Computation of density currents in estuaries

Calibration for homogeneous flow in a tidal flume

Report on mathematical investigation

R 897 part V

August 1979
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NOTATION

$A_0, A_k$  Fourier coefficients
$a_0$  tidal amplitude at sea
$B_0, B_k$  Fourier coefficients
$b$  width
$C$  Chézy coefficient
$c$  concentration
$c_{\text{max}}$  concentration at sea
$\bar{c}$  depth averaged concentration
$c_g, c_n, c_r$  coefficients in the transition function
$D_x, D_z$  diffusivity in the $x$- and $z$-direction respectively
$D_{nx}$  numerical diffusivity in the $x$-direction
$E$  estuary number
$E_x$  truncation error
$F_{nx}$  numerical viscosity in the $x$-direction
$f(z)$  velocity distribution at the upstream boundary
$g$  gravitational acceleration
$g(t,z)$  transition function
$H$  depth
$k$  wave number
$L$  length
$L_F$  fictive length
$L_i$  intrusion length
$l_m$  mixing length
$M$  linearity coefficient
$N_t$  number of time steps
$N_x, N_z$  number of step sizes in the $x$- and $z$-direction respectively
$O$  order symbol of Landau
$P_t$  tidal prism
$p$  pressure
$Q$  discharge
$Q_r$  fresh water discharge
$Q_t$  tidal discharge
$R$  hydraulic radius
$t$  time
$T$  tidal period
$u$  velocity in the $x$-direction
$u_0$  maximal flood velocity
$u^*$  shear velocity
$w$  velocity in the $z$-direction
NOTATION (continued)

x  longitudinal direction
z  vertical direction
zb  position of the bottom
z0  coefficient for roughness length

H.W.S.  high water slack
L.W.S.  low water slack

M.E.V.  maximal ebb velocity
M.F.V.  maximal flood velocity

α  flood number
Δx, Δz  step size, in the x- and z-direction respectively
Δρ  density difference between river and sea water
ΔB0  variation of the mean level
ΔB1  variation of the amplitude of the vertical tide
Δφ1  variation of the phase of the vertical tide
ΔA1  variation of the amplitude of the tidal discharge
ΔΨ1  variation of the phase of the tidal discharge
Δφ  variation of the phase difference between upstream and downstream boundary condition
εx, εz  turbulent diffusion coefficient in the x- and z-direction respectively
κ  von Karman coefficient
ρ  density
τ  time step
τv  time step for momentum equation
τd  time step for diffusion equation
ζ  position of the free surface
ζ0  position of the free surface at x = 0.0 m
ω  frequency
COMPUTATION OF DENSITY CURRENTS IN ESTUARIES

Calibration for homogeneous flow in a tidal flume

1 Introduction

The present Report is the second of a series of three in which the calibration of the vertical two-dimensional density currents model for tidal flume conditions is presented.

In this Report the calibration for homogeneous conditions is presented, while the first Report dealt with the numerical accuracy and the third Report will deal with the calibration for inhomogeneous conditions [3,6].

In the present Report first a review of the measurements is given. Next the influence of several physical parameters is tested, and the results of these tests are used in the calibration with tidal flume data, which is also presented. Finally, conclusions are drawn about the model for homogeneous tidal flume circumstances.

This Report has been drawn up by Mr. M. Karelse (Chapter 2) and Mr. P.A.J. Perrels. It is the result of a study which is incorporated in a basic research programme T.O.W. (working group "Stromen en transportverschijnselen") executed by Rijkswaterstaat (Public Works and Water Control Department), the Delft Hydraulics Laboratory and other research institutes.
2 Review of the measurements

2.1 Description of the flume

The lucite flume used for the experiments has a rectangular cross-section 0.67 m wide and 0.50 m high. Two straight sections and the bend between them have a total length of 101.5 m (Figure 1). Downstream the flume ends in a sea basin 8 m long, 6 m wide, and a bottom 1.1 m below the bottom of the flume. By means of a control valve any periodic tidal movement of the water level can be generated. In the test used for calibration of the two-dimensional model rhodamine WT was used as tracer. The rhodamine-concentration of the sea water was kept constant by means of a circulation system which pumps water with the desired concentration into the basin through perforated tubes on the bottom. At the upstream end of the flume there is equipment to supply separately a constant and a variable discharge of fresh water (with rhodamine concentration c = 0). This makes it possible to simulate a fictive flume which has a greater length than the present flume (see Figure 1). The variable discharge of fresh water is programmed according to one-dimensional tidal computations for flumes with lengths larger than 101.5 m. For a detailed description of the flume, see Van Rees et al [7] and Rigter [8].

2.2 Test used for calibration

Several tests with plates (2 x 2 cm) on the bottom of the flume arranged in a diagonal pattern to obtain the desired roughness are available (see report M 896 - 38A) [4]. One of these tests was used for the calibration of the two-dimensional numerical model in homogeneous tidal circumstances, and the boundary condition and flow parameters of this test are given in the following table:
<table>
<thead>
<tr>
<th>quality</th>
<th>symbol</th>
<th>test T22</th>
</tr>
</thead>
<tbody>
<tr>
<td>depth (averaged over T)</td>
<td>H</td>
<td>0.216 m</td>
</tr>
<tr>
<td>physical length of the flume</td>
<td>L</td>
<td>100.65 m</td>
</tr>
<tr>
<td>fictive length of the flume</td>
<td>L_f</td>
<td>179.34 m</td>
</tr>
<tr>
<td>Chézy coefficient</td>
<td>C</td>
<td>19 m$^{1.5}$ s$^{-1}$</td>
</tr>
<tr>
<td>tidal period</td>
<td>T</td>
<td>558.75 s</td>
</tr>
<tr>
<td>tidal amplitude at sea</td>
<td>a_0</td>
<td>0.025 m</td>
</tr>
<tr>
<td>fresh water discharge</td>
<td>Q_r</td>
<td>0.0029 m$^3$s$^{-1}$</td>
</tr>
<tr>
<td>density differences between river and sea water</td>
<td>Δρ</td>
<td>0 kg m$^{-3}$</td>
</tr>
<tr>
<td>Rhodamine concentration sea water</td>
<td>c</td>
<td>0.9 × 10$^{-5}$ kg m$^{-3}$</td>
</tr>
</tbody>
</table>

Table 1  Boundary conditions and flow parameters

The tidal motion in this test can be characterised by the following values of the estuary parameters:

the flood number $\alpha = \frac{Q_r T_r}{P_t} = 0.36$

the estuary number $E = \frac{u_0^2}{g H} \frac{P_t}{Q_r T_r} = 0.034$

in which $P_t = $ tidal prism, the volume of sea water penetrating into the flume during the flood period (≈ 4.5 m$^3$)

$u_0 = $ maximal flood velocity

$g = $ gravitational acceleration

An inhomogeneous estuary with these values of the estuary parameters would be called a partly mixed estuary.

At 16 stations, at distances $n\Delta x$ (with $n$ going on from 1 to 16 and $\Delta x = 3.66$ m) from the mouth of the flume, the water level has been measured as a function of the time during a tidal period. In 8 stations at distances $\Delta x$, $3\Delta x$, $5\Delta x$, $7\Delta x$, $9\Delta x$, $11\Delta x$, $13\Delta x$ and $16\Delta x$ from the mouth of the flume velocities and concentrations have been measured. In each station:

- the velocities were measured at 12 positions in the vertical (with distances
$\Delta z = 1/13 \ H$ between each other.

- the rhodamine concentration was measured at 4 positions in the vertical at distances $2\Delta z$, $5\Delta z$, $8\Delta z$ and $11\Delta z$ from the bottom.

2.3 One-dimensional analysis of the test

In report M 896-38A it is shown that the tidal motion realized in the tidal flume differs from the tidal motion calculated with the one-dimensional model that was used to compute the upstream boundary condition (variable discharge at $x = L$; see Paragraph 2.1). The analysis of these differences is important for the comparison of flume tests and the two-dimensional model. The difference between measured and calculated tidal motion is caused by:

a) the limited accuracy of the measurements and of the adjustment of the flume. At each measured value a statistical deviation from this value ought to be added.

b) systematical differences between the numerical model and the tidal flume like:

- the different positions at which the sea boundary condition is fixed. In the tidal flume at a distance of 8 m from the mouth of the flume a vertical, sinusoidal tidal motion is generated, while in the numerical models this tidal motion is generated in the mouth of the flume.

- the amplitude and phase of the variable discharge at the upstream boundary (at $x = L$) differ somewhat from the desired values (following from a one-dimensional computation with a long flume, length $L_f$).

- the roughness for tidal flow in the flume is assumed to be the same as for permanent flow, but possibly the effective roughness in a tidal flow differs from that in a permanent flow.

