Near Field Thermal Radiation Distance Sensing

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Abstract

Near field thermal radiation is the enhancement of thermal radiation by photon tunnelling when the separation gap between the emitting and the receiving object is below the wavelength of thermal radiation. This is caused by evanescent waves on the surface of the object, by surface plasmon polaritons and by surface phonon polaritons. In recent years researchers have modelled and experimentally measured near field thermal radiation.

The near field radiative heat transfer between two objects of materials supporting surface phonon polaritons is a strong function of the separation gap. Below 1 µm the slope of the distance-heat flow curve is $d^{-a}$, where $a$ depends on the geometry of both objects. The goal of this thesis was to determine if near field thermal radiation can be used as a measurement signal to determine the size of the separation gap between two objects.

Using the principles of fluctuational electrodynamics models were made to calculate the near field radiative heat flow between various materials (dielectrics, metals, semiconductors and magnetic materials). For curved objects the proximity approximation is used to calculate radiative heat flows. It was discovered that the amount of near field heat radiation between two objects strongly depends on the types of material involved. The heat transfer between two SiO$_2$ plates is expected to be over 1000 times as high as the black body radiation limit in submicron gaps. The heat transfer between an SiO$_2$ plate and a gold plate remains well below the black body radiation limit for the same separation gap.

An experimental setup was designed and build to demonstrate that near field
thermal radiation can be used to measure the gap between two objects made of a material supporting surface phonon polaritons. Because of its thermal sensitivity silicon nitride cantilevers with a layer of gold on them were used as thermal sensors, a glass microsphere glued on the free end of the cantilever provides the surface phonon polariton supporting material. The substrate used was a glass microscope slide.

The experiments where the setup was placed in air confirmed that the setup is sensitive to heat flows between the sphere and the substrate, in atmospheric conditions this heat flow is dominated by gas conduction, in vacuum this is dominated by near field thermal radiation. These experiments also revealed that the setup is sensitive to drift, likely to be caused by temperature changes in the environment. When the experiment was done in vacuum the results were inconclusive, the contact point between the sphere and the substrate could not be determined and therefore it is not known where the size of the separation gap is zero. More measurements are needed to confirm whether or not near field thermal radiation can be used to measure distances between objects.
Acknowledgments

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# Contents

Abstract I

Acknowledgments III

List of Figures VIII

List of Tables XVI

Abbreviations XVIII

List of Symbols XIX

1 Introduction 1

  1.1 Scope .................................................. 1

  1.2 Goal .................................................. 2

2 Thermal Radiation Physics 4

  2.1 Origins of Thermal Radiation .......................... 4

  2.2 Far Field Thermal Radiation .......................... 4

  2.3 Near Field Thermal Radiation ........................ 8

    2.3.1 Evanescent Waves ............................... 9

    2.3.2 Surface Phonon Resonance ..................... 10

    2.3.3 Surface Plasmon Resonance .................... 15

    2.3.4 Conclusion .................................. 16

3 Near Field Thermal Radiation Experiments and Applications 17
5.3.5 Laser ......................................................... 71
5.3.6 Position Sensitive Detector ................................. 72
5.3.7 Vacuum Chamber ........................................... 73
5.3.8 Vacuum Pump .............................................. 75
5.3.9 Capacitive Sensor .......................................... 76
5.3.10 Piezo Stage ............................................... 77
5.3.11 Data Acquisition .......................................... 77
5.3.12 Objective Lens ............................................ 77
5.3.13 Pressure Sensor ........................................... 78
5.3.14 Temperature Sensors ...................................... 79

6 Thermomechanical Modelling of the Experimental Setup 81
6.1 Heat Balance for the Cantilever with Sphere ............... 81
6.1.1 Absorbed Laser Power .................................... 82
6.1.2 Heat Conduction in the Cantilever ....................... 82
6.1.3 Radiative Losses from the Cantilever and Sphere ....... 82
6.1.4 Gas Conduction between Sphere and Substrate ........ 83
6.1.5 Heat Lost in Air ........................................... 86
6.2 Thermal Expansion of the Cantilever ....................... 87
6.3 Resonance Frequency Shift by Addition of the Sphere .... 88
6.4 Expected Cantilever Deflection by Near Field Thermal Radiation . 89
6.5 Van der Waals Forces ........................................ 90
6.6 Electrostatic Forces ......................................... 92

7 Measurements ..................................................... 95
7.1 Glueing Microspheres on Cantilevers ...................... 95
7.2 Experiments in Air .......................................... 99
7.2.1 Noise and Drift ......................................... 100
7.2.2 Thermal Noise .......................................... 105
7.2.3 Step Response ......................................... 106
7.2.4 Cantilever Rotation versus Laser Power in Air ....... 107
7.2.5 Cantilever Rotation versus Distance in Air ........... 110
7.3 Vacuum Experiments ...................................... 116
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Radiation emitted from a surface of area $dA$ through solid angle $d\Omega$.</td>
<td>5</td>
</tr>
<tr>
<td>2.2</td>
<td>Wien’s law for room temperature objects.</td>
<td>6</td>
</tr>
<tr>
<td>2.3</td>
<td>View factor determination between a sphere and a large flat surface (left) and between two spheres (right).</td>
<td>7</td>
</tr>
<tr>
<td>2.4</td>
<td>On the left the radiation is partially refracted and the other part is reflected. When the angle of incidence is larger than the critical angle the radiation is completely reflected as shown on the right.</td>
<td>9</td>
</tr>
<tr>
<td>2.5</td>
<td>Impression of propagating waves and evanescent waves. In the top image a planar wavefront hits the interface between a high index material and a lower index material. The wave is refracted and propagates into the low index material. The reflected waves are not shown. The dark and bright areas correspond to differences in polarity of the EM field. Arrows show the direction of propagation. In the bottom image the angle of incidence is higher than the critical angle. The evanescent wave propagates along the interface. The decay length is also shown.</td>
<td>11</td>
</tr>
<tr>
<td>2.6</td>
<td>Same as in Figure 2.5 but now another high index material (blue) is brought close to the surface of the first high index material. When the gap between them is large the evanescent wave coupling is negligible. In the bottom picture the gap is very small and the evanescent wave coupling increases.</td>
<td>12</td>
</tr>
<tr>
<td>2.7</td>
<td>Chain of atoms linked by interatomic forces.</td>
<td>12</td>
</tr>
<tr>
<td>2.8</td>
<td>Transverse optical and acoustic phonons in an chain of oppositely charged ions, notice the separation of charge on the optical phonon.</td>
<td>14</td>
</tr>
</tbody>
</table>
2.9 Dispersion relation for optical and acoustic phonons in a diatomic chain of atoms. LO is the longitudinal optical branch, TO is the transverse optical branch, LA is the longitudinal acoustic branch and TA is the transverse acoustic branch. The material properties used are hypothetical.

2.10 Impression of surface phonon polariton evanescent waves on the interface of two materials. Again the contrast shows the difference in polarity, the arrow shows the direction of propagation.

3.1 Test setup as used in Ref. [1].

3.2 Left: MEMS device used in Ref. [2]. The membranes are partially transparent to the electron beam, the bottom membrane can be seen through the top one. Right: The setup as used in Ref. [3].

3.3 Left: Measurement results from Ref. [4] and Ref. [5]. The solid lines are the theoretical values. Right: Schematic of the setup as used by Shen [4], using an AFM with a microsphere on it. A laser beam is reflected off the cantilever.

3.4 Experimental setup used by Jones and Raschke, from Ref. [6]. Both AFM tip and substrate are heated. The scattered radiation passes through a Michelson interferometer before it is captured by a MCT detector. Inset i shows a typical measured signal and ii shows the shape of the AFM tip. The substrate consists of a material with enhanced near field thermal emission by SPhPs or molecular vibrations.

3.5 Working principle of a TPV device.

4.1 Relative permittivity of SiC.

4.2 Relative permittivity of SiO$_2$ according to the measured values in Ref. [7] and fitted to an oscillator model as in Ref. [8]. On the top the real part is shown, the bottom plot is the imaginary part.

4.3 Relative permittivity of gold.

4.4 Incident, refracted and reflected waves on the interface between material 1 and vacuum. The horizontal component, $k_x$, is also depicted.
4.5 The contribution of the current at position $r'$ to the EM-field at position $r$ is shown. The propagating wavevectors in vacuum and medium 1 ($k_0$ and $k_1$) and their horizontal and vertical components are shown. The polarization vectors $\hat{s}$, $\hat{p}_0$ and $\hat{p}_1$ are shown. The x- and z-directions are determined by the unit vectors $\hat{x}$ and $\hat{z}$.

4.6 Radiative heat transfer between two plates with $\varepsilon = 11.7$. The top plate is at 295 K and the bottom at 275 K. The black body limit is also shown.

4.7 Same as Figure 4.6 but the heat flow is divided into its propagating and evanescent components and into s- and p-polarization components.

4.8 Radiative heat transfer between two SiO$_2$ plates. The top plate is at 295 K and the bottom at 275 K.

4.9 Spectral distribution of the heat flux between two SiO$_2$ plates with a gap of 100 nm in between them.

4.10 Same as Figure 4.8 but the heat flow is divided into its propagating and evanescent components and into s- and p-polarization components.

4.11 Radiative heat transfer between two gold plates. The top plate is at 295 K and the bottom at 275 K.

4.12 Radiative heat transfer between a 295 K SiO$_2$ and a 275 K gold plate.

4.13 Radiative heat transfer between a 295 K SiO$_2$ and a 275 K Si plate.

4.14 Radiative heat transfer between a 295 K SiO$_2$ and a 275 K 10$^{20}$ n-type doped Si plate.

4.15 Radiative heat transfer between a 295 K SiO$_2$ plate and a 275 K SiO$_2$ plate, full solution and approximation.

4.16 Radiative heat transfer between a 295 K array of metallic wires and split-ring resonators and a 275 K plate of the same material.

4.17 Radiative heat transfer between a 295 K array of metallic wires and split-ring resonators and a 275 K plate of the same material, divided into evanescent, propagating and polarization terms.

4.18 Proximity approximation, from Ref. [9].
4.19 Phase space plots of the accuracy of four approximation methods compared to a numerical solution for a sphere and a plate, from Ref. [10]. In the lightest areas the error is 3%, in the darkest areas the error is 450%. The gap size is \( d \), the diameter of the sphere is \( a \). The sphere is at 321 K, the plate at 300 K.

4.20 Total heat transfer as a function of distance for variously shaped objects and a plate, from Ref. [11]. The red dots are the results from Ref. [12].

4.21 Normalized spatially resolved heat transfer between object and plate, from [11].

4.22 Radiative heat transfer between two SiC plates. The top plate is at 295 K and the bottom at 275 K. The green line shows the influence of limiting the wavevector to a realistic value.

4.23 Radiative heat flow between a flat plate and a smooth sphere (red) and between a flat plate and rough sphere (magenta), all made of SiO\(_2\). The roughness is described by a Gaussian height distribution function with a mean of 20 nm for the upper magenta line and 30 nm for the lower magenta line and a standard deviation of 10 nm. The spheres are 50 \( \mu \)m in diameter. The vertical axis contains the total heat flow minus the heat flow at 300 nm. Above 300 nm the PA is not accurate in this case. The value of \( \alpha \) on the y-axis is 0.2558 nW and the temperatures are 0 K and 300 K. Image adapted from Ref. [13].

5.1 Schematic design of the experiment, seen from the top.

5.2 Schematic design of the experiment, seen from the side.

5.3 Floor vibrations do not have a major influence on the alignment of the system. If either the substrate or the right side of the picture moves up and down with respect to the other the horizontal distance between them does not change. The red dot is the laser spot on the cantilever, the laser path is perpendicular to the paper.
5.4 Photograph of the cantilever holder on the left and the cantilever with microsphere on the right. A close-up of the cantilever with sphere is in the bottom left. 72

5.5 Splitting of the laser beam in the Maypa OPS. From the manufacturer’s website, http://www.maypatech.com/. 73

5.6 Vacuum chamber without and with protective cage. 74

5.7 The reflected beam is shifted with a distance $\Delta x$ by the bending of the cantilever. Image adapted from Ref. [14]. 78

5.8 On the left a cantilever with a sphere on it is brought close to the aluminium substrate holder, without a substrate on it. Because of the smooth reflective surface of the aluminium the mirror image of the sphere is seen on the right side of the left picture. This way the distance between the sphere and the substrate holder can be estimated by comparing the distance between the sphere and its mirror image to the size of the sphere. In the right image a cantilever plus sphere is brought close to a glass substrate. The glass is highly transparent but the outline reveals the outer surface of the microscope slide. 79

6.1 Different heat losses in a perfect vacuum (left) and in air or partial vacuum (right). 82

6.2 Heat transfer coefficients for gas conduction (black) and radiation (red) between two infinitely large plates. 85

6.3 Left: illustration of the different length scales between the sphere and the plate, there is no clear heat transfer regime defined. Right: for an overestimate of the gas conduction between the sphere and substrate the sphere is modelled as a disk with a separation equal to the minimum separation distance between the sphere and the substrate. 86

XII
6.4 Gas conductance between a 100 µm disk and an infinite plate (colored) and experimental data on the near field thermal radiative conductance between a 100 µm sphere and an infinite plate (black). The different colors show the influence of lowering the pressure on the gas conductance. The steps in the gas conductance show the discrepancies between the heat flow model of the transition regime and that of the rarefied regime. Each black line is constructed with a few points from the original sources. 

6.5 Rotation of the cantilever at the free end caused by near field heat radiation as a function of separation gap between sphere and substrate. 

6.6 Two different ways in which the sphere can be attached to the cantilever. 

6.7 Rotation of the cantilever at the free end caused by van der Waals forces as a function of separation gap between sphere and substrate. 

6.8 Rotation of the cantilever at the free end caused by electrostatic forces as a function of separation gap between sphere and substrate. The potential difference between sphere and substrate is 100 V, one of them is grounded. 

7.1 These are the steps to place a few microspheres on the very edge of a microscope slide. Step 1: Dispense a few spheres on the slide. Step 2: Hold another microscope slide to the edge of the first one and tilt. Step 3: The spheres will roll towards the edge. Step 4: When the other microscope slide is removed some spheres will fall off the first one and some will remain on its very edge. 

7.2 Used setup for gluing the spheres on the cantilever. The cantiliver chip is in its holder right side of the picture, the microscope slide is on the left. 

7.3 Closeup of Figure 7.2, the white dots on the left are 100 µm spheres. One of the spheres is attached to the cantilver. 

7.4 View from the optical microscope on a PNP-DB cantilever with a 20 µm sphere (left) and on one with a 100 µm sphere (right).
7.5 The glueing process for an MLCT cantilever and 100 µm spheres as seen through the optical microscope.

7.6 Three different intensity spots, the measured center position by the OPS is illustrated by a black cross.

7.7 Photograph of the measurement setup in air. The temperature sensor that measures the air temperature is held by the clamp. The laser is blocked by a piece of paper in front of the PSD.

7.8 A 1 hour startup measurement of \( x_{\text{pos}} \) and \( x_{\text{sum}} \). The laser is switched on at \( t = 9 \) seconds.

7.9 Temperature developments during the start-up experiments.

7.10 A 1 hour drift measurement, without substrate.

7.11 Thermal noise of the longest MLCT cantilever with a 100 µm sphere.

7.12 Turning the laser on and off during a period of 100 seconds.

7.13 Zooming in on one of the steps in Figure 7.12, the deflection and intensity signals are now normalized to their maximum values.

7.14 The position signal as a function of the intensity signal when the gap is 5.16 µm. The zero-power \( x_{\text{pos}} \) is where the red line crosses the vertical axis.

7.15 Zero laser power \( x_{\text{pos}} \) as a function of separation gap \( d \).

7.16 Raw data of the \( x_{\text{pos}} \) signal for one of the experiments where the sphere was brought into contact with the surface (top). The sphere touches the surface at 25.3 seconds. Raw data of the capacitor from the same experiment (below).

7.17 Three measurements on three different days where the sphere was brought into contact with the substrate for the first time that day. For each of these measurements a new cantilever plus sphere was used.

7.18 Comparison of results for a few approaches before the sphere was first brought into contact with the sphere (blue), for when the sphere touched the substrate for the first time (red) and for a few approaches after that (green). All data was fitted to power law functions successfully.
7.19 The experimental setup placed in the vacuum chamber. The capacitive sensor is removed. The two cables provide the actuation voltage of the piezo and the other carries the feedback signal from the strain gauge.

7.20 Laser path with the optical viewport (left). In this picture the optical power meter is in the place where the PSD normally is. A mirror was laid on top of the 45 degree mirror (middle). An alignment tool that allows accurate alignment of the laser beam within the Thorlabs 30 mm cage system (right).

7.21 Pumping down the hose in 15 minutes, the pump is turned on 9 seconds after the measurement starts.

7.22 The cantilever rotation signal $x_{\text{pos}}$ plotted against the sum signal $x_{\text{sum}}$ when the pressure in the surrounding air is 0.1 mbar.

7.23 Measure of $x_{\text{pos}}$ versus the strain gauge elongation $d_{\text{gauge}}$. 

XV
List of Tables

2.1 Types of thermally induced EM-waves ........................................... 16
4.1 Polariton conditions ................................................................. 56
4.2 Dependence of $Q$ on $d$ in near field thermal radiation between two
$SPhP$ supporting materials. ......................................................... 57
5.1 The pros and cons of different shapes of the sensor head. ............ 61
5.2 Thermal and mechanical properties of thin films.......................... 67
5.3 Cantilever sensitivity analysis. The shape is either rectangular or
triangular, no tip means it is a tipless cantilever. Material 2 is the
reflective layer. The geometrical parameters are the total thickness
$t$, thickness of the reflective layer $t_2$, the width $w$ and length $L$. The
bending sensitivity to heat input by the laser is $S_Q$, the bending
sensitivity to temperatures is $S_T$. ............................................. 68
5.4 Different cantilevers on the MLCT and the PNP-DB cantilever chip. 69
5.5 Three available microscope objectives. ................................. 78
7.1 Measured power meter readings during different lighting situations. 101
7.2 The measured drift in $x_{pos}$ over 1 hour periods, twice with the sub-
strate close to the sphere and twice without the substrate. ............ 103
7.3 Correlations between $x_{pos}$ and the other measured signals during
the drift measurements. ............................................................. 104
7.4 All the resonance frequencies measured during this project. The
values in the last two rows are each for a different cantilever with a
100 $\mu$m sphere on it. .............................................................. 106
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFM</td>
<td>Atomic Force Microscope</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>DGF</td>
<td>Dyadic Green’s Function</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>EMT</td>
<td>Effective Medium Theory</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared</td>
</tr>
<tr>
<td>LA</td>
<td>Longitudinal Acoustic</td>
</tr>
<tr>
<td>LO</td>
<td>Longitudinal Optical</td>
</tr>
<tr>
<td>MEMS</td>
<td>Microelectromechanical System</td>
</tr>
<tr>
<td>MCT</td>
<td>Mercurium Cadmium Telluride</td>
</tr>
<tr>
<td>MPA</td>
<td>Modified Proximity Approximation</td>
</tr>
<tr>
<td>NEMS</td>
<td>Nanoelectromechanical System</td>
</tr>
<tr>
<td>PA</td>
<td>Proximity Approximation</td>
</tr>
<tr>
<td>PSD</td>
<td>Position Sensitive Detector</td>
</tr>
<tr>
<td>RTD</td>
<td>Resistance Temperature Detector</td>
</tr>
<tr>
<td>SPhP</td>
<td>Surface Phonon Polariton</td>
</tr>
<tr>
<td>SPP</td>
<td>Surface Plasmon Polariton</td>
</tr>
<tr>
<td>TA</td>
<td>Transverse Acoustical</td>
</tr>
<tr>
<td>TIR</td>
<td>Total Internal Reflection</td>
</tr>
<tr>
<td>TO</td>
<td>Transverse Optical</td>
</tr>
<tr>
<td>TPV</td>
<td>Thermophotovoltaic</td>
</tr>
<tr>
<td>UHV</td>
<td>Ultra High Vacuum</td>
</tr>
<tr>
<td>UV</td>
<td>Ultra Violet</td>
</tr>
</tbody>
</table>
## List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>amplitude noise at resonance frequency (V)</td>
</tr>
<tr>
<td>$A$</td>
<td>Hamaker constant (J)</td>
</tr>
<tr>
<td>$a$</td>
<td>atomic spacing (m)</td>
</tr>
<tr>
<td>$A_j$</td>
<td>area of object $j$</td>
</tr>
<tr>
<td>$A_{white}$</td>
<td>white noise amplitude (V)</td>
</tr>
<tr>
<td>$B$</td>
<td>near field radiative heat transfer constant (W/K)</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field (T)</td>
</tr>
<tr>
<td>$C$</td>
<td>spring constant (N/m)</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light in vacuum (2.998 x 10^{-8} m s^{-1})</td>
</tr>
<tr>
<td>$c_{pj}$</td>
<td>thermal capacity of of material $j$ (JK^{-1})</td>
</tr>
<tr>
<td>$D$</td>
<td>force term (N)</td>
</tr>
<tr>
<td>$D$</td>
<td>electric displacement field (C m^{-2})</td>
</tr>
<tr>
<td>$d$</td>
<td>separation gap (m)</td>
</tr>
<tr>
<td>$d_0$</td>
<td>reference distance (m)</td>
</tr>
<tr>
<td>$d_{cap}$</td>
<td>measured distance by capacitor (m)</td>
</tr>
<tr>
<td>$d_m$</td>
<td>molecular diameter (m)</td>
</tr>
<tr>
<td>$E$</td>
<td>elastic modulus (Pa)</td>
</tr>
<tr>
<td>$E$</td>
<td>energy (J)</td>
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<tr>
<td>$E$</td>
<td>electric field (V m^{-1})</td>
</tr>
<tr>
<td>$e$</td>
<td>electron charge (1.6022 x 10^{-19} C)</td>
</tr>
<tr>
<td>$E_{vdW}$</td>
<td>van der Waals interaction energy (J)</td>
</tr>
<tr>
<td>$F$</td>
<td>volume filling fraction (-)</td>
</tr>
<tr>
<td>$f$</td>
<td>frequency (Hz)</td>
</tr>
<tr>
<td>$F_c$</td>
<td>electrostatic force (N)</td>
</tr>
</tbody>
</table>
\( F_{ij} \) view factor from object \( i \) to object \( j \) (-)
\( F_v dW \) van der Waals force (N)
\( F_{o} \) Fourier number (-)
\( f_{res} \) resonance frequency (Hz)
\( G \) thermal conductance (W K\(^{-1}\))
\( \overline{G} \) dyadic Green’s function (m\(^{-1}\))
\( G_{ij} \) thermal conductance from object \( i \) to object \( j \) (W K\(^{-1}\))
\( G_{i,j} \) components of Dyadic Green’s function in direction \( i,j \) (m\(^{-1}\))
\( H \) magnetising field (A m\(^{-1}\))
\( h \) heat transfer coefficient (W m\(^{-2}\) K\(^{-1}\))
\( h \) Planck constant (6.626 \( \times \) 10\(^{-34}\) J s)
\( h \) reduced Planck constant (\( h/2\pi \))
\( h_{air} \) conduction heat transfer coefficient in rarefied air (W m\(^{-2}\) K\(^{-1}\))
\( h_{ij} \) heat transfer coefficient between object \( i \) and \( j \) (W m\(^{-2}\) K\(^{-1}\))
\( h_{pp} \) near field radiative heat transfer coefficient between two infinite plates (W m\(^{-2}\) K\(^{-1}\))
\( h_{radiation} \) radiation heat transfer coefficient (W m\(^{-2}\) K\(^{-1}\))
\( h_{spec} \) spectral heat transfer coefficient (W s rad\(^{-1}\) m\(^{-2}\) K\(^{-1}\))
\( h_{vis} \) heat transfer coefficient in viscous regime(W m\(^{-2}\) K\(^{-1}\))
\( I \) second moment of inertia (m\(^4\))
\( i \) imaginary number (\( \sqrt{-1}\))
\( J \) current density (A m\(^{-2}\))
\( j_j \) current density in direction \( j \) (A m\(^{-2}\))
\( K \) cantilever constant (-)
\( k \) wavenumber (rad m\(^{-1}\))
\( k \) wavevector (rad m\(^{-1}\))
\( k_{air} \) thermal conductivity of air (W m\(^{-1}\) K\(^{-1}\))
\( k_B \) Boltzmann constant (1.307 \( \times \) 10\(^{-23}\) J K\(^{-1}\))
\( k_{ij} \) wavevector component in direction \( i \) in medium \( j \) (rad m\(^{-1}\))
\( k_j \) wavevector component in direction \( j \) (rad m\(^{-1}\))
\( k_j \) thermal conductivity of material \( j \) (W m\(^{-1}\) K\(^{-1}\))
\( k_j^p \) \hspace{1em} p-polarized wavevector component in direction \( j \) (rad m\(^{-1}\))

