beta \cdot \cos \beta} \cdot e_b \quad [m^3/m \cdot s] \quad (83) $$

$$ q_{sb} \leq \zeta_2 \cdot (1 - p) \cdot D_{50} \cdot v; \quad \zeta_2 \approx 2 \quad (84) $$

with:
- $\nu$ - average flow velocity in the breach cross-section [m/s]
- $\Delta$ - relative density of soil particles $\Delta = \left( \rho_s / \rho_w \right) - 1$
- $e_b$ - bed-load efficiency coefficient, $e_b = 0.13$
- $\phi$ - internal friction angle [deg]

The volumetric suspended sediment transport $q_{ss}$:

$$ q_{ss} = \frac{\tau_0 \cdot \nu}{\rho_w \cdot g \cdot \Delta \cdot \frac{w_s}{v \cos \beta}} \cdot \frac{e_s}{\left( \cos \beta \right)^2} \quad [m^3/m \cdot s] \quad (85) $$

with:
- $e_s$ - suspended load efficiency coefficient, $e_s = 0.01$
- $w_s$ - settling velocity [m/s]

The total volumetric sediment transport:

$$ Q_{st} = P \cdot (q_{sb} + q_{ss}) \quad [m^3/s] \quad (86) $$

with:
- $P$ - wet perimeter of the breach channel [m]

2. Bagnold-Bailard formula:

the volumetric bed-load sediment transport $q_{sb}$ is calculated as:

$$ q_{sb} = \frac{\tau_0 \cdot \nu}{\rho_w \cdot g \cdot \Delta \cdot \tan \phi - \tan \beta} \cdot e_b \quad [m^3/m \cdot s] \quad (87) $$

while the volumetric suspended sediment transport is calculated as:

$$ q_{ss} = \frac{\tau_0 \cdot \nu}{\rho_w \cdot g \cdot \Delta \cdot \frac{w_s}{v \cos \beta}} \cdot e_s \quad [m^3/m \cdot s] \quad (88) $$
**Bottom shear stress**

The bottom shear stress $\tau_0$ is a function of the flow characteristics and surface roughness calculated as:

$$\tau_0 = g \cdot \rho_w \cdot h \cdot J$$  \hspace{1cm} (89)

with:

- $h$ - flow depth
- $J$ - energy slope.

The energy slope can be calculated selecting one of the two following formulae which are implemented in the model:

1. Manning formula:

$$J = \frac{n^2 \cdot v^2}{R^{4/3}}$$  \hspace{1cm} (90)

with:

- $n$ - Manning roughness (typically $n = 0.02$ for sand surfaces)
- $R$ - hydraulic radius

2. Chezy formula

$$J = \frac{v^2 \cdot C_f}{g \cdot R}$$  \hspace{1cm} (91)

with:

- $C_f$ - Chezy’s friction coefficient calculated as:

$$C_f = \frac{0.4}{[\ln(12R/z_0)]^2}$$

- $z_0$ - roughness height calculated as:

$$z_0 = 3 \cdot \theta \cdot D_{90}$$  \hspace{1cm} (92)

The Shields parameter $\theta$ is calculated as:

$$\theta = \frac{C_f \cdot v^2}{\Delta \cdot g \cdot D_{50}} \hspace{1cm} \text{for } \theta \geq 1$$

$$\theta = 1 \hspace{1cm} \text{for } \theta < 1$$  \hspace{1cm} (93)

The roughness height describes the influence of the high concentration of the sediments near the breach bottom as the viscosity of the water-sediment mixture is significantly larger than the viscosity of water (D’Eliso, 2005). When $\theta$ is greater than one, the friction coefficient $C_f$ is calculated iteratively.

**Sediment fall velocity**

The sediment fall velocity $w_s$ can be calculated using one of the following approaches, which can be freely chosen by the user. The formula of van Rijn has already been implemented in available models of dike breaching due to overflow. The formula of Ahrens is suggested for use, as it is much simpler and derived explicitly for coastal areas.

1. Van Rijn formula (Van Rijn, 1993):

$$w_s = \frac{\Delta \cdot g \cdot (D_{50})^2}{18 \cdot v} \hspace{1cm} \text{for } 0.001 \leq D_{50} \leq 0.1$$

$$w_s = \frac{10 \cdot v}{D_{50}} ((1 + 0.01 \cdot \Delta \cdot g (D_{50})^{1/3} \cdot v^{-2})^{0.5} - 1) \hspace{1cm} \text{for } 0.1 \leq D_{50} \leq 1$$
\[
\begin{align*}
\text{w}_i &= 1.1 (\Delta \cdot g \cdot D_{50})^{0.5} \quad \text{for } D_{50} > 1 \\
\text{where the kinematic viscosity of water } \nu \text{ [m/s] depends on the water temperature } T \text{[°C]} \text{ and is calculated as (NBS,1975)}: \\
\nu &= \frac{\rho_w + 1505}{2500 \cdot \rho_w} \cdot 10^{\left(\frac{13}{10 - 0.081(20 - T)^{-4.3}}\right)} \quad \text{for } 0 < T < 20 \\
\nu &= \frac{\rho_w + 1505}{2500 \cdot \rho_w} \cdot 10^{\left(\frac{13 \cdot (20 - T)}{T + 104}^{-3}\right)} \quad \text{for } 20 < T < 40
\end{align*}
\]

2. Ahrens formula (Ahrens, 2000)

\[
\text{w}_i = C_1 \cdot \frac{\Delta \cdot g \cdot D_{50}^2}{\nu} + C_2 \sqrt{\Delta \cdot g \cdot D_{50}}
\]

with:
- \( C_1 = 0.055 \cdot \tanh(12 \cdot A_h^{-0.59} \exp(-0.0004 \cdot A_h)) \)
- \( C_2 = 1.06 \cdot \tanh(0.016A_h \cdot A_h^{-0.50} \exp(-120/A_h)) \)
- \( A_h = \frac{\Delta \cdot g \cdot D_{50}^3}{\nu^2} \)

The kinematic viscosity of water \( \nu \text{[m/s]} \) is calculated as a function of water temperature \( T \text{[°C]} \)

\[
\nu = 10^{-4} (0.0000069 \cdot T^2 - 0.000529 \cdot T + 0.0182)
\]

### 3.4.5 Updating of the breach cross-section

#### Calculation of the dike erosion rate

In order to calculate the total rate of erosion at each dike section, the 1D Exner equation is applied:

\[
\frac{dQ_{st,i}}{dx} + (1 - p) \frac{dA_i}{dt} = 0
\]

where
- \( A \) - breach cross-section
- \( p \) - porosity of the soil

Eq.((98)) is solved using the following finite difference scheme (D'Eliso, 2005):

\[
\frac{1}{2} \cdot \left( \frac{Q_{st,i+1} - Q_{st,i}}{\Delta x_{i+1,i}} + \frac{Q_{st,i} - Q_{st,i-1}}{\Delta x_{i,i-1}} \right) + (1 - p) \cdot \frac{A_{i+1} - A_{i}}{\Delta t} = 0
\]

\[
A_{i+1} = A_i - \frac{\Delta t}{2 \cdot (1 - p)} \cdot \left( \frac{Q_{st,i+1} - Q_{st,i}}{\Delta x_{i+1,i}} + \frac{Q_{st,i} - Q_{st,i-1}}{\Delta x_{i,i-1}} \right)
\]

where:
- \( Q_{st,i} \) - total sediment transport rate for the \( i \)-th Section
- \( A_i \) - breach cross-section for the \( i \)-th Section
- \( \Delta x_i \) - grid space for the \( i \)-th Section
- \( \Delta t \) - time step

At the first and the last section of the dike the following equations are applied (D'Eliso, 2005)
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\[ A_{i,j+1} = A_{i,j} - \frac{\Delta t}{1 - p} \left( \frac{Q_{i,j+1} - Q_{i,j}}{\Delta x_{i,j}} \right) \]  
\[ A_{N,j+1} = A_{N,j} - \frac{\Delta t}{1 - p} \left( \frac{Q_{N,j+1} - Q_{N,j-1}}{\Delta x_{N,j}} \right) \]  