As shown in report M 896 - 38A the (relative) accuracy of the measured water level was much larger than the accuracy of the discharges computed from the measured velocities, so the vertical tidal motion is used to compare the measured and the computed tidal motions. To study the influence of the systematical differences between the numerical models and the tidal flume a one-dimensional analysis has been used. The one-dimensional model (with vertical tidal variation at $x = 0$ and horizontal tidal variation at $x = L$) has been used to determine the sensitivity of the tidal motion for variations in the roughness and the boundary conditions (see report M 896 - 38A). The result of this study was
a best approximation of the homogeneous tidal flow in test T22 with a one-
dimensional model with modified boundary conditions and roughness.
Because the water level variation is periodic in time, it is obvious to use
harmonic analysis to analyse the water level variations in the flume:

$$
\zeta(x,t) = B_0(x) + \sum_{k=1}^{K} B_k \sin(k\omega t + \phi_k)
$$

The most important information from this analysis for comparison of measure-
ment and computation is:
the difference in the mean level $B_0(x) - B_0(1)$
the amplitude $B_1(x)$
the phase difference $\phi_1(x) - \phi_1(1)$

In the following table a comparison is given between test T22 and its one-di-
Mensional approximation:

<table>
<thead>
<tr>
<th>stat.</th>
<th>$\Delta B_0$ (mm)</th>
<th>$B_1$ (mm)</th>
<th>$\Delta \phi_1$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>test T22</td>
<td>comp.</td>
<td>test T22</td>
</tr>
<tr>
<td>1</td>
<td>1.98</td>
<td>1.84</td>
<td>23.21</td>
</tr>
<tr>
<td>7</td>
<td>3.26</td>
<td>2.74</td>
<td>19.99</td>
</tr>
<tr>
<td>16</td>
<td>4.00</td>
<td>3.57</td>
<td>21.50</td>
</tr>
</tbody>
</table>

Table 2 Comparison vertical tidal motion in T22 with that in computation B43

In the above one-dimensional computation there have been used:
- a phase shift of 10.8° in the upstream boundary condition for the variable
discharge
- a Chézy coefficient of 20.5 m$^1$s$^{-1}$
- an amplitude of the vertical tidal motion at $x = 0$ of 0.97 $a_0$
The accuracy of the measured values is:

$$
\sigma_{\Delta B_0} = 0.2 \text{ mm}
$$

$$
\sigma_{B_1} = 0.2 \text{ mm}
$$
\( \sigma_{\Delta \phi_1} = 2^\circ \) (from report M 896 - 38A)
This means that the comparison given in Table 2 shows a good agreement between the amplitude and the phase and a systematic deviation in the mean level \( B_0 \) which is considered of less importance.

2.4 Concentration distribution

As mentioned in Paragraph 2.2 at 4 points in the vertical in the stations \((2n+1) \Delta x\) (with \(n\) going on from 0 to 5 and \(\Delta x = 3.66\) m) the Rhodamine concentration has been measured by pumping water from the measuring point to the fluorimeter (travelling time \(\approx 40\) s).
The accuracy of the concentration measurement itself is in the order of \( \sigma_c / c \approx 0.02 \) (\( \sigma_c \): standard deviation in concentration \( c \)).

The variation of the concentration in the vertical direction during times when large concentration variations in time appear (concentration changing in about 30 s from the minimum value to the maximum value) are caused by the inaccuracy of the phase.

With the one-dimensional model, besides the tidal motion the concentration distribution for test T22 has been computed. A reasonable simulation of the measured distribution has been obtained using a longitudinal dispersion coefficient of \( D_x = 0.05 |u|^3 H \) (for more information see report M 896 - 38A).
3 Description of the mathematical model

After integration over the width and with the shallow-water approximation, the equations for vertical two-dimensional homogeneous currents read [1]:

\[
\frac{\partial u}{\partial t} + \frac{1}{b} \frac{\partial (bu^2)}{\partial x} + \frac{\partial (uw)}{\partial z} - \varepsilon \frac{\partial^2 u}{\partial x^2} - \frac{\partial}{\partial z} \left( \varepsilon \frac{\partial u}{\partial z} \right) = -g \frac{\partial c}{\partial x}
\]  
(3.1)

\[
\frac{\partial c}{\partial t} + \frac{1}{b} \frac{\partial}{\partial x} \{ b \int_{z_b}^{\zeta} w \, dz \} = 0 
\]  
(3.2)

\[
\frac{1}{b} \frac{\partial (bu)}{\partial x} + \frac{\partial w}{\partial z} = 0 
\]  
(3.3)

\[
\frac{\partial c}{\partial t} + \frac{1}{b} \frac{\partial (bcu)}{\partial x} + \frac{\partial (wc)}{\partial z} - \frac{1}{b} \frac{\partial}{\partial x} \left( b \frac{\partial c}{\partial x} \right) - \frac{\partial}{\partial z} \left( D \frac{\partial c}{\partial z} \right) = 0
\]  
(3.4)

For \( \varepsilon_z \) a mixing length approximation is employed, which reads:

\[
\varepsilon_z = K^2 \left( z + z_0 \right)^2 \left| \frac{\partial u}{\partial z} \right|
\]  
(3.5)

in which \( z_0 \) is a measure for the roughness length.

An extended derivation of these equations can be found in [1].

The boundary conditions are:

at the bottom, \( z = z_b \): \( u = 0 \) \hfill (3.6a)

\( w = 0 \) \hfill (3.6b)

\( D \frac{\partial c}{\partial x} \frac{\partial z_b}{\partial x} - D \frac{\partial c}{\partial z} = 0 \) \hfill (3.6c)

at the surface, \( z = \zeta \): \( \frac{\partial u}{\partial z} = 0 \) \hfill (3.7a)

\( D \frac{\partial c}{\partial x} \frac{\partial \zeta}{\partial x} - D \frac{\partial c}{\partial z} = 0 \) \hfill (3.7b)

at the upstream end, \( x = L \): \( u = f(z) \ Q(t) \) \hfill (3.8a)

\( c = 0 \) \hfill (3.8b)
at the downstream end, \( x = 0 \): \[ \frac{\partial^2 u}{\partial x^2} = 0 \quad (3.9a) \]

\[ c = c_{\text{max}} g(t, z), \text{ if } u > 0 \quad (3.9b) \]

\[ \frac{\partial^2 c}{\partial x^2} = 0, \text{ if } u < 0 \quad (3.9c) \]

\[ \zeta = \zeta_0(t) \quad (3.9d) \]
4 Sensitivity to variations of the physical parameters

4.1 Description of the computations

To find the influence of several physical parameters a test series was set up in which systematical variations were made in one parameter at a time, starting from a reference situation.

The data for this reference situation, which approximates the tidal flume circumstances, are:

- \( L = 96.98 \) m
- \( H = 0.216 \) m
- \( T = 558.75 \) s
- \( Q_r = 0.0029 \) m\(^3\)s\(^{-1}\)
- \( \tau_x = 0.37 \) m\(^2\)s\(^{-1}\)
- \( D_x = 2|\mathbf{u}^x|^b + 0.005 \) m\(^2\)s\(^{-1}\)
- \( N_x = 13 \)
- \( N_z = 12 \)
- \( N_e = 1200 \)
- \( \tau = 1.8625 \) s
- \( C = 22.3 \) m\(^3\)s\(^{-1}\) (in the first 63.41 m)
- \( = 24.0 \) m\(^3\)s\(^{-1}\) (in the last 33.57 m)

The boundary condition for \( \zeta \) at \( x = 0.0 \) m reads:

\[ \zeta(t,0) = 0.2160 + 0.02425 \cos (\omega t - 1.571) \]

The boundary condition for \( Q \) at \( x = L \) reads:

\[ Q(t,L) = -0.0029 + 0.01470 \cos (\omega t - 1.3525) + 0.00315 \cos (2\omega t - 3.1751) + 0.00173 \cos (3\omega t - 1.9864) + 0.00093 \cos (4\omega t - 4.0749) + 0.00010 \cos (5\omega t - 2.0980) + 0.00016 \cos (6\omega t - 3.6467) + 0.00006 \cos (7\omega t - 3.2067) + 0.00018 \cos (8\omega t - 4.4373) \]
In the numerical model a bottom roughness was introduced by setting:

\[ l_m = \kappa (z + z_0) \]  
(4.1)

in which \( z_0 \) is a measure for the roughness length. In a steady uniform flow \( z_0 \) is related to \( C \) by:

\[ C = 18 \log \left( \frac{12 R}{33 z_0} \right) \]  
(4.2)

in which \( R \) is the hydraulic radius. Because the model only represents bottom roughness, \( R \) was taken equal to \( H \).

In order to quantify the influence of the variations a norm was defined to which the influence can be related and which makes it possible to compare the influence of different variations.

The norm is defined as the order of the differences that occur, in tide and concentration, due to a variation of 5% in \( C \).

