From Point Cloud to Shape Grammar to Grammatical Transformations

Using Terrestrial Laser surveying to develop and compare shape grammars

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Abstract. This paper describes a generative design approach integrating real building data in the process of developing a shape grammar. The goal is to assess to which extent it is feasible the use of a reverse engineering procedure to acquire actual building data and what kind of impact it may have on the development of a shape grammar. The paper describes the use of Terrestrial Laser Scanning (TLS) techniques to acquire information on the São Vicente de Fora church, then the use of such information to develop the corresponding shape grammar, and finally the comparison of this grammar with the grammar of Alberti’s treatise, to determine the grammatical transformations that occurred between the two grammars.

Keywords. Alberti, shape grammar, shape recognition, design automation, transformation in design.

INTRODUCTION
This paper is centered on the construction of the shape grammar of a Portuguese church called São Vicente de Fora. For this propose a point cloud from a TLS surveying was used and a part of a church element (a Doric base) was then closely analyzed.

This research is part of a wider project aimed at decoding Alberti’s treatise De Re Aedificatoria by inferring the corresponding shape grammar using the computational framework provided by description grammars (Stiny, 1981) and shape grammars (Stiny and Gips, 1972). The goal is to compare the grammar of the treatise with the grammar of actual buildings to determine the extension of Alberti’s influence on Portuguese architecture in the counter-reform period (Kruger et al., 2011) to determine the grammatical transformations (Knight, 1983) that occurred from the original Albertian grammar to the actual buildings grammars.

Established in 1147 by King Afonso Henriques both the monastery and its church of São Vicente de Fora had their reformation by King Filipe I in the 16th century.

It is believed that these renovations followed drawings of Juan de Herrera who was in Lisboa by 1580-1583 and the drawings of Filipe Terzi (So-romenho, 1995). The Portuguese architect Baltazar
Álvares was in charge of directing the construction from 1597 to 1624.

The church has a unique nave, with a transept and a deep main chapel. The walls have pilasters ornamented with a Doric base, a plain shaft and an almost Corinthian Capital with one level of leaves and in the center of the symbols of Saint Vincent and Saint Sebastian. The entablature is Doric with friezes, grooves and mutules. The main nave cradle vault is coffered and its façades have intercommunicating chapels built from 1605 to 1629. There was a dome over the transept that collapsed during the 1755 earthquake.

In the experiment described in this paper, TLS techniques (Mateus, 2012) were used to acquire information from the existing church building and work was focused on the column system. A mesh surface of the church elements was generated from the point cloud obtained using TLS. A Doric column base section line was then detached from that mesh. A process of analysis and evaluation of such line will be showed in the final section of this paper.

**METHODOLOGY**

There was a previous survey of the building using a phase-based laser scanner (FARO Focus 3D) as shown in Figure 1. The scanner was placed in 24 different stations for the acquisition of colored point clouds. After the registration process a colored point cloud model (PCM) was obtained. The registration and decimation of the point clouds were done with the open source software MeshLAB. For the purpose of this work it was considered a local coordinate frame, aligned with the main directions of the church.

The alignment intended to merge pairs of clouds. The final alignment of the PCM was done using their matrix and was the basis for the extraction of ortho images, multiple sections and triangulated models. Several 2D drawings were produced using JRC software. The final project .Aln containing the alignment of the 24 point clouds was sectioned in a dozen of vertical and horizontal planes configuring the multiple planes, sections and elevations of the church. Some of these drawings were then used as raster images for Autocad and Rhinoceros. Finally, after using the 2D drawings from the laser scanning survey in the construction of the grammar, an element was chosen to evaluate and compare with Alberti’s rules for drawing such type of element. Namely, a subset of the point cloud that contained the best column was chosen for the analysis. The information contained in the retrieved data was used to develop the grammar of the column system of São Vicente de Fora. For this purpose we only considered a column and the intercolumn.

This grammar was then compared them with the Column System shape grammar inferred from Albert’s treatise. The deviations were analyzed to identify which rules transformations were needed to obtain the new grammar from the treatise grammar, and then to determine to which extent Alberti’s rules are present in the column system of São Vicente de Fora.

**SÃO VICENTE DE FORA SHAPE GRAMMAR**

In this article we show the rules needed to generate first a colonnade and then the church main nave interior facade by adapting previously developed Doric and Corinthian grammars, which are part of the column system grammar developed after Albert’s treatise.

The Albertian grammar was developed as a parallel grammar encompassing four views: plan, section, elevation, and axonometric (Coutinho et al., 2011). The first three views are developed in the Cartesian product of the algebras U12 and V12, and the fourth of the algebras U13, U33 and V13. The São Vicente de Fora grammar has the same structure. Each grammar rule has a section containing parameters and descriptions and, when needed, a set of functions organized in a way similar to the one used in previous grammars (Duarte et al., 2013). In this article, we show only the elevation view due to space constraints.

