Prepared for:

DG Rijkswaterstaat
Rijksinstituut voor Kust en Zee / RIKZ

Analysis of LISST-data

Report
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ABSTRACT:

The LISST-ST is an instrument for measuring in-situ particle size and settling velocity distribution using laser diffraction and transmission. Data from field experiments carried out by RIKZ showed that the standard software of the manufacturer did not suffice to determine settling velocities. New analysis software has been developed by RIKZ, which is not yet able to describe the data fully consistently, however. An important aspect is that the particle densities derived from Stokes’ law sometimes show unrealistically high values. Therefore the accuracy of the determined settling velocities is uncertain.

The objective of this study is to investigate what are to possible causes for in the inconsistencies in the data. With this knowledge, an advice is to be given on how the accuracy of the LISST-ST data analysis may be improved.

Based on the analysis of the physics of the settling tube of the LISST-ST, it is concluded that the assumption that all particles settle independently in a completely stagnant fluid is often violated. As a result, the calculated particle density distribution, which is presently the only fit parameter in the model, often becomes unrealistically wide to compensate for effects such as convection and particle interaction.

It is therefore advised to include a diffusion term into the analysis software to take these deviations from the assumed ideal behaviour into account. However, if substantial interaction exists between particles with a widely different size, a single diffusion term may not be sufficient to describe the data adequately. A more complicated model would then be required, for example a multi-zone model accounting for a more complicated (e.g. double-peaked) residence time distribution function. Such a model could still be based on the present analysis software, including some additions.

If sufficient measuring data is available, confidence intervals may be calculated with these fit models for the particle density and settling velocity using resampling techniques. A better insight is thus obtained in the accuracy of the measurements.

More insight into the physics in the settling tube can be obtained with laboratory measurements and numerical modelling.

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Samenvatting

De LISST-ST is een instrument dat bestaat uit een valbuis van 30 cm hoog en een laser waarmee de lichtverstrooiing door deeltjes tussen circa 3 en 400 μm wordt gemeten. Dit instrument wordt gebruikt voor de in-situ bepaling van valsnelheid en vlokgrootteverdeling van slib. Tevens kan uit deze gegevens de dichtheidsverdeling van de slibvlokken worden afgeleid met behulp van een valsnelheidswet, die een relatie legt tussen voornoemde parameters. De Wet van Stokes is hiervan een voorbeeld.

Rijkswaterstaat heeft de LISST-ST onder andere ingezet in het Haringvliet en de Noordzee. Hierbij is gebleken dat de dichtheidsverdeling, zoals afgeleid uit de meetgegevens, soms onrealistisch is. De dichtheid van sommige deeltjes bleek soms veel groter te zijn dan 2650 kg/m³, de dichtheid van een massief mineraal kleideeltje. De betrouwbaarheid van de meetgegevens is hierdoor ongewis.

WL | Delft Hydraulics is gevraagd om de eigenschappen van het LISST-ST systeem nader te bestuderen, met name de fysische processen die zich in de valbuis afspelen. Aldus kunnen de oorzaken van de geconstateerde afwijkingen worden opgespoord en kan worden bepaald hoe de analyse van de meetgegevens verder kan worden verfijnd.

De huidige analyse is gebaseerd op de veronderstelling dat alle deeltjes volledig onafhankelijk en ongestoord bezinken. Dit impliceert dat alle deeltjes van gelijke grootte en dichtheid en op gelijk verticaal niveau exact dezelfde verblijftijd hebben in de valbuis. Spreiding in bezinktijd van deeltjes binnen dezelfde grootteklasse wordt volledig toegeschreven aan spreiding in dichtheid.

Een analyse van de fysische processen in de valbuis leert echter dat de aanname van onafhankelijke en ongestoorde bezinking vaak niet juist is. Zo kunnen al bij geringe temperatuurfluctuaties convectiestroming ontstaan met een snelheid die vergelijkbaar is met of zelfs veel groter is dan de bezinksnelheid van kleine deeltjes. Bovendien kan door de fractale structuur van slibvlokken al bij een lage sedimentconcentratie (minder dan 50 mg/l) sprake zijn van onderlinge beïnvloeding van deeltjes. Hiernaast zijn er nog andere effecten die voor enige mate van verstoring kunnen zorgen.

Daarom wordt voorgesteld om de wiskundige beschrijving van de deeltjesbeweging uit te breiden met een stochastische component. Hierin kunnen dan alle verstorende effecten op de eenparige deeltjesbeweging worden verwerkt. Met statistische technieken ("resampling") kan het betrouwbaarheidsinterval van de valsnelheids- en dichtheidsverdeling worden bepaald. Hiermee wordt een 'gladde' verblijftijdsspreiding verkregen.
Een stap verder is om ook deeltjesinteractie mee te nemen, hetgeen niet als een zuiver stochastisch proces kan worden beschouwd. Hiermee kan een ‘gepiekte’ verblijfjijdsspreiding worden verkregen. Dit kan noodzakelijk zijn indien grote deeltjes kleine deeltjes meesleuren, waardoor de verblijfjijd van een deel van de kleine deeltjes mede wordt bepaald door de verblijfjijd van grote deeltjes. Door rekening te houden met deeltjesinteractie ontstaan er voldoende vrijheidsgraden om in de meeste gevallen tot een goede fit te komen van de waargenomen bezinkcurves.

Tenslotte kan ook worden overwogen om een numeriek model te construeren van de beweging van water en deeltjes in de valkolom, waarin de relevante fysische processen afzonderlijk worden gemodelleerd. Het voordeel hiervan is dat het effect van de verstorende processen op de meetresultaten nauwkeuriger kan worden gekwantificeerd.

De constructie van een dergelijk numeriek model moet vergezeld gaan van metingen met de LISST-ST in het laboratorium. Hierbij zouden deeltjes met bekende eigenschappen zoals grootte, dichtheid en valsnelheid kunnen worden gebruikt met diverse verhoudingen van concentratie en grootte. Met deze meetgegevens kan het numerieke model worden gevalideerd en kan de betrouwbaarheid van de meetgegevens als functie van de omgevingscondities beter worden vastgesteld.

Een dergelijk model is waarschijnlijk echter niet geschikt als vervanging van de analyse-software, aangezien het aantal vrijheidsgraden groot is en het model te rekenintensief is voor directe resultaten. De analyse-software is bij uitstek geschikt voor dagelijks gebruik, terwijl het numerieke model en de laboratoriummetingen af en toe kunnen worden ingezet voor calibratie en validatie.
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1 Introduction

The LISST-ST is an instrument for measuring in-situ particle size and settling velocity distribution using laser diffraction and transmission.

Data from field experiments carried out by RIKZ showed that the standard software of the manufacturer did not suffice to determine settling velocities. New analysis software has been developed by RIKZ, which is not yet able to describe the data fully consistently, however. An important aspect is that the particle densities derived from Stokes’ law sometimes show unrealistically high values. Therefore the accuracy of the determined settling velocities is uncertain.

The present analysis software for LISST-ST data is based on the assumptions that 1) all sediment classes behave independently and 2) ambient water in the column is completely stagnant, only sediment particles move. These assumptions may sometimes be violated in the LISST-ST system.