One of the variables investigated was the intrusion length, which was computed by linear extrapolation from the last two points where \( c > 0 \). The formula for the intrusion length reads:

\[ L_i = x_M + \frac{c_M}{(c_M - c_{M-1})} \Delta x \]  
(4.3)

in which \( x_M \) is the coordinate of the last point where \( c > 0 \) and \( c_M \) and \( c_{M-1} \) are the concentrations in \( x_M \) and \( x_{M-1} \) respectively.

4.2 Influence of the bottom roughness

4.2.1 Influence of the roughness length

To test the influence of the roughness length the following computations with different Chézy coefficient were made:

RB 27: \( C = 26.3 \) in the first \( 63.41 \) m  
\[ C = 24.0 \] in the last \( 33.57 \) m

RB 29: \( C = 24.0 \)

RB 16: \( C = 24.0 \) in the first \( 63.41 \) m  
\[ C = 22.3 \] in the last \( 33.57 \) m
RB 28: \( C = 22.3 \)
RB 26: \( C = 22.3 \) in the first 63.41 m
\( C = 20.3 \) in the last 33.57 m

If the roughness length decreases, the amplitude of the vertical tide increases, as can be seen in Figure 2, where the position of the free surface at \( x = L \) is drawn for the computations RB 16, RB 26 and RB 27. The variations in the other two computations were too small to be sketched. In addition to a change of the amplitude, a different roughness also causes a phase shift, as appears from the Fourier analysis of the tidal elevation at \( x = L \):

RB 27: \( \zeta(L) = 0.2211 + 0.0279 \cos (\omega t - 2.67) \)
RB 29: \( \zeta(L) = 0.2215 + 0.0267 \cos (\omega t - 2.80) \)
RB 16: \( \zeta(L) = 0.2216 + 0.0266 \cos (\omega t - 2.83) \)
RB 28: \( \zeta(L) = 0.2219 + 0.0257 \cos (\omega t - 2.95) \)
RB 26: \( \zeta(L) = 0.2221 + 0.0256 \cos (\omega t - 2.99) \)

The influence of the roughness on the velocities is shown in Figure 3, where \( u(t,0) \) has been drawn at M.F.V. and M.E.V. It can be concluded that a higher Chézy number results in higher velocities, especially near the bottom.
In Figure 4 the discharges at \( x = 0.0 \) m have been drawn. A smaller roughness yields higher discharges, as is confirmed by a Fourier analysis of the discharges:

RB 27: \( Q(0) = -0.0029 + 0.0297 \cos (\omega t + 0.56) \)
RB 29: \( Q(0) = -0.0029 + 0.0290 \cos (\omega t + 0.50) \)
RB 16: \( Q(0) = -0.0029 + 0.0290 \cos (\omega t + 0.50) \)
RB 28: \( Q(0) = -0.0029 + 0.0282 \cos (\omega t + 0.50) \)
RB 26: \( Q(0) = -0.0029 + 0.0282 \cos (\omega t + 0.45) \)

The influence of the roughness on the rhodamine concentration is directly correlated to the influence on the velocities.
This means that higher velocities during flood-tide cause higher concentrations and that higher velocities during ebb-tide cause a faster decrease of the concentration. These phenomena are presented in Figure 5.
The influence of the Chézy coefficient on the intrusion length is rather small, as appears clearly from the next list:
RB 27: $L_1 = 39.45$ m  
RB 29: $L_1 = 39.45$ m  
RB 16: $L_1 = 39.33$ m  
RB 28: $L_1 = 39.27$ m  
RB 26: $L_1 = 39.14$ m

Summarising the results of the computations for different $C$, it can be stated that a higher $C$ gives less damping, as can be expected, which causes larger amplitudes of the motion of the free surface and greater velocities, which in turn cause higher fluctuations in the concentrations. The intrusion length, however, increases very little.

### 4.2.2 Quantification of the norm

From the influence due to the variations in $C$ also the 5%-norm, introduced in Paragraph 4.1, can be extracted for tidal flume circumstances.

For the tidal elevation the norm is related to the differences at $x = L$, and for the discharges and the velocities to the differences at $x = 0$. These positions were selected because they are the farthest away from the position where the boundary condition for the tidal elevation and the discharge respectively are prescribed.

For the tidal elevation and the discharge the norm is given in Fourier components:

**Tidal elevation:**  
$\Delta B_0 = 0.0003$ m  
$\Delta B_1 = 0.0007$ m  
$\Delta \phi_1 = 0.093$ rad

**Discharge:**  
$\Delta A_1 = 0.0005$ m$^3$s$^{-1}$  
$\Delta \psi_1 = 0.031$ rad

For the concentrations the norm is related to the deviation at maximal intrusion in the relative depth averaged concentration at $x = 14.92$ m. This position was selected because it is far enough away from $x = 0$ m where the concentration is imposed during flood-tide and it is also far enough away from the point of maximal intrusion. The norm for the concentration reads:

$$\Delta \left( \frac{\bar{c}}{c_{\text{max}}} \right) = 0.015$$
Finally, the norm for the maximal intrusion length is given by:

\[ \Delta L_i = 0.1 \text{ m} \]

4.3 Influence of the upstream boundary conditions

Of the upstream boundary conditions three parameters have been varied, respectively the river discharge, the distribution of the velocity in the vertical direction and the phase-difference between the tidal discharge at the upstream boundary and the tidal elevation at the downstream boundary.

The influence of variations in the boundary condition for the diffusion equation has not been investigated, because the concentrations are supposed to be zero for the present calibrations.

4.3.1 The influence of the river discharge

To test the influence of the river discharge, the following variations have been computed:

- RB 31: \( Q_r = 0.0000 \text{ m}^3\text{s}^{-1} \)
- RB 46: \( Q_r = -0.0026 \text{ m}^3\text{s}^{-1} \)
- RB 16: \( Q_r = -0.0029 \text{ m}^3\text{s}^{-1} \)
- RB 47: \( Q_r = -0.0032 \text{ m}^3\text{s}^{-1} \)
- RB 30: \( Q_r = -0.0058 \text{ m}^3\text{s}^{-1} \)

Considering the harmonic analysis of the free surface elevation at \( x = L \):

- RB 31: \( \xi(L) = 0.2180 + 0.0271 \cos(\omega t - 2.81) \)
- RB 46: \( \xi(L) = 0.2212 + 0.0267 \cos(\omega t - 2.83) \)
- RB 16: \( \xi(L) = 0.2216 + 0.0266 \cos(\omega t - 2.83) \)
- RB 47: \( \xi(L) = 0.2219 + 0.0265 \cos(\omega t - 2.83) \)
- RB 30: \( \xi(L) = 0.2252 + 0.0257 \cos(\omega t - 2.88) \)

It can be seen that a variation of 10% in the river discharge yields a significant variation in the mean water level at \( x = L \), which is of the order of the norm given in Paragraph 4.2. The influence on the amplitude and on the phase of the tidal elevation is small. A variation of 100% yields differences of the order of the norm in the amplitude and the phase and a difference of 10 times the order of the norm in the mean-water level. Further it should be
noticed that even for $Q_r = 0.0$ there is an elevation of the mean water level, due to the non-linear behaviour of the tidal flume. Harmonic analysis of the discharge at $x = 0.0$ m yields:

RB 31: $Q(0) = 0.0000 + 0.0292 \cos (\omega t + 0.51)$
RB 46: $Q(0) = -0.0026 + 0.0290 \cos (\omega t + 0.50)$
RB 16: $Q(0) = -0.0029 + 0.0290 \cos (\omega t + 0.50)$
RB 47: $Q(0) = -0.0032 + 0.0290 \cos (\omega t + 0.50)$
RB 30: $Q(0) = -0.0058 + 0.0285 \cos (\omega t + 0.48)$

The same conclusion that holds for the free surface elevation can also be drawn for the discharges, namely, that a variation of 10% in the river discharge has little influence on phase and amplitude whereas the influence of a 100% variation is of the order of the norm.

It should also be noticed that the influence of the variations increases for larger river discharges. This can be explained by the increasing relative importance of the river discharge compared to the tidal discharge.

In Figures 6 and 7 the free surface elevation and the discharge are sketched respectively. Only the results of the 100% variation are shown, because the differences due to a 10% variation are too small to be sketched.

