Master Thesis Report

Response of a shelf water to a traveling atmospheric pressure disturbance

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"Er zou weinig van mij overblijven indien ik alles moest afstaan, wat ik aan anderen te danken heb."

"Little would be left of me if I had to give back everything I owe to others."

- Goethe
Preface

This report is written as a Master Thesis at the section for Fluid Mechanics, Department of Civil Engineering at Delft Technical University, The Netherlands.

My stay at the TUDelft has been a part of my study to obtain the Danish Master of Science in Engineering degree 'civilingenior, cand. polyt.'.

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Introduction

1.1 Problem description

Every harbour has its own unique set of natural frequencies depending on its geometry. If waves with frequencies close to one of these natural frequencies continue to enter from offshore for some time, the harbour will resonate. This phenomenon is called harbour resonance or seiching and the characteristic oscillations seiches. A more detailed description of seiches and long waves is given in Chapter 2.

The Port of Rotterdam consists of a number of harbours connected through long and narrow channels, see figure 1.1. This system of harbours and channels is very sensitive to the resonance of long waves. The construction of the storm surge barrier in the Nieuwe Waterweg at the entrance of the harbour, along with other major projects in the area, is likely to change this response significantly and to the worse [9].

![Map of Port of Rotterdam](image)

Figure 1.1. Major changes in the future layout of the Port of Rotterdam. From [9].

In recent years a lot of research has been concentrated on determining the effects of these changes more accurately. Little is known about the origin and statistics of the long waves, and often this has been of little or no interest to these studies. Instead, due to the lack of information, the seaward boundary condition, representing the incoming waves, is simply modeled by a harmonic wave. Predictions of seiches are presently made based on the known statistics and by assuming the
worst possible situation that seiching will occur every time there is a storm [13].

For statistical purposes, measurements of seiches from the location Rozenburgesluis (RBS) have been translated to a location outside the harbour and then back again into the harbour at locations where the additional effect of seiching is desired [9]. These statistics are needed to determine the total design load on dikes and other marine constructions.

Recent investigations have shown that also the operation of the storm surge barrier is vulnerable to the seiche motion in the harbour, and that information about seiche activity should be used when operating the barrier. Therefore more accurate statistics and long term predictions of the waves causing seiches are desired. To obtain this, the waves should be observed at a much earlier stage before they actually enter the harbour. The ideal approach would be to follow the waves by means of simulations from where they are generated to the harbour entrance. To be able to do so, the most important generating mechanisms must be known. Many mechanisms have been suggested, among others fluctuations in wind direction, wind velocity and atmospheric pressure. These suggestions have primarily been based on positive correlations between meteorological events and seiche activity [12]. However, there are still too many uncertainties for this information to be used in practice.

1.2 Aim of this study

The aim of this study is to investigate the influence of an atmospheric pressure disturbance on the response of a shelf water, and to find out if such a disturbance could be responsible for the generation of long waves in the North Sea, later causing seiches in the Port of Rotterdam.

1.3 Approach

To achieve this objective, the response of a shelf water to an atmospheric pressure disturbance was investigated. First analytically, then numerically. The approach followed in these two parts is given below.

Analytical study
The analytical study was desired to determine which parameters are of special importance to the problem in general. The applicability of the shallow water equations was therefore analyzed and the equations subsequently simplified to enable a simple one-dimensional analysis. The resulting equation was first analyzed without a pressure field, corresponding to free waves. Then two types of pressure fields were considered, viz. a harmonic, propagating pressure wave and a moving pressure jump. The harmonic, propagating wave is a classical forcing term when analyzing linear differential equations, and the pressure jump should resemble the pressure before and after the passage of a steep cold front.

The numerical study
The numerical study was split up in a sensitivity study and a study of a real meteorological event, the passage of a cold front that occurred in the period 18th to 23rd of April 1993. Later this event caused some seiche activity in the Port of Rotterdam [12]. In the sensitivity test a simple one-dimensional model was set up. With this model a number of experiments were carried out to test the sensitivity of parameters like time step, grid spacing and
bottom friction. In these experiments a harmonic, propagating pressure field like in the analytical study was used with wave length, period, velocity and amplitude based on the (expected) characteristics of the pressure field of the event mentioned above. The primary aim of the sensitivity test was to find a setup of the model for the calculations of the real event. In the cold front simulation the measured air pressure was used to simulate the passing cold front. Again some aspects of modeling this event were investigated to finally choose an appropriate model setup. The results of the final simulations were then compared to observations presented by Veraart [12] and the analytical results.

*Outline of the report*

In Chapter 2 a general description of seiches and long waves is given. Different generating mechanisms are summed up, and finally the specific generating mechanism considered in this study is described in detail. In Chapter 3 the basic equations are described and the one-dimensional shallow water equations are derived. In Chapter 4 the analytical results of this study are presented, first for free waves and then for pressure generated waves in unlimited and limited areas respectively. In Chapter 5 the numerical experiments carried out in the sensitivity test of the simple 1D model are described. The result of the sensitivity test, the setup of the model to be used to simulate the real event, is presented at the end of the chapter. The numerical simulations of the real event are described in Chapter 6, along with the results of these. In Chapter 7 the conclusions of this study are drawn, and recommendations are given for further work.
Seiches and long waves

In this chapter the seiche phenomenon is described, and a definition of long waves is given. Different generating mechanisms are mentioned and the generating mechanism considered in this study is described in more detail.

2.1 Long waves

The seiches described in the next paragraph are part of a larger group of waves called long waves. Long waves have been defined in many ways, depending on the actual situation considered, with the broadest definition as everything between wind waves and tides, i.e. waves with periods between 20 seconds and 12 hours [15]. The narrowest definition encountered in this study is waves that may cause seiches in the Port of Rotterdam, i.e. waves with periods between 15 minutes and 2-3 hours [9, 12]. In the following the latter definition will be used.

Usually long waves are connected with shallow water waves. Shallow water waves are long period waves in areas of small depths. In practice the shallow water approximation is often accepted for waves with a relative water depth $h/L$ less than 1/20, e.g. [2]. Using this limit and the shallow water wave velocity $c = \sqrt{gh}$, this requires that $T > 20h/\sqrt{gh}$. In the North Sea this limit is identical to periods larger than approximately 1 min. The waves mentioned above are thus well above this limit.

Another implication of the shallow water approximation is that the horizontal particle velocity is virtually constant over the depth. This can be used to simplify the general Navier-Stokes equations as described in Chapter 3.

2.2 Seiches in lakes

The word seiche is French and originally means octopus. It was first used as a name for long period oscillations observed at Lake Geneva, probably due to an old legend saying that the oscillations were caused by a giant, mad octopus living in the lake. However, seiche has now become an internationally used scientific term for resonant oscillations in lakes, in harbours and in continental shelf waters [14].

As an example, consider a steady wind blowing over an enclosed water body. The wind shear stress acting on the water surface will induce a current, and if the wind continues for some time, a setup will develop. This setup balances the wind shear forces by means of a surface gradient, see figure 2.1.
Figure 2.1. Wind setup in a lake. From [2].

If the wind suddenly ceases or changes its direction, the forces are unbalanced, and gravity will act to level the surface. Because inertia is large and friction low, the surface overshoots its equilibrium position, and the whole body of water has started a free oscillation around the equilibrium state. This kind of seiche primarily occurs in lakes, and they are therefore called lake seiches. The only damping of the oscillation is due to bottom friction, and it may therefore continue for a long time.

The lake seiche is also known as a half-wave oscillator, because the wave length of the longest wave that fits the basin is twice the length of the basin, see figure 2.2 a. If the length of the basin is \(l\), the wave length of the standing wave is \(\lambda = 2l\). If the depth is \(h\), the fundamental period becomes

\[
T_1 = \frac{2l}{\sqrt{gh}}
\]  

(2.1)

This is called Merian’s formula. From this formula it can be seen that longer and more shallow water body have longer natural periods. Also standing wave patterns with more than one node may develop, as shown in figure 2.2 b.

Figure 2.2. The half-wave oscillator. From [2].

For a standing wave with \(n\) nodes, the basin length is \(l = nL/2\), and Merian’s formula may be extended to
\[ T_1 = \frac{2l}{\sqrt{gh}} \quad T_n = \frac{1}{n} T_1 \] (2.2)

Above, a lake seiche was illustrated with a steady wind shear stress. Other generating mechanisms that may cause lake seiches are fluctuations in air pressure, local solid-earth events such as earthquakes, volcanic eruption and landslides, electrical attraction between water and clouds, and rain-drop impact [14].

### 2.3 Coastal seiches

As mentioned earlier the word seiche is also used for harbour resonance. To distinguish this type from the lake seiches, they are called *harbour seiches* or *coastal seiches*. The difference between lakes and harbours is that harbours are usually connected to a larger body of water through an open boundary. This has two obvious consequences:

At the open boundary waves are only partially reflected, so that if a seiche as described above were to occur, it would quickly dissipate through the open boundary. An external force is thus needed to keep the oscillation going.

On the other hand, the open boundary also exposes the harbour to waves coming in through the open boundary. Consider the harbour shown in figure 2.3. Assume that waves of a given frequency are entering the harbour. At the closed boundary they are completely reflected, and again (partially) at the open boundary. If this happens simultaneously with the next wave entering the harbour, the incoming and reflected waves will be in phase, and the harbour will resonate.

![Figure 2.3. Illustration of incoming waves causing harbour resonance. From [2].](image)

The open boundary will act as a node, due to the large capacity of the ocean. The largest natural period of the harbour in figure 2.3 is then given by

\[ T_1 = \frac{4l}{\sqrt{gh}} \quad T_n = \frac{1}{2n-1} T_1 \] (2.3)

Though these equations are only very rough estimates in case of a complex system like the Port of Rotterdam, they yield results of the correct order of magnitude [13].
In practice of course, advanced numerical models are used to determine the response of a harbour to incoming waves of different frequencies, and for the Port of Rotterdam this has been done numerous times. The results are often presented graphically in an amplification spectrum as in figure 2.4, showing the response during storm surges at the location Rozenburgsesluis (RBS), where seiches are most pronounced.

![Amplification spectrum for Port of Rotterdam (RBS), in different future situations. Present situation. Future situations: d--d quays inundated, --- quays not inundated. From [9].](image)

From this figure it is clear that the amplification of long waves will increase once the ongoing constructions are completed, and especially for wave periods between 50 and 120 min, i.e. for frequencies between approximately 0.15 and 0.5 mHz. According to de Looff [9] 95% of the seiches at RBS have periods in exactly this interval. It can also be seen that the amplification is larger for more frequent events, because quays are then not inundated. So the peaks for periods 16 and 20 min are relevant only for more frequent situations.

2.4 Seiche generating mechanisms

Many different seiche generating mechanisms have been suggested in the literature. Of these, tsunamis, storm surges, surf beat and trapped edge waves are the most referred to. Just recently it has been discovered that also tide generated internal waves may be responsible for generating coastal seiches. The process is however still not totally understood [14, 20].

In this study the effect of an atmospheric pressure disturbance is investigated, and in particular the effect of a passing cold front, since this has also been suggested as a possible long wave generating mechanism [23]. In [19] and [24] seiching in inlets of the Balearic Islands and the Port of Plymouth respectively was explained by fluctuations in air pressure.

In 1957 Wemelsfelder reported two types of long waves, namely squall-oscillations and gust bumps. They are presumably caused by respectively squalls and squall-lines during stormy weather. The characteristics of these waves are [23]:

Squall-oscillations (Dutch: bui-oscillaties) are irregular long period oscillations of the sea surface. They are thought to be caused by squalls, i.e. sudden violent
winds, and are named after these. The periods of these oscillations lie between a few minutes and more than an hour, with an average of 35 min, and the amplitudes may be up to half a meter. There is no correlation between the waves at different locations along the Dutch coast, i.e. the horizontal extension is small (< 20 km). On the other hand, they can clearly be followed in through the Port of Rotterdam, where they travel at the shallow water velocity, and they are thus progressive waves. Squall-oscillations of smaller amplitudes are present under almost all kinds of meteorological circumstances. This fact together with the stochastic nature indicate that they could be a result of the normal macroscopic turbulence in the lower atmosphere.

*Gust bumps* (Dutch: buistoten) are single elevations of the sea surface and are named after the gusty winds of e.g. a cold front. Gust bumps of up to 50-60 cm have been recorded, and elevations larger than 15 cm occur once or twice a year, especially in connection with storm surges. However, these elevations often disappear in the 'noise' of squall-oscillations. The horizontal extension of a gust bump is large, since the same elevation can be followed along the entire Dutch coast. These facts indicate that they could be generated by a passing front or by heavy showers. The probability of gust bumps developing are thought to be the highest if the front speed is about the same as the mean shallow water velocity in the North Sea [22].

### 2.5 Cold fronts

A front is defined as the narrow transition zone between two regions of air with different temperature, humidity or pressure. The difference in these quantities reduces the mixing of air across the transition zone, which makes the front relative stable. Due to difference in air density, the front will not be vertical, but lean towards the region with largest density. The declination of a front is usually not larger than 1:100 - 1:25, and it is therefore always highly exaggerated in illustrations.

Different types of fronts can be defined, depending on the temperatures on either side of the front. One of these is the cold front shown in figure 2.5 along with its opposite, the warm front.

![Figure 2.5. Temperature and pressure around a cold and a warm front. Based on [21].](image-url)
As indicated on the right in figure 2.5, the passage of a warm front is usually connected with a drop in air pressure, and the passage of a cold front with an increase in air pressure. Warm fronts are in general slower and less steep than cold fronts, and the wind and pressure fluctuations are usually stronger and the turbulence more intense around a cold front.

A distinction can be made between two types of cold fronts, namely active and passive cold fronts. The active cold front is steep and moves fast. This is caused by strong winds perpendicular to the front line. Often a relative narrow area with heavy showers precedes the active cold front. The passive cold front, on the other hand, is less steep and moves slowly. In case of showers, these are spread over a large area behind the front.

Comparing the description of cold fronts with the one of squall-oscillations and gust bumps, it seems likely that fast cold fronts are most capable of generating long waves, and it was therefore chosen to investigate the effect of the change in air pressure around a cold front in this study.
The basic equations

3.1 Applicability of the shallow water equations

The general three-dimensional Navier-Stokes equations and the continuity equation are sufficient to describe most practical problems in hydraulics. However, setting up and running a three-dimensional numerical model is an expensive and time consuming task. Therefore, if the problem under consideration has a predominantly horizontal character, the general equations can be simplified by depth integration to a set of two-dimensional equations, the so-called shallow water equations. This can formally be done if certain assumptions are met [5]. For the physical problem considered here, the most important assumptions are:

1. The fluid is incompressible.
2. The surface tension can be neglected.
3. The bottom configuration is constant on the time scale of interest.
4. The bottom is impermeable.
5. The density and viscosity of the fluid are constant.
6. The gravitational acceleration and the Coriolis effect are constant.
7. The horizontal length scale of the simulated phenomenon is much larger than the characteristic vertical length scale.

The compressibility of water is only important to the description of surface waves in situations where the fluid particle velocity approaches the speed of sound. For water this is about 1500 m/s, and assumption 1 is thus true in almost any case [2]. The compressibility of water is important to other problems, however, for instance to the propagation of sound.

The surface tension only influences the propagation of capillary waves with wave lengths less than 3-5 cm [2],[3]. Assumption 2 is thus no real restriction when dealing with long waves.

Since the time scale of the long waves considered is measured in hours, assumption 3 may be considered valid. The sea bed is usually not impermeable, but the energy loss due to percolation is usually a lot smaller than that due to bottom friction [2]. Furthermore the energy loss due to percolation is largest for short period waves, whereas that due to bottom friction increases with the wave period. Assumption 4 may therefore be considered valid for long period waves.

Some differences in density must be expected, since this is a function of salinity and temperature. Also differences in the content of sediment may influence the density locally [4]. However, since the depth of the southern North Sea is 20-50 m, these differences are probably small compared to those in the oceans, due to tidal mixing. The viscosity consisting of more components, see section 3.3, is per definition not constant. However, it is often modeled as a constant, lacking better alternatives. Furthermore the assumption of constant density and viscosity is a generally accepted assumption in ordinary hydraulics [5], and so also here.
The basic equations.

In an area the size of the North Sea variations in gravity have no practical influence on the accuracy of the results required in this study [5]. The area under consideration stretches from approximately 51° to 56°N. The corresponding error in the Coriolis coefficient, being a function of degree latitude, is less than about three percent. Assumption 6 is thus no limitation.

The characteristic vertical scale, the depth, is as mentioned in the order of 20-50 m. The wave length of the waves considered in this study is in the order of 15 km based on the shallow water wave velocity \( c = \sqrt{gh} \) and the relation \( L = cT \), assuming periods larger than 15 min. Assumption 7 is thus valid.

If the above mentioned assumptions can be considered valid, the two-dimensional shallow water equations may be used to describe the problem. These equations read [6]:

The continuity equation:

\[
\frac{\partial H}{\partial t} + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0
\]  \hspace{1cm} (3.1)

The momentum equation in the \( x \)-direction:

\[
\frac{\partial Q_x}{\partial t} + \frac{\partial (Q_x^2 / D)}{\partial x} + \frac{\partial (Q_x Q_y / D)}{\partial y} - \frac{\partial}{\partial x} (D \nu \frac{\partial (Q_x / D)}{\partial x}) - \frac{\partial}{\partial y} (D \nu \frac{\partial (Q_y / D)}{\partial y}) + gD \frac{\partial (H + P)}{\partial x} + Fr |Q_1| Q_x / D^2 - C_0 Q_x - W_x = 0
\]  \hspace{1cm} (3.2)

The momentum equation in the \( y \)-direction:

\[
\frac{\partial Q_y}{\partial t} + \frac{\partial (Q_y^2 / D)}{\partial y} + \frac{\partial (Q_x Q_y / D)}{\partial x} - \frac{\partial}{\partial y} (D \nu \frac{\partial (Q_y / D)}{\partial y}) - \frac{\partial}{\partial x} (D \nu \frac{\partial (Q_x / D)}{\partial x}) + gD \frac{\partial (H + P)}{\partial y} + Fr |Q_1| Q_y / D^2 + C_0 Q_y - W_y = 0
\]  \hspace{1cm} (3.3)

where  
- \( t \) is time 
- \( x, y \) horizontal coordinates 
- \( g \) gravitational acceleration 
- \( H \) water level with respect to a chosen datum 
- \( D \) total water depth 
- \( Q_x \) depth integrated velocity in the \( x \)-direction 
- \( Q_y \) depth integrated velocity in the \( y \)-direction 
- \(|Q|\) magnitude of the \( Q \)-vector, \(|Q| = \sqrt{Q_x^2 + Q_y^2}\) 
- \( P \) atmospheric pressure divided by water density and gravity, i.e. by \( \rho g \) 
- \( W \) wind shear stress divided by water density \( \rho \) 
- \( Fr \) bottom friction coefficient 
- \( C_0 \) Coriolis coefficient 
- \( \nu \) viscosity coefficient
The terms in the momentum equation are, in order of appearance: local acceleration, convective acceleration, viscosity (2×), surface and atmospheric pressure gradients, bottom friction, Coriolis force and wind shear stress.

### 3.2 Physical parameters

In the shallow water equations, three physical parameters are present. These are:

**Bottom friction coefficient, \( Fr \).**

The bottom friction coefficient, \( Fr \), is defined by

\[
\tau = -Fr \, \rho \, U \, |U|
\]  

(3.4)

It may either be set to a constant or be estimated using a relative roughness value.

**Coriolis coefficient, \( C_0 \).**

The Coriolis force is not a physical force, but a term included in the equations to compensate for the rotation of Earth [8]. \( C_0 \) is given by

\[
C_0 = 2\Omega \sin \phi
\]  

(3.5)

where \( \Omega = 2\pi / T = 7.27e-5 \text{ rad/s} \) is the angular frequency of the Earth rotation, and \( \phi \) is degree latitude. The average Coriolis coefficient in the North Sea then becomes \( C_0 = 1.15e-4 \text{ s}^{-1} \). The Coriolis effect is only important in areas with length scales larger then the Rossby radius \( R_i = \sqrt{gH / C_0} \). For the North Sea this is approximately 150 km, and the Coriolis effect is thus important here.

**Viscosity coefficient, \( \nu \).**

This coefficient is a sum of the three components: molecular, turbulent and dispersive viscosity. The order of magnitude of these terms are \( 10^4 \text{ m}^2/\text{s}, 10^2 - 10^6 \text{ m}^2/\text{s} \) and \( 10^1 \text{ m}^2/\text{s} \) respectively. The dominating component is thus the dispersive viscosity. However, the viscosity is not very important to the problem considered here. This is discussed further in paragraph 5.2. The ratio between the viscous and convective terms forms a sort of Reynolds number

\[
Re = \frac{\nu}{U \Delta x}
\]  

(3.6)

For decreasing currents and grid size, the viscosity thus plays an increasing role compared to the convective acceleration.

### 3.3 The 1D shallow water equations

If the problem under consideration is one-dimensional in nature, equations (3.1-3) can be reduced to two equations, the one-dimensional shallow water equations. They are obtained by setting all quantities and derivatives in the \( \gamma \)-direction to zero. The equations then become:
The basic equations.

The continuity equation:

\[ \frac{\partial H}{\partial t} + \frac{\partial Q}{\partial x} = 0 \]  

(3.7)

The momentum equation:

\[ \frac{\partial Q}{\partial t} + \frac{\partial (Q^2/D)}{\partial x} - \frac{\partial}{\partial x} (D \nu \frac{\partial (Q/D)}{\partial x}) 
+ gD \frac{\partial (H+P)}{\partial x} + Fr |Q/Q/D^2 - W = 0 \]  

(3.8)

using the same notation as above and leaving out index \( y \). Here the terms of the momentum equation are: local acceleration, convective acceleration, viscosity, surface and atmospheric pressure gradients, bottom friction, and the wind shear.

These equations are used for analytical purposes in the next chapter, in a more compact notation, however.
Analytical solutions

4.1 Introduction

Analytical solutions to a simplified problem are often desired to gain insight into the physical problem under consideration and to explain certain behaviors. Which parameters are important to the problem and in which way do they influence the solution. In this chapter the solution to the one-dimensional shallow water equations has been found for an atmospheric pressure field traveling over a channel of constant depth. In the case of an unlimited area the equations are solved for two situations, a) 'no bottom friction' and b) 'linear bottom friction'. In both cases the problem is first solved assuming the pressure variation to be harmonic, then by assuming an abrupt pressure jump represented by a Heaviside step function. In the case of a closed basin the solution becomes more complex due to reflections, and therefore only the frictionless case has been considered.

4.2 Linearizing the 1D equations

The full shallow water equations in one and two dimensions are described in Chapter 3. In one dimension the shallow water equations are the continuity and momentum equation. Letting indices denote partial differentiation, and $t$ and $x$ the independent variables time and position, they may be written

\[ \eta_t + Q_x = 0 \] (4.1)

\[ Q_x + (Q^2/D)_x - (Dv(Q/D)_x)_x + gD(\eta_x + p_x) + Fr|Q/Q|D^2 - W = 0 \] (4.2)

where
- $\eta$ is surface elevation relative to MWL
- $h$ is the water depth relative to MWL
- $D$ is the total water depth given by $D = \eta + h$
- $p$ is the atmospheric pressure divided by $\rho g$

The terms in the momentum equation represent local acceleration, convective acceleration, viscosity, surface and pressure gradients, bottom friction and wind shear.