Sections 1, \( i \) and \( N \) are defined as in Fig.28.

\[ \text{Figure 28: Finite difference scheme} \]

**Calculation of the breach profile change**

The calculation of the change in the breach profile is performed assuming that the breach channel is rectangular and the ratio between lateral (\( d_b \)) and vertical (\( d_z \)) erosion (Fig.29) \( c_{db/dz} \) is given. The following equation for the calculation of the vertical erosion is introduced (D’Eliso, 2007):

\[ d_z = \frac{-B_b + \sqrt{B_b^2 + 8c_{db/dz} \cdot dA}}{4c_{db/dz}} \]  

the lateral erosion is then calculated as:

\[ d_b = d_z \cdot c_{db/dz} \]

\[ \text{Figure 29: Breach channel evolution (modified from D’Eliso, 2007)} \]

More details and derivations can be found in the original publication (D’Eliso, 2007)
4. Model implementation

In order to develop the complete detailed dike breaching model the information on the modeling of flow and erosion processes given in Chapters 2 and 3 are summarized and implemented into a computer program. The existing COBRAS model is used to provide the information on the loading on the dike slope during the grass and clay erosion phases, while the flow models for the front-face erosion of the sand core and for the core wash-out are coded in MATLAB v7.0. The complete morphodynamic module is coded in MATLAB as well. The simplified flow chart of the detailed model is shown in Figure 30. In Section 4.1 the modeling of the grass cover erosion is summarized, Section 4.2 describes the implementation of the clay erosion module, including shear failure simulation in water-filled cracks. The modeling of sand core erosion including the transition phase between erosion of clay and sand is reported in Section 4.3, while the simulation of the sand core wash-out is described in Section 4.4. Finally, the input and output modules are described. Example of dike breach simulation and discussion of the results are provided as well.

![Figure 30: Detailed model - simplified flow chart](image)
4.1 Description of the model

For the proper simulation of the dike breaching, the time series of the breaking wave induced impact pressures and flow field on the outer slope have to be generated by the Cobras model. The obtained outputs are then saved as external *.txt files that can be loaded by the dike breaching model. The hydrodynamic and morphodynamic modules as well as the free surface flow and infiltration models are not coupled.

4.1.1 Discretization

The dike is modeled as the outer contour of a cross-section as a matrix that contains the coordinates of all the nodes at the surface of each layer of the dike. The numerical griding (distance between successive nodes) is uniform over the whole dike cross-section (Fig. 31). The default grid space $\Delta x$ is set to $\Delta x = 0.1m$ while time step $\Delta t$ is set to $\Delta t = 0.1s$. Both values however can be freely changed by the user, but it is suggested to use maximally $\Delta x = 0.2m$ and $\Delta t = 0.2s$, as the loading on the outer slope acts on relatively small areas and during very short periods of time, and may not be properly reproduced if larger steps are used.

![Figure 31: Definition of numerical grid and dike sections](image)

### 4.2 Simulation of the dike cover erosion

Grass erosion phase and clay erosion phase form the entire cover erosion phase which starts with the incipient wave action and ends when the erosion reaches the sand core of the dike. The hydrodynamic module is essentially the same during the erosion of grass and clay and contains the simulation of free surface flow and infiltration. The outputs of the hydrodynamic module are used as the inputs for the morphodynamic module. During the simulation of the grass erosion the focus is on the reproduction of the roots reinforcement properties, while during the clay erosion, the shear failure in cracks is the leading process. In Fig. 32 the flow chart for the simulation of grass and clay cover is given.
4.2.1 Flow simulation

During the erosion of the grass and clay cover only the flow conditions on the outer slope are calculated. The COBRAS model is used to simulate the free surface flow. A MATLAB subroutine for the postprocessing of the defaults output files generated by COBRAS model is delivered together with the main program file. The final output file saved in the *.mat format contains the information on
impact pressure, flow velocity and layer thickness for each node and each time step. For each time step the model calculates the infiltration and saturation water front (Eqs. 2.54-2.59) and the resulting volumetric water content in the soil according to a linear distribution along the vertical infiltration path.

4.2.2 Grass and clay erosion

The erosion of the grass cover is calculated for all the sections of the seaside slope. In the first step, the soil cohesion is calculated as a function of the water content. Applying Eq. 3.1 the root volume ratio is then calculated. Based on this information the erodibility coefficient for impact pressures is calculated (Eqs. 3.3-3.7) and erosion depth for each node and each time step is calculated (Eq.3.2). Simultaneously, for each node, the erosion depth due to flow of run-up and run-down is calculated by applying Eqs. 3.8-3.9, where the erodibility coefficient is calculated using Eqs. 3.10-3.12. For each time step the total erosion depth is calculated and the condition of critical erosion depth is controlled. The assumed critical depth is reached, when the root percentage in the soil volume becomes negligible, i.e. \( RVR \leq 0.44\% \). If this condition is fulfilled, the breach is initiated, the numerical and graphical information on the erosion progress history, development of the dike profile and associated time are stored in an .xls file. At the next time step the Phase 2 (clay erosion) begins. The local clay cover erosion is calculated for all sections of the seaside slope. The erodibility coefficient due to impact pressures is calculated as a function of water content and soil type (Eq.3.7). The depth of erosion resulting from a single wave impact is calculated using Eq. 3.6. Simultaneously, the depth of erosion induced by the wave run-up and run-down flow is calculated by using Eqs. 3.8-3.9 and the erodibility coefficient is calculated by applying Eqs.3.10-3.12. For each time step also the soil cohesion is calculated and the limit state equation for the shear failure in water filled cracks is solved (Eq. 3.19). If a shear failure occurs, the dike profile is updated. For each time step the total erosion depth is calculated and the condition of critical erosion depth \( \left( d > d_c \right) \) is controlled. If this condition is fulfilled for at least one node, the information on the erosion progress history, updated dike profile and time associated with this phase are stored. At the next time step the transition phase between clay and sand erosion begins.

4.3 Sand core erosion - beach profile formation and transition phase

Compared to the preliminary model, an additional phase between clay and sand core erosion was introduced, according to the results of the laboratory experiments performed at the LWI. This transition phase starts when the clay erosion phase ended, and ends when the dimensions of the scour hole have grown up to the point, when the plunge point is located on the uncovered sand core. At that moment, the core erosion starts and ends when the overflow occurs, i.e. when the dike crest is below the mean water level. The complete flow chart for the simulation of the front-face sand core erosion is given in Fig.33.
Figure 33: Flow chart of the detailed morphodynamic module - sand core erosion including the transition phase between clay and core erosion
4.3.1 Flow simulation

During the transition phase the flow parameters are obtained exactly like in the case of cover erosion, i.e. using the information provided by the COBRAS model. After the end of the transition phase for each time step the condition for wave overtopping is controlled by comparing the wave run-up limit (Eq.2.30) and the highest node of the dike. Depending on the fulfillment of the overtopping condition two alternative cases may occur for each time step:

- No overtopping occurs - only the flow on the outer slope is simulated. The energy flux is calculated by using Eqs. 2.8 - 2.15, then the wave height is given by Eqs. 2.16-2.20 and finally the wave energy dissipation along the slope is obtained from Eq. 2.21;
- Overtopping occurs - the flow on the crest and on the inner slope is simulated. After the discharge is calculated with Eqs. 2.30 - 2.38 and 2.40-2.43, the flow parameters within a wave cycle are calculated according to Eq. 2.39.