The influence of a 10% variation on the concentrations is of the order of the norm, while that on the intrusion length is about twice the order of the norm. The influence of a + 100% variation is about 10 times the order of the norm, considering the concentration and the intrusion. The influence of a - 100% variation or, in other words, reducing the river discharge to zero, yields finally a concentration equal to $c_{max}$ everywhere and an intrusion equal to the length of the flume. This behaviour can be seen also in Figure 8 which shows that in the case with $Q_r = 0$ the concentrations did not yet reach an equilibrium. The concentration at $t = 1.0$ is higher than the concentration at $t = 0.0$. Finally, the intrusion lengths are given:

RB 31: $L_i = 54.1$ m (after 4 tidal cycles)
RB 46: $L_i = 39.6$ m
RB 16: $L_i = 39.3$ m
RB 47: $L_i = 39.1$ m
RB 30: $L_i = 37.5$ m
4.3.2 The influence of the velocity distribution

The influence of the upstream velocity distribution on the free surface motion, on the rest of the velocity field and on the concentrations is negligible. From a comparison of the following computations:

RB 16: logarithmic velocity distribution
RB 32: uniform velocity distribution

it appears that the differences in the free surface elevation, the velocities, the concentrations and the intrusion length are at most one-tenth of the order of the norm.

4.3.3 The influence of the phase difference

To test the influence of the phase difference between free surface motion at the downstream boundary and the discharge at the upstream boundary, variations of $+ 0.1$ rad and $- 0.1$ rad have been investigated. If the reference case is indicated by $\Delta \phi = 0$, the following list can be given:

RB 33: $\Delta \phi = 0.1$ rad
RB 16: $\Delta \phi = 0.0$ rad
RB 34: $\Delta \phi = - 0.1$ rad

(variations were imposed at the upstream boundary)

In Figure 9 the influence of $\Delta \phi$ on the position of the free surface is sketched at $x = L$. Changing the phase yields a different amplitude, which is also expressed in the following list of the Fourier components.

RB 33: $\zeta(L) = 0.2217 + 0.0246 \cos (\omega t - 2.82)$
RB 16: $\zeta(L) = 0.2216 + 0.0266 \cos (\omega t - 2.83)$
RB 34: $\zeta(L) = 0.2216 + 0.0286 \cos (\omega t - 2.84)$

The influence on the tidal amplitude is large, about three times the norm given in Paragraph 4.2. The changes in the mean level and the phase are small compared to the norm.

The influence of $\phi$ on the discharge is similar, as is shown in Figure 10. The influence is again concentrated in a change of the amplitude, which is confirmed by a Fourier analysis of the discharges at $x = 0.0$ m:

RB 33: $Q(0) = - 0.0029 + 0.0278 \cos (\omega t + 0.47)$
RB 16: $Q(0) = - 0.0029 + 0.0290 \cos (\omega t + 0.50)$
RB 34: $Q(0) = - 0.0029 + 0.0301 \cos (\omega t + 0.53)$
The order of magnitude of the differences for the amplitude of the tidal discharge is about 2.5 times the norm and for the phase about the same size as the norm.

The concentration is influenced directly via the convective terms and weakly via the dispersion which is a function of the horizontal velocity. The differences in the concentration become the largest during flood-tide and disappear soon after H.W.S. As can be seen in Figure 11 the order of the influence on the concentration is about three times the order of the norm. Also the intrusion length is affected by a change of $\Delta \phi$. The order of the influence is also three times that of the norm, which is in accordance with the differences in the discharge and the concentration. The variations in the intrusion length are given in the next list:

- RB 33: $L_i = 39.00$ m
- RB 16: $L_i = 39.33$ m
- RB 34: $L_i = 39.56$ m

### 4.4 Influence of the downstream boundary conditions

The downstream boundary condition for the momentum equation:

$$\frac{\partial^2 u}{\partial x^2} = 0$$

is left unchanged. The reason for this was twofold:

- From comparisons with measured velocity profiles near the downstream boundary the present condition appeared to give very satisfactory results (see Paragraph 5.2: Calibration with tidal flume data).
- Even if these results had not been so satisfactory a better condition would not have been obvious.

The tidal amplitude is varied and in the boundary condition for the diffusion equation the form of the function, which describes the transition from the ebb-tide to flood-tide, is changed.

#### 4.4.1 The influence of the tidal amplitude

To test the influence of the tidal amplitude the following computations have been performed:
RB 35: $a_0 = 0.02550 \, \text{m}$
RB 16: $a_0 = 0.02425 \, \text{m}$
RB 36: $a_0 = 0.02300 \, \text{m}$

In Figure 12 the influence on the free surface elevation at $x = L$ is shown. It appears that the variations in amplitude are slightly damped, but further similar to the variations at $x = 0.0 \, \text{m}$. This phenomenon is also demonstrated by the Fourier components of the free surface elevation at $x = L$:

RB 35: $\zeta(L) = 0.2218 + 0.0275 \cos(\omega t - 2.80)$
RB 16: $\zeta(L) = 0.2216 + 0.0266 \cos(\omega t - 2.83)$
RB 36: $\zeta(L) = 0.2214 + 0.0257 \cos(\omega t - 2.86)$

Comparison with the norm shows that the differences in the mean level are somewhat smaller than the norm. The differences in the amplitude are slightly larger, and the order of the phase shift is about one-third of the order of the norm.

The influence on the tidal discharge is straightforward, which means that a larger tidal amplitude yields larger velocities and discharges. Figure 13 shows the variation in the discharges at $x = 0.0 \, \text{m}$. A Fourier analysis of the discharges was also made, the results of which are listed below:

RB 35: $Q(0) = -0.0029 + 0.0295 \cos(\omega t + 0.52)$
RB 16: $Q(0) = -0.0029 + 0.0290 \cos(\omega t + 0.50)$
RB 36: $Q(0) = -0.0029 + 0.0283 \cos(\omega t + 0.48)$

The order of the differences in the amplitude of the tidal discharge is 1.0 to 1.4 times the order of the norm, while the order of the phase shift is about two-thirds that of the norm.

The tidal amplitude influences the concentration mainly via the convective terms and weakly via the dispersion.

In accordance to the variations in the discharges the influence of the tidal amplitude on the concentrations appears to be that a larger tidal amplitude yields a more rapid increase of the concentrations and also a more rapid decrease, as is shown in Figure 14.

A larger tidal amplitude appears to give higher concentrations and also a slightly larger maximal intrusion, as is shown in the next list of intrusion lengths.
RB 35: \( L_i = 39.5 \) m
RB 16: \( L_i = 39.3 \) m
RB 36: \( L_i = 39.2 \) m

The order of the differences in the concentration is about the same as the order of the norm, and the variations in the intrusion length have a magnitude of 1.0 to 2.0 times that of the norm.

4.4.2 The influence of the transition function

For the description of the transition of the concentration at the seaward boundary from its value at L.W.S. to \( c_{\text{max}} \), a linear function is adopted of the form:

\[
c = c_r + c_g t \quad \text{if} \quad c < c_{\text{max}}
\]
\[
c = c_{\text{max}} \quad \text{if} \quad c > c_{\text{max}}
\]

\( c_r \) denotes the concentration at low water slack and \( c_g \) denotes the gradient with which the concentration rises during flood-tide. \( c_{\text{max}} \) is the concentration of the sea and therefore the maximal concentration that can occur. The coefficients \( c_r \) and \( c_g \) are both functions of \( z \), and (4.5) is applied at each gridpoint in the vertical.

It is assumed that this approach reflects two important features:
- the phase differences between the reversal of the tide at bottom and free surface
- the imperfect mixing of the fresh water at sea

To test the influence of (4.5) on the concentrations in the rest of the field three variations of \( c_g \) have been used in the computations:

RB 16: \( c_g = c_n \)
RB 23: \( c_g = 4 \cdot c_n \)
RB 37: \( c_g = c_n + 3 \left( \frac{H-z}{H} \right) c_n \)

in which \( c_n \) is the gradient that yields a maximum transition time of 0.4 T (for \( c_r = 0.0 \)). For the present circumstances this means: \( c_n = c_{\text{max}} / 0.4 \) T.

In Figure 15 the fluctuations of the depth-averaged concentrations are shown at \( x = 0.0 \) m. From this figure it becomes clear that both the increase and the decrease of the concentration at \( x = 0.0 \) m are steeper for higher values of \( c_n \). This influences, in addition to the transport, also the vertical and horizontal distribution of the rhodamine concentration and the intrusion length.
In Figure 16 and 17 the concentrations at \( x = 7.46 \text{ m} \) are drawn for M.F.V. and M.E.V. From these pictures it becomes clear that during flood-tide the concentrations near the free surface are higher than near the bottom, due to the higher velocities near the free surface. Further a higher value of \( c_g \) yields greater differences in the vertical concentration distribution. In Figure 16 also the influence of the linear variation of \( c_g \) with \( z \) can be seen. In that case the highest concentration occurs at \( z = 0.5 \text{ H} \). During ebb-tide the reverse situation occurs, which means that the concentrations near the free surface are lower than near the bottom.

From Figure 17 it becomes clear that for \( c_g = c_n \) the differences in the vertical concentration distribution are smaller than in the other two cases.

In the next table the horizontal density distribution is given at maximal intrusion.