\( k_j^s \) \hspace{1em} s-polarized wavevector component in direction \( j \) (rad m\(^{-1}\))

Kn \hspace{1em} Knudsen number (-)

\( L \) \hspace{1em} length (m)

\( l \) \hspace{1em} length (m)

\( M \) \hspace{1em} bending moment (Nm)

\( m \) \hspace{1em} mass (kg)

\( m_{\text{cantilever}} \) \hspace{1em} cantilever mass (kg)

\( m_e \) \hspace{1em} free electron mass \( (9.1096 \times 10^{-31} \text{ kg}) \)

\( m_e^* \) \hspace{1em} effective electron mass (kg)

\( m_h^* \) \hspace{1em} effective hole mass (kg)

\( m_j \) \hspace{1em} mass of object \( j \) (kg)

\( m_{\text{sphere}} \) \hspace{1em} mass of sphere (kg)

\( n \) \hspace{1em} particle density (m\(^{-3}\))

\( n \) \hspace{1em} refractive index (-)

\( N_e \) \hspace{1em} number of free electrons per unit volume (m\(^{-3}\))

\( N_h \) \hspace{1em} hole concentration (m\(^{-3}\))

\( n_j \) \hspace{1em} refractive index of medium \( j \) (-)

\( P \) \hspace{1em} absorbed laser power (W)

\( p \) \hspace{1em} pressure (Pa)

\( Q \) \hspace{1em} Q factor (-)

\( Q \) \hspace{1em} heat flow (W)

\( q \) \hspace{1em} heat flux (W m\(^{-2}\))

\( q \) \hspace{1em} distributed load (N m\(^{-2}\))

\( Q_{\text{air}} \) \hspace{1em} heat transfer to surrounding air (W)

\( Q_{\text{cond}} \) \hspace{1em} heat transfer by conduction (W)

\( Q_{ij} \) \hspace{1em} heat flow from object \( i \) to object \( j \) (W)

\( q_{ij,NF} \) \hspace{1em} near field component of the heat flux between object \( i \) and \( j \) (W m\(^{-2}\))

\( q_{ij,FF} \) \hspace{1em} far field component of the heat flux between object \( i \) and \( j \) (W m\(^{-2}\))

\( Q_{NF} \) \hspace{1em} heat transfer by near field thermal radiation (W)

XXI
\( Q_{pd} \)  heat transfer from plate to dipole (W)
\( Q_{rad} \)  heat transfer by radiation (W)
\( q_{spec} \)  spectral heat flux (W s rad\(^{-1}\) m\(^{-2}\))
\( Q_{sub} \)  heat transfer by gas conduction to the substrate (W)
\( \hat{P}_j \)  p-polarization unit vector in medium \( j \) (m)
\( R \)  radius (m)
\( R \)  spectral radiation intensity (W m\(^{-2}\) rad\(^{-1}\))
\( r \)  distance (m)
\( r \)  position (m)
\( r^p_{ij} \)  reflection coefficient between medium \( i \) and \( j \) in p-polarization (-)
\( r^s_{ij} \)  reflection coefficient between medium \( i \) and \( j \) in s-polarization (-)
\( \mathbf{r}_j \)  position of object \( j \) (m)
\( S \)  Poynting vector (W m\(^{-2}\))
\( S \)  noise spectrum (V)
\( s \)  distance (m)
\( \hat{s} \)  s-polarization unit vector (m)
\( s_{evan} \)  evanescent wave contribution (rad m\(^{-1}\))
\( s_{prop} \)  propagating wave contribution (rad m\(^{-1}\))
\( S_Q \)  rotation sensitivity to heat flows (rad W\(^{-1}\))
\( S_T \)  rotation sensitivity to temperature changes (rad K\(^{-1}\))
\( T \)  temperature (K)
\( t \)  thickness (m)
\( t \)  time (s)
\( T_0 \)  stress free temperature (K)
\( T_{air} \)  air temperature (K)
\( T_{avg} \)  average temperature (K)
\( T_{can} \)  cantilever holder temperature (K)
\( T_{dev} \)  deviation temperature (K)
\( T_{end} \)  temperature at the end of the cantilever (K)
\( t_{ij}^p \) Fresnel transmission coefficient from medium \( i \) to medium \( j \) for p-polarization (-)

\( t_{ij}^s \) Fresnel transmission coefficient from medium \( i \) to medium \( j \) for s-polarization (-)

\( T_j \) temperature of object \( j \) (K)

\( t_j \) thickness of layer \( j \) (m)

\( T_{room} \) room temperature (K)

\( T_{sphere} \) sphere temperature (K)

\( T_{sub} \) substrate temperature (K)

\( u \) spectral energy density \((\text{J s rad}^{-1} \text{m}^{-3})\)

\( v \) propagation velocity (m/s)

\( V \) electric potential (V)

\( V \) volume (m\(^3\))

\( w \) width (m)

\( x \) position along cantilever (m)

\( \hat{x} \) unit vector in x-direction (m)

\( x_j \) position of object \( j \) (m)

\( x_{left} \) sum signal of left quadrants (V)

\( x_{pos} \) x-position signal (V)

\( x_{right} \) sum signal of right quadrants (V)

\( x_{sum} \) sum signal of all quadrants (V)

\( y_{bottom} \) sum signal of bottom quadrants (V)

\( y_c \) displacement of centre coordinate (m)

\( y_{pos} \) y-position signal (V)

\( y_{sum} \) sum signal of all quadrants (V)

\( y_{top} \) sum signal of top quadrants (V)

\( z \) deflection (m)

\( \hat{z} \) unit vector in z-direction (m)

**GREEK SYMBOLS**

\( \alpha \) polarizability \((\text{C m}^2 \text{V}^{-1})\)

\( \alpha_j \) polarizability of particle \( j \) \((\text{C m}^2 \text{V}^{-1})\)
\(\alpha_j\) thermal expansion coefficient of material \(j\) (K\(^{-1}\))

\(\beta\) spring constant (N m\(^{-1}\))

\(\gamma\) damping factor (rad s\(^{-1}\))

\(\gamma_e\) electric dissipation factor (rad s\(^{-1}\))

\(\gamma_m\) magnetic dissipation factor (rad s\(^{-1}\))

\(\delta\) Dirac delta function

\(\delta_{ij}\) Kronecker delta

\(\delta_j\) penetration depth in medium \(j\) (m)

\(\varepsilon\) relative permittivity (-)

\(\varepsilon_0\) permittivity of vacuum \((8.8554 \times 10^{-12} \text{ F m}^{-1})\)

\(\varepsilon_{in}\) intrinsic relative permittivity (-)

\(\varepsilon_j\) emissivity of an object made of medium \(j\) (-)

\(\varepsilon_j\) relative permittivity of medium \(j\) (-)

\(\varepsilon_\infty\) high frequency dielectric constant (-)

\(\Theta\) average energy of a Planck oscillator (J)

\(\theta\) (rotation) angle (rad)

\(\theta_{cr}\) critical angle (rad)

\(\kappa\) thermal diffusivity (m\(^2\) s\(^{-1}\))

\(\kappa\) conductivity (rad\(^2\) s\(^{-2}\))

\(\kappa_j\) conductivity of medium \(j\) (rad\(^2\) s\(^{-2}\))

\(\lambda\) wavelength (m)

\(\lambda_{air}\) mean free path of air (m)

\(\lambda_{\text{max}}\) peak wavelength (m)

\(\mu\) relative permeability (-)

\(\mu_0\) permeability of vacuum \((4\pi \times 10^{-7} \text{ H m}^{-1})\)

\(\mu_j\) relative permittivity of medium \(j\) (-)

\(\nu\) Poisson’s ratio (-)

\(\rho_j\) density of material \(j\) (kg m\(^{-3}\))

\(\sigma\) Stefan Boltzmann constant \((2\pi^5 k_B^4 / 15h^3c)\)

\(\tau\) time constant (s)

\(\tau_e\) free electron scattering time (s)

\(\tau_h\) free hole scattering time (s)
\[ \Omega \quad \text{solid angle (sr)} \]
\[ \omega \quad \text{angular frequency (rad s}^{-1}\text{)} \]
\[ \omega_m \quad \text{magnetic resonance frequency (rad s}^{-1}\text{)} \]
\[ \omega_{ns} \quad \text{resonance frequency without sphere (rad s}^{-1}\text{)} \]
\[ \omega_p \quad \text{plasma frequency (rad s}^{-1}\text{)} \]
\[ \omega_r \quad \text{resonance frequency (rad s}^{-1}\text{)} \]
\[ \omega_{r,j} \quad \text{resonance frequency in medium } j \text{ (rad s}^{-1}\text{)} \]
\[ \omega_s \quad \text{resonance frequency with sphere (rad s}^{-1}\text{)} \]
Chapter 1

Introduction

1.1 Scope

In recent years experiments have confirmed that heat transfer by near field thermal radiation can greatly surpass the black body radiation limit. This heat flow strongly depends on the distance between the emitting and absorbing object, like the capacitance between two metal plates it increases with decreasing distances. The question arose whether or not this phenomenon can be used as a sensing principle to measure the distance between the involved objects.

There are many ways to measure distances with high accuracy. Besides optical methods like interferometry, capacitive sensing, eddy current sensing and inductive sensing are used. Why would anybody use thermal radiation to measure distances? All objects with a temperature higher than absolute zero emit thermal radiation so it benefits any material, not just metals as it does for capacitive, eddy current and inductive sensors. The material of the target also doesn’t have to be reflective as it should when using an interferometer. Sampling thermal radiation requires no contact between the objects, it works for soft samples and it will not contaminate the measured surface in any way. Absorbing some thermal radiation from an object or emitting a small amount of thermal radiation to it is a very minimally invasive method of measuring.
Chapter 1. Introduction

With near field effects prevailing at submicron distances near field thermal radiation sensing can become an important tool in nanomanufacturing as nanomanufacturing requires new nanometerology tools to be developed. One particular application in which such a sensor is valuable is the 'meta-instrument' in development by TNO right now. The meta-instrument is an instrument that positions a lens made of a photonic metamaterial close to its target. These metamaterial lenses allow imaging beyond the diffraction limit. This kind of metamaterial has to be placed in the near field of the substrate to allow subwavelength details of the object to be imaged [15], this translates into a separation gap of in between 20 and 100 nm in this particular application.

1.2 Goal

In a final design of a sensor that can sense the position of another object there are four basic properties that describe its performance. The accuracy of a measurement is high if its uncertainty (often defined as an uncertainty interval like ±5 µm) is low. The precision of a measurement is a measure of its repeatability, it should give the same value again and again when nothing in the system changes. High accuracies and precisions are achieved by a high sensitivity of the sensor to the measured signal. The sensitivity is defined as the change in sensor output divided by the change in the measured quantity. The resolution is the smallest step in measured signal that can be resolved. Finally the speed of the measurement is important as well, this is measured by the response time of the sensor to a change in the measured quantity. When used in a production process high speed measurements are required to achieve a high throughput.

The main goal is to perform a feasibility study on whether or not it is possible to use near field thermal radiation as a sensing principle to measure the position of an object in very close vicinity of the sensor (in the order of micrometers or even nanometers). This requires an experimental proof of concept, a measurement setup that allows the distance between two objects to be determined by measuring the near field thermal radiation between the two objects or by measuring a physical
quantity derived from the near field thermal radiation. The influence of the sensor design on the accuracy, the precision, the resolution and speed of measurements should be studied as well.
Chapter 2

Thermal Radiation Physics

Electromagnetic (EM) radiation is the propagation of electromagnetic fields. A propagating wave, also known as a travelling wave, transports energy in the direction it is travelling. Radiation can also be described in terms of particles, i.e. photons. In this chapter the basic concepts and properties of thermal radiation are explained including near field thermal radiation.

2.1 Origins of Thermal Radiation

Microscopic charges (electrons, ions, polar molecules) inside objects of temperature higher than absolute zero are subjected to thermal oscillations and collisions. According to Maxwell’s equations accelerating charges generate an electromagnetic field. Heat exchange between two objects by radiation is the process in which one object absorbs thermal radiation emitted by the other object.

2.2 Far Field Thermal Radiation

A black body is an object that absorbs all electromagnetic radiation falling onto it, reflecting none. As a consequence, all radiation leaving a black body is emitted
by the surface itself. No surface can emit more radiation than a black body at
the same temperature, it’s an idealized upper limit for an emissive body. For a
black body in thermal equilibrium its emitted thermal radiation is characterized
by Planck’s law [16]. Planck’s law describes the radiation spectrum, the spectral
power of radiation emitted per unit surface area per unit solid angle as a function
of temperature (T) and radial frequency (ω):

\[
R(T, \omega) = \frac{\hbar\omega^3}{4\pi^3c^2} \cdot \frac{1}{e^{\hbar\omega/k_B T} - 1}
\]  

Figure 2.1: Radiation emitted from a surface of area dA through solid angle dΩ.

The wavelength of a photon λ, and its frequency in Hz f, are related to the speed
of light c, as \( \lambda = c/f \). The radial frequency is \( \omega = 2\pi f \). The constants in
the equation are the Boltzmann constant \( k_B \), and the reduced Planck’s constant
\( \hbar = h/2\pi \), where \( h \) is Planck’s constant. A black body of temperature T emits
photons of different wavelengths but has a peak in its distribution at a certain
wavelength, \( \lambda_{\text{max}} \). This peak shifts with temperature and is approximated by
Wien’s displacement law [17]:

\[
\lambda_{\text{max}} \cdot T = 2.989 \times 10^{-3} \text{ K m}
\]

For room temperature objects, the peak wavelength is in the thermal infrared (IR)
at about 10µm. Integrating Planck’s law over a half sphere and over all frequencies
the result is the radiant energy emitted per unit area by a black body as stated in the Stefan-Boltzmann law \[18\]:

\[
q = \frac{\pi^2 k_B^4 T^4}{60h^2c^2} = \sigma T^4
\]

(2.3)

When travelling EM waves emitted by the thermal fluctuations in one material are absorbed by another material thermal energy is exchanged. The net exchange in thermal radiation energy between two black bodies is determined by a view factor \((F_{12})\) and the area of the first object:

\[
Q_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4)
\]

(2.4)

The radiation emitted by a real surface is better described by a grey body model with emissivity \(\epsilon\). The emissivity is the fraction of black body radiation emitted by a grey object of the same temperature. The heat transfer between two enclosing grey bodies (like two infinite surfaces facing each other) is given by \[17\]:

\[
Q_{12} = \left( \frac{1 - \epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \left( \frac{1 - \epsilon_2}{\epsilon_2} \right) \frac{A_1}{A_2} \right)^{-1} A_1 \sigma (T_1^4 - T_2^4)
\]

(2.5)

The heat transfer between two finite grey bodies enclosed by a third surface (the surroundings) that absorbs thermal radiation and emits the same amount of radi-
2.2. Far Field Thermal Radiation

Far field thermal radiation (no energy loss from the system) is:

\[ Q_{12} = \left( \frac{1 - \epsilon_1}{\epsilon_1} + \frac{A_1 + A_2 - 2A_1F_{12}}{A_2 - A_1F_{12}^2} \right) + \left( \frac{1 - \epsilon_2}{\epsilon_2} \right) \frac{A_1}{A_2}^{-1} A_1 \sigma \left( T_1^4 - T_2^4 \right) \] \hspace{1cm} (2.6)

Consequently if \( A_2 \) becomes very large with respect to \( A_1 \) this simplifies to:

\[ Q_{12} = \epsilon A_1 \sigma (T_1^4 - T_2^4) \] \hspace{1cm} (2.7)

Linearising this relation for small temperature differences, using a heat transfer coefficient \( h = Q_{12(\Delta T \rightarrow 0)}/A_1 \Delta T \) and the average temperature of the surfaces \( T_{avg} \) the result is:

\[ Q_{12} = 4 \epsilon A_1 T_{avg}^3 \sigma (T_1 - T_2) = hA_1(T_1 - T_2) \] \hspace{1cm} (2.8)

These relations are often used in engineering practice and are shown here because they will be used later. The emissivity is assumed to be constant over all frequencies.

For geometries other than infinite flat surfaces the view factor becomes important. The view factor between a sphere and an infinite flat surface is \( 1/2 \) \cite{17}. Note that this is not a function of distance between sphere and surface. The view factor between a flat surface and a sphere can be calculated using the reciprocity rule for view factors:

\[ A_1F_{12} = A_2F_{21} \] \hspace{1cm} (2.9)

\[ \begin{array}{c}
\text{Figure 2.3: View factor determination between a sphere and a large flat surface (left) and between two spheres (right).}
\end{array} \]
For two non-touching spheres of equal size the view factor is [17]:

\[ F_{12} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{R^2}{s^2}} \right) \]  

(2.10)

Both spheres have a radius of \( R \), \( s \) is the distance between the centres of the spheres. The view factor tends to \( 1/2 - \sqrt{3}/16 \approx 0.067 \) when the gap becomes really small or when the spheres become very large. For large separations the view factor tends to zero. When the spheres are of unequal size and \( R_2 >> R_1 \) the view factor is [17]:

\[ F_{12} = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{R_2^2}{s^2}} \right) \]  

(2.11)

This is no longer a function of the radius of the smaller sphere. When the gap between the spheres becomes really small the view factor becomes the same as for a sphere-infinite surface geometry, namely \( 1/2 \). The reverse view factor can be found using the reciprocal rule again but tends to be very small since sphere 2 is much larger than sphere 1. With (2.6) the grey body heat flow between these objects can be calculated.

Planck’s law is only valid in a region further than a characteristic distance away from its source, approximately equal to the wavelength of the radiation as given by Wien’s law. This region is termed the ‘far field’. The shorter distance region is the ‘near field’. What happens in the near field region is explained in the next section.

2.3 Near Field Thermal Radiation

In the near field propagating waves are not the only waves contributing to energy transfer, evanescent fields also contribute. The coupled modes of lattice vibrations and EM fields near the surface of an object can greatly enhance near field thermal radiation.
2.3. Near Field Thermal Radiation

2.3.1 Evanescent Waves

Evanescent waves are usually explained using the concept of total internal reflection (TIR) and Snell’s law. If radiation propagating in a medium with a high index of refraction reaches the interface with another material of lower index of refraction and the angle of incidence is larger than the critical angle the radiation will be totally reflected:

$$\theta_{cr} = \sin^{-1}\left(\frac{n_0}{n_1}\right)$$

Figure 2.4: On the left the radiation is partially refracted and the other part is reflected. When the angle of incidence is larger than the critical angle the radiation is completely reflected as shown on the right.

This equation gives the critical angle as a function of the higher index of refraction $n_1$, and the index of refraction for air or vacuum $n_0 = 1$. However, the EM field is not abruptly terminated at the interface, Maxwell’s equations do not allow such a discontinuity. When solving Maxwell’s equations for the case of TIR an EM field appears in the material with the low refractive index that decreases exponentially with increasing distance away from the surface (Figure 2.5). These waves propagate along the interface between the two materials, they are evanescent waves. Evanescent waves decay in the direction normal to the interface with a decay length equal to the wavelength of radiation. Because these waves do not
propagate in the medium of low refractive index there is no radiation or energy exchange in this direction [19]. This can also be explained by looking at the Poynting vector. The Poynting vector is the cross product of the electric and magnetic field vectors. The energy flux by the EM field is equal to the real part of the Poynting vector. The electric field near the surface is purely imaginary, the magnetic field is purely real so there is no energy flux [20]. Thermal radiation can originate from any point in a body of temperature T. Some waves will propagate to the other medium and other waves will experience TIR. It is important to notice that propagating waves do exist in the near field. Close to their source these waves are much more coherent than they are in the far field. Wave-effects like interference and diffraction can occur. Usually the contribution of propagating radiation to the total energy transfer in the near field is small.