RIKZ has asked WL | Delft Hydraulics to advise on this matter and to investigate 1) what are the causes for the inconsistencies in the data and 2) how can the accuracy of the measurements be improved with this knowledge.

2 Objective

The objective of this study is to investigate what are possible causes for the inconsistencies in the data. With this knowledge, an advice is to be given on how the accuracy of the LISST-ST data analysis may be improved.

As the present study is a concise desk study, some of the physical effects influencing the measurements may not be quantified in detail, as elaborate mathematical analysis or laboratory tests may be required for this. In such cases, recommendations will then be given on how to proceed in a possible next phase of the study.

3 Methods

The following approach is adopted in this study. First a short description of the LISST-ST is given, focussing on its settling tube. Secondly, a list of hypotheses on the causes for data inconsistencies is made. Thirdly, the magnitude of potential effects estimated for the conditions in the LISST-ST settling tube using simple analysis and rules-of-thumb. The most important phenomena are subsequently studied in more detail. Finally, an advice is prepared on how to improve the accuracy of the data analysis.
4 Short description of LISST-ST

The LISST-ST consist of a laser, a settling tube and a set of optical disks that detect the forward scattering pattern of the laser beam, from which the particle size distribution and volume concentration can be derived. The settling tube has a diameter of 5 cm (Fig. 1). Its height above the laser beam is 30 cm. The baffles installed to reduce flow circulation are 1 cm apart. The laser beam diameter in the water is 6 mm; its optical path in water is 5 cm. Samples are drawn into the tube using a three-bladed impeller. Its rotation speed is estimated at 3 s\(^{-1}\). Eight vertical doors with a diameter of 2 cm are located both at the top and the bottom of the column. Doors are closed 4 s after turning of the impeller in order to sample sediment less affected by the impeller movement. Measurements start 1 s after closing the sample doors (in 50 ms). The tube and the baffles are made of perspex. The bottom part, where the laser light enters the tube, is made of aluminium. For more details the reader is referred to Agrawal and Pottsmith (2000).

![Figure 1: Left: the settling column, along with the impeller and sliding doors. Top and bottom lids are not shown. The column is 5 cm in diameter, and 30 cm tall from the inlet to the laser beam. Right: LISST-ST mounted on a tripod. After Agrawal and Pottsmith (2000).]
5 Hypotheses

Ideally, the settling velocity, particle size distribution and the volume concentration measured with the LISST-ST should perfectly agree with the real in-situ values. However, three sources of error may cause deviations:

1. sampling,
2. unaccounted physical phenomena in the settling tube,
3. errors in laser diffraction technique.

The main focus of this study is on the physics in the settling tube. The other two potential sources for errors will only be dealt with concisely.

The following potential sources of errors are identified based on a literature study and in-and external discussion. Between brackets is indicated to which of the three categories discussed above they fall.

- Sediment Sampling (1)
- Initial effects (e.g. non-homogeneous sediment distribution, effect of impeller) (1/2)
- Convection (2)
- Wall effects (2) (including plate)
- (De-) flocculation (2)
- Differential settling (2)
- Hindered settling (2)
- Return flow (2)
- Deviation from Stokes (e.g. higher Reynolds number or non-spherical shape) (2)
- Wave- or current induced vibrations (2)
- Concentration measurement (3)
- Multiple scattering (bias towards smaller fractions in particle size distribution) (3)
- Aliasing at the limits of the particle size range (particles outside the range do scatter light; their contribution to light scattering is added to the smallest and largest size classes) (3)
- Effect of non-spherical particles on light scattering (large flocs may be semi-transparent and have an erratic surface shape) (3)
- Undersampling of the large size-classes because of a limited amount of particles (3)
6 Separate analysis of error sources

6.1 Sediment Sampling

The first source of errors is sediment sampling. Ideally, the sample drawn into the LISST-ST tube should be representative for the local in-situ suspended sediment concentration and properties (size and density distribution, shape etc.). However, it is known that heavier particles are less likely to be drawn into the sampling tube than lighter particles, which creates a bias towards lighter particles.

The way to avoid this is isokinetic suction, for which the suction velocity is equal to the undisturbed velocity of the ambient fluid and the suction tube is placed in the direction of water motion. No acceleration occurs so the sample is directly representative of the concentration (Velden, 1990).

By its design, transverse suction instead of isokinetic suction occurs in the LISST-ST instrument. Because water does not travel by itself into the tube, active suction is necessary in this case. The suction velocity has to be more than approximately three times the flow velocity in order to get reasonably representative samples, although the measured concentration is roughly 75% of the real concentration. Suction tubes with a diameter of 3 mm give fair results; bigger tubes proved to give stagnation problems (Bosman et al., 1987).

The sample door diameter of 2 cm in the LISST-ST will not be problematic in terms of stagnation, however, as the settling column is located directly behind the sample doors and no tube connections are used. On the other hand, the criterion of a suction velocity of three times the flow velocity will often not be met, as the suction velocity is estimated at 0.1 to 0.2 m/s. Substantial sampling effects are then expected; the concentration of larger particles will be underestimated.

In addition, large fragile particles such as mud flocs make break-up during sampling because of turbulent shear. A change in particle size distribution has indeed been observed for a comparison of simultaneous data from the LISST-ST and LISST-100 (without settling tube). Deflocculation is quantified in §6.6. Mud flocs larger than about 200 μm are likely to break up into smaller flocs.

In conclusion, the sediment concentration, particle size and density distributions in the LISST-ST column may deviate substantially from those in the ambient water, although the column samples in-situ. However, inconsistent results derived from settling curves of sediment samples (either being representative or not) in the LISST-ST can not be explained by this.

6.2 Initial effects

For data processing it is assumed that all sediment is homogeneously mixed at the start of the experiment. Here it is investigated whether this is a reasonable assumption.
The sample is drawn in by an impeller with a rotation speed of about 2–5 rps. After the impeller is stopped, the sample doors remain open for 4 s. It is assumed that in this period most of the fluid in the LISST-ST column is replaced with fresh fluid, as Agrawal and Pottsmith (2000) state that the period of 4 s is meant to replace sediment affected by the impeller movement (e.g. by break-up of flocs) with undisturbed sediment.

The impeller Reynolds number is given by \( \text{Re}_i = \frac{d^2 N}{v} \), where \( d \) is the impeller diameter and \( N \) is its rotation speed. For \( 10 < \text{Re}_i < 10,000 \) the flow is in a transition range. Flow is turbulent near the impeller and laminar in remote parts of the tube (Perry and Green, 1997). For \( d = 0.03 \text{ m} \) and \( N = 2 \text{ s}^{-1} \), \( \text{Re}_i = 1800 \).

The time scale for decay of turbulent eddies is proportional to \( t = \frac{d^3}{64v} \). Between the baffles \( (d = 0.01 \text{ m}) \) \( t = 2 \text{ s} \). Near the impeller below the baffles \( (d_{\text{bfb}} = 0.05 \text{ m}) \) \( t = 39 \text{ s} \). It may therefore be assumed that the water beneath the baffles is well mixed, even after the impeller is stopped. However, the water between the baffles is probably not completely mixed. Sediment concentration gradients may therefore occur, but only if the concentration in the ambient water changes markedly during the additional sampling period of 4 s. This is unlikely, although a sudden ‘jump’ in concentration may occur during the passage of a front or turbulent burst.