<table>
<thead>
<tr>
<th>position</th>
<th>0.0 m</th>
<th>7.46 m</th>
<th>14.92 m</th>
<th>22.38 m</th>
<th>29.84 m</th>
<th>37.3 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>number</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RB 16</td>
<td>0.982</td>
<td>0.792</td>
<td>0.545</td>
<td>0.299</td>
<td>0.110</td>
<td>0.024</td>
</tr>
<tr>
<td>RB 23</td>
<td>1.000</td>
<td>1.000</td>
<td>0.983</td>
<td>0.681</td>
<td>0.317</td>
<td>0.080</td>
</tr>
<tr>
<td>RB 37</td>
<td>1.000</td>
<td>1.000</td>
<td>0.868</td>
<td>0.555</td>
<td>0.235</td>
<td>0.056</td>
</tr>
</tbody>
</table>

Table 3. Depth-averaged concentrations at maximal intrusion

From this table it becomes clear that a higher \( c_g \) yields a steeper concentration gradient in the \( x \)-direction.

The intrusion length itself, however, changes only very little, as appears from the following:

- RB 16: \( L_I = 39.3 \text{ m} \)
- RB 23: \( L_I = 39.8 \text{ m} \)
- RB 37: \( L_I = 39.6 \text{ m} \)

The order of the differences in the concentration is 20 to 30 times that of the norm, and the order of the differences in the intrusion length is 3 to 5 times that of the norm.
4.5 The influence of the horizontal exchange coefficients

In the derivation of the mathematical model [2] it appeared from an estimation of the order of magnitude, that the terms which describe the horizontal exchange of momentum and mass could be neglected. Nevertheless they were included for numerical reasons. During the verification of the numerical model it appeared, however, that the term describing the horizontal exchange of mass did have influence, and therefore the influence of the horizontal exchange terms was tested more carefully.

In the first place the influence of the coefficient of momentum exchange was tested with the following two computations:

RB 16: \( \varepsilon_x = 0.37 \text{ m}^2\text{s}^{-1} \)

RB 19: \( \varepsilon_x = 1.48 \text{ m}^2\text{s}^{-1} \)

From these it turns out that the influence of \( \varepsilon_x \) in this range is negligible, which makes the choice of a suitable \( \varepsilon_x \) to ensure stability much easier. It also means that the numerical viscosity will not have great influence, unless very large steps are used, which is confirmed by the results in [3].

Next, the influence of \( D_x \) has been investigated, and therefore an estimate of the order of magnitude of the diffusivity has been made. If an expression similar to the relation of Elder [7] is adopted:

\[
D_x = 2 |u^*| b, \quad (4.6)
\]

then the order of magnitude for tidal flume circumstances is given by:

\[
D_{x,\text{max}} \approx 0.06 \text{ m}^2\text{s}^{-1}
\]

On the basis of the expression for the numerical diffusivity [3], a discretisation was chosen, so that the numerical diffusivity is almost an order smaller than the physical diffusivity:

\[
D_{nx,\text{max}} \approx 0.008 \text{ m}^2\text{s}^{-1}
\]

Next the investigation was focussed on two questions:
- what is the influence of the order of magnitude of \( D_x \)
- what is the influence of the form of the relation adopted for \( D_x \)
To investigate the influence of the order of magnitude two computations have been compared:

RB 42: \( D_x = 0.03 \text{ m}^2\text{s}^{-1} \)
RB 44: \( D_x = 0.06 \text{ m}^2\text{s}^{-1} \)

In Figure 18 the influence of the order of the dispersion coefficient is shown on the depth averaged concentrations, respectively at \( x = 0.0 \text{ m} \) and at \( x = 14.92 \text{ m} \). The smallest dispersion coefficient yields the steepest increase and decrease of the concentrations, which can be understood from the view point that a smaller \( D_x \) yields less diffusion. The same influence is noticed when considering the horizontal depth-averaged concentration distribution (see Table 4) and the intrusion length. A smaller \( D_x \) yields a steeper decrease of the rhodamine concentration at the upstream side and a smaller intrusion length. The order of the differences in the concentration is about the same as the norm, but the order of the differences in the intrusion length, however, is 13 times that of the norm.

For the investigation of the form of the relation adopted for \( D_x \) three computations were compared:

RB 42: \( D_x = 0.03 \)
RB 43: \( D_x = 2|u^x|b + 0.005 \)
RB 45: \( D_x = 2|\bar{u}^x|b + 0.005 \)

Averaged over a tidal cycle the order of \( D_x \) is the same for all three computations. In RB 42, however, \( D_x \) is constant, while in RB 43 and RB 45 it varies with the shear velocity. In RB 43, \( D_x \) depends on the local shear velocity, and so \( D_x \) depends on z. In RB 45, \( D_x \) depends on a shear velocity based on the depth averaged velocity.

In Figure 19 the influence on the depth averaged concentrations is shown. Only the results of RB 42 and RB 43 are sketched, because the differences between the results of RB 43 and RB 45 are too small to be sketched.

From Figure 19 it appears that the influence of the form of the dispersion relation is rather small, compared to the influence of the order shown in Figure 18.

The order of the differences in the concentration is about 0.1 times that of the norm, while the differences in the intrusion length are about the order
of the norm (see Table 4). The differences between the results of RB 43 and RB 45 are negligible, compared to the order of the norm.

<table>
<thead>
<tr>
<th>position number</th>
<th>0.0 m</th>
<th>7.46 m</th>
<th>14.92 m</th>
<th>22.38 m</th>
<th>29.84 m</th>
<th>37.30 m</th>
<th>L (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RB 42</td>
<td>0.999</td>
<td>0.999</td>
<td>0.996</td>
<td>0.858</td>
<td>0.305</td>
<td>0.021</td>
<td>37.8</td>
</tr>
<tr>
<td>RB 43</td>
<td>0.999</td>
<td>0.999</td>
<td>0.993</td>
<td>0.842</td>
<td>0.307</td>
<td>0.024</td>
<td>37.9</td>
</tr>
<tr>
<td>RB 44</td>
<td>0.999</td>
<td>0.999</td>
<td>0.985</td>
<td>0.829</td>
<td>0.394</td>
<td>0.076</td>
<td>39.1</td>
</tr>
<tr>
<td>RB 45</td>
<td>0.999</td>
<td>0.999</td>
<td>0.994</td>
<td>0.846</td>
<td>0.307</td>
<td>0.024</td>
<td>37.9</td>
</tr>
</tbody>
</table>

Table 4  Horizontal concentration distribution by maximal intrusion and the intrusion length

4.6 The influence of the vertical exchange coefficients

To investigate the influence of the vertical exchange coefficients, the coefficients for the exchange of momentum and mass have been varied separately. Firstly, the exchange coefficient for momentum was varied:

\[
R 9: \xi_z = 1 \frac{l^2}{m} \left| \frac{\partial u}{\partial z} \right|
\]

\[
R 19: \xi_z = 1.1 \frac{l^2}{m} \left| \frac{\partial u}{\partial z} \right|
\]

A harmonic analysis of the free surface elevation at \( x = L \) and the discharge at \( x = 0.0 \) m yields:

\[
R 9: \zeta(L) = 0.2220 + 0.0272 \cos (\omega t - 2.916)
\]

\[
Q(0) = -0.0029 + 0.0290 \cos (\omega t + 0.471)
\]

\[
R 19: \zeta(L) = 0.2220 + 0.0270 \cos (\omega t - 2.944)
\]

\[
Q(0) = -0.0029 + 0.0288 \cos (\omega t + 0.463)
\]

It turns out that enlarging the vertical exchange coefficient for momentum results in a reduction of the tidal amplitudes of the free surface elevation at \( x = L \) and of the discharge at \( x = 0.0 \) m.

Further, a large \( \xi_z \) yields steeper velocity profiles in the vertical, that is higher velocities near the bottom and smaller velocities near the surface. The differences near the bottom are of the same order as those near the sur-
face, but of opposite sign. The maximal differences are about 1% of the velocity at the surface.
Compared to the norm, the differences in the free surface elevation are less than one-third of the order of the norm, while the differences in the discharge are about two-fifth of the order of the norm. The order of the differences in the concentration is one-fifth of the order of the norm. The variation in the intrusion length is about that of the norm.