4.3.2 Simulation of the sand core erosion

All the essential calculations regarding clay erosion during the simulation of the transition phase between clay and core are calculated as in the previous phase, i.e. by applying Eqs. 2.53-2.56, 3.6-3.7, and 3.19 for the erosion due to impact pressures or Eqs. 3.8-3.12 for the erosion due to wave run-up and run-down. Simultaneously, the progress of sand core erosion in the direction of waves is calculated applying the model of Larson et al. (2004) given by Eq. 3.20. As the progress of sand erosion is significantly faster than that of the clay cover, undermining of the clay revetment occurs. The collapse condition is controlled by the limit state equation (Eq.3.21). If a bending failure occurs, the dike profile is updated and it is assumed that the collapsed block of clay is removed instantaneously by wave action. The transition phase ends, when the dimensions of the scour hole have grown up to the point, when the plunge point (Eq.2.20) is located on the uncovered sand core. In the next time step the numerical and graphical outputs are stored and the next phase, i.e. the sand core erosion starts.

When no overtopping occurs, the Eqs. 2.22-2.29 are used to describe the sediment transport rate and bed profile change on the outer slope. If overtopping occurs, the erosion along the dike profile is calculated using Eqs. 3.8-3.12 for the erosion of the clay revetment or Eqs. 3.22-3.40 for the erosion of the sand core. For each time step the condition of overflow is controlled, and if it is fulfilled, the outputs are stored, and the last phase, i.e. dike core wash-out begins.

4.4 Dike core wash-out

The sand core wash-out module is essentially the same module that has been used in the preliminary model. In this phase the third dimension is introduced (see Section 3.4.3). During the wash-out of the sand core the overflow discharge is calculated as a function of the overflow head using the weir formula of Poleni (Eqs. 2.44-2.47). The distributions of the flow velocity and flow depth in the breach channel are calculated applying the explicit forward solution of the Eqs. 2.48-2.52. The total volumetric sediment transport rate is calculated applying the excess shear approach (Eqs.3.22-3.40) and consequently the breach profile is updated using Eqs. 3.35-3.40. The simulation ends when (i) the maximal time of simulation, given as input parameter, is reached or (ii) the water level in the inundated area has reached the MWL of the sea. The complete flow chart of the core wash-out module is given in Fig.34.
4.5 Example of dike breach simulation

The detailed model was applied for the breaching simulation of a typical prototype dike in order to provide typical outcomes from the model and to explore the main differences between preliminary and detailed model. The simulation of a historical dike damage and failure are performed as well. The results given by the detailed model are compared with the results given by the preliminary model and the verification against the measurements is made.

4.5.1 Breaching of a typical North Sea dike

As the first example simulation, the model was applied for the breaching simulation of a typical North Sea dike (Fig.35).

Figure 35: Geometry of the prototype dike

The same main dike parameters as in the case of preliminary model were used, while the parameters that are used only either in the preliminary or in the detailed model were so adjusted, that they represent the grass and clay of the same quality.
<table>
<thead>
<tr>
<th>INPUT PARAMETER</th>
<th>SYMBOL</th>
<th>PM</th>
<th>DM</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of waves</td>
<td>Waves</td>
<td>Irregular</td>
<td>Irregular</td>
<td>[m]</td>
</tr>
<tr>
<td>Significant wave height</td>
<td>$H_s$</td>
<td>1.4</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Peak period</td>
<td>$T_p$</td>
<td>4.7</td>
<td>4.7</td>
<td>[s]</td>
</tr>
<tr>
<td>Mean water level</td>
<td>mwL</td>
<td>6</td>
<td>6</td>
<td>[m]</td>
</tr>
<tr>
<td>Dike height</td>
<td>$H_D$</td>
<td>10</td>
<td>10</td>
<td>[m]</td>
</tr>
<tr>
<td>Dike crest</td>
<td>$B_D$</td>
<td>2</td>
<td>2</td>
<td>[m]</td>
</tr>
<tr>
<td>Outer slope</td>
<td>$m$</td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Inner slope</td>
<td>$n$</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Dike crest</td>
<td>$D$</td>
<td>10</td>
<td>10</td>
<td>[m]</td>
</tr>
<tr>
<td>Outer slope</td>
<td>$D_B$</td>
<td>2</td>
<td>2</td>
<td>[-]</td>
</tr>
<tr>
<td>Thickness of the clay layer</td>
<td>$d_c$</td>
<td>1</td>
<td>1</td>
<td>[m]</td>
</tr>
<tr>
<td>Polder area</td>
<td>$A_p$</td>
<td>1000000</td>
<td>1000000</td>
<td>[m^2]</td>
</tr>
<tr>
<td>Root volume ratio</td>
<td>$RVR$</td>
<td>-</td>
<td>2.67 - 0.75^{(d-2)}</td>
<td>[%]</td>
</tr>
<tr>
<td>Grass erosion coefficient</td>
<td>$E_{g,max}$</td>
<td>2 - 10^{-6}</td>
<td>-</td>
<td>[m^3s^{-1}]</td>
</tr>
<tr>
<td>Grass cover factor</td>
<td>$C_f$</td>
<td>0.75</td>
<td>-</td>
<td>[-]</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>$w$</td>
<td>2</td>
<td>-</td>
<td>[-]</td>
</tr>
<tr>
<td>Initial infiltration front</td>
<td>$z_{inf}$</td>
<td>-</td>
<td>0.2</td>
<td>[m]</td>
</tr>
<tr>
<td>Initial water content</td>
<td>$\theta_i$</td>
<td>-</td>
<td>0.2</td>
<td>[m^3/m^3]</td>
</tr>
<tr>
<td>Saturated water content</td>
<td>$\theta_s$</td>
<td>-</td>
<td>0.42</td>
<td>[m^3/m^3]</td>
</tr>
<tr>
<td>Clay erosion coefficient</td>
<td>$E_{c,max}$</td>
<td>8 - 10^{-5}</td>
<td>-</td>
<td>[m^3s^{-1}]</td>
</tr>
<tr>
<td>Clay content</td>
<td>$c$%</td>
<td>-</td>
<td>30</td>
<td>[%]</td>
</tr>
<tr>
<td>Mean depth of cracks</td>
<td>$d_{c,mean}$</td>
<td>-</td>
<td>0.4</td>
<td>[m]</td>
</tr>
<tr>
<td>Sediment size</td>
<td>$D_{50}$</td>
<td>0.2</td>
<td>0.2</td>
<td>[mm]</td>
</tr>
</tbody>
</table>

Table 11: Input parameters for the simulation of breaching

The main outcomes from both models and the relative differences are given in Table 12.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>PM</th>
<th>DM</th>
<th>ε</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grass erosion time</td>
<td>$t_g$</td>
<td>h</td>
<td>16.03</td>
<td>18.98</td>
<td>-18%</td>
</tr>
<tr>
<td>Clay erosion time - cracks</td>
<td>$t_{c,c}$</td>
<td>h</td>
<td>-</td>
<td>5.33</td>
<td>-</td>
</tr>
<tr>
<td>Clay erosion time - transition phase</td>
<td>$t_{c,tr}$</td>
<td>h</td>
<td>-</td>
<td>9.22</td>
<td>-</td>
</tr>
<tr>
<td>Total time of clay erosion</td>
<td>$t_c$</td>
<td>h</td>
<td>14.15</td>
<td>14.55</td>
<td>-3%</td>
</tr>
<tr>
<td>Cover failure time</td>
<td>$t_{c,f}$</td>
<td>h</td>
<td>30.18</td>
<td>33.53</td>
<td>-11%</td>
</tr>
<tr>
<td>Time of core erosion and wash-out</td>
<td>$t_s$</td>
<td>h</td>
<td>5.57</td>
<td>13.20</td>
<td>-42%</td>
</tr>
<tr>
<td>Total time of breaching</td>
<td>$t_t$</td>
<td>h</td>
<td>35.75</td>
<td>46.73</td>
<td>-31%</td>
</tr>
<tr>
<td>Peak outflow discharge</td>
<td>$Q_p$</td>
<td>m^3/s</td>
<td>1478</td>
<td>2209.1</td>
<td>-49%</td>
</tr>
<tr>
<td>Final breach width</td>
<td>$B_{b,max}$</td>
<td>m</td>
<td>85.39</td>
<td>102.19</td>
<td>-20%</td>
</tr>
</tbody>
</table>

