Initial imperfection survey on a cylindrical shell at the Ultra-Centrifuge Nederland n.v.

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LIST OF SYMBOLS

\( A_{lo}, A_{kl}, B_{kl} \) Coefficients of the half-wave cosine Fourier representation, see Eq. (1)

\( C_{lo}, C_{kl}, D_{kl} \) Coefficients of the half-wave sine Fourier representation, see Eq. (2)

\( E \) Young's modulus

\( i, k \) Number of half-waves in the axial direction

\( i \) Number of full-waves in the circumferential direction

\( L \) Shell length

\( L_{HA} \) Shell length used for harmonic analysis

\( NC \) Number of data points in the circumferential direction

\( NR \) Number of data points in the axial direction

\( R \) Shell radius

\( t \) Shell thickness

\( w \) Imperfection data, positive outward, in mm

\( \bar{w}, \bar{W} \) Radial imperfection with respect the perfect cylinder, positive outward

\( x, y \) Axial and circumferential coordinates in the middle surface of the shell respectively

\( x_1, y_1 \) Eccentricities (see figure 8)

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) Small angles in radians-inclination of best-fit reference axis (see figure 8)

\( \theta = \frac{y}{R} \) Non-dimensional circumferential coordinate

\( \nu \) Poisson's ratio

\( \xi \) Equivalent initial imperfection amplitude
ABSTRACT

The results of the initial imperfection survey of a circular shell, with an inner- and outer-skin made out of carbon fibres with an aluminium honey-comb in between, are presented. At UCN the shell is called the “Demonstrator Model” shortly the “Demonstrator”.

The modal components of the measured imperfection surface as a function of the circumferential and axial wave numbers are calculated.

The characteristic imperfection distributions associated with the fabrication process used are presented.
1. INTRODUCTION

Whereas the degrading effect of the unavoidable initial imperfections on the load carrying capability of thin walled structures is now universally known, still in general, very little information is available about the shape and the size of initial imperfections produced by the different fabrication processes. If one wants to improve the buckling load predictions and make full use of the non-linear analysis capabilities now available with several of the advanced structural analysis codes, then a detailed knowledge of the actual imperfections that are present in the real structure is a must.

The survey described in this report was carried out on a circular cylindrical shell made out of two sheets of carbon fibres with a honey-comb core (see chapter 2). This sandwich construction results in a very efficient structure with an $R/t_{\text{equivalent}} = 83$, where for $t_{\text{equivalent}} = 12.5$ mm the thickness of the honeycomb and the two sheets was used.

The shell was scanned before loading and the measured initial imperfections were recorded on the test-site (Ultra-Centrifuge Nederland n.v. in Almelo) with the help of the data-aquisition system of the Structures Laboratory, Faculty of Aerospace Engineering of the Delft University of Technology. In this paper the results of this imperfection survey are presented in the form recommended by the Initial Imperfection Data Bank [1] [2], with the exception that the results are not normalized for the wall thickness of the shell.

2. THE "DEMONSTRATOR" SHELL

The Demonstator shell is shown in figure 1 and it consists of a basic cylindrical cfrp-aluminium honeycomb sandwich shell. The inner- and outer-skins of this shell are produced by a wet filament winding technique.

For the inner-skin first a layer with fibre orientation of $\pm 22.5^\circ$ was wound over the full length of a mandril, followed by a layer with fibres in the $90^\circ$ direction (the $0^\circ$ direction being parallel to the longitudinal axis of the shell).

The outerskin, covering the aluminium honeycomb, starts with a layer with fibre orientation of $90^\circ$ followed by winding a layer with fibres in the $\pm 22.5^\circ$ direction. Each skin has a thickness of about 0.225 mm and the honeycomb has a thickness of 12 mm. A lower and upper aluminium endring is bonded to the shell.

Inside the shell a tank must be mounted. In order to be able to introduce the loads into the sandwich structure, the innerskin is locally reinforced with cfrp laminates at the lower tank interface and the lower endring. The outerskin is reinforced at the upper tank interface, lower tank interface and lower
endring. These reinforcements have fibres in the $0^\circ, +/- 45^\circ$ and $90^\circ$ directions. At the location of the upper- and lower-tank interfaces and the lower endring heavier aluminium honeycombs than in the rest of the structure has been used. At the lower tank interface 40 inserts are bonded into the sandwich structure. The total mass of the shell complete with interfacings is 18.78 kg. For more detailed information about the manufacturing process of this shell see reference [3].