To test the influence of the exchange coefficient form mass the following computations were made:

\[ R \, 9: \, D_z = 1.2 \left( \frac{\partial u}{\partial z} \right) \]
\[ R \, 11: \, D_z = 1.1 \left( \frac{\partial u}{\partial z} \right) \]

Of course, a variation in \( D_z \) has no influence on the tidal phenomena. The influence on the concentrations is an order smaller than the influence of \( \varepsilon_z \). The main difference occurs in the vertical concentration distribution. As can be expected, a larger \( D_z \) yields smaller differences between the concentrations at the bottom and at the free surface. An increase of 10% of \( D_z \) yields a reduction of 8.5% of the maximal difference between the concentrations at bottom and surface. The influence on the intrusion length is negligible.

\[ R \, 9: \, L_i = 39.4 \, m \]
\[ R \, 10: \, L_i = 39.3 \, m \]
\[ R \, 11: \, L_i = 39.4 \, m \]

4.7 Summary and conclusions of the test of the physical sensitivity

In the preceding paragraphs the physical sensitivity of the model has been investigated, and ten parameters have been varied. A review of the variations and their influence is given in Table 5.
Summarising the main results of this investigation:
- The tidal motion is mainly influenced by the upstream and downstream boundary conditions and the bottom roughness.
- The concentration is influenced by the tidal motion, but also strongly influenced by the transition function and slightly by the horizontal exchange coefficient for mass.
- The intrusion length is strongly influenced by the tidal motion, the transi-
tion function and the order of the horizontal exchange coefficient for mass.

From the investigation the following conclusions may be drawn:
- The influence of the horizontal exchange coefficient for mass, $D_x$, is significant in accordance with the intrusion length. Therefore a proper formulation has to be found, based on lateral averaging. This formulation must be tested under several circumstances.
- The influence of $e_z$ and $D_z$ is mainly restricted to the vertical velocity respectively concentration distribution. Variation of $e_z$ only makes sense if the bottom roughness is exactly known. Variation of $D_z$ only makes sense if systematically significant deviations occur in the vertical concentration distribution.
- Variation of the parameters $f(z)$ and $e_x$ is not useful, because their influence is negligible.

The results of this sensitivity analysis will be used by the calibration of the tidal flume data. Except a modification of $z_0$, $\Delta \phi$ and $B_0$ this will particularly be a calibration of the relations for $c_e$ and $D_x$. 
5 Discussion of the tests

5.1 Description of the computations

In the first part of this chapter the results of two-dimensional computations will be compared with results of one-dimensional computations. The data for the two-dimensional computations are taken from the one-dimensional computations [4]. These data read:

\[ L = 100.65 \text{ m} \]
\[ H = 0.216 \text{ m} \]
\[ T = 558.75 \text{ s} \]

C is varied per computation

\[ Q_r = 0.0029 \text{ m}^3\text{s}^{-1} \]
\[ \varepsilon_x = 0.37 \text{ m}^2\text{s}^{-1} \]
\[ D_x = 2 \left| u^* \right| b + 0.005 \text{ m}^2\text{s}^{-1} \]
\[ N_x = 11 \]
\[ N_z = 24 \]
\[ N_L = 500 \]
\[ \tau = 4.47 \text{ s} \]

The boundary condition for \( \zeta \) at \( x = 0.0 \text{ m} \) reads:

\[ \zeta(t,0) = 0.216 + 0.025 \cos(\omega t - 1.571) \]

The boundary condition for \( Q \) at \( x = L \) reads:

\[ Q(t,L) = -0.0029 + 0.01470 \cos(\omega t - 1.5408) \]
\[ + 0.00315 \cos(2\omega t - 3.5516) \]
\[ + 0.00173 \cos(3\omega t - 2.5546) \]
\[ + 0.00093 \cos(4\omega t + 1.4495) \]
\[ + 0.00010 \cos(5\omega t - 3.0106) \]
\[ + 0.00016 \cos(6\omega t + 1.5123) \]
\[ + 0.00006 \cos(7\omega t - 4.5887) \]
\[ + 0.00018 \cos(8\omega t + 0.3257) \]

For the verification with tidal flume data the following data were used in the two-dimensional computation:
\[ L = 100.65 \text{ m} \]
\[ H = 0.216 \text{ m} \]
\[ T = 558.75 \text{ s} \]
\[ Q_r = 0.0029 \text{ m}^3\text{s}^{-1} \]
\[ \varepsilon_x = 0.37 \text{ m}^2\text{s}^{-1} \]
\[ D_x = 2 \left| u_x^* \right| b + 0.005 \text{ m}^2\text{s}^{-1} \]
\[ N_x = 55 \]
\[ N_z = 24 \]
\[ N_t = 2400 \]
\[ \tau_v = 0.93125 \text{ s} \]
\[ \tau_d = 0.2328125 \text{ s} \]
\[ C = 21.6 \text{ m}^\frac{1}{s} \text{ in the first 61.30 m} \]
\[ 19.6 \text{ m}^\frac{1}{s} \text{ in the last 39.35 m} \]

The boundary condition for \( \zeta \) at \( x = 0.0 \text{ m} \) reads:

\[ \zeta(t,0) = 0.216 + 0.02425 \cos(\omega t) \]

The transition function for \( c \) at \( x = 0.0 \text{ m} \) reads:

\[ c = c_r + 8c_n t \text{ if } c < c_{\text{max}} \]
\[ c = c_{\text{max}} \text{ if } c > c_{\text{max}} \]

The boundary condition for \( Q \) at \( x = L \) reads:

\[ Q(t,L) = -0.0029 + 0.01470 \cos(\omega t - 1.4525) \]
\[ + 0.00315 \cos(2\omega t - 3.3751) \]
\[ + 0.00173 \cos(3\omega t - 2.2864) \]
\[ + 0.00093 \cos(4\omega t - 4.4749) \]
\[ + 0.00010 \cos(5\omega t - 2.5980) \]
\[ + 0.00016 \cos(6\omega t - 4.2467) \]
\[ + 0.00006 \cos(7\omega t - 3.9067) \]
\[ + 0.00018 \cos(8\omega t - 5.2373) \]
5.2 Comparison of one-dimensional and two-dimensional computations

A comparison between one-dimensional and two-dimensional computations has been made to investigate if the one-dimensional model can provide an accurate approximation for certain boundary conditions of the two-dimensional model. These boundary conditions are:
- the free surface elevation at $x = 0.0 \text{ m}$
- the discharge at $x = L$
- the bottom roughness

If the result of this comparison is favourable it would be possible to make a one-dimensional computation of a network, for example the Northern Delta estuary and then make a two-dimensional computation of part of the network.

In that case the one-dimensional model would yield boundary conditions for the two-dimensional model and the two-dimensional model would yield detailed information on part of the network.

For the comparison a series of 4 one-dimensional and two-dimensional computations has been made.

The first computations were made for a uniform value of $C$ throughout the flume in the one-dimensional computation and a uniform roughness length, based on $C = 19$ and $R = 0.216 \text{ m}$ (see formula 4.2), in the two-dimensional computation.

Comparison of the Fourier analyses of the discharge at $x = 0$ and the motion of the free surface at $x = L$ yields:

I: one-dimensional: $Q(t,0) = -0.0029 + 0.0266 \cos(\omega t - 1.163)$
   $\zeta(t,L) = 0.2218 + 0.0220 \cos(\omega t - 2.936)$

   two-dimensional: $Q(t,0) = -0.0029 + 0.0270 \cos(\omega t - 1.213)$
   $\zeta(t,L) = 0.2218 + 0.0234 \cos(\omega t - 3.013)$

which shows large differences, especially in the amplitude of the free surface elevation (about twice the norm) and in the phase of the discharges (about 2.5 times the norm). Herefore the following reasons can be given:
- The numerical accuracy.

The one-dimensional computations were made with a five times smaller step size $\Delta x$ than the two-dimensional computations. Refining the two-dimensional
computations would give a correction of 0.5 times the order of the norm.
- In the one-dimensional model the convective terms have been omitted, which yields, however, only small deviations.
- In the one-dimensional model the variation of the bottom roughness with the water depth has been omitted. This yields important differences. Future computations, with an improved one-dimensional model, will show exactly how important these differences are.

For the present comparison this difference will further be ignored and attention will be focussed on the reactions of both models on variations of the boundary conditions.

The second computation was made with the same roughness, but now an additional phase shift was imposed between the up- and downstream boundary conditions $\zeta(t,0)$ and $Q(t,L)$. The size of the phase shift was 0.25 rad.

Comparison of the harmonic analysis, of the discharges at $x = 0$ and of the free surface elevation at $x = L$, with the results of I shows:

\[
\begin{align*}
\text{II: one-dimensional: } Q(t,0) &= -0.0029 + 0.0296 \cos (\omega t - 1.094) \\
\zeta(t,L) &= 0.2216 + 0.0272 \cos (\omega t - 2.995) \\
\text{two-dimensional: } Q(t,0) &= -0.0029 + 0.0299 \cos (\omega t - 1.143) \\
\zeta(t,L) &= 0.2216 + 0.0288 \cos (\omega t - 3.049)
\end{align*}
\]

Thus the differences caused by the variation in the boundary condition largely agree. The amplitude of the discharge increases about 11 per cent and there is a phase shift of about 0.07 rad for both the one-dimensional and the two-dimensional models. For the motion of the free surface there is a decrease in the average level of 0.1 per cent for the one-dimensional model as well as for the two-dimensional model. Further, there is an increase of the tidal amplitude of about 20 per cent and a phase shift of about 0.06 rad for the one-dimensional model and of 0.04 rad for the two-dimensional model.

