COASTAL ENGINEERING
Volume II
Harbor and Beach Problems
edited by
W.W. Massie, P.E.

Coastal Engineering Group
Department of Civil Engineering
Delft University of Technology
DELFT
The Netherlands

Spring 1978.
Revised and reprinted Fall 1980.
Reprinted with some corrections, Winter 1985/86.
I hear and I forget.
I see and I remember.
I do and I understand.

Confucius.
# TABLE OF CONTENTS - Volume II

Harbor and Beach Problems

1. Introduction 1

2. Survey of Topics Treated 2
   2.1 Purpose 2
   2.2 Subdivisions 2

3. Ship Motions 3
   3.1 Introduction 3
   3.2 Vertical Movements 3
   3.3 Horizontal Motions 4
   3.4 Encounter Frequency 5
   3.5 Determination of Motion in Waves 5
   3.6 Useful Definitions and Approximations 8
   3.7 Example 9

4. Channel depth 10
   4.1 Introduction 10
   4.2 Approach of the problem 11
   4.3 Ship motions 13
   4.4 Water level variation 14
   4.5 Bottom roughness 16
   4.6 Keel clearance variations 17
   4.7 Properties of Normal and Rayleigh distribution 20
   4.8 Chance of hitting the channel bottom 21
   4.9 Ship traffic density 22
   4.10 Varying storm condition 23
   4.11 Further evaluation steps 24
   4.12 A look back 28a

5. Channel width 29
   5.1 Introduction 29
   5.2 An Idealized Problem 29
   5.3 A Realistic Problem 31
   5.4 Design Methods 31
   5.5 Additional Factors 32

6. Ship Maneuvering Models 34
   6.1 Physical Models 34
   6.2 The Simulation Approach 35
   6.3 Description of Ship Simulator 35
   6.4 Ship Simulator Uses 36
   6.5 Critical Remarks 37

7. Maneuverability Improvement 38
   7.1 Motivation 38
   7.2 Tugboat Assistance 39
   7.3 Bow Thrusters 41
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>Total Channel Optimization</td>
<td>42</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>42</td>
</tr>
<tr>
<td>8.2</td>
<td>Definition of Total Optimum</td>
<td>42</td>
</tr>
<tr>
<td>8.3</td>
<td>Construction Costs</td>
<td>42</td>
</tr>
<tr>
<td>8.4</td>
<td>Damage Costs</td>
<td>43</td>
</tr>
<tr>
<td>8.5</td>
<td>Maintenance Costs</td>
<td>43</td>
</tr>
<tr>
<td>9.</td>
<td>Coastal Sand Transport</td>
<td>45</td>
</tr>
<tr>
<td>9.1</td>
<td>Introduction</td>
<td>45</td>
</tr>
<tr>
<td>9.2</td>
<td>Concept of Formulas</td>
<td>46</td>
</tr>
<tr>
<td>9.3</td>
<td>Simplications of the process</td>
<td>43</td>
</tr>
<tr>
<td>9.4</td>
<td>Plan of Attack</td>
<td>47</td>
</tr>
<tr>
<td>10.</td>
<td>Radiation Stress and its Components</td>
<td>49</td>
</tr>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>49</td>
</tr>
<tr>
<td>10.2</td>
<td>Principal Radiation Stresses</td>
<td>49</td>
</tr>
<tr>
<td>10.3</td>
<td>Radiation Stress Changes</td>
<td>50</td>
</tr>
<tr>
<td>10.4</td>
<td>Radiation Stress Components</td>
<td>51</td>
</tr>
<tr>
<td>10.5</td>
<td>Application to Coastal Engineering Problems</td>
<td>55</td>
</tr>
<tr>
<td>11.</td>
<td>Wave Set-Up</td>
<td>57</td>
</tr>
<tr>
<td>11.1</td>
<td>The Phenomenon</td>
<td>57</td>
</tr>
<tr>
<td>11.2</td>
<td>Solutions to The Differential Equation</td>
<td>57</td>
</tr>
<tr>
<td>11.3</td>
<td>Spilling Breaker Solution</td>
<td>58</td>
</tr>
<tr>
<td>11.4</td>
<td>Plunging Breaker Solution</td>
<td>59</td>
</tr>
<tr>
<td>11.5</td>
<td>Special Remarks</td>
<td>61</td>
</tr>
<tr>
<td>11.6</td>
<td>Example</td>
<td>62</td>
</tr>
<tr>
<td>12.</td>
<td>Radiation Shear Stress Gradient</td>
<td>63</td>
</tr>
<tr>
<td>12.1</td>
<td>Introduction</td>
<td>63</td>
</tr>
<tr>
<td>12.2</td>
<td>Changes Outside the Breaker Zone</td>
<td>63</td>
</tr>
<tr>
<td>12.3</td>
<td>Changes Within the Breaker Zone</td>
<td>64</td>
</tr>
<tr>
<td>13.</td>
<td>Tidal Forces Along A Coast</td>
<td>67</td>
</tr>
<tr>
<td>13.1</td>
<td>Coordinates Used</td>
<td>67</td>
</tr>
<tr>
<td>13.2</td>
<td>The One-Dimensional Tidal Force Component</td>
<td>67</td>
</tr>
<tr>
<td>14.</td>
<td>Turbulent Forces</td>
<td>69</td>
</tr>
<tr>
<td>14.1</td>
<td>Introduction</td>
<td>69</td>
</tr>
<tr>
<td>14.2</td>
<td>Mathematical Description</td>
<td>69</td>
</tr>
<tr>
<td>15.</td>
<td>Bottom Friction Forces</td>
<td>71</td>
</tr>
<tr>
<td>15.1</td>
<td>Introduction</td>
<td>71</td>
</tr>
<tr>
<td>15.2</td>
<td>Friction in Constant Currents</td>
<td>71</td>
</tr>
<tr>
<td>15.3</td>
<td>Friction with Waves Alone</td>
<td>74</td>
</tr>
<tr>
<td>15.4</td>
<td>Friction with Combined Currents and Waves</td>
<td>76</td>
</tr>
<tr>
<td>15.5</td>
<td>Additional Remarks</td>
<td>80</td>
</tr>
<tr>
<td>16.</td>
<td>Longshore Current Computation</td>
<td>81</td>
</tr>
<tr>
<td>16.1</td>
<td>Introduction</td>
<td>81</td>
</tr>
<tr>
<td>16.2</td>
<td>Basic Force Equilibrium</td>
<td>81</td>
</tr>
<tr>
<td>16.3</td>
<td>Effect of Turbulence</td>
<td>83</td>
</tr>
<tr>
<td>16.4</td>
<td>Effect of Irregular Waves</td>
<td>83</td>
</tr>
<tr>
<td>16.5</td>
<td>Example</td>
<td>83</td>
</tr>
<tr>
<td>16.6</td>
<td>Additional Driving Forces</td>
<td>86</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>-----------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>17.</td>
<td>Early Coastal Transport Formulas</td>
<td>89</td>
</tr>
<tr>
<td>17.1</td>
<td>Introduction</td>
<td>89</td>
</tr>
<tr>
<td>17.2</td>
<td>The CERC Formula</td>
<td>89</td>
</tr>
<tr>
<td>17.3</td>
<td>Modern Justification of the CERC Formula</td>
<td>91</td>
</tr>
<tr>
<td>17.4</td>
<td>Variation with Angle of Approach</td>
<td>91</td>
</tr>
<tr>
<td>17.5</td>
<td>CERC Formula Coefficient</td>
<td>93</td>
</tr>
<tr>
<td>17.6</td>
<td>Example of Use of CERC Formula</td>
<td>96</td>
</tr>
<tr>
<td>17.7</td>
<td>Limitations of the CERC Formula</td>
<td>96</td>
</tr>
<tr>
<td>18.</td>
<td>Sand Transport Mechanism</td>
<td>97</td>
</tr>
<tr>
<td>18.1</td>
<td>Introduction</td>
<td>97</td>
</tr>
<tr>
<td>18.2</td>
<td>Basic Concepts</td>
<td>97</td>
</tr>
<tr>
<td>18.3</td>
<td>Bottom Roughness</td>
<td>98</td>
</tr>
<tr>
<td>18.4</td>
<td>Concluding Remarks</td>
<td>101</td>
</tr>
<tr>
<td>19.</td>
<td>Modern Coastal Sand Transport Formulas</td>
<td>103</td>
</tr>
<tr>
<td>19.1</td>
<td>Introduction</td>
<td>103</td>
</tr>
<tr>
<td>19.2</td>
<td>Transport Formulas for Currents Alone</td>
<td>103</td>
</tr>
<tr>
<td>19.3</td>
<td>Influence of Waves on Bed Transport</td>
<td>110</td>
</tr>
<tr>
<td>19.4</td>
<td>Bed Shear Stress Modification</td>
<td>110</td>
</tr>
<tr>
<td>19.5</td>
<td>Bed Load Transport by Waves and Current</td>
<td>112</td>
</tr>
<tr>
<td>19.6</td>
<td>Influence of Waves on Suspended Transport</td>
<td>114</td>
</tr>
<tr>
<td>19.7</td>
<td>Total Sediment Transport</td>
<td>116</td>
</tr>
<tr>
<td>19.8</td>
<td>Critical Comments on Bijker Formula</td>
<td>118</td>
</tr>
<tr>
<td>19.9</td>
<td>Example of Bijker Formula</td>
<td>119</td>
</tr>
<tr>
<td>19.10</td>
<td>Sensitivity of the Bijker Formula</td>
<td>124</td>
</tr>
<tr>
<td>19.11</td>
<td>Comparison to CERC Formula</td>
<td>125</td>
</tr>
<tr>
<td>20.</td>
<td>Coastal Changes with Single Line Theory</td>
<td>127</td>
</tr>
<tr>
<td>20.1</td>
<td>Introduction</td>
<td>127</td>
</tr>
<tr>
<td>20.2</td>
<td>Equation of Continuity</td>
<td>127</td>
</tr>
<tr>
<td>20.3</td>
<td>Equation of Motion</td>
<td>129</td>
</tr>
<tr>
<td>20.4</td>
<td>Solution, Boundary and Initial Conditions</td>
<td>130</td>
</tr>
<tr>
<td>20.5</td>
<td>Application to Breakwater Accretion</td>
<td>131</td>
</tr>
<tr>
<td>20.6</td>
<td>Non-Parallel Accretion</td>
<td>134</td>
</tr>
<tr>
<td>20.7</td>
<td>Transport Past Breakwater Tip</td>
<td>135</td>
</tr>
<tr>
<td>20.8</td>
<td>Critical Evaluation</td>
<td>140</td>
</tr>
<tr>
<td>20.9</td>
<td>Example</td>
<td>141</td>
</tr>
<tr>
<td>21.</td>
<td>Sand Transport Along A Beach Profile</td>
<td>147</td>
</tr>
<tr>
<td>21.1</td>
<td>Introduction</td>
<td>147</td>
</tr>
<tr>
<td>21.2</td>
<td>Two Dimensional Transverse Transport</td>
<td>148</td>
</tr>
<tr>
<td>21.3</td>
<td>Example</td>
<td>150</td>
</tr>
<tr>
<td>21.4</td>
<td>Three Dimensional Transverse Transport</td>
<td>159</td>
</tr>
<tr>
<td>22.</td>
<td>Coastal Changes with Multiple Line Theories</td>
<td>161</td>
</tr>
<tr>
<td>22.1</td>
<td>Introduction</td>
<td>161</td>
</tr>
<tr>
<td>22.2</td>
<td>The Schematization</td>
<td>161</td>
</tr>
<tr>
<td>22.3</td>
<td>Equations of Continuity and Motion</td>
<td>163</td>
</tr>
<tr>
<td>22.4</td>
<td>Initial and Boundary Conditions</td>
<td>165</td>
</tr>
<tr>
<td>22.5</td>
<td>Solution to the Equations</td>
<td>166</td>
</tr>
<tr>
<td>22.6</td>
<td>Future Developments</td>
<td>166</td>
</tr>
</tbody>
</table>
23. Dune Coasts 167
   23.1 Introduction 167
   23.2 Dune Formation 168
   23.3 Short Term Dune Dynamics 171
   23.4 Long Term Dune Dynamics 173
   23.5 Analysis Method 176

24. Shore Protection Works 179
   24.1 Introduction 179
   24.2 Sand Supply 179
   24.3 Groins 181
   24.4 Sea Walls 183
   24.5 Detached Breakwaters 186
   24.6 Accretion Control 187

25. Channel Sedimentation 191
   25.1 Introduction 191
   25.2 Physical Changes 191
   25.3 Bed Load Transport 194
   25.4 Suspended Load Transport 194
   25.5 An Approximate Solution 194
   25.6 More exact sedimentation determination 195

Symbols and Notation 197
   Roman Letters 197
   Greek Letters 200
   Special Symbols 200
   Subscripts 201
   Functions 202
   Dimensions and Units 202

References 203
<table>
<thead>
<tr>
<th>Table number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Contributors to This Volume</td>
<td>1</td>
</tr>
<tr>
<td>4.1</td>
<td>Properties of Normal Distribution</td>
<td>20</td>
</tr>
<tr>
<td>6.1</td>
<td>Characteristics of &quot;Esso Atlantic&quot;</td>
<td>34</td>
</tr>
<tr>
<td>7.1</td>
<td>Tugboat and Ship Performance Data</td>
<td>40</td>
</tr>
<tr>
<td>10.1</td>
<td>Radiation Stress Values</td>
<td>52</td>
</tr>
<tr>
<td>16.1</td>
<td>Longshore Current Distribution</td>
<td>87</td>
</tr>
<tr>
<td>17.1</td>
<td>CERC Formula Coefficients</td>
<td>94</td>
</tr>
<tr>
<td>19.1</td>
<td>Einstein Integral Factors</td>
<td>108</td>
</tr>
<tr>
<td>19.2</td>
<td>Steps in Sand Transport Computation</td>
<td>117</td>
</tr>
<tr>
<td>19.3</td>
<td>Sediment Transport Distribution</td>
<td>121</td>
</tr>
<tr>
<td>19.4</td>
<td>Total Sand Transports</td>
<td>124</td>
</tr>
<tr>
<td>20.1</td>
<td>Shoreline Accretion Parameters</td>
<td>133</td>
</tr>
<tr>
<td>20.2</td>
<td>Corrections to Tip Transport Computations</td>
<td>140</td>
</tr>
<tr>
<td>20.3</td>
<td>Coastline Form Computations</td>
<td>144</td>
</tr>
<tr>
<td>Figure number</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.1</td>
<td>Effect of Squat and Trim on VLCC</td>
<td>4</td>
</tr>
<tr>
<td>3.2</td>
<td>Vertical Ship Motions in Waves</td>
<td>4</td>
</tr>
<tr>
<td>3.3</td>
<td>Horizontal Ship Motions in Waves</td>
<td>4</td>
</tr>
<tr>
<td>3.4</td>
<td>Encounter Frequency Definition Sketch</td>
<td>5</td>
</tr>
<tr>
<td>3.5</td>
<td>Wave and Ship Spectra</td>
<td>7</td>
</tr>
<tr>
<td>4.1</td>
<td>Definition Sketch, Channel Depth Parameters</td>
<td>11</td>
</tr>
<tr>
<td>4.2</td>
<td>Keel clearance variations with definitions</td>
<td>19</td>
</tr>
<tr>
<td>5.1</td>
<td>Channel Design Parameters</td>
<td>31</td>
</tr>
<tr>
<td>5.2</td>
<td>Path Width for Ship</td>
<td>33</td>
</tr>
<tr>
<td>7.1</td>
<td>Stopping Distances for Tankers</td>
<td>39</td>
</tr>
<tr>
<td>9.1</td>
<td>Principle Sketch of Continuity</td>
<td>45</td>
</tr>
<tr>
<td>9.2</td>
<td>Sediment Transport Principle Sketch</td>
<td>47</td>
</tr>
<tr>
<td>10.1</td>
<td>Plan Showing Principal Stresses</td>
<td>51</td>
</tr>
<tr>
<td>10.2</td>
<td>Mohr's Circle Analysis</td>
<td>53</td>
</tr>
<tr>
<td>10.3</td>
<td>Mohr's Circles</td>
<td>54</td>
</tr>
<tr>
<td>10.4</td>
<td>Coastal Plan with Stress Elements</td>
<td>54</td>
</tr>
<tr>
<td>11.1</td>
<td>Element of Coastal Water</td>
<td>57</td>
</tr>
<tr>
<td>11.2</td>
<td>Wave Set-Up with Spilling Breaker</td>
<td>59</td>
</tr>
<tr>
<td>11.3</td>
<td>Wave Set-Up with Plunging Breaker</td>
<td>60</td>
</tr>
<tr>
<td>11.4</td>
<td>Circulation Current in Breaker Zone</td>
<td>61</td>
</tr>
<tr>
<td>15.1</td>
<td>Logarithmic Velocity Profile</td>
<td>73</td>
</tr>
<tr>
<td>15.2</td>
<td>Wave Friction Parameters</td>
<td>75</td>
</tr>
<tr>
<td>15.3</td>
<td>Geometry of Velocity Components</td>
<td>77</td>
</tr>
<tr>
<td>16.1</td>
<td>Simplified Velocity Profile</td>
<td>82</td>
</tr>
<tr>
<td>16.2</td>
<td>Example Velocity Profiles</td>
<td>87</td>
</tr>
<tr>
<td>17.1</td>
<td>Relation Between U' and S</td>
<td>95</td>
</tr>
<tr>
<td>18.1</td>
<td>Bottom Velocity and Shear Stress Variation</td>
<td>97</td>
</tr>
<tr>
<td>18.2</td>
<td>Schematic Representation of Sediment Movement</td>
<td>98</td>
</tr>
<tr>
<td>18.3</td>
<td>Eddy Formation Near Ripples</td>
<td>98</td>
</tr>
<tr>
<td>18.4</td>
<td>Steps in Erosion and Deposition</td>
<td>99</td>
</tr>
<tr>
<td>18.5</td>
<td>Sediment Concentration Curves</td>
<td>101</td>
</tr>
<tr>
<td>19.1</td>
<td>Example Concentration, Velocity, and Transport Profiles</td>
<td>107</td>
</tr>
<tr>
<td>19.2</td>
<td>Suspended Sediment Transport Parameters</td>
<td>115</td>
</tr>
<tr>
<td>19.3</td>
<td>Example Sediment Transport Profiles</td>
<td>123</td>
</tr>
<tr>
<td>19.4</td>
<td>Sensitivity of Bijker Formula</td>
<td>125</td>
</tr>
<tr>
<td>Figure number</td>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>20.1</td>
<td>Beach Profile and Schematization</td>
<td>127</td>
</tr>
<tr>
<td>20.2</td>
<td>Continuity Equation Relationships</td>
<td>128</td>
</tr>
<tr>
<td>20.3</td>
<td>Shore Plan</td>
<td>130</td>
</tr>
<tr>
<td>20.4</td>
<td>Accretion near Breakwater</td>
<td>131</td>
</tr>
<tr>
<td>20.5</td>
<td>Accretion Geometry</td>
<td>133</td>
</tr>
<tr>
<td>20.6</td>
<td>Non-Parallel Accretion Profile</td>
<td>134</td>
</tr>
<tr>
<td>20.7</td>
<td>Profile at Start of Sand Passage</td>
<td>135</td>
</tr>
<tr>
<td>20.8</td>
<td>Validity Zones of Models</td>
<td>139</td>
</tr>
<tr>
<td>20.9</td>
<td>Harbor Entrance Plan</td>
<td>142</td>
</tr>
<tr>
<td>20.10</td>
<td>Beach Accretion Lines</td>
<td>146</td>
</tr>
<tr>
<td>21.1</td>
<td>Influence of Current on Sand Transport</td>
<td>147</td>
</tr>
<tr>
<td>21.2</td>
<td>Schematized Beach Profile</td>
<td>149</td>
</tr>
<tr>
<td>21.3</td>
<td>Beach Profile</td>
<td>151</td>
</tr>
<tr>
<td>21.4</td>
<td>Upper Limit of D Profile</td>
<td>152</td>
</tr>
<tr>
<td>21.5</td>
<td>Lower Limit of D Profile</td>
<td>153</td>
</tr>
<tr>
<td>21.6</td>
<td>Horizontal Scale of Equilibrium Profile</td>
<td>154</td>
</tr>
<tr>
<td>21.7</td>
<td>Profile Characterizing Parameters</td>
<td>155</td>
</tr>
<tr>
<td>21.8</td>
<td>Maximum Transport Along Profile</td>
<td>156</td>
</tr>
<tr>
<td>21.9</td>
<td>Location of Maximum Transport</td>
<td>157</td>
</tr>
<tr>
<td>21.10</td>
<td>Relative Profile Transport</td>
<td>158</td>
</tr>
<tr>
<td>22.1</td>
<td>Beach Profile Schematizations</td>
<td>162</td>
</tr>
<tr>
<td>22.2</td>
<td>Shore Plan and Profile</td>
<td>163</td>
</tr>
<tr>
<td>22.3</td>
<td>Continuity Equation Relationships</td>
<td>164</td>
</tr>
<tr>
<td>23.1</td>
<td>Dunes Encroaching on Highway</td>
<td>167</td>
</tr>
<tr>
<td>23.2</td>
<td>Aerial Photo of Dune Coast</td>
<td>168</td>
</tr>
<tr>
<td>23.3</td>
<td>Types of Dunes</td>
<td>169</td>
</tr>
<tr>
<td>23.4</td>
<td>Dune Coast Profile</td>
<td>170</td>
</tr>
<tr>
<td>23.5</td>
<td>Storm Profile</td>
<td>172</td>
</tr>
<tr>
<td>23.6</td>
<td>Effect of Storm on Dutch Coast</td>
<td>174</td>
</tr>
<tr>
<td>23.7</td>
<td>Dune Changes on Dutch Coast</td>
<td>175</td>
</tr>
<tr>
<td>23.8</td>
<td>History of Dutch Coastal Changes</td>
<td>176</td>
</tr>
<tr>
<td>24.1</td>
<td>North Sea Storm</td>
<td>181</td>
</tr>
<tr>
<td>24.2</td>
<td>Groin Protected Coast</td>
<td>182</td>
</tr>
<tr>
<td>24.3</td>
<td>Seawall Reinforced Dunes</td>
<td>184</td>
</tr>
<tr>
<td>24.4</td>
<td>Seawall protecting Rock Coast</td>
<td>185</td>
</tr>
<tr>
<td>24.5</td>
<td>Detached Breakwater Segments</td>
<td>186</td>
</tr>
<tr>
<td>24.6</td>
<td>Submerged Natural Tombolo</td>
<td>188</td>
</tr>
<tr>
<td>24.7</td>
<td>Sand Escaping from Accretion Area</td>
<td>189</td>
</tr>
<tr>
<td>24.8</td>
<td>Development of Transport Past Tip</td>
<td>189</td>
</tr>
<tr>
<td>25.1</td>
<td>Coastal Plan and Longshore Profile</td>
<td>192</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

W.W. Massie

This second volume of the series on coastal engineering is intended as an amplification of certain topics mentioned in volume I. The organization is much the same as in the first book; two of the four main topic categories get emphasis here: harbors and coastal morphology.

Background material related to radiation stress is introduced briefly in chapter 10. Otherwise, background information is drawn from the first volume.

Again in this volume, we use an American rather than English spelling and the more difficult technical words are also included in the separately available word list.

Figures are drawn to scale unless otherwise noted, and we have sought to use consistent notation throughout this volume, at least, and as much as possible, throughout the entire set of notes.

Literature references are mentioned by author's name and year in the text; a complete listing is included in the back of the volume. Tables of symbols are also included in the back of the volume.

The technical topics to be treated in this volume are briefly summarized in the following chapter of this book. Contributing staff members are listed in table 1.1. Those responsible for the technical accuracy of each of the chapters are listed at the start of each chapter.

Table 1.1 Contributors to this volume.
Prof. dr. ir. E.W. Bijker, Professor
Ir. J.J. van Dijk, Senior Scientific Officer
Ir. J. van de Graaff, Scientific Officer
Ir. L.E. van Loo, Senior Scientific Officer
W.W. Massie, MSc., P.E., Senior Scientific Officer
Ir. P.J. Visser, Scientific Officer

All of the above persons are members of the Coastal Engineering Group, Delft University of Technology, Delft, The Netherlands.

Corrections for the revised edition have been made by the above persons and P.J.M. Lapidaire, Student Assistant.

Several example computations are presented in this book. They are intended to illustrate the process of a computation and the results which come from it. The reader should be careful not to become too involved with the details of the computational procedures; these can be quickly enough "re-discovered" by anyone having a sufficient insight. The development of this insight is the objective of this book.
2. SURVEY OF TOPICS TREATED

2.1 Purpose

The purpose of this volume is to treat certain coastal engineering topics specifically related to harbor approaches and to coastal morphology. Each of these subareas of coastal engineering is adequately defined in chapter 2 of volume I. Harbors and coastal morphology are presented together, here, because of their strong interdependence. The construction of a harbor entrance, or even only the dredging of an approach channel, can (and usually will) upset the existing bottom morphology in the area - along the coast or in the channel. The designer of an optimum harbor entrance, therefore, must consider both shipping and morphological aspects in his design.

2.2 Subdivision

Even though these topics are strongly interrelated, an attempt to separate the subtopics has been made. The following six chapters discuss the proper dimensioning of approach channels considering navigational aspects primarily. The following five chapters provide information on the movement of ships and the utilization of this information for a channel design. In chapter 8 the various aspects of channel design are brought together in order to attempt to make an optimum channel design. Morphological processes involving sediment movements along the coast and in channels are seen to influence this optimum design significantly.

The mechanics of water movement - longshore current - along a coast is carefully unraveled in chapters 9 through 16. Beach material movement along a sandy coast is treated in chapters 17 through 19. The better sediment transport determinations are built up from the knowledge of the longshore current studied in chapter 16. The result of the sediment transport determination is a relatively simple method to predict coastline changes presented in chapter 20. This simple model is refined and improved in chapters 21 through 23.

Two specific applications of sand transport computations conclude this volume. The evaluation of coastal defense works is discussed in chapter 24 and the prediction of channel erosion and sedimentation - chapter 25 - completes this volume and completes a cycle back to harbor approach channel optimizations discussed in chapter 8.

Two other applications of sediment transport computations - prediction of erosion near offshore structures and pipelines on the seafloor - are considered to be a bit specialized for many users of this book.
3. SHIP MOTIONS

W.W. Massie

3.1 Introduction

The displacements (movements) of a ship relative to its position when stationary in still water are of extreme importance in the design of a harbor entrance. Vertical relative movements are of importance for channel depth determinations while horizontal movements about a given desired course line are important for channel width and collision avoidance considerations.

3.2 Vertical movements

Vertical relative displacements of ships can be caused by waves but may also occur as a result of the ship's forward speed in still water. This latter displacement can be split into two components: squat and trim while waves give rise to vertical displacements via pitch, heave, and roll. All of these motion components are defined and described below.

Squat.

Squat is a uniform sinking of the ship - an apparent increase in draft - resulting from pressure changes in the surrounding water. As the ship moves forward, water flows in the opposite direction along the ship from bow to stern. Applying the Bernoulli Theorem reveals that the pressure at a given level in this return flow must be lower than at the same elevation in still water; the surface water level drops and the ship sinks along with it. This phenomenon occurs in all waters, both deep oceans and restricted channels. In restricted channels the return flow velocity will be relatively higher because the same return flow volume must pass through a smaller cross section; the water level lowering and squat are greater, thus, in restricted channels.

Trim.

Trim is a differential sinking of the stern of a ship relative to the bow. Thus, trim is the rotation of the ship about a horizontal crosswise (beam) axis; it results from asymmetry of the return flow patterns at bow and stern. The action of the propeller will increase the effective return flow at the stern of a well streamlined ship form such as a container ship or fast cargo vessel; such ships will trim with their stern deeper than the bow. Bulk carrier or large oil tankers, on the other hand, have a very high block coefficient* and the blunt bow leads to return current concentrations near the bow. This results in a bow-down trim.

* The block coefficient is defined as the ratio of the displaced water volume to the product of length, beam, and draft of the ship.
Figure 3.1 shows quantitative results from model tests carried out at the British National Physical Laboratory reported in an anonymous article in the July 1974 issue of The Motor Ship. The curves show keel clearances at the bow of a 300 m long Very Large Crude Carrier as a function of speed and keel clearance at zero speed.

Wave Induced Motions.

Figure 3.2 illustrates the three wave induced vertical motions of a ship. The scale of this figure is distorted in order to clarify the illustration. The actual vertical motion of some point on the ship is determined by the superposition of heaving, pitching, and rolling motions.

The actual motion of the ship depends upon the size of the ship relative to the waves. In principle, heave, pitch and roll of a ship in waves can each be considered equivalent to a mass-spring dynamic system. We remember from dynamics that such systems have a natural or resonant frequency and at this frequency the displacements can be large even though the exciting forces (waves) are of small amplitude. A rowboat will respond much more violently to a wave 0.5 m high with a period of 2 seconds than will a large bulk carrier. In general, these latter ships are only slightly influenced by head seas. Beam seas, on the other hand, can excite roll motions which can be of significance for determining the maximum draft required. This results from the extreme width of such large ships. For example, if a large tanker with 60 m beam rolls only $3^\circ$ (a hardly noticable amount) its draft at one side will increase by:

$$\frac{60 \sin 3^\circ}{2} = 1.6 \, \text{m}$$

(3.01)

3.3 Horizontal motions

Three horizontal ship motion components caused by waves are illustrated in figure 3.3. Additionally, use of the rudder while steaming in still water will cause a ship to yaw, sway, and roll. This last effect is most pronounced on large ships for which the center of mass is well above the center of lateral resistance. The centripetal acceleration combined with a lateral hydrodynamic resistance cause a roll moment.

Horizontal motion components yaw and sway caused by either rudder action or waves are most important for determining the required maneuvering areas and channel widths for ships underway. Surge, sway, and yaw components are important for mooring forces of ships and roll can be an additional factor in locating fenders on a quay.
3.4 Encounter frequency

A ship moving into head waves (against the direction of wave propagation) will encounter more waves per unit of time than would an observer at a fixed point. If, on the other hand, the ship were travelling with the waves, she would encounter relatively fewer waves per unit of time. A more general situation is shown in plan in figure 3.4.

A formula for encounter frequency can be derived from the figure via kinematics:

$$\omega_e = \omega \left(1 - \frac{v_s}{c} \cos \alpha\right)$$  \hspace{1cm} (3.02)

where $c$ is the celerity of the wave,
$v_s$ is the velocity of the ship,
$\alpha$ is the angle between the positive direction of $v_s$ and $c$,
$\omega$ is the wave frequency, and
$\omega_e$ is the wave encounter frequency experienced by the ship.

Note that in figure 3.4, $\alpha$ is more than $90^\circ$ and thus, $\cos \alpha$ is negative.

The encounter period, $T_e$, can, of course, be computed from the general relation:

$$T_e = \frac{2\pi}{\omega_e}$$  \hspace{1cm} (3.03)

Usually, however, dynamic analyses are done using frequency as an independent parameter.

3.5 Determination of motions in waves

About 1860 Sir William Froude analyzed the motion of sailing warships of that era by assuming that the movement of the ship was the same as the average movement of the equivalent volume of water in the undisturbed wave. An equivalent form of his assumption is that the pressures exerted on the ship's hull surface are the same as those at the same location in an undisturbed wave. Many practical problems can be solved with acceptable accuracy using this simple and crude assumption. If, however, the ship's keel clearance is somewhat restricted or the ship is especially large relative to the wave length, then the disturbance (diffraction) of the on-coming wave by the ship becomes increasingly important and can no longer be neglected.

Naval architects have developed better theoretical models for computing ship motions since the time of Froude. The so-called strip theory is often used for computing heave and pitch in regular waves; the method is well documented by Comstock-editor (1967). These later methods make it possible to include wave diffraction effects and the generation of waves by the moving ship.

When the motion components of a ship are linear (all directly proportional to wave height) then it is possible to determine the total response to waves by superposition of the individual response components. Luckily, most ship response problems can be treated with linear models since the ship dimensions are usually large enough relative to the wave length.
The superposition principle makes it possible to determine the response of a ship to a spectrum of waves using a response function method, just as in many other problems in dynamics. We may remember from dynamics that the response functions needed to transform a force (wave) spectrum to a response (motion) spectrum can be determined by subjecting the ship to a series of constant frequency excitations (waves). Each wave frequency determines a single point in the response function. In many cases these responses can be computed. They can always be determined via model tests, and are usually obtained in the latter way except in deep water.

When the water depth becomes less than about 50 percent more than the ship draft, the ship response to a given wave condition becomes dependent upon the average keel clearance. As the keel clearance becomes smaller, the flow pattern around the ship becomes more disturbed relative to the deep water conditions. In general, this results in a lower response function value for both horizontal and vertical motions; the ship moves less in response to a given force.

Computation of responses in real shallow water situations becomes very difficult; model tests yield the only reliable response data.

An example may make this principle more clear. Figure 3.5.a. illustrates a wave record and its associated spectrum, $S_\eta(\omega)$. In that figure:

- $S_\eta(\omega)$ is the wave energy density (rate of change of wave energy per unit crest length with respect to frequency),
- $\omega$ is the wave frequency, and
- $\eta$ is the water surface elevation at any instant of time.

If this spectrum, $S_\eta(\omega)$ is given for an observer at a fixed point - as it usually is - it must be replotted with a new horizontal scale based upon the encounter frequency, $\omega_e$, using equation 3.02, and shown in figure 3.5b.

Figure 3.5c shows the response function $H(\omega)$ of a ship such as could be determined in a series of model tests using a series of regular waves of various periods.

The resulting spectrum representing the ship motion shown in figure 3.5d results from multiplying ordinates of the spectrum in figure 3.5b with the square of the corresponding ordinates in figure 3.5c. One of the many possible ship motion registrations corresponding to the determined spectrum is also shown. Since the extreme values of the original wave spectrum satisfied a Rayleigh Distribution, the extremes of the ship movement, $S_\eta$, can also be expected to satisfy this distribution.

Response functions will be used in the following two chapters to compute ship motions needed to determine channel depths and widths.
a. Wave record with spectrum.

b. Transformed spectrum.

c. Response function.

d. Resulting spectrum and pitch motion.

FIGURE 3.5
WAVE AND SHIP SPECTRUM.
3.6 Useful Definitions and Approximations

It is often desirable to estimate approximate dimensions of a certain size ship for the purpose of preliminary harbor planning. The following definitions and approximate relationships can be handy for such work; detailed plans must be based upon more accurate data, however.

The **deadweight tonnage** (DWT) of a ship is its total capacity to carry cargo, supplies, and people. It thus includes the mass of crew, passengers, provisions, fuel, water, movable furniture and other supplies as well as cargo.

The **lightweight tonnage** of a ship includes the mass of the ship alone in a totally empty condition - all storage spaces empty.

The **displacement** of a ship is the mass of water displaced by the ship. Since Archimedes Principle applies to floating bodies, this displacement is also equal to the total mass of the loaded ship: the sum of lightweight and deadweight.

Further the following relationship holds:

\[
\text{Displacement} = \rho C_B L B D
\]  
(3.04)

where, 

- \( B \) is the ship beam (width),
- \( C_B \) is a so called block coefficient,
- \( D \) is the ship draft,
- \( L \) is the ship length, and
- \( \rho \) is the mass density of water.

Normal block coefficient values for commercial ships range from about 0.4 for a fast destroyer to nearly 0.9 for a supertanker.

The **gross register tonnage** of a ship is a measure of its internal volume - with certain exceptions, see for example Baker (1952) - measured in units of 100 cubic feet (2.83 m\(^3\)).

The **net register tonnage** of a ship is a measure of the volume available for carrying revenue-earning cargo. Again, 100 cubic feet is the unit of volume used. Note that neither of the register tonnages just described are actual masses; they are actually volume measurements.

For most ships the DWT is about 1.5 times the gross register tonnage and about twice the gross register tonnage for very large crude carriers (VLCC). These relationships are dimensionally inconsistent and are valid for DWT in metric tons and register tonnages in the usual units.

Usually the displacement of a fully loaded ship is about 1.3 to 1.4 times its DWT. Further, the gross register tonnage varies from 1.7 (for freighters) to 1.3 (for VLCC) times the net register tonnage.

For most freighters, the ratio of length to beam varies between 5 and 8. Higher ratios are usually found on the faster ships. The ratio of beam to draft is usually about 2. Draft restrictions of very large ships, however, result in a somewhat higher ratio value; for them, a ratio nearer to 3 is common.
3.7 Example

The information in the previous section can be used to estimate the dimensions of a ship. Estimate, for example, the draft of a 250,000 DWT tanker.

The displacement is about 1.3 times DWT.

\[ \text{Displacement} = 1.3 \times 250,000 = 325,000 \text{ tons.} \]  

(3.05)

The block coefficient is chosen as about 0.9. Since the ship will be draft limited, the beam will be about 3 times the draft:

\[ B \approx 3D \]  

(3.06)

Tankers are not fast ships; their beam is, thus, usually about 1/5 of their length or:

\[ L \approx \frac{5B}{15D} \]  

(3.07)

Substitution of all of this, with \( \rho = 1.030 \text{ tons/m}^3 \) into (3.04) yields:

\[ 325,000 \% (1.030)(0.9)(15D)(3D)(D) \approx 41.72D^3 \]  

(3.08)

or:

\[ D \approx 19.8 \text{ m} \]  

(3.09)

say, the draft is 20 meters.
4. CHANNEL DEPTH

4.1 Introduction

The development of supertankers about a decade ago has led to increases in ship sizes for other cargoes as well. This increase in ship dimensions - including draft - has led to a need for deeper and wider harbor approach channels. The additional depth also means that the channel will be longer, too, in view of the usual sea bed slope near coasts. The volume of material that must be dredged in order to provide one unit of extra channel depth increases rapidly as the channel depth increases. Investment costs for dredging, navigation systems, and channel maintenance also increase sharply as a function of depth while the number of ships actually needing such a great depth provided - and their benefit to the harbor - decreases as the channel depth provided increases. All of these factors combined with the increasing scarcity of capital for such large scale investments makes it necessary to optimize the depth of ship channel chosen.

The basic general principles of such an optimization have already been outlined in chapter 13 of volume 1. The steps a through d first listed in section 3 of that chapter are to be applied here; a quick review of them with our particular problem in mind can help set the stage for the further work in this chapter.

a. A design alternative must be chosen

This involves much more than a channel depth; a design ship or series of different ships, shipspeed, channel width, and channel alignment must also be chosen. The channel geometry will obviously influence its construction costs, but it also influences the ship's response to waves and thus the potential damage costs.

b. Determine the construction costs

The construction costs follow directly from the channel geometry and include both initial and capitalized maintenance dredging costs as well as investment and maintenance of necessary navigational aids.

c. Determine the damage cost

Economic damage remains difficult to evaluate in this application just as in most applications. Damage, now, results from, for example:

- A ship having to be drydocked and repainted after having eroded her bottom paint on a sand bar.

- A ship which cannot maneuver properly in a small channel running aground and having to be rescued or salvaged.

- Ship collisions resulting in damage or possible sinking.

- A ship hitting the bottom, being holed and possibly sinking.
Consequentia damage can also occur such as environmental damage from an oil spill, loss of life, cargo losses, or costs of delays if the channel is blocked or closed.

Costs associated with the above types of damage can be difficult enough to determine, but they must also be multiplied by the chance that such damage actually occurs. The evaluation of this chance will be the major work in the rest of this chapter.

In some cases, the estimate of the damage costs is at best rather arbitrary. Then, instead of trying to determine a complete economic optimum, some designers choose alternatively to reduce the chance of a particular type of damage to an acceptably low level.

d. Repeat these steps for various designs

The large number of independent design parameters - see step a, above - results in a considerable computational effort and associated bookkeeping problem. Examples of optimization computations for breakwaters having at most two independent variables are given in volume III.

4.2 Approach to the problem

Returning now to the problem at hand, we will be interested in evaluating a given channel depth in the light of two criteria:

1. Is the depth great enough so that the ship can maneuver adequately in the channel provided?

2. Is the chance that a ship hits the channel bottom during its passage acceptably low?

Both of these criteria depend upon the keel clearance of the ship. The first of the above depends upon the average value of the keel clearance provided (or, at least, upon that provided during a sufficiently high percentage of the time)\(^*\), while the second criterion depends upon individual instantaneous keel clearance values. Apparently both the average keel clearance as well as its statistical variations will be important in the analysis to follow.

A more vivid picture is the following: A given ship is sailing with a given speed through a given channel. The speed and channel dimensions are causing a squat plus trim of the ship which when combined with the ship draft and channel depth result in an average keel clearance. For the moment we will let a diver swim along under the ship - in the keel clearance space. If this average keel clearance is sufficient, the ship can maneuver acceptably (the first criterium is satisfied!) and our diver swims happily along.

There are more factors involved, however. Water level variations caused by tides or storm surges are causing this keel clearance to change slowly.

\(^*\) This will be assumed to be equal to the average keel clearance for the rest of the discussion in this chapter.
Waves present are causing the ship to move about its average depth position considerably more frequently. Both the water level changes and the ship response to waves are causing the "roof" above our swimming diver to move up and down. The channel bed is not a flat plane, however. Dredging tolerances or inaccuracies as well as sedimentation ripples on the channel bed will cause unevenness here, too; the "floor" under our diver moves up and down as well. These individual movements are not too important, alone, but the safety of our ship (and diver) is dependent upon their combined influence.

Indeed, if the "floor" and "roof" meet then the diver is not too well off and the ship has hit the bottom!

Figure 4.1. shows some of the items involved. The symbols in the figure will be defined in later sections of this chapter. The next three sections will be devoted to discussions of each of the keel clearance components.

Before starting such a discussion it is appropriate to first discuss the suitability of channel depth data. The most well-known sources of depth data are hydrographic charts published for mariners. Because these charts are for mariners, the charted depths are the shallowest depths in the vicinity; the actual sea bed lies below the surface defined by the charted depths. Thus, dredging quantities - especially initial dredging quantities - estimated using such charts tend to be high. Generally, however, better information is also available from the various hydrographic services upon direct request. Data sheets, charting the depths used to draw the hydrographic chart, are often available.
These charts show many more depth measurements, of course. Occasionally, the actual sounding records are available. Since these latter provide an extremely high density - every few meters along each profile measured - of soundings which are often uncorrected for tide, etc.; they are usually more work than they are worth for our purposes.

The datum for such charts is also usually about the lowest water level that can be expected; the astronomical Lower Low Water Spring (LLWS) is often used. This chart datum level may be considerably lower than the average water level in the channel during passage of the ships.

4.3 Ship motions

Once the average channel depth and orientation as well as the design ship and speed have been chosen, several necessary design items can be determined. The influence of squat plus trim can now be determined as already indicated in section 3.2. This squat plus trim, Z, will reduce the average keel clearance accordingly.

The ship draft, D, will depend upon the degree of loading, but also upon the water density in the channel. Such density difference influences can be very important for a deep draft ship; a draft increase of 1 meter when going from sea water to fresh and possibly warmer river water is not exceptional. Obviously, the proper ship draft should be used to determine the average keel clearance available.

The channel orientation (direction of its centerline) will determine the relative direction of wave approach in any chosen storm condition. (For the purposes of this and the next few sections only a single storm condition and a single ship will be considered. These restrictions will be relaxed after section 4.8.) This relative wave direction, storm wave spectrum, and ship speed make it possible to determine the wave spectrum as encountered by the ship.

Naval Architects can provide the necessary basic response functions - one for pitch was used for illustration in figure 3.5. Our interest, however, is not in these component motions but rather in the motion of some point on the ship's bottom that is likely to be critical (have the best chance of hitting the bottom). Once such a critical point has been chosen, then a response function for it, giving its vertical motion per unit wave amplitude versus frequency, can be determined using simple laws of kinematics.

The location of the critical point on the ship's hull is in some cases easy but can also be difficult. The critical point on a modern sailing yacht will obviously be at the deepest point of its relatively short keel. Roll of the yacht will not influence the vertical motion of this point unfavorably (it will even decrease the draft) and since the keel is also near midships, pitch will be unimportant relative to heave.

* The water level needed to determine this depth will be discussed in the next section.
For a supertanker, on the other hand, with a very flat bottom one of several points could be critical. Considering only pitch, the bottom of the bow bulb would probably be the most critical. However, if the ship rolls, then the outer edges of the bottom - the bilges - can be in danger. Often times the above two motions are combined and a forward shoulder point toward the bow but where the ship still has appreciable beam will be the critical point. In case of doubt, the critical depth point must be chosen by trial.

Once the response function for the critical point is known, the response spectrum for this point can be determined just as was done in section 3.5. This response spectrum, which we can denote by $R(w)$, will show the variance of the location of the critical point per unit frequency as a function of frequency. This energy density spectrum will have units of $m^2s$ versus $1/s$. The frequencies in this response spectrum will extend only over those frequencies present in both the input wave spectrum and the relevant response functions; this is inherent in the way the response spectrum is determined.

In the next section, we shall attempt to express the water level variations relative to a chosen average level in a spectrum form as well.

4.4 Water level and its variation

The water level, $L$, relative to the chart datum chosen for channel design purposes is dependent upon many factors. One of the more important factors is the density of traffic of the design ship. If these design ships enter the port only occasionally - every few days, for example - it is usually acceptable to delay their entry until sometime near high tide. This statement is valid, of course, only if other conditions such as currents allow safe navigation at those times.

The designer is often a bit conservative in selecting the high water level for the occasional design ship. If, for example, there is a significant variation in high water levels during a month, he would choose a high water level that would be exceeded every normal day, the Higher High Water Neap (HHWN) as a basis. If shipping delays might still be too costly, a still lower level based upon the Lower High Water Neap (LHWN) might be selected.

Ships of extreme draft entering Rotterdam, for example, receive specific instructions advising them when to enter the outer approach channel relative to H.W. This advice is based upon the actual computed tide curve for the specific day and the ship characteristics.