When a second object is brought close to a high temperature object and the evanescent waves of the high temperature object extend into the second object like in Figure 2.6 they can excite charges inside it, instigating Joule heating. This object should have a higher index of refraction than the medium that forms the gap between them (usually air or vacuum). The evanescent field of the first object is reflected by the other object. The Poynting vector of the coupled evanescent field does have a real component and energy flows between the two objects [21]. It is said that radiation ‘tunnels’ between the gap [22]. The magnitude of evanescent waves can be strongly enhanced by surface resonances and the energy transfer can be increased greatly beyond the energy transfer as predicted by Planck’s law. Without considering these surface resonances the enhancement of black body radiation between two objects of the same lossless material with refractive index n due to evanescent waves is equal to $n^2$ times the black body heat exchange when the vacuum gap between them becomes infinitely small [4].

### 2.3.2 Surface Phonon Resonance

The random motion of particles constitutes the thermal energy of solids. In crystalline materials atoms are ordered in three dimensional lattices with each atom bonded to its neighbours. To gain some understanding into lattice vibrations a
general first step is to model the material as a one-dimensional chain of masses and springs. The masses represent the atom masses and the springs generate the inter-atomic forces [16]. Only nearest neighbour interactions are taken into account, in reality the interatomic forces diminish with increasing distance but are not restricted to nearest neighbours only. The equation of motion is written for the atom at position $x_n$, $\beta$ is the spring constant of the bonding interaction and
Figure 2.6: Same as in Figure 2.5 but now another high index material (blue) is brought close to the surface of the first high index material. When the gap between them is large the evanescent wave coupling is negligible. In the bottom picture the gap is very small and the evanescent wave coupling increases.

$m$ is the mass of each individual atom:

$$m\ddot{x}_n = \beta(x_{n+1} + x_{n-1} - 2x_n)$$  \hspace{1cm} (2.13)

Figure 2.7: Chain of atoms linked by interatomic forces.
2.3. Near Field Thermal Radiation

Using a travelling wave solution the dispersion relation is [16]:

\[ \omega = \sqrt{4\beta/m \sin (2ka)} \] (2.14)

In this equation \( a \) is the atomic spacing. This dispersion relation is the relation between frequency and wavenumber. The wavenumber, \( k \), is the spatial frequency of a wave, the number of waves per unit length so \( kv = f \) with propagation velocity \( v \). Unlike acoustic vibrations in air or surface waves on water the relation between the propagation velocity and frequency is not linear. If the wavelength is long, that is \( ka \ll 1 \), then the macroscopic linear relation appears and the vibration propagates with the velocity of sound in the medium. For a diatomic chain of atoms, i.e. a chain with two different alternating types of atoms, the dispersion relation is found in a similar way and is given as [16]:

\[ \omega^2 = \frac{\beta(m_1 + m_2)}{m_1 m_2} \left(1 \pm \sqrt{1 - \frac{m_1 m_2}{(m_1 + m_2)^2} \sin^2(2ka)}\right) \] (2.15)

This is shown in Figure 2.9. There are two types of vibrations occurring. The upper branch is the optical branch, it supports the highest frequencies. The lower branch is called the acoustic branch, at low wavenumbers it becomes the macroscopic linear dispersion relation. This is illustrated in Figure 2.8. An important distinction between the optical and acoustic branch is that in the optical branch the atoms oscillate in counterphase with their neighbours and in the acoustic branch they oscillate mostly in unison with their neighbours as you would expect for long wavelengths (sound waves). Besides longitudinal vibrations transverse vibrations can also occur chains of linked atoms. This adds a transverse acoustic branch and a transverse optical branch to Figure 2.9. The number of vibration modes is limited or in other words, the number of modes is quantized and depends on the number of atoms in the lattice. The energy of a lattice vibration is also quantized and increases with steps of \( \Delta E = \hbar \omega \) [16]. Because of this particle-like behaviour a quantum of lattice vibration is considered to be a 'quasiparticle' and is called a 'phonon'.

Surface phonons are phonons that exist at the surface of an object. They are different from bulk phonons in the sense that they arise from the termination of the crystal structure at the surface. The differences between bulk phonons and
Figure 2.8: Transverse optical and acoustic phonons in an chain of oppositely charged ions, notice the separation of charge on the optical phonon.

Figure 2.9: Dispersion relation for optical and acoustic phonons in a diatomic chain of atoms. LO is the longitudinal optical branch, TO is the transverse optical branch, LA is the longitudinal acoustic branch and TA is the transverse acoustic branch. The material properties used are hypothetical.

surface phonons is described in more detail in Section 4.2.1. In short, a surface phonon is characterized by its frequency, its propagation speed across the surface and the intensity with which it decays away from the interface.

Because polar materials are made up of pairs of positive and negative charges some oscillations in the lattice can generate an EM field like shown in Figure 2.8.
The EM field generated by surface phonons has evanescent components in both directions normal to the surface as seen in Figure 2.10. This hybrid mode of lattice vibration and the accompanying EM field can also be considered a quasiparticle and is called a surface phonon polariton (SPhP). The frequencies at which lattice vibrations cause a considerable EM field are very specific making this a much more monochromatic source of EM fields than regular thermal radiation. Examples of materials that support SPhP’s are oxides, glasses, sapphire, SiC, BN, III-V semiconductors and II-VI semiconductors [1, 23, 24].

![Figure 2.10: Impression of surface phonon polariton evanescent waves on the interface of two materials. Again the contrast shows the difference in polarity, the arrow shows the direction of propagation.](image)

### 2.3.3 Surface Plasmon Resonance

In a plasma positive and negative charges are not bound to each other. The medium itself has no net charge; the positive charges are balanced by the negative charges. A metal can be considered a plasma because its free electrons are not bound to the positive ion cores. These free electrons form a gas-like cloud of negative charges around the stationary cores that is highly responsive to applied electric fields. If the electrons are displaced with respect to the cores they will experience a Coulomb force to pull them back and they will start to oscillate around the cores. Thermal vibrations can also cause this separation of charges. A plasmon is a quantum of plasma oscillation just like phonons are a quantum of lattice
vibration, both are quasiparticles. The coupling mode between this mechanical vibration and the generated EM field at the surface is called a surface plasmon polariton (SPP). Metals with a lot of free electrons like gold, silver and copper are very susceptible to surface plasmon resonances. Unfortunately, for metals the frequency of plasma oscillations are much higher than thermal frequencies near room temperature, they cannot be thermally excited [25]. Doped semiconductors also exhibit surface plasmon resonances [19]. Because of their lower plasma frequency these surface plasmon resonances can be thermally excited [26].

2.3.4 Conclusion

Both SPPs and SPhPs are evanescent waves. Unlike far field thermal radiation near field radiative heat transfer due to polaritons is highly monochromatic. Now that all contributions in the near field are discussed they are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Region</th>
<th>Type of waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>far field</td>
<td>Propagating waves</td>
</tr>
<tr>
<td>near field</td>
<td>- Propagating waves</td>
</tr>
<tr>
<td></td>
<td>- Evanescent waves (TIR, SPP, SPhP)</td>
</tr>
</tbody>
</table>

**Table 2.1:** Types of thermally induced EM-waves
Chapter 3

Near Field Thermal Radiation Experiments and Applications

In this chapter different experiments are discussed that were designed to measure near field thermal radiation. Thereafter some applications of near field thermal radiation are explored. This overview creates understanding of the current state or research and is also helpful in understanding how different microscale/nanoscale probing instruments are made.

3.1 Past Experiments on Near Field Thermal Radiation

The experiments are organised by their geometrical configuration. In Section 3.1.1 experiments are discussed wherein the radiative heat flux between two parallel plates was measured. In Section 3.1.2 the heat exchange measured is between a conical tip and a surface. In Section 3.1.3 the heat exchange is between a sphere and a surface.
Chapter 3. Near Field Thermal Radiation Experiments and Applications

3.1.1 Experiments with Parallel Plates

The earliest models of near field thermal radiation describe the heat flux between two infinite half spaces with a gap in between them. This model is applicable for two plates with a thickness much larger than the characteristic wavelength of thermal radiation and lengths and widths much larger than the gap size. Experimentally it is very difficult to maintain a micrometer sized gap between two large plates. For a 10 cm square plate a misalignment less than 1 mrad will cause the plates to contact when the gap is supposed to be 1 \( \mu \text{m} \). Curvature and surface roughness will also cause non-uniformity of the gap. A benefit of using two plates is that the total heat flow is large compared to the other geometries and is thus easier to measure.

In the earliest experiments the heat flux between two parallel plates was measured. Domoto \textit{et al.} [27] used copper plates at cryogenic temperatures with gaps in between them ranging from 1 to 10 \( \mu \text{m} \). A benefit of low temperatures is the long characteristic wavelength of thermal radiation. The near field enhancement becomes apparent at larger gaps compared to room temperatures. Hargreaves [28] used chromium plates with temperatures of 130 K and 300 K with gaps in between them ranging from 1.5 to 6 \( \mu \text{m} \). They measured heat fluxes higher than those of far field radiation but less than the black body limit.

A more recent example where the heat flux was measured between two plates is the research by Ottens \textit{et al.} [1]. They measured heat flux between sapphire plates and used stepper motors to adjust the spacing and tilt of the plates. The distance between the plates was measured by capacitors and ranged from 1 \( \mu \text{m} \) to 100 \( \mu \text{m} \). One of the plates was heated by a copper ring. Temperatures of the plates were measured using thermometers attached to the plates. The whole system was placed in vacuum. The results of this experiment were in close agreement with the theory of Section 4.4.2.

Kralik \textit{et al.} measured the heat flow between two tungsten disks [29, 30]. Tungsten was chosen for its high emissivity. The disks were placed in an ultra high vacuum (UHV), liquid helium cooled environment. A resistive heater generated the tem-
3.1. Past Experiments on Near Field Thermal Radiation

Figure 3.1: Test setup as used in Ref. [1].

Hu et al. used polystyrene spheres as spacers in between two glass optical flats to maintain a constant gap [3]. The average size of the spheres was 1 µm but the gap size was determined by the largest spheres and was about 1.6 µm. The heat conduction through the spheres is much less than the radiative transfer between plates. The top plate was heated and the heat flow through the bottom one was measured. The radiative transfer measured agrees very well with the model in Section 4.4.2.

In Ref. [2] an experiment is described where the near field thermal radiation is measured by a micro electromechanical system (MEMS). Using sacrificial layers to create a gap between two membranes the problems of parallelism when positioning two plates with respect to each other was avoided. A pattern of Pt lines was deposited on the SiO₂ membranes to provide Joule heating, this increases the
temperature of one of the membranes with respect to the other. The temperatures of both membranes was measured by measuring the resistance in the Pt lines. The relation between resistance and temperature was linear and described by a temperature resistance coefficient. These measurements were done in vacuum. The measured heat flux was up to 7 times larger than the black body radiation when the gap was 1 µm.

3.1.2 Experiments with Thermal Probing Tips

There have also been experiments in which the radiative heat flow was measured between a small conical tip and a surface. Xu et al. conducted an experiment to measure heat flux between an indium needle 100 µm in diameter and a gold plate [31]. The distance between tip and surface was as low as 12 nm. Their results were rather inconclusive, the resulting heat flux was less than expected. Kittel et al. achieved better results [32]. They used a scanning thermal microscope to measure the heat flux between the tip of a thermal profiler and plates of gold and GaN under UHV conditions. They specifically focused on the divergence of the heat flux at very small gap distances and found that the heat flux saturates
3.1. Past Experiments on Near Field Thermal Radiation

at distances lower than 10 nm, in accordance with their expectations. For larger
the heat flux was in good agreement with existing theories. They used a glass
micropipette with a 50 µm Pt/Ir wire protruding from it. The protruding part
was electrochemically etched to create a sharp tip. The heat flux from the tip to
the surface was measured as a change in temperature of the tip. Temperatures
were measured using a thermocouple integrated into the tip.

3.1.3 Experiments with Bimaterial Cantilevers

In recent years a method of measuring near field radiative heat flow was adopted
from experiments on measuring Casimir forces [4]. Shen and coworkers measured
the near field radiative heat transfer between an SiO$_2$ microsphere and a flat SiO$_2$
surface as depicted in Figure 3.3. They did this by glueing the sphere on the
tip of a bimaterial atomic force microscopy (AFM) cantilever with an ultraviolet
(UV) adhesive. A major advantage of using a sphere is that the geometry of a
sphere is indifferent to rotations, there are no alignment problems. The sphere
was brought close to a substrate. The laser that is used for measuring the bending
of the cantilever was also used to heat the end where the microsphere was glued
on (see Figure 3.3). The heat flow from the laser causes a temperature gradient
over the cantilever. The temperature gradient will cause thermal expansion in
the two materials of the cantilever. Because they have different coefficients of
thermal expansion the cantilever will bend. If the microsphere is brought close to
the surface a distance-dependent heat loss to the substrate will relax the bending
of the cantilever. The cantilever was placed upright to minimize influences of
disturbing forces (van der Waals forces, electrostatic forces). The temperatures
of the substrate and the base of the cantilever remained constant. A piezoelectric
motion controller was used to control the gap size. When the sphere hits the
surface a sharp peak in bending is observed, this is used as a reference value for
the sphere-substrate distance. To prevent heat conduction through air vacuum
conditions were used. It was reported that the resolving power of the system was
as low as 0.1 nW. They found an increase in black body radiaton by three orders
of magnitude. Later Shen et al. repeated their earlier experiments with a gold
sphere and gold substrate. At a gap of 30 nm the measured heat exchange was 8 times as large as the black body radiation [5].

Figure 3.3: Left: Measurement results from Ref. [4] and Ref. [5]. The solid lines are the theoretical values. Right: Schematic of the setup as used by Shen [4], using an AFM with a microsphere on it. A laser beam is reflected off the cantilever.

Rousseau et al. used a similar setup [9]. They used a heating element to heat the substrate to create a temperature difference between sphere and substrate. Both research groups got results wherein the heat flux was larger than the black body limit and followed an inverse dependency on the gap size. Rousseau et al. reported that the surface roughness of the microsphere caused a consistent shift in the heat flow-distance curve. The measurements agreed very well with theory once the surface roughness was accounted for by introducing an appropriate shift in the gap size.

3.2 Applications of Near Field Thermal Radiation

So far interest into near field thermal radiation is mostly academic. Except for near field thermophotovoltaics (TPV) and near field thermal radiation microscopy
no specific applications of near field thermal radiation have been developed. There are many suggestions however for future applications. The central purpose of this thesis is to determine whether near field thermal radiation can be used to measure the distance between two objects at nanometer or micrometer-sized separations. This is believed to be possible because the amount of heat transfer strongly depends on the distance between the objects.

3.2.1 Near Field Thermal Radiation Microscopy

De Wilde et al. developed a near field scanning optical microscope that needs no external illumination [33]. The illumination comes from the surface itself by means of its thermal radiation. A tungsten tip probes the evanescent field and scatters tunneled photons to a detector, a mercury-cadmium-telluride (MCT) infrared camera. The tip oscillates and a lock-in amplifier extracts the near field signal from the detector output. This separates the near field signal from background infrared signals. Thermal emission is increased by the elevated temperature of the substrate. Images created with this setup revealed a resolution beyond the diffraction limit. Kajihara et al. developed a similar system [34]. They used a charge sensitive infrared phototransistor instead of a MCT detector and achieved a resolution of 300 nm. Jones and Raschke used a heated AFM tip instead of a tungsten needle [6]. Their experimental setup is shown in Figure 3.4. They reported a spatial resolution of 50 nm.

What is important to realize is that these methods do work in air, it is the scattered radiation (photons) that is detected, not the heat transfer. In ambient environments gas conduction through air is the dominating mode of heat transfer between non-touching objects even when surface polariton enhanced thermal radiation comes into play (see also Section 6.1.4).
Figure 3.4: Experimental setup used by Jones and Raschke, from Ref. [6]. Both AFM tip and substrate are heated. The scattered radiation passes through a Michelson interferometer before it is captured by a MCT detector. Inset i shows a typical measured signal and ii shows the shape of the AFM tip. The substrate consists of a material with enhanced near field thermal emission by SPhPs or molecular vibrations.

3.2.2 Thermophotovoltaics

Solar cells are photovoltaic devices that use semiconducting material to convert solar radiation directly into electric power. TPV devices convert thermal radiation, generated by a temperature difference between thermal emitter and collector, into electric power. Unfortunately the efficiencies of TPV devices are still low [35]. The radiative transfer and therefore the efficiency can be enhanced if the collector is placed in the near field of the emitter, especially if it takes advantage of surface polaritons. The thermal energy from the emitter should not be converted into lattice or plasmon vibrations however but into creating electron-hole pairs, generating a current.

A particular challenge is to match the surface resonance frequency with the band gap frequency of the semiconductor [35]. Photons with a frequency equal to the band gap frequency will convert all of their energy into the energy of a hole-electron pair. If they have a greater frequency all excess energy is lost as dissipation.

If
3.2. Applications of Near Field Thermal Radiation

Figure 3.5: Working principle of a TPV device.

their frequency is lower than the band gap energy no energy is converted and all energy is lost. So ideally the source emits thermal radiation of one exact frequency but as explained in Section 2.1 black and grey bodies will always emit radiation of a distribution of different wavelengths. Near field sources using surface resonances can be rather monochromatic on the other hand. The use of coatings is a candidate to modify the spectrum of radiation before it is absorbed by the semiconductor. Messina and Ben-Abdallah [35] suggested that the addition of a graphene sheet in front of the collector causes the appearance of new surface resonant modes which couple to the resonant modes from the emitter increasing both the power output and the efficiency of the near field TPV device. For a 16 nm gap the efficiency increases from 10% in the absence of graphene to 20% with the graphene in place.

3.2.3 Other applications

With further decreasing sizes in microelectronics heat dissipation is becoming an increasing problem [36]. Knowledge of near field radiation can aid in measuring temperature and thermal transport at nanoscales. Perhaps near field heat transfer can be used as a solution for thermal management in nanoelectronics, MEMS [2] and nanoelectromechanical systems (NEMS). Conventional methods of temperature probing like infrared thermometry or the use of thermocouples have a resolution in the order of micrometers, this is not enough for nanoelectronics [36]. Nanoscale heat transfer also plays a role in magnetic hard disk drives and AFM
[4]. Scanning thermal microscopy uses a temperature sensor on the end of the tip of a scanning tunneling microscope or AFM tip to measure temperatures on a nm scale. For measuring temperature on a nanometer scale it is important to understand the thermal transfer between sample and probe if the probe is brought close to the sample.

Other applications in which near field thermal radiation may become important are: nanomanufacturing [19], thermal characterization of nanoscale materials, thermal spectroscopy, nanoscale heaters that could be used for heat assisted magnetic recording, heat assisted lithography [9, 11] or new radiative cooling techniques where the cooling element does not touch the object being cooled [37, 1]. It was also suggested that spacecraft waste heat can be converted into electricity effectively using near field thermal radiation [38].
Chapter 4

Modeling Near Field Thermal Radiation

In the near field radiative heat flow between objects increases with decreasing separation between them. The heat flow depends on the optical properties of the participating media. The near field enhancement arises from evanescent waves and surface polaritons. In this chapter mathematical models are developed to model near field thermal radiation for different kinds of materials and different geometries of the involved objects as well.

4.1 Maxwell’s Equations

Although no full derivations are exhibited in this chapter the reader should be familiar with some basic concepts of electromagnetism. Electromagnetic fields are described by Maxwell’s equations. The frequency dependent Maxwell’s equations
for time harmonic fields of the form $e^{-i\omega t}$ read as [39]:

\[
\begin{align*}
\nabla \times \mathbf{E} &= i\omega \mathbf{B} & (4.1) \\
\nabla \times \mathbf{H} &= -i\omega \mathbf{D} + \mathbf{J} & (4.2) \\
\nabla \cdot \mathbf{D} &= \rho & (4.3) \\
\nabla \cdot \mathbf{B} &= 0 & (4.4)
\end{align*}
\]

The electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$, the electric displacement field $\mathbf{D}$, the magnetic flux density $\mathbf{B}$ and the current density $\mathbf{J}$ are all functions of position and frequency: $\mathbf{E} = \mathbf{E}(r, \omega)$. Both $\mathbf{D}$ and $\mathbf{B}$ depend on material properties, for linear materials:

\[
\begin{align*}
\mathbf{D} &= \varepsilon_0 \varepsilon \mathbf{E} & (4.5) \\
\mathbf{B} &= \mu_0 \mu \mathbf{H} & (4.6)
\end{align*}
\]

With $\varepsilon_0$ as the permittivity of vacuum, $\varepsilon = \varepsilon(r, \omega)$ is the relative permittivity, also known as the dielectric function. Similarly, $\mu_0$ is the permeability of vacuum and $\mu = \mu(r, \omega)$ is the relative permeability. It is assumed that the polarization is linearly related to the electric field, non-linear effects are negligible for thermal radiation [21]. The relative permittivity is a complex function consisting of a real and an imaginary part. In a uniform medium, the complex refractive index is the root of the relative permittivity and relative permeability:

\[
n(\omega) = \sqrt{\mu(\omega) \varepsilon(\omega)}
\]

### 4.2 Relative Permittivity

Dielectrics are electrical insulators that are polarized by electric fields, charges inside it will align themselves with the electric field without conduction. Metals are good electrical conductors and semiconductors have an electrical conductivity in between those of conductors and insulators. The permittivity of intrinsic silicon can be considered frequency independent in the thermal IR with a value of $11.7 + 0.0001i$ [7]. For other materials two general models for the relative permittivity and the conditions for which SPPs and SPhPs exist are discussed next. All the used material parameters are tabulated in Appendix A.
4.2. Relative Permittivity

4.2.1 Lorentz Oscillator Model

For polar dielectrics like BN, SiC, SiO$_2$ and Al$_2$O$_3$ the relative permittivity is well described by the Lorentz oscillator model in which the motion of the electrons is modeled by a spring and a damping force bounding them to the atom cores [8]:

\[
\varepsilon(\omega) = \varepsilon_\infty + \frac{\kappa}{\omega_r^2 - \omega^2 + i\gamma\omega} \tag{4.8}
\]

Parameters used here are the resonance frequency $\omega_r$, the conductivity $\kappa$, the damping factor $\gamma$ and the high frequency dielectric constant $\varepsilon_\infty$. In terms of phonon frequencies, $\omega_r$ is the transverse optical phonon frequency and $\sqrt{(\varepsilon_\infty \omega_r^2 - \sigma)/\varepsilon_\infty}$ is the longitudinal optical phonon frequency [40]. Below this resonance frequency the relative permittivity is positive, the material is an electrical resistor but it is polarized by external electric fields. Above the resonance frequency the polarizability of the material saturates, the polarizable elements are too slow to follow the fast switching electric field.