In conclusion, errors in the data analysis because of a violation of the assumption of an initially homogeneous suspension are not very likely, but may occur in environments where sediment concentration is highly variable on short spatial and temporal time scales. If the laser sensor indicates a concentration fluctuation in time, this may be an indication for inhomogeneity.

Sediment particles need some time to adapt to a changing flow situation. For example, if stationary particles are released in a fluid, it will take some time before they reach their terminal settling velocity. This is expressed by the Stokes number, defined as the ratio between the grain response time and the hydrodynamic time scale (see for example Van Rhee, 2002). For particles of 50 \( \mu \text{m} \), equilibrium is reached within 2 ms, for particles of 200 \( \mu \text{m} \) within 20 ms. Particle inertia effects may therefore be neglected in the LISST-ST settling tube.

6.3 Convection

Convection may be generated as a result of a temperature difference between the fluid inside the settling tube and the ambient fluid. Temperature differences generate density differences which drive convective currents.

The time scale for heat transfer can be estimated as follows. The penetration thickness for unsteady heat transfer in a semi-infinite slab of material is given by (Bird et al, 1960):

\[
\delta_r = 4\sqrt{\alpha r},
\]

(6.1)

where \( \alpha = k/\rho C_v \) is the thermal diffusivity of the medium. For water at \( T = 20 ^\circ \text{C} \), \( \alpha = 1.4\times10^{-7} \text{ m}^2/\text{s} \). This means that after \( t = 280 \text{ s} \), the temperature increase in the centerline of
the settling tube has changed with 1% of the original temperature difference between centerline and cylinder wall.

For large times, the semi-infinite approach is not valid anymore as geometric effects play their part. For a cylinder-shaped object with a diameter of 5 cm, the temperature increase in the centerline of the cylinder is more than 80% of the original temperature difference after $t = 1786$ s.

Based on these numbers, it is estimated that temperature changes in the ambient water at a time scale of 10 min. or less do not penetrate significantly into the settling column. Such changes therefore result in a temperature difference between column wall and core, generating convective flow. However, the settling column does adapt to temperature changes at a larger time scale.

The maximum temperature change within 10 min. of the ambient water is therefore representative for the maximum temperature gradient to be expected in the settling column.

For the Siltman experiments the maximum temperature gradient is estimated at 1 °C/h or 0.17 °C per 10 min. The maximum spatial temperature differences to be expected in the column are in the order of 0.1 °C.

In the following the resulting strength of the convective currents is estimated. To this order, three dimensionless numbers are introduced, the Grasshof, Nusselt and Prandtl number:

\[
\text{Grasshof number: } Gr = \frac{g \beta \Delta T L^3}{v^2}
\]

\[
\text{Nusselt number: } Nu = \frac{hL}{k} = \frac{qL}{k \Delta T}
\]

\[
\text{Prandtl number: } Pr = \frac{C_p \mu}{k}
\]

where $k =$ heat transfer coefficient; $C_p =$ specific heat; $D =$ length scale; $L =$ length of heat transfer surface; $h = q/\Delta T =$ heat transfer coefficient; $g =$ gravitational acceleration; $\mu =$ dynamic viscosity; $v = \mu/\rho$ kinematic viscosity; $\beta =$ cubic expansion coefficient. In the Table 1 the magnitude of these parameters is given for water at $T = 20$ °C and for the settling column conditions ($L = 0.3$ m; $\Delta T = 0.1$ °C). For water at 20 °C, $Pr = 7$.

For $Gr >> Re$ free convection occurs. For $Gr Pr < 10^9$ the flow remains laminar, as is the case in this example. The solution of this so-called free convection problem is given in terms of (a combination of) these three dimensionless numbers.

\[
Nu = a \ (Gr \ Pr)^m
\]

For water at 21 °C $a = 0.59$ and $m = 0.25$ (Perry and Green, 1997). From this the rate of heat transfer $q$ can be calculated.
The velocity profile induced by free convection along a vertical wall is given by Ostrach (1953), see Figure 2. At \( \eta = 0.8 \) the velocity along the wall of the settling tube is maximal (\( \text{Pr} = 7 \)). Note that \( \eta \) is defined in Figure 2. With \( \text{Gr} = 5.6 \times 10^6 \), \( \eta = 0.8 \) occurs for \( y / x = 0.023 \) or, at \( x = 0.3 \text{ m}, y = 7 \times 10^{-3} \text{ m} \). At \( \eta = 5 \) the velocity is nearly zero, or, at \( x = 0.3 \text{ m}, y = 0.04 \text{ m} \). Note that this is larger than the tube radius \( r = 0.015 \text{ m} \). The assumption of a semi-infinite medium is therefore violated. In reality the velocity near the wall will therefore be slightly less because of return flow.

The maximum velocity induced by a 0.1 °C temperature difference is for \( f'(\eta) = 0.15 \). At \( x = 0.30 \text{ m} \) this results in \( u = 2v f'(\eta) \sqrt{\text{Gr}_{x} / x} = 2 \text{ mm/s} \). This is a very substantial velocity compared with the settling velocity of small particles. Note that the maximum velocity is proportional to \( \Delta T^{0.5} \) and to \( L^{1.5} \). Convection is reduced if the column height is decreased. This has as additional advantage that the settling time and measuring time is halved, although the separation between the size classes is reduced and the measurement frequency should be doubled. This can be achieved without problem, however.

It is remarked that in many cases the temperature gradient in the settling column will remain (much) below 0.1 °C. As a result, the convective flow will also be much weaker. Assuming \( \Delta T = 0.01 \) °C, \( \text{Gr} = 5.5 \times 10^5 \) and \( u = 0.7 \text{ mm/s} \). In addition, part of the temperature difference between the water inside the tube and the ambient water is accommodated in the tube wall. Still, a typical convection velocity may be in the order of 0.1 mm/s. This is large compared to the settling velocity of the smallest fractions. Therefore the assumption of undisturbed flow is not valid for the smallest fraction and the determination of the settling velocity is unreliable without taking into account additional dispersion.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>0.6</td>
<td>W m(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>( C_p )</td>
<td>4.18 \times 10^3</td>
<td>J kg(^{-1}) K(^{-1})</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.001</td>
<td>Pa s</td>
</tr>
<tr>
<td>( \nu )</td>
<td>10(^{-6})</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.21 \times 10(^{-3})</td>
<td>K(^{-1})</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.4 \times 10(^{-7})</td>
<td>m(^2) s(^{-1})</td>
</tr>
<tr>
<td>( L )</td>
<td>0.30</td>
<td>m</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>0.1</td>
<td>°C</td>
</tr>
<tr>
<td>( \text{Pr} )</td>
<td>7.0</td>
<td>-</td>
</tr>
<tr>
<td>( \text{Gr} )</td>
<td>5.6 \times 10^6</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: values for water at \( T = 20 \) °C;
Laser light as heat source. A 10 mW diode laser is used as light source in the LISST-ST (Agrawal and Potsmith, 2000). Assuming that 10% of the light is dissipated in the column, the rate of dissipation is 1 mW. At a temperature difference of 0.01 °C between tube wall and core water temperature, the heat loss or gain through the tube wall is 50 mW. Therefore the energy flux from the laser light is judged to be of secondary importance for the generation of convective currents.