Difference between PM and DM: $\varepsilon = 100 \cdot (PM - DM)/PM$

Table 12: Main outputs of the simulation and comparison with the preliminary model
The progress of the grass and clay erosion calculated by the preliminary and detailed model and the indications on the most important points of the erosion course are given in Figure 36.

\[ \text{Figure 36: Progress of the cover erosion - comparison of the preliminary and detailed model} \]

The comparative analysis of the obtained results indicates the following aspects related to the erosion of grass and clay:

- **time of grass erosion** \((t_g)\) calculated by the detailed model is 18% higher than in the preliminary model, due to the completely different method to simulate the erosion process. In the preliminary model the modified equations for the estimation of the residual strength of grass cover were applied. In those formulae, possible weaker points were accounted for in the safety coefficients, resulting in the faster erosion. The detailed model is based on the results of the laboratory experiments on the reinforcement grass properties, and the variations in the grass properties are already directly included as the material properties and taken into account during the simulation. Due to the higher concentration of the roots directly under the surface of the slope, almost no damage is observed in the first 10 hours, which relatively good corresponds to the observations made by Smith et al (1994) during the large-scale tests on the grass erosion made in a wave flume;

- **time of clay erosion** \((t_{cc})\) calculated by the detailed model is almost the same (3% higher) as in the case of the preliminary model. Similarly to the grass erosion, also here an additional safety coefficient was included in the preliminary model. However, no influence of cracks was taken into account in the preliminary model. In the detailed model the cracks in the clay layer that result in faster erosion were included, but on the other hand the safety coefficients are not included anymore. The clay erosion phase is divided into two parts, with two different courses of erosion. During the first part, the rapid progress of erosion after the shear failure occurs that can be seen in Fig. 36 clearly dominates, confirming the observations made by Führbörter (1966) on the crucial importance of cracks. Nevertheless, the distribution of the cracks on the slope, which is purely random, can strongly influence this part resulting either in shorter or longer time of erosion, depending on the number, location and depth of the cracks. In the second part if this phase, the clay erosion decelerates as the sand core erosion and consequently the clay undermining occurs. Till the end of this part the clay layer is stable and the collapse of the undermined clay cover is considered to be the end of the clay erosion phase;

- **warning time** \((t_w)\) which is equal the cover failure time \((t_{ct})\) is slightly (11%) higher for the detailed model than for the preliminary one, which is a logical consequence of the higher grass and clay erosion times.

The **time of front-face core erosion** \((t_s)\) is significantly (42%) higher for the detailed model. The main reason of this difference is the protective function of the remaining parts of the clay cover that is accounted for in the detailed model. In the preliminary model the clay cover was assumed to fail instantaneously as soon as at least one point of the clay cover failed. This assumption was however a strong simplification and it was removed in the detailed model, as during the laboratory experiments performed at the LWI was found not to be valid.
The total breaching time ($t_t$) is 31% higher than in the case of the preliminary model, and the highest time increase was donated by the core erosion time. The increase of the total breaching time results from the removal of the simplifying assumptions made in the preliminary model. Those simplifications, especially assumptions on instantaneous failure of grass and clay cover were always on the safe side, decreasing the time of breaching.

The final breach shape (Figure 37) given by the detailed model is similar to the one obtained from the preliminary model. The largest differences can be noted on the landside, where the breach calculated by the detailed model is wider. This difference is due to the fact, that in the detailed model also the simulation of the inner slope erosion resulting from wave overtopping is included. As a consequence, different initial conditions for the simulation of the overflow are provided. Those initial conditions are also the source of faster increase of the overflow head and resulting from this also higher peak outflow discharge.

![Figure 37: Final breach shape - comparison of the preliminary and detailed model](image)

4.5.2 Simulation of a dike damage

The data on damage of a dike (Fig. 38) caused by impact forces during storm surges was used to test the prediction capability of the detailed model. The obtained results are compared with the predictions given by the preliminary model and with the measured dimensions of the scour hole. Unfortunately, very limited information on this damage is available, and it is only possible to compare the dimensions of the scour hole. In this case the results given by the detailed model are almost identical to those given by the preliminary one and to the measured values. No information on the time needed for this damage to occur is available and therefore there is no point of reference to compare the simulation results with the reality. However, the time of erosion given by the detailed model is 33% higher than the time given by the preliminary model. The source of this difference is the same as in the case of dike breach simulation given in Section 4.1.1.

![Figure 38: Damage of the dike (after Forschungs- und Vorarbeitenstelle Neuwerk) and comparison with the results from preliminary and detailed model](image)
### 4.5.3 Simulation of a dike failure

The available information on the dike breach in Ülversbüller Koog (Wohlenberg, 1963) was compared with the results provided by the preliminary and detailed model (Table 14).

#### Table 13: Main outputs of the simulation and comparison with the preliminary model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured</th>
<th>Preliminary model</th>
<th>Detailed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of the hole</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Depth of the hole</td>
<td>2.0</td>
<td>2.11</td>
<td>2.25</td>
</tr>
<tr>
<td>Storm surge duration</td>
<td>unknown</td>
<td>13.52</td>
<td>18.06</td>
</tr>
</tbody>
</table>

#### Table 14: Main outputs of the simulation and comparison with the preliminary model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Observed</th>
<th>Preliminary model</th>
<th>Detailed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total time of breaching [h]</td>
<td>≈23</td>
<td>27.10</td>
<td>31.64</td>
</tr>
<tr>
<td>Core wash-out time [h]</td>
<td>≈2</td>
<td>3.20</td>
<td>3.85</td>
</tr>
<tr>
<td>Final breach width (min) [m]</td>
<td>≈35</td>
<td>32.8</td>
<td>39.18</td>
</tr>
<tr>
<td>Final breach width (max) [m]</td>
<td>≈80</td>
<td>87.5</td>
<td>101.46</td>
</tr>
</tbody>
</table>

Figure 39: Dike breach in the Ülversbüller Koog (after Landesamt für Wasserwirtschaft - Schleswig-Holstein, 1962)

In the investigated case, the preliminary model gives better results, while the detailed model overestimates all the parameters. Nevertheless, the results given by both models are only indicative, due to the lack of information on the parameters which are essential for the proper breaching simulation. The grass cover quality, clay layer thickness and the mean grain size $D_{50}$ were arbitrary assumed so that the moderate erosion resistance is accounted for.

Generally, the detailed information on the historical dike breaching initiated from the seaside which would allow for a better model validation is lacking. The description of breaching time, progress of erosion as well as the breach widening and deepening rate are usually reported by accidental eye witnesses and therefore rather estimated than measured. Although some pictures of the full breach are available, the shape and final breach dimensions are also roughly estimated as the detailed measurements are lacking.
5. Parameter and model uncertainties

The uncertainty of the results obtained from the model is influenced by two types of uncertainties which are due to the input parameters and model parameters. The uncertainties of the input parameters may be the result of (i) the natural variations in the sea state, soil properties and (ii) the imperfections during the dike construction. The uncertainties in the model parameters may result from (i) the limitations of the empirical formulation, which are always based on a limited number of experimental tests, and (ii) the simplifications made in the numerical simulation of the processes involved. This chapter presents briefly the uncertainties of the detailed model which were identified.

5.1 Hydrodynamic module

In comparison to the preliminary model, which is fully based on the empirical formulae, the simulation of the impact pressures and flow during the breach initiation in the detailed model is based on the numerical model that solves the fundamental equations of the fluid dynamics and is therefore considered to be more reliable. However, a number of uncertainties related to the input parameters can still be identified, as the simulation of infiltration requires a number of input parameters which are case-specific and subject to natural variations of the soil and also to the possible weathering.