![Figure 4.1: Relative permittivity of SiC](image)

The actual optical properties of materials can be found in tables like in Ref. [7]. Because the optical properties of SiO$_2$ will be used a lot in this report they are studied more carefully. The relative permittivity of SiO$_2$ is modelled by fitting
Chapter 4. Modeling Near Field Thermal Radiation

measured data to a double oscillator model as was done in Ref. [8]:

\[ \varepsilon(\omega) = \varepsilon_\infty + \sum_{j=1}^{2} \frac{\kappa_j}{\omega^2 - \omega_j^2 + i\gamma_j \omega} \] (4.9)

Note that the data was made to fit the oscillator model in the thermal IR, whether the fitting is accurate in other areas does not matter for any thermal radiation calculations. This was confirmed by determining the factor \( B \) in (4.41) with measured data and with the oscillator model, both the yielding the same result.

4.2.2 Drude Model

The relative permittivity of a metal can be modeled using the Drude model [41]:

\[ \varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega - i\gamma)} \] (4.10)

The first term is usually 1, \( \omega_p \) is the plasma frequency. The plasma frequency and damping factor can be found in optical property tables like in [42], where measured data in the IR was fitted to a Drude model. The plasma frequency can also be computed with [43]:

\[ \omega_p = \sqrt{\frac{N_e e^2}{m_e \varepsilon_0}} \] (4.11)

In this equation \( N_e \) is the number of electrons per unit volume, \( e \) is the elementary charge and \( m_e \) is the mass of an electron. The permittivity of gold is shown in Figure 4.3. Below the plasma frequency, radiation is reflected by the metal. When the frequency of incident radiation becomes larger than the plasma frequency EM waves propagate into the material, the metal has become transparent.

The relative permittivity of doped silicon near room temperatures can also be described by a Drude model, the plasma frequency can be tuned by the doping concentration [26, 44]:

\[ \varepsilon(\omega) = \varepsilon_{in} - \frac{N_e e^2 / \varepsilon_0 \mu_e^*}{\omega^2 + i\omega/\tau_e} - \frac{N_h e^2 / \varepsilon_0 \mu_h^*}{\omega^2 + i\omega/\tau_h} \] (4.12)

The term \( \varepsilon_{in} \) represents the relative permittivity of intrinsic silicon in the infrared. The second term is the contribution to the permittivity by free electrons and the
Relative Permittivity

![Graph of relative permittivity](image)

Figure 4.2: Relative permittivity of SiO₂ according to the measured values in Ref. [7] and fitted to an oscillator model as in Ref. [8]. On the top the real part is shown, the bottom plot is the imaginary part.

third one is the contribution by free holes, \(N_e\) and \(N_h\) are their carrier concentrations, \(m_e^* = 0.27m_e\) and \(m_h^* = 0.37m_e\) are their effective masses with the free electron mass \(m_e\); \(\tau_e\) and \(\tau_h\) are the scattering times of free electrons and holes respectively. For example, an n-type dopant concentration of \(10^{20}\text{cm}^{-3}\) gives a relative permittivity at 300 K of [45, 46].

\[
\varepsilon(\omega) = 11.7 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} 
\] (4.13)
The plasma frequency is $4.05 \times 10^{14}$ rad s$^{-1}$ and $\gamma = 1.0132 \times 10^{14}$ s$^{-1}$.

### 4.3 Surface Resonance Conditions

In Figure 4.4 a schematic representation of an EM wave incident on the interface of material 1 and vacuum is shown using a wavevector representation. The magnitude of a wavevector is equal to the wavenumber of the wave and the direction is equal to the direction in which the wave is travelling.

From the solutions to Maxwell’s equations at the interface between material 1 and vacuum the wavevector in the direction along the surface is [47]:

$$ k_x = k_0 \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + 1}} $$

This is the dispersion relation at the interface between the two materials. From the definition of the refractive index: $k_1 = n_1 k_0$. Evanescent waves exist if the angle of incidence is larger than the critical angle (2.12). Computing $\theta_i$ from the figure:

$$ \theta_i = \sin^{-1} \left( \frac{1}{n_1} \cdot \frac{k_x}{k_0} \right) $$
Using (4.14) the condition for TIR is:

$$\theta_i = \sin^{-1} \left( \frac{1}{n_1} \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + 1}} \right) > \theta_{cr}$$  \hspace{1cm} (4.16)

This implies that $\varepsilon_1 < -1$. Looking back at (4.14) there is a clear resonance in the wavevector at $\varepsilon_1 = -1$. At this resonance the energy at the surface becomes very high as reported in the next section.

### 4.4 Analytical Methods

In this section the basics of a fundamental method to describe near field thermal radiation are provided using the methods of fluctuational electrodynamics and the fluctuation dissipation theorem. The basic results are given here for later reference, the focus is on the assumptions used in the models and on physical interpretations of the results. In Refs. [20, 19, 47, 48, 19, 21] the interested reader can find more extensive explanations of this general approach.
It should be mentioned that analytical solutions are only available for simple geometries like semi-infinite one-dimensional layers or spheres. First radiation from and between flat plates is discussed. Then radiation between small spheres (dipoles) and between a small sphere and a plate is discussed. For larger spheres the proximity approximation (PA) is employed. Finally some results of numerical modelling methods and an overview of the limitations of the used methods are presented. Recently another approach for modelling radiative heat transfer was proposed using a thermokinetic approach [49]. So far this method has not gained traction among other authors.

### 4.4.1 Thermal Radiation from an Infinite Plate

The thermal radiation emitted by a plate is described in this section. The thickness of the plate is assumed to be much larger than the penetration depth of evanescent waves inside it, such that it can be described as a half-space, also known as a semi-infinite solid. Remember that the penetration depth is \( \delta \approx |k_{z1}| \). The plate consists of medium 1; medium 0 is vacuum as can be seen in Figure 4.5.

It is customary to use the dyadic Green’s functions (DGF) for solving the Maxwell’s equations in near field analysis. Using this the electric and magnetic fields can be written as [21]:

\[
\begin{align*}
\textbf{E}(\mathbf{r}, \omega) &= i\mu_0\omega \int_V \overline{\textbf{G}}(\mathbf{r}, \mathbf{r}', \omega) \cdot \textbf{J}(\mathbf{r}', \omega) d\mathbf{r}' \\
\textbf{H}(\mathbf{r}, \omega) &= \int_V \nabla \times \overline{\textbf{G}}(\mathbf{r}, \mathbf{r}', \omega) \cdot \textbf{J}(\mathbf{r}', \omega) d\mathbf{r}'
\end{align*}
\] (4.17) (4.18)

The DGF \( \overline{\textbf{G}} \) depends on the geometry of the system. It provides the electric field at point \( \mathbf{r} \) due to the point source \( \textbf{J} \) at position \( \mathbf{r}' \) [39]. The DGF for a half-space is given as [41]:

\[
\overline{\textbf{G}}(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{4\pi^2 k_{z1}} \int_0^\infty (\hat{s} t_{10} \hat{\mathbf{s}} + \hat{p} t_{10} \hat{\mathbf{p}}) e^{i(k_{z0}z - k_{z1}z')} e^{i(k_{z0}z - k_{z1}z')} e^{i(k_{z0}z - k_{z1}z')} k_x dk_x
\] (4.19)
4.4. Analytical Methods

With the Fresnel transmission coefficients for s- and p-polarization [19]:

\[ t^s_{10} = \frac{2k_{z1}^0}{k_{z0} + k_{z1}} \]  
\[ t^p_{10} = \frac{2n_1k_{z1}^0}{k_{z1} + \varepsilon_1k_{z0}} \]  

(4.20) \hspace{1cm} (4.21)

The directions of polarization are given by the unit vectors:

\[ \hat{s} = \hat{x} \times \hat{z} \]  
\[ \hat{p}_0 = \frac{k_{z0}\hat{z} \times k_{z0}\hat{x}}{k_0} \]  
\[ \hat{p}_1 = \frac{k_{z1}\hat{z} \times k_{z1}\hat{x}}{k_1} \]  

(4.22) \hspace{1cm} (4.23) \hspace{1cm} (4.24)

The s-direction, also known as transverse-magnetic, is out of the paper towards the reader. The p-direction, also known as transverse-electric, is perpendicular to the wavevector and s-direction, see figure 4.5.

The current sources in the material are caused by thermal movements and are stochastic in nature. The time average of the current is zero and therefore the time average of both the electric field and the magnetic field are also zero. However, the power of heat flux is given by the time-averaged Poynting vector [21]:

\[ \langle S(r, \omega) \rangle = \frac{1}{2} \text{Re}\{\langle E(r, \omega) \times H^*(r, \omega) \rangle} \]  

(4.25)

This is not necessarily equal to zero. The ensemble averages are denoted with angle brackets, the \(*\) denotes a complex conjugate.

The fluctuation-dissipation theorem states that thermal movements of charges in materials with a temperature higher than absolute zero generate randomly fluctuating currents. These currents are the source of the EM-fields that constitute thermal radiation. It is assumed that the body is in local thermal equilibrium, its material is nonmagnetic, isotropic and it has a local relative permittivity. This means that the relative permittivity is a function of frequency only and not of wavevector. The fluctuation dissipation theorem gives the cross spectral density between two current densities [41]:

\[ \langle j_m(r, \omega)j_n^*(r', \omega') \rangle = \frac{4\omega\varepsilon_0\Theta(\varepsilon)}{\pi} \Theta(\omega, T)\delta(r - r')\delta(\omega - \omega')\delta_{mn} \]  

(4.26)
Figure 4.5: The contribution of the current at position \( r' \) to the EM-field at position \( \mathbf{r} \) is shown. The propagating wavevectors in vacuum and medium 1 (\( k_0 \) and \( k_1 \)) and their horizontal and vertical components are shown. The polarization vectors \( \mathbf{s}, \mathbf{p}_0 \) and \( \mathbf{p}_1 \) are shown. The x- and z-directions are determined by the unit vectors \( \mathbf{x} \) and \( \mathbf{z} \).

The cross spectral density is the Fourier transform of the cross-correlation. The \( j_m \) and \( j_n \) terms are components of the current vector \( \mathbf{J} \). It is seen from the Dirac delta function \( \delta(\mathbf{r} - \mathbf{r}') \) that two current densities are not correlated when they are in different positions. Current densities are not correlated when they are at different frequencies either: \( \delta(\omega - \omega') \). The Kronecker delta \( \delta_{mn} \) is 1 if \( m = n \) and zero otherwise. This means that the current density in one direction is not correlated to the current density in another orthogonal direction, this is true for isotropic materials. The \( \Theta \) in the equation is the average energy of a Planck oscillator and is [16]:

\[
\Theta(\omega, T) = \frac{\hbar \omega}{e^{\hbar \omega/k_B T} - 1}
\]  

(4.27)

With these relations the spectral energy density can be found, which is the elec-
4.4. Analytical Methods

tromagnetic energy per unit volume per unit frequency [19]:

\[
u(\omega, T) = \frac{\Theta(\omega, T)\omega}{2\pi^2 c^2} \left[ \int_0^{\omega/c} \frac{k_x}{k_{z0}} \left( \frac{1 - |r_{01}^s|^2}{2} + \frac{1 - |r_{01}^p|^2}{2} \right) dk_x \right.
\]

\[
+ \int_{\omega/c}^{\infty} \frac{k_x^3}{k_{z0}^2|k_{z0}|} \left( \text{Im}(r_{01}^s) + \text{Im}(r_{01}^p) \right) e^{-2|k_{z0}|z} dk_x \right] (4.28)
\]

In this equation \( r_{01}^s \) and \( r_{01}^p \) are the Fresnel reflection coefficients for s-polarization and p-polarization respectively [41]:

\[
r_{01}^s = \frac{k_{z0} - k_{z1}}{k_{z0} + k_{z1}} \quad (4.29)
\]

\[
r_{01}^p = \frac{\varepsilon_1 k_{z0} - k_{z1}}{\varepsilon_1 k_{z0} + k_{z1}} \quad (4.30)
\]

As can be seen in (4.28), the solution for the spectral energy density consists of two terms. If there is a propagating wave in vacuum then \( k_0 = \omega/c \) because \( c \) is the speed of light and radiation propagates with the speed of light. As a result, \( k_x \) has to be less than \( \omega/c \). The first part of the solution is identified as the region where the wave is propagating in vacuum. If \( k_x > \omega/c \) then there is no propagation in the vacuum, only an evanescent field. The spectral power is purely imaginary here and clearly decays exponentially with increasing distance, \( z \), away from the surface.

The heat flux emitted by a single plate can be calculated using (4.25). It was found that [19]:

\[
q(T) = \int_0^{\infty} \frac{\Theta(\omega, T)}{4\pi^2} \int_0^{\omega/c} k_x (2 - |r_{01}^s|^2 - |r_{01}^p|^2) dk_x d\omega \quad (4.31)
\]

The heat flux emitted by the surface is only determined by the propagating waves. The evanescent fields do not contribute. Identifying the spectral emissivity as \( \epsilon_\omega = (2 - |r_{01}^s|^2 - |r_{01}^p|^2)/2 \) and assuming axial symmetry about the direction normal to the surface, \( 2\pi k_x dk_x = \cos(\theta) d\Omega \), the previous equations changes to [19]:

\[
q(T) = \int_0^{\infty} \epsilon_\omega(\omega, \theta) R(\omega, T) \int_0^{2\pi} \cos(\theta) d\Omega d\omega \quad (4.32)
\]

This is in accordance with classical radiation theory. Since a black body does not reflect, setting the reflection coefficients to zero leads to a heat flux of \( \pi R(T, \omega) \).
Chapter 4. Modeling Near Field Thermal Radiation

4.4.2 Thermal Radiation between Two Infinite Plates

Now another plate is added, consisting of medium 2 above plate 1 with a gap \( d \) between them. The material between the two plates is still vacuum. Both plates are assumed to be homogeneous, isotropic and nonmagnetic. The DGF for the two plate geometry is the same as for the half-space, only the transmission coefficients are different \([19]\).

\[
\mathbb{G}(\mathbf{r}, \mathbf{r'}, \omega) = \frac{i}{4\pi^3 k_{z1}} \int (\hat{s} t_{12}^s \hat{s} + \hat{p} t_{12}^p \hat{p}) e^{i k_x (x-x')} e^{i (k_{z2} z - k_{z1} z')} k_x dk_x \tag{4.33}
\]

\[
t_{12}^s = \frac{t_{13}^s t_{32}^s e^{i k_{zod}}}{1 - r_{13}^s r_{32}^s e^{2i k_{zod}}} \tag{4.34}
\]

\[
t_{12}^p = \frac{r_{13}^p t_{32}^p e^{i k_{zod}}}{1 - r_{13}^p r_{32}^p e^{2i k_{zod}}} \tag{4.35}
\]

Using this the resulting heat transfer between the two plates can be written as \([21]\):

\[
q_{12} = \frac{1}{\pi^2} \int_0^\infty (\Theta(\omega, T_1) - \Theta(\omega, T_2)) \int_0^\infty s(\omega, k_x) dk_x d\omega \tag{4.36}
\]

The function \( s(\omega, k_x) \) can be split into a sum of two parts, one for propagating waves (\( 0 < k_x < \omega/c \)) and one for evanescent waves (\( \omega/c < k_x < \infty \)) \([21]\):

\[
s_{\text{prop}}(\omega, k_x) = \frac{k_x(1 - |r_{01}^s|^2)(1 - |r_{02}^s|^2)}{4|1 - r_{01}^s r_{02}^s e^{2i k_{zod}}|^2} + k_x(1 - |r_{01}^p|^2)(1 - |r_{02}^p|^2) \tag{4.37}
\]

\[
s_{\text{evan}}(\omega, k_x) = \frac{\Im(r_{01}^s)\Im(r_{02}^s) k_x e^{-2d \Im(k_{zod})}}{|1 - r_{01}^s r_{02}^s e^{-2d \Im(k_{zod})}|^2} + \frac{\Im(r_{01}^p)\Im(r_{02}^p) k_x e^{-2d \Im(k_{zod})}}{|1 - r_{01}^p r_{02}^p e^{-2d \Im(k_{zod})}|^2} \tag{4.38}
\]

In the equations \( d \) is the separation gap between the plates. The total heat transfer is a sum of the contribution of propagating waves and those of evanescent waves. Each of these are a sum of contributions from s-polarized waves and p-polarized waves. Using (4.36), (4.37) and (4.38) a script was written in MATLAB to calculate heat fluxes between two infinite parallel plates. Drude and Lorentz oscillator models with the parameters from Refs. \([42, 40, 7]\) for SiC, GaAs, cBN, SiO\(_2\), Au, Cu, Al, Pt, Ti, Si were used. The script takes the temperatures of each plate and the choice of material for each of the plates as input. First, consider the radiative
heat transfer between two plates with a constant purely real permittivity of 11.7, this is the permittivity of Si if the imaginary part is ignored. The enhancement over black body radiative heat transfer when the gap sizes approaches zero is indeed \( n^2 = \varepsilon \) as seen in Figure 4.6.

\[
\begin{array}{|c|c|}
\hline
\text{gap size (m)} & \text{q (W/m}^2) \\
\hline
10^{-9} & 10^4 \\
10^{-8} & 10^3 \\
10^{-7} & 10^2 \\
10^{-6} & 10^1 \\
10^{-5} & 10^0 \\
10^{-4} & 10^1 \\
\hline
\end{array}
\]

Figure 4.6: Radiative heat transfer between two plates with \( \varepsilon = 11.7 \). The top plate is at 295 K and the bottom at 275 K. The black body limit is also shown.

For two SiO\(_2\) plates the heat flux shows a \( 1/d^2 \) dependence in the near field (Figure 4.8). This divergence is caused by the evanescent p-polarized waves. For large distances the radiative heat flux tends to a constant value. When looking at the spectral distribution of the heat flow the monochromatic nature of surface polariton enhanced near field thermal radiation is revealed. Compare the distribution in Figure 4.9 to that of a black body in Figure 2.2 to see the difference between the broadband spectrum of the black body and the narrow peaks in Figure 4.9.

For metals the influence of evanescent s-polarized waves is much larger in the nanometer regime, but on even smaller scales the heat flux also diverges because of the evanescent p-polarized component (Figure 4.11).

Generally speaking large near field enhancements occur when top and bottom plate have similar functions for their permittivities [21]. The heat flux between a polar
Chapter 4. Modeling Near Field Thermal Radiation

Figure 4.7: Same as Figure 4.6 but the heat flow is divided into its propagating and evanescent components and into s- and p-polarization components.

Figure 4.8: Radiative heat transfer between two SiO$_2$ plates. The top plate is at 295 K and the bottom at 275 K.

dielectric and a metal is much less than between two dielectric plates or between two metal plates (Figure 4.12). The heat flow between SiO$_2$ and undoped Si is
4.4. Analytical Methods

**Figure 4.9:** Spectral distribution of the heat flux between two SiO$_2$ plates with a gap of 100 nm in between them.

**Figure 4.10:** Same as Figure 4.8 but the heat flow is divided into its propagating and evanescent components and into s- and p-polarization components.

enhanced past the black body limit in the submicrometer regime (Figure 4.13). The enhancement is a lot larger between silicon dioxide and highly doped silicon (Figure 4.14). The heat transfer between gold and silicon is much less than the
Chapter 4. Modeling Near Field Thermal Radiation

Figure 4.11: Radiative heat transfer between two gold plates. The top plate is at 295 K and the bottom at 275 K.

black body limit.

Figure 4.12: Radiative heat transfer between a 295 K SiO₂ and a 275 K gold plate.

For small distances an asymptotic expansion of (4.36) for the heat transfer coeffi-
4.4. Analytical Methods

Figure 4.13: Radiative heat transfer between a 295 K SiO$_2$ and a 275 K Si plate.

Figure 4.14: Radiative heat transfer between a 295 K SiO$_2$ and a 275 K 10$^{20}$ n-type doped Si plate.

cient $h_{pp} = q_{\Delta T \rightarrow 0}/\Delta T$ is given in [24] as:

\[
h_{pp} = \int_0^\infty h_{spec}(\omega) \, d\omega
\]

\[
\approx \int_0^\infty \frac{k_B}{d^2} \left( \frac{\hbar \omega}{k_B T} \right)^2 \frac{\Im[\varepsilon_1] \Im[\varepsilon_2]}{[1 + \varepsilon_1]^2 [1 + \varepsilon_2]^2 (e^{\hbar \omega/k_B T} - 1)^2} \, d\omega
\]

\[
= B \frac{d^2}{d^2}
\]
Only the evanescent p-polarized waves are taken into account here, this is also known as the single polarization approximation. If the relative permittivity has no imaginary part $B$ is zero. For two SiO$_2$ plates $B \approx 7.37 \times 10^{-12} \text{ W K}^{-1}$. For a large separation of surfaces, the far field contribution to the heat flux between two grey bodies is (2.5):

$$q_{12,FF} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$  \hspace{1cm} (4.42)

These are not functions of the spacing between the plates. The total heat flux can be estimated as the sum of the near and far field contribution:

$$q_{12} = q_{12,NF} + q_{12,FF}$$  \hspace{1cm} (4.43)

For SiO$_2$ with an emissivity of 0.835 the comparison of the full equation and the approximation is shown in Figure 4.15. The approximation coincides with the full equation except in the area around 1 $\mu$m, there the approximation is an underestimate. This is because in this area the evanescent s-polarized contribution is significant, which is ignored in the approximation.