Another way to illustrate the limited influence of the laser light is to calculate how long it takes before the laser beam volume (diameter $d = 6$ mm, length $l = 5$ cm) is heated up 0.01 °C. This is:

$$t = \frac{\pi d^2 l \rho C_p \Delta T}{P_{\text{laser}}} = 60 \text{ s}$$ (6.6)

For $w > 10^{-4}$ m/s particles and surrounding fluid have been replaced in this period of time. Again the influence is small compared to convective currents generated by a typical heat flux through the tube wall.
6.4 Wall effects

Wall effects can be important if the particle diameter is not small compared to the diameter of the settling column. In the LISST-ST, the perspex baffles are spaced at 1 cm, whereas the largest measured particle diameter is 350 μm, i.e. 30 times smaller. Wall effects are therefore expected to be of minor importance.

For values of diameter ratio $\beta = \text{particle diameter} / \text{cylinder diameter}$ less than 0.05 (Perry, 1997, p. 6-54):

$$k_w = \frac{1}{1 + 2.1\beta} \quad (6.7)$$

where $k_w$ is the correction factor for the Stokes' law ($Re_p < 1$). For the example above $k_w = 0.93$, i.e. a deviation of 7% from Stokes' law. See also Bernhardt (1994, §3.2.3).

It is noticed that at low concentrations, the number of flocs between the plates at a certain level is limited to only a few. For example, with $d = 350$ μm and $c_{\text{bloc}} = 10$ mg/l, the average floc distance is 0.5 cm or half the gap width. Flocs are then much more likely to interact with the side walls than mutually. Depending on the physical and chemical properties of the side walls, flocs may remain stuck after contact with the side walls. This has been observed during VIS measurements (Video In Situ, Van Leusen, 1994 and Cornelisse, pers. com.). In these cases the concentration decay under the baffles will proceed slower, as the residence time of the flocs is increased with the time that they remain stuck at the side walls.

This effect is only expected to be important for small numbers of large flocs. For small particles this effect is negligible. Note that wall effect from temperature gradients are excluded from this analysis. They are dealt with in the previous section.

6.5 Particle interaction by differential settling

Differential settling is the process that particles with different diameter or density have different settling velocities. Depending on the sediment concentration, particles of different classes may interact. Interaction may imply coagulation after collision (flocculation) or just influencing the settling velocity of smaller particles in the trajectory of bigger ones. Flocculation is probably unimportant in the LISST-ST, see §6.6.

Here the fractal approach is adopted (see Kranenburg 1994 for more explanation). The excess density of a floc is expressed as:

$$\Delta\rho_f = \rho_f - \rho_w = (\rho_s - \rho_w) \left( \frac{D_p}{D_f} \right)^{3-n_f} \quad (6.8)$$

where $n_f$ is the fractal dimension, $D_f$ the floc diameter, $D_p$ the diameter of the primary particle, $\rho_s$ the solids density (2650 kg/m$^3$), $\rho_w$ the water density (1000 kg/m$^3$) and $\rho_f$ the floc density. The volume fraction of flocs $\phi_f$ is expressed as:
\[ \phi_f = \left( \frac{\rho_s - \rho_w}{\rho_f - \rho_w} \right) \frac{c}{\rho_s} \left( \frac{D_f}{D_p} \right)^{3-n_f} \]  
\[ (6.9) \]

where \( c \) is the dry mass concentration of the flocs.

The number of particles per m\(^3\) is calculated from the ratio between dry weight \( c \) and dry weight per particle \( \Delta \rho_f \rho_w \pi D_f^3/(6(\rho_s - \rho_w)) \):

\[ n = \frac{6c}{\pi D_f^3 \rho_s} \left[ \frac{D_f}{D_p} \right]^{3-n_f} \]  
\[ (6.10) \]

The average particles distance \( l \) scales with \((1/n)^{1/3}\):

\[ l = \left( \frac{1}{n} \right)^{1/3} = D_f \left[ \frac{\pi \rho_s}{6c} \left( \frac{D_p}{D_f} \right) \right]^{1-n_f} \]  
\[ (6.11) \]

The relative particle distance \( l_r = l/D_f \) is expressed as:

\[ l_r = \left[ \frac{\pi \rho_s}{6c} \left( \frac{D_p}{D_f} \right) \right]^{1-n_f} \]  
\[ (6.12) \]

This fractal approach implies that the relation between settling velocity \( w_s \) of a single mud floc in still water, and floc size \( D_f \) does not follow Stokes' law but (e.g. Winterwerp, 1998):

\[ w_{s,r} = \frac{\alpha}{18 \beta} \left( \frac{\rho_s - \rho_w}{\mu} \right) \left( \frac{g D_p^{3-n_f}}{D_f} \right) \left[ \frac{\alpha' g \Delta \rho_p}{\nu} D_f \right] \]  
\[ (6.13) \]

in which \( \mu \) (\( \nu \)) is the dynamic (kinematic) viscosity of the fluid and \( Re_p = w_s D_f/\nu \) is the particle Reynolds number. For spherical \((\alpha = \beta = 1)\), Euclidean \((n_f = 3)\) particles in the Stokes' regime, for which \( Re_p \ll 1 \), the well-known Stokes' formula for a stationary settling particle is obtained:

\[ w_{s,r} = \frac{(\rho_s - \rho_w) g D_f^2}{18 \mu} \]  
\[ (6.14) \]

Note that for \( n_f = 2 \), the settling velocity is proportional to the floc diameter \( D_f \).

The area of flocs \( A_f \) in m\(^2\)/m\(^3\) is expressed as:

\[ A_f = \frac{3c}{2D_f \rho_s} \left[ \frac{D_f}{D_p} \right]^{3-n_f} \]  
\[ (6.15) \]

For \( n_f = 2 \) the area becomes independent of the floc size and this equation can be written as
\[ A_f = \frac{3c}{2D_p \rho_s} \] (6.16)

For a column with height \( L \) the surface fraction occupied by flocs is:

\[ x = \frac{3cL}{2D_p \rho_s} \] (6.17)

The fraction of primary particles that will be influenced at least for a short period of time by flocs passing by is given by surface fraction of flocs \( x \). With \( L = 0.3 \text{ m} \) and \( D_p = 3 \mu\text{m} \) a substantial percentage of primary particles (\( P = 15\% \)) is in the pathway of flocs already at \( c > 1 \text{ mg/l} \). Note that this is very low concentration, which is caused by the fractal nature of the flocs (\( n_f = 2 \)). For massive particles (\( n = 3 \)) with \( D_f = 350 \mu\text{m} \) the interaction probability is much smaller at the same concentration \( c = 1 \text{ mg/l} \): \( P = 0.13\% \).

In Kuijper et al (1993) it is stated that the volume concentration should be less than 0.05 to 0.5\% to prevent mutual particle interaction in a settling column. The lower value is equivalent with a floc concentration of about 10 mg/l (\( n_f = 2; D_f = 350 \mu\text{m} \)).

For concentrations typical for the range of operation of the LISST-ST the assumption of independent settling behaviour of the eight different classes is reasonable for unflocculated material, but invalid for flocculated material. The importance of the interaction depends on whether particle aggregation occurs or not.