5.1.1 Input parameters uncertainties

The following types of input parameters are required by the hydrodynamic module:

- Sea parameters - wave height ($H_s$ or $H$), wave period ($T_p$ or $T$) and mean water level (MWL). Those parameters are the most important and also most uncertain parameters. Only direct monitoring of the nearshore hydrodynamics could reduce them;
- Dike geometry, including dike height ($H_d$), crest width ($B_d$), outer ($m$) and inner ($n$) slopes. In their case only small uncertainties may occur, as the process of dike construction is controlled. Differences due to settling can occur, but they are of negligible importance, as focus is on the forces on the outer slope. The polder area behind the dike is also an input parameter for the overflow calculation, but it is strongly case-specific and the related uncertainties will be not quantified;
- soil parameters like saturated water conductivity and initial infiltration front as well as the clay and grass layer thicknesses are needed for the calculation of water infiltration into the dike. Those parameters are very case-specific and subject to significant uncertainties;
- the occurrence of cracks, their number, depth and location on the slope strongly affect the infiltration and have to be taken into account as a source of significant uncertainties.

Table 15 provides more information on the identified uncertainties and their range.

<table>
<thead>
<tr>
<th>INPUT PARAMETER</th>
<th>SYMBOL AND UNIT</th>
<th>MEAN</th>
<th>$\sigma$</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Significant wave height</td>
<td>$H_s [m]$</td>
<td>2.50</td>
<td>0.325</td>
<td>0.13</td>
</tr>
<tr>
<td>Peak period</td>
<td>$T_p [s]$</td>
<td>8.00</td>
<td>1.6</td>
<td>0.20</td>
</tr>
<tr>
<td>Mean water level</td>
<td>$MWL [m]$</td>
<td>7.00</td>
<td>1.05</td>
<td>0.15</td>
</tr>
<tr>
<td>Dike height</td>
<td>$H_d [m]$</td>
<td>10.00</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>Crest width</td>
<td>$B_d [m]$</td>
<td>2.00</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>Outer slope</td>
<td>$m [-]$</td>
<td>6.00</td>
<td>0.30</td>
<td>0.05</td>
</tr>
<tr>
<td>Inner slope</td>
<td>$n [-]$</td>
<td>3.00</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>Grass cover thickness</td>
<td>$d_g [m]$</td>
<td>0.08</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td>Initial saturated water front</td>
<td>$z_{swf} [m]$</td>
<td>0.25</td>
<td>0.005</td>
<td>0.20</td>
</tr>
</tbody>
</table>
### 5.1.2 Model uncertainties

The detailed hydrodynamic module consists of a number of models, that are subject to different uncertainties. The applied models can be briefly summarised as follows:

1. RANS-VOF model for the calculation of the impact pressures, flow velocities and layer thickness on the outer slope
2. simplified infiltration model
3. wave overtopping model
4. overflow model

Impact pressures, flow velocities and layer thickness on the outer slope are calculated using the model that solves the fundamental equations of the flow dynamics, so that only small (\(\text{CoV}<0.05\) for the calculations of free surface flow) uncertainties of the model outputs may occur. However, as the parameters are different for each wave, sufficiently long simulations time are needed to obtain results that are suitable for the application in the dike breaching model.

The empirical coefficients (as \(\alpha\) in Eq. 2.53 or \(\beta\) in Eq. 2.54) incorporated in the simplified model for the calculation of the infiltration front are a source of uncertainties (\(\text{CoV}=0.10\)) and it is suggested to implement more process-oriented model for the calculation of water infiltration.

The wave overtopping and wave overflow models include coefficients calibrated on laboratory tests, such as the friction coefficient \(f\) (Eq. 2.34) with the \(\text{CoV}=0.10\), or the discharge coefficients \(\mu_{\text{comb}}\) (Eq. 2.42) and \(\mu_{\text{over}}\) (Eq. 2.44), both with \(\text{CoV}=0.20\).

### 5.2 Morphodynamic module

All models implemented in the detailed morphodynamic module are empirical models and therefore a subject to large uncertainties. The highest coefficients of variation are observed in the case of parameters describing the properties of the grass, including root percentage in the soil and coefficients describing the reinforcement properties of those roots. The following subsections describe briefly the key parameters for all simulation phases, while Table 16 summarizes the information on the most important uncertainties and their range.

#### 5.2.1 Grass erosion module

**Input parameters uncertainties**

The parameters describing the root volume ratio (RVR) in Eq. 3.1 are the most important and the most uncertain input parameter in the grass erosion simulation. This parameter depends on the grass type, soil type, maintenance and even season of the year. Therefore, it is strongly recommended to perform in-situ tests for the specific case. The same concerns also the grass cover factor \(Cf\) (Eq. 3.4) and total Manning roughness (Eq. 3.5).

**Model uncertainties**

The model coefficients in Eq.3.2 have been calibrated based only on a limited number of tests, therefore they are considered as very uncertain and should be used with caution. The total Manning roughness (Eq. 3.5) is also strongly dependent on the data set used for the validation.
5.2.2 Clay erosion module

Parameters uncertainties

The input parameters for the clay erosion simulation due to impact pressures are calculated based on the definition of three clay types: weak, moderate and strong. Those definitions cover however a wide range of soil types and are therefore subject to large uncertainties. It is recommended to perform in-situ tests for the specific cases instead of assuming the soil type. Some of the parameters (k_d in Eq. 3.7, for instance) are a function of water content, that depends on the assumed initial water content and applied infiltration model, which are both subject to large uncertainties.

Model uncertainties

The model coefficients in Eq. 3.6 have been calibrated using quite large sets of experimental tests covering all three mentioned types of soil, but are still subject to quite large uncertainties. The formula used for the calculation of the total Manning roughness (Eq. 3.5) is also strongly dependant on the data set used for the validation.

5.2.3 Core erosion module

Parameter uncertainties

The leading parameters of the core erosion process (D_{50} and n) should have controlled values of the construction materials and are subject to relatively well known and small uncertainties. However, due to their significant influence on the erosion process they should be also included in the analysis.

Model uncertainties

The semi-empirical model for the simulation of the beach profile formation was calibrated using a large number of data sets, but its uncertainties are relatively large (CoV=0.28). The ratio between vertical and lateral erosion as well as the initial breach channel width are derived based on a limited number of field observations or laboratory experiments. They should be therefore applied with caution.

<table>
<thead>
<tr>
<th>INPUT PARAMETER</th>
<th>SYMBOL AND UNIT</th>
<th>MEAN</th>
<th>σ</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean root volume ratio</td>
<td>RVR[-]</td>
<td>0.55</td>
<td>0.41</td>
<td>0.74</td>
</tr>
<tr>
<td>Root reinforcement coefficient</td>
<td>b[-]</td>
<td>5.00</td>
<td>1.20</td>
<td>0.24</td>
</tr>
<tr>
<td>Grass cover factor</td>
<td>C_f[-]</td>
<td>0.75</td>
<td>0.1</td>
<td>0.13</td>
</tr>
<tr>
<td>Critical grass erosion depth</td>
<td>d_{crit}[m]</td>
<td>0.08</td>
<td>0.02</td>
<td>0.25</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>w[-]</td>
<td>2.5</td>
<td>0.5</td>
<td>0.20</td>
</tr>
<tr>
<td>Saturated water content</td>
<td>θ[m^3/m^3]</td>
<td>0.42</td>
<td>0.06</td>
<td>0.15</td>
</tr>
<tr>
<td>Clay percentage</td>
<td>c[%]</td>
<td>30.00</td>
<td>7.50</td>
<td>0.25</td>
</tr>
<tr>
<td>Internal friction angle</td>
<td>φ^0[°]</td>
<td>32.00</td>
<td>3.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Sediment size</td>
<td>D_{50}[mm]</td>
<td>0.20</td>
<td>0.02</td>
<td>0.1</td>
</tr>
<tr>
<td>Internal friction angle</td>
<td>φ^0[°]</td>
<td>32.00</td>
<td>3.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Soil porosity</td>
<td>n[-]</td>
<td>0.40</td>
<td>0.112</td>
<td>0.28</td>
</tr>
<tr>
<td>Initial breach channel width</td>
<td>B_{tm}[n \cdot r_n]</td>
<td>2.00</td>
<td>0.5</td>
<td>0.25</td>
</tr>
<tr>
<td>SBeach coefficient</td>
<td>K[m^4/N]</td>
<td>1.4 \times 10^4</td>
<td>0.4 \times 10^4</td>
<td>0.28</td>
</tr>
</tbody>
</table>

*Table 16: Uncertainties related to the input parameters for the morphodynamic module (partially after D'Eliso, 2007 and Kortenhaus, 2003)*
6. Sensitivity analysis

The variability of the input parameters influences the results given by the model. In order to investigate this influence, a two level approach is applied for the sensitivity analysis:

- Level I analysis - only one parameter varies, while the other are constant. The tests are performed five times, and if it is necessary, i.e. the outputs of variations are significant, the number of tests is increased

- Level II analysis - a set of parameters varies, according to their assumed distribution.