For this occasional ship problem, the value of $L$ will usually be positive (depending upon the datum level, of course) and it will probably not vary too much during the passage of the ship.
If, on the other hand, the design ship must enter the harbor channel very frequently - one can think of a ferry boat with a fixed time schedule entering many times each day - then the designer might choose a water level that can be guaranteed with almost perfect certainty. This level will probably be lower than even LLWS in order to allow for extra-ordinary conditions such as set down. This set down could be caused by a strong wind blowing from the shore, for example. Since such a low water level would be reasonably well defined, its associated variance would be small, just as with the "occasional ship problem." The choice of such a low design water level, while important for individual ship passages would be conservative for an overall channel optimization evaluation in this case; after all, most of the time a ship would be entering the channel when the water level was considerably higher.

An overall channel evaluation would, then, better be based upon a water level equal to the mean sea level and a corresponding (large) variance, which included the entire tide as well as other influences. Such an approach will lead to a better overall evaluation of this "frequent ship problem" which is distinct from the "occasional ship problem", described previously.

For either type of problem the channel depth and thus the average keel clearance is fixed once the average water level has been chosen. The water level variance, \( \sigma_L^2 \), about this chosen mean level will depend upon many factors.

Considering first the occasional ship entering near high tide, the channel water level will vary to some extent as the result of tides and possibly wind set-up during the time period in which the ship is in the channel. This time duration is dependent upon the channel length and the ship speed. In any case, a small (relative to that of the frequent ship problem) water level variance usually will exist. If the channel is extremely long or the ship exceedingly slow, however, then tides may even cause considerable water level variations and the corresponding mean water level during the ship's passage must be lower; the water level variance will now increase correspondingly. In the limit - when a ship needs more than one tide period to pass through a channel - this occasional ship problem becomes the same as a frequent ship problem as far as water levels are concerned.

Mean sea level is probably the best water level to choose as basis for the frequent ship problem and an overall channel evaluation. Since ships can be entering at any time, the water level variance will now include the entire tide as well as storm surge influences.

The water level variance, \( \sigma_L^2 \), can be determined easily enough from the now known water level changes relative to the chosen mean level. If this level change is schematized as a sine wave (pretty good for a full tide, at least) with amplitude \( A_L \), then

\[
\sigma_L^2 = \frac{1}{2} A_L^2
\]  

(4.01)
Now, we need only to convert this value of $O_{L^2}$ to a spectrum value to get the same form as that used for the ship response, $R(\omega)$. This can be done by realizing that water level variations occur with very low frequencies - corresponding to the periods of the tidal components; the frequency of a semi-diurnal tide is about $1.4 \times 10^{-4}$ rad/s, for example. Remembering, additionally, that $O_{L^2}$ represents an area under a spectrum curve, $O_{L^2}$ can be converted to a rectangular spectrum of width $\Delta \omega$ and height:

$$L(\omega) = \frac{O_{L^2}}{\Delta \omega}$$  \hspace{1cm} (4.02)

The value of this width, $\Delta \omega$, will not turn out to be so very important. $^*$

$$\Delta \omega = 10^{-4} \text{ rad/s}$$  \hspace{1cm} (4.03)

This value of $L(\omega)$ should obviously be plotted at its proper frequency and just as $R(\omega)$, it has units of m$^2$ s.

4.5 Bottom roughness

The irregularities of the channel bottom can be measured by making an echo sounder profile along the channel. Such a profile has a mean value relative to a chosen datum level; this mean determines the average depth of the channel. This average depth may be somewhat more than that plotted on sea charts as has already been indicated.

The depth variations which have been recorded as a function of distance along the channel are actually encountered (passed over, we hope!) by our ship as a function of time. The time scale is dependent upon the ship speed. With this transformation accomplished, the sea bed can be treated in a way analogous to a wave record and its energy density spectrum, denoted by $r(\omega)$ can be determined. The ship speed will be instrumental in determining the frequencies in this spectrum, of course. As a ship moves faster over a given bottom the spectrum $r(\omega)$ will be shifted toward higher frequencies. Just as with the previous spectra, $r(\omega)$ will have units of m$^2$ s.

Different bottom configurations will have different spectra, of course. A channel bed consisting of long sand waves - megaripples - will have a very low frequency spectrum relative to the ship response spectrum. On the other hand, a very jagged bed - blasted rock, for example - can have most of its variance concentrated at higher frequencies.

Before leaving this subject of bottom roughness, a few additional comments about channel depth measurements might be appropriate. The echo sounder profile used to determine the roughness may well fail to yield a true representation of the bottom. The echo sounder measures distances with reference to a hydrophone in the bottom of a moving boat. Motion of the boat caused by waves cannot be distinguished on the record from sea bed irregularities. The echo sounder measurement is also dependent upon the speed of sound in water, and this, in turn, depends upon the water temperature and salinity.

$^*$ The reason for this will become obvious later in section 6 of this chapter.
All in all, a number of measuring errors are often included along with the actual bottom roughness and will show up in its spectrum. This is not serious, usually, as long as engineers realize that the optimization computations which follow will no longer represent the absolute truth.

4.6 Keel clearance variations

Now that the spectrum of movement of a representative deepest point of a ship, $R(\omega)$, the spectrum of water levels, $L(\omega)$ and the channel bottom roughness spectrum, $r(\omega)$ have all been determined, these can easily be combined to get a keel clearance spectrum, $e(\omega)$:

$$e(\omega) = R(\omega) + L(\omega) + r(\omega) \quad (4.03)$$

The addition is carried out for each frequency individually (ordinates are added) and the resulting keel clearance spectrum is once again a function of frequency. This spectrum describes only the variance of the keel clearance about its mean value and the frequencies involved; it says nothing about the actual average keel clearance provided. Of course, this average keel clearance has been used in determining $e(\omega)$ but it does not show up itself in the resulting spectrum.

In order to carry out the necessary further statistical operations, it will be necessary to express the information contained in the spectrum $e(\omega)$ in a more convenient form. Review of chapter 11 of volume I indicates that some sort of number of encounters, $\eta$, and some characteristic motion ($H_{sig}$ was used there) are needed. An average period of encounter $T_m$ is an acceptable substitute for $N$.

The amplitude of the displacements represented by the spectrum can best be characterized by its standard deviation:

$$\sigma_e^2 = \int_0^\infty e(\omega) \, d\omega \quad (4.04)$$

$\sigma_e^2$ is actually nothing more than the area under the spectrum graph. This area is often denoted by $m_0$, a zeroth moment, in the literature.

The average period between relative maxima of the function represented by $e(\omega)$ is — according to Rice (1944-1945):

$$\frac{1}{T_m} = 2\pi \left( \frac{m_2}{m_0} \right)^{\frac{1}{2}} \quad (4.05)$$

where the $j$th moment, $m_j$, is defined as:

$$m_j = \int_0^\infty \omega^j e(\omega) \, d\omega \quad (4.06)$$

Equation 4.04 is a special case of (4.06).

* Make the computations your slave rather than become a slave to the calculations.
The relative maxima referred to above is defined as all maximum values of the keel clearance variation irrespective of their absolute levels or whether a negative minimum has occurred between them. In this way, all possible occurrences of extreme values in the keel clearance are included. Figure 4.2 illustrates the definition of $T_m$ based upon a suggested keel clearance variation record.

Figure 4.2
KEEL CLEARANCE VARIATION WITH DEFINITIONS

- : maxima
\$\nearrow\$: upward zero crossing
$T_m$ : interval between maxima
$T_o$ : interval between zero crossings

Given $T_m$, the number of encounters of extremes in the keel clearance, $N$, can be determined by dividing the length of time the ship is in the channel by $T_m$.

Moments can be used to determine one additional handy bit of information concerning the keel clearance behavior. This information is the spectrum width, $\varepsilon$:

$$\varepsilon^2 = 1 - \frac{m^2}{m_0 m_4}$$  \hspace{1cm} (4.07)

If $\varepsilon = 1$ then the extremes of the keel clearance record represented by the spectrum $e(\omega)$ can be described by a Normal Distribution. If, on the other hand, $\varepsilon = 0$, then a Rayleigh Distribution is more appropriate for describing the keel clearance extremes.

For the problem at hand, we can expect the value of $\varepsilon$ to be rather near 0 since each of the components on the right hand side of (4.03) individually represents a function whose extremes are pretty much Rayleigh distributed.

It is appropriate to reflect for a moment upon the relative influence of each of the components of (4.03) on the parameters just derived.

Since $\sigma_e$ is determined only by the area under the spectrum curve of $e(\omega)$ and this area is equal to the sum of the areas under each of the component spectra, then the frequency distribution of these spectra (and thus of $e(\omega)$) plays no role in the $\sigma_e$ value.
Indeed, it was not even necessary to resort to the use of a keel clearance spectrum if only a $\sigma_e$ value was desired. The relative importance of each component further is determined by its individual spectrum value relative to the others. A very rough channel bed will obviously play a more important role in the keel clearance variance than a smoother bed.

The two remaining parameters, $T_m$ and $c$, depend upon moments about the line $\omega = 0$ as represented by equation 4.06. Further consideration of the behavior of that equation shows that for $j > 0$ spectrum area elements at relatively high frequencies play a more important role in $m_j$ than do equivalent area elements at low frequencies. Also, this high frequency dominance becomes more pronounced as $j$ increases.

This explains, first of all, why it was suggested back in section 4.4 that it was rather unimportant what value of $\Delta_0$ was used in equation 4.02; the spectrum component area $L(\omega) \Delta_0$ remains constant and is plotted so close to the axis $\omega = 0$ that it usually plays only a negligible role in the determination of $m_j$ for $j > 0$.

With the above knowledge about moments and equations 4.05, we see that the higher frequency spectrum values will be the most influential in determining $T_m$ and then primarily via the $m_4$ factor. Carrying this reasoning through, $T_m$ decreases as the total energy of the keel clearance spectrum shifts toward higher frequencies. Thus, the highest frequency components of $e(\omega)$ determine $T_m$ to a great extent, making this value smaller and thus the number of encounters, $N$, larger.

If the sea bed is very jagged, the highest frequency components of $e(\omega)$ will come from $r(\omega)$ and the bed roughness will greatly influence $T_m$. On the other hand, if the channel bed has only long megaripples, then $T_m$ will be essentially determined by the ship motions.

The value of $c$ in equation 4.07 is a bit more complex to analyze. However, the value of $c$ will increase as the frequency range of the spectrum $e(\omega)$ increases. Since the lowest frequency is always near zero (from $L(\omega)$) the highest frequency in the keel clearance spectrum will, in our case, determine $c$. Unless the bottom roughness introduces relatively high frequencies into $e(\omega)$, it can be expected that $c$ will be pretty close to zero so that the Rayleigh Distribution will adequately describe the statistical properties of the extremes of the keel clearance.

Before examining these extremes, the properties of the Normal and Rayleigh Distributions will be reviewed in the following section.
4.7 Properties of Normal and Rayleigh Distributions

As review, properties the Rayleigh Distributions as well as the Normal Distribution are summarized below. In the equations and in table 4.1 x is a parameter which has been made dimensionless by dividing by the standard deviation, $\sigma$.

For the Rayleigh Distribution:

$$ P(x) = \frac{1}{2} e^{-\frac{1}{2} x^2} $$  \hspace{1cm} (4.08)

and for the Normal Distribution with zero mean:

$$ P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} q^2} dq $$  \hspace{1cm} (4.09)

where in both distributions:

$P(x)$ is the chance that a value x is is equalled or exceeded.

Values of $P(x)$ versus x are listed in table 4.1.

<p>| Table 4.1 Properties of Normal and Rayleigh Distributions |
|----------------------------------|-----------------|</p>
<table>
<thead>
<tr>
<th>$P(x)$</th>
<th>Normal</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>0.50000</td>
<td>1.00000</td>
</tr>
<tr>
<td>0.2</td>
<td>0.42075</td>
<td>0.98020</td>
</tr>
<tr>
<td>0.5</td>
<td>0.30854</td>
<td>0.88250</td>
</tr>
<tr>
<td>1.0</td>
<td>0.15866</td>
<td>0.60653</td>
</tr>
<tr>
<td>1.5</td>
<td>0.06601</td>
<td>0.32465</td>
</tr>
<tr>
<td>2.0</td>
<td>0.02276</td>
<td>0.13534</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00621</td>
<td>0.04394</td>
</tr>
<tr>
<td>3.0</td>
<td>0.00135</td>
<td>0.01111</td>
</tr>
<tr>
<td>3.5</td>
<td>232.6x10^{-6}</td>
<td>0.00219</td>
</tr>
<tr>
<td>4.0</td>
<td>31.7x10^{-6}</td>
<td>335.5x10^{-6}</td>
</tr>
<tr>
<td>4.5</td>
<td>3.40x10^{-6}</td>
<td>40.07x10^{-6}</td>
</tr>
<tr>
<td>5.0</td>
<td>0.287x10^{-6}</td>
<td>3.727x10^{-6}</td>
</tr>
<tr>
<td>6.0</td>
<td>986.6x10^{-12}</td>
<td>0.0152x10^{-6}</td>
</tr>
<tr>
<td>7.0</td>
<td>1.280x10^{-12}</td>
<td>22.90x10^{-12}</td>
</tr>
<tr>
<td>8.0</td>
<td>622.1x10^{-18}</td>
<td>12664.1x10^{-18}</td>
</tr>
<tr>
<td>9.0</td>
<td>0.1128x10^{-18}</td>
<td>2.577x10^{-18}</td>
</tr>
<tr>
<td>10.0</td>
<td>7.62x10^{-24}</td>
<td>192.87x10^{-24}</td>
</tr>
</tbody>
</table>
4.8 Chance of hitting the channel bottom

Now that the statistical properties of the keel clearance variation are known, via $\sigma_e$, and $N$, we can try to determine when the ship will hit the bottom - or our diver gets in a tight spot, if you wish. This will occur whenever the keel clearance becomes zero, or expressed in another way, whenever the instantaneous keel clearance variation, $e(t)$, exceeds the average keel clearance provided, $\bar{c}$, and $N$, of course, only describe the variation of the keel clearance.

An expression for the average keel clearance is - from figure 4.1:

$$\bar{c} = h + \bar{L} - D - Z$$  \hspace{1cm} (4.10)

and the actual keel clearance at any moment is equal to:

$$c(t) = \bar{c} - e(t)$$ \hspace{1cm} (4.11)

where:

- $\bar{c}$ is the average keel clearance provided,
- $h$ is the water depth - to the average bottom level measured from some datum elevation,
- $D$ is the ship draft,
- $\bar{L}$ is the average water level relative to the same datum as $h$, and
- $Z$ is the squat plus trim of the ship's critical bottom point.

Now, all that is needed is to evaluate the chance that $e(t)$ exceeds $\bar{c}$. Before starting this exercise it should be pointed out that we are interested in the chance that the ship hits the bottom at least once during its passage through the channel. It is assumed, here, that if the ship hits the bottom at all the damage will be done; it is, therefore, unimportant how often she hits the bottom. Lastly, from the properties of probability in this application:

$$\text{chance that event happens at least once} = 1 - \text{chance that event never happens}$$ \hspace{1cm} (4.12)

Getting to work, the average keel clearance can be made dimensionless by dividing it by the keel clearance standard deviation:

$$x = \frac{\bar{c}}{\sigma_e}$$ \hspace{1cm} (4.13)

where $x$ is the dimensionless keel clearance limit which can be used in (4.08).

The chance that any single extreme value of the keel clearance variation equals or exceeds $\bar{c}$ is:

$$P(c) = e^{-\frac{1}{2}x^2}$$ \hspace{1cm} (4.14)
The chance that \( c \) is not exceeded is:

\[
1 - P(c) \quad (4.15)
\]

During the ship's passage, she experiences \( N \) extremes in the keel clearance. The chance that none of these \( N \) independent extremes exceeds \( c \) is:

\[
\left[ 1 - P(c) \right]^N \quad (4.16)
\]

Finally, the chance that our ship does hit the channel bottom at least once during her passage will be:

\[
E_1 = 1 - \left[ 1 - P(c) \right]^N \quad (4.17)
\]

\( E_1 \) is the chance that a single ship runs into difficulty in the channel by hitting the bottom during the passage and during the particular storm condition (wave direction as well as wave height) used to determine \( R(\omega) \). It has been assumed implicitly that \( c \) was chosen large enough so that the ship will satisfy other maneuvering criteria.

This exercise is far from completed. Any number of storm conditions can occur and the ship traffic density will influence the optimization as well. These factors will be included in the next section.

4.9 Ship traffic density

Suppose there are two ships in the channel instead of one. How will that influence \( E_1 \)?

A quick but incorrect answer is to say that two ships will experience \( 2N \) extreme values of the keel clearance and that \( N \) in equation 4.17 need only be replaced by \( 2N \). The error in this reasoning is that \( E_1 \) would then be the chance that both ships encounter difficulty but would neglect the possibility that one ship makes a safe passage while the other ship strands somewhere.

The correct reasoning is that if there are two ships in the channel then the chance of a ship getting into difficulty will, on the average, be twice as great. In general, if there are \( m \) ships in the channel, then the chance of damage will be:

\[
E_1 = m \quad (4.18)
\]

where \( m \) is an integer number of ships, of course.

Relation 4.18 can lead to the total chance of ships encountering difficulty becoming more than 1. At first glance this seems ridiculous, but we must remember that this will later be associated with the cost per individual ship accident so that if relation 4.18 becomes greater than one it simply reflects the fact that several ships may now encounter difficulty more or less at the same time.
The traffic density in our channel influences this computation even more, however. Above, the chance of damage for m ships in the channel during a given storm condition is computed, but what is the chance that these m ships actually are present? For a normal sort of traffic pattern, the chance, \( p(m) \), of encountering m ships in the channel at the same time will decrease as m increases. Indeed, for the occasional ship problem, for example, \( p(m) \) is nearly equal to one for \( m = 0 \) ( \( m = 0 \) is perfectly valid here, by the way, and notice what is does to (4.18); this is valid too!)

Values of \( p(m) \) versus \( m \) can usually be derived for a new channel using queuing theory and the expected ship arrival and ship servicing patterns. Generally speaking, \( p(m) \) will become negligibly small for values of \( m \) greater than some finite value \( M' \). Furthermore, it should be obvious from statistics that:

\[
\sum_{m=0}^{M'} p(m) \overset{\text{def}}{=} 1 \quad (4.19)
\]

This information, too, can be combined with the earlier result of the previous section. The chance that a design ship using our channel gets into difficulty in the chosen storm taking into account the traffic pattern expected is:

\[
E_1' = E_1 \sum_{m=0}^{M'} m \; p(m) \quad * (4.20)
\]

The term \( E_1 \) is outside the sum since it is independent of the sum index, \( m \).

If the storm conditions were constant - or alternatively, had no influence on the ships and thus the keel clearance - we would now have completed this part of the exercise. An inland canal would fit this restriction, but for major harbor approach channels, it is totally unrealistic to assume that the ships will always experience the same wave conditions. The influence of varying wave conditions will be discussed in the following section.

4.10 Varying storm conditions

The term "storm conditions" has been used in this title rather than "wave height" because more storm variables are involved in the optimization of the channel depth. Storm conditions needed to determine the ship motions are the wave direction relative to the channel axis and a wave spectrum. This latter item - a Pierson Moskovitch Spectrum for example - has at least two parameters, a wave height (or amplitude) and a wave period. The wave data that is needed for the optimization involves the statistical chance that a chosen combination of wave height, wave period and wave direction occurs.

Such extensive wave data is seldom available, and the designer must use a good deal of common sense when combining independent

\* This sum begins with \( m = 0 \) only for completeness; starting with \( m = 1 \) will not change the result.
statistical data on wave height with that on wave period and that for wave direction. For example, examination of possible wave steepnesses often leads to a correlation between wave height and wave period; the geography of surrounding land masses often leads to a relation between wave height and direction or - when wave diffraction can be important - between wave period and wave direction.

The result of this analysis of storm data will be a table listing combinations of significant wave height, $H_{\text{sig}}$, characteristic wave period, $T_0$, and wave direction, $\phi$, together with the associated chance, $p(H, T, \phi)$ that this combination occurs. This chance, $p(H, T, \phi)$, is the chance that this chosen wave spectrum and wave direction can be found at any arbitrary moment. If we specifically include the degenerate case when $H = 0$ (there are no waves), then:

$$\sum_{i=1}^{N'} p_i(H, T, \phi) = 1 \tag{4.21}$$

where:

- $i$ is an index of the values in the table of storm data, and
- $N'$ is the number of entries in this table.

$N'$ may be a rather large number, by the way.

Once this table of storm statistics has been determined, it can be easily combined with the work already accomplished. The chance that our design ship gets into difficulty in the chosen storm and that that storm occurs is:

$$E_2 = E_1' p(H, T, \phi) \tag{4.22}$$

Of course, $p(H, T, \phi)$ should have been subscripted just as in (4.21) and thus $E_2$ should also have a subscript, $i$. If this was all that was needed, we would be very happy, but such is not the case. Each of the $N'$ different sets of wave conditions causes its own ship response spectrum $R_i(\omega)$. Since the response function for the vertical motion of the critical point on the ship's bottom is also dependent upon the angle of wave approach, $\phi$, we shall also have to use a number of different response functions to determine $R_i(\omega)$ from the set of $N'$ wave spectra. Luckily, the number of response functions needed will usually be much less than $N'$; the response functions themselves depend primarily upon the wave direction and not on wave height or period.

It is also possible that a coupling exists between storm conditions and the average water level, $L$, and its standard deviation, $\sigma_L$. This not only means that the spectrum $L(\omega)$ is also coupled to the storm conditions, but that $\overline{c}$ is coupled to them via equation 4.10 as well. This means, in turn, that the response functions can also depend upon this water level and thus storm condition, since the response functions are dependent upon $\overline{c}$ (at least when $\overline{c}$ is small relative to, say, the ship's beam).

* Moment, here, implies a period of at most a few hours. Ideally, it would be long enough so that a ship could pass through the channel and short enough so that the storm conditions remain constant.
For the frequent ship problem from the harbor point of view, storm caused variations in water level will be absorbed in $Q_L$ and $L$ will remain mean sea level. In turn, $C$ will now remain constant and this last complication will become considerably less complex.

Before leaving the problem of the various storms, we might realize that it is also possible that the ship traffic density can be coupled to the storm conditions. The simplest situation would be that the harbor is closed during severe storm conditions. This might be motivated by seeing that the economic risk to ships in the channel was then too high and that paying the ship to wait is cheaper than paying to have a deeper channel dredged.

A similar coupling of traffic density and storm conditions involves a harbor of refuge into which a large number of boats might flee as a predicted severe storm were building up.

It should be obvious, by now, that the necessary computations are not, of themselves, very complicated, but that the designer is faced more with a large bookkeeping problem in order to keep all of the proper factors together.

Section 4.11 will discuss some ideas which can keep the bookkeeping within reasonable limits.

4.11 Further evaluation steps

Two items will occupy our attention in this section: How can the bookkeeping problem referred to above be reduced, and how must the results of all these computations be utilized.

Beginning with the bookkeeping, we may already realize that the major portion of the actual computation effort is concentrated in the multiple determination of the ship's response. This can be simplified by realizing, first, that the response function for a ship experiencing waves approaching from 30° off the bow, say, is the same regardless of whether these waves approach from port or starboard. This can reduce the work in computing response functions. The number of response functions can be reduced even more by neglecting the influence of small changes in average keel clearance relative to the actual average keel clearance on these response functions. The idea, here, is that a change in average keel clearance of, say, 10% will probably not change the ship's response to a given wave spectrum so very much. (The influence of a 10% change in $C$ on the chance that the ship gets into difficulty can be considerable, however, and this influence on $E_1$ may not be neglected.)

A last step to reduce the number of necessary ship response functions is to compute them for larger steps of wave angle, $\phi$. Steps smaller than 30° are usually an unnecessary luxury, but a step size greater than 45° becomes rather crude. The bookkeeping is simplest, of course, if the wave angle step used in the storm statistics (or an integer multiple of this step) is used to compute the necessary response functions.

---

* As opposed to the individual ship captain's viewpoint; see section 4.12.
** Errors introduced by such short-cuts will be brought into perspective in section 12 of this chapter.
Switching our attention to other portions of the computation and bookkeeping procedure, waves having a significant influence on the ship motion may only come from a few directions as a result of geographical restrictions, or meteorological restrictions such as monsoons. If (and this is a special case) the ship traffic pattern is totally independent of the storm conditions then the inclusion of \( m \) and \( p(m) \) in the computation can be delayed. The factor,

\[
\sum_{m=0}^{M'} m \cdot p(m)
\]

is now independent on the storms and can be factored out and included only after the sum in equation 4.23, below, has been made.

The savings in effort resulting from changes in this portion of the computation procedure are small, however, relative to those involving the ship response.

What now? The result of all of this work described above is a table containing \( N' \) values of \( E_2 \), each associated with a different storm condition for our chosen design ship. The overall chance that this ship will encounter difficulty, \( E_3 \), during a given short time interval will be:

\[
E_3 = \sum_{i=1}^{N'} E_2^i
\]  

(4.23)

Since the economists who will use the value of \( E_3 \) usually think in annual terms, the value of \( E_3 \) just found will have to be converted to an annual basis. (It is now for a single time interval during which a given ship is in the channel - see the first footnote in section 4.10.) In principle the chance that something goes wrong is the same during any of the intervals in the year. The number of these intervals, \( M \), depends upon the travel time per ship. For a transit time of, say, 1 day, \( M \) will be 1460.

The chance of damage on an annual basis, \( E_4 \), will now be:

\[
E_4 = M \cdot E_3
\]  

(4.24)

Why don't we use the annual number of design ships using the channel instead of \( M \) (as defined above) in equation 4.24? This idea is tempting but incorrect; the influence of the total traffic density and its distribution as a function of time has already been included via \( m \) and \( p(m) \) in equation 4.20. Using the number of ships again now would include the traffic density influence twice.

Just as with equation 4.18, \( E_4 \) can also be greater than 1. The explanation for this given in section 4.9 remains valid here.

Now that the overall annual chance of damage to the design ship is known, it is a simple matter* to determine the equivalent annual damage cost:

\[
\text{Annual damage cost} = E_4 \cdot \text{individual accident cost}
\]  

(4.25)

This cost will recur each year as a result of damage to our chosen design ship.

* Provided that the damage cost associated with a single accident has been determined.
What about other ships? Their dimensions, average keel clearance and response to wave action will all be different, as will the expected cost resulting from a single occurrence of damage. In theory, we should carry out all of the above computations for each type of ship entering the harbor, but a bit of common sense will lead quickly enough to a restriction to at most a few types of ship. After all, the chance that a rowboat hits bottom in a supertanker channel is near enough to zero for engineering purposes.*

The total annual cost of the channel will be the sum of the annual costs for each of the ships involved plus the annual cost of maintenance dredging for this channel.

In order to evaluate the total design (including construction costs) these annual costs will have to be converted to a lump sum amount of capital, which if invested at an interest rate, \( i \), would provide just enough money to pay the above annual costs over the lifetime, \( \xi \), of the channel. From finance, these annual costs can be converted to such a lump sum amount (capitalized) by multiplying the annual cost by a present worth factor:

\[
\text{pawf} = \frac{(1 + i)^{\xi} - 1}{i (1 + i)^{\xi}}
\]

(4.26)

where:

\( i \) is the interest rate per period expressed as a decimal, and

\( n \) is the number of payment periods.

The total cost of the chosen channel design will be the sum of its construction cost and its capitalized damage plus maintenance costs.

Now that the total cost of our (single!) chosen channel design has been determined we are still a long way from the optimum design. We must repeat all of the above work for other design choices. There are a legion of possible design variables that can be used for the optimization; the short list of possibilities which follows is not complete but, hopefully, illustrative.

The most obvious parameter in a channel depth optimization is the channel depth, itself. A depth change \( \delta h \) can do much more than cause a corresponding change in \( \bar{c} \) as might be expected via equation 4.10. Indeed, the depth change will change the squat plus trim as well as the response function for the ship so that both the average keel clearance and the keel clearance variance will change. Also, we can expect the channel dredging costs (for both construction and maintenance) to increase with increasing channel depth.

Another possibility is to change the channel alignment. This will primarily change the wave and current pattern relative to the channel. Once again, the ship motions will be changed changing the keel clearance statistics. A longer or shorter channel, by the way, will also change the number of extreme values of the keel clearance, \( N \). Construction as well as maintenance dredging costs can be expected to change as a function of channel alignment as well.

* Even if she did hit, she would have to have sunk before she hit and not as a result of hitting!
A third constructional alternative could be the construction of a breakwater. This will change the wave pattern in the channel and thus reduce the ship motion and thus the keel clearance variance. A breakwater will also change the current patterns and may reduce the costs of maintenance dredging appreciably. On the other hand, the capitalized costs of the breakwater should now be charged to the channel in order to keep the optimization comparisons honest.

Changes in channel utilization are also possible. What happens if the ships are required to travel more slowly? This will reduce squat and trim and increase average keel clearance. The wave spectrum as encountered by the ship will also change just as the frequencies in the bed roughness spectrum. The transit time will increase as will the number of extremes of the keel clearance for each ship. Some of these changes reduce the chance of damage while others increase that chance and the net result of a speed change is hard to predict beforehand.

A last independent change that will be listed here is to close the channel to certain types of traffic under certain conditions. Assuming that the total traffic remains the same, this means that this diverted traffic must be allowed to enter at some other time, modifying the ship traffic statistics.

If the channel closure conditions involve water levels (this is quite likely) then the average water level and water level variance will change as well influencing, in turn, the keel clearance statistics, while the ship delays represent a damage cost for the channel.

4.1.2 A look back.

It may appear from the above discussions that we are now capable of carrying out a "perfect" design of a ship channel. Unfortunately this is not yet the case and a number of potentially relevant considerations have not yet been included.

The channel bed in this analysis has been schematized by a single length profile while the ship passes over a band on the channel bed that is at least as wide as the ship.

Another limitation has been the assumption that one and only one point on the ship will ever hit the bottom. While this may be true for some ships - a modern sail yacht with a short deep keel - this is probably not the case for a symmetrical flat-bottomed barge.

On the latter type of ship several points may be equally critical in terms of keel clearance; this influence should be included, too. Another, and possibly related factor which has been neglected is the influence of ship length on the number of extreme values in the keel clearance. A ship of length $L$ moving with speed $V$ will be above a given (high) point on the sea bed for a time duration of $L^\prime / V$ seconds. Several keel clearance extremes could occur in this time interval, all of which are caused by the single hump on the channel bed.
Still another problem can arise if, for example, the ship motions occur at a much lower frequency than that at which bottom roughness peaks are encountered (and we hope, passed over). Another way of expressing the same idea is to consider the channel bed to have a roughness with a very short wavelength—think of a comb with its teeth pointing upward if it helps the visualization. The determination of the keel clearance statistical properties as carried out in section 4.6 includes the possibility that the ship will hit the bottom somewhere between the top points of the bottom roughness. This is usually completely correct, by the way. In the situation at hand now, however, the physical dimensions of the ship prevent it from hitting the bottom other than on the peaks of the bed roughness. Also, when the ship now hits the bottom there is at least a reasonable chance that several peaks will be more or less "mowed off" in one pass.

One way to include these influences could be to define a new "effective bed" in this case. This "effective bed" would pass smoothly through the successive peaks of the actual bed and would, of course, have entirely different statistical properties.

Why have the above-mentioned influences not been included in the earlier discussion? The general reason is that no one is sure how, exactly, these influences should be described or included; this problem occupies a significant place in the research program of the Coastal Engineering Group.

Until now, we have examined the optimization problem from the point of view of the harbor owner. There is no reason "that an individual ship captain cannot carry out a set of computations just like those up through section 4.8 in order to assess the risk to his particular ship during a particular channel passage.

This ship captain's problem is obviously an occasional ship problem, even though the channel may have been optimized as a frequent ship problem.

Why is this distinction between captain's analysis and harbor-master's analysis important? Even though the overall risk to the harbor resulting from the channel use may be quite acceptable, this risk will not be evenly distributed over the ships using the channel. The captain entering under bad conditions—a heavy storm near low water with spring tides—runs a much greater risk than he would with the same ship on a calm day at high tide. Since the captain carries the ultimate responsibility for his ship he may refuse to pass through the channel under extremely adverse conditions, even though the harbor-master has not closed the channel to him.

If many ship captains refuse to use the channel as outlined above then there must be something wrong with the optimization. The damage costs to that ship will possibly be lower—based only on a delay, now—which means that the capitalized damage cost of the channel as built will be lower than was computed during the optimization. Apparently this saving in damage costs should have been invested in a slightly deeper channel, initially.

* Try placing your finger between the teeth of the comb, above, without hitting the points.

** There is no reason in theory, at least; he may have difficulty obtaining the necessary data.
5. CHANNEL WIDTH

5.1 Introduction

In principle, an optimization technique similar to that suggested in the previous section - using a total cost basis - could be applied to the selection of an optimum channel width. Once again, the optimum would be sought by attempting to minimize the sum of construction, maintenance, and total damage costs; all of these costs should be interpreted broadly. For example, if a large tanker runs astray in a narrow channel, hits one edge and swings broadside to the channel grounding on the other edge and then sinks, the total damage cost will include the cost of:

- clearing up wreckage of the tanker,
- clearing up spilled oil,
- possible damage to fisheries from the oil, and
- costs to other ships, the harbor, and whole economy resulting from blockage of the approach channel.

These last costs may be much greater than the first items on the list and be much more difficult to predict.

The cost determinations are not the only difficulty, however. The horizontal movements of a ship underway are determined to a large extent by the actions of the helmsman - by an unpredictable (in extreme cases) human control device. This makes a correct mathematical description of the problem even more difficult than for channel depth. Even so, some attempts have been made as are indicated in the remaining sections of this chapter.

5.2 An idealized problem

In order to arrive at a reasonable mathematical description of the phenomena involved, let us consider a ship moving up a channel of constant depth and infinite width; the sea bottom is a horizontal plane covered by a constant depth of water. The helmsman's orders are to hold the ship on a path along a given straight line. Currents will, of course, influence the ship, but there are no waves. Also, there are no other ships nearby. By eliminating the edges of the channel and other ships we eliminate extraneous inputs to the pilot which might cause a panic type reaction.

The ship's position relative to the desired course line could probably be expected to follow a normal distribution, the same distribution used to describe the water surface elevation in irregular waves. The average position of the ship would correspond to the desired course line and the degree of variation in position relative to this line could be measured in terms of a standard deviation.

If a normal distribution adequately describes the ship's position, then, just as with waves, we can expect the distances between extremes of a ship's path to be described by a Rayleigh distribution. Thus, knowing the standard deviation of the ship's path and the number of extreme values to be expected during a given passage, we can calculate
the chance that a ship will exceed some given path width using the Rayleigh distribution. The mathematics follows that of equations 4.10 through 4.13 in the previous chapter. The chance that the ship's path goes more than a distance $B$ away from the course line is:

$$P(B) = e^{-\frac{B^2}{2 \sigma_p^2}}$$  (5.01)

where: $B$ is the excursion from the course line, and

$\sigma_p$ is the standard deviation of the ship's path.

The chance that $B$ is not exceeded each time is:

$$1 - P(B)$$  (5.02)

and the chance that this is not exceeded in a series of $N$ extreme values is:

$$[ 1 - P(B) ]^N$$  (5.03)

Finally, the chance that the ship goes further than a distance $B$ from the desired course line is:

$$E_1 = 1 - [ 1 - P(B) ]^N$$  (5.04)

Oldenkamp (1973) has conducted an analysis of a limited amount of data available and found that for that limited data, the Rayleigh distribution did not describe the distribution of the extreme values exactly. He observed that the ships often tended to sail close and parallel to the desired course line rather than make the extra effort to reach the desired path exactly. For his data, a parameter $\epsilon = 0.92$ described the best-fit distribution while $\epsilon = 1.00$ for a Rayleigh distribution - see Allersma, Massie (1973).

How will the keel clearance influence the ship's behavior? In general, as the keel clearance becomes smaller, the ship becomes more difficult to turn. The implication of this is that the ship becomes more "course stable" as the keel clearance decreases - the ship has a greater tendency to maintain the path it happens to be on.

This would, in turn, lead logically to a smaller number of course extremes during the passage of a given distance while these fewer extremes could be expected to be of greater magnitude as well. Thus, the standard deviation of the course would be greater, too.

This discussion has been limited to an extremely schematized problem. Factors which influence a real problem are described in the next section.

---

*This is true whether the path is correct or not!"
5.3 A realistic problem

What are the factors influencing the actual path of a given ship under real conditions? Only after all of these influences are known and adequately described mathematically will it be possible to make meaningful statistical calculations based upon theory alone.

The most obvious unrealistic limitation of the problem in the previous section comes from the channel width. How does the edge of a channel influence the ship's path? First, as the ship nears a channel edge its hydrodynamic properties change in response to the proximity of the channel slopes; its steering characteristics change. This is in addition to changes in steering ability caused by changes in average keel clearance in the channel itself. Secondly, there can be a psychological human reaction - a panic - when the ship seems about to run aground by approaching too close to the channel markers.

Other ships either moving or moored in the vicinity will influence the behavior of our ship, again in both of the ways just described above. This problem is well known to the river engineers specialized in canal navigation.

Waves can also influence the horizontal movements of ships. Granted, their influence on the largest ships will probably be minimal, but their influence on a smaller ship such as a ferry or fishing boat can be appreciable.

Finally, a very important but unpredictable influence is the skill and disposition of the pilot.

The methods available to arrive at a somewhat responsible channel design in spite of all these difficulties are outlined in the following section and in chapter 6.

5.4 Design methods

In spite of the practical difficulties of an exact mathematical description of the physical processes involved in the determination of a ship's path, three methods have been developed which can help lead to a responsible channel design.

The oldest and most widely used technique for predicting the performance of a channel bases a prediction upon experience with similar ships in similar channels. Figure 5.1, based upon figure 7 by Eden, Jr. (1971) shows acceptable and unacceptable channel depth and width combinations for a 250,000 DWT tanker. This figure is based upon simulation data - see chapter 6. Kray (1973) summarizes the state of the art nicely. The figure gives some indication of the acceptable channel dimensions based upon the dimensions of the design ship. The method worked very well in the age when ship sizes did not increase rapidly. Extrapolation of data represented by graphs such as figure 5.1 is dangerous. In the past after a slight extrapolation had been made for a new, larger ship, new data on channel adequacy was obtained and added before a design for a still larger ship was needed. The accelerated growth of ship sizes in the last decades has made such an approach useless; the experience gained with large ships has not kept pace with the demand to design channels for even larger ones. Other
methods have been sought and found to attack this large ship problem. There is, of course, enough data available to make it possible to design channels for smaller ships using past experience. The techniques described below will only be needed for small ships under special conditions.

A second method of approaching the design problem is, to reconstruct the actual proposed situation in a physical model. Such models, if large enough (scales of up to 1:50 are used), can reproduce the hydraulic situation almost exactly. The human pilot, however, cannot be reproduced on the proper scale. Both his time scale and scale of distance perception are distorted in a physical model. Even so, physical models have been used and will most likely continue to be used to at least determine the hydrodynamic characteristics of ships in channels. These characteristics may then be used as input to other analysis methods.

The third method available to evaluate a channel design makes use of a ship simulator, a device in which a pilot reacts with a computer in the same way that he would react with his environment in a real ship navigation situation. These ship simulators will be described in more detail in the following chapter.

5.5 Additional factors

So far, this discussion has been limited to the movement of the center of mass of a ship. Often, a ship sweeps out a path somewhat broader than the beam of the ship. For example, if a ship entered a harbor with a speed and cross current as indicated in figure 5.2, the actual swath of the ship would range - outside the breakwater - from a width of 110 m to a maximum of 232 m while the ship is only 60 m wide and 300 m long, itself.

For a large ship, the athwartships force from a cross current or wind can be substantial. The following orders of magnitude are realistic for a ship of about 120,000 deadweight tons.

A cross current of 1 knot can cause an athwartships force of about $14 \times 10^5$ N. A cross wind of 20 m/s (Beaufort force 8, from volume I, chapter 4) can cause an athwartships force of about $1.2 \times 10^5$ N on a loaded ship and $8 \times 10^5$ N on an empty ship.
Figure 5.2
PATH WIDTH FOR SHIP
300M LONG, 60 M BEAM

* Value increased from 30° by moment resulting from current change.
6. SHIP MANEUVERING MODELS

6.1 Physical models

The most obvious way to study the maneuverability of a ship at sea or in a harbor is with a full scale ship under natural conditions. Since this is often impractical for a proposed ship or harbor facility, physical models are often used. Some of the requirements for such models and their associated shortcomings are described below.

Since the time scale is distorted in any physical model, (time is not reproduced on a one-to-one basis) it is impossible to include this scale in the perceptive capability of the human pilot. Additionally, because of his size, the pilot cannot always be located in the proper relative position aboard his model ship. His visual impression will be different from that which he would experience standing on the bridge of a full scale ship. If he is located outside the model ship (on the shore, for example) he will not notice small course changes as easily but will have an advantage of having a better overall view of the total situation. Even when the pilot’s head has been brought into the proper relative location on a ship model, he still has a relative advantage because his visual depth perception is more sensitive at the shorter distances encountered in the model. With normal binocular vision, distances can usually be estimated rather well up to about 200 m. Obviously, much more distance information is present within a range of 200 m on a model than in prototype. This binocular vision benefit in a model can be compensated by covering one of the pilot’s eyes.

Table 6.1 illustrates the characteristic dimensions of a model of the supertanker “Esso Atlantic”. One sees that these ships are large - even on a model scale of 1:50; they have a higher displacement than many pleasure yachts. On the other hand, such supertankers have relatively little power, a yacht of size comparable to the 1:50 model would have a few hundred times as much power.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Deadweight (tons)</th>
<th>Displ. (tons)</th>
<th>Length (m)</th>
<th>Beam (m)</th>
<th>Depth (m)</th>
<th>Draft (m)</th>
<th>Power (kw)</th>
<th>Speed (kt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>508731</td>
<td>670000</td>
<td>406.6</td>
<td>71.0</td>
<td>31.2</td>
<td>25.0</td>
<td>33570</td>
<td>16</td>
</tr>
<tr>
<td>25</td>
<td>32.56</td>
<td>42.88</td>
<td>16.2</td>
<td>2.84</td>
<td>1.25</td>
<td>1.0</td>
<td>.430</td>
<td>3.2</td>
</tr>
<tr>
<td>50</td>
<td>4.07</td>
<td>5.36</td>
<td>8.13</td>
<td>1.42</td>
<td>0.62</td>
<td>0.5</td>
<td>.038</td>
<td>2.26</td>
</tr>
</tbody>
</table>

*On a large tanker, even the bow of the ship is more than 200 m away.*
6.2 The simulation approach

Ship simulators are machines upon which a pilot can experience the maneuvering of a given ship in a given situation. While physical models, just described in the previous section, satisfy this definition, we shall restrict ourselves, now, to machines upon which the human element can be handled on a natural time scale. A similar, perhaps better known simulator of the type of interest here, is the aircraft flight simulator used to train airline and military pilots. The basic property of all simulators of this type is that the pilot is made to think he is piloting an actual ship under real conditions. How this is accomplished with less than an actual ship is explained in the following section.

6.3 Description of ship simulator

The description which follows gives the general features of the more advanced ship maneuvering simulators available. There are as many differences in detail as there are individual simulators; no particular simulator is described here.

The most obvious part of a ship simulator is a full sized ship's bridge complete with all the amenities such as chart table, compass, radar, other navigation instruments, and perhaps even the coffee pot; all of the instruments work. For the moment, our pilot is on a real ship's bridge on a moored ship in a thick fog for nothing can be seen - yet - looking off from the bridge.

A second, much less obvious (in fact seldom seen), component of the simulator is a large computer. Both hybrid and pure digital computers have been used in the past; digital computers have developed sufficient computational speed recently to be winning the application contest at the moment. The computer is connected to all of the bridge instruments gathering input from the ship's bridge controls and guiding the read-out instruments accordingly. For example, when the helmsman turns the steering wheel to port, the rudder angle indicator is changed accordingly by the computer. Further, the control measures ordered by the pilot are used as input for the computation of changes in the speed and heading of the ship. These changes are also reported - at a proper time scale - to the pilot via his instruments on the bridge. The coordinates on the position determining system and the compass and speed indicator all reflect the ship's response to the pilot's orders. How is this response determined for a given ship in a given channel? This is just exactly the relationship or set of relationships needed for a direct statistical analysis suggested back in chapter 5! Because these relationships cannot, in general, be analytically predicted, the only hope is to determine them empirically based upon either prototype or physical model studies. Indeed, a series of standard tests, such as a zig-zag test, routinely carried out during trials of a new ship can be used to provide many of the necessary coefficients or relationships. More specific effects such as the influence of the proximity to the channel side slopes are best determined by conducting a series of "standard" maneuvers with a
physical model. With all of these relationships available, the computer can determine the path of the ship and change the instrument readings accordingly. The specific empirical nature of the resulting relationships makes theoretical statistical analysis unrewarding. The pilot is now navigating his ship but is still operating in a dense fog; he can see nothing looking out from the bridge.

Some use only the two components described above. The more sophisticated simulators, however, add a spatial dimension to the surroundings. The more successful of these additions to the system project an image of the surroundings - buoys, coast, navigation lights, etc. on a screen surrounding the bridge. This projected image is, of course, also modified continuously to correspond to that seen by someone standing on an actual moving ship bridge. The best simulators generate this image of the surroundings by projecting the shadow image of a physical model on to the surrounding screen. A small but intense light source occupies the same position in the model as the ship's bridge. The light moves relative to the model (actually the model is moved relative to the fixed light) in order to modify the projected image. Thus, a turn of the ship results, in fact, in an opposite rotation of the model. Such a projected image removes the pilot from the "fog bank" and provides valuable extra visual data to the pilot. The realism is made even more complete by projecting an image of the bow of the ship - as seen from the bridge - onto the same surrounding screen. In principle, it is simple enough to project images of other ships in order to simulate various ship traffic conditions.

The computer controls this projection model as well, of course. In addition, the computer can compile statistics of the simulation run during its progress. The standard deviation of the actual ship's path relative to the described path can be determined, for example.

6.4 Ship simulator uses

Such a ship maneuvering simulator has many uses. The most obvious is probably the training of new harbor pilots just as the airlines use flight simulators. Unfortunately, the high expense of such a simulation facility have prevented its getting much use for this purpose.