Assuming that the volume of water affected by the floc movement is 10 times the floc volume (wake etc.), the fraction of small particles affected is \( 10 \phi_f \). For floc concentrations of 10 mg/l and less the fraction of small particles affected at the same time will be less than 1\%. Without aggregation or capturing, the duration of the interaction between a large floc and a primary particle will be limited to a period in the order of 1 s. This is a small period compared with the total settling time of the primary particle (10\(^4\) s). In addition, the number of interactions will be limited to a few per particle at a mass concentration about 10 mg/l.

Summarizing, although particle interaction does occur for flocs at a concentration about 10 mg/l, a substantial effect on settling velocity of primary particles is only to be expected if the floc and the primary particle travel together, e.g. after aggregation, for a substantial time compared with the settling time of the floc. The likelihood for this is discussed in the next section.

Without aggregation, particle interaction starts to have an effect near \( c = 100 \text{ mg/l} \). At \( c = 1 \text{ g/l} \), particle interaction will be very important (for flocs with \( n_f = 2, D_f = 350 \mu\text{m} \)).

### 6.6 Flocculation and deflocculation

Flocculation is defined as the continuous process of aggregation of particles after collision. Particle collisions may occur e.g. because of velocity gradients (shear) and differences in settling velocity. Deflocculation is defined as the break-up of particles caused by turbulence.
At the start of the experiment, when water is drawn into the settling tube by impeller force, deflocculation may occur because of turbulent shear stresses. Given the short duration of this period (order of second), flocculation will not be important.

Aggregate size is limited by the Kolmogorov length scale for turbulence. Larger aggregates are destroyed as they cannot resist turbulent shear, which increases with length scale. The Kolmogorov length scale is calculated according to:

\[ l_k = \left( \frac{v^3}{\varepsilon} \right)^{\frac{1}{3}} \]  

(6.18)

where \( \varepsilon \) is the turbulent dissipation rate per unit of mass. Assuming that in the period of 4 s between switching off the impeller and closing the sample doors all water in the tube is replaced once, a velocity in the tube of 0.1 m/s is estimated. Between the baffles (1 cm apart), a typical shear rate is 20 s\(^{-1}\). In terms of Reynolds number, the Kolmogorov length scale is expressed as (Perry and Green, 1997):

\[ l_k = 4D \text{Re}^{-0.38} \]  

(6.19)

With \( D = 0.01 \text{ m} \) and \( \text{Re} = 1000 \) follows that \( l_k = 0.2 \times 10^{-3} \text{ m} \) (200 \text{ \mu m}). Therefore the largest size class of the LISST-ST (359 \text{ \mu m}) is likely to be affected by deflocculation: the measured volume concentration of large flocs is smaller than the in-situ volume concentration. Note that the flow tends to be laminar between the baffles, whereas close to the impeller the flow is turbulent and also smaller flocs may be destroyed. This occurs only during sampling with the impeller switched on.

Flocculation may occur by: 1) turbulence or 2) differential settling. Flocculation by turbulence is unlikely as the time scale for turbulence decay is short (2 - 40 s). The effect of wave- or current-induced vibrations is judged to be minor (see §6.10). Flocculation by differential settling is discussed in Stolzenbach and Elimelech (1994). The chance that particles come close together in the settling tube is substantial, but the change that they aggregate is much smaller, depending on size, density and porosity. It is judged that flocculation is not important in the LISST-ST, but this assessment is not beyond doubt, especially for large, porous, fractal aggregates. Porous aggregates are discussed by Li and Logan (1997). Further investigation is beyond the scope of the present study.

### 6.7 Hindered settling

Hindered settling is normally encountered in concentrated slurries. Below 0.1 percent volumetric particle concentration, there is less than a 1 percent reduction in settling velocity. The hindered settling velocity is expressed as:

\[ u_t = u_{t_0} (1 - \phi)^n \]  

(6.20)

where \( u_t \) is the terminal settling velocity, \( u_{t_0} \) is the terminal settling velocity of a single sphere (infinite dilution) and \( n \) is a function of the particle Reynolds number. For \( \text{Re}_p < 0.3 \)

\( n = 4.65 \).
For rigid particles (fractal dimension \( n_f = 3 \)) the volume concentration exceeds \( \phi = 0.1\% \) only at a dry mass concentration of 3 g/l. For small and/or rigid particles, hindered settling may be neglected in the LISST-ST. However, for flocs with \( n_f = 2 \) and a diameter of 100 times that of its primary particles, the same volume concentration is already reached at \( c = 30 \) mg/l (see Equation 1). Therefore hindered settling may be important in the LISST-ST if:

1. large aggregates are present in high concentrations (order 100 mg/l and more), and
2. these aggregates have a frail structure and therefore a low excess density.

### 6.8 Return flow

Return flow is a result of the continuity requirement: in a closed system the transport of a volume \( V \) in downward direction will result in the transport of an equal volume \( V \) in upward direction.

The volume flux of each particle class is calculated from the product of the settling velocity and the volume fraction:

\[
F_i = w_{vp,i} \phi_i = \frac{(\rho_s - \rho) g D^2 c}{\mu} \quad (6.21)
\]

Note that the settling volume flux is proportional to the sediment concentration \( c \) and to the particle diameter squared \((D^2)\), but independent of the fractal dimension. Herein it is assumed that all water within the aggregate travels with the aggregate. However, it is known that for very open-structured aggregates (probably not relevant for estuarine mud) a certain through-flow occurs (e.g. Johnson et al., 1996). This will reduce the displaced volume, but enhance the settling velocity of the floc.

The resulting average return velocity is \( w_{vp} = F_i/(1-\phi_{tot}) \). For particles with \( D = 350 \) \( \mu \)m and \( c = 250 \) mg/l, \( w_{vp} = 10^{-5} \) m/s, which is equal to the undisturbed settling velocity of a particle with a diameter of 3.5 \( \mu \)m. This illustrates that for high concentrations of large particles, return flow has an important effect on the settling velocity of small particles. Small particles and large particles at a concentration less than 50 mg/l do not cause substantial return flow, however.

More details on the settling behaviour of polydisperse particle systems can be found in Van Rhee (2002, §3.5.).

### 6.9 Deviation from Stokes’ Law

Stokes’ Law is valid for rigid spherical particles at \( \text{Re}_p < 0.1 \), where \( \text{Re}_p = u_d d_p / v \). For spheres with \( \rho = 2650 \) kg/m\(^3\) this criterion is violated for \( d > 50 \) \( \mu \)m. However, for fractal aggregates the excess density decreases with increasing particle diameter. For \( n_f = 2 \) the criterion \( \text{Re}_p < 0.1 \) is only violated for \( d > 180 \) \( \mu \)m. At \( d > 350 \) \( \mu \)m, \( \text{Re}_p = 0.4 \), resulting in a reduction of the settling velocity compared with Stokes’ Law of 4\% according to:
\[ u_t = \sqrt{\frac{4gd_p (\rho_p - \rho_w)}{3\rho_w c_d}} \quad \text{with} \]
\[ c_d = \left( \frac{24}{Re_p} \right) \left( 1 + 0.14 \frac{Re_p^{0.6}}{1 + Re_p^{0.6}} \right) \quad 0.1 < Re_p < 1,000 \]  

(6.22)

For rigid particles (e.g. sand) with \( d = 350 \, \mu m \) and \( \rho = 2650 \), Stokes' Law already overestimates the settling velocity with a factor of 1.7.