6.1 Level I analysis

During the level I analysis the influence of the variations in a single parameter on the variations in the model outputs is investigated. The parameters that were identified either to be subject to the largest uncertainties (see Chapter 5) or as the leading parameters together with the related phases and investigated range of variation are listed in Table 17. For each parameter given in Table 17 five tests are performed. The considered parameter varies in the given range, while all the other parameters are kept constant.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Unit</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root Volume Ratio</td>
<td>A</td>
<td>[-]</td>
<td>2.65</td>
<td>0.265</td>
<td>GE</td>
</tr>
<tr>
<td>Grass cover factor</td>
<td>Cf</td>
<td>[-]</td>
<td>0.75</td>
<td>0.1</td>
<td>GE</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>w</td>
<td>[-]</td>
<td>2.5</td>
<td>0.5</td>
<td>GE</td>
</tr>
<tr>
<td>Initial infiltration front</td>
<td>z_{inf}</td>
<td>[m]</td>
<td>0.25</td>
<td>0.05</td>
<td>GE</td>
</tr>
<tr>
<td>Initial water content</td>
<td>\theta</td>
<td>[m^3/m^3]</td>
<td>0.2</td>
<td>0.05</td>
<td>GE,CE</td>
</tr>
<tr>
<td>Saturated water content</td>
<td>\theta_s</td>
<td>[m^3/m^3]</td>
<td>0.42</td>
<td>0.02</td>
<td>GE,CE</td>
</tr>
<tr>
<td>Clay percentage</td>
<td>c%</td>
<td>[-]</td>
<td>30</td>
<td>7.5</td>
<td>GE,CE</td>
</tr>
<tr>
<td>Minimal depth of cracks</td>
<td>d_{c,min}</td>
<td>[m]</td>
<td>0.15</td>
<td>0.05</td>
<td>CE</td>
</tr>
<tr>
<td>Maximal depth of cracks</td>
<td>d_{c,max}</td>
<td>[m]</td>
<td>0.4</td>
<td>0.1</td>
<td>CE</td>
</tr>
<tr>
<td>Sediment size</td>
<td>D_{50}</td>
<td>[mm]</td>
<td>0.2</td>
<td>0.02</td>
<td>SE</td>
</tr>
<tr>
<td>Sand density</td>
<td>\rho_s</td>
<td>[kg/m^3]</td>
<td>1800</td>
<td>60</td>
<td>SE</td>
</tr>
<tr>
<td>Internal friction angle</td>
<td>\phi_s</td>
<td>[°]</td>
<td>30</td>
<td>2</td>
<td>SE</td>
</tr>
</tbody>
</table>

GE - grass erosion, CE - clay erosion, SE - sand erosion

Table 17: Input parameters for the Level I analysis

6.1.1 Grass erosion time

Time of grass erosion is influenced by the parameters of clay and protective properties of the grass cover. Figures 40 - 43 show the effect of the key parameters on the grass erosion time.

The root volume ratio RVR and the damping coefficient \( w \) are the leading parameters that describe the grass erosion due to impact pressures. Already a small increase in root network density can significantly increase the time of grass erosion (Fig. 40) and a similar increase can be also observed during the analysis of the influence of the changes in the damping coefficient \( w \) on the model outputs (Fig.41).
The grass cover factor \( C_f \) and the clay content in the subsoil are identified as the key parameters describing the erosion due to the wave run-up and run-down flow. The influence of the \( C_f \) on the grass erosion time is given in Fig. 42. The grass erosion time increases together with the increase of the grass cover factor \( C_f \), but the course of this increase is rather linear, especially when compared to the exponential course of the RVR influence given in Fig. 40, which confirms that the erosion due to the impact pressures dominates the grass erosion phase. The influence of the clay contents on the subsoil seems to be negligibly small (Fig. 43).
During the performed analysis the parameters describing water conditions in the soil and therefore influencing the erosion properties of the subsoil were also taken under consideration. The effect of the initial infiltration front $z_{inf}$ (Fig. 44) and the initial volumetric water content $\theta_i$ (Fig. 45) is relatively small. The main reason of the negligible importance of those parameters is the high infiltration rate of the vegetated cover. The water content in the grass cover rapidly increases, and reaches in very short time the saturated state resulting in a very high effect of the saturated water content $\theta_s$ (Fig.46).
6.1.2 Time of clay erosion

The following parameters are identified as the leading ones during the clay erosion:

- clay percentage in the soil ($c\%$);
- initial infiltration front ($z_{inf}$);
- initial ($\theta_i$) and saturated ($\theta_s$) water content;
- minimal and maximal depth of the cracks in the clay layer.

The tests were performed five times for each of the listed parameters. During all tests, one parameter varies in the given range, while all others take their mean value and are kept constant. The results of the simulations are presented in Figures 47 - 52. The influence of clay percentage in the soil and the initial infiltration front on the erosion time (Figure 47 and Figure 48, respectively) is relatively small. The time of clay erosion is however very sensitive to the changes in both the initial (Fig.49) and saturated (Fig.50) volumetric water content. This results from the fact, that the erodibility coefficient of the clay depends nonlinearly on the changes on the water content in the soil (Eq. (65)).
The variation in the depth of the cracks, especially the minimal crack depth (Fig. 51) influences significantly the clay cover failure time. Two mechanisms lead to this effect: (i) local increase of permeability at the locations of cracks, therefore faster increase of the water content and consequently decrease of the erosion resistance and (ii) rapid local erosion, when the shear failure in water-filled cracks occurs.
6.1.3 Core erosion and wash-out time

The key parameter during the erosion and wash-out of the sand core is the median sediment size $D_{50}$. The influence of this parameter on the peak outflow discharge and time of core failure was investigated.

Figure 53: Influence of the sediment size on the core failure time
Figure 54: Influence of the sediment size on the peak outflow discharge

The performed tests however show that the influence of the sediment size on both the core failure time (Fig.53) and peak outflow discharge (Fig.54) is relatively small. The higher sediment size results in both longer time of core erosion and smaller peak outflow discharge.

6.1.4 Summary of the level I analysis

In the level I analysis the effect of the variations in single parameters on the range of model outputs was investigated. During the grass erosion phase the root volume ratio RVR, the damping coefficient w and the saturated volumetric water content $\theta_s$ were found to have the largest effect on the model outputs. Simultaneously, the same parameters are subject to largest uncertainties, which results in high variations in the prediction of the warning time, when the model is applied practically. The influence of the initial infiltration front $z_{inf}$ and the clay percentage in the soil c% is relatively small during the grass erosion and clay erosion phases. The effect of the initial volumetric water content $\theta_i$ on the grass erosion time is negligibly small, but it becomes of the crucial importance when the clay erosion time is considered. The permeability of the clay layer is at least one order of magnitude smaller than the permeability of the grass, so that the initial water content becomes more important than the saturated one. The presence and especially minimal depth of cracks in the clay layer strongly increase the soil permeability which leads to the increase of the water content and decrease of the clay erodibility coefficient and consequently the clay erosion time. The outputs from the core erosion module are rather constant, the influence of the median sediment size is relatively small.