The simulator can be used to evaluate ship movements during approaches to a proposed harbor. This is, indeed, how we as harbor designers can utilize the simulator most effectively. Alternatively, the behavior of a new type of ship approaching an existing harbor can be simulated. The data obtained from a number of "trials" with such a simulator can provide much valuable data for the evaluation of a whole harbor layout as well as the approach channel.

A simulator has even been used in the offshore industry to develop an optimum tugboat deployment and operation strategy to position a large gravity structure and hold it in position in the North Sea while it was sinking on to the sea bed.
Even situations not commonly encountered (luckily) such as mechanical failures of the steering gear can be simulated. "What to do if." strategies can be developed from experience gained in this way.

The data reported by Oldenkamp (1973), referred to in the previous chapter, was even obtained on a simulator.

6.5 Critical remarks

Ship maneuvering simulators have made it possible to obtain much useful data on the behavior of a given ship under given conditions. This can be invaluable for the evaluation of harbor designs. Unfortunately, the simulation results still have some limitations.

Not all possible inputs to a simulation model are included. Effects of wave action, so important for smaller ships especially, are seldom if ever included, for example. Most simulators do not provide for the pilot to call on tugboats for assistance. It is doubtful whether the effects of, for example, varying the side slopes of an approach channel are accurately enough determined in a physical model for inclusion in successive simulation runs.

Even if their results do not represent the "absolute truth," simulations can, of course, still provide valuable information to the designer.

In some cases, a designer will be tempted to improve the handling characteristics of ships using his proposed harbor as an alternative to designing a much wider channel or harbor entrance. Data on what can be expected from tugboat assistance is provided in chapter 7.
7. MANEUVERABILITY IMPROVEMENT

7.1 Motivation

Often a harbor designer is faced with a decision concerning two design alternatives: A "modest" channel in which at least some ships will present too high a risk of mishap caused by maneuverability problems, or a "spaciously" dimensioned channel in which all ships can navigate safely. The second alternative may look very attractive until the capital costs of such an extensive harbor and channel area are computed and the port facilities planner starts protesting.

Methods to make the first of the above alternatives attractive are the subject of this chapter. A couple of methods have already been hinted at: modify the channel alignment to make maneuvering easier by, for example, reducing cross currents. A second, but expensive alternative solution is to construct breakwaters in order to block or re-direct cross currents in order to reduce their detrimental effects. In some places even special wind screens have been built - but only within a harbor - to reduce wind effects on maneuvering, slow-moving ships.

Why do ships encounter maneuvering difficulties? As has already been pointed out in chapter 5, a cross current can cause a ship to sweep out a wider path than normal. As the ship speed through the water decreases, this influence becomes more pronounced. The relatively low forward speed of ships in and near harbors makes their rudders less effective as well. This reduction is even worse when the propeller is stopped in order to slow down the ship more rapidly. Since the rudder is often located just aft of the propeller, loss of its jet also reduces rudder effectiveness. If a ship throws the propeller into reverse to slow down even faster, there is a good chance that all steerability will be lost. Indeed, when a large tanker (200,000 DWT) makes an emergency stop from a speed of about 15 knots (7.7 m/s), it will have a stopping distance of about 2.5 nautical miles (4.6 km) and will most certainly not remain on course*. A much more practical alternative where there is adequate space, is for a tanker captain to call for full speed ahead and put the helm hard over making a controlled U turn. In this way, he can successfully avoid an obstacle only a little more than a kilometer ahead. Obviously, such a maneuver is not practical in a channel.

---

*The stern of a ship equipped with a right-hand (clockwise turning) propeller will tend to swing to port when the propeller is turning in reverse. Thus, the bow of the ship swings to starboard if the ship is still moving ahead.
7.2 Tugboat assistance

One alternative open to a ship captain navigating in a restricted waterway is to enlist the assistance of tugboats. Only small ships can utilize tugboats as "brakes" effectively. For large ships (the definition of large depends somewhat on the tugboats available, but 50,000 DWT is always a large ship) tugboats can be most efficiently used to counteract cross-current influences and hold the ship on course, generally. With the steering task taken over by the tugs, the ship can reverse its propeller, if necessary, in order to decelerate more rapidly.

Figure 7.1 shows stopping distance data for three types of tankers based upon field observations at Rotterdam.

How much tugboat power is needed? In Europort, large tankers are usually supplied with a total of about $7 \times 10^5$ N total pulling force usually distributed over at least four tugboats. This means that the tugs will be rather large - as harbor tugs go - somewhat more than 2000 kw power. Table 7.1 lists some reference data on various types of tugboats. The data listed for a supertanker indicates how underpowered they are!

What are the operational problems with tugs? The most important handicap of most tugs is that a towing line must be passed between the ship and the tug underway. The highest possible speed at which a tugboat captain dares attempt such an operation is about 6 kt (3 m/s) under ideal conditions. The speed must be reduced still further to about 3 kt (1.5 m/s) before the tugs can assist the ship effectively. Because of their extreme maneuverability, tugboats with Voith-Schneider propellers are somewhat more effective at higher speeds. Indeed, such tugs can move at full speed or pull in any direction; this advantage offsets their relative inefficient use of power reflected in table 7.1.
Table 7.1  Tugboat and ship performance data

<table>
<thead>
<tr>
<th>Type</th>
<th>Towing force</th>
<th>Power displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>power</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(N/kw)</td>
<td>(kw/ton)</td>
</tr>
<tr>
<td>tug with normal propeller</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>tug with propeller in tunnel</td>
<td>210</td>
<td>4</td>
</tr>
<tr>
<td>tug with Voith-Schneider Propeller</td>
<td>135</td>
<td></td>
</tr>
<tr>
<td>large sea tug</td>
<td>170</td>
<td>5</td>
</tr>
<tr>
<td>&quot;Smit Rotterdam&quot;</td>
<td>-</td>
<td>3.76</td>
</tr>
<tr>
<td>supertanker</td>
<td>-</td>
<td>0.075</td>
</tr>
<tr>
<td>Navy Destroyer</td>
<td>-</td>
<td>11.5</td>
</tr>
</tbody>
</table>

The operation of transferring a towline to a tug also costs time which can be translated into distance progressed along the channel. A recent Japanese invention allows the tug to make fast directly to the side of the ship using a giant suction cup. It remains to be seen whether this will prove to be effective and sufficiently dependable.

The problem of reducing the necessary stopping distance within a harbor might be alleviated by extending tugboat assistance offshore so that deceleration could be started sooner. Unfortunately, wave action can handicap the operation of a harbor tug at sea. Not only is the transfer of a tow line more dangerous and time-consuming, the motion of the tug in the waves can impose undesirable shock loads on the towing cable and its hardware. Kroese and Nieuwenhuysse(1974) have examined this problem in some detail.

Also, of course, tugboats must be available to be utilized and they cost money to rent. When the travel time of a ship is of high importance and harbor maneuvers are carried out frequently, an alternative to the use of tugs is often economical.
7.3 Bow thrusters

Fast ships carrying expensive cargos calling at many ports can find the delays and rental costs of tugboats to be too much of an economic burden on their overall operation. Container ships are an example of the type of ship involved.

Such ships often have bow thrusters in addition to twin propellers, both of which can improve maneuvering characteristics when compared to single screw sister ships. A bow thruster is a propeller mounted on an axle placed crosswise of the ship in a tube extending through the ship deep in the water well forward. Such a thruster can exert a direct crosswise force at the bow to help in maneuvering and berthing operations. Of course, such bow thrusters represent a capital investment for a ship; they are usually only economical on ships such as container ships.

In the following chapter, we re-examine the entire problem of optimum channel design working from the components presented in this and the previous chapters.
8. TOTAL CHANNEL OPTIMIZATION

8.1 Introduction

The previous four chapters have discussed aspects of harbor approach channel design with primary emphasis on depth in chapter 4 and on width in the remaining three chapters. In fact, the depth and width of a channel are very strongly interrelated; it is the purpose of this chapter to point out this interrelationship and develop the insight into a total channel optimization.

8.2 Definition of total optimum

In general in a competitive society, one seeks the cheapest supplier for particular goods and services on the marketplace. Wise investors consider more, however, than just initial cash expense, especially when an investment is at all substantial. Thus, an optimum channel design will be that one which is cheapest over a long term - its useful life - and not necessarily the design representing the lowest initial investment. As has, in fact, already been indicated the optimum design will involve a summation, on some legitimate basis such as capitalized value, of the costs of constructing, maintaining, and insuring against damage to shipping. Each of these topics is treated separately below with emphasis on the interrelationships between factors presented in chapters 4 through 7.

8.3 Construction costs

Construction costs of a particular channel are largely determined by the site conditions such as soil material, wave and tidal conditions, and method and location of dredge spoil disposal. While these factors are very important to a project, they usually do not vary significantly as alternative designs for the same project at the same site are being compared. Granted, the construction of a breakwater will make the wave climate in the channel more moderate and this in turn can reduce the unit cost of dredging, but such savings alone are not sufficient to justify the capital cost of the breakwater. Additional savings, accruing from reduced maintenance dredging or a smaller channel can, of course, justify the breakwater.

Even without a breakwater, both the depth and width of a channel influence its initial cost as well as its maintenance cost. It is conceivable that a relatively wide and shallow channel can be built for about the same initial investment as a deeper but narrower channel.
8.4 Damage costs

The insurance model for damage costs has been adequately described in chapter 4. Of importance, here, is only the realization that these costs, too, are related to channel width and depth in a rather complex way.

The relatively wide shallow channel suggested in the last sentences of the previous section would need its extra width, perhaps, to compensate for the poor maneuverability of the ships resulting from the low average keel clearance. Increasing the depth of the channel would improve maneuverability most likely making a width reduction possible while maintaining the same annual damage cost.

8.5 Maintenance costs

In addition to the construction and damage costs – the only two mentioned in chapter 13 of volume I we must, for a channel, include the costs of routine dredging necessary to maintain the channel dimensions. Since we can intuitively feel that the quantity of maintenance dredging will be related to the channel geometry, a prediction of this maintenance dredging will be necessary in order to complete the evaluation of particular channel alternatives.

The prediction of such sedimentation is an extremely complex problem in itself. If we consider a sediment-laden current crossing a channel the abrupt change in hydraulic conditions – waves, currents, depth, perhaps even water salinity near a river mouth – will cause some sort of time dependent morphological change in the channel. Local sedimentation or perhaps even erosion can be expected. It should be obvious that the classical sediment transport formulas used by river engineers will be insufficient to predict channel bottom changes in a marine approach channel. Most of the remainder of this book is devoted to the proper prediction of coastal morphological changes in general. Chapter 25 will discuss the state of the art relative to channel sedimentation.

It would be a mistake to conclude from the above that the dredging and sedimentation of an approach channel are the only morphological factors influencing a harbor design; the construction of a new harbor on anything but a solid rock coast will trigger morphological changes along a whole segment of coast. While it is not usually necessary to involve all of these changes and associated costs in an evaluation of an approach channel design, they must most certainly be included in the evaluation of the total harbor project.
9. COASTAL SAND TRANSPORT

9.1 Introduction

Sediment transports are of utmost importance in coastal engineering. In many coastal engineering problems the quality of a proposed solution is dependent upon quantitative estimates of erosion and accretion.

Waves and currents along with the physical properties of the bottom material, together, determine the rate of material transport in the coastal zone. This transport rate, its variations and resulting coastal changes, are of importance for the prediction of both natural coastline changes and the influence of man-made structures on the coastal zone. Even away from the coastal zone, sediment transport problems can be important; scour occurring near offshore structures or pipelines can play a significant role in their stability.

The sediment transport process may, in general, be divided into three steps:

a. The stirring-up of bottom material bringing it into suspension in the water above, or to loosen this material from the bottom.
b. The horizontal displacement of these particles by the water, and
c. The sedimentation of these particles once again.

Usually, of course, we are interested in the effects of sediment transport on some given bottom area. It should be obvious that a continuity principle can be applied to a volume extending from the given bottom area to the water surface as shown in figure 9.1. The resulting erosion or accretion of the bottom can be determined once the resulting sediment transport through the vertical boundary of the volume is known. Combining this knowledge with the steps a to c mentioned above, we see that only step b is really important; in principle we need not concern ourselves with either of the other two steps as separate problems. Our main interest is, then, the horizontal displacement of individual material particles through a given cross section in a given time.

![Figure 9.1: Principle Sketch of Continuity](image-url)
For many problems the distribution of the sediment transport over a vertical profile is immaterial for the resulting bottom changes; then, knowing the material properties such as void ratio as well, a sediment transport can be expressed in terms of volume of material per unit width per unit time - [L$^3$/LT].

It would be wonderful to get a simple theoretical expression for such a sand transport in terms of physical wave, current, and material parameters. Unfortunately, no one has yet (1977) been entirely successful at this; we can, however, develop a conceptual model via which sediment transport formulas can be derived.

9.2 Concept of formulas

Figure 9.2 illustrates the problem to be solved. We wish to determine the volume rate of sediment transport through a unit width of the y-z plane extending from the bottom, z = -h, to the water surface, z = η. In general, neither the wave nor the current direction need coincide with the given axes.

The sediment transport through the plane shaded in figure 9.2 can be expressed as:

$$S_x = \frac{1}{t'} \int_{-h}^{\eta} \int_{0}^{t'} c(z,t) \cdot u_p(z,t) \, dt \, dz \tag{9.01}$$

where $c(z,t)$ is the instantaneous concentration of material in suspension expressed in units of volume of deposited bottom material per unit volume of (flowing) water. Wave action causes rapid variations in $c$, while bottom elevation changes affect it more slowly.

$h$ is the local water depth,

$S_x$ is the sediment transport rate expressed in units of volume per unit width and time,

$t$ is the time,

$t'$ is a period over which the integration is carried out, and

$u_p(z,t)$ is the instantaneous $x$ component of the velocity of the sediment particles passing through the plane; this results from both wave and current influences, and

$\eta(x,y,t)$ is the instantaneous water surface elevation.

In the above, any variations in the parameters over the unit width have been averaged out. The time, $t'$, used in equation 9.01 should be long enough to average out the effects of irregularities in the waves and is, thus, much longer than a single wave period.
The principle just expressed is simple enough; major difficulties arise, however, when we try to evaluate the functions \( c(z,t) \) and \( u_p(z,t) \) for substitution into equation 9.01. Indeed, much of the rest of this book will be devoted to the determination of acceptable means of predicting the two above functions in terms of known, measurable parameters.

### 9.3 Simplifications of the Process

Equation 9.01 includes both \( c(z,t) \) and \( u_p(z,t) \) within the integral, since, in general both terms are functions of both variables. However, it can often be practical for real problems to attempt to simplify equation 9.01. Such a simplification is possible, for example, when longshore transport in a surf zone is considered.

Within the surf zone of a beach the angle of wave attack, \( \phi_{br} \), is always small. Even though the wave angle in deep water, \( \phi_0 \), may be large, refraction outside the breaker zone will reduce this angle considerably. For example, with a deep water wave height, \( H_0 \), of 2m, an angle, \( \phi_0 \), of 30° and a period of 7 s, \( \phi_{br} \) is only 13.3°.
This results in a velocity pattern in the breaker zone shown qualitatively in figure 9.3. If attention is focused on the longshore sediment transport direction then it is obvious that the waves cause only a small variation in the longshore current velocity and thus $u_p(z,t)$ in (9.01) is essentially independent of time, $t$. This simplification allows (9.01) to be written as:

$$S_x = \int_{-h}^{h} u_p(z) \left[ \frac{1}{T} \int_0^T c(z,t) \, dt \right] \, dz \quad (9.02)$$

The concentration, $c(z,t)$ also presents problems. Little is known about the concentration in a wave field varies during a wave period; one of the major problems is the lack of reliable measuring instruments. Luckily, much more is known about the time average concentration, $\bar{c}(z)$, in waves. Introduction of a time average concentration into (9.02) yields:

$$S_x = \int_{-h}^{h} u_p(z) \bar{c}(z) \, dz \quad (9.03)$$

which is much more convenient than (9.01). The work in the next chapters is indeed based upon this simpler equation form.

9.4 Plan of attack

It may seem to the uninitiated reader that we shall be wandering far from our objective in the course of the next few chapters. This is not really so as we point out here below.

Chapters 10 deals with radiation stress, a wave phenomenon which contributes significantly to the hydrodynamics in the coastal zone. After the general discussion of chapter 10, a specific radiation stress component is examined in chapter 11; it is responsible primarily for an increase in the still water level along a beach. In special cases, however, this wave set-up can also result in a longshore force component which influences a current along the coast in the breaker zone - the longshore current. The discussion of these special cases is postponed to chapter 16, however.

Chapter 12 treats another radiation stress component which is nearly always a significant contributor to the driving force of the longshore current. Other, usually less significant force components, needed for dynamic equilibrium of the water in the breaker zone are discussed briefly in chapters 13 and 14.

Chapter 15 details how the bed friction force under a combination of waves and currents can be evaluated. This resulting friction force is especially important in the breaker zone.

Chapter 16 attempts to solve the problem of determining the longshore current in the breaker zone via an equilibrium using the results of chapters 12 through 15. This current velocity is essentially the $u_p$ needed for equation 9.01.

Chapters 17 and 18 provide historical and background information for the determination of sand transport presented in chapter 19. It should be obvious from equation 9.01 that currents found in chapter 16 will appear again in chapter 19 which finally answers the question posed in the previous section of this chapter.
10. RADIATION STRESS AND ITS COMPONENTS

E.W. Bijker

10.1 Introduction

This chapter is devoted to a presentation of the concept of radiation stress and its components which play a significant role in coastal morphological processes. This presentation will be brief; more detailed descriptions are available in the literature - Longuet-Higgins and Stewart (1962, 1964), Dorrerstein (1961), and Battjes (1977).

The theoretical results presented here will be applied to particular coastal problems in the following chapters.

10.2 Principal radiation stresses

Radiation stress is a pressure force in excess of the hydrostatic pressure force caused by the presence of waves. In reality, the radiation stress is neither a true stress (force/area) nor a true force (as implied in the previous sentence) but a force per unit length. (This results from the integration of a force per unit area over the water depth). Even so, transformations applicable to true stresses can still be used on the radiation stress; this will be demonstrated in section 10.4. Unlike hydrostatic pressure, the radiation stress is not isotropic; indeed, just as with stresses, it is associated with a given direction or plane. In this discussion, these planes will be vertical and perpendicular to the two horizontal axes, X oriented in the direction of wave propagation and Y along the wave crest. This will yield the principal stresses.

According to Newton’s second law of motion, a force is equivalent to a rate of change of momentum. A stress is equivalent to a momentum flux, and the radiation stress is determined by integrating this momentum flux of the waves over the depth. When we carry out this integration - it is a considerable task - over the depth on a plane perpendicular to the X axis, then the result is:

\[ S_{XX} = \left( \frac{2kh}{\sinh 2kh} + 1/2 \right) E \]  (10.01)

where:
- \( S_{XX} \) is the principal radiation stress component in the direction of wave propagation,
- \( h \) is the water depth,
- \( k \) is the wave number \( = \frac{2\pi}{\lambda} \),
- \( \lambda \) is the wave length, and
- \( E \) is the wave energy given by (from volume I chapter 5):

\[ E = \frac{1}{8} \rho g H^2 \]  (1-5.09)  (10.02)

in which:
- \( g \) is the acceleration of gravity,
- \( H \) is the wave height, and
- \( \rho \) is the density.
Using equation 5.07 of volume I, equation 10.01 can be expressed in an equivalent form:

\[ S_{XX} = (2n - 1/2) E \]  
(10.03)

where \( n = \frac{c_g}{c} \) is the ratio of wave group velocity to wave celerity. This latter form is often more convenient in practical use. Computation of the second principal radiation stress component acting on a vertical plane perpendicular to the wave crests yields:

\[ S_{YY} = \frac{kh}{\sinh 2kh} E \]  
(10.04)

or, expressed in terms of \( n \),

\[ S_{YY} = (n - 1/2) E \]  
(10.05)

Application of the usual approximations for deep water explained in volume I chapter 5 yields:

\[ S_{XX} = \frac{1}{2} E \]  
(10.03a)

and

\[ S_{YY} = 0 \]  
(10.05a)

In shallow water these stresses become:

\[ S_{XX} = \frac{3}{2} E \]  
(10.03b)

and

\[ S_{YY} = \frac{1}{2} E \]  
(10.05b)

10.3 Radiation stress changes

What are the factors that influence the radiation stress? Obviously, the most important parameter is the wave height, via the wave energy. In deep water, this is the only influencing factor. In intermediate water depths, the water depth, \( h \), and wave length, \( \lambda \), (via \( k \)) or simply \( n \) are important as well. In shallow water, it appears that the radiation stress depends, once again, only upon the wave energy. This is not the whole story, however, since the wave energy is now very dependent upon the water depth when wave breaking occurs.

If we now consider a rectangular element of water enclosed by four vertical principal planes shown in plan in figure 10.1, then, if the wave conditions and depth at all four planes 1, 2, 3, 4 are identical, the radiation stress components on opposite sides of the "block" shown in the figure are identical and there is no resulting force. Only if the wave conditions vary between planes 1 and 2 or 3 and 4 in that figure will there be a resultant force. Thus, we can expect the radiation to influence physical processes only in areas where wave conditions change. Such areas would, therefore, be at locations where wave refraction, diffraction, shoaling, or breaking occur.
The following example illustrates these changes in the principal radiation stresses caused, in this case, only by shoaling and breaking of the waves as they approach a coast.

A constant bottom slope, \( m \), of 1:100 will be assumed and a wave with a deep water height, \( H_0 \), of 5 m will be assumed to approach with its crest parallel to the coast. (refraction and diffraction do not enter the computation). The wave period is 12 seconds.

The wave period yields a deep water wave length of 225 m. The breaker type parameter (chapter 8, volume I) is:

\[
\frac{H_0}{\lambda_0} = 222
\]

so that the breaking parameter, \( p \), is small, implying that spilling breakers will be present. A breaker index, \( \gamma \), of about 0.5 is used, therefore.

Table 10.1 shows the computations for a series of depths.

Notice how the stresses "grow" as the waves approach the coast outside the breaker zone, and how breaking limits and reverses this growth process.

10.4 Radiation stress components

If we wish to know the radiation stress components upon a plane other than a principal plane, the usual methods of plane stress analysis can be used. The graphical Mohr's Circle analysis or its equivalent mathematical form is such a method. Such transformations will be very useful when it becomes necessary to determine stress components on planes parallel to a coastline approached obliquely by waves.
Table 10.1 Radiation stress values

\( H_0 = 5.00 \text{ m}, \ T = 12 \text{ sec}, \ y = 0.50 \)

<table>
<thead>
<tr>
<th>( h ) (m)</th>
<th>( \frac{h}{H_0} )</th>
<th>( H )</th>
<th>( E )</th>
<th>( n )</th>
<th>( S_{XX} ) (N/m)</th>
<th>( S_{YY} ) (N/m)</th>
<th>( \text{dist. from coast} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0.6670</td>
<td>0.9983</td>
<td>4.99</td>
<td>31325</td>
<td>0.5019</td>
<td>15782</td>
<td>60</td>
</tr>
<tr>
<td>125</td>
<td>0.5558</td>
<td>0.9945</td>
<td>4.97</td>
<td>31087</td>
<td>0.5064</td>
<td>15941</td>
<td>199</td>
</tr>
<tr>
<td>100</td>
<td>0.4447</td>
<td>0.9839</td>
<td>4.92</td>
<td>30428</td>
<td>0.5203</td>
<td>16449</td>
<td>618</td>
</tr>
<tr>
<td>80</td>
<td>0.3557</td>
<td>0.9666</td>
<td>4.83</td>
<td>29307</td>
<td>0.5476</td>
<td>17443</td>
<td>1395</td>
</tr>
<tr>
<td>60</td>
<td>0.2663</td>
<td>0.9380</td>
<td>4.69</td>
<td>27655</td>
<td>0.6020</td>
<td>19469</td>
<td>2821</td>
</tr>
<tr>
<td>40</td>
<td>0.1779</td>
<td>0.9142</td>
<td>4.57</td>
<td>26270</td>
<td>0.6947</td>
<td>23364</td>
<td>5115</td>
</tr>
<tr>
<td>30</td>
<td>0.1334</td>
<td>0.9160</td>
<td>4.58</td>
<td>26373</td>
<td>0.7569</td>
<td>26737</td>
<td>6775</td>
</tr>
<tr>
<td>25</td>
<td>0.1112</td>
<td>0.9250</td>
<td>4.62</td>
<td>26894</td>
<td>0.7917</td>
<td>29137</td>
<td>7845</td>
</tr>
<tr>
<td>20</td>
<td>0.0889</td>
<td>0.9343</td>
<td>4.72</td>
<td>27974</td>
<td>0.8292</td>
<td>32406</td>
<td>9209</td>
</tr>
<tr>
<td>15</td>
<td>0.0667</td>
<td>0.9778</td>
<td>4.89</td>
<td>30052</td>
<td>0.8688</td>
<td>37192</td>
<td>11083</td>
</tr>
<tr>
<td>12.5</td>
<td>0.0556</td>
<td>1.0055</td>
<td>5.02</td>
<td>31747</td>
<td>0.8993</td>
<td>41227</td>
<td>12677</td>
</tr>
<tr>
<td>10</td>
<td>0.0445</td>
<td>--</td>
<td>5.00</td>
<td>31432</td>
<td>0.9105</td>
<td>41522</td>
<td>12903</td>
</tr>
<tr>
<td>8</td>
<td>0.0356</td>
<td>--</td>
<td>4.00</td>
<td>20116</td>
<td>0.9278</td>
<td>27270</td>
<td>8606</td>
</tr>
<tr>
<td>6</td>
<td>0.0267</td>
<td>--</td>
<td>3.00</td>
<td>11315</td>
<td>0.9454</td>
<td>15738</td>
<td>5040</td>
</tr>
<tr>
<td>4</td>
<td>0.0178</td>
<td>--</td>
<td>2.00</td>
<td>5029</td>
<td>0.9633</td>
<td>7175</td>
<td>2330</td>
</tr>
<tr>
<td>3</td>
<td>0.0133</td>
<td>--</td>
<td>1.50</td>
<td>2829</td>
<td>0.9724</td>
<td>4087</td>
<td>1336</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0111</td>
<td>--</td>
<td>1.25</td>
<td>1964</td>
<td>0.9770</td>
<td>2856</td>
<td>937</td>
</tr>
<tr>
<td>2</td>
<td>0.0089</td>
<td>--</td>
<td>1.00</td>
<td>1257</td>
<td>0.9815</td>
<td>1839</td>
<td>605</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0067</td>
<td>--</td>
<td>0.75</td>
<td>707</td>
<td>0.9860</td>
<td>1041</td>
<td>344</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0044</td>
<td>--</td>
<td>0.50</td>
<td>314</td>
<td>0.9908</td>
<td>466</td>
<td>154</td>
</tr>
</tbody>
</table>

Figure 10.2a shows the Mohr's Circle for the stresses at some intermediate water depth for the element shown in figure 10.2b. Those, familiar with the pole method of using the Mohr's Circle will recognize that the pole is at \( S_{XX} \) and the stresses on a plane located at an angle \( \theta \) can be found by passing a line having the same relative orientation through the pole. The mathematical description can more easily be obtained either from a force equilibrium on the element in figure 10.2b or from the geometry of the circle. In either case, the results are:

\[
S_{XX} = \frac{S_{XX} + S_{YY}}{2} + \frac{S_{XX} - S_{YY}}{2} \cos \theta \tag{10.06}
\]

\[
S_{YY} = \frac{S_{XX} + S_{YY}}{2} - \frac{S_{XX} - S_{YY}}{2} \cos \theta \tag{10.07}
\]

\[
S_{XY} = \frac{S_{XX} - S_{YY}}{2} \sin \theta \tag{10.08}
\]

The Mohr's Circles corresponding, associated with the principal stresses computed in table 10.1 are shown in figure 10.3. The numbers adjacent to the circles give the water depth for which the circle is valid. Some of the circles are shown dashed in order to assist in differentiating them in the figure.

* Wave height governed by breaking!
Figure 10.2
MOHR'S CIRCLE ANALYSIS
Figure 10.4
COASTAL PLAN WITH STRESS ELEMENTS
10.5 Application to coastal engineering problems

Since the coastal processes to be studied in later chapters of this book can be split into components parallel and perpendicular to the coastline, it is convenient to work with radiation stress components along these axes. Figure 10.4 shows a plan view of a coastal area with principal stresses acting on an element oriented parallel to the wave crests, and normal and shear stresses on an element parallel to the coastline.

In the following few chapters various individual radiation stress components will be examined in more detail in order to explain certain specific coastal phenomena.
11. WAVE SET-UP

11.1 The phenomenon

Waves approaching a coast undergo changes resulting from refraction, diffraction, shoaling, and breaking. Since the radiation stress components are directly expressed in terms of wave parameters, we can also expect radiation stress changes and some influences of these changes. One of the simpler influences of the radiation stress changes is a change in the average water level along a profile perpendicular to the coast.

Figure 11.1 shows such a profile in which the waves approach from the left with crests parallel to the coast. (Consideration of such a special approach direction makes the mathematics considerably simpler and illustrates the principles equally well.) With this restriction, the radiation stress component of interest will be the larger principal stress, $S_{XX}$. Changes in this principal stress will exert a net resultant force on a vertical water element shown in figure 11.1. This radiation stress resultant is counteracted by a static horizontal pressure gradient resulting from a water surface slope just as a Coriolis force was held in equilibrium in volume I chapter 3. This equilibrium between radiation stress change and average water level slope yields the following first order ordinary differential equation:* 

$$\frac{dS_{XX}}{dx} + \rho g (h + h') \frac{dh'}{dx} = 0 \quad (11.01)$$

where:

- $g$ is the acceleration of gravity,
- $h$ is the water depth relative to still water at point $X$,
- $h'$ is the average water level change at point $X$ caused by the waves,
- $S_{XX}$ is the principal radiation stress component,
- $X$ is the horizontal coordinate in the direction of wave propagation, and in this case, perpendicular to the coast, and
- $\rho$ is the mass density of water.

11.2 Solutions to the differential equation

How, then, does the principal radiation stress component $S_{XX}$ vary as waves proceed from deep water to a shore? Since changes in this component are of interest, we examine the derivative of $S_{XX}$ with respect to $X$. Direct differentiation of equation 10.01 is difficult since all three variables, $k$, $h$ and $E$ can be dependent upon the horizontal coordinate $X$ for this problem. Battjes (1977) shows the algebra involved and finds the following solution for 11.01 provided that the waves have not yet broken:

* The normal force from the sloping bottom must be included!
\[ h' = -\frac{k E}{\rho g \sinh 2kh} = -\frac{1}{8} \frac{k H^2}{\sinh 2kh} \] (11.02)

where:

- \( k \) is the wave number.

Equation 11.02 is valid for the region outside the breaker zone. The resulting water level change at the outside of the breaker zone follows from the substitution of shallow water approximations and breaking conditions into (11.02) - see volume 1 chapter 5:

\[ h'_{br} = -\frac{1}{16} \frac{H_{br}^2}{h_{br}} \] (11.03)

where the subscript \( br \) refers to conditions at the outer edge of the breaker zone. The wave height and mean water depth are often proportional in the breaker zone:

\[ H_{br} = \gamma h_{br} \] (11.04)

Where the influence of the set-down \( h'_{br} \) has been neglected since \( h'_{br} \ll h_{br} \). With (11.04), (11.03) becomes:

\[ h'_{br} = -\frac{1}{16} \gamma H_{br} \] (11.05)

Thus, at the outer edge of a breaker zone there is an average water level reduction - a wave set-down - proportional to \( \gamma \) and \( H_{br} \). See figures 11.2 and 11.3. For a given deep water wave height, \( H_0 \), the exact value of this set-down will still depend upon several parameters such as beach slope and wave period via the breaker index, \( \gamma \) - see volume 1 chapter 8.

11.3 Spilling breaker solution

When spilling breakers occur, the direct relationship between wave height and water depth remains valid throughout the breaker zone. The energy decrease of the waves due to breaking must be included, however. Using the shallow water approximation for \( S_{XX} \) (equation 10.03b), and defining \( E \) as:

\[ E = \frac{1}{8} \rho g \gamma^2 (h + h')^2 \] (11.06)

the derivative of the principal radiation stress becomes:

\[ \frac{d S_{XX}}{dx} = \frac{3}{8} \rho g \gamma^2 (h + h') \frac{d(h + h')}{dx} \] (11.07)

where: \( \frac{d(h + h')}{dx} \) is the slope of the water surface relative to the beach.

Substitution of (11.07) into (11.01) and integration over the width of the breaker zone yields:

\[ \Delta h' = \frac{3}{8} \gamma H_{br} \] (11.08)

where \( \Delta h' \) is the change in average water level across the breaker zone.
Since $\Delta h'$ is positive, a water level increase toward the shore can be expected. Remembering that the average water level at the outer edge of the breaker zone is lowered (equation 11.05), the absolute average water level at the beach line relative to the condition without waves is:

$$h'_{bs} = \frac{5}{16} \gamma H_{br}$$

(11.09)

for spilling breakers, where $h'_{bs}$ is the wave set-up at the beach caused by spilling breakers. This is shown in figure 11.2.

---

11.4 Plunging breaker solution

Swart (1974) studied the form of breaking waves near a coast. He found that a "pure" plunging breaker seldom if ever occurred and introduced a parameter, $p$, to describe breakers which are partially spilling and partially plunging - see volume I chapter 8.

If we assume as a limit case that a complete plunging breaker does exist, then the entire energy of the approaching wave is transformed at once as the breaker plunges at the outer edge of the breaker zone. Just as with spilling breakers, the change in principal radiation stress is counteracted by a water level change. This time, however, this level change occurs abruptly at the plunge point (in this ideal case). A simple equilibrium yields:
\[ \Delta h' = \frac{3}{16} \gamma H_{br} \]  
(11.10)

Again including the water level drop outside the breaker zone, we find the absolute set-up at the beach line to be:

\[ h'_{bp} = \frac{1}{8} \gamma H_{br} \]  
(11.11)

where \( h'_{bp} \) is the wave set-up at the beach caused by plunging breakers. Note that this value is lower than that found for spilling breakers - equation 11.08. Figure 11.3 shows an average water level profile.

As already been indicated, a pure plunging breaker is essentially non-existent in nature. More usually, a less pronounced plunging will occur and a breaking wave will continue to propagate toward the coast from the plunge point. This will yield a wave set-up pattern more like that described for spilling breakers described in the previous section and illustrated in figure 11.2.
11.5 Special remarks

The wave set-up just discussed should not be confused with the wind set-up discussed in chapter 3 of volume I. The two phenomena are entirely different and may or may not occur simultaneously. As the names imply, wind set-up is dependent upon the presence of a wind field (with or without waves) while waves alone - on ocean swell, for example - cause a wave set-up. Further, wind set-ups occur over a longer fetch of the wind while wave set-up is purely a coastal phenomena.

If the wave conditions vary along a coast, then, of course, the wave set-up will also vary along the coast. The variation in wave conditions along the coast could be caused by refraction or diffraction or even by differences in breaker type caused, for example, by coastal slope variations. The water level differences between points on the coast will yield, obviously, a pressure gradient along the coast. This pressure gradient can form an important contribution to the driving force for the longshore current at locations where the wave conditions vary rapidly along the beach. See, also, Bakker (1971).

In addition to a wave set-up, the breaking waves set up a circulation current in the breaker zone. This phenomenon is exposed by examining the distribution of the momentum flux which yields the radiation stress over a vertical profile. Since the orbital wave motion is maximum at the surface, we can expect the momentum flux there to be greater than at the bottom. The resisting hydrostatic pressure is evenly distributed over the depth on the other hand. This yields a net coastward force at the surface and a net seaward force near the bottom. The resulting circulation is shown in figure 11.4.

Many experimental measurements of wave set-up have not agreed well with theoretically predicted values. Several explanations have been offered. Battjes (1974) ascribes some of the discrepancy to the influence of the air entrained in the water by the breaking waves. The resulting mixture of air and water has, therefore, a lower density.
Another possible influencing factor is a friction force acting between the bottom and moving water. Even though the circulation currents mentioned above are low, instantaneous friction forces, resulting from the wave orbital motion, can have a non-zero time average, and thus, can contribute an additional net horizontal force component.

The approach to the wave set-up problem solution with waves approaching at an angle to the coast is, in principle, the same as that for the case without refraction influences. Instead of the principal radiation stress component, $S_{XX}$, the normal stress component on a plane parallel to the coast, $S_{XX}'$, will be needed in equation 11.01. In the solution of that equation, one must also remember that the angle of attack, $\phi$, is also a function of distance to the shore; this makes the algebra a bit more complicated.

### 11.6 Example

Compute the wave set-up generated by the waves used to compute table 10.1 and figure 10.3. The regular waves had a deep water height, $H_0$, of 5.0 m, a period, $T$, of 12 seconds, and approached parallel to the coast. The breaker index, $\gamma$, was found to be about 0.5.

A bit of trial and error work with tables of wave functions is needed in order to determine the location of the breaker line. The results are:

$$h_{br} = 10.4 \text{ m} \quad (11.12)$$

and:

$$H_{br} = 5.2 \text{ m} \quad (11.13)$$

Knowing these values, the wave set-up at the outer edge of the breaker zone can be computed using equation 11.05:

$$h'_{br} = \left( \frac{1}{16} \right) (0.5) (5.2) = -0.163 \text{ m} \quad (11.14)$$

The resulting water level change is a set-down of 16.3 cm.

The water level change across the breaker zone follows from equation 11.08 for a spilling breaker.

$$\Delta h' = \left( \frac{3}{32} \right) (0.5) (5.2) = 0.975 \text{ m.} \quad (11.15)$$

The absolute water level at the coastline relative to a condition without waves is, then, about 81 cm.


This concludes for now our discussion of phenomena occurring in a profile perpendicular to a coast.

In the next chapters we concentrate attention on forces working along a coast and the longshore currents and sand transports which result.

* In real situations, the waves will be asymmetrical leading to the non-zero average.

** An iterative program can be conceived for a small programmable pocket calculator to simplify the computations. A generalized version of this procedure is presented in section 16.5.
12. RADIATION SHEAR STRESS GRADIENT

12.1 Introduction

In this and following three chapters we consider force components which act parallel to a coast and, as such, define the dynamic equilibrium of a water mass moving along the coast - the longshore current. The first of these force components arises out of changes in the shear stress component of the radiation stress. As was pointed out in chapter 10, we shall be interested in changes in this shear stress as the waves approach the coast under some angle, \( \phi \). Expressed in equation form, we are interested in:

\[
\frac{d S_{xy}}{dx} = f(x, H_o, T, \phi_0)
\]  

(12.01)

where:
- \( H_o \) is the deep water wave height,
- \( T \) is the wave period,
- \( x \) is the horizontal coordinate perpendicular to the coast,
- \( \phi_0 \) is the angle of attack in deep water, and
- \( f(\cdot) \) denotes some function of (\cdot).

The exact nature of changes in \( S_{xy} \) will be discussed in the following sections.

12.2 Changes outside the breaker zone

Since the waves approaching a coast first begin to change in intermediate water depths, we shall first examine the changes in the shear stress component outside the breaker zone. Bowen (1969) did this and shows the algebra involved in more detail; the basic steps will be shown here in order to develop our insight into the problem.

Adaptation of the results of chapter 10 yields:

\[
S_{xy} = \frac{S_{xx} - S_{yy}}{2} \sin 2\phi
\]

(10.08)  

(12.02)

Using trigonometry and substituting for \( S_{xx} \) and \( S_{yy} \) from (10.03) and (10.05) yields:

\[
S_{xy} = E_n \sin \phi \cos \phi
\]

(12.03)

From refraction theory:

\[
E_n c b = \text{constant}
\]

(1-9.02)  

(12.04)

or, in particular:

\[
E_n c b = E_o n_0 c_0 b_0
\]

(12.05)

where \( c \) is the phase velocity, \( b \) is the distance between orthogonals, and the subscript \( o \) refers to deep water conditions which are known and constant. Equations 12.04 and 12.05 are valid only in the region outside the breaker zone. See volume 1 chapter 9. Also from that chapter, equations 9.05 and 9.06:

*This convention will be changed later. See chapter 13.*
Substituting (12.06) in (12.05) and comparing that to (12.03) yields the startling result:

\[ S_{xy} = E_{o} n_{o} \sin \phi_{o} \cos \phi_{o} = \text{constant!} \]  \hspace{1cm} (12.07)

and hence, the driving force component proportional to \( \partial S_{xy} / \partial x \) is identically zero even though the wave conditions change outside the breaker zone. Since equations 12.04 and 12.05 are valid only outside the breaker zone, we must make a new analysis for the breaker zone; this is done in the next section.

12.3 Changes within the breaker zone

Within the breaker zone we shall begin, again, with the general relationship expressed in equation 12.03:

\[ S_{xy} = E n \sin \phi \cos \phi \]  \hspace{1cm} (12.03)

Using equation 9.05 of volume I, this becomes:

\[ S_{xy} = E n c \cos \phi \frac{\sin \phi_{o}}{c_{o}} \]  \hspace{1cm} (12.08)

Remembering the definition of \( E \) - equation I - 5.09 - and that equation 11.04 now governs the wave breaker height, (12.08) becomes:

\[ S_{xy} = \frac{1}{8} \frac{\sin \phi_{o}}{c_{o}} \rho g \gamma^{2} [ h^{2} n c \cos \phi] \]  \hspace{1cm} (12.09)

Since only the terms within the brackets are dependent upon \( x \), a brute force differentiation can be carried out:

\[ \frac{\partial S_{xy}}{\partial x} = \frac{1}{8} \frac{\sin \phi_{o}}{c_{o}} \rho g \gamma^{2} \left[ 2h n c \cos \phi \frac{dh}{dx} + h^{2} c \cos \phi \frac{dn}{dx} + h^{2} n \cos \phi \frac{dc}{dx} - h^{2} n c \sin \phi \frac{dd}{dx} \right] \]  \hspace{1cm} (12.10)

This result holds only within the breaker zone. It can be simplified, however, by making the usual substitutions for shallow water parameters described in section 5.5 of volume I. These are summarized as follows:

\[ n = 1 \hspace{0.5cm} ; \hspace{0.5cm} \frac{dn}{dx} = 0 \]

\[ \cos \phi = 1 \hspace{0.5cm} ; \hspace{0.5cm} \frac{d\phi}{dx} = 0 \]  \hspace{1cm} (12.11)

\[ c = \sqrt{\frac{g}{h}} \]

From the last line of (12.11):

\[ \frac{dc}{dx} = \frac{1}{2} \frac{g}{h} h^{-1/2} \frac{dh}{dx} \]  \hspace{1cm} (12.12)
With (12.11), the second and fourth terms in the brackets in (12.10) are zero. Substitution of (12.11) and (12.12), then yields:

\[
\frac{\partial S_{xy}}{\partial x} = \frac{1}{8} \rho g y^2 \sin \frac{\phi}{C_o} \left[ 2 h \sqrt{\frac{g}{h}} \frac{dh}{dx} + \frac{1}{2} h^2 \frac{g}{h} \frac{dh}{dx} \right] \tag{12.13}
\]

or, with a bit of algebra:

\[
\frac{\partial S_{xy}}{\partial x} = \frac{1}{8} \rho g y^2 \sin \frac{\phi}{C_o} \left[ 2.5 h \sqrt{\frac{g}{h}} \frac{dh}{dx} \right] \tag{12.14}
\]

\[
= \frac{5}{16} \rho g y^2 (g h)^{3/2} \sin \frac{\phi}{C_o} m \tag{12.15}
\]

where \( m \) is the beach slope, \( \frac{dh}{dx} \).

This last equation gives, then, the contribution of the radiation stress to the driving force parallel to the coast on an element of water of differential thickness, \( dx \), and height, \( h \).

In later chapters we shall be using a different coordinate system in order to better agree with literature on coastal morphology. This, however, will have no fundamental influence on the right hand side of equation 12.15.

* Note that \( \frac{dh}{dx} \) is negative, here, since \( x \) is positive toward the beach.
13. TIDAL FORCES ALONG A COAST

13.1 Coordinates used

In this and the following chapters, processes occurring along a coastline will be of especial significance. Until this point, however, attention has primarily been focused on phenomena occurring along a profile perpendicular to a coast or in the direction of wave propagation. A new coordinate system is chosen for the remainder of this book in order to achieve better agreement with the reference literature. The axis system can therefore be described as follows:

The x axis is horizontal and parallel to the shoreline. It is directed positively to the right for an observer standing on the beach looking out at the sea.

The y axis is also horizontal, perpendicular to the shoreline and positive in the direction of the sea. Waves approaching with crests parallel to the coast are travelling along the y axis in the negative direction, therefore. The x-y plane is usually placed at the still water level.

The z axis is directed upward from the still water level; its definition has not changed.

Axis-dependent equations picked up from earlier work will be transformed to the new coordinate system; a note reminding us of this will be included.

13.2 The one-dimensional tidal force component

The equation of motion of a tidal wave propagating along a coast line follows from long wave theory:

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} - g \frac{\partial Z}{\partial x} = \frac{g V |V|}{C^2 h}
\]  

(13.01)

where: 
- C is the Chézy friction factor,
- g is the acceleration of gravity,
- h is the water depth,
- V is the average velocity over the depth,
- x is the ordinate along the coast,
- Z is the tidal elevation, and
- t is time.

In this equation, the first three terms represent driving forces while the fourth term is a frictional resistance term.

The driving force component to be included in a longshore current determination comes from the integration of the first three terms of equation 13.01 over the depth, h, and multiplied by the density, ρ:

\[
F_{\text{tide}} = -\rho (h \frac{\partial V}{\partial t} + V h \frac{\partial V}{\partial x} + g h \frac{\partial Z}{\partial x})
\]  

(13.02)

Continuing this one-dimensional approximation, the tidal elevation can be written as:
\[ Z = \tilde{Z} \cos (\Omega t - Kx) \]  

(13.03)

where: \( \tilde{Z} \) is the tidal amplitude, 
\( K \) is the wave number \( = \frac{2\pi}{\lambda_{\text{tide}}} \), 
\( \lambda_{\text{tide}} \) is the wave length of the tide, 
\( \Omega \) is the tidal frequency \( = \frac{2\pi}{T'} \), and 
\( T' \) is the tidal period.