3. EXPERIMENTAL PROGRAM

The dimensions of the shell are given in figure 1. The shell is positioned in an upright position on a turn-table and the reference beam placed parallel to it on an adjustable tripod. Figure 2 shows the experimental set-up. As can be seen from this figure 4 linear voltage displacement transducers (L.V.D.T.) are used to carry out the measurements. Each transducer was connected to its own signal conditioner. The analog output of the signal conditioner for transducer number 1 is 0.1 V per mm and for the transducers 2,3 and 4 the output is 1 V per mm.

The fixed pickup 1 is bearing against the outer machined diameter of the turn-table, and is used to record the rigid body motion relative to the fixed reference beam when turning the shell in a new angular position. The upper disk of the turn-table is machined carefully and the maximum variation in diameter is about 0.06 mm. This disk acts as the reference ring described in reference [4].

The other two fixed pick-ups (2 and 3) are used for measuring the shape of the top and bottom ring of the shell, respectively. During this survey the transducer number 2 was bearing against the inner diameter of the top ring.

The transducer 4 is installed on a carriage which is moved along the beam by an electric drive. It bears against the surface of the shell and is used to record the shape of the shell generator. Upon completion of an axial scan the shell is rotated to a new angular position, followed by another axial scan. The process is continued until the whole surface has been surveyed and recorded. The axial scan starts just below the upper end ($d_A = 37.5$ mm) and terminated just above the lower end ($d_B = 40.0$ mm) of the shell (see figure 2).

The exact shape of the reference beam has been measured optically (see figure 3) in order to correct for the measured initial imperfections during the imperfection survey.

The axial position of the carriage is recorded by an electro-optical device which scans a strip, attached to the beam, with equally spaced cut-outs. Each time a cut-out is detected an electric pulse is generated. This pulse is used as a signal for the data-acquisition system to digitize and store the signal from transducer 4 at that moment. This system makes it possible to record the imperfection data every
2.5 mm in axial direction. The position in circumferential direction (in degrees) is used as the scan number.

The data-acquisition system consists of an HP 9000/370 computer, an HP 3497A data acquisition/control unit (for scanning and A to D conversion) and a digital X-Y plotter. The resolution of the A to D convertor of the HP 3497A was set to 0.1 mV. After each scan the measured shape can be recorded on the X-Y plotter for quick look examination.

Back at the university the imperfection data is copied in two files one containing the imperfection data and the other one the additional information needed during the imperfection survey.

4. DATA REDUCTION

4.1. Preliminary work

Before processing of the data can start the imperfection data in file ucndm1 (the raw data file) has to be brought into a standard shape. Especially for this shell one has to remove the data of the reinforcement at the bottom part of the shell.

Furthermore the imperfection data of the second reinforcement for the upper tank interface must be corrected to get a continuous shape pattern. To do so a special computer program called ucn00 was made for this purpose. This program was run three times, each time adding an extra correction process to the preceding step. All the time the data file ucndm1 was used as the input data file.

After each processing step the result (stored in file ucndm1b) was compared with the preceding result to check the correctness of the last correction step.

Step 1: All the measured imperfection data was formatted in such a way that other data reduction programs easily can read it.

After this step a 3-dimensional plot was made to show the raw imperfection data (see figure 4).

Step 2: Same formatting as in step 1 but now removed the reinforcement at the bottom of the shell (about 172.5 mm was removed).

The result of this step can be examined in figure 5.

Step 3: This processing step includes step 2, and all the raw imperfection data of the second reinforcement was given the arbitrary value of 99.999 mm.

The final result of step 3 (see figure 6) was recorded in file ucndm1b which was used as the start data file for the data reduction process as described below. For more detailed information of the data reduction process see reference [4].
4.2. Correction program

The next program called xx01 was run. With this program errors made during the measurement session were corrected. For this shell two corrections had to be made:

1. Translation of the shell during the imperfection survey.
2. Correction for the reinforcement for which, in the preceding program, the imperfection data was set to 99.999 mm.

The way the corrections were done is described below.

Translation of the shell

During the measurement session, the relative position of the shell with respect to the measurement device was constantly monitored. With the information obtained from transducer number 1 (bearing against the outer machined diameter of the turn-table) this program made the correction for this translation of the shell.