![Figure 4.15: Radiative heat transfer between a 295 K SiO$_2$ plate and a 275 K SiO$_2$ plate, full solution and approximation.](image-url)
4.4.3 Thermal Radiation from Metamaterials

Because of the development of metamaterials at TNO and use of metamaterials in the meta-instrument it was deemed worthwhile to explore the use of metamaterials and their unusual properties in near field thermal radiation experiments. Metamaterials are engineered composite materials with minute features exhibiting properties not seen in nature like a negative index of refraction [50, 51] or a magnetic response from nonmagnetic composite materials. They derive their electromagnetic properties from their physical structure, not only from their chemistry. With engineered materials it is possible to tune their electromagnetic response by tuning these feature sizes. The possibility of tuning the thermal emission by metamaterials allows increasing the efficiency of TPV devices.

Using effective medium theory (EMT) the electromagnetic properties are assumed to be constant over the material, they are not a function of position. The photon wavelength should be much larger than the size of the miniature features that make up the metamaterial for EMT to be applicable. The effectiveness of EMT can be checked for multilayered metamaterials [46] by comparing it to analytical solutions for multilayered materials [52, 53].

The near field thermal radiation exchanged between metamaterials can be described by the same formalisms used in the previous section although the effects of magnetism should be included in the fluctuational electrodynamics approach: the relative permeability, \( \mu = \mu(\omega) \), is not necessarily equal to 1. Besides a DGF for electric fields generated by an electric current source and magnetic fields generated by an electric current source, a DGF for electric fields by a magnetic current and one for magnetic fields generated by a magnetic current is invoked [54]. Besides the correlation for electric currents in (4.26) there is also one for magnetic currents. There is no correlation between magnetic and electric currents. But essentially what it all comes down to is that the heat transfer equation is the same as in (4.36) for magnetic materials [54]. However, the wavevectors and the Fresnel reflection coefficients should include permeability terms [55]:

\[
r_{01}^s = \frac{\mu_1 k_{z0} - k_{z1}}{\mu_1 k_{z0} + k_{z1}}, \quad r_{01}^p = \frac{\varepsilon_1 k_{z0} - k_{z1}}{\varepsilon_1 k_{z0} + k_{z1}}
\]  

(4.44)
Chapter 4. Modeling Near Field Thermal Radiation

The wavevectors should also include permeability terms in accordance with the definition of the refractive index, (4.7). After making these amendments to the MATLAB script it is able to work with magnetic materials as well. In Refs. [55] and [56] examples of engineered metamaterials are given: an array of metallic wires and split-ring resonators. The permittivity and permeability read as:

\[
\varepsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma_e)}, \quad \mu = 1 - \frac{F\omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega} \tag{4.45}
\]

The values used for the volume filling fraction \(F\), the magnetic resonance frequency \(\omega_m\), the electric dissipation factor \(\gamma_e\), the magnetic dissipation factor \(\gamma_m\), and the plasma frequency are listed in appendix A. The heat transfer between two such materials is even larger than it is for \(\text{SiO}_2\), see Figure 4.16. Polaritonlike transfer only occurs in p-polarization for nonmagnetic materials, for materials that are effectively magnetic and electric, surface waves exist in p- and in s-polarization as well as can be seen in Figure 4.17. The heat transfer mostly occurs at two particular frequencies where \(\varepsilon = -1\) and where \(\mu = -1\) if the imaginary parts are sufficiently small. This can be seen from the full dispersion relations for both polarizations:

\[
k^p_x = k_0 \sqrt{\frac{\varepsilon(\varepsilon - \mu)}{\varepsilon^2 - 1}}, \quad k^s_x = k_0 \sqrt{\frac{\mu(\mu - \varepsilon)}{\mu^2 - 1}} \tag{4.46}
\]

The heat flow is proportional with the inverse second power of gap size for submicrometer gap sizes. Negative refraction has no influence on the heat flow [55]. The direction of heat flow is still from hot to cold obviously.

4.4.4 Dipole Heat Radiation

When modeling the heat flow from and to small spheres with diameters less than about 100 nm they can be considered point-like dipoles. Since the characteristic wavelength of thermal radiation is about 10 \(\mu\)m, the electromagnetic field is more or less uniform inside the sphere. The polarizability of a dipole is expressed by the
4.4. Analytical Methods

Figure 4.16: Radiative heat transfer between a 295 K array of metallic wires and split-ring resonators and a 275 K plate of the same material.

Figure 4.17: Radiative heat transfer between a 295 K array of metallic wires and split-ring resonators and a 275 K plate of the same material, divided into evanescent, propagating and polarization terms.

Clausius Mosotti relation [57]:

\[
\alpha(\omega) = 4\pi R^3 \frac{\epsilon(\omega) - 1}{\epsilon(\omega) + 2}
\]  

(4.47)
Using this the total radiative heat transfer between two nanoparticles is defined in Ref. [19] as:

\[ Q = \int_0^\infty \frac{3 \Im[\alpha_1(\omega)] \Im[\alpha_2(\omega)]}{4\pi^3} \left[ \Theta(\omega, T_1) - \Theta(\omega, T_2) \right] d\omega \quad (4.48) \]

The imaginary part of the polarization can be written as [19]:

\[ \Im[\alpha(\omega)] = \frac{12\pi R^3}{\epsilon(\omega)} \left| \frac{\Im[\epsilon(\omega)]}{\epsilon(\omega) + 2} \right|^2 \quad (4.49) \]

If the imaginary part of the relative permittivity is small there is a very high spectral heat flow at \( \epsilon(\omega) = -2 \), this is the condition for volume polaritons to exist.

The dipole approximation can also be used to describe the heat flow between a small particle and a plate. When the scattered power by the particle is considered negligible the heat radiated by the bulk and absorbed by the particle is found as [57]:

\[ Q_{pd} = \int_0^\infty \frac{\omega^4}{c^4} \frac{\Im[\epsilon_p(\omega)] \Im[\alpha(\omega)] \Theta(\omega, T_p)}{\sum_{i,j=x,y,z} \int_V |G_{i,j}(r_d, r', \omega)|^2 dr' d\omega} \quad (4.50) \]

Herein \( \epsilon_p \) is the permittivity of the plate, \( \alpha \) is the polarizability of the dipole, \( V \) is the volume of the plate, \( G_{i,j} \) are the components of the DGF of a half-space (4.19), \( r_d \) is the position of the dipole and \( r' \) is the position of a point in the plate. An asymptotic expansion of the previous equation gives the following expression at short distances [57]:

\[ Q_{pd} \approx \int_0^\infty \frac{R^3}{\pi d^3} \frac{3 \Im[\epsilon_d(\omega)] \Im[\epsilon_p(\omega)]}{|\epsilon_d(\omega) + 2|^2 |\epsilon_p(\omega) + 1|^2} \Theta(\omega, T_p) d\omega \quad (4.51) \]

This equation also shows the conditions for which surface polaritons and volume polaritons exist. In the far field the heat transfer tends to a constant value independent of distance between dipole and surface.
4.5 Proximity Approximation

A straightforward method to approximate the radiative heat flow in the near field between gently curved surfaces is the usage of the proximity approximation (PA). The curved surface is approximated by an infinite series of infinitesimally small flat surfaces. The analytical solution for parallel surfaces is then used to solve the problem. An illustration of this method is shown in figure 4.18.

![Diagram of Proximity Approximation](https://example.com/proximity-diagram.png)

**Figure 4.18: Proximity approximation, from Ref. [9]**

Using the near field approximation for parallel surfaces, (4.15), and a conductance defined as $G = \frac{Q_{\Delta T \rightarrow 0}}{\Delta T}$, the near field conductance between plate and sphere is calculated as [47]:

$$G_{ps} \approx 2\pi R \int_{r=d}^{\infty} h_{pp}(r)dr = 2\pi Rd(B/d^2) = 2\pi RB/d \quad (4.52)$$

The radius of the sphere is $R$, $d$ is the minimal distance between the surface of the sphere and the plate. Compared to experimental results, Shen *et al.* [4] concluded that the PA only gave a correct order of magnitude for the heat flux. Experiments by Rousseau *et al.* [9] however gave results that are in very good agreement with the PA. The PA only accounts for near field thermal radiation. Adding a far field term to it increases this approximations validity at larger separation gaps:

$$G_{ps} \approx 2\pi RB/d + Q_{FF} \quad (4.53)$$

This equation is the modified proximity approximation (MPA). The far field term can be calculated with (2.5).
The same approach can be used to find the near field conductance between two spheres of equal radius with the PA [48]:

\[ G_{ss} \approx \pi d Rh = \pi RB/d \] (4.54)

Because the PA is only applicable for gently curved objects this equation holds for spheres whose radius is much larger than the gap in between them. Using numerical calculations Narayanaswamy proved the inverse gap dependency of the near field heat flow for large spheres and also the inverse proportionality to the sixth power of gap size for very small spheres [58]. In between the two there is a gradual change between both dependencies. The conductance between spheres is a function of gap size, sphere size and optical properties whereas for parallel plates it was a function of gap size and optical properties of the materials. Later it was shown that the PA approaches the exact results at small separation for a sphere and a plate [12] and also for two spheres [59, 60].

### 4.6 Numerical Methods

In recent years numerical methods have been developed to solve near field thermal radiation problems for configurations involving objects other than spheres or flat plates like periodic structures or even arbitrarily shaped 3D objects. Numerical methods themselves are not discussed here but some of the results of these studies are shown. Overviews and detailed explanations of these methods can be found in Refs. [61, 62, 63, 64, 65].

Otey and Fan [10] used a general scattering approach and fluctuational electrodynamics to find the heat flow between a dielectric sphere and dielectric plate. Then results were compared with the dipole approximation, PA, MPA and a far field only model. As can be seen in Figure 4.19 the dipole approximation works best for submicron spheres at distances more than 100 nm away from the plate. The far field only approach is valid for gaps larger than 10 microns. For spheres larger than a micrometer in diameter the MPA gives accurate results over all distances.
Figure 4.19: Phase space plots of the accuracy of four approximation methods compared to a numerical solution for a sphere and a plate, from Ref. [10]. In the lightest areas the error is 3%, in the darkest areas the error is 450%. The gap size is \( d \), the diameter of the sphere is \( a \). The sphere is at 321 K, the plate at 300 K.

McCaulay et al. [11] use a numerical scattering technique and fluctuational electrodynamics to calculate the heat transfer between a doped silicon object and a dielectric (silica) plate. They use scattering theory formalisms and a boundary-element method that describes the surface as a mesh to compute scattering matrices. Figure 4.20 shows the total heat transfer between the object and the plate as
a function of distance between them. For a sphere these results coincide with the analytical solution found in [12], for the cylinder a $d^{-2}$ appears when the gap size goes to zero just like for the parallel plates. The cone shows a logarithmic relation between heat flow and distance as was predicted qualitatively by using the PA in [13]. The concentration of the heat flow on the plate was also studied (Figure 4.21). The heat flux is most concentrated for a cylinder. Surprisingly the heat flow from the cone to the substrate is spread over a larger area than the cylinder or sphere and shows a local minimum directly below the cone. For smaller cone angles the dip in heat flux becomes even larger.

Figure 4.20: *Total heat transfer as a function of distance for variously shaped objects and a plate, from Ref. [11]. The red dots are the results from Ref. [12].*
4.7 Limits of Near Field Thermal Radiation

With the used models the near field radiative heat flow diverges when \( d \to 0 \), of course this cannot describe physical reality. The heat transfer mechanism changes to conduction when both surfaces are in contact. In order to incorporate a limit into the near field radiative heat flow two methods are discussed here. When the gap between two objects decreases the evanescent energy transfer increases the parallel wavevector component \( k_x \), larger wavevectors implying shorter wavelengths. Because the thermal energy in polar dielectrics is transferred to surface phonons the shortest wavelength achievable in them should be in the order of inter-atomic distances in the crystal lattice. This was first proposed by Volokitin and Persson [18] and effectively cuts off the integration limit in (4.36) to a finite value for \( k_x \) [66]. Using the MATLAB script in Appendix B it can be shown that this method causes the heat flow to level off. Setting the maximum wavevector number to \( 2\pi/\lambda \) with the wavelength equal to half the lattice constant for SiC (0.44 nm) the heat flow shows a maximum near 0.5 nm (Figure 4.22).

Chapuis et al. [67] proposed using a non-local relative permittivity instead of a

Figure 4.21: Normalized spatially resolved heat transfer between object and plate, from [11].
Chapter 4. Modeling Near Field Thermal Radiation

Figure 4.22: Radiative heat transfer between two SiC plates. The top plate is at 295 K and the bottom at 275 K. The green line shows the influence of limiting the wavevector to a realistic value.

local one. Just like the frequency dependency of the permittivity expresses the non-locality in time (the response to electric fields is not instantaneous), a non-locality in space causes the electric displacement at one point to depend on the electric field at other nearby points as well. The permittivity effectively becomes a function of the wavevector now: $\varepsilon = \varepsilon(\omega, k)$. Non-locality can become important near the interface between two media. Using a non-local modification of the Drude model for modeling the near field radiative transfer between metals the $1/d^2$ dependency disappeared and the heat flow saturated at short distances.

One last thing to take into account when experimenting with near field thermal radiation is that real surfaces are not perfectly flat. Surface roughness limits the minimum gap size between both objects. There are a few studies in which this was investigated. Biehs and Greffet used second order perturbation theory to study the heat flow between a particle and a rough plate [68] and between two rough plates [69]. The heat flows were found to increase compared to flat surfaces, at smaller distances this increase is larger. Kruger et al. [13] used the PA to determine the influence of surface roughness. The benefit of using the PA is that
it can be combined with a PA for spherical objects. Though the PA might not give accurate results it presents qualitative insight into the influence of roughness. It was shown that the effects of roughness introduces a logarithmic divergence of the heat-distance curve decreasing the heat flow at small separations (Figure 4.23). The figure shows the $1/d$ dependence of the smooth sphere and the $-\log d/d_0$ dependency of the rough sphere. The zero distance was defined as the ”touching distance” where the largest roughness peaks touch the other surface. The heat transfer is much less for the rough sphere than it is for the smooth sphere. Perhaps a lot can be learned from theory and experiments on Casimir forces, in which the effects of surface roughness have been studied extensively [70, 71].

**Figure 4.23:** Radiative heat flow between a flat plate and a smooth sphere (red) and between a flat plate and rough sphere (magenta), all made of SiO$_2$. The roughness is described by a Gaussian height distribution function with a mean of 20 nm for the upper magenta line and 30 nm for the lower magenta line and a standard deviation of 10 nm. The spheres are 50 µm in diameter. The vertical axis contains the total heat flow minus the heat flow at 300 nm. Above 300 nm the PA is not accurate in this case. The value of $\alpha$ on the y-axis is 0.2558 nW and the temperatures are 0 K and 300 K. Image adapted from Ref. [13].
All in all, very little experimental research has been done on where the near field enhancement reaches its limits. Only Kittel et al. claimed to have observed some saturation at distances smaller than 10 nanometers between a gold probe and gold surface and between a gold probe and a GaN surface [32]. They assumed the tip of their thermal probe could be modeled using the dipole approximation.

4.8 Conclusion

The conditions for the existence of surface polaritons are summarized in Table 4.1. It was determined that the radiative heat transfer between two materials highly depends on the optical properties of both materials. The heat flow between a metal and a polar dielectric is much less than between equal materials. This is unfortunate since the premise of this research was to find a method that works equally well for all kinds of materials. Metamaterials can be engineered for high near field radiative heat flows. When doing so, the magnetic properties of the metamaterial should be taken into account as well.

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \varepsilon(\omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface</td>
<td>(-1)</td>
</tr>
<tr>
<td>Dipole</td>
<td>(-2)</td>
</tr>
</tbody>
</table>

**Table 4.1: Polariton conditions**

When considering the heat flux between two polar dielectric objects like SiC or SiO\(_2\), the dependency of the near field heat flow on the distance between them is listed in Table 4.2 for different geometries. Whether these dependencies uphold for nanometer range gaps sizes remains uncertain, experimental evidence should clarify this.
4.8. Conclusion

Table 4.2: Dependence of $Q$ on $d$ in near field thermal radiation between two SPhP supporting materials.

<table>
<thead>
<tr>
<th>configuration</th>
<th>dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>plate-plate</td>
<td>$1/d^2$</td>
</tr>
<tr>
<td>large sphere-plate</td>
<td>$1/d$</td>
</tr>
<tr>
<td>dipole-plate</td>
<td>$1/d^3$</td>
</tr>
<tr>
<td>cone-plate</td>
<td>$-\log d/d_0$</td>
</tr>
<tr>
<td>sphere-sphere</td>
<td>$1/d$</td>
</tr>
<tr>
<td>dipole-dipole</td>
<td>$1/d^6$</td>
</tr>
</tbody>
</table>
Chapter 5

Experiment Design

In this chapter the design of the experiment is explained. First some of the alternatives that were studied are discussed. Then the final design is explained in detail. The choices of equipment used in the setup are explained afterwards.

5.1 Design Alternatives

Designing a setup that measures the amount of tunnelled infrared photons from a substrate to a sensor head like in Ref. [33, 34, 6] was not considered feasible. Very little equipment was available to build something similar. This leaves the option to measure the absorbed heat to a sensor head by measuring its change in temperature. Unlike the tunnelled infrared photon detector this does not work in air because in air the gas conduction is a significant disturbance. Five possible shapes of the sensor head were listed in the previous chapter. The sensor head can have the shape of a very small sphere (a dipole), a rectangular plate parallel to the substrate (plate-plate), a cylinder with the one of the flat sides facing the substrate (cylinder-plate), a cylindrical cone with the sharp tip towards the substrate (cone-plate) or a large sphere near the substrate (sphere-plate).

While the heat flow between a dipole and a plate results in the highest sensitivity \((1/d^3)\) to the distance in between the two no practical solution for measuring the
tiny amount of heat flow to a nanosphere or nanoparticle was found.

To prevent a square plate from touching another larger plate below it the tilt of the top plate has to be controlled very precisely. For example, for a 1 cm square plate hovering 10 nm above another plate a rotation of 1 \( \mu \)rad will cause them to touch. Surface roughness and surface curvature also complicate matters. In order to control the tilt of the top plate it needs to be measured and actuated during the experiment. To minimize this sensitivity the top plate (the sensor head) should be made smaller, unfortunately MEMS production methods were not available for this project. Another drawback is that the top plate absorbs heat from a large area, it is not laterally sensitive and is unlikely to be used as a topology scanning tool.

The proposed cylinder also has a \( 1/d^2 \) relation of the heat flow like the parallel plates. This is because the bottom of the cylinder resembles the the flat area of the parallel plate [11]. This also means that the sensitivity to tilt also exists for the cylinder-plate geometry.

Measuring the heat flow from a plate to a conical tip (like the tip of a needle) is not as sensitive to the distance in between them as it is for any of the other geometries. The horizontal resolution is also less, see Section 4.6. It doesn’t absorb heat from the zone below the sharp tip but from a ring around it.

Another important issue is the response time of the system. For a system that detects scattered infrared photons the response time is only limited by the response time of the radiation detector. The response time for heat conduction through solid objects is deduced from the dimensionless Fourier number that characterizes transient heat conduction rates:

\[
Fo = \frac{t}{\tau} = \frac{\kappa t}{L^2}
\]  

(5.1)

The time constant \( \tau \) is governed by the thermal diffusivity \( \kappa \) and the characteristic length of the diffusion path \( L \). Furthermore the thermal diffusivity is a function of thermal conductivity \( k \), density \( \rho \), and specific heat capacity \( c_p \):

\[
\kappa = \frac{k}{\rho c_p}
\]  

(5.2)
For a plate-plate system the temperature sensor which measures the temperature of the top plate should be as close to the surface facing the bottom plate as possible. If the sensor head is made from a piece of 1 mm thick glass microscope slide and the temperature sensor is placed directly on top of it the time constant is already 3 seconds. At that time the heat flow is only 63% developed, it takes five times as long before the heat flow is more than 99% developed. Glass microspheres are widely available in numerous sizes. For a 20 µm glass sphere the time constant for heat conduction is 0.3 ms, for a 100 µm sphere it is 7 ms. As a typical time response for a bimaterial cantilever used as a thermal sensor Lai et al. measured a time constant of 0.2 ms for a 200 µm long triangular silicon nitride cantilever [72]. This leaves the sphere-plate geometry as the only viable candidate. Most importantly because the sphere is invariant to tilt. Because of its spherical shape it can also get closer to the substrate, the minimal contact area makes sure the influence of roughness peaks is less than it is for the large contact area between parallel plates. Another reason to go for the sphere is that there already is a very sensitive temperature sensor available for it, the bimaterial cantilever. Resolutions of $10^{-6}$ K are possible for them [72]. This is much better than any conventional temperature sensor. In Section 4.6 it was reported that the PA is a very accurate method for approximating the heat flow between a sphere and a plate. This means that any experimental results from a sphere-plate experiment can easily be translated into a prediction for a similar instrument that works with parallel plates.

The comparison of the alternatives is summarized in Table 5.1.

5.2 Design Baseline

The final design of the experiment is depicted schematically in Figures 5.1 and 5.2. The optical readout system is the same as in an earlier TNO project [14], it is designed to accurately measure angle changes of a reflective surface. The setup was characterised to have a high bandwidth (40 MHz), low drift (2 nm deflection over 1000 s) and a low noise 15 fm/√Hz. Starting from the left a laser is generated and fed through a fibre to the collimator. There the laser light is collimated into
### Pros Cons

<table>
<thead>
<tr>
<th>IR radiation detection</th>
<th>very fast, works in air, high vertical sensitivity, complex, expensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>dipole-plate</td>
<td>very fast, high sensitivity</td>
</tr>
<tr>
<td></td>
<td>does not work in air, heat flow very small, unpractical</td>
</tr>
<tr>
<td>plate-plate</td>
<td>high vertical sensitivity</td>
</tr>
<tr>
<td></td>
<td>slow, tilt sensitive, no horizontal sensitivity, does not work in air, working range limited by surface roughness and curvature</td>
</tr>
<tr>
<td>cone-plate</td>
<td>-</td>
</tr>
<tr>
<td>cylinder-plate</td>
<td>high vertical sensitivity</td>
</tr>
<tr>
<td></td>
<td>does not work in air, tilt sensitive, slow, difficult to make</td>
</tr>
<tr>
<td>sphere-plate</td>
<td>fast, not tilt sensitive, high horizontal sensitivity, working range down to nanometers, very sensitive temperature sensor available does not work in air, moderate vertical sensitivity</td>
</tr>
</tbody>
</table>

| Table 5.1: The pros and cons of different shapes of the sensor head. |

a parallel beam. Some laser light goes to the beam dump and some to the power meter before it is reflected through an optical viewport into the vacuum chamber. The waveplates and polarizing beam splitters make sure that the amount of light going to the beam dump is minimal because the transmission of the polarizing beamsplitter depends on the laser polarization. The light absorbed by the beam dump has no use. When the parallel laser bundle is in the chamber it is reflected
to the microscope lens. There it converges to a small spot onto the far end of a cantilever with a sphere glued on top of it. The laser is reflected back off the cantilever and passes the objective again into a parallel bundle. On its way back through the optical path the majority of the laser bundle is split into a part that goes to the position sensitive detector (PSD) and one that goes to the camera.