Similarly, the tidal velocity can be written as:

\[ V = \tilde{V} \sin (\Omega t - Kx - \phi) \]  

(13.04)

where: \( \tilde{V} \) is the amplitude of the pure tidal current, and 
\( \phi \) is the phase angle between vertical and horizontal tide - see volume I chapter 20 for an example of this phase shift.

The necessary partial differentiations of equations 13.03 and 13.04 can be carried out easily. These results can be substituted into 13.02 yielding:

\[ F_{\text{tide}} = -\rho h \tilde{V} [\Omega - K\tilde{Z} \sin (\Omega t - Kx - \phi)] \cos (\Omega t - Kx - \phi) \]

\[ -\rho g K \tilde{Z} \sin (\Omega t - Kx) \]  

(13.05)

Since this is a bit complicated, it can be simplified in an approximation by retaining only the first and last terms which are usually an order of magnitude greater than the other terms. Thus, the influence of the water surface slope along the coast and inertia are usually the most important of the tidal force terms, and (13.05) becomes:

\[ F_{\text{tide}} = -\rho h \{g K\tilde{Z} \sin (\Omega t - Kx) + \tilde{V} \cos (\Omega t - Kx - \phi)\} \]  

(13.06)

The parameters involved in equation 13.06 cannot be evaluated from tidal height measurements at a single location. Simultaneous measurement of tide level and tidal current will yield the necessary parameters. One must be careful with tidal current measurements in the coastal zone, however; as will be shown in chapter 15, the presence of waves will influence the bottom friction force acting on a steady current. This implies that the only dependable measurements of tidal currents - influenced only by tidal forces - can be made when no waves are present, or a correction procedure must be used.

The tidal force which then results from the analysis of tidal elevation and current data can be one of the driving force components for a coastal longshore current. After considering other force components in the following two chapters, several of these components will be combined in chapter 16 to determine the resulting current velocity along a coast.

* This force may be positive or negative at any given moment dependent upon the actual flow direction at that instant.
14. TURBULENT FORCES

14.1 Introduction

The previous two chapters have discussed driving force components for the longshore current; chapter 15, on the other hand, discusses a resistance force component. This current chapter concentrates on the action of turbulent forces which both drive and resist fluid motion; they tend to smooth out sharp, steep velocity profiles. We will concern ourselves, here, with a horizontal, turbulence-caused dispersion of momentum through a vertical plane parallel to the coast (the x axis) resulting from a gradient of the velocity in the y-direction, \( \frac{\partial u}{\partial y} \), where \( u \) is the x component of the velocity. This momentum transfer may be expressed as a shear stress acting on this plane.

14.2 Mathematical description

Using the theory of turbulent momentum diffusion one can express the turbulent shear stress as:

\[
\tau_{yx} = \rho \left( u' v' + \rho \varepsilon_y \frac{\partial u}{\partial y} \right)
\]

where:
- \( u \) is the velocity component along the x axis parallel to the coast,
- \( u' \) is the turbulent velocity fluctuation in the x direction,
- \( v' \) is the turbulent velocity fluctuation in the y direction,
- \( y \) is the coordinate perpendicular to the coast,
- \( \varepsilon_y \) is the turbulent diffusion coefficient, sometimes called the "eddy viscosity", and
- \( \rho \) is the mass density of water.

The "eddy viscosity" is often defined in terms of a so-called mixing length:

\[
\varepsilon_y = v' \delta_y
\]

where \( \delta_y \) is the horizontal mixing length.

Thornton (1970) explains the whole problem in much more detail. He relates both \( v' \) and \( \delta_y \) to the wave motion - the horizontal wave orbital velocity and particle displacement, respectively. (In the surf zone, both of these lie approximately along the y axis). Battjes (1975, 1976), on the other hand, relates \( \varepsilon \) to the normal viscosity of a current having velocity, \( v \).

Model measurements carried out by Swart (1974) indicate, in any case, that \( \varepsilon_y \) should have a value in the order of \( 10^{-2} \) m\(^2\)/s for model conditions. Using model scale laws, it is, in principle, possible to convert this to a prototype value.
15. BOTTOM FRICTION FORCES

15.1 Introduction

This fourth force component acting on the water in the longshore current results from the bottom friction of the beach on the water element above it; this friction force is related to the velocity in the element. However, in a breaker zone, the instantaneous velocity of the water there is dependent upon both the more or less constant longshore current and the rapidly varying velocity components in the breaking waves.

The proper description of these wave velocity components is, in itself, an impossible task; every mathematical description is only an approximation, at best. We shall later assume that the orbital velocity components can be described by simple sine functions. Even with this (probably crude) assumption it proves difficult to describe the bottom friction phenomena accurately. An attempt is made, here, to explain this rather complex phenomena. First, we shall examine the development of friction under a constant current without waves. Later in this chapter we look at friction under waves alone and finally, using the insight gained, attack combined waves and currents.

15.2 Friction in constant currents

The normally encountered expression for bottom friction in steady flow is, from elementary fluid mechanics:

\[ \tau_c = \rho \, g \, \frac{V^2}{C} \]  \hspace{1cm} (15.01)

where \( C \) is the Chézy friction factor,
\( g \) is the acceleration of gravity,
\( V \) is the current velocity averaged over the flow cross-section,
\( \rho \) is the mass density of the fluid, and
\( \tau_c \) is the (constant) bottom shear stress acting against the flow.

More generally, the shear stress acting across any horizontal plane in the fluid is:

\[ \tau = \rho \, \lambda_z \left[ \frac{d V(z')}{dz'} \right]^2 \]  \hspace{1cm} (15.02)

where \( \lambda_z \) is the mixing length,
\( z' \) is a vertical coordinate axis with origin at the bottom, and
\( V(z') \) is the current velocity at elevation \( z' \).

By making the special assumption that:

\[ \lambda_z = k \, z' \sqrt{\frac{h-z'}{h}} \]  \hspace{1cm} (15.03)

Prandtl (1926) and Von Kármán (1930) were able to solve equation 15.02 to get the well known Prandtl - Von Kármán logarithmic velocity distribution law:

* This represents a change of vertical axis origin. \( z' = 0 \) corresponds to \( z = -h \).
\[ V(z') = \frac{1}{K} \frac{V}{z'} \ln \left( \frac{z'_0}{z'} \right) \]  \hspace{1cm} (15.04)

where: \( V_\star \) is a velocity often called the "shear velocity" at some elevation (see below),
\( z'_0 \) is the elevation at which the velocity is zero, and
\( K \) is the Von Kármán constant which has been found by experiment to be equal to 0.4.

\( V_\star \) is somewhat difficult to interpret physically.

It is the velocity occurring at elevation:

\[ z' = z'_0 e^{K} \]  \hspace{1cm} (15.05)

having magnitude:

\[ V_\star = V \frac{\sqrt{g}}{T} \]  \hspace{1cm} (15.06)

which is of no special significance. Another relationship involving \( V_\star \) is:

\[ V_\star = \sqrt{\frac{\tau}{\rho}} \]  \hspace{1cm} (15.07)

and appears quite often. We shall attempt to avoid further use of \( V_\star \).

The elevation \( z'_0 \) has been related to the bottom roughness, \( r \), experimentally:

\[ z'_0 \approx \frac{r}{33} \]  \hspace{1cm} (15.08)

Below this elevation, equation 15.04 yields negative — very unrealistic! — values. Therefore, velocities near the bottom are usually described by a linear velocity profile from the origin (\( z' = 0, V(0) = 0 \)) tangent to the profile described by equation 15.04; the result is shown in figure 15.1. The elevation of the point of tangency, \( z'_t \), turns out to be:

\[ z'_t = e z'_0 = \frac{er}{33} \]  \hspace{1cm} (15.09)

where \( e \) is the base of natural logarithms. For convenience, we shall denote the velocity at this elevation by \( V_t \). From the figure, the velocity gradient at an elevation \( z'_t \) above the bottom is:

\[ \frac{dV(z'_t)}{dz'} \bigg|_{z' = z'_t} = \frac{V_t}{z'_t} \]  \hspace{1cm} (15.10)

At this same elevation, the mixing length is:

\[ L_{z'_t} = \sqrt{\frac{h-z'_t}{h}} \]  \hspace{1cm} (15.11)

using equation 15.03.

Since \( z'_t \ll h \), equation 15.11 is approximately:

\[ L_{z'_t} \approx \frac{z'_t}{h} \]  \hspace{1cm} (15.12)
Substitution of (15.12) and (15.10) into equation 15.02 yields an alternate expression for the bottom shear stress:

\[ \tau_c = \rho \kappa^2 \frac{V_z^2}{t} \]

(15.13)

* This is actually the shear stress at an elevation \( z' = z'_t \) above the bottom. This is universally accepted as equivalent to that at the actual bottom, however.
The reader may get the impression from the above formulas that a bed shear stress can easily be determined. This is not the case, since one of the more important physical parameters involved, the bed roughness, \( r \), is very difficult to measure in practice.

On the one hand, it seems logical to relate the bed roughness, \( r \), to the ripple height. On the other hand, there is evidence that in some cases a much larger scale roughness is also present than is predicted from the dimensions of individual ripples.

For sinusoidal ripples, Hinze (1962) found that the bed roughness was equal to half the ripple height. This relationship is quite different for a saw-tooth form, however.

Equation 15.13 expresses the bottom shear stress in terms of the velocity occurring very near the bottom. This will be useful for current determination both with and without waves as well as for sediment transport determinations - chapter 19.

Of course, we can express \( V_t \) in terms of \( V \) in this case if we wish. Using equation 15.06 in 15.04 yields, ultimately:

\[
V_t = \frac{\sqrt{2} \tau}{R} V \tag{15.14}
\]

### 15.3 Friction with waves alone

Jonsson (1966) carried out experiments to determine bed shear stresses under waves. He found that this shear stress, \( \tau_w \), could be described by:

\[
\tau_w = \frac{1}{2} f_w \rho u_b^2 \tag{15.15}
\]

where: \( f_w \) is a dimensionless coefficient, and
\( u_b \) is the instantaneous water velocity near the bottom.

Jonsson derived an empirical relation for \( f_w \) in terms of more readily measurable parameters: the bottom roughness, \( r \), and the amplitude of the water displacement near the bottom, \( a_b \). His relationship as rewritten by Swart (1974) is:

\[
f_w = \exp \left[ -5.977 + 5.213 \left( \frac{a_b}{r} \right) - 0.194 \right] \tag{15.16}
\]

This relation is also shown graphically in figure 15.2 and is only valid for \( 1.47 < \left( \frac{a_b}{r} \right) < 3000 \). For values of \( \frac{a_b}{r} \leq 1.47 \), \( f_w \) has a constant value of 0.32.

Both \( u_b \) and \( a_b \) are easily evaluated using short wave theory. The velocity at the bottom, \( u_b \), follows from a substitution of \( z = -h \) into equation 5.01 of volume I:

\[
u_b = \frac{\omega H}{2} \frac{1}{\sinh kh} \sin \omega t \tag{15.17}
\]

Similarly, \( a_b \) - denoted by \( \delta \) in volume I* - follows from equation 5.03 of that volume:

* The reason for this notation change will become obvious in section 15.4.
Sometimes it is acceptable to use the shallow water approximations for the two above equations. The effects of the use of the shallow water approximations will be demonstrated in the example in this chapter and again in chapter 19.

\[ a_b = \frac{H}{2} \frac{1}{\sinh kh} \]  

(15.18)
The computations of \( u_b \) and \( a_b \) just carried out neglect boundary layer effects. According to boundary layer theory we should expect the actual velocity to be zero at the bottom. A boundary layer can be expected to develop in a thin region near the bottom; the fact that it does not have time enough to develop a velocity profile over the entire depth is not important to us. Jonsson (1966), thus measured logarithmic velocity profiles in his experiments and assumed that a linear portion would develop near the bottom just as with constant currents in this same region. Continuing with an approach entirely parallel to that in the previous section, we can assume that the actual water velocity in the wave at an elevation \( z'_t \) above the bottom will be directly proportional to the bottom velocity computed in equation 15.17:

\[
\begin{align*}
    u_t &= p \, u_b \\
    \text{where } p & \text{ is a dimensionless parameter discussed further below.}
\end{align*}
\]

Bijker (1967) plugged this into an equation like (15.13) and found:

\[
\tau_w = p \, k^2 \, (p \, u_b)^2
\]

\[ (15.20) \]

where \( \tau_w \) is the bottom shear stress under the wave. This shear stress has amplitude \( \tau_w' \).

Bijker (1967) assumed that \( p \) was constant and, from a series of model tests, he indeed found a nearly constant value of 0.45 for \( p \). Later, comparison of the work of Bijker to that of Jonsson (1966) indicated that \( p \) should be a variable. It can be evaluated by equating (15.15) and (15.20):

\[
\begin{align*}
    p &= \frac{1}{k} \left( \frac{f_w}{z} \right) \\
    \text{where } f_w & \text{ is the maximum value of } p \text{ physically possible is } 1.00. \\
    \text{Values of } p \text{ as a function of } z_b \text{ are also shown in figure 15.2.} \\
    \text{Before attempting to combine the influences of waves and currents, it is useful to summarize the results so far.} \\
    \text{Figure 15.1A shows the horizontal velocity distributions under waves and in a constant current. The "standard" expressions for bottom shear stress - equation 15.01 for currents and equation 15.15 for waves - have been re-worked to equations in which the velocities at a given elevation, } z' = e \, z'_0, \text{, are used. This results in equation 15.13 for currents and 15.20 for waves.} \\
    \text{The combined effects of waves and currents will be considered in the following section.}
\end{align*}
\]

15.4 Friction with combined current and waves

In the two previous sections we have developed expressions for the bottom shear stress under currents and waves in terms of the velocities at a distance \( z'_c \) above the bottom - equations 15.13 and 15.20 respectively.

* We need only to assume that it develops in the region \( 0 \leq z' \leq z'_c \).

** \( p = 1 \) corresponds to the maximum value of \( f_w = 0.32 \) given above.
Bijker (1967) extended this to include combinations of currents and waves. He let the wave crests approach the constant current direction under an angle $\phi$. Figure 15.3 shows the constant current velocity, $V_t$ and the wave particle velocity, $p u_b$, in the plane $z' = z'_t$. Bijker simply added these velocities as vectors to obtain a time dependent resultant velocity, $V_r$, also shown. From the figure:

$$V_r = \sqrt{V_t^2 + (p u_b)^2 + 2 p u_b V_t \sin \phi}$$  \hspace{1cm} (15.22)

and:

$$\cos \theta = \frac{V_t + p u_b \sin \phi}{V_r}$$  \hspace{1cm} (15.23)

![Geometry of velocity components at elevation $z'_t$ above bottom](Figure 15.3)

Proceeding just as in the previous sections, we can now determine the shear stress under the combination of currents and waves, $\tau_{cw}$:

$$\tau_{cw} = \rho \kappa^2 \sqrt{V_t^2 + (p u_b)^2 + 2 p u_b V_t \sin \phi}$$  \hspace{1cm} (15.25)

This shear stress is now directed along the time-varying line of action of $V_r$. The current, flowing in the $x$ direction will be influenced primarily by the $x$ component of this stress, $\tau_{cwx}$. In chapter 19 the total shear stress, $\tau_{cw}$, will be important for the sediment transport; for now, however, we shall concentrate exclusively on the friction shear stress component $\tau_{cwx}$, important for the current. This component can be expressed as:

$$\tau_{cwx} = \tau_{cw} \cos \theta$$  \hspace{1cm} (15.26)

or, using (15.22) through (15.24) in (15.25):

$$\tau_{cwx} = \rho \kappa^2 \sqrt{V_t^2 + (p u_b)^2 + 2 p u_b V_t \sin \phi} \left[ V_t + p u_b \sin \phi \right]$$

in which $u_b$ is a function of time. For our purposes, however, it is sufficient to determine the time average of $\tau_{cwx}$. We do this by first writing out $u_b$:
and for convenience introducing equation 15.14 for \( V_t \). Doing this and carrying out a lot of algebra yields the following expression for the time average \( x \) component of the bottom shear stress:

\[
\overline{\tau}_{cwx} = 2 \frac{T}{4} \int_{-T/4}^{T/4} \left( 1 + \frac{Q_b}{V} \sin \omega t \sin \phi \right) \sqrt{1 + \left( \frac{Q_b}{V} \sin \omega t \right)^2 + 2 \frac{Q_b}{V} \sin \omega t \sin \phi} \, dt
\]

(15.28)

where \( \xi \) is a collection of parameters:

\[
\xi = \frac{D \cdot k \cdot C}{\sqrt{g}} = \frac{C \sqrt{f_w}}{\sqrt{2g}}
\]

(15.29)

Equation 15.28 is an elliptic integral and is, as such, not conducive to analytical solution. Bijker (1967) used a numerical procedure to evaluate the above integral for various realistic values of the independent variables, \( V, Q_b, \xi \) and \( \phi \). Then, by fitting an equation to the results obtained, he determined that for \(|\phi| < 20^\circ\):

\[
\overline{\tau}_{cwx} = \tau_c \left[ 0.75 + 0.45 \left( \frac{Q_b}{V} \right) \right]^{1.13}
\]

(15.30)

or, introducing the definition of \( \tau_c \) from equation 15.01:

\[
\overline{\tau}_{cwx} = \frac{C}{\xi} V^2 \left[ 0.75 + 0.45 \left( \frac{Q_b}{V} \right) \right]^{1.13}
\]

(15.31)

Equation 15.31 is the relation between bed shear stress and velocity that should be used in the dynamic equilibrium of - for example - a wave-driven longshore current. Unfortunately, equation 15.31 is rather inconvenient* for this since \( V \) is then an unknown.

In order to arrive at a handier solution, we introduce a restriction that the angle between wave crests and the current be small. This is usually not too bad an assumption within the breaker zone. With this basis a still cruder assumption is made, namely:

\[
\sin \phi = 0
\]

(15.32)

instead of the more common one:

\[
\sin \phi = \phi
\]

(15.33)

Introduction of equation 15.32 into 15.28 yields:

\[
\overline{\tau}_{cwx} = 2 \frac{T}{4} \int_{-T/4}^{T/4} \sqrt{1 + \left( \frac{Q_b}{V} \sin \omega t \right)^2} \, dt
\]

(15.34)

* This inconvenience will be demonstrated in chapter 16; it will lead to solutions by successive approximations.
which can be further simplified by making the assumption (usually valid in the breaker zone) that:

$$\xi u_b >> V$$  \hspace{1cm} (15.35)

Doing this allows us to carry out the integration directly yielding:

$$\frac{1}{T_{CWX}} = \frac{2}{\pi} \frac{\xi b}{V^2}$$  \hspace{1cm} (15.36)

Equation 15.36 can be expressed in another form by introducing equations 15.01 and 15.29 into 15.36 yielding:

$$\frac{1}{T_{CWX}} = \frac{D}{\pi C} \sqrt{2 g \sqrt{V} \xi b}$$  \hspace{1cm} (15.37)

A still further simplification can be introduced by using a shallow water approximation to evaluate $\xi b$ - see volume I chapter 5. Such an approximation yields:

$$\xi b = \omega \frac{H}{2 k h}$$  \hspace{1cm} (I-5.01b) (15.38)

Using the relation between breaker height and water depth:

$$H = \gamma h$$  \hspace{1cm} (15.39)

and:

$$\frac{2 \pi}{k} = \lambda = \sqrt{g h} \frac{T}{\lambda} = \sqrt{g h} \frac{2 \pi}{\omega}$$  \hspace{1cm} (15.40)

equation 15.38 becomes:

$$\xi b = \frac{\gamma}{2} \sqrt{g h}$$  \hspace{1cm} (15.41)

Substituting this relationship into equation 15.37 then results in:

$$\frac{1}{T_{CWX}} = \frac{D g}{\sqrt{2 \pi C} \sqrt{V} \sqrt{g h} \sqrt{V}}$$  \hspace{1cm} (15.42)

Equation 15.42 can be used in a dynamic equilibrium balance in order to derive a simple equation for the longshore current in the breaker zone. As already mentioned earlier, equation 15.31 would yield a more accurate result assuming that equation 15.17 were used to evaluate $\xi b$. Comparative results are shown in section 16.5.

Both equations 15.42 and 15.31 remain - indirectly - dependent upon the bottom roughness, $r$, and all of its uncertainties as indicated earlier in this chapter. Swart (1976) studied this problem for combined waves and currents by assembling a large collection of data. Two empirical relationships resulted:

$$\frac{r}{\Delta r} = 25 \frac{\Delta r}{r}$$  \hspace{1cm} (15.43)

where: $\Delta r$ is the ripple height in m,

$\lambda_r$ is the ripple length in m,

$r$ is the apparent bed roughness in m.
These equations should be used with great prudence, but they may indicate a general tendency. On the other hand, it can be just a difficult to measure $\Delta r$ and $\lambda_r$ in the breaker zone as it is to estimate $r$ directly.

15.5 Additional remarks

The equations and philosophy just presented has been used with reference to a longshore current in the breaker zone. Except for certain limitations imposed as a simplification - $\varphi \ll 1, \xi_0 \gg V$ - the procedure is general. Other important applications can be found wherever waves influence the local current velocities near the bottom. Thus, for example, the wave influence on currents in wide river mouths can also be studied using the procedure presented. The influence of waves on tidal currents in shallow seas or bays can also be evaluated; the validity of the simplifying assumptions made must, of course, be checked in each case.

In the previous sections we concerned ourselves only with the component of the bottom shear stress in the $x$ direction indicated in figure 15.3. It is perhaps surprising that, in general, the resulting time-averaged bottom shear stress also has a component in the $y$ direction. Only for the special cases where $\varphi$ is an integer multiple of $\pi/2$ is this not the case.

The reason for this resultant $y$ component of the bottom shear stress follows from the time average of $\tau_{cw}$ based upon $v_r^2$. Since the velocity vector pattern in figure 15.3 is not symmetrical about the $x$ axis (unless $\varphi$ is an integer multiple of $\pi/2$), the pattern of $v_r^2$ and hence $\tau_{cw}$ is also unsymmetrical and the stress component parallel to the $y$ axis has a non-zero average. As a result of this phenomenon, the current direction will change relative to a no wave condition. In the breaker zone, where the angle $\varphi$ is usually small, this effect is not noticeable. Outside the breaker zone where, for example, a river current intersects a wave field, $\varphi$ is no longer limited in value and a wave influence on the current direction can sometimes be observed.

In the following chapter various force components will be combined to predict the longshore current in the breaker zone.
16. LONGSHORE CURRENT COMPUTATION

E.W. Bijker
J. v.d. Graaff

16.1 Introduction

The previous four chapters have been devoted to discussions of various force components acting on an element of water in the breaker zone. As long as the wave conditions and shore geometry remain constant along the coast, these are the only force components acting on this water element; additional force components which can result when this limitation is not applied will be discussed in a later section of this chapter.

Rather than attempt to formulate a general current formula based upon a dynamic equilibrium of all four of the force components involved, we begin more simply by formulating an equilibrium between only two force components which are nearly always present in the breaker zone. This solution is expanded in succeeding sections of the chapter.

16.2 Basic force equilibrium

Since the bottom friction and the radiation stress gradient are always present in the breaker zone, it seems appropriate to begin a prediction of the resulting longshore current velocity with an equilibrium of these two forces.

From chapter 12, the driving force is:

\[ \frac{\partial S_{yx}}{\partial y} = \frac{5}{16} \rho \gamma^2 (gh) \frac{3}{2} \sin \phi_0 \frac{a m}{c_0} \]  

where \( c_0 \) is the wave velocity in deep water, 
\( g \) is the acceleration of gravity, 
\( h \) is the water depth, 
\( m \) is the beach slope = \( \frac{dh}{dy} \), 
\( \gamma \) is the breaker index, 
\( \rho \) is the mass density of water, and 
\( \phi_0 \) is the angle of wave approach in deep water.

The stress component from the friction force follows from equation 15.31 if we wish to be precise:

\[ \frac{\tau_{CWX}}{c_0} = \frac{\partial a v^2}{c_0} \left[ 0.75 + 0.45 \left( \xi \frac{\vartheta_0}{V} \right) \right]^{1.13} \]

where \( C \) is the Chézy coefficient, 
\( \vartheta_0 \) is the wave-caused water velocity amplitude near the bottom, 
\( V \) is the (unknown) current velocity averaged over the depth, and 
\( \xi \) is a coefficient defined in equation 15.29.

*Note that the notation has been changed to correspond to the new axis notation introduced in chapter 13. Thus, both \( \frac{\partial S_{yx}}{\partial y} \) and \( m \) are now positive.
Equation (16.01) to (16.02) and solving for $V$ yields the desired expression for the velocity at a given point in the breaker zone. Unfortunately, because of the nature of (16.02) an explicit solution is impossible; the best that can be achieved is:

$$0.75V^2 + 0.45\left(\xi_0 b\right)^{1.13}V^{0.87} = \frac{5\sqrt{2}}{16}\frac{\gamma^2 c^2 \sin \phi_0 m}{c_0} h^{3/2} \quad (16.03)$$

which can only be solved iteratively for $V$. (A Runge-Kutta method is sufficient).

In order to obtain more insight into the velocity distribution in the breaker zone, we shall start again, however this time using the simpler, more approximate friction stress relation (15.42):

$$\tau_{CWX} = \frac{\rho g \gamma \sqrt{h}}{2 \pi} V \quad (16.04)$$

where $f_w$ is the wave friction factor evaluated using equation 15.16 or figure 15.2.

The approximate velocity distribution as a function of distance from the coast, $y$, can be determined equating (16.01) and (16.04):

$$\frac{\sqrt{2}}{2 \pi} \frac{\rho g \gamma \sqrt{h}}{V} = \frac{5}{16} \rho \gamma^2 (gh)^{3/2} \frac{\sin \phi_0}{c_0} h \quad (16.05)$$

Solving this for $V$ yields:

$$V = \frac{5 \pi}{8 \sqrt{2}} \frac{\sin \phi_0}{c_0} \frac{\gamma}{\sqrt{f_w}} \frac{h}{m} \quad (16.06)$$

In this equation:

$$\frac{5 \pi}{8 \sqrt{2}} = 4.349 \text{ is a constant,}$$

$$\frac{\sin \phi_0}{c_0} \text{ depends only upon the deep water wave conditions,}$$

$$\gamma \text{ depends upon wave conditions and beach slope,}$$

$$\frac{\gamma}{\sqrt{f_w}} \text{ is a friction term dependent upon bottom roughness, water}$$

$$\text{depth, and the local wave conditions.}$$

$h$ and $m$ are functions of the distance to the shore.

The dependence of the friction term, above, upon the water depth - even for constant roughness-complicates the problem a bit; therefore, many investigators have assumed that this friction term is constant throughout the breaker zone.

If we accept the above simplifying assumption and further stipulate that the beach is of uniform slope, then the longshore current velocity turns out to be a linear function of the water depth, $h$, within the breaker zone; the maximum velocity in the profile shown in figure 16.1b occurs at the outer edge of the breaker zone shown in figure 16.1a. The fact that the velocity outside the breaker zone is zero follows directly from the fact that $\frac{d V}{d y} = 0$, there, as shown in section 12.2.
16.3 Effect of turbulence

In chapter 14 the effect of turbulent forces was found to be dependent upon the velocity gradient, thus upon \( \frac{dv}{dy} \) in the notation of this section. Since the velocity gradient is infinite at the outside edge of the breaker zone in figure 16.1, we can expect the velocity profile to be most influenced there. Indeed, the horizontal transfer of momentum will decrease the velocities within the outer portion of the breaker zone and provide the driving force for a current in the same direction just outside the breaker zone. Longuet-Higgins (1971) and Battjes (1974) have recently theoretically predicted the velocity distribution resulting from including the turbulent forces in a dynamic equilibrium along with the radiation shear stress gradient and friction.

16.4 Effect of irregular waves

All of the discussion presented so far has been based upon an assumption that regular waves are present. In practice, this will, of course, not be the case; the wave heights will vary and the outer edge of the breaker zone will not be as well defined as is suggested in figure 16.1. The largest waves will break in deeper water than the smaller waves. Battjes (1974) attacked the problem of computing the longshore current distribution by starting with a reasonable description of the irregular wave field within the breaker zone. He computed the resulting radiation shear stress gradient from the (known) wave height distributions at various locations within the breaker zone. Then, he determined the lateral velocity profile from the distribution of this radiation stress component. The effect of this wave irregularity is much like that of the lateral turbulence; the velocity profile becomes wider and less sharply peaked than that shown in figure 16.01. A quantitative comparison will be made in the next section.

Longshore current velocity profiles on real coasts can vary significantly - see figure 16.2. Tidal influences, width of the wave spectrum, variations in bottom roughness, irregular bottom slopes and variations in wave direction and height all contribute to modifying the velocity profile.

16.5 Example

Determine the distribution of the average (over the depth) longshore current velocity in a breaker zone as a function of distance from the shore. Regular waves with a period of 7 seconds approach the coast from deep water with a wave height, \( H_0 \), of 2.0 m and an approach angle, \( \phi_0 \), of 30°. A breaker index, \( \gamma \), of 0.8 and a beach slope, \( m_b \), of 1:100 are to be used. The bottom roughness, \( r \), is assumed to have a constant value of 0.06 m across the entire beach. The first step of the solution is to define the outer edge of the breaker zone. The non-linear character of the problem makes a direct analytical solution impossible; instead, the following iterative scheme can be used, however.
1. Guess a breaker depth, \( h_{br} \), and compute \( h_{br}/\lambda_o \).

2. Using tables of wave functions (or computations) determine the shoaling coefficient, \( K_{sh} \) and the ratio of wave speeds, \( c/c_o \).

3. Determine the angle \( \phi_{br} \) using:
   \[
   \sin \phi_{br} = \frac{c}{c_o} \sin \phi_o
   \]
   (16.07)

4. Compute the breaker height, \( H_{br} \) from:
   \[
   H_{br} = H_o K_{sh} \sqrt{\cos \phi_o / \cos \phi_{br}}
   \]
   (16.08)

5. Compute a new value of \( h_{br} \) from the known values of \( \gamma \) and the computed \( H_{br} \). Return to step 1.

Applying the above procedure to the problem at hand and using tables of wave functions for step 2 yields:

\[
\begin{align*}
H_{br} &= 2.07 \text{ m}, \\
h_{br} &= 2.59 \text{ m}, \text{ and} \\
\phi_{br} &= 13.3^0
\end{align*}
\]
(16.09)

Table 16.1 shows computations for a series of points within the breaker zone. The computation is illustrated below for the point 259 m from the shore; the outer edge of the breaker zone.

The wave length follows from:
   \[
   \lambda = c \frac{T}{\gamma} \sqrt{\frac{h}{\gamma}} \quad T = \sqrt{\frac{g h}{(9.81)(2.59)}}
   \] (7)
(16.10)

The amplitude of the motion on the bottom, \( a_b \) - see chapter 15, follows equation 5.03b of volume I, with (16.10) and results in:

\[
\begin{align*}
a_b &= \frac{\gamma}{4 \pi} \frac{\lambda}{(4)(\pi)} \frac{0.8}{35.3} = 2.25 \text{ m}
\end{align*}
\]
(16.11)

The friction term can then be computed from equation 15.16 or taken from figure 15.2 knowing that the bottom roughness, \( r \), is 0.06 m.

\[
f_w = \exp \left[ -5.977 + 5.213 \left( \frac{2.25}{0.06} \right) -0.194 \right] = 0.034
\]
(16.12)

The Chézy coefficient, \( C \), is determined in the usual way using:

\[
C = 18 \log \left( \frac{12 h}{R} \right) \quad = 48.9 \text{ m}^{1/2}/\text{s}
\]
(16.13)

(16.14)

Then, \( V \) follows by substitution in equation 16.06:

\[
V = \frac{5 \pi \sqrt{3}}{8 \sqrt{2}} \frac{\sin 30^0}{(1.56)(7)} \left( \frac{48.9}{0.034} \right) (2.59) \left( \frac{1}{100} \right) \] (16.15)
\[
= 1.09 \text{ m/s}
\]
(16.16)

This value can be found in the column labeled \( V_1 \) in table 16.1.

\* The shallow water approximations are being used throughout the rest of this computation. This is in keeping with the approximations made earlier in the determination of \( C_{WX} \).
Several different resulting velocity profiles are listed in this table; each is described below. $V_1$ is found from the tabulated computations based on each velocity value on a locally computed friction term. The approximate friction term (15.42) and shallow water wave approximations are used throughout. $V_2$ includes none of the simplifying assumptions inherent in the shallow water approximations. Thus, equation 12.10 is used instead of equation 12.15 to determine the driving force, and the friction force is determined using the more exact relation equation 15.31. Also, intermediate depth wave theory is used for all computations. Only the resulting values are shown; they are seen to be lower by as much as 20%.
$V_3$ in the table results from including a turbulent friction force in the velocity equation. The use of Battjes (1974) approach for regular waves yields the results shown within the breaker zone. The technique may not be applied officially outside the breaker zone, but has been applied there for comparison purposes in this example. Longuet Higgins (1971) uses a different approach from that of Battjes. Longuet Higgins proposes the use of a large range of turbulent lateral friction forces. Use of his method with its largest lateral friction results by the values denoted by $V_4$ in table 16.1.

Still another approximate attempt at reality schematizes the velocity profile by a triangle extending over a width equal to $1.6 \ y_{br}$. Its peak value occurs at $y = \frac{2}{3} \ y_{br}$ from the shore, and its peak value is found by stipulating that:

$$\int_0^{1.6 \ y_{br}} \frac{1}{c_{wx}} \ dy = S \ y_x \ \bigg|_{y = y_{br}}$$

(16.17)

Carrying out the integration yields a peak value of 0.55 m/s denoted by $V_5$ in the table. This type of velocity distribution can be assumed to include some lateral turbulent friction effects.

Finally, if an irregular wave having the same total energy as the regular wave ($H_{rms} = 2.0 \ m$ in this case) is used, then the method suggested by Battjes (1974) neglecting lateral friction yields the velocities denoted by $V_6$.

All of these profiles are compared in figure 16.2.

16.6 Additional driving forces

In the work so far in this chapter variations in wave conditions between places along the coast have not been considered; all wave properties have been assumed to be independent of the location, $x$, along the coast. This is seldom the case in a real situation.

Since the depth contours along a coast are seldom parallel, variation in refraction will cause the wave height to vary as we travel along some depth contour. Partial obstructions such as capes, spits, or even breakwaters will cause additional wave height and direction changes as we again follow a given depth contour along the coast.

The necessity of including a $\frac{\partial H}{\partial x}$ and a $\frac{\partial h'}{\partial x}$ in the longshore current computations should be obvious. Perhaps less obvious is that two additional driving forces need some explanation; both result from the longshore gradient of the wave height and angle of approach.

In chapter 11 we examined the wave set-up, $h'$, resulting from waves approaching a coast. This was found to be dependent upon the wave height, $H$. If the wave height and angle now vary along the coast then we can expect the wave set-up and set-down to vary as well resulting in a slope of the average water level along the coast, $\frac{\partial h'}{\partial x}$. This water surface slope will then provide an additional driving force for the dynamic equilibrium of a water element.

* In general, it is also dependent upon $\varphi$. 

Figure 16.2
EXAMPLE VELOCITY PROFILES
NO. SYMBOL REFERENCE
1 ○ BATTJES
2 □ BATTJES
3 ○ LONGET HIGGINS
4 △ LONGET HIGGINS
5 ◼ BATTJES

FURTHER EXPLANATION OF CURVES:

<table>
<thead>
<tr>
<th>NO.</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regular waves, no lateral friction and approximate ( \tau ) and ( S )</td>
</tr>
<tr>
<td>2</td>
<td>Regular waves, no lateral friction and more accurate ( \tau ) and ( S )</td>
</tr>
<tr>
<td>3</td>
<td>Regular waves with lateral friction</td>
</tr>
<tr>
<td>4</td>
<td>Regular waves with increased lateral friction</td>
</tr>
<tr>
<td>5</td>
<td>Approximation</td>
</tr>
<tr>
<td>6</td>
<td>Irregular waves, no lateral friction</td>
</tr>
</tbody>
</table>
The second additional driving force results from a gradient in the normal stress acting on a plane perpendicular to the coastline. This would be denoted by $S_{xx}$ using the notation of this chapter; in chapter 10 it was denoted by $S_{yy}$ - see figure 10.2. Our element of water experiences a driving force proportional to $\frac{\partial S_{xx}}{\partial x}$ (back in currently popular notation) which is dependent upon both $\frac{\partial \phi}{\partial x}$ and $\frac{\partial H}{\partial x}$.

While both of the forces indicated above have been called driving forces, this does not imply that they always both act in the same direction or in the same direction as $\frac{\partial S_{yy}}{\partial y}$. Obviously, their proper directions must be assigned in a force balance and these directions must be determined for each case separately.

Tides, discussed in chapter 13, can also, of course, influence the longshore current velocity. Because of their more universal occurrence, tidal influences can be found in more and larger areas than the other influences just mentioned above. Indeed, because of tidal phase and amplitude differences occurring along the Dutch coast, for example, tidal influences play an important role in the coastal sand transport process, especially in the region immediately outside the breaker zone.

This concludes our discussion of longshore currents. In the next three chapters we focus attention upon the prediction of sediment movements along the coast.
17. EARLY COASTAL TRANSPORT FORMULAS

17.1 Introduction

This chapter begins a new phase of our study of coastal changes. The previous five chapters have been devoted to the determination of the coastal current, one of the input parameters for a general sediment transport formula such as was suggested in chapter 9.

Here, we begin to consider the movement of sediment instead of water. Before attacking sediment transport via a prediction of sediment concentration and sediment velocity - the method suggested in chapter 9 - we sharpen our insight by first considering one of the first coastal sediment transport formulas.

Since most coastal sediments are sands, most formulas have been developed for sand beaches. Luckily, sand is one of the more predictable soil materials; it has negligible cohesion and a fairly constant shear strength property (angle of internal friction). Finer materials, silts and clays, on the other hand, do not have such simple properties. Because of its simplicity and common occurrence the sediment transport formulas are usually derived for sand; they are even often called sand transport formulas.

The formula presented in the remainder of this chapter was developed from prototype and model measurements long before much of the longshore current theory was available. Indeed, the formula - the so-called CERC formula - was apparently developed soon after World War II by the Beach Erosion Board, the predecessor of the U.S. Army Coastal Engineering Research Center.

17.2 The CERC formula

Observations in both prototype and models made in the decade following World War II indicated a correlation between the volumetric sand transport rate along a coast $[L^3/T]$ and a "component of the approaching wave energy". This sand transport was found to be more or less concentrated in the breaker zone. Expressed as a formula, this sand transport rate, $S$, is:

$$S = A U'$$  \hspace{1cm} (17.01)

where $A$ is coefficient and units conversion factor, and

$U'$ is a component of the energy flux or power entering a unit length of the breaker zone.

The power or energy flux in a unit crest length of wave train approaching the coast was given in volume I:

$$U = E c_g$$  \hspace{1cm} (I-5.10)  \hspace{1cm} (17.02)

where $E$ is the wave energy, and

$c_g$ is the wave group velocity.

* It is a mystery how scalar quantities, energy and power, can have components. Both Longuet-Higgins (1971) and Battjes (1974) show that this concept has no physical interpretation.
U is a perfectly valid scalar physical parameter. Its component - invalid parameter! - along the coast (in the x direction) at the outer edge of the breaker zone is:

\[ U_x = U \sin \phi_{br} \]  \hspace{1cm} (17.03)

where \( \phi_{br} \) is the angle between the wave crests and the coast at the outer edge of the breaker zone. Similarly - and just as incorrectly - the power component perpendicular to the coast is:

\[ U_y = U \cos \phi_{br} \]  \hspace{1cm} (17.04)

This yield a similarly non-interpretiable parameter:

\[ U' = \frac{U_x U_y}{U} = U \sin \phi_{br} \cos \phi_{br} \]  \hspace{1cm} (17.05)

or equivalently:

\[ U' = C_g \sin \phi_{br} \cos \phi_{br} \]  \hspace{1cm} (17.06)

Using refraction theory (volume I, chapter 9) and appropriate approximations:

\[ U' = \frac{1}{16} \rho g H_o^2 C_0 K_{rbr}^2 \sin \phi_{br} \cos \phi_{br} \]  \hspace{1cm} (17.07)

where: 
- \( C_0 \) is the deep water wave speed,
- \( g \) is the acceleration of gravity,
- \( H_o \) is the deep water wave height,
- \( K_{rbr} \) is the refraction coefficient at the outer edge of the breaker zone, and
- \( \rho \) is the mass density of water.

The evaluation of all these variables at the outer edge of the breaker zone can be a bit cumbersome; it would be convenient to avoid extra work. Remembering from refraction theory that the power transmitted between wave orthogonals is constant outside the breaker zone, we see that a portion of equation 17.07

\[ \frac{1}{16} \rho g H_o^2 C_0 K_{rbr}^2 \cos \phi_{br} \]

does remain constant outside the breaker zone so that the \( br \) subscript is not needed for these terms.

The remaining term in (17.07)

\[ \sin \phi_{br} \]

cannot be explained via refraction theory. This means, therefore, that the \( U' \) as a whole does vary outside the breaker zone and that \( \sin \phi_{br} \) at least must be evaluated at the outer edge of the breaker zone.

Substituting (17.07) into (17.01) and substituting a (not dimensionless) value for \( A \) (determined from both model and prototype measurements) yields:

\[ S = 0.014 H_o^2 C_0 K_{rbr}^2 \sin \phi_{br} \cos \phi_{br} \]  \hspace{1cm} (17.08)
which is exactly the same as equation 26.04 in volume I. If consistent units are used, the coefficient, 0.014, is dimensionless. However, it is often convenient to express $S$ in volume per year while $c_o$ remains units of length per second. In such a case, the coefficient is not dimensionless and the equation becomes:

$$S = 0.44 \times 10^6 H_o^2 c_o K \sin \phi_{br} \cos \phi_{br}$$  \hspace{1cm} (17.09)

which also appears in volume I as equation 26.05.

There remains some disagreement as to the proper wave height to use to represent an irregular wave train and the proper value for the coefficient in the two equations immediately above. This will be discussed in detail in section 17.5 after a better physical explanation for the CERC formula is presented in the following section.

17.3 Modern justification of the CERC formula

More recent developments such as the formulation for the radiation stress make it possible to give a more reasonable explanation for the CERC formula in terms of "correct" physical phenomena.

The radiation shear stress for all points outside the breaker zone is constant - see chapter 12. This shear stress, sometimes called lateral wave thrust, is equal to:

$$S_{yx} = E \sin \theta \cos \theta$$  \hspace{1cm} (12.03)$^*$  \hspace{1cm} (17.10)

where $E$ is the ratio $c_o / c_w$.

Since $S_{yx}$ is constant outside the breaker zone, we can choose to evaluate it using wave conditions at the outer edge of the breaker zone:

$$S_{yx} = E_{br} n_{br} \sin \phi_{br} \cos \phi_{br}$$  \hspace{1cm} (17.11)

In the previous chapter, this radiation shear stress provided the driving force for the longshore current within the breaker zone.

Beginning again on a new tack, we can reasonably accept the hypothesis that the waves are the primary factor in stirring sand into suspension for transport by a current. A reasonable characterizing parameter for this stirring can be the wave orbital velocity amplitude near the bottom, $\dot{Q}_b$. If we use shallow water approximations, $\dot{Q}_b$ can be expressed in terms of the wave speed in the breaker zone:

$$\dot{Q}_b = \frac{\gamma}{2} c_{br}$$  \hspace{1cm} (17.12)

In more general terms, $\dot{Q}_b$ is directly proportional to $c_{br}$ within the breaker zone, thus, $c_{br}$ is a legitimate parameter for characterizing the stirring action and hence the sand concentration within the breaker zone.

$^*$ Remember the change of axes!
Now, using the concept expressed in chapter 9, we can develop a sand transport formula by taking a product of $S_{yx}$ (a measure of velocity) with $c_{br}$ (a measure of sand concentration):

$$S_{yx} \cdot c_{br} = E_{br} \cdot n_{br} \cdot c_{br} \cdot \sin \phi_{br} \cdot \cos \phi_{br}$$

This is equivalent to equation 17.06.

### 17.4 Variation with angle of approach

How do changes in the angle between the approaching wave crests and the coastline influence the longshore sediment transport? This can be studied via equation 17.08, but it is more convenient to express the relationship between $S$ and angle of approach, $\phi$, in terms of the deep water angle, $\phi_0$.

Using refraction theory from volume I, chapter 9:

$$K_{rbr}^2 \cos \phi_{br} = \cos \phi_0$$

and:

$$\sin \phi_{br} = \frac{c_{br}}{c_0} \sin \phi_0$$

Equation 17.08 then becomes:

$$S = 0.014 H^2_0 \cdot c_{br} \cdot \sin \phi_0 \cdot \cos \phi_0$$

In order to investigate the effect of changes in $\phi_0$ on $S$, we need to determine which parameters depend upon $\phi_0$. Obviously $\sin \phi_0$ and $\cos \phi_0$ do, but $c_{br}$ does as well, since the wave height at the edge of the breaker zone depends upon the refraction coefficient. This variable wave height means that the outer edge of the breaker zone shifts as $\phi_0$ changes. The fact that $c_{br}$ is dependent upon $h_{br}$ completes the argument.

Thus, the behavior of:

$$f(\phi_0) = c_{br} \cdot \sin \phi_0 \cdot \cos \phi_0$$

must be studied. Unfortunately, this function, $f(\phi_0)$, cannot be written out in a simple algebraic form.

Rather than present a rather cumbersome numerical procedure to evaluate $f(\phi_0)$, we shall be content only to discuss the results of such a study of $f(\phi_0)$ found by evaluating the function for a whole series of values of $\phi_0$ and the wave period. The factor $\sin \phi_0 \cdot \cos \phi_0$ is by far the most important in the function $f(\phi_0)$. Thus, a graph of $f(\phi_0)$ looks much like one of $\sin \phi_0 \cdot \cos \phi_0$. Just as $\sin \phi_0 \cdot \cos \phi_0$, $f(\phi_0)$ is zero for $\phi_0 = 0^0$ and $\phi_0 = 90^0$. In contrast to $\sin \phi_0 \cdot \cos \phi_0$ which is symmetrical about the line $\phi_0 = 45^0$, $f(\phi)$ is asymmetric; values of $f(\phi)$ for $0 < \phi < 40^0$ are generally higher than corresponding values of $f(90^0 - \phi_0)$. This is most pronounced for relatively small values of $\phi_0$. Lastly, the peak value of $f(\phi_0)$ occurs for $\phi_0 < 45^0$ - usually somewhere between $40^0$ and $45^0$.