For non-spherical particles shape factors have to be taken into account. For \( Re_p < 0.05 \),

\[ u_t = K_1 \frac{gd_p^2 (\rho_p - \rho_w)}{18 \mu} \quad \text{with} \]
\[ K_1 = 0.843 \log \left( \frac{\Psi}{0.065} \right) \quad \Psi > 0.67 \]  

(6.23)

where \( \Psi \) is the sphericity, the surface area of a sphere having the same volume as the particle, divided by the actual surface area of the particle. The reduction in settling velocity compared with Stokes' Law is about 15% for \( \Psi = 0.67 \). For more information on shape factors, see also Bernhardt (1994, §3.1.3).

It is often assumed that pore water in non-rigid particles moves along with the particles (e.g. Winterwerp, 1999). However, the higher the porosity, the more likely it becomes that water flows through the pores. In those cases, the drag coefficient is reduced. Johnson et al. (1996) observed settling velocities of fractal aggregates that were 4 to 8 times higher than those of spheres with identical mass and cross-sectional area predicted with Stokes' Law. Even at \( \eta_f = 2 \) this behaviour is observed. They attribute this to a heterogeneous distribution of primary particles in a fractal aggregate, creating larger pores as the aggregates increase in size. These large pores produce a smaller overall drag per total cross-sectional area for the fractal aggregate than calculated for an impermeable of permeable spherical aggregate. However, they remark that the reduced drag coefficient may not be applicable to all types of aggregates, since many aggregates contain a variety of polymers, filaments and other material that can clog pores and alter the flow conditions around and within settling aggregates. For more information of the settling velocities of fractal aggregates is referred to Chen and Kai (1999) and Kim and Yuan (2003).

It is therefore concluded that using Stokes' Law to derive the effective particle density from its settling velocity, substantial errors may result. Neglecting shape factors, particle bulk densities may be underestimated. Neglecting flow through aggregates, particle bulk densities may be strongly overestimated.
6.10 Wave- or current-induced vibrations

Vibrations can be characterised with their amplitude \( a \) and frequency \( f \). The resulting acceleration is proportional to \( af^2 \). If this acceleration is much smaller than \( 0.1 \text{g} \), mixing effects by vibration are judged to be of minor importance, assuming that the sedimentation tube is completely filled with water and sediment. If a water-air interface is present, wave- or current-induced vibrations will have a stronger effect.

Assuming a vibration frequency of \( f = 1 \text{ s}^{-1} \) and amplitude \( a = 10^{-3} \text{ m} \), the resulting acceleration is \( 10^{-3} \text{ m}^2/\text{s} \) or \( 0.01 \text{g} \). Effects of vibrations are therefore expected to be minor if no air is encapsulated.

6.11 Concentration measurements

The LISST-ST measures the volume concentration of sediment. It is known that the light extinction due to particles with a different diameter is different, even at an equal volume concentration.

In principle, this is taken into account in the diffraction model. From the extinction pattern as a function of the angle of diffraction the volume concentration of different particle size classes can be calculated. However, this correction is not perfect, as is shown in Figs. A.1–3 (Appendix A).

There seems to be a strong correlation between the settling curve of the small and large size classes for high volumetric floc concentration (Fig. A.3). Until \( t = 10^2 \text{ s} \), when the coarse fraction with a high volume concentration has settled out, the (low) volume concentration of the smallest size class steadily increases. This may be caused by masking of the weak diffraction signal (at low volume concentration) of the smallest size class by the strong diffraction signal (at high volume concentration) of the largest size class.

Either this artefact has to be corrected, or measurements at high concentrations should be discarded. Probably the addition of diffusion alone (see Chapter 8) is insufficient to correct for these artefacts.

6.12 Multiple scattering

At large concentrations multiple scattering of the laser beam will occur. Multiple scattering is expected at a mass concentration over 100 mg/l. Multiples scattering results in a bias towards smaller size classes, as light scattered twice on large particles may have the same angle of deflection as light scattered once on a small particle. It is therefore advised to apply the LISST-ST at modest concentrations only.

6.13 Aliasing

Particles smaller than the smallest size class (<3 µm) or larger than the largest size class (>350 µm) still have an impact on the diffraction pattern. As a result, the detected volume concentration of the smallest and largest classes may be too high compared with reality. Also, the apparent concentration decay will start too early for the largest size class, whereas
for the smallest size class the concentration curve will show a (too) long tail, reflecting the respective settling times of the fractions >350 μm and <3 μm.

6.14 Non-sphericity

In the applied model for particle scattering it is assumed that the particles are spherical. However, flocs may deviate substantially from a spherical shape. See Boxman (1992), Hage et al. (1991) and Barber and Massoudi (1982) for more details on the scattering theory for non-spherical particles. Irregularly-shaped particles (e.g. with small-diameter protrusions) may diffract light in a similar way as a smaller particle. The concentration of small particles would then be overestimated. The importance of this effect is unknown. For particles close to a spherical shape little effect is expected, but for very irregular particles the effect may be substantial.

6.15 Undersampling

Number of particles present in the laser beam (length \(l = 5\) cm, diameter \(d = 6\) mm) is calculated from the number \((n_d\) defined in Eq. 2\) of particles per m³ according to:

\[
n_{\text{beam}} = \frac{1}{4} \pi d^2 l n_d
\]  

(6.24)

It is thus calculated that for \(d = 350\) μm and fractal dimension \(n_f = 2\), only about 20 particles are present in the laser beam at \(c = 10\) mg/l. For \(n_f = 3\), only 0.2 particles are present at the same concentration.

These small numbers of particles result in fluctuations of the scattering image. At small diameters \((d = 3\) μm) the number of particles in the laser beam is of the order of \(10^5\). A smooth scattering image is the result.
7 Integrated analysis of error sources

If the relative error sources are intercompared, distinction should be made between measurements at high and low concentrations. Some disturbances may occur at any concentration.

All concentrations

Convection
Deviation from Stokes’ Law
Sediment Sampling
Non-sphericity
Aliasing at the limits of the particle size range
Concentration measurement
Initial effects
Wave- or current induced vibrations

Low concentration

Undersampling
Wall effects

High concentration

Interaction by differential settling
Multiple scattering
Hindered settling
Return flow
(De-) flocculation

Error sources judged to be most important are printed in bold. Without knowledge about the specific conditions in which the LISST-ST is deployed, it is hard to indicate which error source is dominant. Sediment concentration, size distribution and (fractal) properties are key factors in this respect.

In the present data-processing model, it is assumed that the shape of the concentration decay curves (see Appendix A) is completely determined by the particle bulk density distribution, assuming Stokes’ Law. Especially assuming a distribution with two peaks, satisfactory fits of the decay curves have been obtained. However, this approach has the disadvantage that all disturbances listed above are lumped into the density distribution. This distribution thus becomes a fitting parameter, whereas at the same time it is meant as a physical output parameter. As a result, it is unclear which part of the density distribution makes physical sense and which part is an artefact resulting from the violations of the model assumptions.
As alternative, it is proposed to take the disturbances into account in the data analysis model by introducing one or more additional parameters. The model may be improved with different levels of sophistication:

1. addition of a simple diffusion term,
2. inclusion of more complicated residence time distribution (RTD) functions,
3. including of all important physical processes by the construction of a hydrodynamic numerical model of the settling tube, including thermal effects and particle-particle interactions.