Reinforcement

To make the influence of this part as small as possible, this area was "filled in" by a linear interpolation. The correction was performed on each axial scan seperately and followed the following steps:

1. The scan was copied to an "approximation scan".
2. All data in the "approximation scan" higher than 3.0 mm or smaller than -3.0 mm was replaced by the nearest value in axial direction that was in between 3.0 and -3.0 mm.
3. A one dimensional Fourier analysis was done on the "approximation scan" calculating the first 6 Fourier coefficients.
4. With those lower order coefficients a "recalculated scan" was composed.
5. The original axial scan was then compared to the "recalculated scan". When a data point in the scan deviated more than 2.0 mm from the "recalculated scan" the data of this point was corrected. The correction was done by removing this incorrect part from the scan and fitting in the corresponding part of the "recalculated scan" in such a way that the substituted part fits to the correct part of the scan.
The corrected data was written to the file ucndm1c. From the final result of this program (xx01) a pseudo 3-dimensional plot is shown in figure 7.

4.3. Best-fit

Before an initial imperfection can be determined, it is necessary to define what is considered as the perfect shell. Here this was done by finding the best-fit circular cylindrical shell to all the measured data of the initial imperfection scans. Using the method of the least-squares one first computes the sums of the squares of normal distance from the measured points in space to the surface of the assumed best-fit cylinder (see figure 8). Minimizing the sum of the residuals $S$ with respect to the unknown parameters $x_1$, $y_1$, $e_1$, $e_2$, and $R$ yields then 5 simultaneous algebraic equations in five unknowns. For $x_1$, $y_1$, $e_1$, and $e_2$ small, these equations can be linearized and then solved to determine the best-fit or "perfect" cylinder.

Next the measured displacements are recalculated with respect to the newly found "perfect" cylinder. The process described above was done by the data reduction program xx02. The final results were stored in file ucndm1f and a pseudo 3-d plot was made (see figure 9). For a detailed description of this best-fit procedure see reference [5].

4.4. Harmonic Analysis

Next by a double harmonic analysis the Fourier coefficients of the measured imperfection surface (stored in file ucndm1f) was obtained. The half-wave cosine representation in the axial direction involves the determination of the two sets of harmonic components $A_{k\ell}$ $(A_{i0})$ and $B_{k\ell}$, where:

$$
\overline{w}(x, \theta) = t \overline{W}(x, \theta) = t \sum_{i=0}^{N} A_{i0} \cos \frac{i \pi x}{L} + t \sum_{k=0}^{N} \sum_{\ell=1}^{k} \cos \frac{k \pi x}{L} (A_{k\ell} \cos \ell \theta + B_{k\ell} \sin \ell \theta)
$$

and $\theta = y/R$. Similarly, the half-wave sine representation in the axial direction also involves the determination of two sets harmonic components $C_{k\ell}$ $(C_{i0})$ and $D_{k\ell}$, where:

$$
\overline{w}(x, \theta) = t \overline{W}(x, \theta) = t \sum_{i=1}^{N} C_{i0} \sin \frac{i \pi x}{L} + t \sum_{k=1}^{N} \sum_{\ell=1}^{k} \sin \frac{k \pi x}{L} (C_{k\ell} \cos \ell \theta + D_{k\ell} \sin \ell \theta)
$$
The double integrals involved in the determination of the required coefficients were carried out numerical using the trapezoidal rules by the program xx03.

Upon completion of the data reduction the following output is available:

1. The measured initial shell displacements representing deviations from the perfect cylinder at zero load (output of the best-fit process in file ucndm1f).
2. The coefficients of the two different Fourier representations of the measured initial imperfections. The half-wave cosine representation can be found in file ucndm1hc and the half-wave sine representation in file ucndm1hs.

5. DISCUSSION OF THE TEST RESULTS

The 3-dimensional plot of the measured initial imperfections with respect to the best-fit cylinder of the UCN cylindrical shell is shown in figure 9. From the Solution vector E (one of the results from the best-fit process) one can obtain the following information (see also figure 8):

\[
\begin{align*}
  x_1 & : 0.2202 \text{ mm} \\
  y_1 & : 0.8996 \text{ mm} \\
  \cos(\epsilon_1) & : -0.0004 \\
  \cos(\epsilon_2) & : -0.0004 \\
  \tan(\epsilon_3) & : -0.0003 \\
  R \text{ at } Z=0 & : 1035.5248 \text{ mm}
\end{align*}
\]

From \(\cos(\epsilon_1)\) and \(\cos(\epsilon_2)\) can be calculated that the centers at top and bottom ring are 0.8 mm out of position. From \(\tan(\epsilon_3)\) can be seen that this shell is a very accurate cylinder.

The calculated coefficients of the half-wave cosine and the half-wave sine double Fourier series are displayed in tables 1 and 2. For clearer representation any amplitude smaller than 0.0001 is replaced by zero. Figure 10 shows the variation (in mm) of the calculated half-wave cosine Fourier coefficients as a function of the circumferential wave number \(l\) for selected axial half-wave numbers \(k\), whereas figure 11 shows similar plots or the half-wave sine Fourier coefficients.