![Figure 5.1: Schematic design of the experiment, seen from the top.](image)

The laser is focused on the far end of the cantilever. The intensity signal of the PSD will assure whether the substrate is blocking a part of the laser bundle or not. If it does it has to be moved to the right in Figure 5.1. The cantilever absorbs some heat of the laser (about 4%). This causes the cantilever to bend. A piezo motor moves the substrate towards the sphere, the cantilever is stationary so the laser stays focused on it. The majority of the returning laser beam goes to the PSD and to the camera. The PSD measures the angle change of the cantilever. The capacitive sensor measures the distance to the substrate holder. When the sphere is moved closer and closer to the substrate it will release more heat to it and its bending will decrease. When the sphere hits the surface the cantilever will
bend sharply, the point where this happens is the point of zero distance between sphere and substrate.

The setup is designed so that floor vibrations have a minimal influence on it. In Figure 5.3 any up and down motion of the substrate or the holder for the cantilever and capacitive sensor does not change the distance between the substrate and the sphere.

In an earlier design of the setup the whole optical path was placed inside the vacuum chamber, this is not a good idea because it requires a lot of feedthroughs, not just for the electrical equipment but also for the laser fibre. The stages and adjustable mirrors in the optical path allow aiming of the laser bundle. If these parts are all placed in the chamber there they are not accessible after the chamber is evacuated. Deformations of the bottom plate of the chamber caused by the pressure difference will cause the laser to be misaligned. For a flat disk of thick-
Figure 5.3: Floor vibrations do not have a major influence on the alignment of the system. If either the substrate or the right side of the picture moves up and down with respect to the other the horizontal distance between them does not change. The red dot is the laser spot on the cantilever, the laser path is perpendicular to the paper.

iness $t$ and radius $R$ simply supported at its boundary and loaded by a constant distributed load $q$ perpendicular on the entire surface the maximum displacement of the centre is [73]:

$$y_c = \frac{qR^5}{62D(1 + \nu)}$$  \hspace{1cm} (5.3)

The term $D$ with elastic modulus $E$ and Poisson’s ratio $\nu$ is:

$$D = \frac{Et^2}{12(1 - \nu)}$$  \hspace{1cm} (5.4)

For a $t = 2$ cm thick stainless steel disk and a 1 atm pressure difference this results in a displacement of the centre of 30 $\mu$m. Deformations like these can misalign the laser beam path and need to be corrected for after the chamber is evacuated.

5.3 Sub-Modules

The choices for all non-obvious equipment used for the experiment are explained in this section.
5.3.1 Cantilevers

The most important property of the cantilever is how much it bends when it absorbs heat from the laser. Before this is discussed some more practical considerations are listed.

- The cantilever should consist of two layers with a different thermal expansion coefficient. The top layer should be reflective so that the laser light is reflected.

- If it is possible to glue a sphere onto a tipless cantilever these should be used. The sphere should be glued centred on the free end of the cantilever. If so the electrostatic and van der Waals forces will only stretch the cantilever and will not bend it. A thicker cantilever is less sensitive to stretching by the attractive forces of the substrate and also less sensitive to bending moments.

- Some vendors sell cantilevers with spheres already attached to them. Unfortunately these have their spheres below the cantilever where the tip normally is and not attached to the far end of the cantilever. No information was found on the microstructure (surface roughness) of these spheres.

- The cantilever should have a fast response time, this means the time constant for thermal relaxation should be low [72]:

\[
\tau = \frac{L^2 \rho_1 c_{p1} t_1 + \rho_2 c_{p2} t_2}{3 \left( k_1 t_1 + k_2 t_2 \right)}
\] (5.5)

Here \( t_j \) is a layer thickness and \( L \) is the length of the cantilever. The material properties in the equation are the density \( \rho_j \), the thermal conductivity \( k_j \) and the heat capacity \( c_{pj} \). Generally the thermal response of these kind of cantilevers is very fast, in the order of microseconds.

For a bimaterial cantilever heated at its free end and clamped at the other end placed inside a perfect vacuum the following differential equation governs its de-
flection $z$ [47, 74]:

$$\frac{\partial^2 z}{\partial x^2} = 6(\alpha_2 - \alpha_1) \frac{t_1 + t_2}{t_2^2 K} (T(x) - T_0)$$

(5.6)

$$= 6(\alpha_2 - \alpha_1) \frac{t_1 + t_2}{t_2^2 K} \left( \frac{Px}{Lw(k_1 t_1 + k_2 t_2)} + T_{dev} \right)$$

(5.7)

The position along the cantilever is $x$, $T(x)$ is the temperature distribution and $T_0$ is the stress free temperature, $T_{dev}$ is how much the temperature of the base of the cantilever deviates from the stress free temperature, $\alpha_j$ is the thermal expansion coefficient of layer $j$. The dimensionless constant $K$ is defined as a function of thicknesses and elastic moduli $E_j$: 

$$K = 4 + 6 \frac{t_1}{t_2} + 4 \left( \frac{t_1}{t_2} \right)^2 + \frac{E_1}{E_2} \left( \frac{t_1}{t_2} \right)^3 + \frac{E_2 t_2}{E_1 t_1}$$

(5.8)

The rotation of the cantilever is given as:

$$\theta(x) = - \int \frac{\partial^2 z}{\partial x^2} \partial x = 3(\alpha_1 - \alpha_2) \frac{t_1 + t_2}{Kt_2^2} \left( \frac{Px^2}{w(k_1 t_1 + k_2 t_2)} + T_{dev} \right)$$

(5.9)

Because the near field heat flow from the sphere to the substrate can be considered a decrease in the power absorbed by the cantilever the sensitivity of bending to the near field heat flow is given as:

$$S_Q = \frac{\partial \theta(L)}{\partial P} = 3(\alpha_1 - \alpha_2) \frac{t_1 + t_2}{Kt_2^2} \frac{L^2}{w(k_1 t_1 + k_2 t_2)}$$

(5.10)

From this it can be seen already that $L$ and $\Delta \alpha = |\alpha_2 - \alpha_1|$ should be maximized for increased sensitivity. On the other hand $w$, $k_1$ and $k_2$ should be minimized. For $t_1$, $t_2$, $E_1$ and $E_2$ it is not so clear how the sensitivity of our instrument can be increased; but after choosing some commercially available cantilevers that meet the requirements it is easy to compare their sensitivities.

Bimaterial cantilevers are also very sensitive temperature sensors. Even though there is barely any heat transfer to the surrounding air because of its low pressure, changes in the temperature of the chip change $T_{dev}$, causing drift. The sensitivity to base temperature changes is:

$$S_T = \frac{\partial \theta(L)}{\partial T_{dev}} = 3(\alpha_2 - \alpha_1) \frac{t_1 + t_2}{Kt_2^2} L$$

(5.11)
As a figure of merit the ratio of both sensitivities is used:

\[
\frac{S_Q}{S_T} = \frac{L}{w(k_1t_1 + k_2t_2)} = G^{-1}
\]

(5.12)

This is the inverse of the conductance \( G \) of the cantilever.

Thin film thermal and mechanical properties can differ from bulk properties [75]. In Table A1 are thermal and mechanical material properties from studies on thin film properties of similar thicknesses and production processes as for the cantilevers studied in this report.

<table>
<thead>
<tr>
<th>material</th>
<th>( \alpha ) ((10^{-6} \text{ K}^{-1}))</th>
<th>( E ) ((10^9 \text{ N m}^{-2}))</th>
<th>( k ) ((\text{W m}^{-1} \text{ K}))</th>
<th>( \rho ) ((\text{kg m}^{-3}))</th>
<th>( c_p ) ((\text{J kg}^{-1} \text{ K}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al</td>
<td>23.6 [77]</td>
<td>80 [72]</td>
<td>94 [78]</td>
<td>2702 [72]</td>
<td>905 [77]</td>
</tr>
<tr>
<td>Si(_3)N(_4)</td>
<td>3.8 [76]</td>
<td>180 [77]</td>
<td>13 [79]</td>
<td>2865 [76]</td>
<td>700 [80]</td>
</tr>
</tbody>
</table>

**Table 5.2:** Thermal and mechanical properties of thin films.

For 11 different commercially available cantilevers by different producers their sensitivities are compared in Table 5.3. Their materials, shape and sizes are also listed.
<table>
<thead>
<tr>
<th>type</th>
<th>producer</th>
<th>shape</th>
<th>tip</th>
<th>material 1</th>
<th>material 2</th>
<th>( t ) (( \mu \text{m} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNP-TR-TL-Au</td>
<td>NanoWorld</td>
<td>△</td>
<td>no</td>
<td>SiN</td>
<td>Au/Cr</td>
<td>0.5</td>
</tr>
<tr>
<td>Arrow\textsuperscript{TM} TL1Au</td>
<td>NanoWorld</td>
<td>□</td>
<td>no</td>
<td>Si</td>
<td>Au/Ti</td>
<td>1.0</td>
</tr>
<tr>
<td>PNP-DB</td>
<td>NanoWorld</td>
<td>□</td>
<td>yes</td>
<td>SiN</td>
<td>Au/Cr</td>
<td>0.5</td>
</tr>
<tr>
<td>PPP-CONTR</td>
<td>NanoSensors</td>
<td>□</td>
<td>yes</td>
<td>Si</td>
<td>Al</td>
<td>2</td>
</tr>
<tr>
<td>PPP-CONTAuD</td>
<td>NanoSensors</td>
<td>□</td>
<td>yes</td>
<td>Si</td>
<td>Au</td>
<td>2</td>
</tr>
<tr>
<td>PPP-RT-CONTR</td>
<td>NanoSensors</td>
<td>□</td>
<td>yes</td>
<td>Si</td>
<td>Al</td>
<td>2</td>
</tr>
<tr>
<td>PPP-ZEILR</td>
<td>NanoSensors</td>
<td>□</td>
<td>yes</td>
<td>Si</td>
<td>Al</td>
<td>4</td>
</tr>
<tr>
<td>ContAl-G</td>
<td>BudgetSensors</td>
<td>□</td>
<td>yes</td>
<td>Si</td>
<td>Al</td>
<td>2</td>
</tr>
<tr>
<td>SiNi</td>
<td>BudgetSensors</td>
<td>△</td>
<td>yes</td>
<td>SiN</td>
<td>Au/Cr</td>
<td>0.52</td>
</tr>
<tr>
<td>MLCT</td>
<td>Bruker</td>
<td>△</td>
<td>yes</td>
<td>SiN</td>
<td>Au/Ti</td>
<td>0.55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t_2 ) (nm)</th>
<th>( w ) (( \mu \text{m} ))</th>
<th>( L ) (( \mu \text{m} ))</th>
<th>( S_Q ) (W(^{-1}))</th>
<th>( S_T ) (10(^{-5}) K(^{-1}))</th>
<th>( S_Q/S_T ) (10(^4) K W(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PNP-TR-TL-Au</td>
<td>35</td>
<td>2.28</td>
<td>200</td>
<td>114</td>
<td>26.7</td>
</tr>
<tr>
<td>Arrow\textsuperscript{TM} TL1Au</td>
<td>35</td>
<td>100</td>
<td>500</td>
<td>14.3</td>
<td>31.7</td>
</tr>
<tr>
<td>PNP-DB</td>
<td>70</td>
<td>40</td>
<td>200</td>
<td>176</td>
<td>60.6</td>
</tr>
<tr>
<td>PPP-CONTR</td>
<td>30</td>
<td>50</td>
<td>450</td>
<td>3.64</td>
<td>9.41</td>
</tr>
<tr>
<td>PPP-CONTAuD</td>
<td>30</td>
<td>50</td>
<td>450</td>
<td>2.28</td>
<td>6.00</td>
</tr>
<tr>
<td>PPP-RT-CONTR</td>
<td>30</td>
<td>50</td>
<td>450</td>
<td>3.64</td>
<td>9.41</td>
</tr>
<tr>
<td>PPP-LFMR</td>
<td>30</td>
<td>48</td>
<td>225</td>
<td>8.53</td>
<td>19.2</td>
</tr>
<tr>
<td>PPP-ZEILR</td>
<td>30</td>
<td>55</td>
<td>450</td>
<td>0.391</td>
<td>2.33</td>
</tr>
<tr>
<td>ContAl-G</td>
<td>30</td>
<td>50</td>
<td>450</td>
<td>3.64</td>
<td>9.41</td>
</tr>
<tr>
<td>SiNi</td>
<td>70</td>
<td>2.30</td>
<td>200</td>
<td>107</td>
<td>55.5</td>
</tr>
<tr>
<td>MLCT</td>
<td>45</td>
<td>2.20</td>
<td>310</td>
<td>331</td>
<td>45.0</td>
</tr>
</tbody>
</table>

**Table 5.3:** Cantilever sensitivity analysis. The shape is either rectangular or triangular, no tip means it is a tipless cantilever. Material 2 is the reflective layer. The geometrical parameters are the total thickness \( t \), thickness of the reflective layer \( t_2 \), the width \( w \) and length \( L \). The bending sensitivity to heat input by the laser is \( S_Q \), the bending sensitivity to temperatures is \( S_T \).
Based on the information in Table 5.3 the PNP-DB and MLCT cantilever chips were purchased. Both of them have multiple cantilevers on their chip. The ones in Table 5.3 are the largest cantilevers on the chips. The longest PNP-DB cantilever are expected to have a time constant of 0.5 ms, the longest of the MLCT cantilevers is expected to have a time constant of 1.8 ms. The real time constant of the cantilevers will be determined experimentally. An overview of all the cantilevers on the chips is in Table 5.4.

<table>
<thead>
<tr>
<th>cantilever</th>
<th>$f_{res}$ (kHz)</th>
<th>shape</th>
<th>$C$ (N m$^{-1}$)</th>
<th>$L$ (µm)</th>
<th>$w$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLCT A</td>
<td>22</td>
<td>△</td>
<td>0.07</td>
<td>175</td>
<td>22</td>
</tr>
<tr>
<td>MLCT B</td>
<td>15</td>
<td>△</td>
<td>0.02</td>
<td>210</td>
<td>20</td>
</tr>
<tr>
<td>MLCT C</td>
<td>7</td>
<td>△</td>
<td>0.01</td>
<td>310</td>
<td>20</td>
</tr>
<tr>
<td>MLCT D</td>
<td>15</td>
<td>△</td>
<td>0.03</td>
<td>225</td>
<td>20</td>
</tr>
<tr>
<td>MLCT E</td>
<td>38</td>
<td>△</td>
<td>0.1</td>
<td>140</td>
<td>18</td>
</tr>
<tr>
<td>MLCT F</td>
<td>125</td>
<td>△</td>
<td>0.6</td>
<td>85</td>
<td>18</td>
</tr>
<tr>
<td>PNP-DB 1</td>
<td>67</td>
<td>△</td>
<td>0.48</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>PNP-DB 2</td>
<td>17</td>
<td>△</td>
<td>0.06</td>
<td>200</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 5.4: Different cantilevers on the MLCT and the PNP-DB cantilever chip.

5.3.2 Microspheres

With regard to the choice of microspheres the following points are noted:

- Larger spheres are less likely to stick to surfaces during the glueing process than smaller spheres.
- The spheres should be made of SiO$_2$ or a similar material that supports surface polaritons in the thermal infrared.
- Microspheres dissolved in water can have some residue on them after the water is evaporated [81].
- Near field heat radiation from a sphere to a plate increases proportionally with $R$, far field radiation scales with the area of the sphere, that is with $R^2$. 
So for a small sphere the influence of the near field becomes apparent at a larger separation then for large spheres.

- Large spheres release more heat to the substrate and create a larger change in bending and thus a higher measured signal.

For the experiments three different kinds of microspheres were available, 100 µm spheres by Corpuscular inc., 100 µm spheres by Thermo Scientific and 20 µm spheres by Thermo Scientific. The first ones are pure SiO$_2$ microspheres, dissolved in water. Bao reported that these kind of spheres were very smooth, with a peak to peak roughness of about 30 nm and an rms roughness of 2.5 nm [82]. Van Zwol’s study on microsphere roughness [83] revealed that the 100 µm spheres by Thermo Scientific, made of borosilicate glass, also have a peak to peak roughness of 30 nm. The smoothest spheres in van Zwol’s study were the 20 µm spheres by Thermo Scientific, with a roughness of 0.7 nm, made of soda lime glass. The spheres by Thermo Scientific are supplied dry, not in a solution. According to the manufacturer’s data sheets the soda lime glass spheres contain 60-72.5% SiO$_2$, the borosilicate glass spheres contain 52.5% SiO$_2$.

### 5.3.3 Adhesive

To attach the sphere to the cantilever an adhesive is needed. It is important to choose an adhesive that has a long curing time so that it does not cure during the process of attaching the sphere to the cantilever. Van Zwol used an epoxy glue (Bison Kombi 5 minutes) [81]. Bao used a silver epoxy and bakes it at 120 degrees celsius for 30 minutes to cure it [82]. Shen used a UV adhesive that is cured under UV light after the sphere is attached. This has the advantage that there is no time limit in which the process has to be completed. For this project Norland Optical Adhesive 61 was used, cured by a UV lamp.
5.3.4 Substrate

As substrate a microscope slide made by Menzel-Glazer made of extra white glass, containing 72.20% SiO\textsubscript{2} was used. Additionally, discarded pieces of silicon from the Kavli Nanolab were available as substrates. The doping levels of these pieces were unknown, of the ones with oxide layer grown on them the thickness was unknown. The pieces with oxides on them cannot be considered full SiO\textsubscript{2} samples because their oxide thickness is probably much less than the penetration depth of near field thermal radiation, about 10 \( \mu \)m. Because of time constraints only the microscope slide was used as a substrate. Drawback of the microscope slide is that it is not known if its microstructure is just as flat as a thermally grown oxide. A rough surface can lower the near field thermal radiation as was shown in Section 4.7. The microscope slide was glued with a thin layer of UV adhesive on the aluminium substrate holder. The whole contact area was covered with glue to make sure the thermal resistance between substrate and substrate holder was as small as possible. The substrate was cleansed with acetone and then quickly wiped with isopropanol before the experiment. A custom made aluminium substrate holder connects the substrate to the piezo motor. This polished piece of aluminum is also the target for the capacitive sensor.

5.3.5 Laser

The laser beam is generated by an LP635-SF8 pigtailed laser diode by Thorlabs, the diode is driven by a Pro8000 laser controller with current and temperature control. The laser wavelength is 635 nm, it is carried by a single mode fibre to a collimator that sends a parallel beam through the waveplates and beam splitters. In preparation of the measurements a Newport 1830-C optical power meter was used to measure the laser power at different stages along the optical path. This way the optimal rotation for the waveplates was determined.
Figure 5.4: Photograph of the cantilever holder on the left and the cantilever with microsphere on the right. A close-up of the cantilever with sphere is in the bottom left.

5.3.6 Position Sensitive Detector

The position sensitive detector (PSD) used is the Maypa OPS with a 40 MHz bandwidth. The measurements in this project take place at much lower frequencies than that. The Maypa OPS is different from an ordinary quadrant photo detector in the sense that it employs a beam splitter and two reflective prisms to separate the laser beam before it reaches the photodiodes, see Figure 5.5. This eliminates the dead space that is in between the quadrants of an ordinary quadrant photo detector.

The output configuration of the Maypa OPS is the same as that of a quadrant photodiode. The intensity signal is the sum of the voltages of all quadrants. The x-position voltage is the sum of the voltages of the two quadrants on the left minus the sum of the voltages of the quadrants on the right. The y-position voltage is the sum of voltages of the two quadrants on top minus the sum of the voltages of the quadrants on the bottom.
\[ x_{pos} = \frac{(x_{right} - x_{left})}{x_{sum}} \]  
\[ x_{sum} = x_{right} + x_{left} \]  
\[ y_{pos} = \frac{(y_{top} - y_{bottom})}{y_{sum}} \]  
\[ y_{sum} = y_{top} + y_{bottom} \]

Figure 5.5: Splitting of the laser beam in the Maypa OPS. From the manufacturer’s website, http://www.maypatech.com/.

5.3.7 Vacuum Chamber

A glass bell jar with an inner diameter of \(\sim 45\) cm and a height of \(\sim 60\) cm was placed on a stainless steel baseplate. The baseplate contains different KF flanges to connect the vent, the valve, the pressure sensor, the optical viewport and the electrical feedthrough. The optical viewport is highly transmissive for red light, using the optical power meter the viewport was found to absorb \(\sim 5\%\) of the laser light for a single pass. The electrical feedthrough consists of a flange with a 15 pin D-sub connector. The pump is connected to a valve by a stainless steel flexible hose. The vacuum chamber is placed on an optical table. The protective cage that is placed over the bell jar and the baseplate of the chamber is connected to
an electrical ground. The four legs of the chamber have elastomeric dampers on the bottom.