These items are discussed in more detail in the next chapter.

It is remarked that the effect of disturbances may be reduced either by making stricter demands on environmental constraints such as sediment concentration or temperature fluctuation or by changing the design of the settling tube. For example, convection may be reduced by improving the thermal insulation of the column, or a stable density gradient may be created by applying a vertical temperature gradient over the cylinder wall with the higher temperature at the top.

Inaccuracies because of a high sediment concentration may be controlled by a reconfiguration of the entrance geometry such that the number of particles entering the space between the baffles is limited, for example by a deflection of particles from the centreline of the column.
8 Alternative approach for data analysis

8.1 Addition of diffusion term

The present data analysis of the LISST-ST data is based on the assumptions that 1) all sediment classes behave independently and 2) ambient water in the column is completely stagnant, only sediment particles move.

Because of a number of reasons (e.g., convection, particle interactions, optical effects) these assumptions are often violated. Particles with identical properties and initially at the same height may have a different residence time in the settling column. It is therefore proposed to add a diffusion term to the settling equations, which models the above-mentioned effects. The LISST-ST is then mathematically described as a plug flow reactor with axial dispersion:

\[ \frac{\partial c_i}{\partial t} + w_j \frac{\partial c_i}{\partial z} = \frac{\partial}{\partial z} D_a \frac{\partial c_i}{\partial z} \]  

(8.1)

where \( D_a \) is the effective diffusion coefficient and subscript \( i \) refers to the particle fraction class number. It is noted that it is implicitly assumed that all disturbances have a stochastic character, which in reality is not completely true. This is further discussed in §8.2.

The solution for Equation (8.1) depends on the magnitude of the Péclet number \( Pe = w_j L / D_a \), where \( L \) is the height of the tube (0.3 m). For \( Pe \rightarrow \infty \) plug flow exists, whereas for \( Pe = 0 \) the tube is perfectly mixed. By tuning \( D_a \), the actual residence time distribution (RTD) of particles in the tube may be approximated, yielding more realistic results than assuming plug flow.

The small particles (\( w_j \), small) yield small Pe numbers, whereas large particles yield large Pe numbers. The effective diffusion coefficient is expected to be similar for all fractions. The width of the transition zone between high and zero concentration is inversely proportional to Pe or directly proportional to the axial dispersion coefficient \( D_a \).

The advection-diffusion mechanism may also be approximated with a cascade of stirred tank reactors (CSTR). For \( n = 1 \) (a single reactor) the system is ideally mixed, for \( n \rightarrow \infty \) plug flow occurs. There is no exact way to compare the tank-in-series and dispersion models, because the responses are never identical. However, a useful relationship is obtained from equating the variances for the two models (Froment and Bischoff, 1990):

\[ \sigma^2 = \frac{1}{n} \cdot \frac{2}{Pe} \left[ 1 - \frac{1}{Pe} \left( 1 - e^{-Pe} \right) \right] \]  

(8.2)

or,

\[ n = \frac{Pe}{2} + \frac{1}{2} \quad \text{Pe} > 2 \]  

(8.3)
Results obtained with the axial dispersion model and the tanks-in-series model are shown in Figures 3, 4 and 5 for different values for \( n \) and \( \text{Pe} \). In Fig. 3 results are shown on a real time axis. Note the similarity between Fig. 3 and Fig. A1. In Figs. 4 and 5 the dimensionless time \( \theta = t \omega / L \) is shown on the x-axis (note the logarithmic x-axis in Figure 5). On the y-axis the dimensionless concentration response \( F = C/C_0 \) is shown. Note that the curves for \( n = 32 \) and \( \text{Pe} = 100 \) are similar. More information on axial dispersion models can be found in Froment and Bischoff (1990), Brenner (1963) and Bernhardt (1994, p. 50).

Figure 3: Concentration decrease in time for axial dispersion model with \( D = 10^{-5} \text{ m}^2/\text{s} \) and \( \omega_r = 0.02, 0.05, 0.1, \)
\( 0.2, 0.5 \) and \( 1 \text{ mm/s} \) (respectively \( \text{Pe} = 0.6, 1.5, 3, 6, 15 \) and \( 30 \)). \( L = 0.3 \text{ m} \) (note: curves based on infinite pipe approach, curves for finite pipes are similar, see Brenner 1963).

Figure 4: Concentration decay in LISST-ST for particles with equal size and density. Obtained with axial dispersion model (characterised with \( \text{Pe} \)) and tanks-in-series model (characterised with \( n \))
Figure 5: concentration decay in LISST-ST for particles with equal size and density (note logarithmic time axis). Obtained with axial dispersion model (characterised with Pe) and tanks-in-series model (characterised with n).

It is remarked that the relative concentration $F$ at $\theta = 1$ equals 0.5 for $Pe \rightarrow \infty$ ($n \rightarrow \infty$), whereas $F = 0.3679$ ($= 1/e$) for $Pe = 0$ ($n = 1$). A simple model that calculates the average settling velocity according to $w_s = L / t (F = 0.5)$ therefore overestimates the settling velocity up to 40% for large diffusion coefficients.

### 8.2 Extended models

Although the diffusion model described above can be used to handle a wide variety of non-ideal situations, the settling tube may contain elements that do not satisfy the fundamental basis of a diffusion-like model: the overall mixing should be the result of a large number of small random events (Froment and Bischoff, 1990).

For example, if a substantial fraction of small particles is dragged along with large particles, this may result in an additional point of inflexion in the concentration curve. Such behaviour cannot be reproduced with a simple diffusion model. Then, particle interactions have to be taken into account, or distinct regions in the settling column should be considered (Table 2).

Using these types of models, a proper fit of most concentration decay curves should be obtained without the necessity to ‘pollute’ the parameters for particle density and settling velocity with correction for non-ideal behaviour. They capture the essence of the disrupting effects on the particle residence time distribution with a limited number of parameters, without the necessity to model the physics in the settling tube in detail.

If sufficient measuring data is available, confidence intervals may be calculated for the particle density and settling velocity using resampling techniques. A better insight is thus obtained in the accuracy of the measurements.
<table>
<thead>
<tr>
<th>model type</th>
<th>equation of motion (per particle)</th>
<th>transport equation (ensemble)</th>
</tr>
</thead>
<tbody>
<tr>
<td>present</td>
<td>$dz = -w_z dt$</td>
<td>$c(z,t) = c_0 H(z - w_z t)$</td>
</tr>
<tr>
<td>with diffusion</td>
<td>$dz = -w_z dt + D dw_z$</td>
<td>$\frac{\partial c}{\partial t} + w_z \frac{\partial c}{\partial z} = \frac{\partial}{\partial z} D \frac{\partial c}{\partial z}$ (8.1)</td>
</tr>
<tr>
<td>extended</td>
<td>$dz_1 = -w_{z_1} (z_1 \ldots z_N) dt + D (z_1 \ldots z_N) dw_{z_1}$</td>
<td>(?)</td>
</tr>
<tr>
<td></td>
<td>$\vdots$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$dz_N = -w_{z_N} (z_1 \ldots z_N) dt + D (z_1 \ldots z_N) dw_{z_N}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Overview of basic model formulations for LISST-ST data analysis software.