Equations (4.1) and (4.2) are not suited for simple analytical solution, so the momentum equation is simplified in the following way:

In this study only the effect of an atmospheric pressure field is investigated, so the wind friction is left out, i.e. the wind shear stress divided by water density $W = 0$.

The depth is set to a constant, $h$, and the surface elevation $\eta$ is assumed to be much smaller than
the depth, so that \( D \) may be approximated by \( h \).

For progressive shallow water waves the current and the surface elevation are connected through, see e.g. [2]

\[
\frac{\mathcal{Q}}{h} = U = c \frac{\eta}{h}
\]  
(4.3)

where \( c = \sqrt{gh} \) is the shallow water wave velocity. The order of magnitude of the terms in the momentum equation may then be estimated by

\[
O(\mathcal{Q}_t) = \frac{c}{T} \eta \sim 2 \cdot 10^{-3} \text{ m}^2 \text{s}^{-2}
\]  
(4.4)

\[
O((\mathcal{Q}^2/D)_x) \sim \frac{c^2}{hL} \eta^2 \sim 5 \cdot 10^{-6} \text{ m}^2 \text{s}^{-2}
\]  
(4.5)

\[
O(\nu \mathcal{Q} \mathcal{Q}_x)_x \sim \frac{\nu c}{L^2} \eta \sim 5 \cdot 10^{-8} \text{ m}^2 \text{s}^{-2}
\]  
(4.6)

\[
O(gD \eta_x) \sim \frac{c^2}{L} \eta \sim 2 \cdot 10^{-3} \text{ m}^2 \text{s}^{-2}
\]  
(4.7)

\[
O(Fr \mathcal{Q} \mathcal{Q}_x^2) \sim Fr \frac{c^2}{h^2} \eta^2 \sim 10^{-5} \text{ m}^2 \text{s}^{-2}
\]  
(4.8)

where the last approximation is based on \( T = 1000 \text{ s}, L = 15 \text{ km}, h = 32 \text{ m} \) and \( \eta = 0.1 \text{ m} \) (see paragraph 5.2). From these estimates it follows that the convective acceleration is small compared to the local acceleration, and that dissipation due to viscosity is small compared to dissipation due to bottom friction. The convective acceleration and viscosity may thus be neglected. In the first analysis of free waves, the viscosity is included for completeness though, and then later neglected.

Finally the friction term is linearized to give the set of linearized 1D shallow water equations

\[
\eta_t + \mathcal{Q}_x = 0
\]

\[
\mathcal{Q}_t - \nu \mathcal{Q}_{xx} + c^2 \eta_x + c^2 p_x + \alpha \mathcal{Q} = 0
\]  
(4.9.a-b)

where \( \alpha \) is a new friction coefficient given by \( \alpha = Fr \mathcal{Q}/D^2 \). This is of course not constant, but is assumed constant in the following. The momentum equation is differentiated with respect to \( x \), and then reads

\[
\mathcal{Q}_{xt} - \nu \mathcal{Q}_{xxt} + c^2 \eta_{xt} + c^2 p_{xt} + \alpha \mathcal{Q}_x = 0
\]  
(4.10)
By appropriate differentiation of the continuity equation (4.9.a) and substitution into the momentum equation (4.10), the latter can be rewritten

\[ c^2 \eta_{xx} - \eta_{tt} + \nu \eta_{xx} - \alpha \eta_x + c^2 p_{xx} = 0 \] (4.11)

This is the one-dimensional shallow water wave equation including linear bottom friction and viscosity. It will be solved in the following paragraphs.

### 4.3 Solutions in unlimited areas

At first it is assumed that no vertical boundaries are present, i.e. the area for which a solution should be found is an unlimited channel. This is done to avoid the influence of reflection at the boundaries.

#### 4.3.1 Free waves

First the shallow water wave equation without forcing terms is solved.

In the no friction case, the simplest version of the wave equation is obtained from (4.11) by neglecting pressure, friction and viscosity

\[ c^2 \eta_{xx} - \eta_{tt} = 0 \] (4.12)

The solution to this equation is given by

\[ \eta(x,t) = \psi(x - ct) + \phi(x + ct) \] (4.13)

where \( \psi \) and \( \phi \) are arbitrary functions. The solution is thus a sum of two solutions traveling in each direction. This essentially states that any initial surface profile propagating with the shallow water wave velocity \( c \), will continue undisturbed at this speed.

The solution to equation (4.12) with the initial conditions

\[\begin{align*}
\eta(x,0) &= \psi(x) + \phi(x) \\
\eta_t(x,0) &= c \phi'(x) - c \psi'(x)
\end{align*}\] (4.14.a-b)

was found by d'Alembert, and can be written, see e.g. [1]

\[\eta(x,t) = \frac{1}{2}[f(x + ct) + f(x - ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(s) \, ds \] (4.15)

where \( f \) and \( g \) are the initial conditions \( \eta \) and \( \eta_t \), respectively. This is easily demonstrated by insertion of \( \eta(x,0) \) and \( \eta_t(x,0) \) from (4.14). A well-known solution to (4.12) is the harmonic, propagating surface wave

\[ \eta = \eta_0 \sin(k(x - ct)) \] (4.16)
where \( c = \omega/k \). Here \( \omega \) is the frequency and \( k \) the wave number. By insertion in (4.15) it may again be demonstrated that this wave will travel undisturbed through the model. This was here done using the MapleV programme, see Appendix A.

Assume however, that a wave is present with velocity \( v \neq c \). Such a wave may then be written

\[
\eta(x,t) = \eta_0 \cos(k(x-ct))
\]  
(4.17)

and inserted in d’Alembert’s solution. The result is a changed wave profile given by

\[
\eta(x,t) = \frac{1}{2} \eta_0 \cos(k(x-ct)) + \frac{1}{2} \eta_0 \cos(k(x+ct)) + \gamma \eta_0 \sin(kx)\sin(kct)
\]  
(4.18)

where \( \gamma = v/c \) is the relative velocity of the original wave. This expression can for all \( \gamma \) be rewritten to

\[
\eta(x,t) = \gamma \eta_0 \cos(k(x-ct)) + (1-\gamma) \eta_0 \cos(kx)\cos(kct)
\]  
(4.19)

and

\[
\eta(x,t) = \eta_0 \cos(k(x-ct)) + (\gamma - 1) \eta_0 \sin(kx)\sin(kct)
\]  
(4.20)

The first expression is the most logical for \( \gamma < 1 \), the second for \( \gamma > 1 \). They are both combinations of an ordinary harmonic wave, propagating with the shallow water velocity \( c \), and a standing wave. An example of the resulting wave envelope is shown in figure 4.1. By inserting \( kx = 0 \) and \( \pi \) respectively in the expressions, it can be found that for \( \gamma < 1 \), the envelope varies between \([\gamma \eta, \eta] \), while for \( \gamma > 1 \) it will vary between \([\eta, \gamma \eta] \).

![Wave envelope - transformed wave](image)

Figure 4.1. Resulting wave envelope of propagating and standing wave (\( \gamma = 0.9 \)).

A situation where this phenomenon could arise, is when the forcing from e.g. wind or pressure suddenly disappears, or when forced waves are reflected.

Another implication of d’Alembert’s solution is that the only solution, in case of initially no waves, is the null-solution, i.e. free waves are not generated spontaneously.

Including linear friction and viscosity the solution to equation (4.11) is assumed to be of the form
\[ \eta = \eta_0 e^{i(\omega t - kx)} \]  (4.21)

Inserting this in the wave equation yields

\[ -c^2k^2 \eta + \omega^2 \eta - i\alpha \omega \eta - i\nu \omega k^2 \eta = 0 \]  (4.22)

\( \eta \) disappears so

\[ \omega^2 - i[\alpha + \nu k^2] \omega - c^2k^2 = 0 \]  (4.23)

which is a quadric equation in \( \omega \). This is solved to give

\[ \omega_{1,2} = \frac{i}{2} \left[ \alpha + \nu k^2 \right] \pm \sqrt{c^2k^2 - \frac{1}{4} [\alpha + \nu k^2]^2} \]  (4.24)

Neglecting viscosity, the solution then becomes

\[ \eta = \eta_0 e^{-\frac{\alpha}{2} t} e^{i(\omega_d t - kx)} \quad \omega_d = \sqrt{\omega^2 - \alpha^2/4} \]  (4.25)

Neglecting friction, the solution becomes

\[ \eta = \eta_0 e^{\frac{-\nu k^2}{2} t} e^{i(\omega_d t - kx)} \quad \omega_d = \sqrt{\omega^2 - (\nu k^2)^2/4} \]  (4.26)

where \( \omega_d \) is the frequency of the damped solution. Both friction and viscosity reduce the amplitude exponentially in time, and result in a lower frequency than that of the free waves of the same wave length, and thus in a lower free wave velocity.

Comparing \( \alpha \) and \( \nu k^2 \) in equation (4.25) and (4.26) by inserting typical values, it may again be shown that friction dominates viscosity.

### 4.3.2 Harmonic pressure, no friction

Including the forcing from a moving atmospheric pressure field and neglecting friction and viscosity, the equation to be solved is

\[ c_s^2 \eta_{xx} - \eta_t + c_s^2 \eta_{xx} = 0 \]  (4.27)

where the shallow water wave velocity is now denoted by \( c_s \) to distinguish it from the velocity of the pressure field \( c_p \).

The solution to this non-homogenous equation can be found using complex notation. A propagating pressure wave and a propagating water wave can then be expressed by
\[
\begin{align*}
  p &= p_0 e^{i(\omega t - kr)} = p_0 e^{i\theta} \\
  \eta &= -\eta_0 e^{i(\omega t - kr - \varphi)} = -\eta_0 e^{i\theta} e^{-i\varphi}
\end{align*}
\] (4.28.a-b)

where \(\omega\) is the angular frequency and \(k\) the wave number of the pressure disturbance. The velocity of the pressure is then given by \(c_p = \omega/k\). \(\varphi\) is a possible phase lag of the surface elevation. The minus on the surface elevation is added to make positive pressure correspond to negative surface elevation, and thus zero phase lag in the static case. This is more intuitive than the straightforward definition. Inserting the expressions for \(p\) and \(\eta\) in equation (4.27) gives

\[
c_s^2 k^2 \eta_0 e^{-i\varphi} - \omega^2 \eta_0 e^{-i\varphi} = c_s^2 k^2 p_0
\] (4.29)

where the factor \(e^{i\theta}\) has disappeared. Dividing by \(c_s^2 k^2\) and rearranging the terms gives the complex equation

\[
\eta_0 e^{-i\varphi} (1 - \gamma^2) = p_0
\] (4.30)

in the two unknowns \(\eta_0\) and \(\varphi\). Here \(\gamma\) is given by

\[
\gamma = \frac{c_p}{c_s}
\] (4.31)

Equation (4.30) can be solved to give the amplification \(\zeta = \eta_0/p_0\)

\[
\zeta = \frac{1}{|1 - \gamma^2|} \quad \varphi = \begin{cases} 0 & \text{for } \gamma < 1 \\ \pi & \text{for } \gamma > 1 \end{cases}
\] (4.32.a-b)

This solution is illustrated on figure 4.2.

![Amplitude portrait, no friction](image)

![Phase portrait, no friction](image)

Figure 4.2. Solution to the shallow water wave equation without friction in an unlimited area.

In case of a relative slowly moving pressure field, i.e. for \(\gamma < 1\), a positive pressure corresponds to
negative elevation, just like in the static case, and the phase lag is zero. For fast moving pressure fields, i.e. for $\gamma > 1$, a positive surface elevation occurs, where the pressure is positive, in other words the surface is in opposite phase. See figure 4.3.

![Air pressure wave](image.png)

**Figure 4.3.** Air pressure wave contra surface waves in phase and opposite phase. H and L indicate relative high and low pressure.

Note that the amplification, according to the solution, approaches infinity when the pressure wave velocity and the free wave velocity coincide. This is a result of neglecting the damping terms. Furthermore it should be mentioned that when resonance becomes large, so does damping and other non-linear effects, and the solution is thus not reliable close to the peak.

### 4.3.3 Harmonic pressure, linear friction

Usually the bottom friction is modeled by a quadric expression in $Q$, but to keep the equation linear it is here taken into account through a linear term, as mentioned earlier. The wave equation to be solved for this case is then

$$c_s^2 \eta_{xx} - \eta_t - \alpha \eta_t + c_s^2 \eta_{xx} = 0 \quad (4.33)$$

Again we insert $p$ and $\eta$ using complex notation to obtain

$$\eta_0 e^{-i\phi} (c_s^2 k^2 - \omega^2 + i \alpha \omega) = c_s^2 k^2 p_0 \quad (4.34)$$

or rearranging and using $\gamma$

$$\eta_0 e^{-i\omega} \left[ (1 - \gamma^2) + i \left( \frac{\alpha}{\omega} \gamma^2 \right) \right] = p_0 \quad (4.35)$$

20
Again the two unknowns are the amplitude $\eta_0$ and the phase $\varphi$. Taking the absolute value on both sides of the equation, the amplification can be determined as

$$\zeta = \frac{1}{\sqrt{(1 - \gamma^2)^2 + \left(\frac{\alpha}{\omega}\right)^2 \gamma^2}}$$

(4.36)

The phase $\varphi$ is determined by demanding the same arguments on both sides of the equation, leading to

$$\varphi = \arctan \left(\frac{\alpha}{\omega} \frac{\gamma^2}{1 - \gamma^2}\right)$$

(4.37)

This formulation returns $\varphi$-values in the interval $[0, \pi/2[$ for $\gamma$-values in the interval $[0, 1[$ and $\varphi$-values in the interval $]-\pi/2, 0[$ for $\gamma$-values in the interval $]1, +\infty[$. To get all phases in the interval $[0, \pi]$, $\pi$ should be added to all negative phases. The results are illustrated for $\alpha = 3e-5$ s$^{-1}$ in figure 4.4 and 4.5. The value of $\alpha$ is estimated from results obtained during the 1D simulations.

![Amplification](image)

Figure 4.4. Wave amplification as a function of $\gamma$ and pressure wave period, $T$. $\alpha = 3e-5$ s$^{-1}$. Note the wide $T$-interval.

![Phase](image)

Figure 4.5. Phase portrait as a function of $\gamma$ and pressure wave period, $T$. $\alpha = 3e-5$ s$^{-1}$. Note the narrow $\gamma$-interval.
In figure 4.6-9 the influence of bottom friction and pressure wave period on the amplification and the phase is illustrated. In equation (4.36) the second term in the denominator contains the influence of the bottom friction. This term is proportional to $\alpha^2$ and $T^3$. This term is important in the following cases:

1) when $\gamma$ approaches 1. The first term in the denominator of equation (4.36) disappears, and the amplification approaches

$$\zeta = \frac{2\pi}{\alpha T}$$

(4.38)

It is thus inversely proportional to $\alpha$ and $T$.

2) for all $\gamma$ and large $T$. The influence of bottom friction increases for increasing wave period as seen in figure 4.6. Furthermore, for large periods the approximation above is valid in a broader area around $\gamma = 1$. This is seen by comparing figure 4.7. and 4.8. Note the different horizontal scales.

![Influence of friction](image)

Figure 4.6. Wave amplification as a function of wave period for selected (a) $\alpha = 2e-5 \text{ s}^{-1}$ and (b) $\alpha = 4e-5 \text{ s}^{-1}$. Friction becomes more important with increasing wave period.

3) for all $\gamma$ and large $\alpha$. The value of $\alpha$ is limited, so in practice large friction alone cannot prevent large amplification.
Figure 4.7. Wave amplification as a function of $\gamma$ for selected $\alpha$-values. $T = 1000s$. (a) $\alpha = 4e-5 \ s^{-1}$, (b) $\alpha = 3e-5 \ s^{-1}$ and (c) $\alpha = 2e-5 \ s^{-1}$. Note the narrow $\gamma$-interval.

Figure 4.8. Wave amplification as a function of $\gamma$ for selected $\alpha$-values. $T = 10000s$. (a) $\alpha = 4e-5 \ s^{-1}$, (b) $\alpha = 2e-5 \ s^{-1}$ and (c) $\alpha = 1e-5 \ s^{-1}$.
Figure 4.9. Phase portrait as a function of $\gamma$ for selected $\alpha$-values. $T = 1000s$. On the figure corresponding, approximate $Fr$-values are indicated. Note the narrow $\gamma$-interval.

The expression for $\eta_0$ in the no friction case can be derived from the formulas above by letting $\alpha$ approach zero. The matching phases for $\gamma$ greater than or less than unity can be found by letting $\alpha$ approach zero and $\gamma$ be a constant in $[0, 1]$ and $[1, \infty]$ respectively.

4.3.4 Pressure jump, no friction

In this paragraph the Laplace transformation is used to solve the wave equation. This method along with some characteristic properties are described in Appendix B.

Again the wave equation without friction is written

$$c_s^2 \eta_{xx} - \eta_t + c_s^2 p_{xx} = 0$$

(4.39)

Using the formulas for Laplace transforming derivatives with respect to time, the wave equation becomes

$$c_s^2 H_{xx} - (s^2 H - s \eta(x,0) - \eta_t(x,0) + c_s^2 p_{xx} = 0$$

(4.40)

where $H$ and $P$ are the Laplace transforms of $\eta$ and $p$ respectively. With the initial conditions $\eta_t(x,0) = 0$ and $\eta(x,0) = 0$, two terms drop out and the equation becomes

$$c_s^2 H_{xx} - s^2 H + c_s^2 P_{xx} = 0$$

(4.41)

Since no $s$-derivatives occur, this can be seen as an non-homogeneous ordinary differential equation in $x$, with $s$ as a parameter. The general solution to the corresponding homogenous equation is given by

$$H_h = A(s) e^{-\frac{\xi}{c_s}} + B(s) e^{\frac{\xi}{c_s}}$$

(4.42)
where the usual arbitrary constants $A$ and $B$ may be functions of $s$.

\[ p = p_0 \left( t - \frac{x}{c_p} \right) h(t) , \quad h(t-a) = \begin{cases} 1 , & t > a \\ 0 , & t < a \end{cases} \]  \hspace{1cm} (4.43)

The term $-x/c_p$ is the translation of the Heaviside function in the positive $x$-direction. The second Heaviside function defines the pressure difference to be zero at all times $t < 0$. From the definition of the Laplace transform it is, however, seen that this last part of $p$ has no influence on the Laplace transform, since this is unity for all times larger than the lower limit of integration. The transformed pressure $P$ therefore becomes

\[ P = p_0 e^{-x/c_p} \frac{s}{s} \]  \hspace{1cm} (4.44)

and thus the derivative

\[ P_{xx} = \frac{s^2}{c_p^2} P \]  \hspace{1cm} (4.45)

In order to find a particular solution to the non-homogeneous equation, $H$ is assumed to be proportional to $P$, i.e. $H = kP$. Insertion of $H$ and $P$ in the equation yields

\[ k \left[ P_{xx} - \frac{s^2}{c_p^2} P \right] = -P_{xx} \]  \hspace{1cm} (4.46)
and thus
\[ k \left[ \frac{s^2}{c_p^2} - \frac{s}{c_s} \right] P = -\frac{s^2}{c_p^2} P \]  
(4.47)

Here \( s^2 \) is canceled out, and multiplying by \( c_p^2 \) the factor \( k \) can then be determined to
\[ k = \frac{-1}{1 - \gamma^2} \]  
(4.48)

with \( \gamma \) defined as before. The complete solution to the transformed non-homogeneous equation then becomes
\[ H = A(s) e^{-\frac{z}{c_s} t} + B(s) e^{-\frac{z}{c_s} t} - \frac{P_0}{1 - \gamma^2} \frac{e^{-\frac{z}{c_p} t}}{s} \]  
(4.49)

Since \( \eta(x,t) \) is zero when there is no pressure, also \( H(x,s) \) should be zero. This implies that both \( A(s) \) and \( B(s) \) are zero. Left is the solution
\[ H = -\frac{P_0}{1 - \gamma^2} \frac{e^{-\frac{z}{c_p} t}}{s} \]  
(4.50)

The inverse Laplace transform of this function is again a Heaviside given by
\[ \eta = -\frac{P_0}{1 - \gamma^2} h(t-x/c_p) \]  
(4.51)

Comparing this result with the one obtained using a complex harmonic pressure, it can be seen that the amplification is identical. Only the shapes are different, and they each correspond to the shape of the forcing pressure field.

Though the solution does not explicitly contain a phase, a logical phase could be defined to be zero when \( \eta \) and \( p \) have opposite signs, corresponding to the static situation, and to be \( \pi \) when \( \eta \) and \( p \) have the same sign. With this definition, also the phase is identical to the one found using a complex harmonic pressure.

Thus, the amplitude and phase portraits are identical to those shown in figure 4.2.

It should be noted that the solution found above could also have been found by assuming \( \eta \) proportional to \( p \), i.e. \( \eta = kp \) and entering this directly into equation (4.39). This includes differentiation of Dirac's delta function \( \delta(t) \).
4.3.5 Pressure jump, linear friction
Including the linear friction term the equation to be solved is again

\[ c_s^2 \eta_{xx} - \eta_t - \alpha \eta_x + c_s^2 P_{xx} = 0 \]  \hspace{1cm} (4.52)

The Laplace transformed equation then becomes

\[ c_s^2 H_{xx} - (s^2 H - s \eta(x,0) - \eta_t(x,0)) - \alpha(s H - \eta(x,0)) + c_s^2 P_{xx} = 0 \]  \hspace{1cm} (4.53)

Using the same initial conditions - the surface at rest - three terms are canceled out, and the transformed wave equation can be written

\[ H_{xx} - \frac{1}{c_s^2} (s^2 + \alpha s) H = - P_{xx} \]  \hspace{1cm} (4.54)

To find a particular solution \( H \) is again assumed to be proportional to \( P \), i.e. \( H = kP \), and \( H \) and \( P \) are inserted in the equation. Rearranging, the constant is given by

\[ k \left[ (1 - \gamma^2) - \frac{\alpha \gamma^2}{s} \right] = -1 \]  \hspace{1cm} (4.55)

The complete solution then becomes

\[ H = A(s)e^{-\frac{s - \sqrt{s^2 + \alpha s}}{c_s}} + B(s)e^{\frac{s - \sqrt{s^2 + \alpha s}}{c_s}} - \frac{P_0}{s(1 - \gamma^2) - \alpha \gamma^2} e^{-\frac{s}{c_p}} \]  \hspace{1cm} (4.56)

for \( s > 0 \). Using the same argumentation as before \( A(s) \) and \( B(s) \) must be zero, and the solution can be rewritten to

\[ H = -\frac{P_0}{1 - \gamma^2} \frac{1}{s - \alpha \frac{\gamma^2}{1 - \gamma^2}} e^{-\frac{s}{c_p}} \]  \hspace{1cm} (4.57)

The inverse Laplace to this solution is

\[ \eta = -\frac{P_0}{1 - \gamma^2} \frac{1}{t - x/c_p} e^{\frac{\gamma^2}{1 - \gamma^2}(t-x/c_p)} \]  \hspace{1cm} (4.58)

This expression consists of three factors. The two first are seen to be identical to the solution without friction and the third is therefore a modification to this solution. This becomes more clear if the solution is written as
\[
\eta = \begin{cases} 
- \frac{P_0}{1 - \gamma^2} e^{\frac{-\gamma}{1 - \gamma^2} \psi} & \psi > 0 \\
0 & \psi < 0 
\end{cases}
\tag{4.59}
\]

where \(\psi = t - x/c_p\). The amplitude is seen to diverge for \(\gamma\) approaching 1, as in the no friction case. For \(\gamma < 1\), the modification factor, and therefore also \(\eta\), will increase exponentially with time (or \(\psi\)). For \(\gamma > 1\), \(\eta\) may also approach infinity, but now the third term damps \(\eta\) exponentially in time. This is illustrated on figure 4.11.