These two figures shows that the initial imperfections of this shell are dominated by low order modes, that is, the amplitudes of the Fourier coefficients calculated from the experimentally measured imperfections decay with increasing wave number \(l\) with distinct peaks at \(l=2\) and \(l=8\). The peak at \(l=2\) is
caused by the out of roundness of the shell and the peak at $l=8$ can be explained with the imperfection pattern seen at the top of figure 9 and can be seen as a result of the fabrication process.

The variation of the calculated half-wave cosine Fourier coefficients as a function of the axial half-wave number $k$ for selected circumferential wave number $l$ is shown in figure 12, whereas figure 13 displays similar plots for the half-wave sine Fourier representation. Here a rapid decay of the amplitude of the imperfections with increasing half-wave number $k$ can be observed.

The amplitude of the axisymmetric imperfections are in the same order as the asymmetric imperfections. Figure 14 show the variation of the half-wave cosine axisymmetric Fourier coefficients as a function of the axial half-wave number $i$ whereas figure 15 show a similar plot for the half-wave sine axisymmetric Fourier coefficients.

6. REFERENCES
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**Table 1** Fourier coefficients of the half-wave cosine representation

\[
A_{k\ell} = \cos \frac{k \pi x}{L} \cos \frac{\ell y}{R}
\]

\[
B_{k\ell} = \cos \frac{k \pi x}{L} \sin \frac{\ell y}{R}
\]
Table 2  Fourier coefficients of the half-wave sine representation

| $K$ | $L$ | $|C_{K}^{m}|$ | $|D_{K}^{m}|$ |
|-----|-----|----------------|----------------|
| 1   | -1   | 0.0473          | -0.0073        |
| 2   | -0.0079 | -0.0226          | -0.0226        |
| 3   | 0.0018 | 0.0046          | 0.0046         |
| 4   | 0.0153 | 0.0401          | 0.0401         |
| 5   | 0.0473 | -0.0073         | -0.0073        |
| 6   | 0.0226 | 0.0046          | 0.0046         |
| 7   | 0.0046 | 0.0046          | 0.0046         |
| 8   | 0.0046 | 0.0046          | 0.0046         |
| 9   | 0.0046 | 0.0046          | 0.0046         |
| 10  | 0.0046 | 0.0046          | 0.0046         |
| 11  | 0.0046 | 0.0046          | 0.0046         |
| 12  | 0.0046 | 0.0046          | 0.0046         |
| 13  | 0.0046 | 0.0046          | 0.0046         |
| 14  | 0.0046 | 0.0046          | 0.0046         |
| 15  | 0.0046 | 0.0046          | 0.0046         |
| 16  | 0.0046 | 0.0046          | 0.0046         |

Note: The table entries for $|C_{K}^{m}|$ and $|D_{K}^{m}|$ are given in radians.
Fig. 1  Dimensions of the "Demonstrator Model"

**NOTE:**
Upper tank 1/5 ring including the inserts shall be provided in phase IV for test purposes if necessary.
Fig. 2  Sketch of the experimental setup
Fig. 3  Shape of the reference beam for different vertical positions
Fig. 4  Measured initial shape of the Demonstrator (result of formatting step 1)

Fig. 5  The initial shape of the Demonstrator without the imperfections of the lower tank interface (result of formatting step 2)
Fig. 6  The initial shape of the Demonstrator, the imperfections of the upper tank interface removed (result of formatting step 3)

Fig. 7  The initial shape of the Demonstrator after correction for the upper tank interface and the translation of the shell
**Solution Vector:**
- \( E(1) = X_i \)
- \( E(2) = Y_i \)
- \( E(3) = \cos(\varepsilon_1) \)
- \( E(4) = \cos(\varepsilon_2) \)
- \( E(5) = \tan(\varepsilon_3) \)
- \( E(6) = R \) at \( Z = 0 \)

**Fig. 8**  Best-fit cone and cylinder reference axis
Fig. 9  Initial shape of the Demonstrator with respect to the best-fit cylinder
Fig. 10  Circumferential variation of the half-wave cosine Fourier representation

Fig. 11  Circumferential variation of the half-wave sine Fourier representation
Fig. 12  Axial variation of the half-wave cosine Fourier representation

Fig. 13  Axial variation of the half-wave sine Fourier representation
Fig. 14  Axial variation of the half-wave cosine axisymmetric Fourier representation

Fig. 15  Axial variation of the half-wave sine axisymmetric Fourier representation