Resampling is a generic technique for the assessment of the uncertainty in an estimate of some statistic. The idea is to construct a large number of new data sets from the original data set. For each so-called ‘resample’ of the original data set the estimation procedure is repeated. This leads to an ensemble of estimates for the statistic, and from this ensemble uncertainties can be determined. The uncertainty can be in the form of a spread or standard deviation, or a confidence interval. In this way resampling can be used within model calibration as well, and provides estimates for the uncertainties in the model parameters and model predictions.

For the calculation of the density a settling law has to be applied a priori. For irregularly shaped, tenuous floc, the deviation from Stokes’ Law may be substantial. The measurements would become more reliable if independent data on floc density would be acquired. For example, a sedimentation balance may be integrated into the settling column. Another possibility is to mount a video camera on the column to obtain direct information on settling velocity and floc shape and structure (e.g. Maggi and Winterwerp, 2003).

### 8.3 Numerical model on movement of water and particles

The next level of sophistication is the construction of a numerical model of the hydrodynamics and the sediment movement in the column. Such a model would help to improve the understanding of the system. Ideally, the model should be validated with laboratory experiments with the LISST-ST (see next section).

However, it should be realised that such a model is probably not suited to include in the data analysis software, as its number of variables may be too large to get a well-defined fit and it is computationally too expensive to get instant results. Such a model would still contain a number of empirical relationships, especially on particle-particle interaction and on particle-fluid interaction beyond the level of a settling law.

The settling velocity is derived directly from the settling time and column height. Although the apparent width of the settling velocity distribution is influenced by distorting effects such as convection and particle interaction (captured in one or more diffusion-like terms), the measured median settling velocity is not very sensitive to these effects (Figures 4 and 5).
The error bands around the LISST-ST settling velocity data are therefore expected to be modest.

However, in the derivation of the excess density from the settling velocity and particle size, errors start to accumulate. The settling velocity and particle size measurements contain errors, and a settling law has to be applied a priori. The error bands of the particle bulk density data are therefore expected to be substantially wider than those of the settling velocity.

### 8.4 Laboratory measurements

It is therefore advised to make independent, direct measurements on the mass concentration in the LISST-ST settling tube in the laboratory, e.g. with a settling balance. These will help to determine the error bands of the indirect method to estimate mass concentration, i.e. derivation from the particle diameter and settling velocity. Independent data on floc size, structure and settling velocity can be obtained with a video camera (for flocs > 20 μm, depending on camera resolution). These images will help to improve the understanding of the system.

Other laboratory measurements on the LISST-ST system could consist of:

- Measurement without particles or a low concentration of tracer particles only. From these measurements the importance of convection can be identified.
- Measurements with monodisperse particles with known size and concentration (with both rigid, Euclidian particles and flocs). Concentration and particle size should be varied.
- Measurements with mixtures of monodisperse particles of different sizes. Concentration and ratio between size classes should be varied.

As the properties of individual particles (size, density, concentration, settling velocity) are known, the settling curves can be reconstructed. Differences between the observed and calculated settling curves can be attributed to disturbances such as convection, particle interaction etcetera.

In this way the combination of laboratory measurements and numerical modelling can be applied to calibrate and validate the LISST-ST system more accurately than presently achievable without these tools. For daily use in the field the data analysis software is better suited.
9 Conclusions and recommendations

Based on the analysis of the physics of the settling tube of the LISST-ST, it is concluded that the assumption that all particles settle independently in a completely stagnant fluid is often violated. As a result, the calculated particle density distribution, which is presently the only fit parameter in the model, often becomes unrealistically wide to compensate for effects such as convection and particle interaction.

It is therefore advised to include a diffusion term into the analysis software to take these deviations from the assumed ideal behaviour into account. However, if substantial interaction exists (either real or ‘virtual’ because of optical artefacts) between particles with a widely different size, a single diffusion term may not be sufficient to describe the data adequately. A more complicated model would then be required, for example a multi-zone model accounting for a more complicated (e.g. double-peaked) residence time distribution function. Such a model could still be based on the present analysis software, including some additions. A total reconstruction is not required.

If sufficient measuring data is available, confidence intervals may be calculated with these fit models for the particle density and settling velocity using resampling techniques. A better insight is thus obtained in the accuracy of the measurements. As the settling velocity is directly derived from the measured (volume) concentration decrease in time, relatively small error bands are expected. However, the density distribution is derived from the settling velocity and particle diameter assuming certain a settling law (Stokes). Much larger error bands are therefore expected for the density distribution.

It is recommended to make a comparison between the particle bulk density calculated from the settling velocity and particles size (indirect method) and direct measurements in the laboratory. This is especially important for floc-like particle aggregates. With laboratory tests a better estimate of the accuracy of the indirect method to determine density is obtained than based on statistical analysis of LISST-ST field data only. At present, deployment of an independent technique to measure sediment mass concentration simultaneously is also recommended for in the field.

More insight into the physics in the settling tube can be obtained with a combination of laboratory measurements and numerical modelling. A numerical model is probably not suited as a substitution of the fit model, as the number of parameters tends to be large and it is computationally too expensive to get instant results.

The length of the settling tube can be made smaller (for example 15 cm instead of 30 cm) without important consequences for the accuracy. This has as advantage that the settling time and measuring time is halved, although the separation between the size classes is reduced and the measurement frequency should be doubled. This can be achieved without problem.
At present, only measurements of short duration (0.5 s) are made on a logarithmically spaced time interval. During most of the 12 to 24h measuring period, no data are recorded. It is recommended to increase the frequency or duration of sampling, enabling more averaging and statistical analysis.

With the present data analysis software, the reliability of results may be improved if only measurements at low concentrations are considered, or only the last part of the settling curve is taken into account, when the larger size classes have settled out. Particle interaction is then unimportant. This is easily done if a clear distinction occurs between the classes in terms of settling velocity. However, for very small fractal dimensions \(D = 1\) the settling velocity is independent of the particle diameter. In such cases the approach does not work. Such low fractal dimensions are unlikely to occur, however. This approach also does not work for an inhomogeneous initial condition. In §6.2 it was shown that sediment initially tends to be homogeneously distributed over the column.

A fundamental problem is that the sediment concentration cannot be controlled in situ, which sets a limit to the applicability of the instrument. Measurements in highly turbid environments tend to be unreliable. A system for sub-sampling within the tube may minimise this problem by reducing the sediment concentration between the baffles.
References


Appendix: Figures

LISST-ST data analysed RIKZ 1 cmp. model, SANDPIT 24 juli 2003, sample: 12

Fig. A1: LISST-ST results at low floc concentration (30 ppm)
LISST-ST data analysed RIKZ 1 cmp. model, SANDPIT 24 juli 2003, sample: 15

Fig. A2: LISST-ST results at medium floc concentration (150 ppm)
Fig. A1: LISST-ST results at high floc concentration (1000 ppm)
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