* The wave period enters the computation via the value, $c_0$, used in the refraction computation.
17.5 CERC formula coefficient

As was already indicated in section 17.2, certain disagreements exist with regard to the proper coefficient value to use in equation 17.08 or 17.09. The choice of the proper wave height (\(H_{\text{sig}}\) or \(H_{\text{rms}}\)) introduces an additional complication.

The early model tests used to determine equation 17.09 were conducted using regular waves for which \(U'\) can easily be evaluated. The significant wave height, \(H_{\text{sig}}\), was most likely used to characterize the waves in the prototype upon which the equation was checked even though the physically proper characterizing wave height is the root-mean-square wave height, \(H_{\text{rms}}\). This error leads to an error of a factor \(2^{\frac{1}{2}}\) in \(U'\) and hence the coefficient - see volume I chapter 10.

Additionally, there is further discussion about the proper coefficient value stemming from the various sets of model and prototype data. Study of the literature on this topic is confusing since a given set of data is often presented in various different hopefully equivalent manners by different investigators.

Figure 17.1 shows some experimental data relating \(S\) to \(U'\) with \(U'\) based upon \(H_{\text{rms}}\). If a linear relationship between these parameters is assumed as in the CERC formula, then a least-squares fit using all of the data points results in line 1 plotted in the figure. If, on the other hand, the one point given by Moore and Cole is neglected, line 2 results which indicates that the sand transport, \(S\), should be nearly four times as great for the same wave conditions!

The disagreement is further illustrated in table 17.1 where coefficients for the CERC formula determined by various investigators are compared. When all coefficients are related to the same characteristic wave, the Shore Protection Manual gives a coefficient yielding a sand transport nearly 6 times that suggested by line 1 in figure 17.1!

The discussion is far from ended........

\[ H_{\text{sig}}^2 = 2 H_{\text{rms}}^2 \]

\[ U' \]

** The data in the figure is plotted on a logarithmic scale for convenience, only. **
Table 17.1  CERC formula coefficients

<table>
<thead>
<tr>
<th>Investigator</th>
<th>Coefficient</th>
<th>Characterizing Wave Height</th>
<th>Corresponding coefficient in (17.09)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigator</td>
<td>Coefficient</td>
<td>Characterizing Wave Height</td>
<td>Corresponding coefficient in (17.09)</td>
</tr>
<tr>
<td>Original CERC</td>
<td>0.014</td>
<td>$\text{H}_{\text{sig}}$</td>
<td>0.44x10^6</td>
</tr>
<tr>
<td>Shore Protection Manual (1973)</td>
<td>0.028</td>
<td>$\text{H}_{\text{rms}}$</td>
<td>0.88x10^6</td>
</tr>
<tr>
<td>Komar (1976)</td>
<td>0.025</td>
<td>$\text{H}_{\text{sig}}$</td>
<td>0.79x10^6</td>
</tr>
<tr>
<td>Svašek (1969)</td>
<td>0.049</td>
<td>$\text{H}_{\text{rms}}$</td>
<td>1.55x10^6</td>
</tr>
<tr>
<td>figure 17.1:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not recommended</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line 1</td>
<td>0.008</td>
<td>$\text{H}_{\text{rms}}$</td>
<td>0.25x10^6</td>
</tr>
<tr>
<td>line 2</td>
<td>0.036</td>
<td>$\text{H}_{\text{rms}}$</td>
<td>1.13x10^6</td>
</tr>
<tr>
<td>Delft Univ. of Tech. Computer Program</td>
<td>0.039</td>
<td>$\text{H}_{\text{rms}}$</td>
<td>1.23x10^6 †</td>
</tr>
</tbody>
</table>

† These coefficients are really variables in this computer program. Commonly used values of these coefficients are listed.
17.6 Example of use of CERC formula

Since a computation of sand transport using the CERC formula is so straightforward it is not deemed necessary to illustrate its use here. Instead, a computation using the CERC formula is postponed until section 11 of chapter 19 where the results are compared to those of other sand transport determinations.

17.7 Limitations of the CERC formula

The CERC formula with proper coefficient (whatever that is) is surprisingly trustworthy for many more or less routine applications. It does, however, have some limitations which make it unsuitable for certain problems.

Only a total sand transport rate is computed. No information on the distribution of this transport over the width of the breaker zone is obtained. This can be a serious limitation when coasts having several offshore bars or short groins are being studied.

The formula takes no account of the bottom material properties. It was derived for beaches having rather uniform sand with average diameters ranging 175 μm to 1000 μm (1 mm). The presence of similar beach sand is a prerequisite to the use of this formula.

The beach slope or breaker zone width do not enter the CERC formula.

Only driving forces resulting from waves which have the same properties at all points along the coast are considered. The formula will then fail where other driving forces play a significant role – see chapter 16 for a further discussion of this.

The CERC formula is not applicable to shoals, dumping grounds, or near dredged channels.

Sválek (1969) has attempted to overcome the first limitation and has modified the CERC formula to yield a distribution of the sand transport across the breaker zone. His approach is to assume that the sand transport occurring across some element of width of breaker zone, is directly proportional to the loss of power by the waves crossing that same width. This assumption, although plausible, has not been proven rigorously.

In another attempt to eliminate most all of these limitations of the CERC formula Bijker (1967) attacked the problem afresh modifying a sediment transport formula for constant currents to include wave effects. The details of this will be presented in chapter 19; first, however, we need to review the physical sand transport phenomena in the next chapter.
18. SAND TRANSPORT MECHANISM

18.1 Introduction

An insight into the physical process of stirring up of bottom material, its transport and re-deposition by waves and currents will be helpful for the understanding of the philosophy of the more modern coastal sediment transport formulas. In this chapter, we examine these physical processes occurring near a sandy bottom over which waves are propagating.

18.2 Basic concepts

Waves travelling in all except deep water cause a horizontal oscillating water movement near the bottom. The water there moves with a time-dependent velocity, $u_b$.

As has already been shown in chapter 15, the shear stress near the bottom increases as the velocity near the bottom increases. This is true universally - for waves or currents or a combination of them. When this shear stress exceeds a certain critical value (corresponding to a velocity $u_{bcr}$ at the bottom) sand grains on a plane bed will begin to move with the water. Since the individual sand grains have so little mass they usually attain a velocity essentially equal to that of the water very quickly. Thus, the grains can be assumed to remain at rest when $u_b < u_{bcr}$ and to move with velocity $u_b$ when $u_b > u_{bcr}$. This assumption becomes invalid, however, if in a special case, $u_b \approx u_{bcr}$ for an extended time period.

If a graph of the bottom velocity, $u_b$, versus time, $t$, is a bit asymmetrical with respect to its zero value, a resultant transport of bottom material could be expected. Such a transport is suggested by figure 18.1.

![Figure 18.1](image-url)
The velocity diagram shown in the figure would lead to a net bottom transport in the positive $u_b$ direction. Sand grains will move back and forth, as suggested in figure 18.2, with a net forward movement. The asymmetry of the velocity diagram shown in figure 18.1 is nearly always present in shallow water. The simple linear wave theory is totally inadequate to describe the water motions accurately even though it is often used because of its relative simplicity.

18.3 Bottom roughness

Slight irregularities of a sandy plane bed will initiate the formation of a wave-like rippled bottom profile. This rippled bottom disturbs the flow pattern near the bottom; separations occur at various points on each ripple profile at different times in the wave period. For example, when the bottom velocity is positive - defined as the direction of wave propagation in this chapter - a separation and resulting eddy will occur on the front of the ripple as shown in figure 18.3a. The opposite pattern - figure 18.3b - will be generated one half wave period later. This discussion implicitly defines the front and back of a ripple as shown in figure 18.3.

It seems obvious that the sand transport will be strongly influenced by these ripples and resulting eddies. Two separate mechanisms of sand transport can be distinguished; the first of these results directly from the presence of the eddies.

As has been indicated above, a primary eddy will form on the front side of a ripple when the bottom velocity, $u_b$, is positive - figure 18.3a. The high local velocities resulting from this eddy cause local erosion as is indicated in figure 18.4a; this sand remains in suspension within the eddy. A short time later, indicated in figure 18.4d, the flow stops and the eddy "explodes" dispersing the entrained sand upward - figure 18.4b.

Still later, this sand falls back to the bottom on the "upstream" side of the ripple from which it was eroded as indicated in figure 18.4c. This last process is very dependent upon the details of the water motion, the exact shape of the ripples, and the bottom material properties.

Obviously a mirror-image situation with a secondary eddy exists during the other half of the wave period. Again, asymmetry of the ripples as well as the wave motion will guarantee that a net sand transport results. Note that the material eroded by the primary eddy during the presence of positive velocities is moved in the negative direction and vice-versa.
Figure 18.4
STEPS IN EROSION AND DEPOSITION
The irregularities in the waves are usually such that the maximum positive velocity - generating the primary eddy - is greater than the maximum magnitude of the negative velocities - generating the secondary eddy. This, in turn, can lead us to expect a relatively strong primary eddy having a higher eroded sand concentration than the secondary eddy. From the discussion of figure 18.4, then, we can conclude that a negative net sand transport will result - see Bijker et al (1976). In some cases it is even possible that a small positive resulting water movement (current) superimposed on the waves will increase the magnitude of the negative sand transport. How can this happen? The constant current component strengthens the effect of the primary eddy and thus erodes more material during phase a in figure 18.4. As long as a negative velocity is still present during phase c of that figure, a large volume of material will be moved in the negative direction. Conversely, the positive average current weakens the secondary eddies and reduces the positive transport of sand; an increased net transport in the negative direction results.

The above discussion also points out one of the significant difficulties in carrying out experimental sand transport investigations of this type: the net sand transport (usually the important item) results from the difference between two other absolute sand transport quantities which are much larger than the value we are seeking. As one may remember from numerical analysis, small errors in either of these large quantities can change the value of their difference (the net sand transport) drastically.

It would seem that in order to make much progress with studies of sediment transport, it is essential to determine the exact eddy pattern and sediment concentration, both as a function of time. While this is easily suggested, it is extremely difficult to determine these items even in a laboratory situation. Kennedy and Locher (1972) were among the first to measure such sediment concentrations successfully in a model; an example of their results is shown in figure 18.5. The unequal peaks in the sediment concentration are easily explained in the light of the asymmetry of the wave and ripples.

A second influence of the presence of the ripples is a local contraction of the streamlines near the ripple crests. The higher resulting current velocities near the ripple crests can cause local erosion of material which is "deposited" where the streamlines are more widely spaced - in the following valley.

These two processes just outlined may not be seen separately. Certainly some of the material eroded from the ripple crest will be caught in the eddy just downstream (whatever direction the water is flowing at that instant). That portion of the crest material caught in the eddy will then be transported in a direction opposite to that described in the above paragraph. Generally speaking, when waves are present, the eddy formation and its consequences dominate the sand transport process; the ripple crest erosion usually plays only a relatively minor role.
18.4 Concluding remarks

It should be obvious from the previous section that a sediment transport concept based exclusively on a principle of exceedance of a critical bottom shear stress will most likely run into difficulty. A newer theoretical/experimental approach is to attempt to predict the eddy and ripple formations and local sediment concentrations in terms of more readily measured or predicted parameters such as water velocities above the ripples and the bed material properties. It is even possible that the bed shear stress may remain an adequate parameter for the description of the net effect of the much more complex phenomena occurring in the immediate vicinity of the bottom ripples. Detailed studies of the phenomena involved are just beginning. The literature already cited in this chapter represents some of the first results. Research is continuing on a rather intensive basis; members of the Coastal Engineering Group of the Delft University of Technology are among the investigators.
This chapter has concentrated on a very detailed examination of the sand transport in a very small region near a portion of an individual ripple on a sandy bed. Our more immediate concern in practice, however, is the prediction of sand movements on a much larger scale - within a portion of a coastal breaker zone, for example. In the remaining chapters, therefore, we return to this much larger scale problem, and in the next chapter, for example, consideration of individual eddies is completely neglected; the best large scale sediment movement description available now (1980) relates this transport to a bed shear stress.
19. MODERN COASTAL SAND TRANSPORT FORMULAS

J. v.d. Graaff

19.1 Introduction

Now that some of the details of the sand transport mechanism have been considered, we can attempt to formulate the more modern sand transport formulas for transport caused by waves plus currents. As one might expect from the introduction presented way back in chapter 9, the modern formulas, in general, determine a concentration of material, \( c(z,t) \), multiply this by a particle velocity, \( u_p(z,t) \), integrate over the depth and average over time in order to determine a sediment (sand) transport, \( S_x \). Equation 9.01 expresses this in a mathematical form. As was indicated in the previous chapter, it is universally assumed that sediment particles in motion move with essentially the same horizontal velocity component as the surrounding water. (This certainly is not so in the vertical direction, however, because of gravity influences.)

Since the water velocities in a breaker zone have already been determined, the main emphasis of this chapter will be on the determination of a sediment concentration profile \( c(z,t) \) in the most general sense.

Many sediment transport formulas make a distinction between sediment transported along the bottom - bed load transport, \( S_b \) - and sediment transport carried in suspension well above the bottom, \( S_s \). The total transport is obviously the sum of these two terms.

Before discussing coastal sediment transport formulas, we shall first review some formulas developed for steady currents alone such as might be found in rivers.

19.2 Transport formulas for currents alone

Most of the sediment transport formulas reviewed here are also discussed in detail in courses and literature on (river) sediment transport. This will not be duplicated here; only results will be presented along with some insight into their meaning for our coastal application.

One of the earlier of the modern formulas was formulated by Frijlink (1952) using data and concepts of Kalinske (1947). In its most convenient form, the Kalinske-Frijlink formula for a unit channel width is:

\[
S_b = B \frac{C}{D} \left[ \frac{\Delta}{\mu V^2} \right] \exp \left( -0.27 \frac{C^2 D}{\mu V^2} \right)
\]

where:
- \( B \) is a dimensionless coefficient,
- \( C \) is the Chézy friction factor,
- \( D \) is mean sediment grain size,
- \( \exp \left( \cdot \right) \) denotes the exponential function, \( e^ \cdot \),
- \( g \) is the acceleration of gravity,
- \( S_b \) is the bed load transport,
- \( V \) is the average - constant - velocity,
- \( \mu \) is a so-called "ripple factor", and
- \( \Delta \) is the relative sediment density defined by:
\[ \Delta = \frac{\rho_s - \rho}{\rho} \]  
(19.02)

where: \( \rho_s \) is the mass density of the sediment particles, and \( \rho \) is the mass density of water.

In this formula, the coefficient, \( B \), usually has a value of about 5. Bijker (1967) in contrast to Frijlink did not include the ripple factor, \( \mu \), in the first part of the equation. This empirical factor indicates the influence of the form of the bottom roughness on the bed load transport; the actual roughness, \( r \), is incorporated in the Chézy coefficient, \( C \), of course.

The relation between equation 19.01 and the movement of bed material can be made more obvious by changing some parameters in equation 19.01. Expressing the Chézy coefficient in terms of the bed shear stress:

\[ \frac{C^2}{V^2} = \frac{\rho g}{\tau_c} \]  
(19.03)

where \( \tau_c \) is the bed shear stress - see equation 15.01. The exponential term in equation 19.01 then becomes:

\[ \exp \left[ - 0.27 \frac{\Delta D \rho g}{\mu \tau_c} \right] \]  
(19.04)

which is often referred to as the "stirring parameter" in the Kalinske-Frijlink formula. Note, as well, that this term is dimensionless.

The remaining terms in equation 19.01:

\[ B D \frac{V}{C} \sqrt{g} \]  
(19.05)

are often collectively called the "transport parameter" which has units of volume per unit width per unit time.

A more or less physical explanation for the appearance of the dimensionless parameter \( \sqrt{g} \) in the transport parameter is that the bed load transport is related to the velocities near the bed, and

\[ V \frac{\sqrt{g}}{C} = V_* \]  
(19.06)

is the velocity occurring at elevation:

\[ z' = z_0 + \frac{h}{r} \]  
(19.06)  
(19.07)

- see section 15.2. Thus, \( V_* \) is more typical of the velocities near the bottom in the layer where the bed load transport takes place.

The bottom roughness, \( r \), influences this velocity via its influence on \( C \):

\[ C = 18 \log \frac{12h}{r} \]  
(16.13)  
(19.08)

where \( h \) is the water depth.
The Kalinske-Frijlink formula was developed for rivers in which the major portion of the sediment transport took place in a small zone near the bed - as bed load transport. As such, the formula neglects the influence of sediment carried in suspension. Along a beach, however, we can expect the high turbulence in the breaking waves to hold a relatively large quantity of sand in suspension; we cannot generally neglect suspended sediment transport in the breaker zone.

Einstein (1950) approached the problem of sediment transport in a river having both suspended and bed load transport. He approached the problem in the same fundamental way as was expressed in chapter 9 by determining a total transport:

$$ S = \int_{0}^{h} c(z') V(z') \, dz' \tag{19.09} $$

where: $c(z')$ is the concentration of sediment at an elevation $z'$, and $V(z')$ is the horizontal velocity at that same elevation. He split this total transport into two parts: a bed transport occurring in a layer of thickness $a$, near the bed:

$$ S_b = \int_{0}^{a} c(z') V(z') \, dz' \tag{19.10} $$

and a suspended transport:

$$ S_s = \int_{a}^{h} c(z') V(z') \, dz' \tag{19.11} $$

Einstein (1950) used the Prandtl-Von Kármán logarithmic velocity distribution - see section 15.2 - to describe $V(z')$. He described the concentration, $c(z')$, using a diffusion equation modified to include the effects of gravity on the particles:

$$ W c(z') + \varepsilon_z \frac{d}{dz} \frac{c(z')}{dz} = 0 \tag{19.12} $$

where: $W$ is the fall velocity of the particles in water, and $\varepsilon_z$ is a diffusion coefficient (eddy viscosity).

The fall velocity can be difficult to determine. The following empirical relations have been found for sand in pure water at the given temperatures. They are valid for average grain diameters, $D_{50}$, in the range

$$ 50 \, \mu m < D_{50} < 300 \, \mu m \tag{A} $$

* This concentration, $c$, is expressed in terms of volume of deposited sediment per unit volume of water. As such, it includes the voids in the deposited sediment. This is of extreme importance when calculating sediment transports based upon measured sediment concentrations expressed in units of mass per unit volume.

** This thickness was in the order of 2 to 3 times the bed material grain diameter in Einstein's work.
For a temperature of 18°C:
\[
\log \frac{1}{\rho} = 0.4949 (\log D_{50})^2 + 2.4113 \log D_{50} + 3.7394 \tag{B}
\]
and for 10°C:
\[
\log \frac{1}{\rho} = 0.47584 (\log D_{50})^2 + 2.1795 \log D_{50} + 3.1915 \tag{C}
\]
The diffusion coefficient can be related to the same parameters as those used in the logarithmic velocity distribution. The result is that \( c_z \) is a specific function of \( z' \):
\[
c_z = \kappa V_x z' \left( \frac{h-z'}{h} \right) \tag{19.13}
\]
where \( \kappa \) is the Von Kármán coefficient = 0.4. Substitution of (19.13) into (19.12) and solving for \( c(z') \) - not too easy a task! yields:
\[
c(z') = c(b) \left( \frac{h-z'}{z'} \right) ^{z_x} \tag{19.14}
\]
where: \( c(b) \) is the concentration at some chosen elevation \( z' = b \) above the bottom, and
\[ z_x = \frac{W}{\kappa V_x} \tag{19.15}
\]
By choosing \( b \) in (19.14) to be the elevation of the boundary between bed and suspended transport layers, \( z' = a \), and combining (19.14) and (15.04) in (19.11) yields:
\[
S_s = \int_0^h c(a) \left( \frac{h-z'}{z'} \right) a \frac{z'}{h-a} \frac{V_x}{\kappa} \ln \frac{z'}{z'^0} \, dz' \tag{19.16}
\]
Einstein determined \( c(a) \) from the bed load transport using his own bed load transport formula. As will be shown later, Blijker (1968) applied the same principle, but used the Frijlink-Kalinske bed load transport formula instead.
Einstein further solved the integral in equation 19.16 in terms of two other integrals, \( I_1 \) and \( I_2 \). This resulted in a suspended transport formula looking like:
\[
S_s = 11.6 \sqrt{\frac{c}{\rho}} a c(a) \left[ I_1 \ln \frac{33h}{r} + I_2 \right] \tag{19.17} \]
where:
\[
I_1 = 0.216 \frac{A(z_x-1)}{(1-A)^{z_x}} \int_0^1 \left( \frac{1-\zeta}{\zeta} \right) ^{z_x} \, d\zeta \tag{19.18}
\]
\[
I_2 = 0.216 \frac{A(z_x-1)}{(1-A)^{z_x}} \int_0^1 \left( \frac{1-\zeta}{\zeta} \right) \ln(\zeta) \, d\zeta \tag{19.19}
\]
in which: \( A \) is a dimensionless roughness, \( \frac{r}{h} \), and \( \zeta \) is a dimensionless elevation \( z' \).

* Some investigators substitute the "shear velocity", \( V_x \), for \( \sqrt{\frac{c}{\rho}} \).
Einstein (1950) provided graphs and tables of the functions $I_1$ and $I_2$ for various values of $z^*$ and $A$. Later investigators - Bakker and Bogaard (1977) for example - have evaluated the entire term in brackets in equation 19.17 instead of working with the two integrals $I_1$ and $I_2$. Values of this term:

$$ Q = [ I_1 \ln \left( \frac{33h}{r} \right) + I_2 ] $$

(19.20)

are listed in table 19.1 as a function of $\frac{r}{h}$ and $z^*$. (The significance of other parameters listed there will be explained later).

Figure 19.1 shows an example of a concentration profile, $c(z')$ for $z^* = 1$, $r = a = 0.06$ m and $h = 3$ m. The associated logarithmic velocity profile and resulting transport profile are also shown. All three profiles have been made dimensionless by dividing by appropriate parameters as indicated on the axis of the figure.

Figure 19.1
EXAMPLE CONCENTRATION, VELOCITY AND TRANSPORT PROFILES
($z^* = 1, r = a = 0.06$ m, $h = 3$ m)
Table 19.1  Values of Einstein integral factors (all items are dimensionless)

<table>
<thead>
<tr>
<th>r/h</th>
<th>( z_*=0 )</th>
<th>( z_*=0.20 )</th>
<th>( z_*=0.40 )</th>
<th>( z_*=0.60 )</th>
<th>( z_*=0.80 )</th>
<th>( z_*=1.00 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x10^{-5}</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
<td>Q, S_<em>/S_b, S_</em>/S_b</td>
</tr>
<tr>
<td>3.03x10^5</td>
<td>5.54x10^5</td>
<td>5.54x10^5</td>
<td>3.28x10^4</td>
<td>6.00x10^4</td>
<td>6.00x10^4</td>
<td>3.88x10^3</td>
</tr>
<tr>
<td>1.44x10^5</td>
<td>2.63x10^5</td>
<td>2.63x10^5</td>
<td>1.79x10^4</td>
<td>3.27x10^4</td>
<td>3.27x10^4</td>
<td>2.43x10^3</td>
</tr>
<tr>
<td>5.36x10^4</td>
<td>9.80x10^4</td>
<td>9.80x10^4</td>
<td>7.98x10^3</td>
<td>1.46x10^4</td>
<td>1.46x10^4</td>
<td>1.30x10^3</td>
</tr>
<tr>
<td>2.53x10^3</td>
<td>4.63x10^3</td>
<td>4.63x10^3</td>
<td>4.32x10^3</td>
<td>7.90x10^3</td>
<td>7.90x10^3</td>
<td>803.</td>
</tr>
<tr>
<td>1.19x10^4</td>
<td>2.18x10^4</td>
<td>2.18x10^4</td>
<td>2.33x10^3</td>
<td>4.26x10^3</td>
<td>4.26x10^3</td>
<td>496.</td>
</tr>
<tr>
<td>4.36x10^3</td>
<td>7.98x10^3</td>
<td>7.98x10^3</td>
<td>1.02x10^3</td>
<td>1.87x10^3</td>
<td>1.87x10^3</td>
<td>260.</td>
</tr>
<tr>
<td>2.03x10^3</td>
<td>3.72x10^3</td>
<td>3.72x10^3</td>
<td>545.</td>
<td>998.</td>
<td>999.</td>
<td>158.</td>
</tr>
<tr>
<td>940.</td>
<td>1.72x10^3</td>
<td>1.72x10^3</td>
<td>289.</td>
<td>529.</td>
<td>530.</td>
<td>95.6</td>
</tr>
<tr>
<td>336.</td>
<td>615.</td>
<td>616.</td>
<td>123.</td>
<td>226.</td>
<td>227.</td>
<td>48.5</td>
</tr>
<tr>
<td>0.01</td>
<td>153.</td>
<td>280.</td>
<td>281.</td>
<td>63.9</td>
<td>117.</td>
<td>118.</td>
</tr>
<tr>
<td>0.02</td>
<td>68.9</td>
<td>126.</td>
<td>127.</td>
<td>32.8</td>
<td>60.0</td>
<td>61.0</td>
</tr>
<tr>
<td>0.05</td>
<td>23.2</td>
<td>42.4</td>
<td>43.4</td>
<td>13.1</td>
<td>24.0</td>
<td>25.0</td>
</tr>
<tr>
<td>0.10</td>
<td>9.84</td>
<td>18.0</td>
<td>19.0</td>
<td>6.28</td>
<td>11.5</td>
<td>12.5</td>
</tr>
<tr>
<td>0.20</td>
<td>3.90</td>
<td>7.13</td>
<td>8.13</td>
<td>2.80</td>
<td>5.13</td>
<td>6.13</td>
</tr>
<tr>
<td>0.50</td>
<td>0.836</td>
<td>1.53</td>
<td>2.53</td>
<td>0.716</td>
<td>1.31</td>
<td>2.31</td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>r/h</td>
<td>$z_a = 1.50$</td>
<td>$z_a = 2.00$</td>
<td>$z_a = 3.0$</td>
<td>$z_a = 4.0$</td>
<td>$z_a = 5.0$</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q$</td>
<td>$S_s/S_b$</td>
<td>$S_t/S_b$</td>
<td>$Q$</td>
<td>$S_s/S_b$</td>
<td>$S_t/S_b$</td>
</tr>
<tr>
<td>1x10^{-5}</td>
<td>2.33</td>
<td>4.26</td>
<td>5.26</td>
<td>0.973</td>
<td>1.78</td>
<td>2.78</td>
</tr>
<tr>
<td>2x10^{-5}</td>
<td>2.31</td>
<td>4.23</td>
<td>5.23</td>
<td>0.973</td>
<td>1.78</td>
<td>2.78</td>
</tr>
<tr>
<td>5x10^{-5}</td>
<td>2.28</td>
<td>4.17</td>
<td>5.17</td>
<td>0.967</td>
<td>1.77</td>
<td>2.77</td>
</tr>
<tr>
<td>1x10^{-4}</td>
<td>2.25</td>
<td>4.11</td>
<td>5.11</td>
<td>0.967</td>
<td>1.77</td>
<td>2.77</td>
</tr>
<tr>
<td>2x10^{-4}</td>
<td>2.21</td>
<td>4.04</td>
<td>5.04</td>
<td>0.967</td>
<td>1.77</td>
<td>2.77</td>
</tr>
<tr>
<td>5x10^{-4}</td>
<td>2.13</td>
<td>3.90</td>
<td>4.90</td>
<td>0.962</td>
<td>1.76</td>
<td>2.76</td>
</tr>
<tr>
<td>1x10^{-3}</td>
<td>2.05</td>
<td>3.76</td>
<td>4.76</td>
<td>0.951</td>
<td>1.74</td>
<td>2.74</td>
</tr>
<tr>
<td>2x10^{-3}</td>
<td>1.96</td>
<td>3.58</td>
<td>4.58</td>
<td>0.940</td>
<td>1.72</td>
<td>2.72</td>
</tr>
<tr>
<td>5x10^{-3}</td>
<td>1.78</td>
<td>3.26</td>
<td>4.26</td>
<td>0.907</td>
<td>1.66</td>
<td>2.66</td>
</tr>
<tr>
<td>0.01</td>
<td>1.62</td>
<td>2.96</td>
<td>3.96</td>
<td>0.869</td>
<td>1.59</td>
<td>2.59</td>
</tr>
<tr>
<td>0.02</td>
<td>1.42</td>
<td>2.59</td>
<td>3.59</td>
<td>0.809</td>
<td>1.48</td>
<td>2.48</td>
</tr>
<tr>
<td>0.05</td>
<td>1.10</td>
<td>2.02</td>
<td>3.02</td>
<td>0.694</td>
<td>1.27</td>
<td>2.27</td>
</tr>
<tr>
<td>0.10</td>
<td>0.836</td>
<td>1.53</td>
<td>2.53</td>
<td>0.568</td>
<td>1.04</td>
<td>2.04</td>
</tr>
<tr>
<td>0.20</td>
<td>0.552</td>
<td>1.01</td>
<td>2.01</td>
<td>0.414</td>
<td>0.758</td>
<td>1.76</td>
</tr>
<tr>
<td>0.50</td>
<td>0.174</td>
<td>0.319</td>
<td>1.32</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Many other investigators have proposed sediment transport formulas. Engelund and Hansen (1967), for example, suggest the following formula which they based upon certain prototype river measurements:

\[ S = 0.05 \frac{C_5}{\rho g \frac{5}{2}} \frac{a^2}{\Delta z} D_{50} \]  

(19.21)

where: \( D_{50} \) is the grain size exceeded by 50% (by weight) of the bed material sample, and

\( S \) is the total sediment transport - sum of bed and suspended transports.

Another sediment transport formula was proposed by White and Ackers (1973). Details of this formula can be found in the literature or in more specific courses on sediment transport.

19.3 Influence of waves on bed transport

It would seem logical to include the influence of waves on sediment transport in a manner more or less analogous to the way their influence was included in the longshore current friction force determination - see chapter 15. Indeed, Bijker (1967) has done this in a way which demonstrates a clear insight into the phenomenon involved. The approach of Bijker was to introduce the wave influence via a modification of the bottom shear stress in a sediment transport formula already available for currents. He chose the Kalinske-Frijlink formula - equation 19.01 - for the bed load transport and coupled this on the Einstein formula for suspended sediment transport - equation 19.17.

The instantaneous velocity component caused by the waves can be significant in the breaker zone even though the time average of this component is small relative to the longshore current velocity. This observation leads to an hypothesis that the waves contribute primarily to the stirring up of material from the bottom rather than the transport. Working out this idea, Bijker modified the bottom shear stress term in the stirring parameter of the Kalinske-Frijlink formula. The details of this modification of \( \tau_c \) in this stirring term are presented in the following section.

19.4 Bed shear stress modification

It was indicated in chapter 18 that the bed shear stress is important for the movement of sediment on a shore or in a channel. The influence of waves on the bed shear of a longshore current has also already been explained in chapter 15; there, the component of the bed shear stress in the current direction was averaged in order to determine a resultant steady state equivalent shear stress.

Without thinking, we might substitute this same bed shear stress into our sediment transport formulas. The error in such an approach is revealed by the answer to the question: what bed shear stress component determines when a bottom material particle starts to move? Expressed less formally: in what direction must we "kick" a
bottom material particle in order to stir it loose so that it may be transported? The answer to these questions is that it does not matter one bit in which direction the force - bed shear stress - acts in the stirring term of the Kalinske-Frijlink formula.

The shear stress which must be used in this stirring term has already come up in chapter 15; it is:

\[ \tau_{CW} = \rho \kappa^2 V_r^2 \]  \hspace{1cm} (15.24)
\[ \tau_{CW} = \rho \kappa^2 V_r^2 \]  \hspace{1cm} (19.22)

where \( V_r \) is the instantaneous resultant velocity.

The background of this term can be found in section 15.4. In contrast to the further work in that chapter, we shall continue working with \( \tau_{CW} \) instead of its \( x \) component, \( \tau_{CWx} \). No absolute value has been taken in equation 19.22 since all terms are non-negative, anyway.

Just as in chapter 15, we shall need to compute an average value \( \tau_{CWx} \) of this instantaneous shear stress. Its direction no longer plays a role; we need only consider the magnitude of the (vector) quantity. Since the only time variable in equation 19.22 is \( V_r \), it is sufficient to compute the average magnitude of the square of the resultant velocity, \( V_r^2 \).

Recalling the definition of \( V_r \) from chapter 15:

\[ V_r^2 = V_t^2 + (p u_b)^2 + 2 p u_b V_t \sin \phi \]  \hspace{1cm} (15.22)
\[ V_r^2 = V_t^2 + (p u_b)^2 + 2 p u_b V_t \sin \phi \]  \hspace{1cm} (19.23)

where \( p u_b \) is the wave current velocity at height \( z_t \) above the bottom,

\( V_t \) is the constant current velocity at the same elevation, and

\( \phi \) is the angle between the wave crests and (constant) current.

A more complete discussion of these can be found in chapter 15.

The value of \( \phi \) will not be restricted since it is desirable to derive a formula for general application in any combination of waves and current.

In equation 19.23 only \( u_b \) is a function of time. Picking up:

\[ u_b = u_b \cos \omega t \]  \hspace{1cm} (15.27)
\[ u_b = u_b \cos \omega t \]  \hspace{1cm} (19.24)

and remembering that:

* We should be aware that we are making a potentially serious fundamental error here; we are proposing the substitution of an average value of an independent variable, \( \tau_{CWx} \), into a non-linear relationship, the stirring term of the Kalinske-Frijlink formula, in order to obtain an "average" result. This is fundamentally wrong. In order to be fundamentally correct we must first substitute the instantaneous value of \( \tau_{CW} \) into the stirring term and then take the average. This fundamental error has been accepted in the interest of avoiding a monumental problem in mathematics.
\[
\frac{1}{2\pi} \int_0^{2\pi} \cos x \, dx = 0 \quad (19.25)
\]

and:
\[
\frac{1}{2\pi} \int_0^{2\pi} \cos^2 x \, dx = \frac{1}{2} \quad (19.26)
\]

(19.23) becomes, simply:
\[
\overline{v_r^2} = v_t^2 + \frac{1}{2} (p \frac{\partial u_b}{\partial t})^2
\]
\[
= v_t^2 \left[ 1 + \frac{1}{2} \left( \frac{p \frac{\partial u_b}{\partial t}}{v_t} \right)^2 \right] \quad (19.27)
\]
\[
(19.28)
\]

Substituting this last result into (19.22) yields:
\[
\overline{\tau_{cw}} = \rho \kappa^2 v_t^2 \left[ 1 + \frac{1}{2} \left( \frac{p \frac{\partial u_b}{\partial t}}{v_t} \right)^2 \right]
\]
\[
(19.29)
\]

in which we recognize:
\[
\rho \kappa^2 v_t^2 = \tau_c \quad (15.13) \quad (19.30)
\]
as the shear stress under a current alone. Substituting (19.30) and equation 15.20 into (19.29) yields a very simple form:
\[
\overline{\tau_{cw}} = \tau_c + \frac{1}{2} \xi \tau_w
\]
\[
(19.31)
\]
Another convenient form expresses the ratio of \(\overline{\tau_{cw}}\) to \(\tau_c\) in terms of common parameters. Using equations 15.14 and 15.29 along with 19.30 in equation 19.29 yields the desired result:
\[
\overline{\tau_{cw}} = \tau_c \left[ 1 + \frac{1}{2} \left( \frac{p \frac{\partial u_b}{\partial t}}{v_t} \right)^2 \right] \quad (19.32)
\]
which is somewhat different from equation 15.30.

19.5 Bed load transport by waves and current

The result of the previous section can be substituted directly into the stirring term of the Kalinske-Frijlink formula shown in (19.04). Using (19.32) to modify \(\tau_c\) in (19.04) and multiplying by (19.05) yields:
\[
S_b = \frac{B \Delta V \sqrt{g}}{C} \exp \left[ \frac{-0.27 \Delta D \rho g}{\mu \tau_c \left[ 1 + \frac{1}{2} \left( \frac{p \frac{\partial u_b}{\partial t}}{v_t} \right)^2 \right]} \right] \quad (19.33)
\]
or equivalently using equation 19.03:
\[
S_b = \frac{B \Delta V \sqrt{g}}{C} \exp \left[ \frac{-0.27 \Delta D \rho C^2}{\mu v^2 \left[ 1 + \frac{1}{2} \left( \frac{p \frac{\partial u_b}{\partial t}}{v_t} \right)^2 \right]} \right] \quad (19.34)
\]
It is obvious from these relationships that the presence of the waves \( (a_b) \) increases the sediment transport. Further, since \( \phi \) does not enter the equation, the increase in sediment transport is independent of the wave direction provided the current velocity is maintained. This seems logical in light of the earlier remarks concerning the direction of the bed shear stress relative to the stirring of bed material.

Bijker (1967) assumed that the bottom transport occurred in a bottom layer having thickness equal to the bottom roughness, \( r \). The concentration of bed material in this layer, \( c_b \), (assumed to be constant over the thickness) is, then:

\[
c_b = \frac{S_b}{r} \int_{0}^{r} V(z') \, dz'
\]

(19.35)

The integral is evaluated from the velocity profile of the current - see chapter 15, especially figure 15.1b:

\[
\int_{0}^{r} V(z') \, dz' = \frac{1}{2} z_t' \, V_t + \frac{1}{r} \, \frac{1}{c_p} \int_{z_t'}^{r} \ln \frac{z'}{z_o} \, dz'
\]

(19.36)

Using the definitions of \( z_t' \), etc. in terms of \( r \) and carrying out the integration yields:

\[
\int_{0}^{r} V(z') \, dz' = 6.34 \sqrt{\frac{c}{\rho}} \, r = 6.34 \, V_r \, r
\]

(19.37)

With this result equation 19.35 becomes:

\[
c_b = \frac{S_b}{6.34 \sqrt{\frac{c}{\rho}} \, r}
\]

(19.38)

This concentration is assumed to be constant over the entire thickness, \( r \), of the bed transport layer. Also, as pointed out earlier, this concentration is expressed in units of volume of deposited sediment per unit volume of water and thus includes the voids in the deposited sediment.

* We are here converting this modified Frijlink bed load formula to a form corresponding to equation 19.10. This may seem strange at first sight.
19.6 Influence of waves on suspended transport

Since the concentration distribution of the suspended sediment depends upon the bed shear stress - via \( z^*_b \) (equation 19.15) in equation 19.14 - Bijker (1968) simply accounted for the influence of waves by modifying the shear stress term. Reasoning that the shear stress in (19.14) acts in the same physical way as in the stirring term of the bed transport formula, he modified the shear stress via equation 19.32. Also, choosing \( a = r \) and selecting \( c(a) \), then, equal to \( c_b \) yields:

\[
c(z') = c_b \left( \frac{r}{h-r} \right) ^{\frac{h-z'}{z'}} \sqrt{\frac{W}{\rho}} \sqrt{\frac{1 + \frac{5}{2} \left( \frac{d_b}{V} \right)^2}{\tau_c}} \tag{19.39}
\]

The suspended load transport then follows from:

\[
S_S = \int_r^h c(z') V(z') \, dz' \tag{19.11} \tag{19.40}
\]

where: \( c(z') \) is determined in equation 19.39, and \( V(z') \) is defined in equation 15.04.

The result, after substitution of (19.38), (19.39) and (15.04) in equation 19.40, successful completion of a lot of algebra, and use of (19.20) is:

\[
S_S = 1.83 \, Q \, S_b \tag{19.41}
\]

which shows that the suspended load transport is directly proportional to the bed load transport. This is logical, considering the direct relationship between \( c_b \) and both \( S_b \) and \( S_S \). Values of:

\[
\frac{S_S}{S_b} = 1.83 \, Q \tag{19.42}
\]

have been included in table 19.1 and are plotted in figure 19.2 as a function of the two independent parameters, \( A \) and \( z^*_b \). Of course, the shear stress used to compute \( z^*_b \) must be modified; equation 19.15 becomes:

\[
z^*_b = \frac{W \sqrt{\rho}}{\kappa \sqrt{\tau_c \left( 1 + \frac{5}{2} \left( \frac{d_b}{V} \right)^2 \right)}} \tag{19.43}
\]
Figure 19.2
SUSPENDED SEDIMENT TRANSPORT
PARAMETERS

\[ A = r/h \]
19.7 Total sediment transport

Now that both the bed load transport, \( S_b \), and the suspended load transport, \( S_s \), are known, the total transport, \( S \), follows by addition. Additionally, since \( S_s \) is directly related to \( S_b \), an especially simple relationship results:

\[
S = S_b + S_s = S_b \left( 1 + 1.83 \frac{Q}{S_b} \right)
\]

In this equation, \( S_b \) is evaluated using either equation 19.33 or 19.34, and \( Q \) must be evaluated using the modified value of \( z^* \) given in equation 19.43. Values of the term in parantheses in equation 19.44 - \( \frac{S}{S_b} \) - are also included in table 19.1 and can also be found by adding 1.0 to the values in figure 19.2.

The procedure just outlined is often referred to as the Bijker formula since he was the one who first modified the bottom shear stress in the way just outlined.

The theoretical work is now completed. The only remaining problem is that of evaluating all the parameters involved in terms of known or measurable quantities.

It turns out that only the ripple factor, \( \mu \), needs further definition. It is usually defined via an empirical relation:

\[
\mu = \left( \frac{C}{C'} \right)^{3/2}
\]

where: \( C \) is the Chézy coefficient evaluated via equation 19.08, and
\( C' \) is another Chézy coefficient based upon the bed material properties:

\[
C' = 18 \log \frac{12}{D_{90}}
\]

in which \( D_{90} \) is the soil grain diameter allowing 90% (by weight) of the soil to pass.

Table 19.2 shows the steps necessary to compute the sediment transport occurring along a unit width of beach with water depth, \( h^* \). The distribution of the sand transport across the breaker zone can be determined by carrying out steps 7 through 19 in that table for various chosen values of \( h \) ranging up to the depth at the outer edge of the breaker zone, \( h_{br} \). Such a computation, obviously involves a lot of work; digital computer programs are available. If necessary, the computations could be carried out using a series of programs for a pocket computer. A sample computation will be shown in section 19.9.

\* Some variation in the sequence of steps may be appropriate depending upon the nature of the given data or perhaps the computation method used.
Table 19.2  Steps in coastal sand transport computation

<table>
<thead>
<tr>
<th>Step</th>
<th>Determination/evaluation</th>
<th>equation</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Determine deep water wave conditions $H_o$, $T$, $\phi_o$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Compute deep water wave speed, $c_o$ and wave frequency, $\omega$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Determine oceanographic and hydrographic data: bathymetry, soil sample, water density, $\rho$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Laboratory analysis of soil: $\rho_s$, D, W, $D_0$</td>
<td></td>
<td>soil sample</td>
</tr>
<tr>
<td>5</td>
<td>Compute relative density, $\Delta$</td>
<td>(19.02)</td>
<td>$\rho_s$, $\rho$</td>
</tr>
<tr>
<td>6</td>
<td>Determine breaker index, $\gamma$</td>
<td>(Vol I)</td>
<td>$H_b$, $T$, bath.</td>
</tr>
<tr>
<td>7</td>
<td>Choose water depth, $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Compute local wave conditions</td>
<td>(Vol I)</td>
<td>$H_b$, $\omega$</td>
</tr>
<tr>
<td></td>
<td>(include refraction, diffraction)</td>
<td>(15.18)</td>
<td>$h$, $\phi_o$</td>
</tr>
<tr>
<td>9</td>
<td>Estimate roughness, $r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Compute: $A = \frac{r}{h}$</td>
<td>(19.08)</td>
<td>$h$, $r$</td>
</tr>
<tr>
<td></td>
<td>$C$</td>
<td>(19.46)</td>
<td>$h$, $D_0$</td>
</tr>
<tr>
<td>11</td>
<td>Compute $f_w$</td>
<td>(15.16) or fig. 15.2</td>
<td>$a_b$, $r$</td>
</tr>
<tr>
<td>12</td>
<td>Compute $p^*$</td>
<td>(15.21) or fig. 15.2</td>
<td>$f_w$</td>
</tr>
<tr>
<td>13</td>
<td>Compute $V$</td>
<td>(16.03) or (16.06)</td>
<td>$\phi_o$, $c_o$, $\gamma$, bath.</td>
</tr>
<tr>
<td></td>
<td>(only for wave driven longshore current; otherwise, field measurement or other computation method.)</td>
<td></td>
<td>$C$, $f_w$</td>
</tr>
<tr>
<td>14</td>
<td>Compute $\mu$</td>
<td>(19.45)</td>
<td>$C$, $C'$</td>
</tr>
<tr>
<td>15</td>
<td>Compute $\zeta$</td>
<td>(15.29)</td>
<td>$p$, $C$</td>
</tr>
<tr>
<td></td>
<td>$\tau_c$</td>
<td>(19.03)</td>
<td>$\rho$, $V$, $C$</td>
</tr>
<tr>
<td>16</td>
<td>Compute $z^*$</td>
<td>(19.43)</td>
<td>$\rho$, $W$, $\phi_b$, $V$, $\tau_c$</td>
</tr>
<tr>
<td>17</td>
<td>Compute bed transport $S_b$</td>
<td>(19.33) or (19.34)</td>
<td>$\Delta$, $D$, $C$, $V$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\mu$, $\xi$, $\phi_b$</td>
</tr>
<tr>
<td>18</td>
<td>Determine: $Q$</td>
<td>table 19.1 or fig. 19.2</td>
<td>$A$, $z^*$</td>
</tr>
<tr>
<td>19</td>
<td>Compute $S$</td>
<td>(19.44)</td>
<td>$S_b$, $Q$</td>
</tr>
</tbody>
</table>

* In his original work, Bijker (1967) assumed that $p$ was a constant equal to 0.45.
19.8 Critical comments on Bijker formula

The whole method of adjusting the bed shear stress to account for the wave presence is based upon relationships for a constant current. In particular, the mixing length distribution (15.03) has been assumed leading to the logarithmic Prandtl-Von Kármán velocity distribution (15.04). This velocity distribution has been assumed to be valid for the current alone as well as for the combination of waves and current.