![Figure 4.11. Response to a moving pressure jump in the linear friction case. a) \(\gamma < 1\), b) \(\gamma > 1\).](image)

The fact that \(\eta \to \infty\) for \(\gamma \to 1\) may seem to indicate no damping. However it should be noted that the spectrum of the Heaviside function includes infinite frequencies, and that the response found for a complex harmonic pressure in the linear friction case also approaches infinity when the frequency approaches infinity.

The behavior of the solution for \(\gamma\) less than unity is somewhat non-physical and different from what would be expected. Often this will be the actual situation, and one would not expect the amplification to explode in a quasi-stationary situation, \(\gamma \ll 1\).

To check if the solution found is really a solution to the shallow water equation, the \(\eta\) found by Laplace transforming the equation was inserted in the original equation. For this purpose the computational algebra programme MapleV was used. Since this programme tends to produce a large amount of output, the printout, including explaining text, is placed in Appendix A.

A physically more logical solution, obtained by changing the sign in the exponential function, was tested as well. This solution would produce a damped response in the quasi-stationary case. This possible solution proved to be incorrect.

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4.4 Closed basin

So far an unlimited area has been considered and subsequently solutions have been determined without the use of boundary conditions. In nature the presence of boundaries will result in reflections, and the previously found solutions can therefore only exist for a short period of time, until reflections occur. In the following paragraphs the effect of closed boundaries will be investigated for a one-dimensional channel of constant depth. Adequate boundary conditions are introduced at the closed boundaries to determine the specific solution. The equation is solved using a Fourier expansion method. The method is described in [11].

4.4.1 Free waves

In the free wave case there is no driving pressure, i.e. \( p = 0 \). Further neglecting bottom friction, the wave equation to be solved is

\[
c^2 \eta_{xx} = \eta_{tt} \quad 0 < x < l
\]  \hspace{1cm} (4.60)

where \( c \) is the shallow water velocity. Considering a closed basin of length \( l \), the boundary conditions at both ends are determined by setting \( Q \) to zero in the linearized momentum equation. In general, the boundary conditions are

\[
\eta_x(0,t) = -p_x(0,t) \quad \eta_x(l,t) = -p_x(l,t)
\]  \hspace{1cm} (4.61)

For free waves they reduce to the Neumann conditions

\[
\eta_x(0,t) = 0 \quad \eta_x(l,t) = 0
\]  \hspace{1cm} (4.62)

A separated solution to (4.60) is

\[
\eta(x,t) = X(x) T(t)
\]  \hspace{1cm} (4.63)

Insertion of this solution into equation (4.60) yields

\[
- \frac{T''}{c^2 T} = - \frac{X''}{X} = \lambda
\]  \hspace{1cm} (4.64)

where \( \lambda > 0 \). From these two differential equations, \( X \) and \( T \) are determined to

\[
X(x) = C \cos \beta x + D \sin \beta x
\]
\[
T(t) = A \cos \beta c t + B \sin \beta c t
\]  \hspace{1cm} (4.65)

where \( \beta^2 = \lambda \). Now the boundary conditions in (4.62) require that \( X(0) = X(l) = 0 \). Differentiating the expression for \( X \) and inserting the first condition, \( D \) is found to be zero. Inserting the second condition we get
\[-C \beta \sin \beta l = 0\]  \hspace{1cm} (4.66)

To avoid the obvious solution \( C = 0 \), \( \beta \) should be a solution to \( \sin \beta l = 0 \), i.e.

\[\beta = \frac{n\pi}{l} \hspace{1cm} n = 1, 2, 3, \ldots \]  \hspace{1cm} (4.67)

There are thus infinitely many solutions to equation (4.60). The value \( \lambda = 0 \) should be checked separately. To do so, it is inserted in equation (4.64). This yields

\[X''(x) = 0 \hspace{1cm} T''(t) = 0\]  \hspace{1cm} (4.68)

and thus

\[X(x) = C + Dx \hspace{1cm} T(t) = A + Bt\]  \hspace{1cm} (4.69)

Differentiation and insertion of the Neumann conditions for \( X \) yields that \( X \) is equal to \( C \), i.e. \( \lambda = 0 \) is an eigenvalue. The complete solution is then written

\[\eta(x, t) = A_0 + B_0 t + \sum_{n=1}^\infty \left(A_n \cos \frac{n\pi ct}{l} + B_n \sin \frac{n\pi ct}{l}\right) \cos \frac{n\pi x}{l}\]  \hspace{1cm} (4.70)

This solution is the typical standing wave pattern with discrete wave lengths and wave periods that may be found from equation (4.67) and \( L = cT \). The two first terms represent the mean water level and a constant source term, and are thus of no interest here.

The exact solution described by the coefficients, depends on the initial conditions. In case of initially no waves, the result is again the null-solution.

If linear friction is included, the wave equation may be written

\[c^2 \eta_{xx} = \eta_{tt} + \alpha \eta_t \hspace{1cm} 0 < x < l\]  \hspace{1cm} (4.71)

The boundary conditions are the same as in equation (4.61-62), the Neumann conditions, and again the separated solution (4.63) is inserted. This yields the two differential equations

\[-\frac{T'' + \alpha T'}{c^2 T} = -\frac{X''}{X} = \beta^2\]  \hspace{1cm} (4.72)

The solution to the equation in \( X \) including boundary conditions is shown above. The equation in \( T \) becomes

\[T'' + \alpha T' + c^2 \beta^2 T = 0\]  \hspace{1cm} (4.73)
and the solution for $\alpha < 2c\pi/l$ and $\beta > 0$ is given by

$$T(t) = e^{-\frac{\alpha}{2}t} \left( A \cos \omega_d t + B \sin \omega_d t \right)$$

$$\omega_d = \sqrt{c^2 \beta^2 - \alpha^2/4}$$  \hspace{1cm} (4.74)

This result is seen to be analogue to the one obtained for free waves in unlimited areas, as given in equation (4.25). For $\beta = 0$, the solution to (4.73) is

$$T(t) = A + B e^{-\alpha t}$$  \hspace{1cm} (4.75)

The complete solution may then be written

$$\eta(x,t) = A_0 + B_0 e^{-\alpha t} + \sum_{n=1}^{\infty} e^{-\frac{\alpha}{2}t} \left( A_n \cos \omega_{d,n} t + B_n \sin \omega_{d,n} t \right) \cos \frac{n\pi x}{l}$$  \hspace{1cm} (4.76)

The solution in the linear friction case is thus also a standing wave pattern, with exponentially damped amplitude.

As in the unlimited area case, also viscosity can be included, and it enters the equations the same way as friction does.

### 4.4.2 Fourier expansion method

Including the forcing of a pressure field, the equation to be solved, including linear friction for generality, is

$$c_s^2 \eta_{xx} - \eta_{tt} - \alpha \eta_t + c_s^2 p_{xx} = 0 \hspace{1cm} 0 < x < l$$  \hspace{1cm} (4.77)

The two boundary conditions are

$$\eta_x(0,t) = -p_x(0,t) \hspace{1cm} \eta_x(l,t) = -p_x(l,t)$$  \hspace{1cm} (4.78)

Furthermore the initial conditions are set to

$$\eta(x,0) = 0 \hspace{1cm} \eta_t(x,0) = 0$$  \hspace{1cm} (4.79)

Again the harmonic pressure field is chosen as forcing term, and it is written

$$p(x,t) = p_0 \cos (\omega t - kx)$$  \hspace{1cm} (4.80)

where $k$ is defined as $2\pi/L$.

In the method of Fourier expansion all quantities present in the equation are expanded in Fourier cosine series. The cosine expansion should be chosen because the solution to the homogeneous
(4.87) \( (j)^n u \frac{1}{p} \int \frac{dx}{x} = \int \frac{dx}{x} \cos u \int \frac{dx}{x} \cos u \int \frac{dx}{x} = (j)^n u \) \\

and likewise for \( v \nabla \).

(4.86) \( (j)^n u \frac{1}{p} \int \frac{dx}{x} \cos u \int \frac{dx}{x} \cos u \int \frac{dx}{x} = (j)^n u \)

Integration by parts from equation (4.86) yields

\( \frac{1}{p} \int \frac{dx}{x} \int \frac{dx}{x} \cos u \int \frac{dx}{x} = \int \frac{dx}{x} \int \frac{dx}{x} \cos u \)

Leibnitz's rule for differentiation of integrals

(4.84) \( (i)^m \frac{\partial}{\partial x} = (i)^m u - (i)^m v \frac{\partial}{\partial x} \)

This can be written

\( [i]\int_0^1 \frac{dx}{x} \cos u \int \frac{dx}{x} \cos u \int \frac{dx}{x} = \int \frac{dx}{x} \cos u \int \frac{dx}{x} \cos u \int \frac{dx}{x} = (i)^m u \)

Now, multiply the wave equation with the cosine term and integrate over the interval \([0,1]\)

(4.83) \( \frac{1}{x} \int \frac{dx}{x} \cos u \int \frac{dx}{x} = (i)^m \)

The same is done to the pressure to obtain

(4.82) \( \frac{1}{x} \int \frac{dx}{x} \cos u \int \frac{dx}{x} = (i)^m \)

where the coefficients \( u \) and \( v \) are determined by

(4.8) \( \frac{1}{x} \cos u \frac{1}{x} \int \cos u \frac{1}{x} \int \cos u \frac{1}{x} = u \)

The problem was expanded this way. In this case we get

Analytical solutions
The expression for \( w_n \) is rewritten using Green's second identity
\[
\int_a^b \left( -X_1''X_2' + X_1'X_2'' \right) dx = \left[ -X_1'X_2 + X_1X_2' \right]_a^b
\] (4.88)

Using \( \lambda_n \) for \( n\pi/l \), \( w_n \) can be manipulated to
\[
w_n(t) = -\frac{2}{l} \int_0^l \eta \lambda_n^2 \cos \lambda_n x \, dx - \frac{2}{l} \left[ -\eta_0(x, t) \cos(n\pi) - \eta(x, t) \lambda_n \sin(n\pi) \right]_0^l
\]
\[
= -\lambda_n^2 \eta_0(t) - \frac{2}{l} \left( -\eta_0(l, t) \cos(n\pi) + \eta_0(0, t) \right)
\] (4.89)
\[
= -\lambda_n^2 \eta_0(t) - \frac{2}{l} \left( \eta_0(0, t) - (-1)^n \eta_0(l, t) \right)
\]

Entering the expressions for \( u_n \), \( v_n \) and \( w_n \) in the wave equation yields
\[
\eta_n''(t) + \alpha \eta_n'(t) + c_s^2 \lambda_n^2 \eta_n(t) = c_s^2 \rho \eta(t) + c_s^2 \frac{2}{l} (-\frac{n^2 \pi^2}{l^2 - n^2 \pi^2}) \left( \eta_0(l, t) - \eta_0(0, t) \right)
\] (4.90)

The last term contains the boundary conditions. They would have been left out if the expansion of \( \eta \) had just been entered into the equation, followed by differentiation by term.

The boundary conditions are found from the original expression for the pressure, i.e.
\[
\eta_0(0, t) = -p_s(x, t) \bigg|_{x=0} = -p_0 k \sin(\omega t)
\]
\[
\eta_0(l, t) = -p_s(x, t) \bigg|_{x=l} = -p_0 k \sin(\omega t - kl)
\] (4.91)

The Fourier coefficients of \( p_{sn} \) can be found by direct integration of (4.83). This yields
\[
p_n(t) = 2p_0 k^2 \frac{kl}{k^2 l^2 - n^2 \pi^2} \left( (-1)^n \sin(\omega t - kl) - \sin(\omega t) \right)
\] (4.92)

After some rearrangements, the equation to be solved can be written
\[
\eta_n''(t) + \alpha \eta_n'(t) + c_s^2 \lambda_n^2 \eta_n(t) = a \left( \sin(\omega t) - (-1)^n \sin(\omega t - kl) \right)
\] (4.93)

with initial conditions \( \eta_n(0) = 0 \) and \( \eta_n'(0) = 0 \). The constant \( a \) is given by
\[
a = -2 c_s^2 p_0 \frac{k}{l} \frac{n^2 \pi^2}{k^2 l^2 - n^2 \pi^2}
\] (4.94)

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4.4.3 Solution for no friction

Equation (4.93) and the initial conditions were solved in the no friction case using MapleV, see Appendix A. The solution can be written

\[ \eta_n(t) = A \left[ (1 - (-1)^n \cos kl) \sin \omega t + (-1)^n \sin kl \cos \omega t \right. \\
\left. - (-1)^n \sin kl \cos \lambda_n c t + \frac{\omega}{\lambda_n c} \left((-1)^n \cos kl - 1\right) \sin \lambda_n c t \right], \]

(4.95)

\[ A = -2c^2 p_0 \frac{k}{l} \frac{n^2 \pi^2}{k^2 l^2 - n^2 \pi^2} \frac{1}{\lambda_n^2 c^2 - \omega^2}, \quad \lambda_n = \frac{n \pi}{l} \]

Introducing \( \Delta = l/L \) and \( \gamma = c_r/c \), it is rewritten

\[ \eta_n(t) = A \left[ (1 - (-1)^n \cos 2\pi \Delta) \sin 2\gamma \Delta \omega_1 t + (-1)^n \sin 2\pi \Delta \cos 2\gamma \Delta \omega_1 t \right. \\
\left. - (-1)^n \sin 2\pi \Delta \cos n \omega_1 t + 2\Delta \gamma \frac{n}{n} \left((-1)^n \cos 2\pi \Delta - 1\right) \sin n \omega_1 t \right], \]

(4.96)

\[ A = \frac{4p_0}{\pi} \frac{n^2}{4\Delta^2 - n^2} \frac{\Delta}{4\gamma^2 \Delta^2 - n^2}, \quad \omega_1 = \frac{2\pi \cdot c}{2l} \]

Note that \( \omega = 2\gamma \Delta \omega_1 \), since it is determined by \( \omega = ck \). The expression between parenthesis is a harmonic variation in time of limited magnitude, and \( A \) is a factor depending on \( n, \gamma \) and \( \Delta \).

First, consider only the factor \( A \). It can be seen that resonance may not only occur when

\[ 4 \Delta^2 = n^2 \quad \Rightarrow \quad L = \frac{1}{n} 2l \]

(4.97)

i.e. when the wave length of the pressure field coincide with a 'natural wave length' (standing waves), but also if

\[ 4\gamma^2 \Delta^2 = n^2 \quad \Rightarrow \quad T = \frac{1}{n} T_1 \]

(4.98)

i.e. when the frequency of the pressure coincide with one of the natural frequencies of the basin. This corresponds to the formulas for natural frequencies mentioned in Chapter 2.

For free shallow water waves, these two situations are inseparable, because wave length and wave period are connected through \( L = cT \), where \( c \) is constant. For the atmospheric pressure wave, however, we are (in principle) free to choose both wave length and wave period, because the velocity \( c_r \) is not necessarily constant, and the two situations may thus occur independently. On the other hand, for both situations to occur simultaneously, the pressure wave velocity must
correspond to the shallow water velocity, i.e. $\gamma$ must be 1.

The factor $A$ has been plotted as a function of $n$ and $\gamma$ in figure 4.12.

![Figure 4.12. The factor $A$ as a function of $n$ and $\gamma$. $\Delta = 10.5$.](image)

The two lines of resonance, $n = 2\Delta$ and $n = 2\Delta \gamma$, described by equation (4.97-98) are clearly present in the figure ($\Delta = 10.5$, $n = 21$ and $n = 21\gamma$).

Now consider the whole expression (4.96). In order to compare it to factor $A$, it has been plotted the same way in figure 4.13.

![Figure 4.13. An example of the full expression for $\eta_0(t)$ plotted as a function of $n$ and $\gamma$. $\Delta = 10.5$.](image)
From this figure it is clear that considering the factor $A$ alone is not sufficient. The two lines, $n = 2\Delta$ and $n = 2\Delta \gamma$, are still visible, but now resonance only occurs at the point of intersection. Thus, for resonance to occur the pressure wave should still travel approximately at the shallow water velocity.

### 4.5 Conclusions of the analysis

The most important conclusion that may be drawn from the found analytical solutions is, that if the atmospheric pressure disturbance travels at a velocity near the free shallow water wave velocity, i.e. when the parameter $\gamma$ is close to unity, resonance is likely to occur. The influence of friction on the amplification is largest for long period pressure waves. On the other hand, for long period pressure waves significant resonance occurs for a wider interval of $\gamma$-values.

From the solutions to the case of a moving pressure jump and the case of a closed basin, no additional conclusions could be drawn. The solutions found were unrealistic and uncondensed respectively.
Sensitivity test for numerical model

In the previous chapter, analytical solutions to the linearized one-dimensional shallow water wave equations were found. The purpose of this was to gain some first ideas of the physical behavior of the pressure generated waves in a simple model.

In this chapter a similar one-dimensional numerical model DUCHESS is used to investigate some physical and numerical aspects of the simulation of pressure generated long waves. The sensitivity of the model as well as assumptions made, will be tested so that later results can be evaluated.

5.1 Approach

This chapter is aimed at the following two subjects:

A model setup based on rules of experience and the waves expected to develop is made. The numerical and physical model parameters estimated this way define a standard model setup that is tested to finally choose an appropriate model setup for use in later simulations.

The importance of different pressure field characteristics will be investigated to determine which characteristics should be given special attention later on. The speed and the frequency of the atmospheric pressure wave have in the analytical case been shown to be important, and it will be investigated if this statement holds in the numerical case.

To realize these two aims the following approach is used:

A standard experiment is designed using the DUCHESS programme. The setup of this standard experiment is based on some general rules of experience, the analytical results and some rough estimations. The results of the standard experiment will be used to test the sensitivity of the chosen model parameters by comparing them with those of alternative experiments.

The numerical and physical model parameters to be varied are:

- Grid spacing. The largest possible grid spacing without an unacceptable loss of accuracy should be chosen. Since the total processing time and the size of input files are proportional to the number of grid points, this is a very important parameter.

- Time step. The largest acceptable time step should be chosen. Since processing time is inversely proportional to the time step, also the time step is important when trying to keep down processing time. Also the size of input files may depend on the time step.

- Bottom-roughness. To determine the effect of bottom friction on the generated waves. The results can be compared to the analytical results.
- Viscosity. In the analytical model these terms were left out. It is therefore of interest to determine the effect of the viscosity on the generated waves.

- Convective terms. Also these terms were left out in the analytical model, because they were expected to be of little influence.

### 5.2 Setup of the model

#### 5.2.1 Geometry

Though the DUCHESS programme solves the full two-dimensional set of equations, a simple one-dimensional model can be setup by choosing only one (internal) grid point per cross section and closing the longitudinal boundaries on both sides.

The length of the model is chosen to 500 km, i.e. about the size of the southern North Sea. The same grid size is used in both directions. The width of the model then depends on the grid spacing and thus the number of grid points in the longitudinal direction. This fact has no influence on the results. The bottom level is set constant to about 32 m, approximately the average depth of the southern part of the North Sea.

At both ends the boundaries are closed, i.e. the model simulates a closed basin. In the initial situation the surface is at rest.

#### 5.2.2 The pressure field

The driving force in all simulations is a moving atmospheric pressure field covering the whole area. The characteristics of the pressure field were chosen based on an article by Gomis et al. [19] reporting the presence of non-dispersive pressure waves during seiche events in the inlets of the Balearic Islands, and the real event of a passing cold front described in the next chapter. The pressure was thus assumed to be a harmonic non-dispersive wave moving from the left boundary towards the right, see figure 5.1, with a speed of 15 m/s and a period of 1000 s or 16.7 min. This results in a wave length of 15 km. The amplitude of the pressure wave was set to 5 cm water column or approximately 5 mbar. Using the harmonic wave furthermore facilitates comparison with the analytical results.

#### 5.2.3 Numerical parameters

Considering the analytical solution, the generated waves are assumed to have wave lengths and periods of at least the same order of magnitude as the pressure wave generating them. The characteristics of the pressure wave are therefore used to estimate the numerical model parameters.

A first estimate of the smallest number of grid points per wave length is about 15. This number applies in the time domain as well as in space. With a wave length of 15 km and a period of about 17 min, this rule of thumb results in the first estimates $\Delta x = 1$ km and $\Delta t = 1$ min.

For this combination the one-dimensional Courant number $C_r$ defined as

$$C_r = \frac{c \Delta t}{\Delta x}$$  \hspace{1cm} (5.1)

is less than about 1.5. The DUCHESS scheme is unconditionally stable, but for reasons of accuracy
C, should not be taken too large. In the user manual it is recommended not to use Courant numbers larger than about 10. In [5] a Courant number smaller than 2 is recommended for large scale simulations.

The total simulation time is set to 24 hours, including a warm-up period. This is more than three times the time needed for a free wave to travel all the way through the model, and it is therefore possible to observe what happens, when waves are reflected in the model.

5.2.4 Physical parameters
In the DUCHESS programme different physical effects can be included. This is done through the following three coefficients:

- Coriolis coefficient, \( C_n \). It is possible to take the Coriolis effect into account in DUCHESS through this coefficient. In a one-dimensional model however this has no meaning and in the standard experiment it is therefore set to zero (default). The Coriolis coefficient is further described in paragraph 3.2.

- Friction coefficient, \( Fr \). The bottom friction can be modeled using different formulations. In the sensitivity tests, the friction coefficient is set constant, and in the standard experiment \( Fr = 0.005 \) is used.

- Viscosity coefficient, \( \nu \). By changing this coefficient, more or less diffusion can be entered in the model. The influence of this parameter is often neglected in simulations of long waves [10] or it is as standard set to \( \nu = 10 \text{ m}^2/\text{s} \) [5]. In the standard experiment \( \nu \) is set to 10 m\(^2\)/s.

5.2.5 Model output
During computation different output can be requested in DUCHESS. In this study the simulation results are analyzed based on different series of the surface elevation. Obviously the surface elevation will vary in time as well as space. It was therefore chosen to extract time series in five stations equally spread out over the area. These stations are numbered from the left boundary and shown in figure 5.1.

![Station numbers diagram](image)

Figure 5.1. The 1D model with station numbers.

To get an even better view of how the surface develops in time and space, a surface profile of the whole basin was requested at every time step. From these profiles a 'movie' of the sea state could be made. For this purpose a MATLAB programme was developed to actually let the waves come alive. This way of visualizing the results is of course not suited for presentation in a report, but it has been a very good support, when interpreting the results obtained from the ordinary times
series. A printout of the MATLAB programme 'movie2d.m' is shown in Appendix E.

5.3 Experiments

In this paragraph the experiments carried out, in order to determine a suitable setup of the later models, are described. The standard DUCHESS command file is shown in Appendix E. Figures with prefix C are collected in Appendix C.

5.3.1 Standard experiment
The standard experiment is based on the model setup described in the previous sections. It is used as a reference throughout the sensitivity analysis to compare alternative model setups to the standard one.

Figure 5.2. Unsteady response as a result of cold-starting the model.