For the third pair of the computations a variation in the roughness was made. In the first 62.22 m C was taken equal to 22.1 and in the last 38.43 m equal to 19.0, in the one-dimensional computations. Because of the larger step size in the two-dimensional model, the best approach of the one-dimensional computation was $C = 22.1$ in the first 59.47 m and $C = 19.0$ in the last 41.18 m. For this computation no additional phase shift between $\zeta(t,0)$ and $Q(t,L)$ was in-
The results of the harmonic analysis of the discharge $Q(t,0)$ and the free surface elevation $\zeta(t,L)$ are:

III: one-dimensional: $Q(t,0) = -0.0029 + 0.0280 \cos (\omega t - 1.083)$
$\zeta(t,L) = 0.2213 + 0.0240 \cos (\omega t - 2.739)$

two-dimensional: $Q(t,0) = -0.0029 + 0.0283 \cos (\omega t - 1.128)$
$\zeta(t,L) = 0.2213 + 0.0253 \cos (\omega t - 2.841)$

Comparison of these results with the results of I shows that the discharges at $x = 0.0$ m react similarly in the one-dimensional and two-dimensional model, with an increase of the tidal amplitude of about 5 per cent and a phase shift of about 0.08 rad. Considering the vertical tide, the mean water level decreases about 0.2 per cent and the amplitude increases about 10 percent for both computations. The phase shift in the one-dimensional model is about 0.2 rad and in the two-dimensional model 0.17 rad. Finally in the last pair of computations the two variations of the above have been combined.

The data for the one-dimensional computation are:

$C = 20.5 \text{ m$^3$s}^{-1}$ in the first 62.22 m
$C = 19.0 \text{ m$^3$s}^{-1}$ in the last 38.43 m

and an additional phase difference $\Delta\phi = 0.188$ rad between $\zeta(t,0)$ and $Q(t,L)$.

For the two-dimensional model this becomes:

$C = 20.5 \text{ m$^3$s}^{-1}$ in the first 59.47 m
$C = 19.0 \text{ m$^3$s}^{-1}$ in the last 41.18 m

and an additional phase difference $\Delta\phi = 0.188$ rad between $\zeta(t,0)$ and $Q(t,L)$.

A harmonic analysis of the discharge $Q(t,0)$ and the free surface elevation $\zeta(t,L)$ yields:

IV: one-dimensional: $Q(t,0) = -0.0029 + 0.0293 \cos (\omega t - 1.074)$
$\zeta(t,L) = 0.2213 + 0.0262 \cos (\omega t - 2.900)$

two-dimensional: $Q(t,0) = -0.0029 + 0.0296 \cos (\omega t - 1.127)$
$\zeta(t,L) = 0.2211 + 0.0276 \cos (\omega t - 2.971)$
Comparison of the results of I and IV yields a good agreement in the variation of the discharges and in the variation of the amplitude of the free surface elevation. Summarising the results if this comparison of two-dimensional computations with one-dimensional results, the conclusion must be drawn that qualitatively they react similarly. Also the reactions on the variations largely agree. Quantitatively they show, however, significant differences. Until an explanation of these differences is found, they exclude the possibility that the one-dimensional model will provide accurate boundary conditions for the two-dimensional model. This can also be concluded from a comparison of the best one-dimensional approximation [4] and the best two-dimensional approximation (see Paragraph 5.3) of tidal flume data.

5.3 Calibration with tidal flume data

For the calibration with tidal flume data special measurements have been made in the Delft tidal flume, which are described in Chapter 2 of this report. The data for the numerical computation are given in part 5.1 of this chapter. The comparison between measurements and numerical results is firstly directed to the motion of the free surface. This is because the accuracy, with which the free surface elevation is measured is better than the measurement of velocities and discharges. Harmonic analysis of the measured free surface elevation yields the following results:

<table>
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<tr>
<th>position (m)</th>
<th>measurements</th>
<th>computations</th>
</tr>
</thead>
<tbody>
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<td>$B_0$ (m)</td>
<td>$B_1$ (m)</td>
</tr>
<tr>
<td>3.66</td>
<td>0.2158</td>
<td>0.0232</td>
</tr>
<tr>
<td>10.98</td>
<td>0.2168</td>
<td>0.0220</td>
</tr>
<tr>
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<td>0.0203</td>
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<td>32.94</td>
<td>0.2183</td>
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</tr>
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<td>40.26</td>
<td>0.2191</td>
<td>0.0200</td>
</tr>
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<td>47.58</td>
<td>0.2194</td>
<td>0.0204</td>
</tr>
<tr>
<td>58.56</td>
<td>0.2199</td>
<td>0.0215</td>
</tr>
</tbody>
</table>
From a comparison of the results of the harmonic analysis it can be seen that the mean level and its slope agree well. Also the amplitude agrees very well and so does the phase. In terms of the norm, defined in Paragraph 4.1 and quantified in Paragraph 4.2, the maximal difference in the mean level is about 0.7 times the norm, the maximal difference in the amplitude is about 0.15 times the norm and the maximal difference in phase is 0.25 times the norm. Considering the uncertainties in the measurements (see Chapter 2) this is a good agreement of computations with measurements. The behaviour that can be seen from the harmonic analysis is also shown by the Figures 20 and 22, where $\zeta(10.98)$ and $\zeta(47.58)$ have been drawn respectively. In the Figures 21 and 23 the discharges have been drawn at $x = 10.98$ m and $x = 47.58$ m respectively.

The amplitude of the measured tidal discharge is systematically larger than the amplitude of the computed tidal discharge. This can be explained by the fact that the flow in the flume is not exactly two-dimensional. There is a small influence of the wall, which causes higher velocities in the middle of the flume than near the wall. So the discharge measured in the middle of the flume will be higher than the laterally averaged discharge computed with the numerical model. This behaviour is confirmed by a harmonic analysis of the discharge at $x = 10.98$ m and $x = 47.58$ m. For the measurements the result of the harmonic analysis is:

\[
\begin{align*}
Q(10.98) &= -0.0033 + 0.0290 \cos (\omega t + 0.376) \\
&\quad + 0.0038 \cos (2\omega t - 0.159) \\
Q(47.58) &= -0.0031 + 0.0242 \cos (\omega t + 0.200) \\
&\quad + 0.0040 \cos (2\omega t - 0.479)
\end{align*}
\]

For the computations the result is:

\[
\begin{align*}
Q(10.98) &= -0.0029 + 0.0278 \cos (\omega t + 0.356) \\
&\quad + 0.0036 \cos (2\omega t + 0.687) \\
Q(47.58) &= -0.0029 + 0.0237 \cos (\omega t + 0.213) \\
&\quad + 0.0040 \cos (2\omega t + 0.899)
\end{align*}
\]

The harmonic analysis also shows the inaccuracy of the measured discharges, which hampers the comparison of measured and computed velocities and discharges. With the result of the harmonic analysis also the second harmonic is given, not to support the accuracy but merely to show the importance of the
second harmonic, compared to the first, which is an indication of the importance of the convective terms.
In Figures 24 to 31 the velocity profiles at $x = 10.98$ m and at $x = 47.98$ m are drawn at four characteristic times: M.F.V., H.W.S., M.E.V. and L.W.S. For the measurements the confidence interval is also presented. As confidence interval is taken: $2 \sigma_u$ (standard deviation in $u$). At slack-tide the same confidence interval is used as at M.E.V. In fact around slack-tide a wider interval should be used. Considering the confidence interval there is a good agreement between computations and experiments.
A third subject to verify was the Rhodamine concentration. In Figures 32 to 35 the concentration distributions at $x = 3.66$ m are drawn at four characteristic times: just after L.W.S., at M.F.V., at H.W.S. and at M.E.V. It appears from Figures 32 and 35 that the largest differences occur during the increase and decrease of the Rhodamine concentration. These differences become even more clear from Figure 37, where the depth-averaged concentrations have been drawn. It appears that the computed concentration lags behind the measured concentration during the increase of the concentration. The time-lag is about 0.02 $T$, which is an order larger than the phase shift that appeared in the harmonic analysis of the discharges, so the latter cannot explain these differences. Another difference appears in the vertical concentration distribution. Figures 32, 35 and 36 show for the computations a higher concentration near the surface during flood-tide, due to the higher velocities near the surface, and higher concentrations near the bottom during ebb tide, also due to the higher velocities near the surface. In Figure 32 this behaviour cannot be seen from the measurements, but in Figure 35 there seems to be a similar behaviour in the measurements. Considering, however, the accuracy of the rhodamine measurements, which is about 0.02 $c_{\text{max}}$, the differences in the measured concentrations in Figure 35 are hardly significant. In Figure 36, which shows that concentration distribution at $x = 18.3$ m just after H.W.S., the measured concentrations show a significantly larger difference in the vertical than the computed concentrations. This difference can be explained only for a minor part from a density difference, due to temperature differences, which were registered during the measurements. Density differences could also be an explanation for the differences shown in Figure 37. A density difference which causes vertical uniform concentration profiles will also change the course of the depth-averaged rhodamine concentration. Figure 37 shows that the start of the computed course is more gradual than the measured one. A density differ-
ence would alter the computed course and make the start less gradual. A third significant difference that appears from Figure 37 is the decrease of \( \bar{c} \) for increasing \( x \). This decrease is more gradual for the computations than for the measurements. The difference between the maximal measured and the maximal computed concentration at \( x = 25.62 \, \text{m} \) can be explained from the difference at \( x = 3.66 \, \text{m} \), in fact, too few rhodamine enters the flume. Which influence a steeper increase of the concentration at \( x = 3.66 \, \text{m} \) would have on the concentration at \( x = 32.94 \, \text{m} \) is difficult to predict. The results of the variations in the transition function indicate, however, that a faster increase of the concentration at the downstream boundary yields a steeper concentration gradient in the \( x \)-direction, which might improve the calibration results at \( x = 32.94 \, \text{m} \) too.
6 Conclusions