As already indicated, Bijker assumed that the bed transport layer had a thickness equal to the bottom roughness, \( r \), and that the sediment concentration in that layer is constant. For practical problems where the actual roughness is unknown, Bijker suggested using a roughness equal to one half the height of the ripples on the bottom. These ripples could often be measured directly, especially in a model.

More recent studies have indicated that the bottom roughness is often much more than that suggested by Bijker; roughness values of two to four times the bottom ripple height are more acceptable at present.

This drastic increase in bottom roughness then increases the thickness of the bed load transport layer. This, in turn, makes it less acceptable to assume that the bed load material concentration remains constant over the entire thickness of this layer. Recent measurements in the laboratory indicate that concentration variations do exist in this layer. This has consequences, of course, for the reference concentration in the suspended sediment concentration equation.

Further, there is even some doubt about the validity of the diffusion-type concentration relationship used by Einstein when applied to waves. Indeed, it neglects any mixing that might occur as a result of the vertical velocities under the wave. Measurements by Kennedy and Locher (1972) and in an anonymous report from the Delft Hydraulics Laboratory (1976) indicate that several concentration distribution models more or less adequately fit the measured data.

In spite of these limitations - some of which are of very principle nature - the Bijker formula usually gives good results. When, for example, it is applied to beaches having rather uniform sand and a wave-driven longshore current, it yields total results which usually agree rather well with those of the CERC formula presented in chapter 17. This is not necessarily true of other formulas.*

The principle of modifying the shear stress in a sediment transport formula can be applied, in principle, to any sediment transport formula. Often times, however, the insight into the physical process involved is difficult to detect making a correct shear stress modification difficult.

The Bijker-Kalinske-Frijlink formula takes no account of a critical shear stress as defined in chapter 18. In the above mentioned formula, the existence of any current and bed shear stress will lead to a sediment transport while in chapter 18 bed transport could exist only during times when a certain critical shear stress was exceeded.

For field conditions, the suspended sediment transport usually far exceeds the bed load transport – ratios of suspended to bed load of 50 to 1 are normal.

* See van de Graaff and van Oerveen (1979).
There is much discussion about the proper value of the coefficient, B, to be used in the bed load transport formula. Values ranging between 1 and 5 have been suggested. This disagreement reflects the possible in-accuracy of such a sand transport computation. Even though many parameters are involved in the final resulting formulas, errors of more than ten percent are common in practice; in other words, computed sediment trans-ports are often wrong even in the first significant figure! Unfortunately, no great improvement of this situation can be expected until a mass of trustworthy field data with actual measured sediment trans-ports is available. The example, following, illustrates this as well.

19.9 Example of Bijker formula

The following example is intended to demonstrate several principles: First, a computation such as is outlined in table 19.2 is illustrated. Second, the influence of the longshore current velocity distribution is demonstrated by computing sand transport distributions for the various longshore current distributions illustrated in chapter 16. Third, the influence of other parameters such as beach slope and particle grain size is investigated for a given wave and current distribution model. Lastly, a comparison computation using the CERC formula is presented.

The same offshore wave and beach bathymetry conditions assumed in section 5 of chapter 16 will be retained here. These are:
- Wave period, T : 7.0 s
- Wave height, H₀ : 2.0 m
- Approach angle, φ₀ : 30°
- Breaker index, γ : 0.8
- Beach slope, m : 1:100
- Bottom roughness, r : 0.06 m

Additionally a sand bed consisting of sand with a mean diameter, D, of 200 μm is used. The diameter passing 90% of the sample is D₉₀ = 270 μm. Further laboratory analysis yields that the water density, ρ, is 1000 kg/m³ * and that of the sand, ρₛ, is 2650 kg/m³. The particle fall velocity is W = 0.0252 m/s.

The computations involved follow more or less the procedure out-lined in table 19.2, although some short cuts will be taken. Table 19.3 lists the computation values. Six columns of values - y, h, a_b, c, f_w, and V₁ - have been taken directly from table 16.1. The computations for the row y = 259 m will again be illustrated in detail just as was done in section 16.5. Results from that section will be freely used here.

The orbital velocity amplitude at the bottom can be computed using equation 5.01b of volume I, but can more quickly be found from:

\[ a_b = \omega a_b \]  
\[ a_b = \left( \frac{2\pi}{f} \right)(2.25) = 2.02 \text{ m} \]  

* Apparently the beach in question is on a large lake!
The parameter $A$ is simply:

$$A = \frac{r}{R} = \frac{0.06}{2.59} = 0.0232$$

(19.49)

The value of $C'$ comes directly from equation 19.46:

$$C' = 18 \log \left( \frac{12(2.59)}{270 \times 10^{-6}} \right) = 91.1 \text{ m}^3/\text{s}$$

(19.50)

Since $f_w$ and $V_1$ are taken from table 16.1, the next parameter to calculate is the ripple factor. Using its empirical definition (19.45) directly yields:

$$\mu = \left( \frac{48.9}{91.1} \right)^{3/2} = 0.39$$

(19.51)

The parameter $\xi$ can be computed using equation 15.29:

$$\xi = \frac{48.9 \sqrt{0.034}}{\sqrt{2}(9.81)} = 2.04$$

(19.52)

The parameter $z_\star$ is computed with equation 19.43. $\tau_c$ must be computed first, however, using (19.03):

$$\tau_c = \frac{(1000)(9.81)(1.09)^2}{(48.9)^2} = 4.88 \text{ N/m}^2$$

(19.53)

$\tau_{cw}$ is then, from (19.32):

$$\tau_{cw} = \frac{5 \xi q_b}{V}$$

(19.32)

$$\tau_{cw} = 4.88 \left( 1 + \frac{1}{2} \left( \frac{(2.04)(2.02)}{1.09} \right)^2 \right) = 39.75 \text{ N/m}^2$$

(19.54)

The parameter $z_\star$ is then simply:

$$z_\star = \frac{W \sqrt{D}}{k \sqrt{\tau_{cw}}}$$

(19.55)

$$z_\star = \frac{(0.0252)(\sqrt{1000})}{(0.40)(\sqrt{39.75})} = 0.316$$

(19.56)

Knowing $\tau_{cw}$, $S_b$ can most conveniently be computed using equation 19.33 instead of 19.34:

$$S_b = B D \sqrt{\frac{V}{C}} \exp \left[ -\frac{0.27 \Delta \theta p q}{\mu \tau_{cw}} \right]$$

(19.33)

Using the currently (1977) popular value of 5.0 for $B$,

$$S_b = (5.0)(200 \times 10^{-6}) \sqrt{9.81} \left( \frac{1.09}{48.9} \right) \exp \left[ -\frac{0.27 (1.65)(200 \times 10^{-6})(1000)(9.81)}{0.39}(39.75) \right]$$

(19.57)

$$= 6.60 \times 10^{-5} \text{ m}^3/\text{sm}$$

(19.58)
### Table 14.3: Sediment transport computations and results

<p>| | | | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>h</td>
<td>a_b</td>
<td>u_b</td>
<td>A</td>
<td>C</td>
<td>C'</td>
<td>f_w</td>
<td>V_l</td>
<td>μ</td>
<td>ξ</td>
<td>z_x</td>
<td>S_b1</td>
<td>Q</td>
<td>S_1</td>
<td>S_2</td>
<td>S_3</td>
</tr>
<tr>
<td>(m)</td>
<td>(m)</td>
<td>(m/s)</td>
<td>(m/s^2)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(m/s)</td>
<td>(m/s)</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(m^2/s)</td>
<td>(-)</td>
<td>(m^2/s)</td>
<td>(m^2/s)</td>
<td>(m^2/s)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.00</td>
<td>--</td>
<td>--</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>25</td>
<td>.25</td>
<td>0.70</td>
<td>0.63</td>
<td>.240</td>
<td>30.6</td>
<td>72.8</td>
<td>.065</td>
<td>.048</td>
<td>0.272</td>
<td>1.76</td>
<td>.783</td>
<td>2.99 x 10^-6</td>
<td>0.945</td>
<td>8.16 x 10^-6</td>
<td>1.29 x 10^-6</td>
<td>2.54 x 10^-6</td>
</tr>
<tr>
<td>50</td>
<td>.50</td>
<td>0.99</td>
<td>0.89</td>
<td>.120</td>
<td>36.0</td>
<td>78.2</td>
<td>.052</td>
<td>.126</td>
<td>0.312</td>
<td>1.85</td>
<td>.618</td>
<td>8.37 x 10^-6</td>
<td>2.32</td>
<td>4.39 x 10^-5</td>
<td>2.14 x 10^-5</td>
<td>4.14 x 10^-5</td>
</tr>
<tr>
<td>75</td>
<td>.75</td>
<td>1.21</td>
<td>1.09</td>
<td>.080</td>
<td>39.2</td>
<td>81.4</td>
<td>.047</td>
<td>.216</td>
<td>0.334</td>
<td>1.92</td>
<td>.527</td>
<td>1.44 x 10^-5</td>
<td>3.88</td>
<td>1.17 x 10^-4</td>
<td>7.27 x 10^-5</td>
<td>1.23 x 10^-4</td>
</tr>
<tr>
<td>100</td>
<td>1.00</td>
<td>1.40</td>
<td>1.26</td>
<td>.060</td>
<td>41.4</td>
<td>83.7</td>
<td>.043</td>
<td>.318</td>
<td>0.348</td>
<td>1.94</td>
<td>.474</td>
<td>2.09 x 10^-5</td>
<td>5.53</td>
<td>2.32 x 10^-4</td>
<td>1.57 x 10^-4</td>
<td>2.41 x 10^-4</td>
</tr>
<tr>
<td>125</td>
<td>1.25</td>
<td>1.56</td>
<td>1.40</td>
<td>.048</td>
<td>43.2</td>
<td>85.4</td>
<td>.040</td>
<td>.430</td>
<td>0.360</td>
<td>1.95</td>
<td>.439</td>
<td>2.77 x 10^-5</td>
<td>7.25</td>
<td>3.95 x 10^-4</td>
<td>2.78 x 10^-4</td>
<td>4.09 x 10^-4</td>
</tr>
<tr>
<td>150</td>
<td>1.50</td>
<td>1.71</td>
<td>1.53</td>
<td>.040</td>
<td>44.6</td>
<td>86.8</td>
<td>.039</td>
<td>.539</td>
<td>0.368</td>
<td>1.99</td>
<td>.404</td>
<td>3.43 x 10^-5</td>
<td>9.18</td>
<td>6.11 x 10^-4</td>
<td>4.37 x 10^-4</td>
<td>6.12 x 10^-4</td>
</tr>
<tr>
<td>175</td>
<td>1.75</td>
<td>1.85</td>
<td>1.66</td>
<td>.034</td>
<td>45.8</td>
<td>88.0</td>
<td>.037</td>
<td>.663</td>
<td>0.375</td>
<td>1.99</td>
<td>.379</td>
<td>4.17 x 10^-5</td>
<td>11.21</td>
<td>8.97 x 10^-4</td>
<td>6.35 x 10^-4</td>
<td>8.59 x 10^-4</td>
</tr>
<tr>
<td>200</td>
<td>2.00</td>
<td>1.97</td>
<td>1.77</td>
<td>.030</td>
<td>46.8</td>
<td>89.1</td>
<td>.036</td>
<td>.785</td>
<td>0.381</td>
<td>2.00</td>
<td>.359</td>
<td>4.88 x 10^-5</td>
<td>13.37</td>
<td>1.24 x 10^-3</td>
<td>8.71 x 10^-4</td>
<td>1.14 x 10^-3</td>
</tr>
<tr>
<td>225</td>
<td>2.25</td>
<td>2.09</td>
<td>1.88</td>
<td>.026</td>
<td>47.8</td>
<td>90.0</td>
<td>.035</td>
<td>.915</td>
<td>0.387</td>
<td>2.02</td>
<td>.339</td>
<td>5.62 x 10^-5</td>
<td>15.76</td>
<td>1.68 x 10^-3</td>
<td>1.14 x 10^-3</td>
<td>1.33 x 10^-3</td>
</tr>
<tr>
<td>250</td>
<td>2.50</td>
<td>2.21</td>
<td>1.98</td>
<td>.024</td>
<td>48.6</td>
<td>90.8</td>
<td>.034</td>
<td>1.05</td>
<td>0.391</td>
<td>2.02</td>
<td>.324</td>
<td>6.38 x 10^-5</td>
<td>18.19</td>
<td>2.19 x 10^-3</td>
<td>1.46 x 10^-3</td>
<td>1.13 x 10^-3</td>
</tr>
<tr>
<td>259</td>
<td>2.59</td>
<td>2.25</td>
<td>2.02</td>
<td>.023</td>
<td>48.9</td>
<td>91.1</td>
<td>.034</td>
<td>1.09</td>
<td>0.393</td>
<td>2.04</td>
<td>.316</td>
<td>6.60 x 10^-5</td>
<td>19.26</td>
<td>2.39 x 10^-3</td>
<td>1.59 x 10^-3</td>
<td>--</td>
</tr>
<tr>
<td>275</td>
<td>2.75</td>
<td>2.35</td>
<td>2.07</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4.69 x 10^-4</td>
<td>5.39 x 10^-4</td>
<td>5.61 x 10^-4</td>
<td>1.02 x 10^-3</td>
</tr>
<tr>
<td>300</td>
<td>3.00</td>
<td>2.45</td>
<td>2.12</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4.45 x 10^-4</td>
<td>5.02 x 10^-4</td>
<td>--</td>
<td>1.47 x 10^-4</td>
</tr>
<tr>
<td>350</td>
<td>3.50</td>
<td>2.59</td>
<td>2.30</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>7.70 x 10^-5</td>
<td>5.46 x 10^-5</td>
<td>2.11 x 10^-4</td>
<td>4.29 x 10^-5</td>
</tr>
<tr>
<td>400</td>
<td>4.00</td>
<td>2.75</td>
<td>2.45</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4.29 x 10^-5</td>
<td>--</td>
<td>9.6 x 10^-5</td>
<td>--</td>
</tr>
<tr>
<td>450</td>
<td>4.50</td>
<td>3.00</td>
<td>2.60</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4.29 x 10^-5</td>
<td>--</td>
<td>--</td>
<td>9.6 x 10^-5</td>
</tr>
<tr>
<td>500</td>
<td>5.00</td>
<td>3.25</td>
<td>2.75</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>4.29 x 10^-5</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

**Note:** (1) Data taken directly from table 16.1.
The value of $Q$ can be found approximately by entering figure 19.2 with a value of $A$ and by interpolating between curves of $z^*$ values. Alternatively, an interpolation can be carried out in table 19.1. With either method, this yield $Q = 19.26$.

Knowing $Q$, the total transport can be found using equation 19.44:

$$S = 6.60 \times 10^{-5} \left[ 1 + 1.83 (19.26) \right]$$  \hspace{1cm} (19.59)

$$= 2.39 \times 10^{-3} \text{ m}^3/\text{s.m}$$  \hspace{1cm} (19.60)

The total sediment transport can be found by integrating the values of $S$ across the width of the breaker zone. Integrating the values of $S_1$ using the trapezoidal rule and remembering that the last interval, $\Delta y$, is only 9 m. yields:

$$S_1 = 0.179 \text{ m}^3/\text{s}$$  \hspace{1cm} (19.60)

$$= 5.64 \times 10^6 \text{ m}^3/\text{year}$$  \hspace{1cm} (19.61)

This resulting value seems high on a yearly basis, but on the other hand, a deep water wave height of 2.0 m is about twice as high as a year-averaged North Sea wave. Secondly, one has the erroneous tendency to compare the figure in equation 19.61 to net sand transport along the Dutch Coast which is much smaller.

Values of sediment transport rates computed using the other longshore current profiles listed in table 16.1 are also listed in table 19.3.

Values of $S_2$ are found by applying the technique just described for $S_1$, except that intermediate water depth wave theory is used throughout the sand transport computation. (It had already been used along with a more exact force balance to determine the longshore current velocity, $V_2$, in chapter 16).

The remaining sand transports, $S_3$ through $S_6$, all result from the use of the Bijker formula with the correspondingly numbered velocity profile from table 16.1.

All of these results as well as their associated velocity profiles from chapter 16 are compared in figure 19.3. Note that when an intermediate peak in the velocity profile occurs such as with $V_3$ through $V_6$, the corresponding peak in the sediment transport occurs seaward of the velocity peak.

Also, from the computations shown in table 19.3, we can conclude that the suspended transport becomes relatively more important as the water depth increases. This follows from the higher values of $Q$ associated with greater depths in the table.

The local variations in sand transport between the various transport profiles seems rather great. However, when the total sand transports are computed by integrating the curves shown in figure 19.3b, remarkably consistent results are obtained. These are indicated in table 19.4.

A computation using the CERC formula is shown in section 19.11 for comparison. In the following section we investigate the sensitivity of the Bijker formula.
Figure 19.3a
EXAMPLE VELOCITY PROFILES
(copy of figure 16.2)

Figure 19.3b
EXAMPLE SAND TRANSPORT PROFILES

<table>
<thead>
<tr>
<th>NO.</th>
<th>SYMBOL</th>
<th>REF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>o</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>□</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>◇</td>
<td>BATTJES</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
<td>LONGUET-HIGGINS</td>
</tr>
<tr>
<td>5</td>
<td>△</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>×</td>
<td>BATTJES</td>
</tr>
</tbody>
</table>
Furthermore, D appears in the numerator of the exponent of (19.34). An increase in D causes a decrease in transport.

19.10 Sensitivity of the Bijker formula

In the previous example specific values of such parameters as bottom roughness, r, particle grain size, D, beach slope, m, and breaker index, y, have been used. Figure 19.4 shows the total sand transport found using the Bijker formula in combination with the velocity distribution denoted by $V_6$, as a function of bottom roughness, r, for various grain sizes and beach slopes. Offshore wave conditions were held constant and the same as in the previous section. Once again, the results from the CERC formula are shown for comparison. Note that the CERC formula is completely insensitive to the parameters being discussed here.

The bottom roughness influences the total sand transport in 2 ways: As the bottom roughness increases, the longshore current velocity decreases - see chapter 16; secondly, for a given current velocity, the Bijker formula usually gives a lower sediment transport as the roughness is increased. These two influences reinforce each other to yield the decreasing total sediment transport with increasing roughness.

The influence of increasing the average bed material grain size is also obvious from figure 19.4. Increasing the bottom material grain size has decreased the total sediment transport in this example. This may seem surprising in light of the direct relationship between D and $S_b$ in the transport term of equation 19.34. The error in this oversimplified examination is that the grain size, D, also influences the fall velocity, $W_f$ (for the suspended load transport) and even the ripple factor, $u$, indirectly. Thus, the influence of the bed material gradation on the sediment transport is complex, indeed.

Increasing the beach slope tends to increase the longshore current velocity. (This is demonstrated in a very simple case by equation 16.06.) This increased velocity will yield a higher sediment transport per unit width. The increasing beach slope narrows the breaker (and transport) zone, however, so that the total sediment transport on a steep, narrow beach is little different from that on a flatter, wider beach.

* This result is shown in section 19.11.
19.11 Comparison to CERC formula

The application of the CERC formula is illustrated here in order to compare its results to those found using the Bijker formula. The same conditions and parameter values used in sections 16.5 and 19.9 will be used here as well.

The CERC formula - from chapter 17 - is:

\[
S = 0.014 \frac{H_o^2}{c_o} K_r \left( \frac{b_r}{b} \right) \sin \phi_{br} \cos \phi_{br} \cos \theta \quad (17.08) \quad (19.62)
\]

Instead of using the original coefficient in equation 19.62, we shall use the coefficient associated with line 2 in figure 17.1 which is listed in table 17.1. This coefficient value is 0.036.

The necessary data are:

- \( H_o = 2.0 \) m
- \( \phi = 30^0 \)
- \( \phi_{br} = 13.3^0 \)
- \( T = 7.0 \) s

From volume I chapter 5:

\[
c_o = \frac{g}{2\pi} \frac{n}{T} \quad (1-5.05a) \quad (19.63)
\]

\[
= (1.56)(7) = 10.93 \text{ m/s} \quad (19.64)
\]

and from volume I chapter 9:
\[ K_r^2 = \frac{\cos \theta_o}{\cos \theta_{br}} \]  

(19.65)

\[ = \frac{\cos 30^\circ}{\cos 13.3^\circ} = 0.890 \]  

(19.66)

Substituting values into (19.62) yields:

\[ S = (0.036)(2)^2(10.93)(0.890)(\sin 13.3^\circ)(\cos 13.3^\circ) \]  

(19.67)

\[ = 0.314 \text{ m}^3/\text{s} \]  

(19.68)

The result from this CERC formula has already been compared to those of the Bijker formula in table 19.5 and figure 19.4. Note that it brackets the other results rather well. Thus, one can conclude - correctly - that the Bijker formula will solve any problem which the CERC formula will also solve. Why bother, though? The CERC formula is much simpler to apply as has just been demonstrated.

Indeed, the power of the Bijker formula lies in its adaptability to any current condition. The concept of the Bijker formula - the adjustment of the bottom shear stress to account for the waves - can be applied much more universally. Alternatively, the current, \( V \), included in the Bijker formula may be driven by any combination of forces and subjected to all sorts of local influences. For example, the Bijker formula can be used to predict sedimentation in shipping channels in which there are no breaking waves; the CERC formula would yield no result in such a case. This specific problem of channel sedimentation comes up again in chapter 25.

Now that we can compute longshore sediment transport rates for a given set of conditions, we are in a position to attack the problem of predicting coastal changes. The first application of sediment transport computations to predict coastal changes is the topic of chapter 20.
COASTAL CHANGES WITH SINGLE LINE THEORY

20.1 Introduction

The previous chapters have been devoted to the determination of the sediment transport at a given location on the coast. In this chapter we shall apply the knowledge of sediment transport rates to the prediction of coastal changes. As has been pointed out in section 1 of chapter 1-28, only a change in sediment transport as we progress along a coast will cause erosion or accretion of a coast.

The method to be presented here was, in principle, developed by Pelnard-Considère (1954). Although it is old and poorly suited for many problems – it involves some very limiting assumptions – it is one of the few methods available suitable for hand computation. As such, it retains its value.

The profile characterizing the beach to be studied is assumed to move horizontally over its entire height as a result of accretion or erosion. The beach slope does not change, therefore. Such a beach and its schematization have already been illustrated in volume I, figure 26.1. That figure is reproduced here from completeness. The area between and the horizontal displacement of the solid and dashed lines is the same for the schematization and the actual profile. In practice, this profile usually extends somewhat farther seaward than the breaker zone and includes the entire nearshore area. Often, the toe of the profile can be defined as the point where the beach slope becomes essentially horizontal.

Two equations will be necessary in order to predict the coastal changes: an equation of motion and a continuity equation; these will be discussed in the following sections.

20.2 Equation of continuity

Consider a segment of a beach which is changing – either eroding or accreting. If we examine a portion of length dx for a time dt, we shall find that the coastline has moved a distance dy. From figure 20.2, we see that if the depth over which the coastal changes take place is h, then:
Figure 20.2
CONTINUITY EQUATION RELATIONSHIPS

\[ S_x \, dt - (S_x + d S_x) \, dt = dx \, dy \, h \]  \hspace{1cm} (20.01)

where: $h$ is the depth over which the change takes place, $S_x$ is the sand transport along the coast at location $x$, and $S_x + d S_x$ is the sand transport along the coast at location $x + dx$.

In words, the inflow minus the outflow is the volume of material accumulated.

Also:

\[ d S_x = \frac{\partial S_x}{\partial x} \, dx \]  \hspace{1cm} (20.02)

and

\[ dy = \frac{\partial y}{\partial t} \, dt \]  \hspace{1cm} (20.03)

Substitution of these last two relationships into equation 20.01 yields, after simplification:

\[ \frac{\partial S_x}{\partial x} + h \frac{\partial y}{\partial t} = 0 \]  \hspace{1cm} (20.04)

which is the resulting equation of continuity.

Our primary practical interest is in the change in the coastline as a function of time, thus indirectly in $\frac{\partial y}{\partial t}$. If we can evaluate $\frac{\partial S_x}{\partial x}$ in equation 20.04, then we can determine the coastal changes via an integration. This necessary first term of (20.04) is examined in the following section.
In the previous section, we were left with the problem of evaluating \( \frac{\partial S_x}{\partial x} \). What changes along a coast will cause \( S_x \) to change? The most important variables which can change as we proceed along a coast are the wave height and the angle of wave attack relative to the coastline. Of these, we shall restrict ourselves, here, to changes in the angle of wave attack; this implies that there is no diffraction and the offshore wave conditions do not change along the coast.

In section 17.4 we carried out an investigation of the relationship between changes in the angle of wave approach relative to the coast and the resulting sand transport, \( S_x \). There, we examined \( S_x \) for various values of wave attack relative to a fixed coast. We could just as well have examined \( S_x \) for a fixed wave direction and a varying coastline orientation relative to the waves. Thus, by varying \( \phi \) slightly in a sand transport equation, we can then empirically determine \( \frac{\partial S_x}{\partial \phi} \). (This can be done with any longshore sand transport formula). Also, if we restrict our changes in angle of attack relative to a changing coastline to small changes, we can assume \( \frac{\partial S_x}{\partial \phi} \) to be constant:

\[
\frac{\partial S_x}{\partial \phi} = S_x \quad (20.05)
\]

This is our desired equation of motion.

We can transform this known function \( \frac{\partial S_x}{\partial \phi} \) to our unknown function \( \frac{\partial S_x}{\partial x} \) via the Chain Rule:

\[
\frac{\partial S_x}{\partial x} = \frac{\partial S_x}{\partial \phi} \frac{\partial \phi}{\partial x} \quad (20.06)
\]

If, as we have assumed, \( \partial \phi \) is small, then \( \partial \phi \) is equivalent to \( \frac{\partial y}{\partial x} \) and:

\[
\frac{\partial \phi}{\partial x} = -\frac{\partial^2 y}{\partial x^2} \quad (20.07)
\]

The negative sign results from the fact that a positive (increase in) \( \frac{\partial y}{\partial x} \) results in a decrease in \( \phi \).

** We shall discuss the location at which the wave conditions should be taken in more detail in a later section.

** Such an assumption implies that a segment of the entire function relating \( S_x \) to \( \phi \) has been replaced by a straight line. This is not too bad an assumption as long as the changes in \( \phi \) are not too large.

In the following it will be obvious that \( \phi = 0 \) should be included in the solution. We are, thus, restricted in our analysis to small values of \( \phi \).
The angle $\phi$ used here has not been specifically defined; it is the angle of wave attack in some water depth before the coast. This depth must correspond to the depth at the toe of portion of the coast which is modified by the longshore sand transport. This corresponds, thus, to the depth, $h$, in figure 20.2. We denote this angle by $\phi'$ as shown in figure 20.3 when it is measured relative to the original coast (x axis). The angle we need in the equation of motion is, however, the angle between the wave crest at depth $h$ and the instantaneous shoreline at some time, $t$. Thus, also from figure 20.3, we can define $\phi$ as:

$$\phi = \phi' - \frac{\partial y}{\partial x}$$

(20.08)

Note that the result above - that $\phi$ must be measured at a depth $h$ - is in contrast to that presented in chapter 17 where $\phi_o$ was used in the CERC formula. That was valid, there, because we implicitly assume that the beach slope continued to deep water, thus $\phi' = \phi_o$ in that case. The present definition, using $\phi'$, is more general; it is also valid, for example, when the toe of the beach slope is on a horizontal sandbank.

What about the wave height we are going to use to determine the sand transport? (The wave height enters any coastal sand transport formula in some way). Just as with the angle of attack, it is safest to evaluate the wave height (or heights) in the area where the coastal changes are to be predicted. Use of deep water wave data will yield incorrect results if breaking occurs on intermediate offshore bars.

20.4 Equation solution, boundary and initial conditions

Equation 20.05, the equation of motion, and 20.04, the continuity equation, can be combined by substitution of (20.05) and (20.07) into (20.06):

$$\frac{\partial S_x}{\partial x} = -S_x \frac{\partial^2 y}{\partial x^2}$$

(20.09)

and substituting this into (20.04), yielding:

$$-S_x \frac{\partial^2 y}{\partial x^2} + h \frac{\partial y}{\partial t} = 0$$

(20.10)
which can be reduced to a standard form by substituting:

\[ a = \frac{\frac{S}{h}}{\frac{S}{h}} \quad (20.11) \]

so that the final result is:

\[ a \frac{3}{3x^2} - \frac{3y}{3t} = 0 \quad (20.12) \]

The last step in equation 20.11 follows from equation 20.05.

Both initial conditions and boundary conditions are needed in order to solve equation 20.12 for a specific problem. One initial condition - the coast form at time \( t = 0 \) - and two boundary conditions - sand transports as a function of time at two different places - are usually specified. Initial and boundary conditions for a specific problem, that of accretion against an impermeable (for sand) breakwater, are given in the following section along with the resulting shoreline solution.

20.5 Application to breakwater accretion

The construction of a breakwater to protect a harbor approach channel from wave action also upsets the equilibrium of coastal sediment transport. Figure 20.4 shows a sketch plan of such a breakwater. Coastlines for various times, \( t \), are shown.

The initial condition is the shape of the coastline at time \( t = 0 \). This is expressed by:

at \( t = 0 \): \( y = 0 \) for all \( x \) \quad (20.13)
One boundary condition is that at a great distance from the 
breakwater, $x = -\infty$, the sand transport remains constant and equal 
to its value initially on the undisturbed coast:

$$\text{at } x = -\infty: S_x = S \text{ for all } t$$  \hspace{1cm} (20.14)

The second boundary condition is imposed by the breakwater; it 
is impervious to sand. Thus:

$$\text{at } x = 0: S_x = 0 \text{ for all } t > 0$$  \hspace{1cm} (20.15)

This last boundary condition implies, using (20.08) and remembering 
the dependence of $S_x$ on $\theta$ that:

$$\text{at } x = 0: \frac{\partial S}{\partial x} = \theta' \text{ for all } t > 0$$  \hspace{1cm} (20.16)

In other words, the beach accretion progresses seaward always making 
an angle $\theta'$ with respect to the $x$ axis at the breakwater.

The resulting solution to equation 20.12 is:

$$y = \theta' \sqrt{\frac{4at}{\pi}} \left[ e^{-u^2} - u \sqrt{\pi} \theta \right]$$  \hspace{1cm} (20.17)

where: $u = -\frac{x}{\sqrt{4at}}$,  \hspace{1cm} (20.18)

$x$ is the distance along the beach - fig. 20.4, and

$$\theta = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} e^{-u^2} du$$  \hspace{1cm} (20.19)

$\theta$ has the form of a probability integral.

$$\theta = \frac{2}{\sqrt{\pi}} \left[ \int_{0}^{\infty} e^{-u^2} du - \int_{0}^{u} e^{-u^2} du \right]$$  \hspace{1cm} (20.20)

$$\theta = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{u} e^{-u^2} du$$  \hspace{1cm} (20.21)

This last parameter can be evaluated using tables of the normal 
probability distribution*. Some values of $\theta$ and $e^{-u^2} - u \sqrt{\pi} \theta$ are 
listed in table 20.1.

Since $\theta \approx 0$ for $u > 2.5$ we can conclude, using equation 20.18, that 
the breakwater has little influence at distances more than $5\sqrt{at}$ "up- 
stream". ($x = -5\sqrt{at}$).

The outward growth of the coastline at the breakwater, $L(t)$, at 
$x = 0$ is:

$$L(t) = \theta' \sqrt{\frac{4at}{\pi}} = 2 \sqrt{\frac{\theta S}{\pi h}} \sqrt{t}$$  \hspace{1cm} (20.22)

from (20.17) using (20.11). The progress of the coast is proportional 
to the square root of time; all other parameters in equation 20.22 are 
constant for a given problem.

* See, also, equation 4.09 and table 4.1.
Table 20.1  Shoreline accretion parameters.

<table>
<thead>
<tr>
<th>u</th>
<th>0</th>
<th>(e^{-u^2} - u\sqrt{\pi}o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.10</td>
<td>0.8875</td>
<td>0.8327</td>
</tr>
<tr>
<td>0.20</td>
<td>0.7773</td>
<td>0.6852</td>
</tr>
<tr>
<td>0.30</td>
<td>0.6714</td>
<td>0.5569</td>
</tr>
<tr>
<td>0.40</td>
<td>0.5716</td>
<td>0.4469</td>
</tr>
<tr>
<td>0.50</td>
<td>0.4795</td>
<td>0.3538</td>
</tr>
<tr>
<td>0.60</td>
<td>0.3962</td>
<td>0.2764</td>
</tr>
<tr>
<td>0.70</td>
<td>0.3222</td>
<td>0.2128</td>
</tr>
<tr>
<td>0.80</td>
<td>0.2579</td>
<td>0.1616</td>
</tr>
<tr>
<td>0.90</td>
<td>0.2031</td>
<td>0.1209</td>
</tr>
<tr>
<td>1.00</td>
<td>0.1573</td>
<td>0.0890</td>
</tr>
<tr>
<td>1.25</td>
<td>0.0771</td>
<td>0.0388</td>
</tr>
<tr>
<td>1.50</td>
<td>3.389x10^{-2}</td>
<td>1.529x10^{-2}</td>
</tr>
<tr>
<td>1.75</td>
<td>1.333x10^{-2}</td>
<td>5.418x10^{-3}</td>
</tr>
<tr>
<td>2.00</td>
<td>4.680x10^{-3}</td>
<td>1.726x10^{-3}</td>
</tr>
<tr>
<td>2.50</td>
<td>4.084x10^{-4}</td>
<td>1.208x10^{-4}</td>
</tr>
<tr>
<td>3.00</td>
<td>2.216x10^{-5}</td>
<td>5.581x10^{-6}</td>
</tr>
<tr>
<td>3.50</td>
<td>7.430x10^{-7}</td>
<td>1.759x10^{-7}</td>
</tr>
</tbody>
</table>

Some handy geometrical relations, valid if \(\phi\) is sufficiently small, are listed below and shown in figure 20.5.

\[
\frac{\text{distance } OB}{\text{distance } OC} = \phi' \quad (20.23)
\]

\[
\frac{\text{distance } OA}{\text{distance } OC} \approx 2.5 \sqrt{\pi} = 4.43 \quad (20.24)
\]

Also, obviously, from continuity, the total surface area, OAB is:

\[
\frac{St}{h} = \text{a } \phi' t \quad (20.25)
\]

Figure 20.5
ACCRETION GEOMETRY
20.6 Non-parallel accretion

In the previous analysis it was assumed that the entire beach profile at any given point, \( x \), moved forward uniformly. While this assumption simplifies the mathematics, it is often difficult to justify in practice. It would be handy, therefore, to have a solution usable for a situation where the accreting beach profile slope differed from that of the original profile.

Van Hijum (1972) attacked the problem for which the accretion at the toe of the slope progressed more slowly than at the top. In the schematized profile in figure 20.6, the original beach has a slope \( m \) while the accretion zone moves forward at slope \( m' \). Here, \( m \) and \( m' \) are the tangents of the slope angles.

![Figure 20.6](image)

From figure 20.6:

\[
h(y) = \frac{m m'}{m' - m} y
\]  

(20.26)

The equation of continuity (20.04) now gets the form:

\[
\frac{\partial}{\partial x} S_x + \frac{m m'}{m' - m} y \frac{\partial y}{\partial t} = 0
\]  

(20.27)

which results in an equation for the coastline (corresponding to equation 20.10) of:

\[
- S_x \frac{\partial^2 y}{\partial x^2} + \frac{m m'}{m' - m} y \frac{\partial y}{\partial t} = 0
\]  

(20.28)

After much work, van Hijum was only able to find an approximate solution to the above equation:

\[
y \approx 1.59 M^3 \phi' \left(0.724 - \frac{x}{M - x}\right)^2
\]  

(20.29)

where:

\[
M = \left[ \frac{6(m' - m)S t}{m m' (\phi')^2} \right]^{1/3}
\]  

(20.30)

At the breakwater (\( x = 0 \)):

\[
L = \left[ \frac{1.5 (m' - m)S t \phi'}{m m'} \right]^{1/3}
\]  

(20.31)
Comparison of (20.31) with (20.22) shows that accretion at the breakwater progress faster in the initial stages with the non-parallel accretion. This is logical in view of figure 20.6; less sand is needed to form the initial stages of the accretion.

The solution method just presented is, of course, only valid as long as the toe of the accreting slope continues to progress along the original slope. When this accreting toe reaches the bottom of the original slope, the situation reverts to that of parallel accretion outlined in the previous section.

20.7 Transport Past Breakwater Tip

The shoreline development equations in the previous sections were dependent upon an impervious boundary condition at the breakwater. Assuming once again that there is parallel beach accretion, the accretion at the breakwater is given by equation 20.22 in which \( y \) increases indefinitely as long as \( t \) increases. Since it is uneconomical and even irresponsible to build an infinitely long breakwater, a breakwater of a given, finite length can only be expected to stop the longshore sand transport for a finite time. Two important questions can be asked: "How long will a given breakwater completely obstruct the longshore sediment transport?", and "What happens after that time?"

Figure 20.7 shows a shore profile immediately on the accretion side of the breakwater. Since the major portion of the sand transport takes place in the breaker zone, no appreciable transport will take place around the end of the breakwater as long as the breakwater extends through the breaker zone. This implies, in the figure, that transport around the breakwater tip can be expected to start when the depth on the accreted slope at the end of the breakwater has decreased to \( h_{br} \), the depth at the outer edge of the breaker zone.

![Figure 20.7](image)
The accretion shown in figure 20.7 at depths greater than \( h_{br} \) has been transported along the coast within (or very close to) the breaker zone and then moved down the slope of the beach to the toe. This transverse transport along the beach profile will be treated in detail in chapter 21.

The accretion distance, \( L \) in the figure, can be computed knowing the breaker depth and the slopes of the beach and breakwater. Knowing this length, \( L \), the time, \( t_1 \), before sand escapes around the breakwater tip can be computed using equation 20.22:

\[
T = \frac{\pi L^2 h}{4 S \sigma} = 0.785 \frac{L^2 h}{S \sigma}
\]  

(20.32)

For the non-parallel accretion - equation 20.31:

\[
t_1 = \frac{m m' L^3}{1.5 (m' - m) \sigma S}
\]  

(20.33)

In practice, \( L \) will probably be so long that a solution using equation 20.33 will not be valid; the water will not be deep enough for this non-parallel accretion to extend so far.

The above equations answer the first of the two questions posed earlier. Note that at this time, \( t_1 \), the toe of the accretion slope extends beyond the breakwater tip! Because there is little longshore sand transport so deep on the profile, this will not lead to noticeable transport around the breakwater tip, however.

In order to answer the second question about the sand transport around the breakwater tip after time \( t_1 \), we need to formulate a new set of boundary and initial conditions and generate a new solution to the differential equation.

When material is passing around the breakwater tip, our boundary condition:

\[
\text{at } x = 0: S_x = 0 \text{ for all } t > 0
\]  

(20.15)

is no longer valid. Instead, this boundary condition should now become:

\[
\text{at } x = 0: y = L \text{ for all } t \geq t_1
\]  

(20.34)

The other boundary condition:

\[
\text{at } x = -\infty, S_x = S \text{ for all } t
\]  

(20.14)

remains perfectly valid, of course.

The most convenient initial condition would now be:

\[
\text{at } t = t_1, y \text{ is given by equation 20.17 evaluated as a function of } x \text{ for } t = t_1.
\]

This is the actual coastline profile determined using the previous solution.

---

*Strictly speaking \( y \) must be greater than \( L \) for sand transport to take place past the breakwater tip.*
Unfortunately, a workable analytic solution to the differential equation with boundary and initial conditions outlined above has not yet been found; the problem cannot be conveniently solved. This forces us to modify the conditions so as to make an analytic solution possible.

An initial condition which does allow analytical solution of equation 20.12 is:

\[ \text{at } t = 0, \quad y = 0 \quad \text{for } x < 0 \]  
\[ \text{with:} \]
\[ \text{at } t = 0, \quad y = L \quad \text{for } x = 0 \]

This is an initial condition following AOB in figure 20.5 instead of AO as in section 20.5 and curve AB as suggested above. The present initial condition (20.35 and 20.36) implies that the beach bends sharply at \( x = 0 \) and proceeds perpendicular to the coast along the breakwater! Further, the angle between the waves and the "breakwater beach" at the end of the breakwater is initially negative and we can expect sand to be transported to the \textit{left} around the breakwater tip in the initial stages of shoreline development! Since no supply for this sand exists, this solution approach seems rather unrealistic; we can, however, salvage the situation by restricting ourselves by agreeing to use this new solution only when there is a \textit{positive} transport to the \textit{right} around the breakwater tip.

With (20.14), and (20.34) through (20.36) the solution to equation 20.12 is:

\[ y = L \theta \]  

where \( \theta \) is defined just as in equation 20.19.

In order to compute the sand transport, \( S_x \), at points along the accretion coast, we need to evaluate \( \frac{3y}{3x} \) for substitution into equation 20.08 to determine \( \phi \), and hence \( S_x \). Using (20.19) and (20.18) in (20.37) and differentiating:

\[ \frac{3y}{3x} = \frac{L}{\sqrt{\alpha t}} \exp \left[ -\frac{x^2}{4\alpha t} \right] \]  

In particular, at the breakwater, \( x = 0 \):

\[ \left. \frac{3y}{3x} \right|_{x=0} = \frac{L}{\sqrt{\alpha t}} = \beta \]  

where the notation \( \beta \) has been introduced for convenience.

The sand transport at the tip of the breakwater is, now:

\[ S_{\text{tip}} = S(1 - \frac{\beta}{\theta}) = S(1 - \frac{L}{\theta \sqrt{\alpha t}}) \]
Indeed, as long as \( \beta \) is larger than \( \phi' \), there is a sand transport in the negative direction at \( x = 0 \) (to the left around the end of the breakwater). This confirms our earlier observation based upon the initial conditions.

How, then, must we compute the entire coastal development? We split the solution into two phases. The first phase begins when the breakwater is built, is described in section 20.5, and is valid until time \( t = t_1 \) from equation 20.32. The volume of sand accretion at that time will be:

\[
v_1 = S t_1 = \frac{\pi L^2 h}{4 \phi'}(20.41)\]

from equations 20.25 and 20.32 knowing that there is parallel accretion.

The equations for the second phase are developed in this section. The volume of sand accumulated is now:

\[
v_2 = \int_0^{t_2} (S - S_{\text{tip}}) \, dt \quad (20.42)\]

which, with (20.40) works out to be:

\[
v_2 = \frac{S}{\phi'} \int_0^{t_2} \beta \, dt \quad (20.43)\]

\[
= 2 L h \sqrt{\frac{a t_2}{\pi}} \quad (20.44)\]

after a bit of algebra. The subscript, \( 2 \), has been added to \( v \) and to \( t \) to emphasize that it results from the second coastline solution.

The proper time, \( t_2 \), to start using the second coastline solution may be found by stipulating that the volumes of accreted material be equal when the shift is made. This does not mean that \( t_2 \) will be equal to \( t_1 \); indeed, \( t_2 < t_1 \) since the second model allows (fictitious) sand supply from the tip of the breakwater. Equating \( v_1 \) and \( v_2 \) yields:

\[
\frac{\pi L^2 h}{4 \phi'} = 2 L h \sqrt{\frac{a t_2}{\pi}} \quad (20.45)\]

This yields:

\[
t_2 = \frac{\pi^3}{64} \frac{L^2 h}{S \phi'} \quad (20.46)\]

Introducing \( t_1 \) from (20.32) yields:

\[
t_2 = \frac{\pi^2}{16} t_1 = 0.617 t_1 \quad (20.47)\]

* Another, different solution would result by stipulating \( \beta = \phi' \) at time \( t_2 \); in other words, there is no sand transport at the tip.
This confirms our observation about the relative values of $t_1$ and $t_2$. This means, in practice, that we must "start" the second solution at some time later than the time $t = 0$ when the phase 1 solution started. This is shown diagrammatically in figure 20.8. In that figure data pertaining to the first model is indicated above the time axis, that for the second model is below the axis. As we can see from the figure, the time axis for the second solution model is shifted to the right relative to the origin of the original time axis. Since this time origin shift can be inconvenient, we can make an appropriate correction to the equations for phase 2 to allow substitution of times based upon the original time scale. The time scale for phase 2 is found by shifting times from the original scale by $0.383 t_1$ as shown in the figure. Making this time origin shift in equation 20.40 yields:

$$S_{\text{tip}} = S \left[ 1 - \frac{L}{\phi' \left( \frac{ma}{(t - 0.383 t_1)^{1/2}} \right)} \right]$$

(20.48)

or, using (20.32):

$$S_{\text{tip}} = S \left[ 1 - \frac{2}{\pi \sqrt{\frac{t}{t_1} - 0.383}} \right]$$

(20.49)

which describes the sand transport past the breakwater tip for all times $\geq t_1$ using the original time scale.

As a check, we might well determine the sediment transport past the breakwater at time $t = t_1$; this should be zero. With $t = t_1$ (20.49) yields:

$$S_{\text{tip}} = S \left( 1 - \frac{2}{\pi \sqrt{1-0.383}} \right)$$

(20.50)

$$S_{\text{tip}} = 0.189 S$$

(20.51)
This error results from the fact that the details of the shorelines are different for each of the models even when the total volumes of accumulated sediment are equal. The differences can be seen by plotting the two coastline profiles - equations 20.17 and 20.37.

Bakker has determined a correction to be applied to the computed values of transport at the breakwater tip. These are listed in table 20.2.

The merits of all of this work will be discussed in the following section; an example of its application is given in section 20.9.