6.2 Reliability analysis

The main objective of this section is to come up with the uncertainties of the model outputs described in terms of the Coefficient of Variation (CoV). In order to achieve this goal, the random sampling technique (Monte Carlo method) is applied. In this commonly used method (i) the most uncertain input parameters together with the respective probability density functions are identified, (ii) the number of realisations ($N$) is selected and (iii) the model is run $N$ times, each time with the input parameters randomly extracted according to their probability density functions. The results given by each realisation are stored and after all realisations are done the histograms are plotted. For the simulation of dike breaching the following key outputs are selected:

- time of grass erosion $t_g$;
- time of clay erosion $t_c$;
- total time of breaching $t_b$;
- final breach width $B_f$;
- peak outflow discharge $Q_p$;
The simulation is performed for both preliminary and detailed model. The uncertain parameters together with their mean values and standard deviations are given in Table 17. All the parameters are assumed to be normally distributed. For each model N=10000 realisations are performed.

6.2.1 Results of the analysis

Although the input parameters are assumed normally distributed, due to the nonlinearity of equations implemented in the model, the probability density functions of all outcomes are asymmetric with a wider right tail. Generally, the detailed model provides longer times of grass erosion and total breaching times. It should be noted, that the times of clay failure given by the preliminary and detailed model cannot be compared directly, as the models are differently sub-divided, with two additional transition phases included in the detailed model.

![Figure 55: Monte Carlo simulation results for the grass failure time](image)

The mean **time of grass erosion** (Fig. 55) - is longer in the detailed model (\( \mu = 28.57h \)) when compared to the results given by the preliminary model (\( \mu = 13.61h \)). This difference occurred most probably due to a new model for the calculation of the grass root reinforcement and grass erosion resistance that was applied in the detailed model. The coefficients of variation are however very similar (\( \sigma' = 0.74 \) for the preliminary model and \( \sigma' = 0.72 \) for the detailed one) indicating similar relative level of uncertainties. The uncertainties of the outputs given by the detailed model however are related rather to the input parameters than to the model parameters and the detailed model itself is considered to be more reliable.

![Figure 56: Monte Carlo simulation results for the clay failure time](image)
The mean values of the clay erosion time (Fig. 56) - obtained from the preliminary and the detailed model cannot be directly compared, as a different phase subdivision is used. In the detailed model the transition phases between (i) grass and clay and (ii) clay and sand erosion are included. However, even assuming that the transition phases between clay and sand erosion are included in the clay failure time, in the detailed model ($\mu = 4.12h$) it is still significantly shorter than in the preliminary model ($\mu = 11.06h$). The most possible reason of this phenomenon are the cracks in the clay layer. In the preliminary model they were fully neglected, while their presence and the calculation of possible shear failure due to impact pressures are implemented in the detailed model. In fact, the dominant role of the cracks on the clay failure was already observed during the performed laboratory tests. Moreover, the prediction of the clay cover erosion time given by the detailed model is subject to relatively smaller uncertainties ($\sigma’ = 0.37$ compared with $\sigma’ = 0.81$ for the preliminary model).

The mean total breaching times (Fig.57) obtained from the preliminary and detailed model are comparable ($\mu = 36.54h$ for the preliminary model and $\mu = 38.68h$ for the detailed model). The standard deviation and consequently the coefficient of variation that are observed in the case of the detailed model ($\sigma’ = 0.49$) indicate that it is subject to slightly smaller uncertainties, compared with $\sigma’ = 0.59$ for the preliminary model. However, the levels of the uncertainties indicated by both models are similar, which suggest that rather the variations in the input parameters, than the model formulation have the most effect on the overall model performance.

![Figure 57: Monte Carlo simulation results for the time of the full breaching](image)

A significant improvement of the prediction of the two last investigated output parameters, i.e. final breach width (Fig. 58) and peak outflow discharge (Fig. 59) achieved in the detailed model can be observed. The coefficient of variation for the final breach width is reduced from $\sigma’ = 0.26$ obtained from the preliminary model to $\sigma’ = 0.05$ given by the detailed model. The same is observed in the case of the peak outflow discharge where the coefficient of variation is reduced $\sigma’ = 0.29$ to $\sigma’ = 0.05$. The median sand particles diameter, that was identified as the key parameter during the core wash-out and breach widening and deepening for both preliminary and detailed model is subject to relatively small uncertainties. During the level I analysis the influence of the changes in the sediment size was also found to be rather small (c.f. Figures 53 and 54). Furthermore, the same set of equations is used in both preliminary and detailed model, so that the most probable source of this improvement is better and more reliable prediction of the boundary conditions for the overflow simulation. In the preliminary model they were assumed (with a given range of variation), while in the detailed model they are directly calculated, including the changes of the inner slope profile due to the erosion which results from wave overtopping.
Summary

Although all input parameters are assumed normally distributed, the model outputs are strongly asymmetric, with longer right tail, due to nonlinearity of the governing equations. The following can be stated after the analysis of the results given by the preliminary and detailed model (Table 18):

- although the predicted of grass erosion is significantly shorter in the preliminary model, the uncertainties related to the grass erosion time are in fact the same for both preliminary and detailed model and occur rather due to the input uncertainties than due to the model uncertainties;
- large improvement of the CoV related to the clay erosion time was achieved in the detailed model. The transition phase between clay and sand erosion is driven not only by relatively uncertain clay properties, but also by quite certain properties of the sand, which results in the overall reduction of the input parameter uncertainties for this phase;
- huge improvement in the prediction of the core failure time was achieved, pointing out the importance of the boundary conditions for the core erosion phase which are directly calculated in the detailed model. Similar effect of the boundary conditions on the CoV improvement is also observed in the case of peak outflow discharge and final breach width.
Table 18: Main outcomes from the Monte Carlo simulation

The model outcomes given by both preliminary and detailed model are affected mostly by the uncertainties related to the material properties therefore a significant reduction of the uncertainties related to the dike breaching model can be achieved only if specific information on the material properties is available.

7. Summary, conclusions and implications

The detailed computational model reported in this study represents the second and most important part of the tiered modeling approach for the simulation of sea dike breaching induced from the seaside by breaking wave impact. The application of the preliminary model revealed its most important limitations (Stanczak et al, 2007g). In order to remove at least some of those limitations a number of laboratory experiments on the grass and clay erosion were performed (Stanczak et al, 2007f). Finally, the derived models were selected for the implementation in the detailed dike breaching model. In the detailed model, a number of simplifications made in the preliminary model are removed and the processes that were identified during the laboratory tests are also included. Moreover, the focus of the detailed model is on the detailed description of the breach initiation phase, as it was found to be of the crucial importance regarding the prediction of the warning time. The Monte Carlo simulations show encouraging results concerning the prediction of the breaching times and outflow hydrograph.

7.1 Improvements in comparison to the preliminary model

Compared to the preliminary model, the improvements made in the detailed model concern both the hydrodynamic module and morphodynamic module. The most important improvements are summarised in Table 19).

7.1.1 Improvements in the hydrodynamic module

The most important improvement made in comparison to the preliminary model is the application of a solver of the Reynolds Averaged Navier-Stokes Equations based on the Volume-of-Fluid method (RANS-VOF model COBRAS) instead of a set of empirical formulae thus reducing the uncertainties related to the prediction of the loading on the slope. The impact forces on the slope, the shape and location of the impact area, the velocity field induced by wave run-up and run-down as well as the layer thickness on the slope during the grass and clay erosion phase as well as during the transition phase between clay and sand erosion are therefore calculated by solving the fundamental equations of fluid dynamic and not with an empirical formulae.