The model was first cold-started, i.e. the amplitude of the pressure wave was set to the maximum value over the whole area from the beginning. This resulted in the unsteady response shown in figure 5.2, because the sudden change in air pressure causes the stabilizing and driving forces to be unbalanced. The situation is equal to a sudden impact on the surface. To prevent this unsteady response, the amplitude was gradually increased from zero to the maximum value over ten pressure wave periods, using a third order polynomiun with zero gradients at both ends. Applying this warm-up procedure the surface has time to respond to the pressure field. The resulting time series for station 1 to station 5 are shown on figure C.2.1-5 in Appendix C. The time series of station 1 (at the left boundary) is shown on figure 5.3.
Figure 5.3. Time series of the surface elevation in station 1.

In the beginning the wave height grows to a moderate level of about 3 cm. For comparison, the simple analytical expression for an unlimited area and no friction predicts an amplitude of about 12 cm in the actual case. The moderate growth can be explained by the presence of the closed boundary. The waves are not generated instantaneously, but need some distance to grow. After a little more than 30,000 seconds the first waves reflected at the right boundary arrive. Again a gradual growth to a maximum value is observed, but this time the growth is much larger, even larger than the amplitude expected from the analytical expressions. This is probably due to standing waves. After approximately 65,000 seconds the amplitude increases again. This is probably due to the return of waves generated at the left boundary after being reflected at the right boundary.

Figure 5.4. Time series of the surface elevation in station 5.

The time series for station 5 (at the right boundary) is shown in figure 5.4. Here the waves gradually grow to a height of 35 cm, or more than twice the expected. This is due to standing waves developing here right from the beginning.
The time series for station 2 to station 4 (approximately 1/4, 1/2 and 3/4 points) are shown on figure C.2.2-4. These time series are less clear, but a difference in phase speed resulting in time varying nodes may be responsible for the alternating amplification and reduction of amplitude.

So far the analysis was based on the ordinary times series. Considering the simplicity of the problem, actually very little new information could be extracted from these time series, and a number of the conclusions were based on expectations of well-known phenomena like reflection and standing waves. To improve the understanding of how the model responds to the pressure field, the next step is to actually visualize the waves in the model. As described this can only be done 'on screen', but a sample of seven wave profiles at equally spaced time steps, is shown in figure C.3.1-7. Two characteristic 'frames' are shown in figure 5.5.

![Figure 5.5. Two characteristic frames from a 'movie' of the standard experiment.](image)

These profiles confirm two important conclusions based on the times series:

Close to the left boundary the wave height remains low and grows to a maximum value over the first 50-60 grid points, i.e. about 50-60 km. In the beginning this value decreases slightly, when moving further inside the model and stays constant until near the right boundary. Later this changes due to the developing standing wave pattern.

Close to the right boundary a standing wave pattern develops early in the computation. This pattern spreads from the right into the model area. Finally the whole area is participating in a mixed, standing-wavelike motion. For an unlimited area this is not realistic, and when comparing with the analytical results, this is compensated for, see paragraph 5.4.

In the following paragraphs the results of different sensitivity experiments are presented. In this presentation, the standard reference experiment is denoted with a zero, i.e. $a_0$, $b_0$, etc., and these are thus not new experiments.
5.3.2 Grid variation
To find a suitable grid spacing, $\Delta x$, to be used, the following experiments were carried out:

- a0) Standard experiment, $\Delta x = 1$ km
- a1) Standard experiment, but with $\Delta x = 250$ m
- a2) Standard experiment, but with $\Delta x = 500$ m
- a3) Standard experiment, but with $\Delta x = 2$ km
- a4) Standard experiment, but with $\Delta x = 4$ km

The analysis of the grid sensitivity has again been based on series of the surface elevation. The first 6 hours of the ordinary time series resulting from these experiments have been plotted and compared for station 5. These plots are shown in figure C.4.1-4. From these plots the following observations can be made:

There is practically no difference in amplitude or phase of the signal between the two experiments a1 ($\Delta x = 250$ m) and a2 ($\Delta x = 500$ m).

There is a small phase difference between the standard experiment a0 (Standard, $\Delta x = 1$ km) and experiments a1 and a2. This difference seems to stabilize quickly, after only a few periods. This may be explained by considering the analytically determined phase difference. This is, in the linear friction case, a time-independent function of the free wave velocity. Since this in turn is depending on the setup of the model, one should expect a phase difference between all experiments. The deviation in phase is thus an indication of difference in the free wave velocity. There is also a small difference in amplitude between these two experiments, but this is acceptable to reduce the simulation time.

There is a considerable phase and amplitude difference between experiment a0 and experiment a3 ($\Delta x = 2$ km). The difference between experiments a0 and a4 ($\Delta x = 4$ km) is very large. This might be a result of differences in the node-anti-node pattern of the standing waves developing.

Figure 5.6. Plot of the overall maximum amplitude as a function of the distance from the left boundary. Experiment a0-a4.
Therefore the overall maximum amplitudes in time as well as space were calculated over the first 6 hours. This was done from the surface profiles used for the wave movies. A plot of the overall maximum amplitudes as a function of the distance is shown in figure C.5.1-4. From these plots the following conclusions can be drawn:

The bad performance of experiment a3 and a4 as observed from the times series is supported by the overall maximum amplitude plots. The differences are thus not (only) a result of differences in the standing wave pattern. It should be noted that the close-up in figure C.5.2 does not include station 5.

It would be desirable to use the 2 km grid of experiment a3 to save computational time, if a reduction of the time step could enhance the performance. Therefore these additional experiments were carried out:

a3) Standard experiment, but with $\Delta x = 2$ km
a5) Standard experiment, but with $\Delta x = 2$ km and $\Delta t = 30$ s
a6) Standard experiment, but with $\Delta x = 2$ km and $\Delta t = 15$ s

Again the overall maximum amplitude as a function of time and space is calculated. These are shown in figure C.5.5 and C.5.6. It can be seen that the reduction of time step does not sufficiently compensate the larger grid spacing. Thus a 2 km grid cannot handle these waves satisfactorily.

The conclusion of experiments a0 through a6 is that a grid spacing of $\Delta x = 500$ m would be ideal, but since this is primarily an explanatory study of the phenomenon, a grid spacing of $\Delta x = 1$ km can be accepted, in order to save computational time.

5.3.3 Time step variation
The second most important parameter affecting the simulation time is the time step $\Delta t$. To find a suitable time step, the following experiments were carried out:

b0) Standard experiment, $\Delta t = 1$ min
b1) Standard experiment, but with $\Delta t = 15$ s
b2) Standard experiment, but with $\Delta t = 30$ s
b3) Standard experiment, but with $\Delta t = 2$ min
b4) Standard experiment, but with $\Delta t = 4$ min

Also the analysis of the time step sensitivity is based on series of the surface elevation. The first 6 hours of the ordinary time series have been plotted and compared for station 5. These plots are shown in figure C.6.1-4. From these plots the following observations can be made:

There is practically no difference in amplitude and phase between experiments b1($\Delta t = 15$ s) and b2 ($\Delta t = 30$ s).

The difference in amplitude between experiment b0 (Standard, $\Delta t = 1$ min) and experiments b1 and b2 is very small and varies in time. There is practically no difference in phase.

Experiments b3 ($\Delta t = 2$ min) and b4 ($\Delta t = 4$ min) both differ considerably from b0, in amplitude and phase. For b3 the difference in amplitude varies in time.

Again the overall maximum amplitude is calculated for all experiments. The results are shown in
figure C.7.1 and C.7.2, and they support the conclusion of experiments b0 through b4 that a time step of $\Delta t = 1$ min is sufficient to handle the waves generated.

If one should choose between decreasing grid spacing or time step to increase the accuracy, figure 5.7 is useful. It shows that using half the time step is more efficient than using half the grid spacing. The same applies to choosing between increasing grid spacing or time step, but then figure C.5.1 and C.7.1 should be compared. It shows that increasing the time is less hazardous than increasing the grid spacing.

![Overall maximum amplitude](image)

Figure 5.7. Overall maximum amplitude as a function of the distance from the left boundary. Comparing experiment a0-a2 and b0-b2.

### 5.3.4 Bottom roughness

The bottom roughness parameter has been tested to determine the importance of bottom friction. If the numerical scheme is able to utilize the simplifications obtained from neglecting these terms, this could be desirable to save computational time. To check the influence of friction the following experiments have been carried out:

1) Standard experiment, $Fr = 0.005$
2) Standard experiment, but with $Fr = 0.000$
3) Standard experiment, but with $Fr = 0.003$
4) Standard experiment, but with $Fr = 0.007$

The results from station 3 are shown in figure C.8.1 and C.8.2. They can be interpreted as follows:

In the beginning no difference whatsoever is observed between the four experiments. At this point the waves have just been generated and not yet traveled any distance. Thus no energy has been dissipated due to friction. When the first reflected waves arrive a considerable amplitude difference is observed, since these waves have traveled 500km through the model. When the second reflected wave train arrives, the difference increases even further, since these waves have traveled two times through the model.
No phase differences are observed, although the bottom friction is theoretically expected to cause a small phase lag according to equation (4.37).

The time needed to run the model with and without bottom friction was recorded to see if leaving out bottom friction saves time. No difference was observed, thus the conclusion of experiments c0 through c3 is that the friction used in the standard experiment may be used.

5.3.5 Viscosity variation
The viscosity term was left out during the analytical solution of the wave equation, since this is non-linear and furthermore regarded to be of little importance to the problem. This statement was tested out by the following experiments:

- d0) Standard experiment, \( v = 10 \text{ m}^2/\text{s} \)
- d1) Standard experiment, but with \( v = 0 \text{ m}^2/\text{s} \)
- d2) Standard experiment, but with \( v = 100 \text{ m}^2/\text{s} \)

The results of these experiments are shown in figure C.9.1 and C.9.2. From these figures it is clear that:

In the beginning there are no differences at all between the three experiments. When the first reflected waves arrive, a very small difference between experiment d0 (Standard, \( v = 10 \text{ m}^2/\text{s} \)) and d1 (\( v = 0 \text{ m}^2/\text{s} \)) occurs. The unphysical viscosity used in experiment d2 (\( v = 100 \text{ m}^2/\text{s} \)) reduces the amplitude considerably (not the phase), but since there is no obvious reason for choosing such a high value, it can be concluded that viscosity has very little influence on the generated waves.

To see if leaving out viscosity could reduce computational time, the time needed to run the model with and without viscosity was recorded. Since no difference in computational time was observed, it may be concluded that viscosity can be included without loss of time.

5.3.6 Convective terms
Also the convective term was left out in the linear analysis of the wave equation to keep the equations simple. To check the influence of such a simplification, and to check if the convective terms could be left out in the two-dimensional model, experiments with and without the convective terms were carried out, i.e.:

- e0) Standard experiment, including convective terms
- e1) Standard experiment, excluding convective terms

The results of these experiments are shown in figure C.10.1 and C.10.2. From these figures it can be seen that:

The convective terms introduce non-linearities in the response, but have only a small effect on the amplitude, unlike the bottom friction and the viscosity terms.

It is noteworthy that a reduction in computational time of about 15% was achieved by switching the convective terms of, and the possibility of doing so should be considered. Here it is chosen to keep the convective terms, because the aspect of time is not predominant.
5.4 Pressure characteristics

In this paragraph some of the characteristics connected to the physical problem are investigated. Again this is done based on the standard setup of the model.

5.4.1 Front speed
The analytical solution to the shallow water equations indicated a strong relation between the amplification and $\gamma$, the speed of the atmospheric pressure wave relative to the shallow water wave velocity. To test if this is also the case in the numerical model, the following experiments were carried out:

- f0) Standard experiment, $\gamma = 0.85$
- f1) Standard experiment, but with $\gamma = 1.10$
- f2) Standard experiment, but with $\gamma = 1.05$
- f3) Standard experiment, but with $\gamma = 0.95$
- f4) Standard experiment, but with $\gamma = 0.90$
- f5) Standard experiment, but with $\gamma = 0.70$

The $\gamma$-value can be varied by either changing the shallow water wave velocity, $c_p$, i.e. by changing the water depth in the model, or the atmospheric pressure wave velocity, $c_p$. Here it is chosen to vary the water depth. This is done because changing the actual pressure wave velocity would lead to a change in either the wave length or the period of the generated waves and thus, in principle, require a new model setup for every experiment including sensitivity tests of all model setups.

When changing the shallow water wave velocity only one parameter has to be changed, the depth.

Based on the results of the experiments, the overall maximum amplitude could be calculated. This is shown in figure 5.8 and in figure C.11.1 and C.11.2.

![Overall maximum amplitude f0-f5](image)

Figure 5.8. Overall maximum amplitude as a function of the distance from the left boundary for experiments f0-f5. Close-up of the first 200 km.

From this plot it can be seen that the maximum amplitude grows to a certain level. If this (first) maximum, which has a different location for the six experiments, is taken as a measure of the amplification, the following table can be filled in
### Table

<table>
<thead>
<tr>
<th>Experiment [yr]</th>
<th>γ [-]</th>
<th>h [m]</th>
<th>Amplification analytical*</th>
<th>Amplitude measured</th>
<th>Amplification measured**</th>
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<td>3.6</td>
<td>0.36</td>
<td>3.6</td>
</tr>
<tr>
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<td>4.8</td>
<td>0.42</td>
<td>4.2</td>
</tr>
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<td>9.7</td>
<td>0.69</td>
<td>6.9</td>
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<tr>
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</tr>
<tr>
<td>f4</td>
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<td>28.3</td>
<td>5.3</td>
<td>0.67</td>
<td>6.7</td>
</tr>
<tr>
<td>f5</td>
<td>0.70</td>
<td>46.8</td>
<td>2.0</td>
<td>0.17</td>
<td>1.7</td>
</tr>
</tbody>
</table>

** Calculated from equation (4.32.a).
* Corrected due to approximately double amplitude of standing waves.

---

**Figure 5.9.** Theoretical and simulated amplification as a function of γ.

In figure 5.9 the amplifications from the simulations are plotted against the ones obtained in the analytical solution (4.32.a). Here it should be noted that the maximum amplitude in the model has been corrected by dividing the actually measured amplification by 2. This is done to roughly take the standing waves into account. The results are seen to be in good agreement with the theoretical results.

#### 5.4.2 Growth distance

Returning to figure 5.8, another important observation can be made. The distance between the left boundary and the place of maximum amplitude, the growth distance, increases with the amplification. This indicates two things:

The waves need a longer distance to reach their maximum. An identical trend can be found for the growth in time, compare figure C.11.2. This is however less obvious due to
the influence of the warm-up procedure.

The standing wave pattern changes dramatically when \( \gamma \) is increased. This was observed in 'movies' of the results.

![Graph showing maximum amplitude vs. distance of maximum](image)

Figure 5.10. Maximum amplitude as a function of the growth distance.

In figure 5.10 the maximum amplitude has been plotted against the growth distance and the relation is seen to be nearly linear.

The growth distance is interesting when determining the minimum size of the simulation area to be used in the 2D simulations. Simply choosing the largest growth distance found in experiments f0 through f5 is not ideal, since, in order to keep down the time and hard disk capacity needed to run the model, the area should not be chosen too large. In contrast to the 1D model, there exist no free stretches in the North Sea where \( \gamma \) remains close to one for a longer period in time or space, so here the waves will not have the opportunity to grow like here. This means that an area of smaller dimensions than first indicated by figure 5.10 may be used.

To avoid undesired reflections, boundaries should on the other hand not be placed too close to the area of primary interest. This counteracts a decrease of the computational domain.

### 5.4.3 Pressure amplitude

The influence of the amplitude of the pressure wave has been investigated, since this parameter could be important to the growth distance. This was investigated by making the following experiments:

- g0) Standard experiment, \( \Delta P = 5 \) mbar
- g1) Standard experiment, but with \( \Delta P = 10 \) mbar
- g2) Standard experiment, but with \( \Delta P = 15 \) mbar

The experiments are set up by simply changing the scaling factor of the pressure in the DUCHESS command file.
The overall maximum amplitude of these experiments are shown in figure C.12.1 and C.12.2. From these plots it can be found that the amplitude is almost linear, and that the growth distance as well as the growth time are nearly independent of the pressure amplitude.

5.5 Conclusion of sensitivity test

The results of varying the relative pressure field velocity, $\gamma$, agree well with the analytical solutions, if the response of the numerical model is compensated for the standing wave pattern.

The generated waves need some distance and time to reach the maximum amplification. The distance is proportional to the amplification, i.e. the larger the amplification, the longer the distance needed for the waves to fully develop.

For the pressure field used in this test, a suitable model setup should have $\Delta x = 1$ km and $\Delta t = 1$ min. Bottom friction, viscosity and convective terms are less important.
Numerical cold front simulations

The first 2D simulations indicated the presence of energy at frequencies higher than the largest measurable frequency, the Nyquist frequency $f_N$, in the original air pressure data. It was therefore decided to continue the simulations in a 1D model to determine the origin of this high frequency energy. The limitations of the 1D model are accepted, since the primary aim was to make some first exploratory simulations. The results of the simulations are presented in this chapter.

6.1 Setup of the model

In this paragraph the standard model setup is described. This setup is later tested in the sensitivity tests described in paragraph 6.3.

6.1.1 Modeling the pressure field

In figure 6.1 a worked-out sketch of the cold front passing on the 21st of April 1993 is shown. Within this period seiching occurred in Rotterdam. The corresponding surface elevation measurements from three locations in the southern North Sea have been analyzed by Veraart [12], and it was therefore chosen to try to simulate this period. As a logical consequence the front characteristics to be used in the simulations were extracted from this sketch. This yields an approximate front speed between 7 and 10 m/s and a front direction of -65 degrees, i.e. more or less normal to the Dutch coast.

Figure 6.1. The meteorological situation on the 21st of April 1993. Passing cold front. The point * indicates the location of Europlatform. The line towards the Dutch coast is the line along which the bottom profile for the 1D model was extracted.
The pressure fields used in the simulations were generated from a pressure time series measured in the period of 18th to 23rd of April at Europlatform located approximately 40 km off the Dutch coast. This period includes the event of the cold front described above. The original time series has a sample frequency of $f_s = 1/600$ Hz and a discretization of 10 Pa (0.1 mbar). A plot of the measured pressure is shown in figure 6.2.

![Measured air pressure](image)

Figure 6.2. The measured air pressure used in the simulations.

The pressure field was modeled by assuming the measurements of Europlatform to be representative for all points in the model. Not at the same time, however, but delayed according to distance and front speed. Popularly speaking, the front was assumed to have the shape of the 'frozen' time series and then displaced at a constant speed, $c_p$, over the model. This is equivalent to displacing figure 6.2 to the left. In the standard simulations the front speed is set to $c_p = 10$ m/s.

It should be noted that to avoid the cold start as described in the previous chapter, the pressure field was assumed constant until $t = 0$. At this point in time, the front 'enters' the model area at the open boundary. This way of starting the model results in a relative time delay equal to the distance between the open boundary and the location of Europlatform divided by the front speed. Where considered important, this delay has been approximately compensated for. This approach is accepted to avoid a cold start and because the exact point in time is not that important at this stage. Neither does the 1D model used include the exact location of Europlatform.

The method of generating the pressure field is investigated in the following. The front speed is subject to separate experiments.

6.1.2 Geometry
In the standard model, the bottom profile along the line in figure 6.3 was extracted from the original, interpolated 2D bottom profile. The chosen line forms an angle of -40 (nautic) degrees, and is 407 km long. This is not consistent with the actual direction of the front, but was chosen in order to have a reasonably long model and to introduce an open boundary in the model.

The model has an open and a closed boundary. At the open boundary the surface condition $h = 0$.
Numerical cold front simulations.

is specified. The effect of this boundary condition was investigated in one of the experiments described below.

![Selected bottom profile](image)

Figure 6.3. Bottom profile of the Southern North Sea and the line along which the bottom profile used in the 1D simulations is extracted.

6.1.3 Model parameters, standard setup
The standard model setup is based on a bottom file extracted from the 2D bottom file as described above. A grid spacing of $\Delta x = 2$ km is used, since this is a practical lower limit for simulations on a PC in the 2D case. For the same reason the time step is set to $\Delta t = 2$ min.

The lateral boundaries are closed, and at the open boundary the surface elevation is set to zero. Initially the surface is at rest.

The remaining numerical parameters are: $v = 10$ m$^2$/s, $Fr = 0.005$. No additional numerical damping is used, i.e. the DUCHESS parameter NUM is set to 0.5.

The total simulation time is set to 432000 s (5 days), i.e. the duration of the whole air pressure time series. This includes a warm-up period of nearly three days before the seiche event and a period of about one day after.

The model output is again sampled in five stations equally spread out along the model. These stations are numbered 1 to 5 beginning at the open boundary and ending near the coast.

6.2 Methods of analysis
In this paragraph the method of spectral analysis is described. This method was chosen to analyze the results of the sensitivity tests described in the following paragraphs, because a visual inspection of the results gives little quantitative information about frequencies of the waves

53
generated.

### 6.2.1 Spectral density function

The surface elevation $\eta(t)$ in a fixed point is often considered to be a sum of a number of harmonic wave components, i.e. to be periodical in time. $\eta(t)$ may then be written

$$\eta(t) = a_0 + \sum_{k=1}^\infty a_k \cos(\omega_k t + \varphi_k)$$  \hspace{1cm} (6.1)$$

where $a_0$ is the mean water level, and $a_k$, $\omega_k$ and $\varphi_k$ are amplitude, angular frequency and phase respectively of the $k$th wave component, with $\omega_k = k\omega_1$.

In general a function $x(t)$ defined on a finite interval $[0,T]$ may be represented by the complex Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{i2\pi f_k t}$$  \hspace{1cm} (6.2)$$

where $f_k = k/T$. This expression thus includes both positive and negative frequencies. Multiplying equation (6.2) by $e^{-i2\pi f_m t}$ and integrating over one period $T$, only terms with $k$ equal to $m$ are non-zero, and the complex coefficients $c_k$ can be determined to

$$c_k = \frac{1}{T} \int_0^T x(t) e^{-i2\pi f_k t} dt$$  \hspace{1cm} (6.3)$$

The values $|c_k|$ are called the amplitude spectrum, and $|c_k|^2$ the energy spectrum. If $x(t)$ is real-valued, $c_k$ and $c_{-k}$ are complex conjugates, and the two spectra are even functions of $k$, i.e. of the frequency. Equation (6.1) then contains the same information as (6.2). The coefficients of (6.1) and (6.2) are connected through

$$a_0 = c_0 , \quad a_k = 2 \cdot |c_k| \quad k = 1, 2, ...$$  \hspace{1cm} (6.4)$$

This is used later to determine the mean square value $\overline{x^2}$ of the signal. If $x$ is considered to be a random variable, the term variance is used for the mean square value. In the following the term variance is used.

In the discrete case, consider a time series $x(t)$ of length $T$, sampled at $N$ equally spaced points a distance $\Delta t$ apart. This means that

$$t_n = n\Delta t \quad T = N\Delta t$$  \hspace{1cm} (6.5)$$

where $t_n$ is the $n$th time step. The discrete version of equation (6.3) then becomes
Numerical cold front simulations.

\[ c_k = \frac{1}{T} \sum_{n=0}^{N-1} x_n e^{-i2\pi k n/N} \Delta t \quad (6.6) \]

Usually \( c_k \) is calculated for the positive frequencies \( f_k = k/T, k = 0, 1, 2 \ldots N-1 \), and equation (6.6) may then be rewritten to

\[ c_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-i2\pi k n/N} = \frac{1}{N} X_k \quad k = 0, 1, 2 \ldots N-1 \quad (6.7) \]

where \( X_k \) are the coefficients of a discrete Fourier transform of \( x \). Note that the discrete version of \( c_k \) is symmetric about \( k = N/2 \). This is seen by observing that

\[ e^{-i2\pi (N-k) n/N} = e^{-i(2\pi - 2\pi k n/N)} = e^{i2\pi k n/N} \]

or

\[ c_{N-k} = c_k^* = c_{-k} \quad (6.8) \]

where \( ^* \) denotes the complex conjugate. Actually \( c_k \) is periodic with period \( N \). This also shows that the \( c_k \) for \( k > N/2 \) may be seen as representing the negative part of the spectrum, and that energy in the signal with frequencies above \( k = N/2 \), equivalent to the Nyquist frequency \( f_c = 1/(2\Delta t) \), is folded back into negative frequency range. This phenomenon is called aliasing, and is (only) a problem if energy is present above the Nyquist frequency.

Now the two-sided spectral density function, \( S(f_k) \), can be defined through

\[ \lim_{\Delta f \to 0} \left[ S(f_k) \Delta f \right] = |c_k|^2 \quad (6.9) \]

In the discrete case, where \( \Delta f = 1/T \), this leads to an estimate of \( S(f_k) \) given by

\[ \hat{S}(f_k) = T|c_k|^2 = \frac{\Delta f}{N} |X_k|^2 \quad k = 0, 1, 2 \ldots N-1 \quad (6.10) \]

with a frequency resolution \( \Delta f = 1/T \). As mentioned \( X_k \) is symmetric about \( k = N/2 \) as a result of using negative frequencies in the expression for \( x(t) \). The negative frequencies are folded into the positive range by taking the first half of \( S(f_k) \) multiplied by two. This yields the one-sided spectral density function

\[ G(f_k) = S(f_k) , \quad G(f_k) = 2 S(f_k) \quad k = 1, 2 \ldots N/2 \quad (6.11) \]

The one-sided spectrum may also be defined directly by
\[ G(0) \Delta f = a_0^2, \quad \lim_{\Delta f \to 0} [G(f_k) \Delta f] = \frac{1}{2} a_k^2 \quad k = 1, 2 \ldots N/2 \] (6.12)

Either way the result is equation (6.11). A plot of a one-sided spectral density estimate is shown in figure 6.4.

![Figure 6.4. Example of a spectral density estimate.]

A practical problem when calculating the spectral density of finite time series is spectral leakage. Spectral leakage is a result of the finite numbers of frequencies and discontinuities at the ends of the time series. To prevent this leakage, the time series are often windowed by e.g. a Hanning window [18]. The Hanning window is given by

\[ w(n) = 1 - \cos^2 \left( \pi \frac{n}{N} \right) \quad n = 0, 1, 2 \ldots N-1 \] (6.13)

This subject is described in detail in [16] and [18]. When the time series is windowed, the discrete Fourier transform should be multiplied by a factor \( k_w \) to correct the magnitude of the estimate. In general this factor is given by

\[ k_w = \frac{\sqrt{N}}{|w(n)|} \] (6.14)

where \( |w(n)| \) is the norm of the windowing function. For the Hanning window, \( k_w \) is \( \sqrt{8/3} \).

A disadvantage of windowing is that information is lost. When analyzing longer time series, most of this information can be regained by splitting up the time series in subseries, analyzing each of these as described above, and averaging them to obtain an average spectrum. To regain the information, the subseries should be overlapping each other. Often 50% overlapping is used [18]. Splitting up the time series reduces the spectral resolution, but increases the reliability of the estimate.

In this study, the results of the sensitivity tests are analyzed by first calculating the two-sided spectral density in (6.10) and from this calculating the one-sided spectrum through (6.11). The Hanning window is used and the subseries are overlapped 50% when using subseries.

6.2.2 Variance and energy

According to Parseval's theorem, the variance \( \sigma^2 \) of the signal may be written
\[ \bar{x}^2 = \sum_{k=-\infty}^{\infty} |c_k|^2 = a_0^2 + \sum_{k=1}^{\infty} \frac{1}{2} a_k^2 \]  

(6.15)

The variance in a given frequency band may thus according to (6.12) be written as

\[ \Delta(\bar{x}^2) = \sum_{k=-k_0}^{k_0} G(f_k) \Delta f \quad k = 0, 1, 2 ... N/2 \]  

(6.16)

Often the term 'energy' is used instead of 'variance', because the average energy is connected to the variance through [17]

\[ E_p = \frac{1}{2} \rho g \bar{\eta}^2 \quad E = \rho g \bar{\eta}^2 \]  

(6.17)

The problem of using the energy as a measure is that it is often unclear, which of the two energies in equation (6.17) is meant. The variance, on the other hand, is a well-defined quantity. The factor \( \rho g \) is therefore left out in the following, and only the variance is considered.

The results of the experiments in paragraph 6.4 are analyzed by calculating the variance through equation (6.16) in different frequency bands as a function of time. More specifically, the spectral density is calculated once every hour, using an 8 hours Hanning window. From this, the spectral variance is calculated and split up into four frequency ranges, viz. [0.1,0.2] mHz, [0.2,0.5] mHz, [0.5, 2.0] mHz and [2.0 mHz, \( f_s \)], corresponding to the ranges used by Veraart [12]. The relatively long time window is needed to obtain reasonably accurate estimates of the variance at the lowest frequencies. An example of a plot of the time-varying variance is shown in figure 6.5

![Spectral variance. Frequency range 0.1 - 0.2 mHz.](image)

**Figure 6.5. Example of the variance as a function of time.**

A MATLAB programme, VAR, has been developed to completely automate the calculations of the spectral density as a function of time, the variance in certain frequency intervals as a function of time, and the average spectral density, as described above and applied in paragraph 6.3 and 6.4.

Also the standard MATLAB programme PSD (Power Spectral Density estimate) has been used because of its built-in facilities. However, inconsistencies with results from the VAR programme led to inspection of the source text, and it was discovered that PSD calculates and plots the first half of the two-sided spectral density given by (6.10), but leaves out the sample-period µ. This means that different time series, containing essentially the same signal but sampled at different frequencies, will have totally different spectral density estimates. This as a warning to unexperienced users. Based on PSD, an improved version PSDLIN has been implemented to calculate the average one-sided spectral density in equation (6.11). Also the plotting facilities are improved.
The programme VAR, PSDLIN, and a modified version of the subroutine PSDCHK needed for running PSDLIN are found in Appendix E and F.

6.3 Sensitivity tests

In this paragraph the results of the sensitivity tests carried out are presented by means of time series, but especially by means of the spectral density estimates described in paragraph 6.2.

Two types of plots are used, viz. plots with logarithmic and with linear vertical scale respectively. These plots both have advantages and disadvantages. Logarithmic plots are useful if more than a few different scales are of interest at the same time. On the other hand the relative scale can better be seen in linear plots. It should be noted that the logarithmic plots are averaged spectra, using a 256-point fast Fourier transform and 50% overlapping, yielding a resolution of 0.03 mHz, whereas the linear plots are 'maximum resolution spectra', i.e. the whole time series has been used to obtain the finest possible resolution, this being approximately 0.002 mHz.

6.3.1 Standard experiment

The results of the standard experiment are shown in Appendix D. In figure D.2.1 the time series of the surface elevation in station 1 through station 5 are shown. From these times series the following can be observed:

As expected, the amplitude grows with time as well as distance. A little unexpected, this latter growth is approximately linear with the distance from the open boundary. This is due to the very large distance needed for the longest waves to grow to a constant level. This fact is more easily seen from the spectral density plots, described later in this section.

The maximum surface elevation is in the order of magnitude 13.5 cm in station 5. This maximum includes the waves of nearly-zero frequencies mentioned above.

The signal is rather noisy, which indicates that the resolution may be too small for the waves actually generated.

The first part of the time series indicates the delay due to difference in distance from the open boundary, where the front enters.

Because of the noisy response, the further analysis is based on the spectral density of the surface elevation.

In figure D.3.1-2 the average spectral density of the time series in figure D.2.1 is shown in a semi-logarithmic coordinate system. In these plots a considerable amount of energy is present above the Nyquist frequency \( f_n \) of the original air pressure data \( (f_n = 0.83 \text{ mHz}) \), which is considered unphysical. Two peaks are present, at 1 mHz and 2 mHz respectively. In the logarithmic plot the second group of peaks seems to be significant, because it covers a wide range of frequencies, and because it is not present in station 5. However, this peak almost disappears in a linear plot, which may be seen in figure D.4.1 or D.4.2, showing constrained and unconstrained linear plots of the spectral density. On the other hand, the first peak around 1 mHz, which appeared less spectacular in the logarithmic plot, now becomes very visible. The problem is that the vertical scale of the
logarithmic plot (automatically) stretches over many decades, in this case no less than 14. This is why logarithmic plots in general should be used with caution.

The following observation can be made from the linear scale plots in figure D.4.1-2. Note the differences in horizontal and vertical scales:

Between 0 and 0.1 mHz the spectral density (energy) grows towards the coast. This accounts for the growth of the maximum surface elevation mentioned above. The lower the frequency the longer the growth continues. The longest waves obviously need the longest distance to grow to a maximum level.

The spectral density between 0.1 and 0.8 mHz remains constant except for station 5. It is here considerably larger than in the other stations. This difference may be due to standing waves or shoaling.

Around 1 mHz, equivalent to the first peak mentioned above, the spectral density remains more or less constant throughout the model.

At frequencies around 2 mHz, equivalent to the second peak mentioned, the spectral density is practically invisible.

6.3.2 Variation of model parameters
The unphysical energy, above 0.8 mHz, might arise from insufficient modeling, so it was decided to test the chosen model setup. The following experiments on the model parameters were carried out on grid spacing (a0-a2), time step (b0-b2) and numerical damping (c0-c4):

a0) Standard experiment, i.e. Δx = 2 km
a1) Standard experiment, but with Δx = 1 km
a2) Standard experiment, but with Δx = 4 km

b0) Standard experiment, i.e. Δt = 2 min
b1) Standard experiment, but with Δt = 1 min
b2) Standard experiment, but with Δt = 4 min

c0) Standard experiment, i.e. NUM = 0.50 (no additional damping)
c1) Standard experiment, but with NUM = 0.51
c2) Standard experiment, but with NUM = 0.55
c3) Standard experiment, but with NUM = 0.60

The results of these simulations, in the form of spectral density plots, are shown in Appendix D.5 to D.7. From these plots the following can be observed:

Grid spacing, experiment a0-a2, figure D.5.1-4:

The energy between 0 and 0.1 mHz is approximately the same for all three simulations.

In the frequency range 0.1 to 0.8 mHz less energy is present in experiment a1 (Δx = 1 km) than in the standard experiment a0 (Standard, Δx = 2 km). In experiment a2 (Δx = 4 km) a lot more energy is present.
The unphysical energy above the Nyquist frequency 0.83 mHz is still present in all three experiments in station 1. In station 5, however, no energy is present above 0.6 mHz in experiment a2 (Δx = 4 km).

Time step, experiment b0-b2, figure D.6.1-4:

Again the spectral densities in the frequency range 0 to 0.1 mHz are nearly identical.

A moderate difference in spectral density is observed between experiment b0 (Standard, Δt = 2 min) and experiment b1 (Δt = 1 min) in the frequency range 0.1 to 0.8 mHz.

The spectral density of experiment b2 (Δt = 4 min) differs considerably from those of experiments b0 and b1 for frequencies above 0.1 mHz.

Numerical damping, experiment c0-c3, figure D.7.1:

The additional numerical damping has an unexpectedly large effect on waves of all frequencies above 0.1 mHz. For instance, increasing the NUM factor to 0.51 reduces the spectral density to two thirds of the one obtained with a NUM factor of 0.50 (default).

The damping has the largest effect on waves with the highest frequencies.

It may be concluded that a model with grid spacing 1 km and time step 1 min seems to be required to simulate the generated waves sufficiently. Changing the front speed, however, may change this, because this is actually equivalent to changing the wave length in the generating pressure waves, which is connected to the frequencies through $L = c_p T$, i.e. increasing the front speed increases the wave length of the pressure wave.

Also it is concluded that additional numerical damping as introduced by the NUM factor in DUCHESS is too efficient, in the sense that when trying to remove the high frequency energy, it also removes energy from the frequencies of interest. This method of removing the high frequency energy is thus not desirable.

6.3.3 Open boundary conditions
The influence of the conditions applied at the open boundary in the standard experiment on the higher frequency waves was unknown, and it was therefore decided to test these conditions as well. The results are of general interest, however.

The following experiments, combining different boundary conditions at the open boundary, were carried out:

- d0) Standard experiment, i.e. a simple $h = 0$ condition
- d1) Standard experiment, but applying energy smoothing at the open boundary, by setting the DUCHESS parameter HBSMTH to 0.5 (max).
- d2) Standard experiment, but defining the open boundary as weakly reflecting. This is in DUCHESS done by using the command NREFL.
- d3) Applying both energy smoothing and using a weakly reflecting boundary.
The results of these simulations are shown in figure D.8.1-4. From these the following observations can be made:

The energy smoothing, as introduced through the HBSTMH parameter in DUCHESS, has no effect at all. The spectra of experiment d0 and d1 are identical. The same applies for the corresponding experiments with a weakly reflecting boundary, i.e. for experiment d2 and d3.

The introduction of a weakly reflecting boundary reduces the energy of all frequencies above 0.1 mHz dramatically. In station 5 the energy above 0.8 mHz has even almost disappeared.

Comparing the spectrum of d0 and the spectrum of two times d2 as shown in figure D.8.3 these are seen to be quite similar up to about 0.8 mHz. This could be an indication that the weakly reflecting boundary efficiently absorbs the waves reflecting from the open boundary when using a simple h boundary.

The conclusion of the tests described above is that energy smoothing at the boundary, as introduced by the HBSTMH parameter, has no effect. Apparently it only influences waves of the highest frequencies, which are not present in the model.

The high frequency energy in station 5 is reduced to a minimum by introducing the weakly reflecting boundary. However, this way of removing the high frequency energy is not desirable.

It is evident that the definition of correct boundary conditions at the open boundary is vital to the energy level of all frequencies. The physically correct boundary condition is a non-reflecting boundary, but this is not implementable in the numerical model. The simple boundary condition applied in experiment d0 and d1 will certainly act as a reflecting boundary, due to the fixed surface. The weakly reflecting boundary applied in experiment d2 and d3 is a linearized version of the exact characteristic relationship, and will thus partially absorb the waves approaching from the interior of the model [6]. The efficiency of this approach is however undocumented, apart from the above mentioned observation, and should maybe be investigated further. Based on the observations above, the weakly reflecting boundary is preferred.

### 6.3.4 Pressure field generation

The model experiments described above provided no useful solution to the problem of the presence of energy above the Nyquist frequency of the original air pressure data. Therefore a number of experiments on the generation of the DUCHESS input file were carried out.

The generation procedure consists of up to three steps as shown in the processing scheme in Appendix D.9, namely interpolation (Step 1) and filtering (Step 2) of the original pressure time series followed by the translation to all other points in the model (Step 3). Once these three steps have been completed, the results may be affected by the fact that DUCHESS considers the pressure to be constant between to readings from the input file. This is important if the time step in DUCHESS is smaller than the time step of the input pressure file as in experiment e0, e1 and e3.

The following experiments were carried out:

- e0) Standard experiment. Here the most straightforward method was applied. The original
pressure data were used, i.e. no interpolation (Step 1) and no filtering (Step 2). When translating the time series to other points in the model (Step 3), linear interpolation was used. The DUCHESS input file was sampled at 1/600 Hz. In DUCHESS, however, a time step of 2 min was used in the computation, and a new pressure value thus not available at every new time step. Then DUCHESS considers the pressure to be constant between two readings (as opposed to e.g. linear interpolation). In Appendix D.9 this is indicated by a 'yes' in the column 'Interaction DUCHESS'.

e1) The same procedure as applied in e0, but now the closest pressure value was used in Step 3 instead of linear interpolation. That is, a more primitive method.

e2) The same as e0, but now sampled at 1/120 Hz (Step 3).

e3) The same as e0, but now the original air pressure signal has been lowpass filtered using a Butterworth lowpass filter with a cut-off frequency of 0.75$f_c$ (Step 2).

e4) The same as e3, but now sampled at 1/120 Hz (Step 3).

e5) First the original air pressure data were interpolated, to a sample frequency of 1/120 Hz, using cubic spline interpolation (Step 1). Then it was lowpass filtered using a Butterworth filter with a cut-off frequency of 0.20$f_c$, equivalent to $f_c$ of the original pressure signal (Step 2).

e6) The same as e5, but now the interpolated signal is not filtered (Step 2).

In figure D.10.1-2 the spectral densities of all seven experiments are plotted using a constrained, linear vertical axis. Corresponding semi-logarithmic plots comparing the different experiments are shown in figure D.10.3-8. From these plots the following is observed:

e1) Choosing the closest value when generating the DUCHESS input file creates a little more energy in general. A considerable amount of energy is present at frequencies above 2 mHz in station 1, which is characteristic for this simulation. It should be noted that this way of generating the input file is most primitive possible, and that the simulation only was included to see the effect of such an approach.

e2) Generating the 1/120 Hz input file from the original 1/600 Hz pressure data removes virtually all energy above the Nyquist frequency $f_c$. Very small amounts of high frequency energy are still present.

e3) Lowpass filtering the original pressure data only removes the energy between 0.75$f_c$ and $f_c/0.75$, i.e. between 0.6 and 1.1 mHz. The energy above 1.1 mHz is still present.

e4) Compared to experiment e2, filtering the original pressure data before generating a 1/120 mHz input file, only removes energy that is actually physical and thus wanted in the model.

e5) Interpolating the original pressure data and filtering it, increases the energy between approximately 0.4$f_c$ and $f_c$, i.e. 0.3 and 0.8 mHz. It practically removes all energy above $f_c$ completely.
e6) This experiment is identical to experiment e5 for frequencies up to \( f_c \). For higher frequencies there is a difference. In station 5, experiment e6 contains more energy between \( f_c \) and 1.0 mHz, which is even visible in the linear plots.

The most important conclusion of these tests is that DUCHESS should not be allowed to interact, i.e. the pressure field should be specified at every time step of the computation. If the DUCHESS routine were changed to interpolating linearly between readings, results like those of experiment e2 would be expected. This would then be a feasible alternative to specifying the pressure field at all time steps. In the simulations above, DUCHESS was asked to 'resample' the pressure signal at a frequency five times higher. There is probably a lower limit, where specifying the pressure field at all time steps is no longer necessary, and it is expected that this limit could be significantly increased by implementing a linear interpolation routine.

6.3.5 Conclusions sensitivity tests

From all the tests described in the previous sections, it is concluded that a finer model is needed to simulate the event of the passing cold front sufficiently accurate. It also appears that a weakly (non-)reflecting boundary condition should be used at the open boundaries.

Furthermore, the pressure field should be specified at every time step in DUCHESS to avoid the generation of unphysical, high frequent energy. The needed interpolation in time may be done as a combination of interpolating the original pressure signal, and interpolating when generating the input file.

6.4 Final 1D simulations

In this paragraph the results of the final simulations are presented. An important difference with the results of the sensitivity tests is the way of analyzing and presenting the time series. In this paragraph the variance is calculated in four different ranges and plotted as a function of time. This approach is also used by Veraart [12], and therefore the results below may be compared to those presented by Veraart. The variance plots are roughly compensated for the delay due to starting the pressure field at the open boundary instead of at the location where it was measured, as described in paragraph 6.1.

6.4.1 Model setup

Based on the conclusions above, a model with grid spacing \( \Delta x = 1 \text{ km} \) and time step \( \Delta t = 1 \text{ min} \) was set up. The pressure field was interpolated to 1/60 Hz from the original 1/600 Hz, without applying any filtering, and from this signal a DUCHESS input file was generated (see also the processing scheme in Appendix D.9).

With this model the following experiments were carried out:

\[
\begin{align*}
\text{f1}) & \quad \text{Front speed } c_p = 5 \text{ m/s} \\
\text{f2}) & \quad \text{Front speed } c_p = 10 \text{ m/s} \\
\text{f3}) & \quad \text{Front speed } c_p = 15 \text{ m/s} \\
\text{f4}) & \quad \text{Front speed } c_p = 20 \text{ m/s}
\end{align*}
\]

As mentioned earlier, the results of these simulations are analyzed by calculating the spectral variance every hour in the four frequency ranges 0.1 to 0.2 mHz, 0.2 to 0.5 mHz, 0.5 to 2.0 mHz.
Numerical cold front simulations.

and above 2.0 mHz (i.e. up to $f_c = 8.3$ mHz), corresponding to the ranges used by Veraart [12]. However, since practically no energy (variance) is present in the last range, this is left out of the presentation.

6.4.2 Standard simulation
As mentioned earlier, the actual front speed in the simulated period is approximately 7 to 10 m/s. Therefore, of the four simulations, f2 is considered to reproduce the real event best, and the results of this simulation are therefore discussed and compared to the results obtained by Veraart from measurements.

Figure 6.6 shows the simulation results, variances and surface elevation, of the simulation with front speed 10 m/s.

![Spectral variance plots and surface elevation graph](image)

Figure 6.6. Plot of the spectral variance calculated every hour, using an 8 hours Hanning window, for station 5 in experiment f2. Front speed 10 m/s.

From measurements, Veraart calculated and plotted the same quantities. His results are shown in figure 6.7. Note the different placing of labels, the different vertical scales, and that Veraart calculated the energy every two hours. Unfortunately a mistake must have been when producing these plots, because obviously the signal analyzed, having a maximum amplitude of about 10-15 cm, cannot possibly contain energy in the order of 1 m². The problem is probably a faulty unit in the plot. Instead of 'energy [m²]', perhaps it should have been 'energy [cm²]', or if energy is
really meant and *not* the variance suggested earlier in his report, it should have been 'energy [J/m²]', which again leaves the question of which energy is meant, potential or total energy, i.e. a factor 2.

![Graphs showing energy distribution over different frequency ranges.](image)

Figure 6.7. The results obtained by Veraart from measurements. The energy is calculated every two hours using a 12 hours Hanning window. Note the unrealistic energy level. The location is unknown.

To overcome this problem, which disables comparison with measurements, the surface elevations of the 20th and 21st of April were analyzed again. The results are shown in figure 6.8 comparing the spectral variance of measured and simulated elevations. From this figure it clearly seen that the order of magnitude of the results of Veraart is incorrect.

65
Figure 6.8. Spectral variance of the re-analyzed measurements (Europlatform) and of the simulation for the two ranges 0.2-0.5 mHz and 0.5-2.0 mHz.

From figure 6.6, 6.7 and 6.8 the following remarks can be made, when bearing in mind the error in the results of Veraart. These are referred to as 'measurements' and the results of this study as 'simulations':

In the first frequency range, 0.1 to 0.2 mHz, a significant amount of energy is present in the simulation. In the measurements, this range is dominated by tidal energy, probably due to spectral leakage, so unfortunately a comparison of results is not possible. This would have been interesting, because the second half of this range includes a peak of the amplification spectrum of Europlatform, see figure 2.4 [9].

In the second frequency range, 0.2 to 0.5 mHz, the shape and timing of the peak on the 21st of April of the simulation are almost identical to the measurements. A difference is the tail following the peak found in the measurements. This seems to have been caused by spectral leakage of tidal energy. The energy of the simulation is a little lower than that of the measurements, but agrees surprisingly well.

Also in the third frequency range, 0.5 to 2.0 mHz, the shape and timing of the peak in the simulation agree well with the measurements, but here the energy level of the simulation is much lower than that of the measurements. For the specific event, Veraart showed that waves in this range were not responsible for the seiches, but in general this range may
contain waves potential of causing seiches. The modeling of waves in this frequency range is thus insufficient.

The last frequency range used by Veraart, 2.0 to 20 mHz (not shown in figure 6.6), is outside the range of interest when considering seiches in the Port of Rotterdam [9], and furthermore, the energy found in the measurements is probably a result of spectral leakage of wind wave energy. This range is therefore not considered further.

It is clear from the observations above that long wave energy as the one actually measured is generated in the simulation.

![Spectral variance. Frequency range 0.1 - 0.2 mHz. ap120](image)
![Spectral variance. Frequency range 0.2 - 0.5 mHz.](image)
![Spectral variance. Frequency range 0.5 - 2.0 mHz.](image)
![Pressure in water-coloumn](image)

Figure 6.9. Plot of the variance of the air pressure, translated into meters water column.

Veraart suggests that the long waves might be generated by the fluctuations in wind speed and direction. Since the driving force in the simulations purely consists of a pressure field, this strongly indicates that the pressure alone may be responsible for the generation of the long waves in the specific situation. This statement is further supported by figure 6.9, showing the same type of spectral analysis applied to the measured air pressure data. Though the energy level is very low compared to that of the generated waves, it is clear that the relatively small fluctuations in the air pressure on the 21st of April must be responsible for the long waves generated in the model.

Even though long waves are generated, the results are not identical to the measurements. It is not a surprise that differences are observed with measurements, but the difference in energy level between simulation and measurements is large, especially in the frequency range 0.5 to 2.0 mHz.
How this can happen, is discussed in the following.

First it should be noted that Veraart used a 12 hour Hanning window to analyze the measured surface elevations, whereas in this study an 8 hour Hanning window was used. This was chosen, because theoretically the smallest possible window that provides sufficient resolution should be used when energy is present in a limited time interval, as is the case here. Using a too wide window, literally includes information from the past and the future outside the time interval of interest, thus actually leaking energy in time. On the other hand, using a narrower window reduces the spectral resolution, and thus the number of spectral points from which the variance in a given interval can be calculated. In other words, the result becomes less reliable. Specifically for this situation, the difference between using a 12 and an 8 hour window is that the variance (or energy) in the first three ranges mentioned above must be calculated from 4, 13 and 65 respectively 3, 9 and 43 spectral points. The effect on the calculated energy level is that a narrower window yields a higher peak in the energy level. The difference is illustrated in figure 6.10.

![Spectral variance. 8 hour vs 12 hour window](image)

Figure 6.10. Leakage of spectral energy in time. The difference between using an 8 hour (top) and a 12 hour (bottom) Hanning window.

Another difference is that for the simulation the variance is calculated every hour instead of every two hours. This has no significant influence on the magnitude, however, but the plots appear different and this should be born in mind when comparing result.

Apart from these differences in the analysis applied, the following remarks about the modeling of the event may help explain the differences between the simulation and measurements:

Obviously the 1D model cannot reproduce the 2D effects important to the problem, as for instance the transport of energy in two directions. The same applies for waves reflected at the closed boundary. This might lead to an increased energy level.

Using the 1D model, the actual locations of measurements and simulation are not the same. As shown by Veraart, the location, i.e. the distance from the coast and the water depth, might influence the energy level.

The way the pressure field is modeled may be too inaccurate. The permanent character of the front in lateral direction, and especially of the fast fluctuations,
could mean that the amplification of the long waves is allowed to continue longer than is really the case. This effect might lead to an increased energy level in the simulations. For 2D calculations, this problem will extend to the modeling of the pressure field in the cross direction as well.

As shown in the following section, the response is very sensitive to the front speed, so if the actual front speed was different or varied during the passage of the front, this would most likely result in a different energy level.

The absence of the wind field in the simulations may lead to either an increase or a decrease in the energy level, since wind and pressure are each other's cause and effect.

Of course the bottom friction coefficient, which in these simulations was set constant, may influence the result in either direction, depending on whether the used constant over- or underestimates the true friction. The friction coefficient used may have been estimated a little too high.

In the simulations carried out, only the pressure generated waves were present. In combination with other large amplitude waves, such as tides, the influence of the bottom friction on the pressure generated waves may change.

6.4.3 Simulations with different front speed
In figure D.11.1–4 the results of the four simulations f1–f4 are shown. Again the exact point in time where the variance is given, has been compensated for the delay due to starting the pressure field at the open boundary as described in paragraph 6.1. The compensation is different for every plot because different front speeds were used.

From these plots the following can be observed:

In experiment f1 (front speed \( c_p = 5 \text{ m/s} \)) the energy level is about three times lower than in f2 (standard, \( c_p = 10 \text{ m/s} \)), and a distinct second peak is present. The response looks more or less like the negative pressure field, equivalent to the inverted barometric effect.

The energy level is certainly sensitive to the front speed, as expected from the analytical study. This can be seen from figure 6.11, where the peak level has been plotted against the front speed used in the simulations. The average depth of the 1D model is 40 m, which yields a shallow water velocity of approximately 20 m/s. The results thus agree qualitatively with the analytical results of Chapter 4.

The generated waves travel with the speed of the front. This is seen from the fact that the timing of the peak is independent of the front speed, when corrected for the delay of the front, and then coincides with the peak of the air pressure itself. This observation is in agreement with the analytical results found in Chapter 4.
6.4.4 Conclusions and discussions of final simulations

The results of the final cold front simulations are very positive. It has been shown that the long waves analyzed by Veraart can be reproduced in the 1D model used by a pressure field alone, at least to the correct order of magnitude. No fluctuations in wind speed or direction is thus needed, as suggested by Veraart, but as mentioned these may of course play an important role as well.

The results agree not only qualitatively with the results of Veraart, but also with the analytical results obtained earlier. The amplification is clearly sensitive to the front speed, and the generated waves travel with the same speed as the front.
Conclusions and recommendations

7.1 Conclusions

The following can be concluded at the end of this study:

Analytical part
Analytical solutions to the one-dimensional shallow water wave equation, including the forcing of an atmospheric pressure field, were found for an unlimited area, assuming no bottom friction and linear bottom friction respectively. Solutions were found for a moving, harmonic pressure wave and a moving pressure jump respectively. The results show that long waves could be strongly amplified if the pressure disturbance is moving with approximately the shallow water velocity. The results obtained for the moving pressure jump are somewhat unphysical in the case of linear friction, since the amplification is unlimited. This is explained by the fact that the moving pressure jump, modeled by Dirac’s δ-function, theoretically contains infinitely high frequencies, for which the solution obtained for a harmonic pressure wave is also unlimited.

Sensitivity test
A numerical 1D model similar to the one treated analytically was set up using the DUCHESS programme. Also the driving pressure field was the same harmonic pressure wave. A number of sensitivity tests were carried out to determine a proper setup for use in later simulations, and some results were compared to the ones of the analytical study. The most important is that the peak in the amplification for pressure waves moving with the shallow water velocity is clearly reproduced.

It was also found that the distance and time needed for the waves to reach their individual maximum level of amplification increases linearly with the amplification level.

Cold front simulations
The intention of this study was to simulate the passage of a cold front on 21st April, 1993, in a 2D numerical model, since measurements from this event had been analyzed by Veraart in an earlier study. In the first simulations energy was present at high frequencies which could not be accounted for, and it was decided to return to a 1D model to determine the origin of this energy.

In this model, the period of 18th to 23rd of April 1993, which includes the passage of a cold front, was simulated. The model was driven by a simple pressure field based on the measured air pressure. Again a number of sensitivity tests of the model setup were carried out. From these tests it could be concluded that a finer model should be used and that a weakly reflecting boundary condition should be applied at the open boundary.

Further tests on the generation of the pressure field revealed that the pressure field should be interpolated at least linearly. In the present version of DUCHESS this means specifying the pressure field at (almost) every time step. If this is not done, DUCHESS ‘interacts’ by keeping the pressure constant, until the next pressure field specification. This is what caused the unphysical, high frequency energy.
Conclusions and recommendations.

With the refined model, the cold front was simulated again, and the variance (energy) of the response calculated every hour. This approach was also used by Veraart to analyse the actual wave measurements, and from comparing the results of the simulations with those of Veraart, it could be concluded that fluctuations in the pressure might indeed be responsible for the generation of the long waves measured.

7.2 Recommendations

The following is recommended for future work:

Analytical studies
Other ways of modeling the cold front than that of an abrupt pressure jump used in this study, could be tried. Doing so, a more 'correct' cold front model could be used, which is bound to require the use of more complex mathematical solution methods. The case of closed or semi-closed basins could obviously be further investigated.

Simulations
In this study the meteorology was implemented in the numerical model by assuming the pressure field to be 'frozen' and moving at constant speed. A more complex model based on the true meteorological processes in a cold front would make the results more reliable.

More events should be simulated, including different meteorological situations, especially events of passing cold fronts with different front speeds and storms. Also simulations using only wind speed and direction, and simulations with pressure and wind, to determine their combined effect, could be made, as well as simulations including the tide. Of course it would be obvious to proceed with 2D simulations, to see if important effects are left out in the 1D model.

The implementation of linear (or cubic) interpolation in DUCHESS would reduce the hard disk capacity needed. Maybe the generation of the pressure field could even be included as a subroutine.

Available data
More and better measurements are needed to investigate the true meteorological situation during seiche events. This could be used to improve the meteorological modeling.

The work by Veraart should be continued to increase the number of events for comparison with simulations.
References


References.


Symbols

\( \alpha \) : bottom friction coefficient [s\(^{-1}\)].
\( \beta \) : constant.
\( \gamma \) : ratio between \( c_p \) and \( c_s \).
\( \delta(t) \) : Dirac's delta function
\( \zeta \) : amplification factor.
\( \eta \) : surface elevation [m].
\( \lambda \) : eigenvalue.
\( \nu \) : viscosity coefficient [m\(^2\)/s].
\( \rho \) : density of water [kg/m\(^3\)].
\( \tau \) : bottom shear stress [N/m\(^2\)].
\( \phi \) : degree latitude [°].
\( \psi, \phi \) : arbitrary functions.
\( \Omega \) : angular frequency of the Earth rotation [rad/s].
\( \omega \) : angular frequency [rad/s].
\( \omega_d \) : angular frequency, damped [rad/s].
\( a_s \) : Fourier coefficient.
\( C \) : Chezy number, bottom friction [m\(^{1/2}\)/s].
\( c, c_s \) : shallow water velocity [m/s].
\( C_e \) : Coriolis coefficient [s\(^{-1}\)].
\( c_r \) : complex Fourier coefficient.
\( c_p \) : front speed [m/s].
\( C_r \) : Courant number.
\( D \) : depth [m].
\( E, E_p \) : mean wave energy density [J/m\(^2\)].
\( f \) : frequency [Hz].
\( f_c \) : Nyquist frequency [Hz].
\( Fr \) : bottom friction coefficient [-].
\( f_s \) : sample frequency [Hz].
\( g \) : gravitational acceleration [m/s\(^2\)].
\( G(f) \) : one-sided spectral density.
\( H \) : depth [m].
\( h \) : surface elevation [m].
\( h(t) \) : Heaviside step function.
\( \text{IHSMT}\) : energy smoothing at h boundary, DUCHESS.
\( i \) : imaginary unit.
\( k \) : wave number [m\(^{-1}\)].
\( k_w \) : scaling factor.
\( L \) : wave length [m].
\( l \) : length [m].
\( N \) : number of samples.
\( n \) : integer.
\( \text{NUM} \) : numerical damping, DUCHESS.
\( P, H \) : Laplace transformed of functions p and h.
\( P, p \) : atmospheric pressure divided by \( \rho g \) [m].
\( p \) : atmospheric pressure [Pa].
\( \dot{Q} \) : depth integrated velocity [m\(^2\)/s].
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<thead>
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<th>Symbol</th>
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<tr>
<td>$R_r$</td>
<td>Rossby radius [m].</td>
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<td>$s$</td>
<td>independent variable of the Laplace transformation.</td>
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<td>$S(f)$</td>
<td>two-sided spectral density.</td>
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<td>$T$</td>
<td>sample period [s].</td>
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<td>time [s].</td>
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<td>average velocity [m/s].</td>
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<td>$x, y$</td>
<td>space coordinates [m].</td>
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<td>phase [rad].</td>
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Appendix A - E:

Programs

D: Results, cold front simulations
C: Results, sensitivity tests
B: The Laplace Transform
A: Maple Calculations
MapleV calculations

Checking the solution found by Laplace transformation

1. The shallow water wave equation with linear friction
2. Insertion of \( p \) and \( \eta \) in the PDE
3. The four terms of the PDE
4. Substituting \( cp = \gamma cp \) into the PDE
5. Testing the solution by numerical evaluation
6. Plotting the solution

Checking the alternative solution (changed sign in \( \exp \)-function)

1. The shallow water wave equation with linear friction
2. Insertion of \( p \) and \( \eta \) in the PDE
3. The four terms of the PDE
4. Substituting \( cp = \gamma cs \) into the PDE
5. Testing the solution by numerical evaluation
6. Plotting the solution
Checking the solution found by Laplace transformation

1. The shallow water wave equation with linear friction

```maple
> restart;
> pde:=(c^2)*diff(eta,x$2)-alpha*diff(eta,t)+c^2*diff(p,x$2)=0;

pde := c^2 \frac{\partial^2 \eta}{\partial x^2} - \alpha \frac{\partial \eta}{\partial t} + c^2 \frac{\partial^2 p}{\partial x^2} = 0
```

```maple
> eta:=(1-lambda^2)*Heaviside(t-x/cp)*exp(-lambda^2*t/(1*lambda+1));

eta := \frac{c^2 \left(\frac{1}{1 - \lambda^2} \left(1 - \frac{x}{cp}\right) - \lambda^2 \left(1 - \frac{x}{cp}\right) \right)}{1 - \lambda^2}
```

NB! Note that \( \lambda \) (lambdax) is used instead of \( \gamma \) ('gamma'), since 'gamma' is a restricted variable in MapleV

```maple
> p:=p0*Heaviside(t-x/cp);

p := p0 \frac{c^2 \left(\frac{1}{1 - \lambda^2} \left(1 - \frac{x}{cp}\right) - \lambda^2 \left(1 - \frac{x}{cp}\right) \right)}{1 - \lambda^2}
```

2. Insertion of \( p \) and \( \eta \) in the PDE

```maple
> pde:=(c^2)*diff(eta,x$2)-alpha*diff(eta,t$2)+c^2*diff(p,x$2)=0;

pde := c^2 \frac{\partial^2 \eta}{\partial x^2} - \alpha \frac{\partial^2 \eta}{\partial x \partial t} + c^2 \frac{\partial^2 p}{\partial x^2} = 0
```

```maple
> p:=p0*Heaviside(t-x/cp);

p := p0 \frac{c^2 \left(\frac{1}{1 - \lambda^2} \left(1 - \frac{x}{cp}\right) - \lambda^2 \left(1 - \frac{x}{cp}\right) \right)}{1 - \lambda^2}
```

A.2
MapleV calculations.

\[ -\alpha \left( \frac{p_0 \text{Dirac} \left( t - \frac{x}{\text{cp}} \right)}{1 - \lambda^2} \% 1 \right) + \frac{p_0 \text{Heaviside} \left( t - \frac{x}{\text{cp}} \right) \lambda^2 \alpha \% 1 \right)}{\left( 1 - \lambda^2 \right) \left( \lambda - 1 \right) \left( \lambda + 1 \right)} \]

\[ \frac{cs^2 p_0 \text{Dirac} \left( 1, t - \frac{x}{\text{cp}} \right)}{\text{cp}^2} = 0 \]

\[ \% 1 := e^{\frac{x^2 \alpha \left( t - \frac{x}{\text{cp}} \right)}{\left( \lambda - 1 \right) \left( \lambda + 1 \right)}} \]

3. The four terms of the PDE

\[
\begin{align*}
> \text{term1} := \cos^2 \text{diff}(\text{eta}, x^2) &; \\
> \text{term2} := \text{diff}(\text{eta}, t^2) &; \\
> \text{term3} := \alpha \text{diff}(\text{eta}, t) &; \\
> \text{term4} := \cos^2 \text{diff}(p, x^2) &;
\end{align*}
\]

4. Substituting \( \gamma \text{ cp} \) into the PDE

\[
\begin{align*}
> \text{assume}(t > x/\text{cp}) &; \\
> \text{pde} := \text{simplify}(\text{subs}(\text{cp} = \lambda \text{cs}, \text{pde})) &;
\end{align*}
\]

\[ \text{pde} := 0 = 0 \]

The \( \eta \) given above is thus a solution to the PDE. The assumption made contains no limitation, since the solution is zero for \( t < x/\text{cp} \)

5. Testing the solution by numerical evaluation

\[
\begin{align*}
> t := 1200; x := 1200; \text{cp} := 14; \text{cs} := 20; \lambda := \text{cs}; \alpha := 0.00003; p_0 := 1; &; \\
> \text{evalf}(\text{term1}) + \text{evalf}(\text{term2}) + \text{evalf}(\text{term3}) + \text{evalf}(\text{term4}) + \text{evalf}(\text{term1}) + \text{evalf}(\text{term2}) + \text{evalf}(\text{term3}) + \text{evalf}(\text{term4}) &;
\end{align*}
\]

\[ -3.433021930 \times 10^8 \]

\[ 1.682180747 \times 10^8 \]

\[ 1.750841186 \times 10^8 \]

\[ 0 \]

\[ 3 \times 10^{17} \]

The error on the sum (the last number) is due to numerical inaccuracy.

6. Plotting the solution

\[
\begin{align*}
> \text{restart} &; \\
> \text{cp} := 15; \text{cs} := 20; \lambda := \text{cs}; \alpha := 0.1; p_0 := 1; &; \\
> \text{eta} := (x, t) -> p_0 / (1 - \lambda^2) \times \text{Heaviside}(t - x/\text{cp}) \times \text{exp}(\lambda \times \text{cs}^2) \times \text{evalf}(\lambda^2 \alpha) \times (t - x/\text{cp}) &;
\end{align*}
\]

A.3
Checking the alternative solution (changed sign in exp-function)

1. The shallow water wave equation with linear friction

```maple
> restart;
> pde:=cs^2*2*Diff(eta,x) - Diff(eta,t) + alpha*Diff(eta,l) + cs^2*2*Diff(p,x) = 0;

pde := cs^2 \left( \frac{\partial^2}{\partial x^2} \eta \right) - \frac{\partial^2}{\partial t^2} \eta - \alpha \frac{\partial}{\partial t} \eta + cs^2 \frac{\partial^2}{\partial x^2} p = 0
```

```maple
> eta:=-1/(lambda)*Heaviside(t-x/cp)*exp(lambda\alpha*(t-x/cp)/((lambda-1)*(lambda+1)));

\eta := - \frac{\lambda^2 e^{\frac{t - x}{cp}}}{1 - \lambda^2}
```

NB! Note that \( \lambda \) ('lambda') is used instead of \( \gamma \) ('gamma'), since 'gamma' is a restricted variable in Maple

```maple
> p:=Heaviside(t-x/cp);

p := Heaviside \left( t - \frac{x}{cp} \right)
```

2. Insertion of \( p \) and \( \eta \) in the PDE

```maple
> pde:=cs^2*2*diff(eta,x)-diff(eta,t)-alpha*diff(eta,l)+cs^2*2*diff(p,x)=0;

pde := cs^2 \left( \frac{\partial}{\partial x} \eta \right) - \frac{\partial}{\partial t} \eta - \alpha \frac{\partial}{\partial t} \eta + cs^2 \frac{\partial}{\partial x} p = 0
```

```maple
\begin{align*}
\text{Dirac} \left( t - \frac{x}{cp} \right) \%61 = \left( 1 - \lambda^2 \right) \frac{\lambda^2 \alpha^2}{cp^2} & + \left( 1 - \lambda^2 \right) \frac{\lambda^2 \alpha^2}{(\lambda - 1)^2 (\lambda + 1)^2} \\
\text{Heaviside} \left( t - \frac{x}{cp} \right) \lambda^4 \alpha^2 & - \left( 1 - \lambda^2 \right) \frac{\lambda^2 \alpha^2}{cp^2} (\lambda - 1)^2 (\lambda + 1) + \left( 1 - \lambda^2 \right) \frac{\lambda^2 \alpha^2}{(\lambda - 1)^2 (\lambda + 1)^2}
\end{align*}

A.5
MapleV calculations.

\[ -\alpha \left( \frac{\text{Dirac}(t - \frac{x}{cp})}{1 - \lambda^2} - \frac{\lambda^2 \text{Heaviside}(t - \frac{x}{cp})}{(1 - \lambda^2)(\lambda - 1)(\lambda + 1)} \right) + \frac{\text{cs}^2 \text{Dirac}(1, t - \frac{x}{cp})}{cp^2} = 0 \]

\[ \frac{\lambda^2 e^{\left(-\frac{x}{cp}\right)}}{(\lambda - 1)(\lambda + 1)} \]

\%1 := e

3. The four terms of the PDE

\[ > \text{term1} := \text{cs}^2 \text{diff}(\text{eta}, x, 2); \]
\[ > \text{term2} := \text{diff}(\text{eta}, t, 2); \]
\[ > \text{term3} := \alpha \text{cs} \cdot \text{diff}(\text{eta}, 0); \]
\[ > \text{term4} := \text{cs}^2 \text{diff}(p, x, 2); \]

4. Substituting \( cp = \gamma \text{ cs} \) into the PDE

\[ > \text{assume}(t > x/cp); \]
\[ > \text{pde} := \text{simplify} \left( \frac{\lambda \text{cs} (t - \frac{x}{cp})}{\alpha (\lambda - 1)(\lambda + 1)} \right) \]
\[ > \text{pde} := \text{subs}(cp = \lambda \text{cs}, \text{pde}); \]
\[ \text{pde} := -\frac{\alpha^2 e^{\left(\frac{\lambda a t - \lambda x}{\alpha (\lambda - 1)(\lambda + 1)}\right)}}{(-1 + \lambda^2) \text{cs}^2 (\lambda - 1)^2 (\lambda + 1)^2} = 0 \]

The proposed \( \eta \) is not a solution to the equation (or MapleV cannot simplify the expression). The assumption made contains no limitations, since the solution is zero for \( t < x/cp \)

5. Testing the solution by numerical evaluation

\[ > t := 1200; x := 1200; \text{cs} := 14; \lambda := \text{cs} / \text{cp}; \alpha := 0.00003; \text{p0} := 1; \]
\[ > \text{evalf} \left( \text{term1} \right); \text{evalf} \left( \text{term2} \right); \text{evalf} \left( \text{term3} \right); \text{evalf} \left( \text{term4} \right); \text{evalf} \left( \text{term1} + \text{evalf} \left( \text{term2} \right) + \text{evalf} \left( \text{term3} \right) + \text{evalf} \left( \text{term4} \right) \right); \]
\[ .3219434131 \times 10^{-8} \]
\[ .1577522725 \times 10^{-8} \]
\[ .1641911408 \times 10^{-8} \]
\[ .3283822814 \times 10^{-8} \]

The error on the sum (the last number) cannot be explained by numerical inaccuracy. The proposed \( \eta \) is not a solution to the PDE

6. Plotting the solution

A.6
MapleV calculations.

> restart;
> cp:=15;cs:=18;lambda:=cp/cs;alpha:=0.1;p0:=1;
> eta:=(x,t)->p0/(1-lambda^2)*Heaviside(t-x/cp)*exp(lambda^2*alpha*(t-x/cp)/((lambda-1)*(lambda+a+1))):
> plot3d(eta(x,t),x=0..100,t=0..10,title="Alternative solution",labels=["","",""],[axes=boxed,color=black,tickmarks=[3,3,3]]);

Alternative solution

A.7
\textbf{d'Alembert solution for harmonic waves}

\begin{itemize}
  \item d'Alembert's solution for a harmonic wave propagating at the shallow water velocity $c$

  It is demonstrated that a harmonic wave propagating with the shallow water velocity $c$ will travel undisturbed

  \begin{verbatim}
  > restart;
  > dalembert := 0.5*(f(x-c*t)+f(x+c*t))*1/(2*c)*int(g(s),s=x-c*t..x+c*t);
  \end{verbatim}

  \[ dalembert := \frac{1}{2c} \int_{-c}^{c} g(s) \, ds \]

  \begin{verbatim}
  > eta:=eta(x,t)->eta0*cos(k*(x-c*t));
  \end{verbatim}

  \[ \eta := (x, t) \rightarrow \eta_0 \cos(k(x-c t)) \]

  \begin{verbatim}
  > f:=unapply(eta(x,0),x);
  \end{verbatim}

  \[ f := x \rightarrow \eta_0 \cos(k x) \]

  \begin{verbatim}
  > g:=unapply(subs(t=0,diff(eta(x,t),t)),x);
  \end{verbatim}

  \[ g := x \rightarrow \eta_0 \sin(k x) k c \]

  \begin{verbatim}
  > dalembert;
  \end{verbatim}

  \[ .5 \eta_0 \cos(k(x-c t)) + .5 \eta_0 \cos(k(x+c t)) + \eta_0 \sin(k x) \sin(k c t) \]

  \begin{verbatim}
  > dalembert:=unapply(collect(combine(simplify(expand(dalembert))),[cos,eta0,c]),x,t);
  \end{verbatim}

  \[ dalembert := (x, t) \rightarrow \eta_0 \cos(-k x + k c t) \]

  \begin{verbatim}
  > check:=expand(eta(x,t)-dalembert(x,t));
  \end{verbatim}

  \[ check := 0 \]

  Thus the original surface profile is unchanged, the solution travels undisturbed.

  \item d'Alembert's solution for a harmonic wave propagating at velocity $v$ different from the shallow water velocity $c$

  The changed profile of the original wave is found using d'Alembert's solution.

\end{itemize}
MapleV calculations.

\( \text{dalembert} := 0.5((f(x+ct)-f(x-c\tau))+1/(2c))^{\text{int}}(g(s),s=x-c\tau..x+c\tau) \):

\[
dalembert := 0.5 f(x - c t) + 0.5 f(x + c t) + \frac{\int_{x-c\tau}^{x+c\tau} g(s) \, ds}{2c}
\]

\( \text{eta} := (x,t) \rightarrow \text{eta0}\cos(k(x-v^*t)) \):

\[
\eta := (x, t) \rightarrow \eta_0 \cos(k(x - vt))
\]

\( f := \text{unapply(eta(x,0),x)} \):

\[
f := x \rightarrow \eta_0 \cos(kx)
\]

\( g := \text{unapply(subs(\{t=0, \text{diff(eta(x,t),t)}\},eta)),x} \):

\[
g := x \rightarrow \eta_0 \sin(kx) \, k \, v
\]

\( \text{dalembert} := \text{unapply(collect(combine(simplify(expand(dalembert))))},[\cos, \text{eta0}, v],x,t) \):

\[
dalembert := (x, t) \rightarrow \left(1 - \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx + k c t) + \left(1 + \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx - k c t)
\]

Checking the solution by insertion

The found solution is checked by insertion in the original PDE.

\( \text{pde} := \text{diff(dalembert(x,t),x,x) - diff(dalembert(x,t),t,t)} = 0 \):

\[
pde := \frac{\partial^2}{\partial x^2} \left( \left(1 - \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx + k c t) \right)^2 - \left(1 + \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx - k c t) \left[ \frac{1}{2} \frac{v}{c}\right] \\
+ \left(1 - \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx + k c t) \left[ \frac{1}{2} \frac{v}{c}\right] + \left(1 + \frac{1}{2} \frac{v}{c}\right) \eta_0 \cos(kx - k c t) \left[ \frac{1}{2} \frac{v}{c}\right] = 0
\]

\( \text{pde} := \text{expand(pde)} \):

\[
pde := 0 = 0
\]

The found solution satisfies the PDE. The solution may also be written:

\( \lambda := \text{v*dalembert2} = (x,t) \rightarrow \lambda \text{eta0}\cos(k(x-v^*t))\left(1-\lambda\text{eta0}\cos(kx)\right)\cos(k(c^*t)) \):

A.9
MapleV calculations.

dalembert2 := (x, t) \rightarrow \lambda_0 \cos(k (x - c \tau)) + (1 - \lambda) \eta_0 \cos(k x) \cos(k c \tau)

NB! Note that 'lambda' (\lambda) is used instead of 'gamma' (\gamma), because this is a protected variable in Maple V.

> check:=expand(dalembert(x,t)-dalembert2(x,t));

\text{check} := 0

The two formulation are thus identical. The formulation above is only logical for \gamma > 1. If \gamma > 1 the following formulation is more logical:

> dalembert3:=(x,t)->eta0*cos(k*(x-c*t))+((lambda-1)*eta0*sin(k*x)*sin(k*c*t));

\text{dalembert3} := (x, t) \rightarrow \eta_0 \cos(k (x - c \tau)) + (\lambda - 1) \eta_0 \sin(k x) \sin(k c \tau)

> check:=expand(dalembert(x,t)-dalembert3(x,t));

\text{check} := 0

This formulation is thus identical as well.
Solution of the pressure jump, no friction case by direct insertion

\[ p := (x, t) \rightarrow p0 \ \text{Heaviside}(t - \frac{x}{cp}) \]

\[ \eta := (x, t) \rightarrow \eta0 \ \text{Heaviside}(t - \frac{x}{cp}) \]

\[ pde := c^2 \text{diff}(\eta(t),t,t) - c^2 \text{diff}(p(t),t,t) + c^2 \text{Dirac}(1, t - \frac{x}{cp}) = 0 \]

Isolating \( \eta0 \) and substituting \( \gamma = cp/c \)

\[ \eta0 \text{ simplify(subs(cp=c*gamma,pdesolve(pde,eta0)))); } \]

\[ \eta0 = \frac{p0}{-1 + \gamma^2} \]

A.11
Fourier expansion of the pressure by direct integration

\[
\begin{align*}
pt &:= \int_{-l}^{l} pxx(x,t) \cos\left(\frac{n \pi x}{l}\right) \, dx \\
 & \quad + \frac{n \pi \sin(kl) \sin(n \pi) \sin(\omega t) + n \pi \cos(kl) \sin(n \pi) \cos(\omega t)}{(kl+\pi)(k-l-\pi)} \\
 & \quad - \frac{\sin(\omega t) \, lk}{(kl+\pi)(k-l-\pi)} \\
\end{align*}
\]

> restart;
> pxx:=int(-2*I*pxx(x,t)*cos(n*pi*x/l),x=0..l);

\[
pxx := (x, t) \rightarrow \cos(\omega t - k \, x)
\]

\[
\begin{align*}
2 \left(-k \sin(kl) \cos(n \pi) \cos(\omega t) + k \cos(kl) \cos(n \pi) \sin(\omega t)
\right.
\left.\quad + n \pi \sin(kl) \sin(n \pi) \sin(\omega t) + n \pi \cos(kl) \sin(n \pi) \cos(\omega t)\right)/(\,(kl+n \pi)(k-l-n \pi)) \\
-2 \frac{\sin(\omega t) \, lk}{(kl+n \pi)(k-l-n \pi)}
\end{align*}
\]

> pxx:=subs(cos(n*pi)=(-1)^n*sin(n*pi)==0), pxx);

\[
\begin{align*}
pxx &:= -2 k \sin(kl) \cos(n \pi) \cos(\omega t)
\left/(kl+n \pi)(k-l-n \pi)\right. \\
& \quad + 2 k \cos(kl) \cos(n \pi) \sin(\omega t)
\left/(kl+n \pi)(k-l-n \pi)\right. \\
& \quad + 2 n \pi \sin(kl) \sin(n \pi) \sin(\omega t)
\left/(kl+n \pi)(k-l-n \pi)\right. \\
& \quad + 2 n \pi \cos(kl) \sin(n \pi) \cos(\omega t)
\left/(kl+n \pi)(k-l-n \pi)\right. \\
& \quad - 2 \frac{\sin(\omega t) \, lk}{(kl+n \pi)(k-l-n \pi)}
\end{align*}
\]

> pxx:=combine(pxx);
Solution of the differential equation in $\eta(t)$ with initial conditions $\eta(0) = 0$ and $\eta'(0) = 0$

\[
\text{ode} := \left( \frac{\partial^2}{\partial t^2} \eta(t) \right) + c^2 \lambda^2 \eta(t) = \sin(\omega t) - (-1)^n \sin(\omega t - k t)
\]

\[
\text{sol} := \frac{1}{2} \left( -\cos(\lambda c t) \sin(\%4) \lambda c - \cos(\lambda c t) \sin(\%4) \omega + \cos(\lambda c t) \sin(\%5) \lambda c 
\right.
- \cos(\lambda c t) \sin(\%5) \omega + \cos(\lambda c t) \sin(\%2) (-1)^n \lambda c + \cos(\lambda c t) \sin(\%2) (-1)^n \omega 
- \cos(\lambda c t) \sin(\%2) (-1)^n \lambda c + \cos(\lambda c t) \sin(\%3) (-1)^n \omega - \sin(\lambda c t) \cos(\%5) \lambda c 
+ \sin(\lambda c t) \cos(\%5) \omega + \sin(\lambda c t) \cos(\%4) \lambda c + \sin(\lambda c t) \cos(\%4) \omega 
+ \sin(\lambda c t) \cos(\%3) (-1)^n \lambda c - \sin(\lambda c t) \cos(\%3) (-1)^n \omega 
- \sin(\lambda c t) \cos(\%3) (-1)^n \lambda c - \sin(\lambda c t) \cos(\%2) (-1)^n \omega 
\left. - 2 \sin(k t) e^{\frac{i\pi n t}{2}} \cos(\lambda c t) \lambda c \omega^2 \right) \%
\]

\[
\text{sol} := \frac{(-1)^n \sin(\omega t) \cos(\lambda c t) - (-1)^n \sin(\omega t) \cos(k t) + (-1)^n \cos(\omega t) \sin(k t)}{\%
\text{sol} := \left( \frac{(-1)^n \cos(\omega t) \sin(k t)}{\%
\right. \}
\right. \}
\}
\]

\[
\text{sol} := \frac{(-1)^n \cos(\omega t) \sin(k t)}{\%
\text{sol} := \left( \frac{(\omega -1)^n \cos(\lambda c t)}{\%
\right. \}
\right. \}
\]

\[
\text{sol} := \left( \frac{\omega}{\lambda c \%1} - \frac{\omega}{\lambda c \%1} \right) \sin(\lambda c t) - \frac{(-1)^n \sin(k t) \cos(\lambda c t)}{\%
A.13
Factor A - from $\eta(t) = A [...]$ - as a function of $n$ and $\gamma$

> restart;
> omega:=2*Pi/1000:Delta:=10.5:
> f:=(n,lambda)->abs(4*Pi*n^2/(4*Delta^2-n^2)*Delta/(4*lambda^2*2*Delta^2-n^2));

$$f := (n, \lambda) \rightarrow \frac{n^2 \Delta}{\pi (4 \Delta^2 - n^2) (4 \lambda^2 \Delta^2 - n^2)}$$

> plot3d(f(n,lambda),n=1..50,lambda=0..2,grid=[50,60],axes=boxed,view=0..3.5,color=black);

The whole expression $\eta(t) = A [...]$ as function of $n$ and $\gamma$

> g:=(n,lambda)->abs(4*Pi*n^2/(4*Delta^2-n^2)*Delta/(4*lambda^2*2*Delta^2-n^2)*((1-n*cos(2*Pi*Delta))*sin(2*lambda^2*2*Delta^2-n^2)+(-1)^n*sin(2*lambda^2*2*Delta^2-n^2)*cos(n*omega)))*sin(2*Pi*Delta)*cos(n*omega)+2*Delta*lambda/n*((1-n*cos(2*Pi*Delta)-1)*sin(n*omega)));

$$g := (n, \lambda) \rightarrow \frac{4 \pi^2 \Delta \left( 1 - (-1)^n \cos(2 \pi \Delta) \right) \sin(2 \lambda \Delta \omega t)}{\pi (4 \Delta^2 - n^2) (4 \lambda^2 \Delta^2 - n^2)}$$

> omega:=2*Pi/1000:Delta:=10.5; t:=100;
> plot3d(g(n,lambda),n=1..50,lambda=0..2,grid=[50,60],axes=boxed,color=black);
The Laplace transform

The Laplace transformation is used in the analytical solution of the shallow water equation, and therefore this method and its most important properties are briefly described in this appendix.

The Laplace transform of a function $f(t)$ with respect to time $t$, is denoted $\mathcal{L}\{f(t)\}$ or $F(s)$, and defined as

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt$$  \hspace{1cm} (B.1)

When solving ordinary differential equations, this transformation can be a very powerful tool, especially if either Dirac’s delta function or Heaviside’s step function is involved. These functions are defined as

Dirac's delta function:

$$\delta(x - x_0) = \lim_{\Delta x \to 0} \begin{cases} \frac{1}{2\Delta x} & \text{for } |x - x_0| < \Delta x \\ 0 & \text{for } |x - x_0| > \Delta x \end{cases}$$  \hspace{1cm} (B.2)

Heaviside's step function:

$$h(x - a) = \begin{cases} 1 & \text{for } x < a \\ 0 & \text{for } x > a \end{cases}$$  \hspace{1cm} (B.3)

The most important property of the Laplace transform is, that it transforms a given ordinary differential equation into a simple algebraic one. Once the latter has been solved, the solution is transformed back, if possible, to obtain the solution to the original problem.

In practice the Laplace transform is usually looked up in a table of Laplace transform pair, and not by calculating equation (B.1).

When solving partial differential equations, like the wave equation, the Laplace transformation can also be used, particular if one of the independent ranges over the positive axis. When applying Laplace transformation to a partial differential equation in two variables $x$ and $t$, the equation is usually transformed with respect to $t$. The transformed equation then becomes an ordinary differential equation in $x$. If the coefficients of the equation do not depend on $t$, the use of Laplace will simplify the problem.

When $f$ is a function of $x$ and $t$, the Laplace transformation with respect to time is defined as
\[ \mathcal{L}\{f(x,t)\} = F(x,s) = \int_0^{\infty} f(x,t) e^{-st} dt \]  \hspace{1cm} (B.4)

Some important properties of this transformation should be mentioned. These concern the transformation with respect to time of the derivatives of functions. These properties are:

\[ \mathcal{L}\{f'(x,t)\} = sF(x,s) - f(x,0) \]  \hspace{1cm} (B.5)

\[ \mathcal{L}\{f''(x,t)\} = s^2F(x,s) - sf(x,0) - f'(x,0) \]  \hspace{1cm} (B.6)

\[ \mathcal{L}\{f_n(x,t)\} = F_n(x,s) \]  \hspace{1cm} (B.7)

Differentiation of \(f\) with respect to \(t\) is thus substituted by multiplication of the transformed with \(s\), while differentiation of \(f\) with respect to space is not substituted.

Other important transformations used in this study are

\[ \mathcal{L}\{h(t-a)\} = \frac{e^{-as}}{s} \]  \hspace{1cm} (B.8)

\[ \mathcal{L}\{\delta(t-a)\} = e^{-as} \]  \hspace{1cm} (B.9)

\[ \mathcal{L}^{-1}\left\{ \frac{1}{s-a} \right\} = e^{at} \]  \hspace{1cm} (B.10)

\[ \mathcal{L}^{-1}\{e^{-at}F(s)\} = \begin{cases} f(t-a) & t > a \\ 0 & t < a \end{cases} \]  \hspace{1cm} (B.11)

Note that the two last are given as inverse Laplace of the Laplace transformed.
## Results, sensitivity test

### C.1 Complete list of simulations

<table>
<thead>
<tr>
<th>Experiment [nr]$^{(1)}$</th>
<th>$\Delta x$ [m]</th>
<th>$\Delta t$ [s]</th>
<th>Fr [-]</th>
<th>$v$ [m$^2$/s]</th>
<th>NOCO [-]$^{(3)}$</th>
<th>$\gamma$ [-]</th>
<th>$C_r$ [-]</th>
<th>$\Delta p$ [mbar]</th>
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<td>5</td>
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$^{(1)}$ Numbers 1 to 6 indicate the variation in the sensitivity test.

$^{(2)}$ The value is fixed.

$^{(3)}$ NOCO: Number of control events.
### Results, sensitivity test

<table>
<thead>
<tr>
<th>Experiment [nr]$^{(1)}$</th>
<th>$\Delta x$ [m]</th>
<th>$\Delta t$ [s]</th>
<th>Fr [-]</th>
<th>$v$ [m$^3$/s]</th>
<th>NOCO [-]$^{(3)}$</th>
<th>$\gamma$ [-]</th>
<th>$C_r$ [-]</th>
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<td>-</td>
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</table>

$^{(1)}$ The experiments a0, b0, c0, d0, e0, f0 and g0 are all the same as the standard experiment, and thus not explicitly included in the table.

$^{(2)}$ A '-' indicates the same value as in the standard experiment. A value indicates a different value than in the standard experiment.

$^{(3)}$ NOCO on switches the convective terms off. NOCO off is default including the convective terms.
Figure C.2.2. Time series, Station 2, Standard Experiment.

Figure C.2.1. Time series, Station 1, Standard Experiment.

C.2 Time Series, Standard Experiment

Results, sensitivity test
C.3 Wave motion 'movie', standard experiment

Figure C.3.1. Wave motion 'movie', time step 100. Standard experiment.

Figure C.3.2. Wave motion 'movie', time step 150. Standard experiment.
Figure C.3.3. Wave motion 'movie', time step 200. Standard experiment.

Figure C.3.4. Wave motion 'movie', time step 250. Standard experiment.
Figure C.3 6. Wave motion movie, time step 350. Standard experiment.

Figure C.3 5. Wave motion movie, time step 300. Standard experiment.

Results, sensitivity test.
C.4 Time series, experiment a0-a4

Figure C.4.1. Time series, station 3, first 6 hours. Comparison of experiments a0 (Standard, 1km), a1 (250m) and a2 (500m).

Figure C.4.2. Time series, station 3, close up. Comparison of experiments a0 (Standard, 1km), a1 (250m) and a2 (500m).
C.5 Overall maximum amplitude, experiment a0-a6

Figure C.5.1. Overall maximum amplitude as function of the distance from the left boundary. Comparison of experiments a0 (Standard, Δx = 1km), a1 (Δx = 250m), a2 (Δx = 500m), a3 (Δx = 2km) and a4 (Δx = 4km).

Figure C.5.2. Close up of the first 200km of figure C.5.1, excluding experiment a4.
Figure C.5.3. Overall maximum amplitude as function of time. Comparison of experiments a0 (Standard, $\Delta x = 1$ km), a1 ($\Delta x = 250$ m), a2 ($\Delta x = 500$ m), a3 ($\Delta x = 2$ km) and a4 ($\Delta x = 4$ km).

Figure C.5.4. Close up of figure C.5.3, excluding experiment a4.
Figure C.5.5. Overall maximum amplitude as function of distance. Comparison of experiments a0 (Standard, $\Delta x = 1km$, $\Delta t = 1min$), a3 ($\Delta x = 2km$, $\Delta t = 1min$), a5 ($\Delta x = 2km$, $\Delta t = 30s$) and a6 ($\Delta x = 2km$, $\Delta t = 15s$). Close up of the first 200km.

Figure C.5.6. Overall maximum amplitude as function of time. Comparison of experiments a0 (Standard, $\Delta x = 1km$, $\Delta t = 1min$), a3 ($\Delta x = 2km$, $\Delta t = 1min$), a5 ($\Delta x = 2km$, $\Delta t = 30s$) and a6 ($\Delta x = 2km$, $\Delta t = 15s$).
C.6

Time series, Experiment D0-b4

Results, sensitivity test.
C.7 Overall maximum amplitude, experiment b0-b4

Figure C.7.1. Overall maximum amplitude as function of the distance from the left boundary. Comparison of experiments b0 (Standard, $\Delta t = 1\text{ min}$), b1 ($\Delta t = 15$), b2 ($\Delta t = 30$), b3 ($\Delta t = 2\text{ min}$) end b4 ($\Delta t = 4\text{ min}$). Close-up of the first 200km.

Figure C.7.2. Overall maximum amplitude as function of time. Comparison of experiments b0 (Standard, $\Delta t = 1\text{ min}$), b1 ($\Delta t = 15$), b2 ($\Delta t = 30$), b3 ($\Delta t = 2\text{ min}$) end b4 ($\Delta t = 4\text{ min}$). Close-up of the first 200km.
C.8 Time series, experiment c0-c3

Figure C.8.1. Time series, station 3. Comparison of experiment c0 (Standard, Fr = 0.005), c1 (Fr = 0), c2 (Fr = 0.003) and c3 (Fr = 0.007).

Figure C.8.2. Time series, station 3. Comparison of experiment c0 (Standard, Fr = 0.005), c1 (Fr = 0), c2 (Fr = 0.003) and c3 (Fr = 0.007).
Figure C.9.1: Time series, station 1. Comparison of experiment 60 (standard, \( \lambda = 10 \text{m/s} \)) and 62 (\( \lambda = 10 \text{m/s} \)).

Figure C.9.2: Time series, station 2. Comparison of experiment 60 (standard, \( \lambda = 10 \text{m/s} \)).
C.10 Time series, experiment e0-e1

Figure C.10.1. Time series, station 3. Comparison of experiment e0 (Standard, with convective terms) and e1 (without convective terms).

Figure C.10.2. Time series, station 3. Comparison of experiment e0 (Standard, with convective terms) and e1 (without convective terms).
C.11 Overall maximum amplitude, experiments f0-f5

Figure C.11.1. Overall maximum amplitude as function of the distance from the left boundary. Comparing experiments f0 (Standard, \(\gamma = 0.85\)), f1 (\(\gamma = 1.10\)), f2 (\(\gamma = 1.05\)), f3 (\(\gamma = 0.95\)), f4 (\(\gamma = 0.90\)) and f5 (\(\gamma = 0.70\)). Close-up of the first 250km.

Figure C.11.2. Overall maximum amplitude as function of time. Comparing experiments f0 (Standard, \(\gamma = 0.85\)), f1 (\(\gamma = 1.10\)), f2 (\(\gamma = 1.05\)), f3 (\(\gamma = 0.95\)), f4 (\(\gamma = 0.90\)) and f5 (\(\gamma = 0.70\)).
C.12 Overall maximum amplitude, experiments g0-g2

Figure C.12.1. Overall maximum amplitude as function of the distance from the left boundary. Comparing experiments g0 (Standard, Δp = 5mbar), g1 (Δp = 10mbar) and g2 (Δp = 15mbar). Close-up of the first 200km.

Figure C.12.2. Overall maximum amplitude as function of time. Comparing experiments g0 (Standard, Δp = 5mbar), g1 (Δp = 10mbar) and g2 (Δp = 15mbar).
## Results, cold front simulations

### D.1 Complete list of simulations

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<th>(\Delta t) [s]</th>
<th>NUM [-]</th>
<th>HBSMTH [-]</th>
<th>NREFL [-]</th>
<th>Pressure(^{(2)}) [s]</th>
<th>(c_p) [m/s]</th>
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<td>-</td>
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\(^{(1)}\) Experiment number

\(^{(2)}\) Additional condition
Results, cold front simulations.

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<th>Experiment (1)</th>
<th>$\Delta x$ [m]</th>
<th>$\Delta t$ [s]</th>
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<th>HBSMTH [-]</th>
<th>NREFL [-]</th>
<th>Pressure (3) [s]</th>
<th>$c_p$ [m/s]</th>
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<td>Standard</td>
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<td>0.0</td>
<td>no</td>
<td>600</td>
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<td>60i</td>
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(1) The experiments a0, b0, c0, d0 and e0 are all the same as the standard experiment and thus not included explicitly in the table.
(2) A '-' indicates the same value as in the standard experiment. A value indicates a different value than in the standard experiment.
(3) Experiments e1 through e6 are described by the time step of the DUCHESS input file and, if applicable, an abbreviation. This abbreviation in turn describes the air pressure data used when generating the DUCHESS input file. Based on the original air pressure data, sampled at 1/600 Hz, the following pressure data were generated:

<No abbreviation>: In the standard simulation the original pressure data were used. Linear interpolation was applied to obtain the desired time step, when generating the DUCHESS input file.

<n>: The same as above, only here the closest pressure value was used instead of linear interpolation.

<i>: The original air pressure was lowpass-filtered using a 15 points Butterworth filter, with a cut-off frequency of 75 percent of the Nyquist frequency $f_c$.

<if>: The original air pressure was first interpolated using cubic spline interpolation and then lowpass-filtered using a 20 points Butterworth-filter with a cut-off frequency of 0.20$f_c$ (i.e. equal to the Nyquist frequency of the original signal).

<i>: The same as above but without filtering.

Appendix D.9 is a processing scheme showing exactly how the input files were generated.
D.2 Time series, standard experiment

Figure D.2.1 Time series surface elevation, standard experiment. Station 1 through 5.
D.3  Spectral density, standard experiment

Figure D.3.1. Spectral density, standard experiment. Station 1 through 5. Logarithmic vertical scale.

Figure D.3.2. Spectral density, standard experiment. Station 1 through 5. Logarithmic vertical scale. Close-up of peak around 2mHz.
D.4  Spectral density, standard experiment

![Spectral density plots for Station 1 to Station 5](image)

Figure D.4.1. Spectral density, standard experiment. Station 1 through 5. Linear vertical scale (unconstrained).

---

Results, cold front simulations.

D.5
Figure D.4.2. Spectral density, standard experiment. Station 1 through 5. Linear vertical scale (constrained).
D.5 Spectral density, experiment a0-a2

Figure D.5.1. Spectral density, experiment a0-a2. Station 1. Linear vertical scale (unconstrained).

Figure D.5.2. Spectral density, experiment a0-a2. Station 1. Logarithmic vertical scale.

D.7
Results, cold front simulations.

Figure D.5.3. Spectral density, experiment a0-a2. Station 5. Linear vertical scale (constrained).

Figure D.5.4. Spectral density, experiment a0-a2. Station 5. Logarithmic vertical scale.
D.6 Spectral density, experiment b0-b2

Figure D.6.1. Spectral density, experiment b0-b2. Station 1. Linear vertical scale (unconstrained).

Figure D.6.2. Spectral density, experiment b0-b2. Station 1. Logarithmic vertical scale.
Results, cold front simulations.

Figure D.6.3. Spectral density, experiment b0-b2. Station 5. Linear vertical scale (constrained).

Figure D.6.4. Spectral density, experiment b0-b2. Station 5. Logarithmic vertical scale.
D.7  Spectral density, experiment c0-c4

Figure D.7.1. Spectral density, experiment c0-c3. Station 5. Linear vertical scale (constrained).
D.8 Spectral density, experiment d0-d3

Figure D.8.1. Spectral density, experiment d0-d3. Station 1. Linear vertical scale (unconstrained).

Figure D.8.2. Spectral density, experiment d0-d3. Station 5. Linear vertical scale (constrained).
Figure D.8.3. Spectral density, experiment d0-d3. Station 1. Logarithmic vertical scale.

Figure D.8.4. Spectral density, experiment d0 and d2. Station 5. Logarithmic vertical scale. 2-d2 is the spectral density of two times the response of experiment d2.
Processing scheme for the generation of DUCHESS input files

Experiments e0-e6:

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<th>Processing of original air pressure time series</th>
<th>Sample frequency</th>
<th>Generation of input file</th>
<th>Sample frequency</th>
<th>Interaction DUCHESS</th>
<th>Sample frequency</th>
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<td>Linear</td>
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<td>-</td>
<td>75% Nyq</td>
<td>1/600</td>
<td>Linear</td>
<td>1/120</td>
<td>No</td>
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<td>75% Nyq</td>
<td>1/600</td>
<td>Linear</td>
<td>1/120</td>
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<td>100% Nyq</td>
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<td>Linear</td>
<td>1/120</td>
<td>No</td>
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</tbody>
</table>

Experiments f1-f4:

| Cubic spline | 1/60 | Linear | 1/60 | No | 1/60 | f1-f4 |

A '*' indicates that no interpolation or filtering respectively is applied.
Figure D.10.1. Spectral density, experiment e0-e6. Station 1. Linear vertical scale (unconstrained).
Figure D.10.2. Spectral density, experiment e0-e6. Station 5. Linear vertical scale (constrained).
Results, cold front simulations.

Figure D.10.3. Spectral density, experiment e0 and e1. Station 1. Logarithmic vertical scale.

Figure D.10.4. Spectral density, experiment e0 and e2. Station 1. Logarithmic vertical scale.
Figure D.10.5: Spectral density, Experiment C3 and C4, Station 1, Logarithmic vertical scale.

Figure D.10.6: Spectral density, Experiment C2 and C4, Station 1, Logarithmic vertical scale.

Results: cold from simulations.
Results, cold front simulations.

Figure D.10.7. Spectral density, experiment e2 and e5. Station 1. Logarithmic vertical scale.

Figure D.10.8. Spectral density, experiment e5 and e6. Station 1. Logarithmic vertical scale.
D.11 Spectral variance, experiment f0-f4

Figure D.11.1. Spectral variance and surface elevation, experiment f1.
Figure D.11.2. Spectral variance and surface elevation, experiment f2.
Results, cold front simulations.

Figure D.11.3. Spectral variance and surface elevation, experiment f3.
Figure D.11.4. Spectral variance and surface elevation, experiment f4.
Programmes

Pascal programme 'pres_1d.pas':

{ This program generates an input file to the DUCHESS program }

Program pres_1d;
Uses crt;
Var i,j,n,mx : integer;
    p,p0,cp,period,step,t,tend,l,fac,sx : real;
    fname : string[40];
    f : text;
    pres : string;

begin
    p0:=5;  { air pressure divided by water density and gravity [cm]
                compensated in DUCHESS input file by SC=0.01 }
    cp:=15;  { front speed [m/s] }
    l:=15000;  { pressure wave length [m] }
    step:=60;  { time step [s] }
    tend:=86400;  { total computational time [s] }
    mx:=501;  { gridpoints [#] }
    sx:=1000;  { gridspacing [m] }
    period=l/cp;
    n:=trunc(tend/step)+1;
    fname:=c:\duchess\input\std.s1';

begin
    clrscr;
    assign(f,fname);
    rewrite(f);
    gotoxy(30,12);write('Time :');
    for i:=0 to n do
        begin
            t:=step*i;
            if (t=0) then
                fac:=0 else
            if (t<10*period) then
                fac:=2*exp(3*ln(t/(10*period)))+3*sqrt(t/(10*period))
            else fac:=1;
            gotoxy(40,12);write(t:6:0);
            for j:=1 to mx do
                begin
                    p:=fac*p0*cos(2*PI*(t/period-sx*j));
                    str(p:1:4,pres);
                    write(f,pres,'');
                    str(p:1:4,pres);
                    writeln(f,pres);
                end;
        end;
    close(f);
end.
end.
Pascal programme 'front1d.pas':

{ This program generates a 1D input file to the DUCHESS programme

Variables used in the programme:

Front parameters:
  cp:  front speed [m/s]
  ts0: begin time, time series [s]
  ts:  time step, time series [s]
  ns:  number of time steps, time series [#]

Numerical parameters:
  td:  time step, computation [s]
  nd:  number of time steps, computation [#]
  tend: total computational time [s]
  dx:  grid spacing x-direction [m]
  mx:  number of grid points in x-direction [#]
  Rx:  pointer in original time series [#]
  Rt:  pointer in original time series [#]

Program front1d;
Uses crt;
Var li,j,code,n,ns0,nd,ns : integer;
  cp,ts,ts0,td,tend,a : real;
  x,t,dx,dRx,dRt,Rx,Rt : real;
  fname1,fname2 : string[40];
  f1,f2 : text;
  pres : string;
  airpres : array[0..7200] of real;
  header : boolean;

begin
  cp:=8; {front speed [m/s]}
  ts:=60; {time step [s], original time series}
  td:=60; {time step [s], Duchess input file}
  ts0:=0; {begin time, time series [s]}
  tend:=432000; {total computational time [s]}
  mx:=153; {gridpoints in y-direction [#]}
  j0:=1; {point of reference, x}
  dx:=1000; {grid spacing x-direction [m]}
  header:=true; {if true, header is printed in output file, if false, no header is printed}

  fname1:='c:/duchess1df65up/ap1201.dat'; {input file}
  fname2:='c:/duchess1df65up/ap1201.08'; {output file}
  ns0:=trunc(ts0/ts);
  nd:=trunc(tend/td)+1;

  assign(f1,fname1);
  reset(f1);

  {seeking begin time series}
  for i:=1 to ns0 do readln(f1,pres);

  {reading input}
  i:=1;
  repeat
    readln(f1,pres);
    val(pres,x,code);
    if code=0 then

Programmes.

begin
  airpres[i]:=x-10130; {normalising to 1atm.}
  i:=i+1;
end
else begin
  clrscr;gotoxy(10,6);
  write('Error in input. Exits program');
  exit;
end;
until cof(1);
ns:=i-1;

{calculating output}
assign(2,'frame2');
rewrite(2);
{header output file}
if header then begin
  writeln(2,'Total time :','tend:7.0, s');
  writeln(2,'Time step :','td:7.0, s');
  writeln(2,'Grid points :','nx:7');
  writeln(2,'Grid spacing :','dx:7.0, m');
  writeln(2,'Front speed :','cp:7.1, m/s');
  writeln(2);
end;

{calculation of pressure in all points}
Rx:=dx/(ts*cp);
Rt:=td/ts;
gotoxy(10,6);write('Time step number: ');for i:=1 to nd do begin
  for j:=1 to mx do begin
    begin
    R:=(j-0)*Rx+(i-1)*Rt;
    if R<1 then begin
      begin
        n:=1;
        x:=airpres[n];
        end
      else if R>ns then begin
        begin
          n:=ns;
          x:=airpres[n];
        end
      else begin
        begin
          a:=R-trunc(R);
          n:=trunc(R);
          x:=a*airpres[n+1]+(1-a)*airpres[n];
        end
        str(x:4.3,pres);
        writeln(2,pres,'*pres');
      end
    end
  end
end
  gotoxy(10,8);write(i);
end
close(f1);
close(2);
end
MATLAB programme 'movie2d.m':

function movie2d(G,t1,t2,dt);

% movie2d(G,t1,t2,dt)

% Shows a movie of the water surface profiles defined in
% the matrix G. t1, t2 and dt are integers specifying begin,
% end and number of time steps between to frames.
% The matrix G must be arranged so that each column
% represent a profile, i.e.
%
%
% [ p  p  p  .. ]
% [ r  r  r ]
% [ o  o  o ]
% [ f  f  f ]
% [ i  i  i ]
% [ l  l  l ]
% [ e  e  e ]
% [ 1  2  3 ]
%

[m,n]=size(G);

% setting default values
if nargin<2,
 t1=1;
end
if nargin<3,
 t2=n;
end
if nargin<4,
 if t2>t1 & t2<10,
  dt=2;
  t2=n;
 else
  dt=1;
 end
end

% preparing window parameters
width1=0.6;
height1=0.45;
width2=6*width1/8;
height2=6*height1/8;
x1=(1-width1)/2;
y1=(1-height1)/2;
x2=(1-width2)/2;
y2=(1-height2)/2;
mx=ceil(max(max(abs(G)))*10)/10;
close
b=axes('position',[x1 y1 width1 height1]);
set(h,'box','on','fontsize',10,'tick',[-mx -mx/2 0 mx/2 mx])
axis([-m/6 7*m/6 -8*m/6 8*m/6])
title('Wave motion')
xlabel('Grid points')
ylabel('Elevation [m]')
t=text(-m/6,1.45*mx,'Time step');
set(t,'fontsize',10);

E.4
MATLAB programme 'onedbot.m':

% Produces a 1d bottom, 'bot1d', from the 2d bottom 'bot'

% coordinates begin point
j=100;
i=65;

% direction of bottom trace
dir=-65;

% Grid spacing 'bot1d'
dx=2000;

% Grid spacing 'bot', x- and y-direction
dx1=2000;
dy1=2000;

dri=dx*cos(dir*pi/180)/dx1;
dry=dx*sin(dir*pi/180)/dy1;
bot1d(1)=bot(j,i);
k=1;
on=0;

% loop continues until depth is zero
while bot1d(k)==0 on==0,
k=k+1;
ri=k*dri;
ry=k*dry;
i1=floor(ri);
j1=floor(ry);

% interpolation between the four closest points
b1=bot(j-j1,i-i1);
b2=bot(j-(j1+1),i-i1);
b3=bot(j-(j1+1),i-(i1+1));
b4=bot(j-j1,i-(i1+1));

bot1d(k)=((ry-ry1)*b2+(1-(ry-ry1))*b1)*((1-(ri-ri1))+(ry-ry1)*b4*(ri-ri1));
if bot1d(k)==0,
Programmes.

```matlab
on=1;
end
end

% two grid points wide bottom
bot1d=[bot1d;bot1d];

% coordinates of begin and end point in 'bot'
y=[j;j+1];
x=[i;i+1];

MATLAB programme 'psdlin.m':

function [G, f] = psdlin(P1, P2, P3, P4, P5, P6, P7)

% PSDLIN One-sided spectral density estimate, linear vertical axis
% Calculates and plots the one-sided spectral density of a vector x
% It is based on the original MATLAB function PSD. It needs a modified
% version of the routine PSDCHK to run.
% For information on optional input variables, see help PSD.

callcheck(1,7,nargin))
[msg, x, y, fft, Fs, window, overlap, p, dflag, nyq] = evaloptargs('psdchk', nargin, ');
callerror(msg)

% compute PSD
x = x(:); % Make sure x is a column vector
window = window(:); % Number of data points
nwind = length(window); % length of window
if n < nwind % zero-pad x if it has length less than the window length
    x(nwind)=0; n=nwind;
end
k = fix((n-overlap)/(nwind-overlap)); % Number of windows
    % (k = fix(n/nwind) for overlap=0)

index = 1:nwind;
KMU = k*norm(window)^2; % Normalizing scale factor \( \Rightarrow \) asymptotically unbiased
% KMU = k^2*sum(window)^2; % alt. Normalizing scale factor \( \Rightarrow \) peaks are about right

Spec = zeros(nfft,1);
for i=1:k
    if strcmp(dflag,'linear')
        xw = window.*detrend(x(index));
    elseif strcmp(dflag,'none')
        xw = window.*x(index));
    else
        xw = window.*detrend(x(index),0);
    end
    index = index + (nwind - overlap);
    Xx = abs(fft(xw,nfft)).^2;
    Spec = Spec + Xx;
end

% Select first half
if ~any(any(imag(x)~=0)), % if x is not complex

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```
```matlab
if rem(nfft,2), nfft odd
select = (1:(nfft+1)/2);
else
select = (1:nfft/2+1);
end
Spec = Spec(select);

% NB

% The correct one-sided spectrum is two times the two-sided
Spec = 2*Spec(select);

% MATLAB method, standard not on. This estimate is not correct!!
% Spec = 4*Spec(select); % double the signal content - essentially
% folding over the negative frequencies onto the positive and adding.

% NB

else
select = (1:nfft);
end
Spec = Spec*(1/KMU);
freq_vector = (select - 1)*Fs/nfft;

% set up output parameters
if (nargout == 2),
    G = Spec/Fs;
    f = freq_vector;
elseif (nargout == 1),
    G = Spec/Fs;
elseif (nargout == 0),
    G = Spec/Fs;
end

% Setting up plot parameters
s=input('Enter line style and color : ', 's');
if isempty(s),
    color=[Y 'm' c 'r' g 'b' w];
    s=color(ceil(7*rand));
end
newplot;

if max(max(freq_vector))>0.1,
    plot(freq_vector,abs(G),s),
xlabel('Frequency [Hz]'),
else
    plot(1000*freq_vector,abs(G),s),
xlabel('Frequency [mHz]')
end

ylabel('Spectral density [mV/Hz]'),
grid off
if nyq==1,
xlabel('Nyquist frequency = 1')
end
end
```

E.7
MATLAB programme 'var.m':

function [VAR, AVGG, G] = var(x,fs,hs,limits);

% VAR [VAR, AVGG, G] = var(x,fs,hs,limits);
%
% Calculates the spectral variance, VAR, every hour of a
% signal x sampled at
% fs Hz, using an hs hours long Hanning window. The variance is split
% up into four
% frequency intervals defined by the vector limits=[f1 f2 f3 f4].
% The variance is
% then split up in the intervals [f1,f2[, ]2,f3[, ]3,f4[ and ]4,fs]).
% The
% default limits vector is limits=[0.1 0.2 0.5 2.0] mHz(). The default window
% length is 8 hours. The sample frequency must be specified.
%
% VAR is as long as x (measured in hours), but only non-zero in
% points where the
% window does not exceed the time series, i.e. the first and last
% hs/2 points
% are zero.
%
% G is the one-sided spectrum as function of time, and AVGG is the average
% spectrum
% over the whole period. G is as long as x (measured in hours) but only
% non-zero
% in points where the window does not exceed the time series, i.e. the first
% and
% last hs/2 points are zero. AVGG is averaged over the non-zero values of G,
% since
% G is undetermined in the other points.
%
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% Checking that input x is a vector,
% [m,n]=size(x);
% if ~(m==1 & n==1),
% error('x must be a vector')
% end
%
% Make sure x is a column vector
% x=x(:);
%
% Checking input and preparing default values
% if nargin<2
% error('Sample frequency, fs, must be specified')
% end
% if nargin<3
% hs=8;
% end
% if nargin<4,
% if length(hs)==1,
% limits=[0.1 0.2 0.5 2.0]/1000;
% elseif length(hs)==4,
% limits=hs;
% hs=8;
% else
% error('Window length, hs, or limits not correct')
% end
% end
%
% Checking input format
% if (nargin==4),
% if length(hs)==4 & length(limits)==1,
% hide=hs;
% hs=limits;
% limits=hide;
% elseif length(hs)==1,
% error('Window length, hs, must be a scalar')
% end

E.8
elseif length(limits)==4, 
    error('Limits must have four elements')
end
end

% Checking limits
if any(limits>fs/2) 
    error('Frequency above Nyquist frequency in limits')
end
if any(limits<0), 
    error('Negative frequency in limits')
end

% Number of hours in record, H
n=length(x);
H=floor(n/(fs*3600));

% Number of FFT points, nfft, equivalent to hs hours
nfft=floor(hs*3600*fs);

% Length of one hour, h
h=(1*3600*fs);

% Hanning time window, window
w=1:nfft;
window=(1-cos(pi*te/max(te)).^2);

% scaling factor due to windowing, k
k=length(window)/norm(window).^2;

% Calculating the spectrum (valid for any type of window)
G=zeros(nfft/2,H);
for i=ceil(hs/2):1:floor(H-hs/2),
    % Discrete Fourier Transformation
    X=sqrt(k)*fftx(ceil(i*h)-(nfft/2-1):ceil(i*h)+nfft/2).*window;
    % Two-sided spectrum
    S=abs(X).^2/(fs*nfft);
    % One-sided spectrum
    G(1,i)=S(1);
    G(2:nfft/2+1,i)=2*S(2:nfft/2+1);
end

% Averaging over all non-zero spectra
AVGG=sum(G)/(H-hs+1);
[m,n]=size(G);

% Intervals from limits
int=limits*nfft/fs+1;

% Checking for corrupted intervals. The calculation is not stopped
% but a flag is set to skip calculation of VAR
badvar=0;
if any(ceil(int(2:4))<floor(int(2:4))) | any(fLOOR(int(2:4))-ceil(int(1:3))<0), 
    disp('!!! Error. Intervals corrupted. Change limits or window length hs !!!'),
    badvar=1;
end

% Calculation of variance in different frequency intervals
if badvar==0,
% Creating goodness of fit

% The following subroutine computes the goodness of fit

% The goodness of fit is defined as:

% \[ g = \sum_{i=1}^{n} \left( \frac{x_i - \hat{x}_i}{\sigma_i} \right)^2 \]
Standard DUCHESS command file 'std.due':

$ DUCHESS command file

PROJECT 'Long waves' 'F96'
  'Sensitivity test'
  'Standard experiment, time series'

CONDITION 2
ALARM

$ MODEL PARAMETERS
GRID 2,501,1000
PLAN ALL
BOTTOM CONSTANT -31.8
FRIC CONSTANT 0.005
VISC 10.0

$ TURNED OFF PARAMETERS
$ NUM 0.5, no damping (default)
$ SET CORR=1.17E-3, Coriolis effect
$ NOCO, No advective acceleration

$ INITIAL CONDITIONS
INIT H CONSTANT 0
INIT QY CONSTANT 0
INIT QX CONSTANT 0

$ BOUNDARY CONDITIONS
BOUNDARY QY CONST 0 2 501
BOUNDARY QY CONST 0 2 1
BOUNDARY QX CONST 0 1 2 5 501
BOUNDARY QX CONST 0 2 2 5 501

$ PRESSURE FILE
STORM FILE 60 PRESSURE SC = 0.01 'c:\duchess\1d\input\std.1's IDLA=3 NHED=0

$ PLOT OF PLAN
$ COMP 0
$ SHOW PLAN

$ TIME SERIES
$ FORMAT: OUTPUT BEGIN INTERV FILENAME TABLE DISK X Y
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh5.1.dat' & TABLE DISK 1000 500000 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh4.1.dat' & TABLE DISK 1000 380000 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh3.1.dat' & TABLE DISK 1000 240000 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh2.1.dat' & TABLE DISK 1000 120000 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh1.1.dat' & TABLE DISK 1000 1000 H

$ PROFILES
$ FORMAT: OUTPUT BEGIN INTERV FILENAME BLOCK DISK DI DII DIV DIII DIII
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh5.bic' & BLOCK DISK 2 2 402 501 1 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh4.bic' & BLOCK DISK 2 2 302 401 1 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh3.bic' & BLOCK DISK 2 2 202 301 1 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh2.bic' & BLOCK DISK 2 2 102 201 1 H
OUTPUT BEGIN 0 INTERV 60 'c:\duchess\1d\data\stdh1.bic' & BLOCK DISK 2 2 101 1 H

$ COMPUTATION
SET 60
COMP 86400