![Figure 5.6: Vacuum chamber without and with protective cage.](image)

The pressure in the vacuum chamber should be lowered until the flow of air inside it is in the molecular flow regime. In the molecular flow regime the air molecules are far more likely to hit the chamber walls or parts of the setup when travelling through space than they are to collide with each other. Thus whether a flow is in the molecular regime depends on pressure and on the characteristic size of its container too. A gas is considered rarefied if its Knudsen number is much larger than one:

$$\text{Kn} = \frac{\lambda_{\text{air}}}{L} \gg 1$$  \hspace{1cm} (5.17)

Herein $\lambda_{\text{air}}$ is the mean free path of air molecules and $L$ a characteristic length of the vacuum setup. In the molecular flow regime an expression for the heat transfer coefficient is given in [19] as:

$$h_{\text{rare}} = \frac{nT(2k_B)^{3/2}}{2\sqrt{\pi mT}}$$  \hspace{1cm} (5.18)

In this equation $n$ is the particle density, $T$ is average temperature of the two objects between which the heat flow takes place, $m$ is the mass of one gas molecule. The mass of one nitrogen molecule, the main constituent of air, is $10^{-26}$ kg. The particle density is found using the ideal gas law:

$$P = nk_BT$$  \hspace{1cm} (5.19)
Note that $n$ here is the amount of particles per volume instead of the number of moles per volume as is often used in the ideal gas law. With this the expression for the heat transfer coefficient becomes:

$$h_{\text{rare}} = P \sqrt{\frac{2k_B}{\pi mT}}$$

(5.20)

This should be well below with the smallest amount of radiative heat transfer in the experiment which is the far field radiation of the sphere to its surroundings. With the definition for the radiative heat transfer from a small object to its surroundings it follows that:

$$\frac{h_{\text{radiation}}}{h_{\text{rare}}} = \frac{4\epsilon \sigma T^3}{P \sqrt{\frac{2k_B}{\pi mT}}}$$

(5.21)

Using 0.81 as the emissivity of silica [17] and an average temperature of 300 K the required pressure in the chamber for a ratio in heat transfer coefficients of 1000 is about $10^{-4}$ mbar. The exact shape of the surroundings does not matter since the heat flow does not depend on distance between sphere and enclosing as long as the flow is in the rarefied regime. The mean free path of air at this pressure can be estimated with [84]:

$$\lambda_{\text{air}} = \frac{k_B T}{\pi \sqrt{2Pd_m^2}} = \frac{6.7 \cdot 10^{-3}}{P}$$

(5.22)

The molecular diameter $d_m$ of nitrogen is 3.7 Å. For a pressure of $10^{-4}$ mbar the mean free path is indeed larger than the size of the vacuum chamber.

### 5.3.8 Vacuum Pump

The minimum pressure in the chamber is determined by the pump and the sealing of the vacuum chamber. The pump used was an oil-sealed single phase induction motor, the SKWS400-1 by Shin Kang Electric Machine Co., Ltd. It is generally used as a roughing pump and is able to pump down to pressures as low as 0.01 mbar. For lower pressures another high vacuum pump should be added.
5.3.9 Capacitive Sensor

The capacitive sensor measures the distance between a capacitive sensor head and the substrate holder. This is not the absolute distance between the sphere and the substrate but a relative measurement. The absolute distance is determined by the contact point of sphere and substrate. The capacitive sensor used is the Physik Instrumente D-510 with the D-510.021 sensor head. The resolution is documented to be 0.2 nm. The capacitive sensor is aimed at a part of the bare substrate holder, the substrate holder is made of polished aluminium. The sensor head is placed on a translation stage that moves the piezo head towards the substrate holder, the sensor head holder also has three rotational degrees of freedom to make sure the sensor head is placed exactly parallel to the target. The target area should be at least twice as large as the sensor head area. The manufacturer also advises to make the target the moving part, not the sensor. Both requirements were met.

Instead of a capacitive sensor an interferometer could have been used. By its sheer size it will increase the metrology loop. The laser in the interferometer is also an unwanted heat source.

To feed the capacitor sensor head cable through the wall of the vacuum chamber it was cut in half. The cable consists of a core and a double mantle, each mantle and the core were soldered to a different pin of the D-sub connectors. When testing if the capacitive sensor still worked after that it exhibited an erratic and strongly nonlinear response compared to the displacement of the target. The target was displaced by a piezo actuator. The piezo actuator has a strain gauge on it that measures the expansion of the piezo. According to the LEDs on the signal conditioner of the capacitive sensor it was still within its 20 µm working range when the sensor head was more than a centimetre away from its target. For this reason the capacitive sensor could not be used in vacuum. The elongation measured by the strain gauge was used as a measure of the displacement of the substrate.
5.3.10 Piezo Stage

The positioning stage used to move the substrate was the MAX301 - NanoMax 3-Axis Stage by Thorlabs. It is a three axis platform with 4 mm stroke differential screws and 20 µm stroke piezo drives for each axis. The piezos are controlled by the BPC303 3-Channel 150 V Benchtop Piezo Controller, also by Thorlabs. The controller provides PID feedback on each channel, the feedback signal is provided by a strain gauge on the piezo stacks. It has a resolution of 5 nm. Because the capacitive sensor was not working when its cable was fed through the electrical feedthrough, the feedback from the strain gauge was used as a relative distance measurement of the sphere-substrate distance during the vacuum experiments. With both the driving voltage cable and the feedback cable connected to the stage through the electrical feedthrough the accuracy and resolution were tested with a (working) capacitive sensor, both were not changed by the fact that the cables were cut, soldered to D-sub plugs and fed through into the vacuum chamber.

5.3.11 Data Acquisition

For data acquisition (DAQ) an NI USB-6251 device by National Instruments is used. It contains 8 BNC inputs and samples 16 bit with a maximum rate of 1.25 megasamples per second. LabVIEW is used to communicate with the DAQ and to collect measurement data.

5.3.12 Objective Lens

To focus the laser on the cantilever a microscope objective is used. Three Mitutoyo Plan Apo Infinity-Corrected Long WD Objectives were tested in [14]. Their properties are listed in table 5.5. This study revealed that the optical readout system with the 10x lens has the highest sensitivity, a longer working distance results in a larger shift of the reflected laser beam, see Figure 5.7.
Table 5.5: Three available microscope objectives.

<table>
<thead>
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<th>Numerical Aperture</th>
<th>working distance (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10×</td>
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<td>33.5</td>
</tr>
<tr>
<td>20×</td>
<td>0.42</td>
<td>20.0</td>
</tr>
<tr>
<td>50×</td>
<td>0.55</td>
<td>13.0</td>
</tr>
</tbody>
</table>

Figure 5.7: The reflected beam is shifted with a distance $\Delta x$ by the bending of the cantilever. Image adapted from Ref. [14].

Unfortunately the 10x objective was not available during the experiments because it was used in another project so the 20x objective lens had to be used. The 20x objective has the benefit that the camera is not needed to focus the laser on the top of the cantilever. With a flashlight providing backlighting on the cantilever and a piece of paper in front of the PSD to capture the projection of the cantilever chip an image of the cantilever chip is acquired (Figure 5.8). For the 10x lens the projected image is a lot smaller and it is more difficult to discern the image.

To focus the image of the cantilever on the CMOS camera a Mitutoyo MT-4 Accessory Tube Lens is placed in front of the camera.

5.3.13 Pressure Sensor

Pirani pressure sensors are used to measure the pressures in medium vacuums. For this experiments a 925C micropirani pressure sensor by the Kurt J. Lesker company
Figure 5.8: On the left a cantilever with a sphere on it is brought close to the aluminium substrate holder, without a substrate on it. Because of the smooth reflective surface of the aluminium the mirror image of the sphere is seen on the right side of the left picture. This way the distance between the sphere and the substrate holder can be estimated by comparing the distance between the sphere and its mirror image to the size of the sphere. In the right image a cantilever plus sphere is brought close to a glass substrate. The glass is highly transparent but the outline reveals the outer surface of the microscope slide.

is used. It can measure from atmospheric pressure down to $2.7 \times 10^{-5} \text{ mbar}$ with an accuracy of 5% in the $1 \times 10^{-3} \text{ mbar}$ to 133 mbar range.

5.3.14 Temperature Sensors

Bimaterial cantilevers are very sensitive temperature sensors themselves. Depending on the type of cantilever a single chip can hold multiple cantilevers, on one side or on both sides like the ones chosen for this project. As such, one of them can be used for measuring the temperature of the chip. Because this requires another optical readout system this was not a valid option. Thermistors and resistance temperature detectors (RTD) have a higher sensitivity than a thermocouple. The fact that RTDs and thermistors have a lower measurement range is not a problem
because temperature deviations are expected to be small, not larger than a few degrees. A potential problem with thermistors is that they generate some heat themselves, altering the measured temperature.

Type K thermocouples were used to measure temperatures in this project. The thermocouples were installed on a DSCP62 thermocouple-to-DC voltage converter with operating temperatures between 0 and 200 °C. The accuracy is 0.1% of the measurement range. These were used because they were readily available.
Chapter 6

Thermomechanical Modelling of the Experimental Setup

In this chapter predictions are made on the different kinds of heat flows in the experiment. These are the heat flow by solid conduction through the cantilever, heat conduction to the surrounding air and heat flows by thermal radiation. The influence of electrostatic forces, van der Waals forces and thermal expansion of the cantilever on the experiment are also studied.

6.1 Heat Balance for the Cantilever with Sphere

In equilibrium the laser power $P$ absorbed by the cantilever in an ideal vacuum is dissipated in three ways. Some heat is conducted inside the cantilever towards the cantilever chip, this is $Q_{\text{cond}}$. Some heat lost by radiation to its surroundings, this is $Q_{\text{rad}}$. For heat transport taking place in low pressures and on micrometer scales buoyancy forces are much smaller than viscous forces [85], heat losses by natural convection are neglected. When the substrate is near the sphere some heat is lost by near field thermal radiation, $Q_{\text{NF}}$. This is displayed on the left side of Figure 6.1. In air or partial vacuum there are more sources of heat loss. The heat loss to the surrounding air is $Q_{\text{air}}$ and the gas conduction between the sphere and
substrate is $Q_{\text{sub}}$, see the right side of Figure 6.1. Next up these heat flows are studied more closely.

![Figure 6.1: Different heat losses in a perfect vacuum (left) and in air or partial vacuum (right).](image)

### 6.1.1 Absorbed Laser Power

The absorptivity of the gold layer on the cantilever is $\sim 4\%$. The laser power is 2 mW at most. This means the cantilever absorbs 80 $\mu$W of heat from the laser.

### 6.1.2 Heat Conduction in the Cantilever

When the substrate is removed ($Q_{\text{NF}} = 0$) and the radiative heat losses are ignored for now ($Q_{\text{rad}} = 0$) the conduction through the cantilever is the only heat loss in vacuum and $P_{\text{laser}} = Q_{\text{cond}}$. The temperature distribution in this case is linear and was given in (5.6):

$$T(x) = T_{\text{base}} + \frac{P_x}{w(k_1t_1 + k_2t_2)}$$  \quad (6.1)

At the tip of the longest MLCT cantilever, where $x = L = 310 \mu$m, the temperature is 31.2 K higher than it is at the base.

### 6.1.3 Radiative Losses from the Cantilever and Sphere

Using the temperature gradient from the previous section the (far field) radiative heat $Q_{\text{rad}}$ losses can be estimated. These are usually ignored because they are only a few percent of the heat conduction through the cantilever [85].
Integrating the black body radiative heat loss of a piece of cantilever with differential length $dx$ and width $w$ over the whole length using a linear temperature distribution of $T_{room}$ at the base and $T_{end}$ at the end of the cantilever and multiplying this by two, once for each side of the cantilever, the resulting heat loss is:

$$Q_{rad-cantilever} = 2 \int_0^L \sigma w \left( \frac{x(T_{end} - T_{room})}{L} + T_{room} \right)^4 - T_{room}^4 \right) dx \quad (6.2)$$

After evaluation of the integral:

$$Q_{rad-cantilever} = \frac{2\sigma L w}{5} \left( T_{end}^4 + T_{room}^3 T_{end}^2 + T_{room}^2 T_{end}^3 + T_{room}^3 T_{end}^2 - 4T_{room}^4 \right) \quad (6.3)$$

Using $T_{room} = 293$ K and $T_{end} = 324.2$ K the resulting heat loss is 2.4 $\mu$W. This is 3% of the $Q_{cond}$. This is an overestimate of the real heat loss because the radiative heat loss will lower the temperatures in the cantilever and the real heat loss is always less than the black body heat loss. If the sphere also has a temperature of 324.2 K its far field radiative heat loss to the environment can be estimated as well. For a 100 $\mu$m sphere the black body heat loss is 4.0 $\mu$W. Both are only a few percent of $P$ so they are ignored in the estimations in this chapter.

### 6.1.4 Gas Conduction between Sphere and Substrate

Consider two infinitely large plates separated by a small gap filled with gas. When the gas is at atmospheric pressures the heat transfer by gas conduction can be compared to the heat transfer by near field thermal radiation. The mean free path of air at atmospheric pressure is 67 nm, see (5.22). When the separation between the plates is much larger than the mean free path of air the heat flow is governed by collision between air molecules [19]. This is the classic (viscous) regime and the heat flow is governed by Fourier’s law. When the separation gap is much smaller than the mean free path of the air molecules it is said that the air is in the rarefied regime (free molecular flow). The heat flow is governed by the collisions between
the air molecules and the walls and thus it is not a function of the separation distance between the walls.

The boundaries between these regimes are chosen as follows in this report: The domain of the viscous regime is \( d > 100\lambda_{\text{air}} \). The rarefied regime is when \( d < 0.1\lambda_{\text{air}} \). The transition regime is in between them.

In the viscous regime the classical definition of the heat transfer coefficient is:

\[
h_{\text{vis}} = \frac{k_{\text{air}}}{d}
\]

(6.4)

The thermal conductivity of air at room temperature and atmospheric pressure is 0.0257 \( \text{W m}^{-1}\text{K} \) [17]. In the transition regime this relation can also be used but the thermal conductivity of air changes with pressure and can be approximated with [86]:

\[
k_{\text{air, low } P} = k_{\text{air}} \left( 1 + 7.6 \times 10^{-5} \frac{T}{Pd} \right)
\]

(6.5)

The heat transfer coefficient of air in free molecular flow was defined in (5.20). With these relations the heat transfer coefficient between the two plates can be mapped over a wide range of separation gaps. This is done in Figure 6.2. The predicted heat transfer coefficient for the near field thermal radiation is also in the figure. Two data points from plate-plate experiments are also plotted. From the figure it can be concluded that the gas conduction is practically always higher than the near field thermal radiation in atmospheric conditions. The radiative heat transfer is expected to saturate at very small gaps. This kind of analysis was also performed for when the gas in between the plates is replaced by argon, helium or nitrogen. This did not yield any results significantly different than those for air.

Predicting the gas conduction between the sphere and the substrate in atmospheric conditions or in partial vacuum is a challenge. When the sphere is close to the substrate the heat flow might not be in just one of the regimes mentioned. When the gap directly below the sphere is smaller than the mean free path of air the heat flow is in the rarefied regime there. Further away from this point the separation gap might be larger than the mean free path. This is illustrated in the left side of Figure 6.3. An overestimate of \( Q_{\text{sub}} \) is obtained if the sphere is modelled as a disk
with a separation equal to the minimum separation distance between the sphere and the substrate. This is displayed on the right side of Figure 6.3. This way the gas conductance can be compared to measured near field thermal radiation data for a sphere and a plate, this was done in Figure 6.4.

Not all of these experiments in Figure 6.4 were done with 100\,\mu m spheres but according to the PA these results the conductance is proportional with $R$ so the results were scaled to 100\,\mu m.
6.1.5 Heat Lost in Air

In most literature on AFM cantilevers the only heat flow through surrounding gas considered is the gas conduction between the cantilever and the substrate, either directly below the tip [89] or between the bottom of the cantilever and the substrate [90, 85]. In these examples the cantilever is parallel to the sample whereas in this project it is perpendicular to the sample.

Preferentially, the influence of $Q_{\text{air}}$ is determined experimentally. Obviously it is expected that $Q_{\text{air}}$ decreases with decreasing pressure. In an ideal vacuum it should be zero. Because the vacuum used in this project is not UHV the following experiment is suggested: Place a cantilever with a sphere on it in the chamber. Target the laser on it and pump the chamber down to the lowest possible pressure. When the rotation of the cantilever does not change any more during pumping this means that $Q_{\text{air}}$ has become negligible compared to $Q_{\text{cond}}$. Because vibrations of the pump might cause the cantilever to vibrate as well it should also be considered to measure the cantilever’s rotation when gas is slowly allowed to re-enter the evacuated chamber.
6.2. Thermal Expansion of the Cantilever

Because thermal expansion of the cantilever changes the separation gap between the sphere and the substrate and the capacitive sensor is not able to measure this change it is worthwhile to investigate if this effect is negligible or not.
When the laser is targeted on the end of the cantilever the temperature at the end will increase. This will cause the cantilever to expand. For a linear temperature gradient of $\Delta T = T(L) - T(0)$ over a beam the differential thermally induced elongation is:

$$d(\Delta L) = \alpha \frac{\Delta T}{L} x dx$$

Integrating both sides over the full length yields:

$$\Delta L = L\alpha \frac{\Delta T}{2}$$

When there is no substrate in close vicinity the sphere has the same temperature as the end of the cantilever because it cannot lose any heat to its environment. When the sphere approaches the substrate the near field thermal radiation will start to cool the sphere and consequently cool the end of the beam too. Using (5.6) again this temperature drop equals:

$$T_{\text{without } Q_{NF}}(L) - T_{\text{with } Q_{NF}}(L) = [T_{\text{without } Q_{NF}}(L) - T_0] - [T_{\text{with } Q_{NF}}(L) - T_0] = \frac{PL}{w(k_1t_1 + k_2t_2)} - \frac{(P - Q_{NF})L}{w(k_1t_1 + k_2t_2)} = \frac{Q_{NF}L}{w(k_1t_1 + k_2t_2)}$$

In an ideal vacuum $\Delta T$ was found to be 31.2 K. Assume the base of the cantilever, the surroundings and the substrate all have the same temperature. For a 100 µm sphere the largest near field conductance measured was $1.1 \times 10^{-8}$ W K$^{-1}$ in Figure 6.4. This way the upper limit of $Q_{NF}$ is determined to be 0.34 µW. This results in a temperature drop of 0.14 K at the free end of the cantilever. With (6.7) this results in a shrinkage of 79 pm if the cantilever is made of pure silicon nitride and 296 pm if the cantilever is made or pure gold.

### 6.3 Resonance Frequency Shift by Addition of the Sphere

Glueing a sphere on the end of the cantilever adds a large mass to it, changing its resonance frequency. When the cantilever is modelled like a simple mass-spring
system the resonance frequency change is:

\[
\omega_{ns} = \sqrt{\frac{C}{m_{cantilever}}} \\
\omega_s = \sqrt{\frac{C}{m_{cantilever} + m_{sphere}}}
\]

With the information from Table 5.3 and a glass density of 2600 kg m\(^{-3}\) the resonance frequency of the longest PNP-DB cantilever changes from 17 kHz to 4.06 kHz with a 20 \(\mu\)m sphere and to 1.66 kHz with a 100 \(\mu\)m sphere. For the longest of the MLCT cantilevers the resonance frequency changes from 7 kHz to 373 Hz with a 20 \(\mu\)m sphere and to 152 Hz with a 100 \(\mu\)m sphere. Measurements on the thermal noises of these cantilevers can confirm this.

### 6.4 Expected Cantilever Deflection by Near Field Thermal Radiation

In previous sections it was shown that the heat lost by near field thermal radiation is not enough to cool the cantilever down to a uniform temperature, \(Q_{NF} < Q_{cond}\). This means that \(Q_{NF}\) will decrease the bending caused by the laser beam but will not cause the cantilever to return to its normal, straight shape. The change in rotation at the free end of the beam can be calculated with the sensitivity to heat flows of the cantilever determined in the previous chapter. Together with the PA and the value for \(B\) found in the previous chapter for the heat flow between two SiO\(_2\) plates the expected cantilever rotation can be determined:

\[
\theta(L) = S_Q Q_{NF} \\
= S_Q 2\pi RB(T_{sphere} - T_{sub})/d
\]

The resulting relation between \(\theta(L)\) and the separation gap is plotted in Figure 6.5.

With the information available only the deflection caused by \(Q_{NF}\) in an ideal vacuum can be predicted because \(Q_{sub}\) cannot be accurately predicted. Remember
Chapter 6. Thermomechanical Modelling of the Experimental Setup

that the setup is not sensitive to the far field radiation from the sphere to the substrate. The far field radiation between a sphere and an infinite plate is not a function of the separation gap so it does not change during the experiment.

The resolution of the measurement depends on a variety of parameters: the length of the cantilever, the laser spot size, the working distance of the objective lens and the resolution of the DAQ system. The resolution is best determined experimentally.

6.5 Van der Waals Forces

The van der Waals free interaction energy between a sphere of radius \( R \) and a plate is [91]:

\[
E_{vdW}(d) = -\frac{AR}{6d}
\]  

(6.13)
The interaction force is obtained by differentiating with respect to the separation gap:

$$F_{vdW}(d) = \frac{\partial E_{vdW}}{\partial d} = \frac{AR}{6d^2}$$  \hspace{1cm} (6.14)

The Hamaker constant $A$, for silica with a vacuum gap in between is $6.5 \times 10^{-20}$ J [92]. For a 100 µm sphere the van der Waals force is 2.7 nN when the separation is 20 nm. More rigorous derivations based on optical properties are given in papers on Casimir forces [93, 94, 82]. When the cantilever is stretched by van der Waals forces this will modify the separation gap between the sphere and substrate. Is this effect negligible? This is important to know because the capacitive sensor does not measure the extension of the cantilever. The stiffness of the cantilever to elongation is that of two springs in parallel, one spring representing the gold layer and the other the silicon nitride layer:

$$C = \frac{w(E_1t_1 + E_2t_2)}{L}$$  \hspace{1cm} (6.15)

Evaluating this with the material properties in the previous chapter the resulting stiffness is $1.2 \times 10^4$ N m$^{-1}$. Together with the value of the force at a 20 nm gap the elongation is less than a picometer. When the separation gap is larger the elongation is even less.