Rule 0 has on its left side an empty set and on its right side a guide line extracted from the site
plot, which goes from point A to point A10. Rule 1 generates recursively a generic structure of the church main compositional elements in elevation. This grid contains a set of labels A, B, C and D inserted in a horizontal line from the bottom to the highest central line of the barrel vault ceiling. Each label has several sublabels from K to Kn-1 being K ∈ {A} and n>1; n ∈ {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and K ∈ {B, C, D} and n>1; n ∈ {1, 2, 4, 6, 7, 8, 9, 10}. The lines containing the set points {A, D} and {A3, D3} are mirror axes. The equation of this set is

\[ L = wr + 4d + 2ic + 3/2IC + M; \]

where the mirrored part of the main nave is \( M = 4d + 2ic + 3/2IC + w1 \); \( wr \) is the church high chorus width, \( d \) is the pilaster width, \( ic \) is the inter chapel’s intercolumn, and \( IC \) is the main intercolumn. \( M \) relates to a quarter of the church’s main nave, measured closer to the transept, \( w1 \) is the remaining width, which goes from the pilaster axis to the beginning of the arch. Finally \( L \) is equal to \( 1/2 \) of the church’s main nave plot minus \( M \). In Rule 2 an insertion point (A2) is given to start the generation of a proto pilaster. This point is obtained from the interior of the church structure both in plan and in section using the previous rule. Rule 3 call previously developed grammars to insert detailed base, shaft and capital. Rule 4 inserts an arch from a lateral chapel and a point B. Rule 5 takes the former arch and inserts a Doric entablature with triglyphs and a point C using point B as a reference. Rule 6 inserts the barrel vault ceiling on the top of the entablature.
and a point D using point C as the insertion point. Rule 7 inserts a half chapel and a pilaster and a line with A3 and C3 points. Rule 8 mirrors the half chapel using points A3 and C3 to define the mirror axis. Rule 9 inserts a half chapel, a sub inter column (ic) and an axis from point A6 to D6. Rule 10 mirrors two sets of pilasters (with labels A9, A8, A4, A2) using the axis with points A and D as the mirror line. Rule 11 generates the space to accommodate the high chorus using the pilasters with labels A9, A8, A4, A2 and A1. Rule 12 (e me ma) erases labels. A 3D model of the main nave was generated by applying the grammar rules. Rules are presented in Figure 2.

NEW SHAPES FROM AN OPTIMIZATION PROTOCOL - THOUGHTS ON EMBEDDING

As mentioned above, the experiment described in this paper relates to the extraction of data from the 3D model generated out of the point cloud model.
and then the analysis of part of it elements.

There are two main concepts that were taken into consideration in the task of evaluating a line extracted from the point cloud. One is the notion of LCS system (Figure 3) mentioned in Alberti’s De Re Aedificatoria and the other is George Stiny’s notion of embedding.

In Book VII, Chapter VII of the Re Aedificatoria (Alberti, 2011), while describing the Bases and the Capitals of the column system, Alberti mentions that these can be constructed from a minimum vocabulary composed by the letters L, C, S, reversed C and reversed S. Is to be noted that Alberti’s original treatise edition contains no drawings.

The combination of these L, C, S elements and their parametric variation generate the moulds Ovule, Channel, Wave and Gulens. In turn, the combination of these moulds gives different column system elements, such as Pedestal, Base, Column (Shaft), Capital and Entablature. Finally, these might be used to obtain different combinations of Doric, Ionic, Corinthian and Composite style elements. The combinations of these column system elements may produce around 900 different columns. This is the size of the language of columns that can be generated from the LCS system, and which might be recognized using the system showed in the next section. In this way, Alberti was providing a proce-
dure to generate almost all the column system with a sub system that was embedded in it. Finally another function of the LCS system was to provide a location for decorative elements like flowers, leaves, and eggs, which are not addressed in this paper.

The column systematization shape grammar that can be developed from the LCS system is a grammar of detail and it is identified as a bottom up shape grammar. The results of the experiment described in this paper support this hypothesis. Apparently the rules from the treatise are to be applied in an almost straightforward fashion. But Alberti’s established that the designer must use them as pleased in order to achieve “concinitas”, that means according to him quantity, proportion, and location.

The rules in use are rules of the type $x \rightarrow x'$ defined by Stiny, meaning that a shape $x$ is transformed in a similar shape with parametric variations (Stiny, 2011). If we get rid of the parameterization, we may obtain rules of the type $x \rightarrow y$, which transform a shape $x$ into another shape $y$.

Both $x$ and $y$ are elements in the index of dimensions $i$ and $j$, where $i \in \{0d, 1d, 2d, 3d\}$, that is, points, lines, planes, solids, and $j \geq i$. In the LCS vocabulary, $C$ is part of the base that is part of the column and so on. A general definition of this embedding feature is $x \rightarrow \text{prt}(x)$.