In the present report, as a complement to [3] in which the numerical accuracy was investigated, an investigation has been made of the sensitivity of the physical parameters. This investigation proved to be very useful for the calibration with tidal flume data. Considering the tidal motion, the calibration shows very good agreement. The differences are of the same order as the uncertainty in the measurements, while the errors due to numerical discretisation have been made one order smaller based on the results of [3]. This implies that for the present measurements no systematic improvement of the numerical results can be attained. So the conclusion may be drawn that the model represents the tidal motion, for homogeneous flume conditions, well. Apparently the downstream boundary condition:

$$\frac{\partial^2 u}{\partial x^2} = 0 \text{ at } x = 0$$

works well, both physically and numerically. Considering the rhodamine concentrations the agreement shows some flaws. In the first place a proper expression for the dispersion due to lateral averaging is needed. The sensitivity investigation showed the significant influence and the horizontal rhodamine distribution shows differences in the calibration with tidal flume data. A second point is the form of the transition function, which needs further investigation. The downstream boundary condition:

$$\frac{\partial^2 c}{\partial x^2} = 0 \text{ at } x = 0 \text{ and if } u < 0$$

also proved to work well.

In addition, a comparison of two-dimensional computations with one-dimensional computations has been made, but this comparison shows systematic differences and needs continuation if an improved one-dimensional model is available. Finally, the sensitivity test would be more complete if the results could be compared with corresponding flume data.

Summarising the results of the homogeneous test, they offer a sound base for the next inhomogeneous investigations.
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* all deviations are expressed in terms of the norm, defined in Paragraph 4.1 and quantified in Paragraph 4.2

Table 5  Review of the variations and their influence
INFLUENCE OF THE ROUGHNESS LENGTH
POSITION OF THE FREE SURFACE AT $x = L$

DELFt HYDRAULICS LABORATORY

R 897 - V FIG. 2
INFLUENCE OF THE ROUGHNESS LENGTH
DISCHARGES AT x=0.0m

DELFt HYDRAULICS LABORATORY

R 897-V FIG. 4
INFLUENCE OF THE BOTTOM ROUGHNESS
RELATIVE DEPTH AVERAGED
CONCENTRATIONS AT $x = 14.92 \, \text{m}$

DELFt HYDRAULICS LABORATORY  R 897-5  FIG. 5
INFLUENCE OF THE RIVER DISCHARGE $Q_r$
POSITION OF THE FREE SURFACE AT $x = L$

DELFT HYDRAULICS LABORATORY
INFLUENCE OF THE RIVER DISCHARGE $Q_r$
RELATIVE DEPTH AVERAGED
CONCENTRATIONS AT $x = 14.92\, m$

DELFt HYDRAULICS LABORATORY
INFLUENCE OF THE PHASE DIFFERENCE: $\Delta \phi$
DISCHARGES AT $x = 0.0$ m
INFLUENCE OF THE PHASE DIFFERENCE $\Delta \phi$
RELATIVE DEPTH AVERAGED CONCENTRATIONS
AT $x = 14.92$ m

DELTFT HYDRAULICS LABORATORY
R 897-V FIG. 11
INFLUENCE OF THE TIDAL AMPLITUDE $a_0$

POSITION OF THE FREE SURFACE AT $x=L$

DELFt HYDRAULICS LABORATORY
INFLUENCE OF THE TIDAL AMPLITUDE $a_0$
DISCHARGES AT $x=0.0\,\text{m}$

DELTJ HYDRAULICS LABORATORY

R 897-Ⅴ FIG. 13
INFLUENCE OF THE TIDAL AMPLITUDE: $a_0$
RELATIVE DEPTH AVERAGED CONCENTRATIONS
AT $x = 14.92$ m

DELFt HYDRAULICs LABORATORY
R 897 - V  FIG. 14
INFLUENCE OF THE TRANSITION FUNCTION
CONCENTRATION AT x = 7.46 m AT M.F.V.
DELFt HYDRAULICS LABORATORY

R 897-Ⅴ FIG. 16
INFLUENCE OF THE TRANSITION FUNCTION
CONCENTRATION AT x = 7.46 m AT M.E.V.

DELT HYDRAULICS LABORATORY

R 897 - V FIG. 17
INFLUENCE OF THE ORDER OF THE DISPERSION COEFFICIENT $D_x$
RELATIVE DEPTH AVERAGED CONCENTRATIONS
AT $x = 14.92$ m

DELFt HYDRAULICS LABORATORY
R 897-Ⅴ FIG. 18
CALIBRATION WITH TIDAL FLUME DATA
POSITION OF THE FREE SURFACE AT X = 10.98 m

DELFt HYDRAULICS LABORATORY

R 897 - V

FIG. 20
CALIBRATION WITH TIDAL FLUME DATA
DISCHARGES AT x = 10.98 m
DELFT HYDRAULICS LABORATORY
CALIBRATION WITH TIDAL FLUME DATA
POSITION OF THE FREE SURFACE AT $x = 47.58$ m

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R 897 - V FIG. 22
CALIBRATION WITH TIDAL FLUME DATA
DISCHARGES AT \( x = 47.58 \text{ m} \)

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R 897 - V FIG. 23
CALIBRATION WITH TIDAL FLUME DATA

VELOCITY $u$ AT $x = 10.98 \text{ m}$ AND $t = 0.16T$

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CALIBRATION WITH TIDAL FLUME DATA

VELOCITY $u$ AT $x = 10.98 \text{ m}$ AND $t = 0.4 \text{ T}$

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CALIBRATION WITH TIDAL FLUME DATA
VELOCITY $u$ AT $x = 10.98$ m AND $t = 0.64$ T

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FIG. 26
CALIBRATION WITH TIDAL FLUME DATA

VELOCITY $u$ AT $x = 10.98$ m AND $t = 0.96$ T

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CALIBRATION WITH TIDAL FLUME DATA

VELOCITY $u$ AT $x = 47.58 \text{ m}$ AND $t = 0.0 \text{ T}$

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VELOCITY $u$ AT $x = 47.58$ m AND $t = 0.16$ T

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VELOCITY $u$ AT $x = 47.58 \text{ m}$ AND $t = 0.44T$

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R 897-Ⅴ FIG. 30
CALIBRATION WITH TIDAL FLUME DATA
VELOCITY $u$ AT $x = 47.58\, m$ AND $t = 0.44\, T$

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CALIBRATION WITH TIDAL FLUME DATA

VELOCITY \( u \) AT \( x = 47.58 \text{ m} \) AND \( t = 0.64T \)

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CALIBRATION WITH TIDAL FLUME DATA
CONCENTRATION AT $x = 3.66m$ AND $t = 0.048T$

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R 897 - V FIG. 32
CALIBRATION WITH TIDAL FLUME DATA
CONCENTRATION AT \( x = 3.66 \text{ m} \) AND \( f = 0.288 \text{T} \)

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CALIBRATION WITH TIDAL FLUME DATA
CONCENTRATION AT x = 3.66 m AND AT t = 0.768 T

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R 897 - V FIG. 35
CALIBRATION WITH TIDAL FLUME DATA
CONCENTRATION AT x = 18.3 m AND AT t = 0.568 T

DELT HYDRAULICS LABORATORY R 897-V FIG. 36
CALIBRATION WITH TIDAL FLUME DATA
RELATIVE DEPTH AVERAGED CONCENTRATIONS

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