<table>
<thead>
<tr>
<th>$t/t_1$</th>
<th>equation 20.49</th>
<th>corrected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.189</td>
<td>0.000</td>
</tr>
<tr>
<td>1.25</td>
<td>0.316</td>
<td>0.298</td>
</tr>
<tr>
<td>1.50</td>
<td>0.398</td>
<td>0.394</td>
</tr>
<tr>
<td>2.00</td>
<td>0.499</td>
<td>0.499</td>
</tr>
<tr>
<td>3.0</td>
<td>0.606</td>
<td>0.606</td>
</tr>
<tr>
<td>4.0</td>
<td>0.665</td>
<td>0.665</td>
</tr>
<tr>
<td>5.0</td>
<td>0.704</td>
<td>0.704</td>
</tr>
</tbody>
</table>

20.8 Critical evaluation

The method of Pelnard-Considère has one major strong feature - it makes a hand computation of coastal changes possible. It can be used for accretion, as done here, and also for erosion on the lee side of a coastal sediment obstruction. Application in such a case will yield a coastal profile that is a mirror image relative to the origin of those found for accretion.

In the development just presented we have allowed the beach to change (accrete in this case) even outside the breaker zone. The motivation for this was not given; it shall become obvious in a later chapter when sediment movements along a beach profile are discussed.

The development of the transfer of phases between the two coastline profile models is quite arbitrary. The assumption that the length of the accretion at the breakwater, $L$ in figure 20.7, remains constant after time $t = t_1$ will not be exactly true in practice; some continued growth in the second phase should be expected.
The assumptions made in order to get an equation of motion are, at best, so restrictive that the approach is primitive. Wave height and direction variations along the coast, tidal influences, and many of the other more sophisticated points of the Bijker formula have had to be neglected. The assumption about the angle of wave attack, $\phi'$, being very small can be very crude, especially since the toe of the zone affected by longshore transport - where $\phi'$ is measured - can be well outside the breaker zone.

The inclusion of an arbitrary coastline profile as an initial condition - as opposed to the straight line used here - is difficult, if not impossible. This makes the modeling of many "real" coasts rather arbitrary. Indeed, we have assumed a straight coast for $\frac{4.43}{\phi'}$ times the length of the breakwater - equation 20.24. If, for example, $\phi' = 10^0 = 0.175$ rad. and the breakwater is 1000 m long, then we assume a straight coast extending over 25 km!

20.9 Example

A harbor entrance is to be built upon a straight sandy coast which is subjected to waves having a period of 13 seconds and a deep water height, $H_{sig}$, of 1.8 meters. The angle of approach of the waves in deep water, $\phi_0$, is $25^0$. (Such a situation can often be found in tropical seas; this example is not too different from the situation on the coast of Ghana. The waves are of very constant period, height and direction throughout the year).

For simplicity, we shall assume that the breaker index, $\gamma$, is 0.8 and that the beach contour lines are parallel.

A harbor entrance is to be built with a breakwater which is curved in plan - a circle with a radius of 1650 m with center at the existing straight beach line. The breakwater slopes are 1:3 and the natural beach slope is 1:100 to a depth of 7.0 m beyond which the sea bottom is considered to be horizontal for a considerable distance. See figure 20.9.

Before we can begin actual sand transport computations, we must determine the wave angle at the toe of the beach slope. This is a straightforward refraction computation:

$$\lambda_0 = (1.56)(13)^2 = 264 \text{ m}$$

$$\frac{h}{\lambda_0} = \frac{7}{264} = 0.02652$$

Using the tables in volume III of the Shore Protection Manual:

$$\frac{h}{\lambda} = 0.06683$$

$$\frac{c}{c_0} = 0.4199$$

and thus:

This is slightly in error; the numerical values in the rest of this example are thus also a bit incorrect.
\[
\sin \theta' = 0.4199 \sin 25^\circ \quad (20.56)
\]
which yields \(\theta' = 10.2^\circ\) \( (20.57)\)

see figure 20.9.

In order to use the CERC formula to determine the longshore sand transport along the undisturbed coast, we must also determine the angle of wave attack at the breaker line. This breaker line is located at the depth at which the root-mean-square wave breaks.

From equation 10.03 of volume I:
\[
H_{rms_0} = \left( \frac{1}{1.8} \right) (1.8) = 1.27 \text{ m} \quad (20.58)
\]

We can now determine the breaker angle, \(\phi_{br}\), using the iterative procedure outlined in section 5 of chapter 16. This results in:
\[
\begin{align*}
H_{rms_{br}} &= 1.80 \text{ m} \\
h_{br} &= 2.26 \text{ m} \\
\phi_{br} &= 5.6^\circ
\end{align*}
\quad (20.59)
\]

The CERC formula (17.09) (with improved coefficient) then yields:
\[
S = 1.23 \times 10^6 \left(1.27^2\right) (1.56\times13) \left( \frac{\cos 25^\circ}{\cos 5.6^\circ} \right) (\sin 5.6^\circ)(\cos 5.6^\circ)
\]
\[
= 3.24 \times 10^6 \text{ m}^3/\text{yr.} \quad * 
\quad (20.60)
\]

* This rather high value results from a combination of relatively high characteristic wave height and respectable angle of attack.
Before this question can be answered directly, we must determine the effective length of the breakwater shown in figure 20.9. A naive approach is to begin assuming the breakwater extends 1650 m (the distance CD) from the coast. However the correct approach is to realize that sand passing point B on the breakwater will be transported further by the waves. Thus, the breakwater should be schematized by one of length OB where B is located at the point where the approaching waves are tangent to the circle. The distance OB is:

\[ OB = 1650 \cos 10.2^\circ = 1624 \text{ m} \]  
(20.61)

Construction of a figure such as that shown in figure 20.7 with the appropriate length, breaker depth, and slope values yields:

\[ L = 1624 - (100-3)(2.26) = 1405. \text{ m} \]  
(20.62)

Knowing \( L \), the time needed for the sand to accumulate to the point that it is no longer obstructed by the breakwater is found using equation 20.32:

\[ t_1 = 0.785 \frac{(1405)^2(7)}{(3.24 \times 10^6)(0.178)} = 18.8 \text{ years} \]  
(20.63)

Note that the angle \( \phi' \) has been expressed in radians in the above expression.

One may argue that this result must be corrected for the fact that a portion of the space to the left of the y axis in figure 20.9 has been occupied by the harbor instead of accumulated sand. The volume occupied by the harbor to the left of the y axis is about \( 10^7 \text{ m}^3 \) which means that the time \( t_1 \) will actually be:

\[ t_1' = 18.8 - \frac{10^7}{3.24 \times 10^6} = 15.7 \text{ years}. \]  
(20.64)

Note that for further computations, a time \( t_1 \) of 18.8 years must be used. This time corresponds to the computed time; the differential equation solution is not aware of the area occupied by the harbor.

The form of the coast after 10 years, at the time sand starts bypassing, and after 30 and 100 years can be plotted using equation 20.17 for the first two cases and equation 20.37 for the last two cases. Since the times are known, we are to plot \( y \) as a function of \( x \); all other parameters are known. The computations are listed in table 20.3. The computations for \( t = 18.8 \text{ years} \) are illustrated.

Three constant factors can be evaluated. From equation 20.11:

\[ a = \frac{S}{\phi' n} = \frac{3.24 \times 10^6}{(0.178)(7)} = 2.60 \times 10^6 \]  
(20.65)

From equation 20.18:

\[ \sqrt{4ac} = \sqrt{4(2.60 \times 10^6)(18.8)} = 13980. \]  
(20.66)

These values are listed under table 20.3.
<table>
<thead>
<tr>
<th>Time (yrs)</th>
<th>Actual</th>
<th>Computed</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>17.7</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>18.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>19.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>19.1</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>19.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>19.3</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t = 100 years</th>
<th>t = 30 years</th>
<th>t = 18.8 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>O</td>
<td>n</td>
</tr>
<tr>
<td>O</td>
<td>C</td>
<td>O</td>
</tr>
<tr>
<td>n</td>
<td>C</td>
<td>O</td>
</tr>
</tbody>
</table>

Table 20.3: Coastline from Computations
Further, the computations for the row of table 20.3 for \( x = -2 \) km will be shown. Using (20.18):

\[
\begin{align*}
u &= \frac{2000}{13980} = 0.143 \\
(20.67)
\end{align*}
\]

\( e \) can be evaluated using table 20.1*; its value is found to be:

\[
\begin{align*}
e &= 0.841 \\
(20.68)
\end{align*}
\]

which yields a value of \( y \) of 1077 m when substituted into equation 20.17.

For times later than \( t = 18.8 \) years we must switch to the equation solution 20.37 but must remember to compute \( u \) based upon the shifted time scale shown in figure 20.8. For comparison purposes, the shoreline profile is computed for \( t_2 = 0.617 \; t_1 \) - the point at which we change to the new solution in figure 20.8 as well as for the two times asked: 30 and 100 years. Note that these times must also be adjusted for use in the computations; the differential equation time values are listed in table 20.3 as well.

Comparison of the two beach lines for the time at which the accretion starts passing the tip show little significant variation.

Time corrections, based upon the volume of sand displaced by the harbor in our schematized model result in actual times needed to produce a given accretion being shorter than those computed via the solution to the differential equation. The actual times corresponding to the times, \( t \), used in table 20.3 are also listed there.

Examination of the values of \( y \) versus \( x \) in table 20.3 confirms the statement (equation 20.24) that the accretion is essentially zero a distance \( 2.5L\sqrt{\pi'/\delta'} \) before the breakwater.

A plot of the accretion on an undistorted scale would be handy for showing the variations in the angle of attack, but impractical in light of the dimensions involved. Figure 20.10 shows the accretion plotted with a \( y \) axis distorted by a factor 10.

Examination of figure 20.10 shows dramatically how the accretion proceeds back along the shoreline as well as outward along the breakwater. An interesting but academic question is: what is the limit situation at \( t = \infty \)? The coast will, of course, ultimately become parallel to the original coast 1405 m seaward of its original location, at least theoretically. Actually, it will be more seaward with only part of the breaker zone being effectively blocked by the breakwater.

* A more complete table of the probability integral was used while compiling this example, however.
21. SAND TRANSPORT ALONG A BEACH PROFILE  

J. v.d. Graaff

21.1 Introduction

Just as waves can contribute to a sediment movement along a coast, they can also contribute to a sediment movement along a beach profile perpendicular to a coast. As has been indicated already in chapter 18, waves can cause a net sand transport even when no other currents are present and the net mass transport of the waves is zero. This net transport results from a product of a time dependent sediment concentration and a time dependent velocity. Indeed, under some conditions - proper material grain size, for example - a negative - opposite the wave propagation direction - sand transport has been observed even when the long term average velocity has been positive. Of course, if such a resulting current in the direction of wave propagation is made large enough, a positive sand transport will result. Figure 21.1 shows a sample of test results from a laboratory flume in which a current was superimposed on the wave action. Positive sand transports and currents are in the direction of wave propagation. In this example, increasing the current, initially, causes the sand transport to become more negative and later to become positive and increase sharply.

The presence of the current paralleling the direction of wave propagation influences the eddy formation near the ripples directly as well as indirectly by influencing the wave length. In another field, this last influence is the basis for the design of air bubble curtain breakwaters - see volume III.

To make matters more complicated, the presence of a longshore current along a beach also influences the sand transport perpendicular to the beach. In light of the development of the longshore sand transport formulas which reflect an interaction of waves and current - chapters 16 and 19 - it is only logical to expect the presence of a longshore current to influence the on and offshore sediment transport caused by waves. If, indeed, the stirring up of bottom material depends upon the total bottom shear stress, as postulated in chapter 19, then the influence of this longshore current on the on and offshore transport is obvious.

Still another variable in the transport of material along a beach profile is the beach slope. It is conceivable that a component of the gravitational force now contributes to the transverse sediment transport. Perhaps even more important, the wave conditions vary from place to place as we proceed across the profile. This introduces even more complications.

Unfortunately, this was an underdeveloped research area until recently. Few model tests and even fewer dependable prototype measurements have been made. The mathematical descriptions available are, therefore, extremely primitive even when compared to the formulations available for longshore transport. Some of these primitive descriptions are presented in the following sections.
21.2 Two dimensional transverse transport

Bakker (1968) examined the transverse sand transport along a coast having groins extending only part way through the breaker zone. After observing that reasonably flat shore areas were built up by material supplied from offshore and steep beach profiles were flattened as material moved offshore, he proposed a simple transverse transport formula based upon the beach slope. He reasoned that for some equilibrium average beach slope, $m_e$, there would be no transverse sand transport; for other slopes, there would be a transport directly proportional to the difference between the actual and equilibrium slopes. Expressed as a formula:

$$S_y = (m - m_e)$$  \hspace{2cm} (21.01)

where: $m$ is the average beach slope, $\frac{dz}{dy}$, $m_e$ is the equilibrium beach slope, $S_y$ is the sand transport per unit width along the beach profile, and $\alpha$ denotes "is proportional to".

The negative sign in (21.01) follows from the definition of the positive axes resulting in both $m$ and $m_e$ being usually negative while an offshore transport is considered positive in agreement with the positive $y$ axis.

Since Bakker was interested in the sand transport along the beach profile partly blocked by a groin he chose to schematize the coast profile as shown in figure 21.2. In that figure, the point at which the transverse transport rate is desired is denoted by A and is located on the surface between the two schematizing "steps". Since the shaded areas are equal this implies that:

$$L_1 = \frac{1}{h_1} \int_{h_1}^{0} y(z) \, dz$$  \hspace{2cm} (21.02)

and

$$L_2 = \frac{1}{h_2} \int_{-h_2}^{-h_1} y(z) \, dz$$  \hspace{2cm} (21.03)

where $y(z)$ describes the actual profile.

The slope of this beach near point A can now be characterized by:

$$\frac{1}{m} = -2 \left( \frac{L_2 - L_1}{h} \right)$$  \hspace{2cm} (21.04)

The distance $L_2 - L_1$ corresponding to the equilibrium slope is often referred to by a symbol, $W$, so that:

$$\frac{1}{m_e} = -2 \left( \frac{W}{h} \right)$$  \hspace{2cm} (21.05)

* In Bakker's special case, this point was also the toe point of the groins.
Putting (21.04) and (21.05) into (21.01) and adding a proportionality constant, $q_y$, yields:

$$S_y = q_y (W - (L_2 - L_1))$$  \hspace{1cm} (21.06)

This gives an equation of motion which is really much like that used by Pelnard - Considère - equation 20.05*.

The parameters $q_y$ and $W$ in equation 21.06 are both dependent upon many variables including the wave parameters, the sand parameters, and unfortunately, the position of the point A (fig. 21.2) along the beach profile. A dependence of the parameter, $W$, on the location of point A is obvious because of the relation between this location and the schematized slope. Indeed the slope of a beach is not constant but varies along the profile. The dependance of $q_y$ on the location of A is less obvious but is apparently related to (among other things) the changes in the wave pattern occurring as the waves cross the beach.

Even so, Bakker assumed that the necessary parameters, $q_y$ and $W$ could relatively easily be determined: $W$ from measurements on an existing equilibrium profile and $q_y$ from field measurements.

* This comparison with equation 20.05 is not perfect. $s_x$ in (20.05) represented the derivative of a sand transport with respect to an angle; here, $q_y$ represents a derivative with respect to a distance. Granted, the constant depth, $h$, assumed makes it possible to interpret this distance as a slope angle.
Swart (1974) attempted to generalize the concept of Bakker (1968) for all locations along the beach profile and to determine values for $q_y^*$ and $W$ in terms of easily measured physical parameters. He carried out a large number of small scale model studies and a few laboratory studies at more-or-less prototype scale. These tests were all carried out with regular waves and erosion was taking place on all the beach profiles; these are serious limitations for the applicability of the results to prototype problems. Swart's results are a large number of empirical relationships involving non-dimensionless parameters.

Even so, there is nothing better available now (1977), and subject to the restrictions already mentioned, Swart's empirical relations do make it possible to at least estimate transverse sediment transports past any given point on a profile as demonstrated in the following section.

As yet, not enough transverse sediment transport data caused by irregular waves have been collected in order to attempt a correlation with the work by Swart. The question of what characterizing wave best represents an irregular sea in a transverse transport computation has not been answered.

### 21.3 Example

This example illustrates the computation procedure proposed by Swart (1974) for the determination of the sand transport along the beach profile at a chosen location. The beach profile shown in figure 21.3 is subjected to regular waves approaching with crests parallel to the coast. The deep water wave height is 2.0 m and the wave period is 6.0 s. The beach has an average grain size of 225 μm and the still water level is 1.0 m above MSL as shown in the figure. The sand transport along the profile at point $O$ (mean sea level) is desired.

We solve the problem using a two line coastal schematization with the evaluation separating the two zones at Mean Sea Level - the chosen elevation where the transverse transport is to be determined. Thus, we are looking for $S_y$ determined using an equation like (21.06).

In order to determine the value of $(L_2 - L_1)$, we must first determine the upper and lower limits of Swart's $D$ profile. Using figure 21.4 and entering with:

$$\frac{H}{D_50} = \frac{20.488}{6.786} = 30.67 \quad (225 \times 10^{-6})$$

yields (indirectly):

$\star$ Swart chose his zone boundary more generally and generalized Bakker's $q_y$ calling it $s_y$. 

```plaintext
100 100 100 100 100 100 100
```
Figure 21.3
BEACH PROFILE
DISTORTION 1:20
Figure 21.4.
UPPER LIMIT OF D-PROFILE
$h_0, H_b, D_{50}$ in m
T in sec.

$h_0 = 0.93$ m and
$\delta_1 = 1.93$ m

The lower limit of the D profile is found using figure 21.5 entering with:

$$
\frac{H_0^{0.473}}{D_{50}^{0.093}} = \frac{2^{0.473}}{6^{0.894}(225 \times 10^{-6})^{0.093}} = 0.61
$$

** The notation used by Swart is reasonably well adhered to in this section even though it is inconsistent with that used in the rest of these volumes. The subscript 0 here, has nothing to do with deep water, in general.

** Equivalent to $h_1$ in equation 21.02.
yields:

\[
\frac{h_m}{\lambda_o} = 0.090
\]  
(21.11)

from which, with:

\[
\lambda_o = 1.56 \ T^2 = 56.2 \ m
\]  
(21.12)

\[
h_m = 5.04 \ m \text{ and } \delta_2 = 4.04 \ m
\]  
(21.13)

These limits, then, yield the height of the D profile, \( \delta \), of:

\[
\delta = \delta_1 + \delta_2 = h_o + h_m = 5.97 \ m
\]  
(21.15)

These limits are sketched in figure 21.3.

Knowing these limits, the distances \( L_1 \) and \( L_2 \) to the schematized shore steps can be computed using (21.02) and (21.03). This results in values for these parameters of 19.27 m and 194.40 m respectively.

\[
(L_2 - L_1) = 194.40 - 19.27 = 175.13 \ m
\]  
(21.16)
The following step is to evaluate the corresponding equilibrium distance, W, for the point in question. This cannot be done directly in Swart's method; instead, we determine, first, the W value for the still water level, $W_r$, using figure 21.6 entering with:

$$\frac{H_0}{\lambda_0} = \frac{0.132}{0.447} \quad \frac{\lambda_0}{H_0} = \frac{0.717}{0.447}$$

$$= \frac{2.0.132}{(225\times10^{-6})(0.447)} \quad 0.717 = 512.$$

(21.17)

yields:

$$m_r \frac{H_0}{\lambda_0} = 6.49\times10^{-4}$$

(21.18)

\[ \text{two-dimensional model test} \]

\[ \text{three-dimensional model test} \]

\[ \text{prototype cases} \]

---

from which:

$$W_r = \frac{\delta}{2} \frac{H_0}{\lambda_0} = \frac{(5.97)(2)}{(2)(56.2)(6.49\times10^{-4})} = 163.71$$

(21.19)

is the equilibrium distance for the waterline.

The ratio, $\frac{W}{W_r}$, can be determined using figure 21.7. Here:

$$\Delta r = \frac{H_0 - \delta}{\delta} = \frac{5.04-4.04}{5.97} = 0.168$$

(21.20)

Then, entering that figure with:

$$\delta_{50} \approx 0.68\times10^4 \delta_{50} = 225\times10^{-6} \quad (0.168)(0.68\times10^4)(225\times10^{-6})$$

$$= 1.47\times10^{-7}$$

(21.21)
(This is effectively zero). yields:

\[
\frac{W}{W_r} - 0.7 \Delta r = 1.0 \tag{21.22}
\]

which, in turn yields:

\[
W = (1 + 0.7)(0.168)) \times 163.71 = 184.35 \text{ m} \tag{21.23}
\]

Luckily, this value of \(W\) is greater than \((L_2 - L_1)\) indicating that an offshore sand transport can be expected - the only condition for which Swart's method has been checked.

![Figure 21.7](image)

**Figure 21.7**

**GENERAL RELATIONSHIP FOR \(W/W_r\)**

(TWO-DIMENSIONAL CASES)

\(D_{50}\) in meters

\(b = 1\) for \(\Delta r > 0\)

\(b = 0\) for \(\Delta r < 0\)

We now know values for \((L_2 - L_1)\) and \(W\) to substitute into equation 21.06; the only remaining problem is that of determining the value of \(s_y^*\) for the desired location on the profile. Unfortunately, this value must also be found in a somewhat roundabout way.

\* Corresponding to \(q_y\) in (21.06).
Using figure 21.8 entering with:
\[ x = H_0 \frac{1.68}{\lambda_0} \left( \frac{H_0}{\lambda_0} \right) - 0.9 \left( \frac{D_{50}}{h_m} \right) - 1.29 \left( \frac{H_0}{h_m} \right) 2.66 \]
\[ = 21.68 \left( \frac{2}{55.2} \right) - 0.9 \left( 225 \times 10^{-6} \right) - 1.29 \left( \frac{2}{50.04} \right) 2.66 \]
\[ = 2.80 \times 10^5 \]
yields:
\[ \frac{s_{ym} \cdot T}{D_{50}} = 0.972 \]
(21.25)

where \( s_{ym} \) is the maximum value of \( s_y \) occurring on the profile. Equation 21.25 then yields:
\[ s_{ym} = (0.972) \left( \frac{225 \times 10^{-6}}{6} \right) = 3.64 \times 10^{-5} \text{ m/s} \]
(21.26)
The location on the profile at which this maximum offshore transport takes place can be found using figure 21.9. Entering with:

\[
H_o^{-0.55} \left( \frac{H_o}{h_m} \right)^{2.69} = (2)^{-0.55} \left( \frac{2}{5.04} \right)^{2.69} = 0.0568
\]  

yields:

\[
\Delta_{2m} \delta = 0.737
\]  

where \( \Delta_{2m} \) is the \( \delta \) value at the point where \( s_{ym} \) occurs. This value of \( \Delta_{2m} \) turns out to be:

\[
\Delta_{2m} = (0.737)(5.97) = 4.40 \text{ m}
\]  

(Delft Hydraulics Laboratory data
x Coastal Engineering Research Center data)

![Figure 21.9](image)

**Figure 21.9**

**POSITION OF \( s_{ym} \)**

This point is, thus, below the still water level but above (onshore from) the section where we wish to know the sand transport along the profile. (\( |\Delta_2 - \Delta_{2m}| = |4.04 - 4.40| = 0.36 \text{ m above mean sea level} - \text{figure 21.3.} \)).

Figure 21.10 relates \( s_y \) - the desired value - to the maximum value \( s_{ym} \): Entering with:

\[
\frac{|\Delta_2 - \Delta_{2m}| \left( \frac{H_o}{h_m} \right)^{2.69}}{5.97} \left( \frac{2}{5.04} \right)^{2.69} = 0.267
\]  

\[
\left| \frac{1}{\frac{2}{5.04}} \right| \left( \frac{2}{5.04} \right)^{-1} = 0.267
\]  

\[
|4.04-4.40| \left( \frac{2}{5.04} \right)^{2} = 0.267
\]
yields, using figure 21.10b (we are offshore from the maximum):

\[
\frac{S_y}{S_{ym}} = 0.935 \tag{21.31}
\]

or:

\[
S_y = (0.935)(3.64 \times 10^{-5}) = 3.40 \times 10^{-5} \text{ m/s} \tag{21.32}
\]

We can now substitute values into equation 21.06. Using the results (21.16), (21.23), and (21.32):

\[
S_y = (3.40 \times 10^{-5})(184.35 - 175.13) \tag{21.33}
\]

\[
= 3.14 \times 10^{-4} \text{ m}^2/\text{s}
\]

or, on an hourly basis:

\[
S_y = 1.13 \text{ m}^2/\text{hr}. \tag{21.34}
\]

Thus a seaward sand transport of a bit more than one cubic meter per hour can be expected through each meter of coastline at mean sea level. This will not continue indeﬁnitely, of course. The transport of material along the proﬁle will modify the proﬁle and change the parameters involved in this sand transport determination - especially the value \((L_2 - L_1)\).
21.4 Three dimensional transverse transport

When we allow a third dimension - along a coastline - to enter our discussion, we are immediately confronted by many additional resulting currents which further complicate the description of the transverse sand transport. As has already been indicated, longshore currents, whatever their cause - see chapters 12 through 16, will influence the transverse transport. Additional current components, such as rip currents, flowing along the beach profile perpendicular to the shore will obviously also be of direct influence.

A special case of current perpendicular to a coast occurs when a groin interrupts the longshore transport. Water flowing parallel to the coast toward the groin is deflected seaward along the groin. Continuity of the longshore current being re-established beyond the groin will lead to a shoreward current on the lee side of the groin. If this longshore current field normally extends seaward of the obstruction, as a tide current acts over a wide zone along a coast, then the flow around the tip of an obstruction will be more concentrated than would be the case if these longshore currents did not exist in the offshore area.

Unfortunately, the insight into the mechanism and characterizing parameters for these special types of currents is seriously lacking. Bowen (1969) has paid much attention to this problem. The incorporation of these currents in the determination of the transverse sediment transport still involves a lot of work.
22. COASTAL CHANGES WITH MULTIPLE LINE THEORIES

22.1 Introduction

The limitations of the single line shoreline development equations are numerous; collectively, they amount to too strong a schematization of reality. This can apply to initial and boundary conditions as well as the physical wave and beach characteristics. In particular, Bakker (1968) was concerned about a coast upon which the longshore sand transport is only partially blocked by groins which were shorter than the width of the breaker zone.

Bakker proposed a so-called two line theory for the solution of his problem; it turns out to be a special case of multiple line theories. Instead of schematizing a coastline with a single curve, a number of curves are used as explained in the following sections.

22.2 The schematization

Figure 22.1 shows beach profiles with one, two and three line schematizations. The choice of reference axis for the y distances is, of course arbitrary. The areas of each of the pairs of shaded "triangles" are equal. Each longshore transport $S_{xi}$ is directed parallel to the coast out of the plane of the paper and describe the littoral sand movement in its zone. The horizontal planes are usually selected at elevations which correspond more or less to flat portions of the total profile. If special structures, such as groins, are involved in the schematization, the schematizing horizontal planes are often chosen so that the limits of the boundary conditions correspond to the limits of a transport zone. This is illustrated in the sketch in figure 22.2.
Figure 22.1

BEACH PROFILE WITH SCHEMATIZATIONS
(Pairs of cross hatched areas are equal)
22.3 Equations of continuity and motion

Just as with the single-line approximation presented in chapter 20, it will be necessary to develop a continuity relationship and equations of motion for each of the schematization zones.

The equation of continuity is a bit more complex than that for a single line theory since there is now supply and removal of sand from two directions. Figure 22.3 shows an element in plan which can be compared to figure 20.2a. Once again, the net transport of material into the element is equal to the volume retained for each of the N elements - one for each line of the schematization. Thus, for $i = 1$ to $N$: 
\[ S_{xi} \, dt - (S_{xi} + dS_{xi}) \, dt + S_{yi-1} \, dx \, dt - S_{yi} \, dx \, dt = dx \, dy \, h_i \quad (22.01) \]

No sand is transported transversely except between the zones so that:

\[ S_{yo} = 0 \quad (22.02) \]

\[ S_{yN} = 0 \]

Using (20.02) and (20.03) which are still valid in (22.01), yields:

\[ \frac{\partial S_{xi}}{\partial x} - S_{y(i-1)} + S_{yi} + h_i \frac{\partial y}{\partial t} = 0 \quad (22.03) \]

which can be compared to equation 20.04. It should be remembered in the above equations that \( S_{xi} \) is the longshore sand transport over the entire zone width, while \( S_{yi} \) is a transverse transport per unit length of beach. For \( N = 1 \) equation 22.03 degenerates to equation 20.04, Equation 22.03 is the general equation of continuity.

The terms \( S_{y(i-1)} \) and \( S_{yi} \) in equation 22.03 can be evaluated using an equation such as 21.06 and the methods of the previous chapter. The method of Swart presented there yields, really, an equation of motion along the profile.

The rate of change of sand transport along the coastline, \( \frac{\partial S_{xi}}{\partial x} \), can be found by extending the methods of section 20.3. Thus, referring to and extending the results there:

\[ \frac{\partial S_{xi}}{\partial x} = \frac{\partial S_{xi}}{\partial \phi_i} \frac{\partial \phi_i}{\partial x} \quad (20.06) \]

and:

\[ \frac{\partial \phi_i}{\partial x} = \frac{\partial^2 y_i}{\partial x^2} \quad (20.07) \]

where \( \phi_i \) is now the instantaneous angle of approach of the waves at the toe of each zone relative to the schematizing line for that zone. This angle of attack is thus:

\[ \phi_i = \phi_i' - \frac{\partial y_i}{\partial x} \quad (22.06) \]
where $\phi^*_i$ is the angle of wave approach at the toe of the $i$th zone measured relative to the $x$ axis. The depth at this $i$th toe will be from figure 22.1:

$$\sum_{n=1}^{i} h^n$$

(22.07)

Obviously, the wave conditions should be evaluated at this same depth. In general, refraction, shoaling, and breaking will all have to be considered.

In practice, values of $S_{x_i}$ can be found by determining the longshore sand transport distribution across the beach using, for example, the Bijker method outlined in chapter 19. Integration of the resulting sand transport curve across each of the schematization zones independently yields a set of values for $S_{x_i}$.

The derivative needed, $\frac{d S_{x_i}}{d \phi^*_i}$, can then be evaluated approximately by evaluating sets of $S_{x_i}$ as outlined above for various slightly different values of $\phi^*_i$. The derivative in question is then approximated by:

$$\frac{\Delta S_{x_i}}{\Delta \phi^*_i} \approx \frac{\Delta S_{x_i}}{\Delta \phi^*_i}$$

(22.08)

22.4 Initial and boundary conditions

Just as with a one-line approach, appropriate initial and boundary conditions must be established consistent with the problem to be solved. Each line of the solution of an $N$ line approach will need an initial and two boundary conditions. Further, boundary conditions for the transverse sediment transport must also be established. For example, a common physical boundary condition for the transverse transport is that no sand enters or leaves the schematized beach in the transverse direction; this led, in fact, to equation 22.02*. Since other initial and boundary conditions in the longshore direction look for each line much like those in chapter 20, they will not be elucidated further, here.

An additional advantage of the numerical solutions is that initial conditions can be much more flexible. For example, the initial condition implying that the beach be straight and of constant slope is no longer necessary. Now, the initial condition $y_i(x)|_{t=0}$ may describe the actual depth contours. This, of course, makes more realistic solutions possible.

* This boundary condition will be relaxed a bit in chapter 23.
22.5 Solution to the equations

Mathematicians have assured us that under appropriate conditions such as constant values for $S_{x1}$, $S_{y1}$, $\phi_1$, and $W_1$ and simple initial conditions $y_1(x) = \text{constant at } t = 0$, an analytical solution exists to the equations of motion represented by (22.04) combined with the continuity relationship (22.03). Rather than find and use these analytical solutions, however, it has proven more popular and realistic to develop numerical integration schemes to solve the combination of equations 22.04 and 22.03 directly by approximate time-stepping techniques. The development of digital computer programs for this work is a major research activity within the Coastal Engineering Group at the Delft University of Technology.

22.6 Future developments

Progressing one step further in our numerical approximation, we can subdivide each of the schematizing lines of our beach into blocks along the beach; we are, then, placing a grid on the plan of the beach. - the $x$, $y$ plane. Usually, these blocks elements will be relatively long (in the $x$ direction) relative to its width. We can now compute wave, current, and sand transport conditions for each of these blocks at the start of the study and compute coastal changes occurring during some time. After some time, changes in either the coastal geometry or - more likely - the offshore wave and current conditions, will make it necessary for the computation parameters for each element to be recomputed. In this way, the development of a coastline and entire beach subjected to given storm conditions can be simulated. Indeed, even the effects of tides and other forces described previously in section 16.6 can also be included.

While all of this sounds very nice in principle, there remain several practical limitations. Continuity of water as well as sand must be provided, and transient situations such as the development of the longshore current on the lee side of a groin must also be accounted for. Perhaps most important, however, there remains an economic question involving the computation costs: "Will the increased accuracy of the solution justify the additional computational effort and cost?".
23. DUNE COASTS

23.1 Introduction

Dunes occur naturally in many parts of the world. In their most untamed state they are associated with exposed dry sand being transported by the wind. In this state, they can migrate with the wind, often becoming quite high and sometimes encroaching upon and disrupting the works of man. They have been known, for example, to cover highways and railroads and to destroy productive agricultural land - see figure 23.1.

While dunes occur throughout the world, they are actually quite frequent along coasts. The shore of The Netherlands is a splendid example of coastal dunes. They can also be found in many other parts of the world, portions of the coast of Ghana, parts of the Oregon coast in the United States (volume I figures 25.4, 29.9 and 29.10, and figures 23.2 and 23.3 in this volume). The remains of an old transatlantic cable can be seen in the lower right in figure 23.3c. Isolated sand dunes are present along the mud coast of Suriname - Allersma (1968). Because of their height (usually), and thus volume of stored sand, coastal dunes can be utilized, where present, in a coastal protection scheme for the benefit of man. The remainder of this chapter will be devoted to the dynamics of a dune-protected coast such as is found in The Netherlands.
23.2 Dune formation

Two basic components are needed to form coastal dunes: a reasonably continuous, slow supply of sand, and a wind blowing at least somewhat toward the shore.

The sand is usually supplied by the sea - the waves - in one of two ways; either by transverse transport from deeper water or by accretion of a beach caused by a decreasing longshore sand transport along a coast. The first of these transport mechanisms is driven by the small net shoreward mass transport near the bottom in waves outside the breaker zone. (This is in contrast to the discussion of transverse mass transport within the breaker zone presented in section 11.5) Longuet Higgins (1953), Bijker, et al (1975) and Battjes (1976) discuss this mass transport in more detail. This is one of the predominant supplies of sand for the Dutch coast. These dunes generally grow slowly but rather steadily.

The second means of sand supply, a decreasing longshore transport, is a more obvious source of sand for dune buildup. Such a supply nourishes the dunes near the end of Cape Cod in The United States - see figure 23.1. Much more rapid dune growth can take place because of the larger supply.

Figure 23.2
AERIAL PHOTO OF DUNE COAST
SANDY NECK, CAPE COD, U.S.A.
Figure 23.3
THREE TYPES OF DUNES
CAPE COD, U.S.A
With either source of supply, the sand must become dry so that the wind can more easily pick it up for transport. (Wet, unsaturated sand has a fictive cohesion caused by the surface tension of the intergranular water. This surface tension can only be overcome by relatively strong winds). Tidal action can be sufficient to allow an upper layer of sand near the high tide line to dry and be transported by the wind. If the sea water is extremely saline the salts left behind as the water evaporates can be sufficient to cement the sand grains together so that they cannot be disturbed by the wind. Such a salt-cemented sand layer is often called caliche, but is not often found on the shores of the major oceans.

Figure 23.4 shows a somewhat schematized cross-section of a dune-protected coast often found in The Netherlands. It compares well to that in volume I figure 25.1. The dry backshore tends to make the dunes appear to be very independent of the rest of the coast even though they are not. Except during storms, changes occur slowly, almost imperceptibly except to expert observers.
This section has described how dunes are formed and nourished. A section in the following chapter will describe how dune forms can be modified for the benefit of man. The build-up process of a dune coast may, however, be interrupted from time to time by erosion. It is even possible that a dune coast is being more or less steadily eroded even though sand is also being supplied naturally from offshore. The condition necessary for this is that the erosion caused by an increasing longshore transport capacity more than offsets the supply of sand in the transverse direction. The following two sections will describe long and short term dune coastal changes based largely upon experience in The Netherlands.

### 23.3 Short term dune dynamics

During severe storms considerably higher water levels can be expected than under normal conditions*. Under superstorm conditions, as shown in figure 23.4, the dunes, themselves, are subjected to direct attack by the sea. Sand will be eroded from the dunes and transported primarily in the transverse direction - toward the sea along the beach profile. While there may, indeed, be a significant simultaneous longshore transport, this will not be of too much concern, especially if the conditions along the coast do not vary much from place to place and since the storm duration is relatively short. Even so, the changes in the coast profile can be spectacular during the short duration of a severe storm.

Even during the historic storm and flood of January-February 1953 in The Netherlands the damage to the dunes was not severe. In this storm which had an average frequency of intensity of about 1 into 250 years, most dune erosion amounted to about 100 m$^3$ of sand per meter beach length. This translates into a recession of the dune toe of a distance in the order of 20 to 30 meters.

Considerably more severe conditions for the Dutch coast can be conceived. There is, in fact, no limiting case; the probability of occurrence only becomes extremely small. The erosion of the dunes would likewise be much more severe. Indeed, most of the dune - protected areas of the Dutch coast can stand an erosion of more than 500 m$^3$ of sand per meter coastline and still prevent flooding of the hinterland. Unfortunately, a few portions of the coastline do not have such a generous reserve of sand in the dunes.

An extremely important question is, then, "How severe a storm can a given dune coast endure and not fail?" Many studies of this have been carried out in The Netherlands and studies are continuing. Some conclusions based upon both model and prototype studies complete this section.

* The extent of such a water level increase is dependent upon many factors. The Dutch coast is especially susceptible to such water level increases.
Since the transverse sediment transport dominates the coastal transport picture during dune erosion, the principle of continuity dictates that the sand eroded from the dunes must remain somewhere along the beach profile. Measurement made along the Dutch coast shortly before and soon after the storm in 1953 indicated that large portions of the coast had developed a so-called storm profile extending from the storm flood level to a depth more or less corresponding to the outer edge of the breaker zone. The profile was found to fit the empirical relationship:

\[ z = -0.415 \sqrt{y} + 4.5 + 0.88 \]  \hspace{1cm} (23.01)

where \( y \) and \( z \) are defined in the usual way and are shown along with a profile in figure 23.5. Note that equation 23.01 is not dimensionless - metric units were used in its determination. This equation is published in an anonymous report by the Ministry of Public Works (1972) and is valid only for the profile below the still water level. Figure 23.5 shows such a profile. Continuity of sand dictates that the area of erosion equal the area of accretion in the figure.

Unfortunately, no-one was brave enough to carry out measurements during the 1953 storm on the coast; we do not know, therefore, how the storm profile in figure 23.5 developed as a function of time. If the storm flood level is assumed to have been reached instantly and then maintained for some time during which the dune erosion takes place, then we can conclude from figure 23.5 that the breaker zone has become much wider during the erosion process. Further, since the storm beach profile is related to the storm flood still water level, the total quantity of dune material eroded is strongly dependent upon this storm level. The duration of the storm seems relatively less important; once the storm profile shown in figure 23.5 has been developed, further changes occur slowly unless the water level or wave height continues to increase. Empirical relationships have been developed, therefore, to predict dune erosion based upon the storm flood...
level. Unfortunately, the large number of limiting assumptions involved in the method makes it more qualitative than quantitative in practice. Since the publication of the above mentioned work, further model studies carried out in the Delft Hydraulics Laboratory indicate that the computation may be a bit conservative – that less material will be eroded than is predicted.

Indeed, before further research has been completed we present here only the few rather qualitative results listed below, rather than the full set of empirical relations.

As stated above, the actual storm flood level is very important for the determination of the ultimate dune erosion. A small increase level can result in a large increase in erosion.

Relatively high dunes supply more sand for each meter of coastline erosion. The resulting coastline recession will be less, but the actual total volume of sand eroded will be more for a higher dune. This can be visualized by moving the storm profile to the right slightly in figure 23.5; the accretion volume increases rapidly. High dunes will minimize the coastal recession; low dunes will minimize the volume of eroded sand. This relation can be useful when artificially stimulating dune formation so that an optimum dune form can be approached.

A second storm occurring soon after an initial storm of equal intensity will cause relatively little additional damage to the dunes. Transverse sand transports of only 10 to 20 percent of that in the first storm have been experienced under the above conditions in The Netherlands.

Relatively severe storms, such as that in The Netherlands in 1953, do not move the dune sand very far; most of it remains within the breaker zone.

Dunes which are otherwise stable will be built up again over the succeeding years. This process is slow relative to the erosion in the storm, but recovery still only takes, at most a few years.

Usually the highest portion of a storm flood is of relatively limited duration. A major portion of the dune erosion takes place in that few hours, however. This is shown in figure 23.6.

23.4 Long term dune dynamics

A row of dunes protecting a coast needs to be stable over a period of years or decades in addition to being able to survive a severe storm. Slow but persistent coastal changes – especially erosion – must be determined and necessary allowances for these less spectacular but continuous processes must be made.

Since the dunes are so important to the coastal protection of The Netherlands, these slow changes in the dunes positions have been carefully followed for decades. Figure 23.7 shows the ten year average displacements of the toe of the dunes at four places along the Dutch coast during a bit more than a century. Only relative changes are shown in that figure; the zero point of the distance scale is quite arbitrary.
Figure 23.6
EFFECT OF STORM ON DUTCH COAST

Figure 23.7a, measured 10 km south of Den Helder shows a consistent erosion of about 1.3 m per year, while 31 km further south - figure 23.7b - the dunes have remained very stable. Near Bloemendaal, 62 km south of Den Helder the dunes are accreting about 0.6 m per year - figure 23.7c. Just north of Scheveningen - figure 23.7d - an initial slow accretion became an erosion of about 1.4 m/year after about 1900. This is interesting in light of the fact that shore protection works - a seawall and groins - were built at Scheveningen around the turn of the century. The measurement point shown in the figure is about 2 km north of the end of those works.

Figure 23.8 shows the movement of the entire coastal region during one century. The letters a through d on the horizontal axis show the locations of the graphs in figure 23.7.

In contrast to the cause of dune erosion during storms, the coastal changes just described are caused primarily by longshore sediment movements. These coastal changes, then, imply a gradient in the longshore transport capacity along the coast. An accretion, obviously, results from a declining transport capacity; erosion implies an increasing longshore transport capacity. The changes just described in figures 23.7 and 23.8 for the toe of the dunes are typical, also, of the entire beach profile. If we assume that that total profile including the dunes is about 20 m high, then a beach and dune change of 1 m per year implies a gradient in the longshore transport capacity of about 20 000 cubic meters per year per kilometer. When this extends over a considerable distance a very large volume of sand can be involved. In 40 kilometers, for example, 1 m/year erosion implies an increase in sediment transport capacity of 800 000 m³/year!
Figure 23.7
DUNE CHANGES AT 4 POINTS ON DUTCH COAST
Erosion, especially, is of great importance for the long-term safety of a dune-protected coast. While we cannot always explain the reason for a slow coastal change or predict its magnitude exactly, it would seem logical, therefore, to attempt to determine existing tendencies, extrapolate them and attempt to cope with the eventual consequences. Often times, a further erosion, predicted sufficiently in advance, is no great problem; shoreline development can be planned with the possible coastal changes in mind. Problems become more difficult and often more emotional when it is too late to plan for natural coastal changes. Artificial shore protection works are then the most often considered solution. There is an alternative, however; abandonment of the area. This second alternative may be more economical on the long term in some situations.

Shore protection works will be discussed in depth in the following chapter. First, however, we conclude this chapter with a discussion of how to predict changes on dune-protected coasts.

23.5 Analysis method

The discussion in the previous sections of this chapter has been limited to the dune behavior at a single cross-section. Now, using an extension of the multiple line method of the previous chapter, we shall indicate how an entire dune coast might be analyzed.

Since the multiple line method involves both longshore and transverse sediment movement, it may be used successfully either for long-term or short-term analysis. It may, however, be a bit cumbersome for a short-term analysis if the coastal morphology is dominated, then, by transverse transport.
The modification of the multiple line model method involves allowing sand to be added to the beach from the dunes. (We may remember from chapter 22 equations 22.02 that no sand was allowed to enter or leave the breaker zone in the transverse direction). We can now allow sand to enter the longshore transport zone nearest the dunes by removing the restriction that $S_{yo}$ always be zero. This boundary condition must, however, be replaced by some feasible description of the dune modification process. This can, of course, be done based upon a slope just as with all of the other transverse transports.

There are however, two important differences:
First, the dunes tend to "cave in" during erosion depositing a rather large volume of sand on the upper beach all at once. Our simulation model must do this, too. This can be done by stipulating, for example, that a given volume of sand be deposited upon the upper beach whenever the "slope" of the dunes becomes too great. This volume of sand can be related to the sand properties and dune height by an approximate slip circle analysis so familiar to foundation engineers. This yields an abrupt sand supply in contrast to Swart's concept.

The second problem, is that sand transport from the beach to the dunes is caused by an entirely different (and independent) phenomena, the wind. While a continuous type of transport function might be appropriate, too little is known about dune accretion to determine the necessary parameters for sand transport toward the dunes.

Studying an eroding coast, therefore, with a slip circle type of dune supply yields a mathematical description of $S_{yo}$ with a special form. Extrapolating the notation of chapter 21, equation 21.06:

\[
\text{If } (W_o - (L_1 - L_0)) < 0 \quad \text{then } S_{yo} = 0 \tag{23.02}
\]

\[
\text{and if } (W_o - (L_1 - L_0)) > 0 \quad \text{then } S_{yo} = S_{dune} \tag{23.03}
\]

where $W_o$ is a distance corresponding to a "just stable" dune slope, $L_0$ is a distance characterizing the dune, and $S_{dune}$ is the volume of sand deposited on the upper beach during a single time interval via a "cave-in".

Since $S_{dune}$ will normally be large, the upper (first) zone of the beach will "spring forward" as a result of the supply from the dunes. This will automatically restore the status (23.02) and initiate increased transverse transport to zones in deeper water. In a calm weather period during which sand supply from the beach rebuilds the dunes, a more continuous type of sand transport function - more like equation 21.06 - should be used instead of (23.02) and (23.03).