In this particular case, LCS shapes are boundary

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**Figure 3**

LCS system shape grammar.
elements of the columns. Let's take then the definition rule \( x \rightarrow b(x) \) to encode the transformations that occurred in the design of an element from an original grammar to a transformed grammar.

**THE GRAMMATICAL TRANSFORMATIONS - EVALUATION PROCESS AND FEEDBACK**

One role of the grammar is to help tracing the influence of Alberti's treatise on the design of the São Vicente de Fora church by verifying whether its elements can be obtained from Alberti's rules or some sort of transformation of such rules.

The Transformations in Design framework proposed by Knight (1983, 1994) -- according to which the transformation of one style into another can be explained by changes of the grammar underlying the first style into the grammar of the second -- will be used as the theoretical background.

According to Knight, there are at least four different ways of transforming a grammar as diagrammed in Figure 4, namely, rule addition, rule subtraction and rule changing, which can be designated by letters A, S, and C, respectively. A fourth transformation type I can be added if we consider that a rule can remain unchanged. This transformation I is important for our study because each time such a transformation is used there is strong evidence that the designer was knowledgeable of Alberti's rules, as seen in the Loggia Rucellai shape grammar by Alberti himself (Coutinho et al., 2013).

In the São Vicente de Fora grammar presented in this paper, \( r_2, r_4, r_5 \) and \( r_7 \in I; r_6, r_{10} \) and \( r_{11} \in A; r_0, r_1, r_8 \) and \( r_9 \in C; \) and \( r_3 \in S. \)

As Knight mentions, in rule change transformations C shapes are defined as transposed shapes, that is, as new shapes or as resized and/or repositioned shapes. Rules 2, 4, 5 and 7 are equal to those found in the original Albertian grammar. In Rule 3 the changes verified are in the constituents of the Capital. The disposition of the capitals are similar to those found on the second level of the Palacio Rucellai's façade. In this case, there is a simultaneous subtraction and resize and reposition transformations. The Shaft, Base and Capital's heights are equal to the ones described in the treatise. Rule 6 adds a new element to the grammar, a barrel vault ceiling. Is to be noted that this element don't belong to the column system. Rule 10 adds a new chapel and Rule 11 the high chorus. Rule 8 and 9 a mirror but changes the axis location. Is to be noted that in the treatise is not specified the notion of mirror but of symmetry. Rule 0 and Rule 1 change the location of labels.

These rules manipulate elements that need to be more closely observed. A technique to recognize and analyze sets of curves was used. This technique consists in a Grasshopper code (Figure 5) whose main goal is to extract and compare section lines from an element of the column (a Doric Base torus), which proceeds automatically in three different steps:

1. The first step is the extraction of a line section from a mesh surface out of the point cloud.
2. The second step consists in comparing this
line with a curve previously embedded in the code through the distances between the control points of both lines, which need to be bigger than 0. In rule \( x \rightarrow b(x) \), \( b \) is such that (in this particular experiment) \( b > 0 \); and \( b \in B \) where \( B \in \text{LCS} \).

Then, the third step analyses the difference in value of linear distance and rejects the ones that are not in the acceptable range. This last stage is not completely implemented yet. A similar process using canonical representation (Keles et al., 2010) graphs are in use in order to better visually understand the differences and similarities between the topology of different lines. As said above, the control points of the target curve and the points of section extracted from the cloud are points contained in parallel lines. The distance measured is the segments of such lines. This process is not completely efficient. It might work well for straight lines contained in parallel planes but not in the case of curves in the 3d space. It is interesting to note that the process used in this experiment reduces the Algebras so that \( U_{33} \rightarrow U_{12} \rightarrow U_{02} \).

**CLOSURE**

The contributions of this paper are threefold: first, it is the first Portuguese grammar obtained from Alberti’s treatise shape grammar rules; second, it uses a laser surveying and the resulting point cloud model (PCM) as a way of transforming the grammar and develop the São Vicente de Fora shape grammar; and third, curves that are sections of parts of the “real” building obtained from the PCM are automatically recognized, analyzed and evaluated suggesting that this technique might be an efficient way of evaluating large data sets.

The use of such survey data to generate the grammar was of great help, particularly, considering the level of accuracy and detail that is possible to achieve from such a method.

The code to automate the shape recognition proved to be helpful but improvements are necessary, namely the generation of mesh surfaces directly from the PCM in a complete automated way.

The process for choosing the curves from the corpus (that are the models to be merged) needs to be optimized and is not completely defined. So far a linear distance is in use but the notion of neighborhood (Krishnamurti and Stouffs, 2004) might be of great help in order to understand the kind of transformations occurred in the application of the rules in different buildings. This task will be the focus of a future research article.

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