In the detailed model during the front-face sand core erosion the numerical model SBeach is implemented for the calculation of the wave height, energy dissipation related to the wave breaking and consequently the associated dike profile change according to the beach profile formation. The formation of a beach profile during the dike core erosion was identified as the leading process during the tests in a wave flume performed at the LWI. Beginning from the phase of front-face core erosion the flow parameters are calculated along the entire dike profile, not only on the outer slope. This results from the removal of one of the most important simplifications made in the preliminary model.
which stated, that during the entire simulation the condition for wave overtopping and resulting possible erosion of the inner slope is not controlled.

In the detailed model two simplified formulae for the estimation of saturated and infiltration front are included, as the calculation of the clay cohesion $c$ and erodibility coefficient $k_d, p$ are based on the water content in the soil according to the results of the laboratory experiments. Although the applied infiltration models are strongly simplified, they represent an improvement in comparison to the preliminary model, where no infiltration was calculated at all and the soil properties were assumed to remain constant during the entire simulation.

### 7.1.2 Improvements of the morphodynamic module

The focus of the improvements made in the detailed model is on the description of the breach initiation process and detailed simulation of the grass and clay erosion. The preliminary grass erosion module was based on a simple linear function of wave height and an empirical grass erosion coefficient, which included the erosion due to impact pressures and flow of wave run-up and run-down. In order to simulate the grass erosion progress in the detailed model (i) first the erosion properties of the soil itself are calculated as a function of soil type and water content, then (ii) the root reinforcement model is applied and the erosion resistance of the entire cover is obtained. The erosion progress is then calculated separately for (i) the impact pressure as a function of the maximal impact pressure and of the damping layer thickness, and (ii) for the flow of wave run-up and run-down as a function of effective shear stress.

The clay erosion module of the preliminary model was based on a simple formula for the residual strength of a clay layer and the soil properties were arbitrarily assumed. In the detailed model, the soil parameters are calculated based on the results of laboratory tests as a function of the soil type and calculated water content. The erosion is then calculated exactly like in the grass erosion module as a function of impact pressure, damping effectiveness of the water layer and of the flow velocity.

In comparison to the preliminary model, where the soil parameters were assumed constant and equal for the entire cover depth, the detailed model calculates the soil properties for each node of the dike using the volumetric water content. Weaker spots in the clay layer and a number of pull-cracks in the cover are also considered. Moreover, in the detailed model, the formulae for the shear failure in water-filled cracks subject to impact pressure that were derived from the laboratory experiments are applied. Contrarily to the preliminary model, where the whole revetment is assumed to fail instantaneously as soon as one point of sand core is reached by the progress of erosion, in the detailed model, the remaining part of the clay cover still protects the core, and the conditions of undermining and stability are controlled.

The shape of the scour hole given by the preliminary model was assumed to be horizontal with a vertical cliff, and the progress of erosion was calculated as a function of the impact force. The detailed model calculates the dike profile evolution as a function of the local energy dissipation rate, providing more reliable results; the sand wash-out module during the overflow is essentially the same for both models.

During the front-face core erosion, the detailed model controls the wave overtopping condition and if overtopping occurs calculates also the wave overtopping flow on the inner slope and associated erosion due to shear stress, which provides better estimation of the initial conditions for the overflow calculation.

In Table. 19 the most important differences between preliminary and detailed model are listed.
<table>
<thead>
<tr>
<th>Module</th>
<th>Parameter</th>
<th>Preliminary model</th>
<th>Detailed model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hydrodynamic</strong></td>
<td>Loading on the outer slope</td>
<td>Calculated applying simple, empirical formulae</td>
<td>Calculated solving fundamental equations of fluid hydraulics (RANS-VOF method)</td>
</tr>
<tr>
<td></td>
<td>Flow parameters on the inner slope due to overtopping and combined overtopping and overflow</td>
<td>Neglected, no flow is assumed on the inner slope during first three phases</td>
<td>Empirical formulae (D’Eliso, 2007)</td>
</tr>
<tr>
<td></td>
<td>Water infiltration into the dike</td>
<td>Neglected, water content assumed to be constant</td>
<td>Simplified models for water infiltration applied (Wang, 2000)</td>
</tr>
<tr>
<td></td>
<td>Overflow</td>
<td>Broad-crest weir formula</td>
<td>Broad-crest weir formula</td>
</tr>
<tr>
<td><strong>Morphodynamic</strong></td>
<td>Grass erosion</td>
<td>Empirical formulae, only basic information given. Erosion due to general wave action</td>
<td>Empirical formulae, important parameters such as influence of root network and soil parameter are included. Erosion due to impact pressures and shear stress of wave run-up and run-down is calculated</td>
</tr>
<tr>
<td></td>
<td>Clay erosion</td>
<td>Empirical formulae, only basic information given. Erosion due to general wave action</td>
<td>Empirical formulae, influence of water content on the erosion coefficient and other important soil parameter are included. Erosion due to impact pressures and shear stress of wave run-up and run-down is calculated</td>
</tr>
<tr>
<td></td>
<td>Clay cover stability</td>
<td>Neglected, after sand core is reached at one point, the whole clay revetment is removed</td>
<td>After the erosion reached the sand core, the remaining part of the revetment still protects the core. The undermining of clay cover and its stability are simulated</td>
</tr>
<tr>
<td></td>
<td>Cracks in clay and weaker points in revetment</td>
<td>Neglected</td>
<td>The shear failure in cracks is calculated, the results of experimental tests are applied. Randomly distributed weaker spots in the grass and clay are assumed and incorporated into the calculations</td>
</tr>
<tr>
<td></td>
<td>Sand erosion - front face</td>
<td>Analytical model for sand dunes adopted, shape of the breach assumed based on the observations</td>
<td>Analytical model for sand dunes adopted, the shape of the scour hole is calculated applying beach profile model (Sbeach)</td>
</tr>
<tr>
<td></td>
<td>Core wash-out</td>
<td>Selected sediment transport models are implemented in the model and can be freely chosen by the user</td>
<td>Selected sediment transport models are implemented in the model and can be freely chosen by the user</td>
</tr>
</tbody>
</table>
7.2 Implications for the future work

The detailed breaching model represents the second, more sophisticated step of the tiered modeling approach. Although a significant number of limitations and simplifications of the preliminary model was removed, some issues still remain unsolved. The most important problems can be summarised as follows:

- the numerical RANS-VOF model which is used is very expensive in terms of computational effort, therefore the surface of the slope is assumed to be smooth during the entire grass and clay erosion phase so that only one RANS-VOF simulation is needed;
- the calculation of water infiltration into the dike is simplified, the application of a reliable numerical model is recommended;
- the model that describes the increase of the soil erosion resistance resulting from the properties of the grass roots was derived using a limited number of laboratory experiments. Further experimental research, if possible with different types of subsoil, different grass maintenance methods and samples collected in autumn and winter is needed;
- the initial breach width which is one of the most essential input parameters for the transformation of the 2D model into a 3D one and consequently for the proper calculation of the breach widening is still subject to simplifications as no field measurements are available;
- the relationship between breach deepening and widening which is one of the crucial parameters allowing the transformation of the 2D problem into a 3D one is still not clear enough and therefore only a simplified method is implemented in the model;
- the applied sediment transport model needs to be verified in the breaching conditions

Although the developed model represents a huge step towards a process-oriented numerical dike breaching model, at the current state it is not yet possible to develop a reliable and feasible numerical model instead of the presented approach. The profound knowledge of all the involved processes is still missing, and although a large number of laboratory experiments were performed, there are still gaps in the understanding of the physical processes involved. Furthermore, the detailed data on the experienced dike breaching events is lacking and there is an urgent need to perform large-scale experimental investigations in a wave flume.
8. References


17. STANCZAK, G., OUMERACI, H., and KORTENHAUS, A. (2007f): Laboratory tests on the erosion of clay revetment of sea dike with and without a grass cover induced by breaking wave impact, LWI-Bericht Nr 935