* Swart (1974) used this approach. It yields a continuous sand supply function.
24. SHORE PROTECTION WORKS

24.1 Introduction

This chapter is concerned primarily with the various ways in which man can influence the natural processes occurring along a beach. The emphasis here will be on the morphological consequences of the various man-made changes rather than on the construction details of the structures themselves. This latter aspect, in general, belongs more to the field of hydraulic structures than to coastal engineering.

The principles of the morphological consequences of various man-made coastal changes are discussed in the following sections. Not suprisingly, most man-made changes involve beaches that are eroding. Indeed, accreting coasts seldom present problems.

24.2 Sand supply

Probably the simplest and most dependable means of maintaining an eroding beach is to supply sand to that beach from other sources; several methods are available.

The most straightforward seeming method is to move sand to a point shoreward of the breaker line via a dredging operation. Since the sand is to be discharged either into shallow water or upon the dry beach some form of hydraulic suction dredge capable of discharging through a pipeline seems most efficient - see volume I chapter 16. Sometimes, the sand supplied can come from a local excavation project carried out for another purpose; dredging to construct or enlarge a harbor is an excellent example of this. In this case the cost of the beach nourishment will probably be minimal since, at most, there will be only an extra cost for a possible longer pipeline.

Another source of supply often used is to dredge sand from a nearby accreting beach. (Oftentimes, erosion of one beach is accompanied by accretion of another local beach). Accretion and erosion on the two sides of a harbor entrance is an example of this. In the past, permanent fixed structures with dredging equipment mounted on them have been built on the accreting beach within the breaker zone to pick up sand moving along the beach and pump it to the eroding beach more or less continuously. At least one such sand by-passing installation is described in volume I of the Shore Protection Manual. Unfortunately, such installations are often less than complete successes. They may not be properly located to obtain a sufficient supply of sand while a severe storm may cause them to "drown" in sand so that they can no longer operate. The discharge pipe from such an installation must often be permanently installed across a harbor entrance; the resulting submerged pipeline - a sort of U tube - can become plugged with sand in the event of an abrupt pumping failure with the discharge pipeline filled with sand-water mixture. Also, such fixed installations tie up quite a bit of investment capital for a single purpose.
It is often more economical and successful, therefore, to move the sand using more conventional floating dredgers. These can then be utilized intermittently for by-passing while possibly doing other work at other times. It is not essential that the sand be picked up within the normal (calm weather) breaker zone. If a floating dredge makes either a deep pit or a trench parallel to the coast near but outside the calm weather breaker, sand will move to this pit as a result of the transverse sand transport along the beach profile. A multiple line transport theory may be used to study such a transport problem. If the pit is dredged more or less continuously, a solution boundary condition is that the coastal geometry at the pit does not change — any sand entering the pit area simply disappears. Likewise, a multiple line model can be used to simulate the behavior of the sand supplied to the eroding beach. If the necessary coefficients can be determined, it is possible via such a simulation to experiment by trial and error with various pick-up and discharge points in order to select the most favorable locations for these.

When no sand is available either on a beach or as part of other dredging operations, it is sometimes possible to dredge sand well offshore. The site must be selected far enough from the coast (usually a few nautical miles) so that the beach processes are not further affected. Often, the sand is dredged with trailing suction hopper dredges over a relatively large area so as to limit the effect on the offshore bathymetry. On the other hand, such a widespread dredging may increase the effect on the local fishery industry.

A disadvantage of any of the sand supply methods listed above is their long term character. Sand will have to be supplied at regular intervals effectively forever. While the initial capital investment can be very low, it may, conceivably, cost more total money in the long run.

Sand can also be supplied overland. While overland transport of sand can be prohibitively expensive on a continuous basis for supplying a slowly eroding beach, it can prove to be economical for the reinforcement of a dune-protected coast. Moving sand around on the dunes can, for example, lower and broaden their profile in order to make them more durable and minimize the possible future sand loss. Raising the dune crests by moving sand forward may be necessary, on the other hand, if the total coastline recession must be minimized — see chapter 23. Obviously, these measures are aimed only at a short-term beach improvement; it is usually too expensive on a long-term basis.

If the dunes along a coast are still slowly being built up by wind-blown sand, construction of wind breaks or the planting of various grasses can be successful in stimulating the dunes to take on or retain a desired form. Volume I of the Shore Protection Manual describes several types of dune protection and stimulation. Such protection methods may even be needed on an accreting shore in order to prevent the wind from transporting the sand inland from the beach where it could cause significant problems for other works of man such as agricultural lands or highways.
A distinct and very real advantage of all of the above beach nourishment schemes is inherent in beach nourishment itself; the operation is most like that of nature and the consequences of the operation for other nearby portions of the coast is probably the least of all the possible protection methods. The significance of this last remark will become more obvious later in this chapter.

24.3 Groins

Groins can prove to be very effective for stabilizing a coast being eroded as a result of a positive longshore sand transport gradient. The groins must extend entirely across the breaker zone with crests above the still water level to be completely effective. Usually, however, only partial interruption of the longshore sediment transport is needed to achieve beach stability; lower and shorter groins will then be acceptable. The groins at Scheveningen, The Netherlands, are of this latter type. During a severe storm there in January 1976 - figure 24.1 - the groins were completely submerged.

![Figure 24.1](image)

**Figure 24.1**
NORTH SEA STORM
SCHEVENINGEN, THE NETHERLANDS
JANUARY 1976.

The spacing of groins in conjunction with their height and length and the wave approach direction is also important for their effectiveness. Since the shore between the groins will orient itself more or less parallel to the approaching wave crests, beaches already subjected to nearly parallel approaching wave crests can be adequately protected by rather widely spaced groins. Figure 24.2 shows such a coast with the groins about 900 m apart. This spacing is extremely wide. Note how the beach between the groins is nearly straight but not parallel to the general coastline; The angle of wave attack (all waves come from one direction here) is obvious.
At the other extreme, groins must sometimes be placed at intervals along the shore about equal to their length. Since the construction of groins is expensive, it is of the utmost importance that they be properly designed themselves and be properly spaced. No simple rules of thumb can be given for the spacing of groins. We can, however, study the morphological consequences of various groin placement schemes using a multiple line simulation in order to determine optimum dimensions for a set of groins. If we wish to be accurate in our simulations, rather complete simulations will be needed; the influence of a groin on the approaching wave pattern will have to be included in the computation - see chapters 16 and 19.

Construction details of a wide variety of groin structures are given in volume I of the Shore Protection Manual. Many of the ideas of breakwater design in volume III of this Coastal Engineering Series can also be applied to the design of rubble mound groins.
What are the consequences of a row of groins for the remainder of a coastline? Sand approaching a set of groins from the "up drift" side along the beach will be stopped by the first groin; accretion can be expected there. After this accretion has reached the outer tip of the first groin, sand will begin passing into the space between the first and second groin, and so on. If the protected shore is very long, however, we can best not count on material being passed along in this way past the entire groin protected region for a very long time.

What, then, happens "downdrift" of the last groin? There, there will be an appreciable sand transport capacity in the breaker zone (remember, it was an increasing longshore sand transport capacity that eroded our beach in the first place, before we built groins) but no sand will be moving past the location of the last groin. Severe erosion will result; all of the sand which was originally picked up along the now-protected coast will be eroded from a relatively short portion of the coast just "downdrift" from the last groin. This eroding coast can, in turn, be protected by additional groins, but however many groins we build, we shall encounter an erosion problem somewhere. The groins simply displace a problem.

Still, it can be very useful to build groins. By stabilizing a portion of a beach the erosion problems can be concentrated in a smaller beach segment. Possibly, erosion of that particular segment is not detrimental. On the other hand, we might choose the location to be near a convenient outside source of sand for use in beach nourishment, there.

It is very important to remember that groins do little to prevent the transverse transport of sediment on or off shore. Indeed, transverse sand transport has caused severe erosion of parts of the Dutch coast during heavy storms even though groins were located at regular intervals along the shore. Structures to limit this transverse transport are discussed in the following section.

24.4 Sea walls

Sea walls are massive structures built parallel to a coastline to prevent the transverse transport of material from the coast toward deeper water. These structures are often monolithic structures such as that built along the Dutch coast at Scheveningen. Rubble protected slopes are also possible; the shore protection of the landfill area on the island of Jersey is an example of this. A special case of a seawall is one built within a row of dunes to limit the maximum extent of dune erosion.

An extremely important seawall design problem is that of predicting the maximum depth of erosion near the toe of such a structure. Such information is vital for the geotechnical analysis of the wall and embankment retained by it. How might we attack such a problem?
Figure 24.3 shows a sketch with a seawall embedded in a sand coast in order to limit the maximum erosion during a single very severe storm. In this case, the seawall has been built well back in the dunes and it will be attacked only after a considerable quantity of material has been eroded. The figure shows the same situation as that in the previous chapter - figure 23.5 - with a seawall added. Our problem is that of predicting the depth at the toe of the wall, $h_t$, in the figure.

![Figure 24.3: Dunes reinforced by seawall. (distortion 1:25)](image)

One possible approach is to first assume that the seawall is not present. The erosion profile can then be computed via the empirical relation (23.01) provided that the design conditions correspond to those on the Dutch coast. The placement of the seawall would then prevent the erosion of a volume of material represented by area $A$ in figure 24.3. If the storm profile were to be maintained, then the extra volume of material eroded from before the seawall - area $B$ in the figure - would be equal to area $A$. The toe depth, $h_t$, can be determined, then, when some profile for the local erosion is assumed. Model studies carried out at the Delft Hydraulics Laboratory have indicated, however, that the extra volume of material eroded in front of the seawall - $B$ in the figure - is greater than the volume $A$. The ratio of the volumes ranged between one and two. In order to maintain continuity of sand when $B$ is larger than $A$, the entire storm profile apparently shifts slightly seaward. Unfortunately, all of the tests just mentioned were carried out with the seawall initially well buried in the dunes; the volume, $A$, was small compared to the total volume of sand moved. Extrapolation of these results to more exposed seawalls is, therefore most likely rather uncertain, at least until more experience is gained.
The slope of the seawall has been found to be an important parameter determining the toe erosion depth, $h_t$. Generally, the erosion depth before a vertical wall was found to be less than below a sloping wall. Most likely this is a result of the standing wave which forms against the vertical wall and the resulting low bottom velocities under the antinode present at the wall. Such a wall orientation may, however, increase the local dynamic forces acting on the wall; these will be more of importance for the structural design rather than the coastal morphology - see volume III for a discussion of these forces.

While theoretical predictions of erosion patterns before seawalls are not yet trustworthy, neither are model investigation techniques sufficiently developed to make dependable erosion predictions for specific cases. Too little is known of the actual processes involved to establish adequate scaling laws needed to relate model and prototype results. Until much further research has been completed, we can evaluate many sea wall type problems only in a qualitative way unless very extensive studies are carried out. Even so, many seawalls have been built in all parts of the world. Their failures can often be attributed to their design conditions being exceeded.

Not all seawalls protect sandy coasts. Figure 24.4 shows one protecting a solid rock coast in Helgoland, Germany.

Figure 24.4 SEAWALL PROTECTING ROCK COAST
HELGOLAND, GERMANY
Not all seawall designs are so uncertain, however, A rubble mound type seawall - a sort of heavy revetment - has been built on the island of Jersey as part of a landfill project. There, no erosion problems are present since the structure is founded upon solid rock. The only sand anywhere in the vicinity is that being used to make the fill behind the seawall! Even though there is most certainly a sand transport capacity of the waves, the bottom material is immovable and no problems have developed.

24.5 Detached breakwaters

The seawalls just described in the previous section were built on or behind the beach. Sometimes, it is more desirable to build a series of detached breakwater segments offshore parallel to the coast. Detached in the context used here refers to their lack of connection to the land rather than their possible subdivision into segments. Figure 28.7 of volume I and 24.5, here, show a series of such breakwater segments built on the United States coast in the decade of the thirties. Portions of the coast of Israel have been similarly protected more recently. How do such breakwaters change the coastal processes?

The group of breakwater segments do not obstruct the longshore current or sand transport as such in a direct way as would groins. Instead, they modify the wave pattern between them and the coast; this influences both the current pattern and the longshore and transverse sand transport components. Since wave heights are reduced behind the breakwater segments by diffraction and later also by refraction, the sand transport capacity behind the breakwater is reduced leading to the deposition of material supplied from "updrift" in the lee of the breakwater. Further, the refraction and diffraction patterns behind the breakwater also modify the angle at which the waves approach a given segment of coast. Indeed, waves may approach from several directions simultaneously as waves diffracting around each end of a breakwater segment "collide" in the shadow zone.

![Figure 24.5](image-url)

**Figure 24.5**
DETACHED BREAKWATER SEGMENTS
WINTHROP BEACH, U.S.A.
The transverse movement of sand will also be restricted in most cases. This has happened, for example, near detached breakwaters on the coast of Israel near Tel Aviv.

In principle, it should be possible to compute coastal changes in such a case using a multiple line simulation. The task is not easy, however. The rapidly changing wave conditions will require an extensive force balance to compute proper longshore current velocities - see section 16.6. Also, it will be difficult to properly modify the bottom friction in a sediment transport formula when a confused wave pattern is present. Lastly, the beach changes, themselves, will influence refraction patterns making repeated computation of the wave conditions necessary.

Another approach is to use a physical model study. This, too, will not be without problems. In order to reproduce the phenomena involved correctly, an undistorted model must be used. Unless such a model is very large, however, other scale effects will present problems.

Under certain conditions which are difficult to predict except via an extensive study, sand will accrete behind a breakwater segment until it reaches the breakwater itself and forms a tombolo - see figure 24.5. These "certain conditions" involve the wave climate as well as the breakwater segment lengths, gap widths, and distance from the original coast. If the accretion reaches the breakwater, then all longshore currents behind the breakwater are stopped. This can lead to an accumulation of floating trash which will degrade any recreational value of the beach. On the other hand, when a coast has accumulated nearly to a breakwater - has not reached it - the resulting longshore current concentration can result in a locally steep beach which can be dangerous for bathers.

In some cases, tombolos do not even extend above the water nor do their "breakwaters". Figure 24.6 shows such a natural tombolo near Plymouth, Massachusetts in the United States. The shoal extending outward and the irregularity in the otherwise straight coastline both result from the submerged rocky outcrop - High Pine Ledge - offshore.

24.6 Accretion control

The previous sections of this chapter have been concerned primarily with eroding coasts and measures to stabilize them. Not all problems originate with erosion, however. Occasionally accretion needs to be controlled in order to prevent its "spilling over" into areas where accretion would be detrimental. An excellent example of this "spilling over" is the movement of sand past the end of a breakwater built to protect a dredged harbor entrance channel. Methods to predict the quantity of sand passing the breakwater using a single-line simulation model were presented in chapter 20. What, however, is the best method to prevent this undesired movement of sand?
Such a problem is illustrated somewhat schematically in figure 24.7. There, accretion has progressed to the point where sand is already by-passing the end of the breakwater at A. This is evidenced by the fact that the angle of the accreted beach is less than \( \phi' \) at that point - see chapter 20.

One possible but rather uninspired method to halt the sand by-passing would be to simply extend the breakwater seaward at A as indicated by the dashed lines there in figure 24.7. This extension might even have been suggested when the original breakwater was built, but discarded because of the high construction cost in the relatively deep water. Such an extension at A would, of course, halt the by-passing immediately and remain effective until the accretion area had once again built out to the new breakwater tip. Figure 24.8 shows the development of the sand transport past the breakwater tip calculated using equation 20.49 in conjunction with table 20.2. If we assume that a breakwater extension at A is long enough to stop sand transport past the breakwater tip until a time equal to 1.5 times the original \( t_1 \), then a new curve labeled A in figure 24.8 can be computed using new time scale values in a way exactly similar to that used to compute the first curve shown.
A more inspired solution to the problem might be to construct a sort of groin on the beach somewhere "updrift" from the breakwater on the accreting beach - point B in figure 24.7. Construction of such a groin will immediately stop the sand transport past point B, but not past point A, as shown for the first, declining part of curve B in figure 24.8. That sand passing A is being eroded from the beach segment between B and A until the angle of the beach at A is once again equal to $\phi'$. Ultimately, of course, sand will pass by the groin tip at B but not all of this sand will pass point A; some of it will be
retained between A and B rebuilding that coastline segment. The increasing portion of curve B in figure 24.8 lies below curve A, therefore. In contrast to the other curves, curve B is shown only qualitatively in figure 24.8; the precise form of the curve depends upon the time at which the extra groin is built (t/t₁ = 1.20 is shown), the length of the groin, and the distance A-B. The multiple line simulation method already presented can be used to predict the exact behavior, however.
25. CHANNEL SEDIMENTATION

25.1 Introduction

All of the morphology problems discussed so far have involved rather slowly varying parameters; the gradients of wave height, or even sand transport with respect to position were small. Except for these slowly varying conditions, our problems have involved only abrupt boundary conditions such as the fact that the sand transport was stopped completely by a breakwater. How, now, must we approach a channel sedimentation problem?

To make the discussion more specific, consider, for example, the sand escaping around the tip of the breakwater, A, in figure 24.7 of the previous chapter. The theory of the previous chapters has been concentrated so far on answering the question of what happens to that accreting beach and how much sand escapes. Indeed, any reader should be able to answer that question by now. Our problem, now, however, is: What is happening with the sand passing around the breakwater tip, A, in figure 24.7? How much of that sand falls into our channel, and how much, if any, of that sand passes across to the other side where it will ostensibly be carried further? The answers to these questions are sought in the remainder of this chapter.

25.2 Physical changes

The physical parameter variations which influence the sediment transport after passing the tip of the breakwater in figure 24.7 are much more, now, than a change in the angle of the shore relative to the approaching wave crests. The most striking additional changes which occur are that the depth increases rather rapidly and the wave conditions change as we cross the channel; additional current components are also present. Generally the channel will be too deep for wave breaking to occur within it. Figure 25.1 illustrates the parameter changes more clearly. The location of a profile drawn along the beach within the breaker zone and across the channel is indicated by the dashed line CD in figure 25.1a. This profile, itself, is shown in figure 25.1b. Note that the depths on each side of the channel are unequal in figure 25.1b; this reflects the influence of the accretion of sand to the left of the breakwater. The depth corresponding to the outer edge of the breaker zone, h_{br}, is also shown in the longshore profile. Figure 25.1b shows a very abrupt change in water depth on each slope of the channel. Also, all wave breaking ceases in the area of interest to the right of the breakwater location. These changes have a multitude of consequences for the sediment transport along and across the channel.
a. Plan adapted from figure 24.7

b. Profile C D (vertical scale distorted)

Figure 25.1
SHORE PLAN AND LONGSHORE PROFILE
A purely physical change is an altered current pattern in and beyond the channel. As soon as the breaker zone ends near the breakwater tip, the driving force for the longshore current - at least the shear stress gradient in the radiation stress - disappears - see chapter 12. A rapid change in wave set-up as well as set-down will also occur between the accreting beach and the channel. This will yield force components along the longshore profile whose direction depends upon the resulting water surface slope direction. - see section 16.6.

Further, wave height changes will occur along profile C-D to the right of the breakwater location. These changes, both in direction of approach and height, are caused by variations in refraction and shoaling resulting from the bathymetry differences. This means, then, that all of the radiation stress components will be changing quite rapidly near the channel.

The physical result of all this is that the physical cross-channel driving force component will certainly not look anything like equation 12.15 which was based only upon a radiation shear stress gradient. A new force balance will have to be formulated in order to determine the current.

Even though the waves are no longer breaking in the channel, the waves will still influence the bottom friction since they still cause velocity components at the sea bed except in very deep water - see chapter 5 of volume I. Equation 15.28 can still be used to evaluate the friction force since the derivation of the friction relationship in chapter 15 is general up to that point. Of course, the correct velocity, \( V \), must be included in that equation. Not only friction and transverse gradients determine this velocity; longitudinal force gradients are very important as well and may even cause a channel velocity to be nearly parallel to the channel axis. The most obvious longitudinal driving force would result for the combined effects of tides and possible fresh water discharge. The force balance needed to predict the velocity distribution is too complex for theoretical treatment here.

An additional physical complication for the currents results from the fact that conditions vary quickly over short distances. This means that currents will be accelerating and decelerating in the area of the breakwater tip; inertia influences will also have to be included in a force balance. These have been avoided completely until now by stipulating that changes occur only slowly so that inertia influences could be neglected.

How does all of this affect the sediment transport? The effects on the two components of sediment transport, bed load, and suspended load, are indicated in the next sections.
25.3 Bed load transport

Bed load sediment transport responds very quickly to changes in physical conditions. The bed load transport is determined almost exclusively by the local conditions of velocity and bed shear stress. The repeated stirring up and re-settlement of sand grains near the bottom during a single wave period are evidence of this - refer to chapter 18.

This "lack of inertia" of the bed load implies that bed load transport rates can be computed relatively easily at any chosen location once the current and wave conditions have been determined. The Bijker approach reflected in equations 19.34 can be used. These relationships are independent of wave breaking and, therefore, may be used anywhere in the region of interest.

Once the bed load transport rate is known as a function of position we might compute bed load erosions and sedimentations. While this would be easy, it would only be of practical value when the suspended load transport remained either negligible or constant. This is the topic of the next section.

25.4 Suspended load transport

Suspended load presents considerably more of a problem than does the bed load. The suspended load is, of course, distributed over the entire depth at any location. Since suspended material will settle out no faster than its fall velocity (it reaches this speed only in perfectly still water) any settling or suspending process will occur gradually. The suspended sediment concentration at some given point will, therefore, be dependent upon the immediately local conditions of turbulence and bed load as well as the past history of these; the suspended sediment transport does demonstrate "inertia".

Since Einstein developed his formula - equation 19.17 - for a steady state condition, the relationships for suspended load transport developed in chapter 19 will be incorrect, now, in this typically non-steady state problem. In general, changes in suspended load transport will occur more slowly than equations such as (19.17) might lead the user to expect.

While it would, in principle, be possible to derive proper transient state concentration and suspended sediment relationships, the effort might not justify the reward; a simpler, very approximate approach is outlined below, the philosophy behind a more detailed approach is indicated in section 25.6.

25.5 An approximate solution

In order to arrive at a workable solution to a channel siltation problem, the following crude method is suggested. Instead of obtaining a single approximate sedimentation value, we shall determine limits between which the channel morphological changes must lie. This will be based upon the sediment transport equations already developed for steady state conditions.
The first step in the procedure is to evaluate the physical conditions at several critical places. Several points just outside the channel and a few points spaced along the channel axis should be sufficient to predict channel morphological changes near the breakwater tip.

The second step is to evaluate the bed load transport and suspended load transport separately at each of the points just chosen. This is done under the incorrect assumption that the situation is only slowly changing.

The sedimentation or erosion of an area by bed load transport can be found rather accurately by examining the changes in transport rates between the chosen points in and around that area—the points chosen in step one. Since bed load transport demonstrates little "inertia", this result is probably quite accurate.

If deposition of suspended material is expected (the suspended sediment transport decreases) then the maximum deposition of this material can be found by comparing the two steady-state suspended sediment transport rates. Similarly, a maximum suspended load erosion can be found by comparing these steady state rates where erosion by suspended material transport is expected. These rates of deposition or erosion are maximum values by virtue of the "inertia" effect of the suspended sediment transport; the actual suspended load transport changes cannot be more than these. Therefore, the bed load transport change plus the maximum suspended load transport change will yield the upper limit erosion or deposition.

At the other extreme and especially with deposition, no significant actual change in suspended load transport may take place until well beyond the channel. This would become more true as the grain size of the suspended sediment or channel width decreased. Thus, a lower limit on erosion or deposition could be found by considering only bed load transport changes.

The background of a more detailed approach is listed in the following section.

25.6 More exact sedimentation determination

A more detailed theoretical description of sedimentation under conditions of variable depth must include more variables than for situations discussed in earlier chapters. In order to make any improved theoretical approach possible, one must assume that no separation occurs on the slopes of the channel—the current streamlines near the bed continue to follow the slope; this will be true unless the channel slopes are rather steep—steeper than about 1:7, say.

Further common assumptions are that the streamlines of the flow remain horizontal and that the rate of turbulence characterized by the diffusion coefficient, $\varepsilon_z$, adapts instantaneously to each new situation. The assumption of horizontal flow implies that suspended sediment is only moved along the streamline by the current which, in turn, can cause no direct sedimentation.
As has already been indicated, the bed load transport adapts essentially instantly to varying flow conditions so that changes in bed load transport can be computed by the methods already available. The difficulty in the problem at hand involves the proper treatment of the suspended transport changes.

Consider a block of water somewhere above the area where the sedimentation is to be predicted. This block has width (perpendicular to the flow direction) of one unit, height dz and length (in the flow direction) dx. Continuity of sediment for this block yields:

\[
\frac{d}{dx} S(x,z) \, dx \, dz + \frac{d}{dz} S_v(x,z) \, dx \, dz = 0
\]  

(25.01)

where:
- \( S(x,z) \) is the horizontal transport of suspended material
- \( S_v(x,z) \) is the vertical (positive downward) transport of suspended material
- \( x, z \) are the coordinates of the point at which the balance is made.

If we examine (25.01) for the special case that \( z \) is the bed elevation, then \( S_v(x, \text{bed}) \) is the rate at which suspended sediment is deposited.

Equation 19.12, an equation of motion for vertical transport, needs only slight modification:

\[
\frac{d}{dx} S_v(x,z) = W c(x,z) + \varepsilon_z \frac{d}{dz} c(x,z)
\]  

(25.02)

where:
- \( W \) is the particle fall velocity, and
- \( c(x,z) \) is the sediment concentration.

In the steady state situation - chapter 19 - \( S_v \) \( \textit{def} \) 0, resulting in equation 19.12.

Considering the horizontal direction, \( S(x,z) \) is determined by the resulting water velocity \( V(x,z) \) and the concentration \( c(x,z) \):

\[
S(x,z) = V(x,z) \cdot c(x,z)
\]  

(25.03)

The water velocity now changes as a function of distance, \( x \), because the depth of flow changes. These velocity changes must satisfy a continuity relationship for the water, of course.

Appropriate boundary conditions for the solution of the set of equations just described are the following:

a. \( S_v(x,z) \) \( \textit{def} \) 0 at the free water surface - no sediment is added or removed there.

b. \( S_v(x,z) \) \( \textit{def} \) 0 before the upstream channel edge is reached - the steady state condition is not yet disturbed.

Verssens and van Rijn (1977) describe the numerical integration procedures needed to solve this set of equations; a rather sophisticated computer is required.
Bijker (1980) sought a more simple solution that could be carried out using a simpler calculator or even by hand. His simplifications involved the following:

a. Equation 25.01 was integrated over the depth yielding:

\[ \frac{d}{dx} S_s(x) + S_{vb}(x) = 0 \]  

(25.04)

where: \( S_s(x) \) is now the total suspended transport at location, \( x \), and \( S_{vb}(x) \) is the vertical sediment transport at the bed elevation - the sedimentation, in fact - at location \( x \).

b. Bijker also assumed that \( e_z \) was constant over the depth instead of using (19.13). He used the form from Coleman (1970):

\[ e_z = 0.16 \frac{vh}{c} \sqrt{g} \]  

(25.05)

where \( g \) is the acceleration of gravity, \( h \) is the water depth, \( C \) is the Chezy coefficient, and \( V \) is the average velocity over the depth. (If waves are important, then it remains possible to modify (25.05) using the theory of section 19.4).

c. An additional simplification was that

\[ S_s = V \bar{c} h \]  

(25.06)

instead of the more exact expression (19.11) used earlier.

d. Lastly, Bijker schematized the channel with sloping sides to a rectangular cross section with vertical sides located at the halfway points of each slope.

These four additional simplifications made it possible to avoid the numerical integrations required in the method of Verssens and van Rijn (1977). Two comparisons of the methods are included in Bijker's paper (1980). He concluded there that the simpler method was acceptable for predicting sedimentations but not for calculating actual sediment transports over a channel.

Lastly, it can be interesting to examine the deposition of suspended material as a function of distance for a rather routine example, taken from Bijker (1980). Sand of 0.2 mm diameter is exposed to a velocity of 1 m/s in an area 5 m deep. Suddenly the depth increases to 10 m in a channel. About one fifth of the total expected sedimentation of suspended material occurs within 25 m of the channel edge; about half occurs within just under 100 meters and about three quarters is deposited within about 175 m. These figures include only suspended transport changes, and indicate that the upper limit of sedimentation computed in section 25.5 will probably seldom be reached, especially when the water is deeper than in this example.
SYMBOLS AND NOTATION

The symbols used in this set of notes are listed in the table. International standards of notation have been used where available except for occasional uses in which direct conflict of meaning would result. Certain symbols have more than one meaning, however this is only allowed when the context of a symbol's use is sufficient to define its meaning explicitly. For example, $T$ is used to denote both wave period and temperature.

Functions are denoted using the British and American notation. The major discrepancy with European continental notation occurs with the inverse trigonometric functions. Thus, the angle whose sine is $y$ is denoted by:

$$\sin^{-1} y$$

Instead of $\arcsin y$.

Possible confusion is avoided in these notes by denoting the reciprocal of the sine function by the cosecant function, csc, or by $\frac{1}{\sin}$. This same rule applies to the other trigonometric and hyperbolic functions as well.

In the table a meaning given in capital letters indicates an international standard. The meaning of symbols used for dimensions and units are also listed toward the end of the table.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Equation</th>
<th>Dimensions</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>energy density</td>
<td>fig. 3.5</td>
<td>$L^2 T^{-1}$</td>
<td>$m^2/s$</td>
</tr>
<tr>
<td></td>
<td>coefficient</td>
<td></td>
<td>$M^{-1} L^2 T^2$</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>amplitude of orbital displacement</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td></td>
<td>integration limit</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td></td>
<td>coefficient</td>
<td></td>
<td>$L^2 T^{-1}$</td>
<td>$m^2/s$</td>
</tr>
<tr>
<td>B</td>
<td>ship's beam</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td></td>
<td>distance from course line</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td></td>
<td>coefficient</td>
<td></td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>b</td>
<td>distance between wave orthogonals</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>C</td>
<td>Chézy friction coefficient</td>
<td></td>
<td>$L^{1/2} T^{-1}$</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>C'</td>
<td>Chézy friction coefficient</td>
<td></td>
<td>$L^{1/2} T^{-1}$</td>
<td>$m^3/s$</td>
</tr>
<tr>
<td>C_B</td>
<td>block coefficient</td>
<td></td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>c</td>
<td>wave celerity</td>
<td></td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td></td>
<td>concentration</td>
<td></td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>D</td>
<td>ship draft</td>
<td></td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td></td>
<td>particle grain size</td>
<td></td>
<td>$L$</td>
<td>$\mu m$</td>
</tr>
<tr>
<td>D_W</td>
<td>deadweight tonnage</td>
<td></td>
<td>ch.3</td>
<td>$kg$</td>
</tr>
<tr>
<td>D_W_T</td>
<td>deadweight tonnage</td>
<td></td>
<td>M</td>
<td>$kg$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Equation</td>
<td>Dimensions</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>----------</td>
<td>------------</td>
<td>-------</td>
</tr>
<tr>
<td>E</td>
<td>chance or probability</td>
<td>4.13</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>e</td>
<td>BASE OF NATURAL LOGARITHMS</td>
<td>4.07</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Ftide</td>
<td>tidal force per unit area</td>
<td>13.02</td>
<td>M L^{-1} T^{-2}</td>
<td>N/m²</td>
</tr>
<tr>
<td>f</td>
<td>friction factor</td>
<td>15.15</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>g</td>
<td>ACCELERATION OF GRAVITY</td>
<td>10.02</td>
<td>L T^{-2}</td>
<td>m/s²</td>
</tr>
<tr>
<td>H</td>
<td>wave height</td>
<td>4.23</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>h</td>
<td>water depth</td>
<td>4.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>h'</td>
<td>wave set-up</td>
<td>11.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>I</td>
<td>extra channel depth allowance</td>
<td>4.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>z</td>
<td>Einstein integral values</td>
<td>19.18</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>i</td>
<td>interest rate</td>
<td>4.26</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>K</td>
<td>wave number of tide</td>
<td>13.04</td>
<td>L^{-1}</td>
<td>m^{-1}</td>
</tr>
<tr>
<td>K_r</td>
<td>refraction coefficient</td>
<td>17.07</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>k</td>
<td>wave number</td>
<td>10.01</td>
<td>L^{-1}</td>
<td>m^{-1}</td>
</tr>
<tr>
<td>L</td>
<td>ship length</td>
<td>3.04</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>water level</td>
<td>4.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>effective breakwater length</td>
<td>20.22</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>distance to beach schematization</td>
<td>21.02</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>Lc</td>
<td>channel length</td>
<td>4.09</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>mixing length</td>
<td>14.02</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>M</td>
<td>coefficient</td>
<td>20.30</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>M'</td>
<td>maximum number of ships</td>
<td>4.14</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>m</td>
<td>index counter</td>
<td>4.17</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>m'</td>
<td>accretion beach slope</td>
<td>20.26</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>N</td>
<td>number of waves encountered</td>
<td>4.12</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>n</td>
<td>number of extreme values</td>
<td>5.03</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>n'</td>
<td>wave velocity ratio</td>
<td>10.03</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>m'</td>
<td>number of payments</td>
<td>4.26</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>pwf</td>
<td>present worth factor</td>
<td>4.26</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Q</td>
<td>integral value</td>
<td>19.20</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>q</td>
<td>rate of change of sand transport per unit width</td>
<td>21.06</td>
<td>L T^{-1}</td>
<td>m/s</td>
</tr>
<tr>
<td></td>
<td>dummy variable</td>
<td>4.07</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>R</td>
<td>response transfer function fig. 3.5</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>R'</td>
<td>roughness allowance</td>
<td>4.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Equation</td>
<td>Dimensions</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>----------</td>
<td>------------</td>
<td>-------</td>
</tr>
<tr>
<td>S</td>
<td>radiation stress component</td>
<td>10.01</td>
<td>$M T^{-2}$</td>
<td>$N/m$</td>
</tr>
<tr>
<td>s</td>
<td>sediment transport</td>
<td>17.08</td>
<td>$L^3 T^{-1}$</td>
<td>$m^3/yr$</td>
</tr>
<tr>
<td>$s'$</td>
<td>sediment transport per unit width</td>
<td>9.01</td>
<td>$L^2 T^{-1}$</td>
<td>$m^2/yr$</td>
</tr>
<tr>
<td>$u$</td>
<td>instantaneous ship position</td>
<td>4.02</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$u'$</td>
<td>rate of change of sediment transport</td>
<td>20.05</td>
<td>$L^3 T^{-1}$</td>
<td>$m^3/yr$</td>
</tr>
<tr>
<td>$u_{199}$</td>
<td></td>
<td>21.06</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$T$</td>
<td>wave PERIOD</td>
<td>4.27</td>
<td>$T$</td>
<td>$s$</td>
</tr>
<tr>
<td>$T'$</td>
<td>tide PERIOD</td>
<td>13.03</td>
<td>$T$</td>
<td>$s$</td>
</tr>
<tr>
<td>$T_e$</td>
<td>PERIOD of encounter</td>
<td>3.03</td>
<td>$T$</td>
<td>$s$</td>
</tr>
<tr>
<td>$t$</td>
<td>TIME</td>
<td>4.02</td>
<td>$T$</td>
<td>$s$</td>
</tr>
<tr>
<td>$U'$</td>
<td>Wave &quot;energy component&quot;</td>
<td>17.01</td>
<td>$MLT^{-3}$</td>
<td>$N/s$</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity component in X direction</td>
<td>15.17</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$u'$</td>
<td>dummy parameter</td>
<td>20.18</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity fluctuation in X direction</td>
<td>14.01</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V$</td>
<td>VELOCITY</td>
<td>13.01</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V_r$</td>
<td>resultant VELOCITY</td>
<td>15.22</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$V_x$</td>
<td>&quot;shear&quot; VELOCITY</td>
<td>15.04</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v$</td>
<td>VELOCITY component in Y direction</td>
<td>20.41</td>
<td>$L^3$</td>
<td>$m^3$</td>
</tr>
<tr>
<td>$v'$</td>
<td>VELOCITY fluctuation in Y direction</td>
<td>14.01</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>ship VELOCITY</td>
<td>3.02</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>W</td>
<td>equilibrium width of schematized beach</td>
<td>21.06</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$W_s$</td>
<td>sediment particle fall velocity</td>
<td>19.12</td>
<td>$L T^{-1}$</td>
<td>$m/s$</td>
</tr>
<tr>
<td>X</td>
<td>COORDINATE in direction of wave propagation</td>
<td>10.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>x</td>
<td>COORDINATE along channel</td>
<td>4.02</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$X$</td>
<td>COORDINATE in direction of sand transport</td>
<td>9.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$x$</td>
<td>COORDINATE along coast</td>
<td>13.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$X$</td>
<td>dummy variable</td>
<td>4.07</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>$Y$</td>
<td>COORDINATE along wave crest</td>
<td>10.04</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$y$</td>
<td>COORDINATE perpendicular to coast</td>
<td>14.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>Z</td>
<td>vertical COORDINATE</td>
<td>13.03</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z$</td>
<td>tide level</td>
<td>4.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z'$</td>
<td>ship squat plus trim</td>
<td>9.01</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z$</td>
<td>vertical COORDINATE</td>
<td>15.02</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z'$</td>
<td>elevation for zero velocity</td>
<td>15.04</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z'_t$</td>
<td>elevation for velocity profile tangency</td>
<td>15.09</td>
<td>$L$</td>
<td>$m$</td>
</tr>
<tr>
<td>$z^*$</td>
<td>dimensionless depth</td>
<td>19.15</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
### Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Equation</th>
<th>Dimensions</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>angle of approach of waves relative to ship</td>
<td>3.02</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \beta )</td>
<td>angle of accretion at breakwater tip</td>
<td>20.39</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>wave breaking index</td>
<td>11.04</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \delta )</td>
<td>relative density of sediment</td>
<td>19.01</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \delta )</td>
<td>parameter</td>
<td>21.09</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>spectrum width parameter</td>
<td>ch 5</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \eta )</td>
<td>eddy viscosity</td>
<td>14.01</td>
<td>( L^2 T^{-1} )</td>
<td>m²/s</td>
</tr>
<tr>
<td>( \eta )</td>
<td>turbulent diffusion coefficient</td>
<td>19.12</td>
<td>( L^2 T^{-1} )</td>
<td>m²/s</td>
</tr>
<tr>
<td>( \theta )</td>
<td>water surface elevation</td>
<td>9.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>( \theta )</td>
<td>angle relative to principal plane</td>
<td>10.06</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>value of integral</td>
<td>20.19</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Von Kármán coefficient</td>
<td>15.04</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \mu )</td>
<td>WAVE LENGTH</td>
<td>10.01</td>
<td>L</td>
<td>m</td>
</tr>
<tr>
<td>( \nu )</td>
<td>ripple factor</td>
<td>19.45</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \xi )</td>
<td>parameter</td>
<td>15.29</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \pi )</td>
<td>constant = 3.1415926536</td>
<td>3.03</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \rho )</td>
<td>mass density of (sea) water</td>
<td>3.04</td>
<td>( M L^{-3} )</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>standard deviation</td>
<td>4.03</td>
<td>depend upon problem</td>
<td></td>
</tr>
<tr>
<td>( \tau )</td>
<td>shear stress</td>
<td>14.01</td>
<td>( M L^{-1} T^{-2} )</td>
<td>N/m²</td>
</tr>
<tr>
<td>( \psi )</td>
<td>angle of wave approach relative to instantaneous coast</td>
<td>12.01</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \phi' )</td>
<td>angle of wave approach relative to initial coast</td>
<td>20.08</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>tidal wave frequency</td>
<td>13.03</td>
<td>( T^{-1} )</td>
<td>1/s</td>
</tr>
<tr>
<td>( \omega )</td>
<td>surface wave frequency</td>
<td>3.02</td>
<td>( T^{-1} )</td>
<td>1/s</td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>wave encounter frequency</td>
<td>3.02</td>
<td>( T^{-1} )</td>
<td>1/s</td>
</tr>
</tbody>
</table>

### Special symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ldots )</td>
<td>time average of ( \ldots )</td>
</tr>
<tr>
<td>( \ldots )</td>
<td>amplitude of ( \ldots )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>is proportional to</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>change in ( \ldots )</td>
</tr>
<tr>
<td>( \infty )</td>
<td>infinity</td>
</tr>
</tbody>
</table>
### Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>bottom</td>
</tr>
<tr>
<td>bc</td>
<td>bottom, current</td>
</tr>
<tr>
<td>br</td>
<td>breaker line</td>
</tr>
<tr>
<td>c</td>
<td>keel clearance</td>
</tr>
<tr>
<td>cr</td>
<td>critical</td>
</tr>
<tr>
<td>cw</td>
<td>current and wave</td>
</tr>
<tr>
<td>i</td>
<td>counter index</td>
</tr>
<tr>
<td>L</td>
<td>still water level</td>
</tr>
<tr>
<td>m</td>
<td>maximum</td>
</tr>
<tr>
<td>min</td>
<td>minimum</td>
</tr>
<tr>
<td>o</td>
<td>deep water (except ch. 21)</td>
</tr>
<tr>
<td>p</td>
<td>particle</td>
</tr>
<tr>
<td>r</td>
<td>roughness</td>
</tr>
<tr>
<td></td>
<td>resultant</td>
</tr>
<tr>
<td></td>
<td>refraction</td>
</tr>
<tr>
<td>rms</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>s</td>
<td>ship</td>
</tr>
<tr>
<td>sig</td>
<td>significant</td>
</tr>
<tr>
<td>t</td>
<td>at point of tangency</td>
</tr>
<tr>
<td>tip</td>
<td>tip of breakwater</td>
</tr>
<tr>
<td>w</td>
<td>wave</td>
</tr>
<tr>
<td>x</td>
<td>x component</td>
</tr>
<tr>
<td>xx</td>
<td>x component of normal stress</td>
</tr>
<tr>
<td>xy</td>
<td>x component of shear stress</td>
</tr>
<tr>
<td>y</td>
<td>y component</td>
</tr>
<tr>
<td>yx</td>
<td>y component of shear stress</td>
</tr>
<tr>
<td>yy</td>
<td>y component of normal stress</td>
</tr>
<tr>
<td>n</td>
<td>wave, water surface</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>used to distinguish similar values actual meaning from context</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Notation</td>
<td>Meaning</td>
</tr>
<tr>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>cos ( )</td>
<td>cosine of the angle ( )</td>
</tr>
<tr>
<td>cosh ( )</td>
<td>hyperbolic cosine of ( )</td>
</tr>
<tr>
<td>exp ( )</td>
<td>e raised to the power ( )</td>
</tr>
<tr>
<td>f ( )</td>
<td>general function of ( )</td>
</tr>
<tr>
<td>ln ( )</td>
<td>natural logarithm of ( )</td>
</tr>
<tr>
<td>P ( )</td>
<td>chance of exceedance of ( )</td>
</tr>
<tr>
<td>p ( )</td>
<td>chance of occurrence in interval characterized by ( )</td>
</tr>
<tr>
<td>sin ( )</td>
<td>sine of the angle ( )</td>
</tr>
<tr>
<td>sinh ( )</td>
<td>hyperbolic sine of ( )</td>
</tr>
<tr>
<td>tan ( )</td>
<td>tangent of the angle ( )</td>
</tr>
<tr>
<td>tanh ( )</td>
<td>hyperbolic tangent of ( )</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>change in</td>
</tr>
<tr>
<td>$\Pi$ ( )</td>
<td>product of ( )</td>
</tr>
<tr>
<td>$\sum$ ( )</td>
<td>sum of ( )</td>
</tr>
</tbody>
</table>

### Dimensions and units

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm</td>
<td>centimeter</td>
</tr>
<tr>
<td>ft</td>
<td>foot</td>
</tr>
<tr>
<td>h</td>
<td>hour</td>
</tr>
<tr>
<td>kg</td>
<td>KILOGRAM</td>
</tr>
<tr>
<td>km</td>
<td>kilometer</td>
</tr>
<tr>
<td>kt</td>
<td>knot = nautical miles per hour</td>
</tr>
<tr>
<td>L</td>
<td>LENGTH DIMENSION</td>
</tr>
<tr>
<td>M</td>
<td>MASS DIMENSION</td>
</tr>
<tr>
<td>m</td>
<td>METER</td>
</tr>
<tr>
<td>mm</td>
<td>millimeter = $10^{-3} \text{ m}$</td>
</tr>
<tr>
<td>$\mu$m</td>
<td>micrometer = $10^{-6} \text{ m}$</td>
</tr>
<tr>
<td>N</td>
<td>newton</td>
</tr>
<tr>
<td>rad</td>
<td>radian</td>
</tr>
<tr>
<td>s</td>
<td>second</td>
</tr>
<tr>
<td>T</td>
<td>TIME DIMENSION</td>
</tr>
<tr>
<td>yr</td>
<td>year</td>
</tr>
<tr>
<td>$^\circ$</td>
<td>degree angle</td>
</tr>
</tbody>
</table>
REFERENCES

The following list includes bibliographic data on most (and hopefully all) of the references used in the previous chapters. Works are listed in alphabetical order by first author and in sequence of publication.


In Dutch, original title: Richtlijn voor de Berekening van Duinafslag ten gevolge van een Stormvloed.


In Dutch, original title: Golfspanning.


Also appeared as: Publication number 50, Delft Hydraulics Laboratory, Delft, The Netherlands.


In French, original title: Discussion des Formules de Débit Solide de Kalinske, Einstein, et Meyer-Peter et Mueller compte tenue des mesures récentes de transport dans les rivières Néerlandaises.


In Dutch, original title: Kustaangroeï voor een Dam bij Niet-Evenwijdige Aangroeï.


In German, original title: Mechanische Äehnlichkeit und Turbulenz.


In Dutch, original title: Bijdrage tot het onderzoek naar de Mogelijkheid van het met Behulp van Grote Havensleepboten te Assisteren bij de Besturing van Mammoetschepen in een Scheepvaartgeul Buitengaats.


In French, original title: Essai de Théorie à l'Evolution des Formes de Rivages en Plages de Sables et de Galets.

In German, original title: Über die Ausgebildete Turbulenz.


Additional References:


