COMPLEXITY ANALYSIS IN THE DOUBLE TRANSFORMED RADON DOMAIN

MARIANNE VISSINGA¹ and JACOB T. FOKKEMA²

¹ Mobil Producing, Netherlands Inc.

² Delft University of Technology, Faculty of Mining and Petroleum Engineering,

PO Box 5028, 2600 GA Delft, The Netherlands.

Abstract

Let $D_{r,s}(x^r; x^s; \omega)$ represent a data set of a split-spread seismic experiment in the frequency domain, organized for different shot and receiver coordinates x^s and x^r , respectively. In our analysis we first map the data into the p^r - p^s domain by applying a double Radon transform with respect to the source and receiver coordinates according to

$$\tilde{D}_{r,s}(p^r; p^s; \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_{r,s}(x^r; x^s; \omega) e^{-i\omega(p^r x^r + p^s x^s)} dx^r dx^s. \quad (1)$$

On the other hand the data can be organized in another way by introducing the mid-point and half-offset coordinates x^m and x^o , given by

$$x^m = \frac{x^r + x^s}{2} \quad \text{and} \quad x^o = \frac{x^r - x^s}{2}. \quad (2)$$

Then the resulting data organization is denoted by $D_{o,m}$ and related to $D_{r,s}$ by

$$D_{o,m}(x^o; x^m; \omega) = D_{r,s}(x^m + x^o; x^m - x^o; \omega), \quad (3)$$

where we have used the relation $x^r = x^m + x^o$ and $x^s = x^m - x^o$. When we apply to $D_{o,m}$ a double Radon transform with respect to x^o and x^m , we obtain the following map in p^o - p^m domain

$$\tilde{D}_{o,m}(p^o; p^m; \omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} D_{o,m}(x^o; x^m; \omega) e^{-i\omega(p^o x^o + p^m x^m)} dx^o dx^m, \quad (4)$$

with the following relation between $D_{r,s}$ and $D_{o,m}$

$$\tilde{D}_{r,s}(p^r; p^s; \omega) = 2 \tilde{D}_{o,m}(p^r - p^s; p^r + p^s; \omega) \quad (5)$$

with

$$p^m = p^r + p^s \quad \text{and} \quad p^o = p^r - p^s. \quad (6)$$

From eq.(6) it follows that the reference frame of the p^o - p^m data organization is rotated by 45 degrees with respect to the reference frame of the p^r - p^s domain (see Fig.1).

When the Earth is horizontally layered, $D_{o,m}$ is independent of the mid-point coordinate x^m . In that case we can take any value of x^m as representative. For convenience we choose $x^m=0$ and eq.(4) becomes

$$\tilde{D}_{o,m}(p^o; p^m; \omega) = \frac{2\pi}{|\omega|} \delta(p^m) \int_{-\infty}^{+\infty} D_{o,m}(x^o; 0; \omega) e^{-i\omega p^o x^o} dx^o. \quad (7)$$

Hence only for $p^m=0$ or $p^r+p^s=0$ there is information, while for other values of p^m the transformed quantity is zero.

Furthermore, we observe that the transformed data is obtained by a single Radon transform with respect to the offset coordinates. Consequently in the case of a non-planar earth, the information of the lateral variation is not lying on $p^r+p^s=0$. By stationary-phase analysis we proved for the acoustic case that the spatial complexity leads to a structured division in the p^r - p^s domain. In particular from that analysis follows that when we represent the discontinuity interfaces by a set of locally dipping segments, the information of that event is mapped onto a set of lines in the p^r - p^s domain. These features are represented in Fig.1 by the dashed lines and given by

$$p^r + p^s = d_n \left[\left(\left(\frac{1}{c_n^{rms}} \right)^2 - p^{r^2} \right)^{1/2} + \left(\left(\frac{1}{c_n^{rms}} \right)^2 - p^{s^2} \right)^{1/2} \right], \quad (8)$$

where c_n^{rms} is the root mean square velocity to

the horizontal reference level of the n -th interface, while d_n is the tangent of the local dip angle of the pertinent segment and related to p^m by

$$d_n = \frac{\left(p^m c_n^{rms}\right)/2}{\sqrt{1 - \left(\left(p^m c_n^{rms}\right)/2\right)^2}}. \quad (9)$$

Fig.2 shows five "time-stacks" in the p^r - p^s domain to display the complexity of a land data set. Each slice is stacked over a different time window, in which (a) is dominated by the direct wave and (c) clearly depicts a dipping event. Slice (e) shows the decreasing aperture with increasing travel time. The slices (b) and (d) are characterized by dispersed energy.

Thus as we have shown, the spatial complexity of the discontinuity interface leads to a topological discretization in the p^r - p^s domain. This in turn can be exploited in interpretation and migration.

Acknowledgements

We thank A.M. Ziolkowski and J.K. Koster who together designed and carried out the field experiment resulting in the data we use. The financial support of the EEC under contract nr. TH/0112/88/NL is gratefully acknowledged.

Fig.1: The location of reference frames of the p^r - p^s , p^o - p^m domains.

Fig.2: A land data set in the p^r - p^s domain with the following time windows: (a) 0.0-0.1 s, (b) 0.2-0.3 s, (c) 0.5-0.6 s, (d) 0.9-1.0 s and (e) 1.5-1.6 s.