When the sphere is attached to the cantilever like in the left of Figure 6.6 a force that pulls the sphere towards the substrate will not bend the cantilever. Because it might not be practically possible to glue the sphere on the cantilever like this the situation where the sphere is below the cantilever should also be considered. This way the lever arm is 50 µm for a 100 µm sphere. The rotation at the free end of a clamped beam due to a bending moment is:

$$\theta(L) = \frac{ML}{EI}$$  \hspace{1cm} (6.16)

The bending stiffness is of a bimaterial cantilever is [73]:

$$EI = \frac{w t_a^3 t_b E_a E_b}{12(t_a E_a + t_b E_b)} K$$  \hspace{1cm} (6.17)

The constant $K$ is defined in (5.8). There are two bending stiffnesses depending on which layer is on the compressed side and which is on the stretched side. However,
the influence of the gold layer is minimal, the bending stiffness is practically the same as it would be for the nitride layer only, \( EI = E_1 w l^3 = 10.0 \times 10^{-8} \text{ N m}^2 \). Figure 6.7 shows the rotation of the cantilever at the free end caused by van der Waals forces as a function of separation gap. The effect of van der Waals forces diminishes when the separation gap is larger than about 10 or 20 nm.

**Figure 6.6:** Two different ways in which the sphere can be attached to the cantilever.

**Figure 6.7:** Rotation of the cantilever at the free end caused by van der Waals forces as a function of separation gap between sphere and substrate.

## 6.6 Electrostatic Forces

The influence of electrostatic forces on the experiment might be more severe than that of van der Waals forces. Dielectric charging is a common problem in MEMS [95]. Tribocharging and the human body are prevailing sources of charging in
exposed dielectric materials [96]. If the electric potential difference between the sphere and the substrate is known the electrostatic force can be computed readily. Unfortunately the potential difference is not known beforehand. It might be significant even when care has been taken to avoid any charging by preventing contact with humans, preventing rubbing against other surfaces and using grounded conductive holders for the cantilever and substrate. Charging is more problematic in dry environments and vacuum is indeed a dry environment.

The electrostatic force between a charged sphere and a grounded plate as a function of the separation gap $d$ and the sphere potential $V$ is [97]:

$$F_e(d) = 2\pi \varepsilon_0 (V)^2 \sum_{q=1}^{\infty} \frac{\coth \psi - q \coth q\psi}{\sinh q\psi}$$

$$\psi = \cosh^{-1} \left( 1 + \frac{d}{R} \right)$$

Using a potential difference of 100 V and an arbitrarily large number for the summation the electrostatic force on the sphere is 0.69 mN when the gap is 20 nm. With the spring stiffness from the previous section the resulting elongation of the cantilever is 57 nm. The rotation of the cantilever by this force as a function of separation is plotted in Figure 6.8. This is much more significant than the influence of van der Waals forces.
Figure 6.8: Rotation of the cantilever at the free end caused by electrostatic forces as a function of separation gap between sphere and substrate. The potential difference between sphere and substrate is 100 V, one of them is grounded.
Chapter 7

Measurements

The measurements are divided into measurements that took place in air on the one hand and into measurements that took place in vacuum on the other hand. Because there was little time to do the experiments in vacuum most of the measurements that could also be performed in air were done in air. In air the main experiments were meant to characterize noise and drift, the response time of the system, electrostatic forces, the relationship between laser power and the cantilever rotation signal and how the cantilever rotation signal changes with separation gap. These measurements in air were also meant as practice runs for the measurements in vacuum. In the vacuum experiments the main goals were to determine whether or not heat losses from the cantilever to the surrounding air molecules are negligible or not, the relation between laser power and cantilever rotation signal and finally how the cantilever rotation signal changes with separation gap.

7.1 Glueing Microspheres on Cantilevers

The following procedure was successfully used to glue 20 and 100 μm spheres onto the largest PNP-DB and MLCT cantilevers. This is also illustrated in the figures in this sections.

- Put the cantilever in its holder and place the holder onto an XYZ stage.
Figure 7.1: These are the steps to place a few microspheres on the very edge of a microscope slide. Step 1: Dispense a few spheres on the slide. Step 2: Hold another microscope slide to the edge of the first one and tilt. Step 3: The spheres will roll towards the edge. Step 4: When the other microscope slide is removed some spheres will fall off the first one and some will remain on its very edge.

- Place a drop of UV adhesive on the edge of a microscope slide and place it near the XYZ stage.

- Move the cantilever into the drop until the cantilever and the drop visually deform, pull cantilever back out.

- Dispense some microspheres on a second microscope slide, hold a third microscope slide perpendicular to the side of the second one and tilt them both so the spheres end up on the very edge of the second microscope slide. This process is illustrated in Figure 7.1.

- Replace the microscope slide with the adhesive on it with the slide that has the microspheres on it.

- Using the XYZ stage the cantilever was moved into a position where it touches one of the spheres.

- The area was flooded with UV light to cure the adhesive for at least 30 seconds.

- Retraction of the cantilever reveals if the sphere is successfully adhered to the cantilever.
7.1. Glueing Microspheres on Cantilevers

Figure 7.2: Used setup for glueing the spheres on the cantilever. The cantilever chip is in its holder right side of the picture, the microscope slide is on the left.

Figure 7.3: Closeup of Figure 7.2, the white dots on the left are 100 μm spheres. One of the spheres is attached to the cantilver.
Figure 7.4: View from the optical microscope on a PNP-DB cantilever with a 20 µm sphere (left) and on one with a 100 µm sphere (right).

Figure 7.5: The glueing process for an MLCT cantilever and 100 µm spheres as seen through the optical microscope.
7.2 Experiments in Air

For the measurement setup the reader is referred to Figure 5.1. The experiments in air were done before the vacuum chamber was available, all equipment was placed in air. No mirrors were needed in the optical path, the objective lens was placed directly after the $\lambda/4$ plate. In these experiments the measured quantities are: the position signal of the laser beam reflected off the cantilever $x_{\text{pos}}$ (V) measured by the Maypa OPS, the intensity signal $x_{\text{sum}}$ (V) measured by the Maypa OPS, the distance measured by the capacitive sensor $d_{\text{cap}}$ ($\mu$m), the temperature of the substrate holder $T_{\text{sub}}$ (°C), the temperature of the cantilever holder $T_{\text{can}}$ (°C) and the temperature of air directly above the cantilever $T_{\text{air}}$ (°C).

During preliminary tests no difference was measured between $x_{\text{pos}}$ and $\sqrt{x_{\text{pos}}^2 + y_{\text{pos}}^2}$, if the measurement system is aligned properly it is not necessary to measure $y_{\text{pos}}$. Measuring the sum signal $x_{\text{sum}}$ during the experiments is very important, if a part of the laser is blocked by the substrate the reflection of the laser spot is also truncated on one side, leading to a wrong value of $x_{\text{pos}}$ since $x_{\text{pos}}$ is the difference of the intensities on the left quadrants and right quadrants of the laser light on the PSD. This can be seen in Figure 7.6. In all the experiments it was confirmed that the laser beam was never blocked by the substrate. When the measured $d_{\text{cap}}$ is corrected for the point of contact ($d = 0$) between the sphere and substrate, the result is the real separation gap $d$ between sphere and substrate.

![Figure 7.6: Three different intensity spots, the measured center position by the OPS is illustrated by a black cross.](image)

The goal here was not to measure the heat flows themselves. When this measurement setup is used for distance measurements its sensitivity is determined by the relation between $x_{\text{pos}}$ and the separation gap $d$ alone. Determining the heat flows to the substrate requires a calibration to find the relationship between its rotation
at the free end $\theta(L)$ and $x_{pos}$. To translate the $x_{pos}$ signal into a cantilever deflection or rotation a calibration experiment needs to be performed that determines the relation between $x_{pos}$ and translation of a laser beam targeted on the Maypa OPS. Together with the working distance of the microscope objective the rotation of the reflective surface can be determined (see Figure 5.7). Such an experiment was not done in this project. A calibration of the relation between $\theta(L)$ and the sensitivity to heat flows from and to the sphere $S_Q$ is also needed then. A method suggested by Shen et al. [98] for this requires a high vacuum chamber.

![Figure 7.7: Photograph of the measurement setup in air. The temperature sensor that measures the air temperature is held by the clamp. The laser is blocked by a piece of paper in front of the PSD.](image)

### 7.2.1 Noise and Drift

In this section several sources of noise and drift in the setup are deliberated.

The laser used in these experiments is red with a wavelength of 635 nm. The Maypa OPS is susceptible to this wavelength so it is also susceptible to light from
the surroundings even though this light has a much lower intensity. With the optical power meter in the position where the Maypa OPS should be it is possible to measure light levels in different lighting situations. The AFM lab where these experiments took place has uncovered windows so light from the adjacent rooms enters the AFM lab even when the lights in the AFM lab were off. Different situations and light levels are listed in Table 7.1. The lowest measured value of the light intensity seems to be the noise floor of the optical power meter. All experiments from here on were performed with a cardboard box over the setup or in the evening hours when all the lights were off in the building.

<table>
<thead>
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<th>measured power</th>
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<tbody>
<tr>
<td>lights on everywhere</td>
<td>61.19 nW</td>
</tr>
<tr>
<td>lights off in lab, on in adjacent rooms</td>
<td>0.1730 nW</td>
</tr>
<tr>
<td>lights on everywhere, cardboard box over setup</td>
<td>0.0585 nW</td>
</tr>
<tr>
<td>lights off everywhere</td>
<td>0.0450 nW</td>
</tr>
<tr>
<td>lights off everywhere, cardboard box over setup</td>
<td>0.0450 nW</td>
</tr>
<tr>
<td>power meter completely blocked</td>
<td>0.0450 nW</td>
</tr>
</tbody>
</table>

Table 7.1: Measured power meter readings during different lighting situations.

To investigate how the measurement setup performs over long periods of time it is important to quantify the drift of the different measurement signals, particularly for the cantilever rotation signal \( x_{\text{pos}} \). Temperature changes are a common source of drift in any experimental setup. It is also important to know how long it takes before any startup transients are gone. In Figure 7.8 the laser was turned on a few seconds after the measurement started and \( x_{\text{pos}} \) and \( x_{\text{sum}} \) are measured for an hour. The temperatures of the cantilever holder, the substrate and of the air directly above the cantilever were also measured, see Figure 7.9. The gap between the substrate and the sphere was later determined to be 10.4 µm. A cardboard box was placed over the setup to minimize any air flows. Right after the laser is turned on \( x_{\text{pos}} \) begins to drift with \( x_{\text{sum}} \) remaining constant. The temperatures do not change considerably during this one hour period.

After this experiment four consecutive one hour measurements on all the signals
Figure 7.8: A 1 hour startup measurement of \( x_{\text{pos}} \) and \( x_{\text{sum}} \). The laser is switched on at \( t = 9 \) seconds.

were performed (Figure 7.10), first two with the substrate near the sphere and later two with the substrate removed from the setup completely. The drift in \( x_{\text{pos}} \) was still considerable in all these measurements (Table 7.2). To find the source of the drift the correlation coefficients between each measured signal and \( x_{\text{pos}} \) was determined, see Table 7.3. There is a strong correlation between the distance measured by the capacitive sensor \( d_{\text{cap}} \) and \( x_{\text{pos}} \). However when these measurements were repeated with the substrate removed there is still drift in \( x_{\text{pos}} \) (drift 3 and drift 4 in Table 7.3) so the drift is not caused by movements of the substrate alone. The correlations between the drift and all the temperatures measured are very low. Either the drift is not caused by temperature variations or the temperature measurements are not accurate enough to measure this. The source of the drift in \( x_{\text{pos}} \) is not well understood.

The experimental setup was designed so that the influence of floor vibrations on \( d_{\text{gap}} \) is minimal. The optical table actively damps vibrations in vertical directions. During working hours the short term variations in \( d_{\text{gap}} \) as measured by the capacitive sensor were 6 nm peak to peak when measured over a period of 100 s. When the piezo actuator is on for prolonged periods \( d_{\text{gap}} \) might drift even though the actuation voltage is kept constant. Because the value of \( d_{\text{gap}} \) is accurately
### 7.2. Experiments in Air

**Figure 7.9:** Temperature developments during the start-up experiments

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$x_{pos}$ drift (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>drift 1 (with substrate)</td>
<td>0.0116</td>
</tr>
<tr>
<td>drift 2 (with substrate)</td>
<td>0.0123</td>
</tr>
<tr>
<td>drift 3 (without substrate)</td>
<td>0.0367</td>
</tr>
<tr>
<td>drift 4 (without substrate)</td>
<td>0.0148</td>
</tr>
</tbody>
</table>

**Table 7.2:** The measured drift in $x_{pos}$ over 1 hour periods, twice with the substrate close to the sphere and twice without the substrate.

measured by the capacitive sensor this is not a problem.

During the experiments a peculiar source of noise was discovered. At seemingly random moments a few intermittent periods of high pitch noise in the value of
Figure 7.10: A 1 hour drift measurement, without substrate.

<table>
<thead>
<tr>
<th>correlation</th>
<th>drift 1 (with substrate)</th>
<th>drift 2 (with substrate)</th>
<th>drift 3 (without substrate)</th>
<th>drift 4 (without substrate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{pos}$</td>
<td>d$_{cap}$: 0.8595</td>
<td>d$_{cap}$: 0.9582</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$x_{sum}$</td>
<td>-0.3275</td>
<td>-0.4080</td>
<td>-0.9023</td>
<td>-0.5607</td>
</tr>
<tr>
<td>$t_{air}$</td>
<td>-0.3428</td>
<td>-0.2615</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$t_{can}$</td>
<td>-0.0519</td>
<td>-0.1019</td>
<td>-0.2741</td>
<td>-0.0563</td>
</tr>
<tr>
<td>$t_{sub}$</td>
<td>-0.4376</td>
<td>0.1817</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 7.3: Correlations between $x_{pos}$ and the other measured signals during the drift measurements.

$x_{pos}$ and $d_{cap}$ were noticed. The existence of this noise was also confirmed by an oscilloscope, hence it was not something generated in the DAQ system. When listening carefully high pitch sounds coming from the ceiling were identified as the source of this interference. This was also confirmed while listening and watching the noisy signals live on the oscilloscope. This explains why the noise was only seen in $x_{pos}$ and $d_{cap}$ signals, two measurements of motion, and not in the sum or temperature. This noise was expected to be from the airconditioning or lighting in the ceiling, in the evening hours when the lights were off no such acoustic
interference was experienced.

When ordering cantilevers there was concern that the addition of the microspheres to the cantilever lowers the resonance frequency into the region where 1/f noise becomes apparent. In all measurements made with this setup 1/f noise crosses the white noise at a frequency of $\sim 200 \text{ Hz}$, lower than any measured cantilever resonance frequency in this project.

### 7.2.2 Thermal Noise

Thermal movements inside the cantilever cause it to vibrate in its resonance frequency [99]. This becomes visible when measuring the rotation of a free standing cantilever over a prolonged period of time. A Fourier transform of the measured rotation reveals a sharp peak on the resonance frequency of the cantilever. The best way to measure thermal noise is to measure $x_{\text{pos}}$ at high sampling rates and average the power spectral density over multiple measurements. The resonance frequency in Hz, $f_{\text{res}}$, is found by fitting the noise spectrum of the $x_{\text{pos}}$ signal to that of a damped oscillator [100]:

$$S(f) = A_{\text{white}}^2 + \frac{A^2 f_{\text{res}}^2}{4Q^2} \left[ (f - f_{\text{res}})^2 + \frac{f_R^2}{4Q^2} \right]^{-1} \tag{7.1}$$

In this equation $Q$ is the Q-factor, $A_{\text{white}}$ is the background white noise amplitude and $A$ is the amplitude noise at the resonance. This method is illustrated in Figure 7.11. The resonance frequencies of all the cantilevers investigated in this project are listed in Table 7.4. These kinds of thermal noise measurements were done before each series of experiments as a check to see if all equipment is working properly and to confirm the presence of the cantilever and the sphere on it.

The resonance frequencies of the cantilevers with a sphere on them are higher than expected. Perhaps it is the glue on the cantilever that stiffens that part of the cantilever when it cures. Or it could be that the formula for the added mass calculation (6.9) is not accurate. The differences between similar cantilevers with a sphere on it could be explained by the fact that not all spheres are the same size. According to the information provided with the spheres their diameter was
Chapter 7. Measurements

<table>
<thead>
<tr>
<th>cantilever</th>
<th>$f_{\text{res}}$ according to manufacturer</th>
<th>measured $f_{\text{res}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>short PNP-DB</td>
<td>67 kHz</td>
<td>66.7 kHz</td>
</tr>
<tr>
<td>long PNP-DB</td>
<td>17 kHz</td>
<td>11.3 kHz</td>
</tr>
<tr>
<td>longest MLCT</td>
<td>7 kHz</td>
<td>7.08 kHz</td>
</tr>
<tr>
<td>long PNP-DB + 100 $\mu$m sphere</td>
<td></td>
<td>691 Hz, 721 Hz, 865 Hz</td>
</tr>
<tr>
<td>longest MLCT + 100 $\mu$m sphere</td>
<td></td>
<td>495 Hz, 514 Hz, 563 Hz</td>
</tr>
</tbody>
</table>

**Table 7.4:** All the resonance frequencies measured during this project. The values in the last two rows are each for a different cantilever with a 100 $\mu$m sphere on it.

97.6 $\mu$m ± 4.9 $\mu$m. The exact position of the spheres where the sphere is glued to the cantilever is also not the same for each cantilever. Also the amount of adhered glue on each cantilever be different.

![Figure 7.11: Thermal noise of the longest MLCT cantilever with a 100 $\mu$m sphere.](image)

**7.2.3 Step Response**

By turning the laser on and off while measuring $x_{\text{sum}}$ and $x_{\text{pos}}$ on the OPS the response time of the system can be determined. Turning the laser on does not result in a perfect step in the intensity signal. Still, from a measurement like this
it can be concurred that it takes about 0.1 s before the deflection signal $x_{pos}$ is fully adjusted to the step in $x_{sum}$ (Figure 7.13). Because $x_{pos}$ is not accurate when $x_{sum}$ is less than about 0.3 V it is better to look at the response of the system when the laser is turned on and not when it is turned off because then the final values of $x_{pos}$ are below 0.3 V. The rise time of the Maypa OPS is documented to be 8.75 ns, much less than the total response time of this measurement system. This experiment was done with a large PNP-DB cantilever and a 100 µm sphere. These measurements and subsequent ones were made at least one hour after the laser was turned on and pointed on the cantilever at full power. All tests were performed in the afternoon or in the evenings when no other people were in the lab.

![Figure 7.12: Turning the laser on and off during a period of 100 seconds.](image)

### 7.2.4 Cantilever Rotation versus Laser Power in Air

The goal of this series of measurements was to measure the effect of electrostatic forces on the $x_{pos}$ signal. In (5.6) a linear relation between the laser power and the cantilever rotation in vacuum was predicted. When the heat losses to the
Figure 7.13: Zooming in on one of the steps in Figure 7.12, the deflection and intensity signals are now normalized to their maximum values.

surrounding air are also linear with the absorbed laser power this should also be true for air. This would imply that the heat losses to air can be written as

$$Q_{\text{air}} = \int_0^L h_{\text{air}} (T(x) - T_{\text{air}}) \, dx$$

with the linear heat transfer coefficient $h_{\text{air}}$.

The rotation caused by electrostatic forces is not a function of laser power. The total rotation of the cantilever is the sum of thermally induced rotation and rotation by electrostatic forces. Unfortunately some laser power is needed to perform a measurement on the rotation of the cantilever. The following experiment is meant to find the zero-laser-power $x_{\text{pos}}$ at different separation gaps. When the value of the zero-laser-power $x_{\text{pos}}$ changes when the sphere approaches the substrate this is caused by electrostatic forces and not by any thermal effects. When the laser is targeted on the cantilever and the laser power is gradually decreased the measured $x_{\text{pos}}$ will decrease as well. When the intensity signal of the Maypa OPS $x_{\text{sum}}$ becomes too low (less than about 0.3 V) the $x_{\text{pos}}$ data is no longer reliable, it needs a certain intensity before it works properly. With this part excluded a linear fit to the data reveals $x_{\text{pos}}$ when the laser power is zero. This can be seen in Figure 7.14. Because the laser power is lowered slowly over a period of 100 seconds transient effects are negligible. The straight line confirms the linearity between heat input...
at the far end of the cantilever and the rotation at that end.

Figure 7.14: The position signal as a function of the intensity signal when the gap is 5.16 µm. The zero-power $x_{\text{pos}}$ is where the red line crosses the vertical axis.

When this experiment is repeated with the substrate at various distances from the sphere the zero-power $x_{\text{pos}}$ as a function of the separation gap can be plotted like in Figure 7.15. Unfortunately performing these experiments takes a long time and the change in the zero-power $x_{\text{pos}}$ as seen in Figure 7.15 is of the same order of magnitude as the drift of $x_{\text{pos}}$ measured in Section 7.2.1 during a similar timeframe, about 0.01 V or 0.02 V over a one hour period. This experiment was performed with a large PNP-DB cantilever and a 100 µm sphere. It was chosen to perform these experiments only in air because of time constraints.

Another way to find out if electrostatic forces play a role in this setup is to measure the thermal noise of the cantilever once without the substrate near the cantilever and once with the substrate very close to the sphere. Any electrostatic forces on the sphere should change the resonance frequency. With a 100 µm sphere attached to the largest of the MLCT cantilevers the thermal noise peak was at 431.2 Hz when the substrate was removed. After the substrate closed in on the sphere
and the separation between them was $\sim 3 \, \mu m$ the thermal noise peak was still at 431.2 Hz. With the combined results of both experiments described in this section it was deemed unlikely that electrostatic forces play a considerable role in the experimental setup.

### 7.2.5 Cantilever Rotation versus Distance in Air

The goal of the experiments in this section was to determine the relation between $x_{\text{pos}}$ and the separation gap $d$ between the sphere and the substrate in air. During these experiments the substrate and the cantilever holder were both grounded to minimize electrostatic potential differences, see Figure 7.7. As a substrate the bare aluminium holder was used, it is expected that the gas conduction only depends on the properties of air and not on the used substrate.

During these experiments it was discovered that the drift in the $x_{\text{pos}}$ signal was...
considerable giving very inconsistent results for measurements taking long time averages. The experiment had to be performed in a short time, but not so short that transient effects become visible. With the total response time determined to be 0.1 second earlier and after a few test runs it was decided that the substrate had to be moved slowly towards the sphere over a period of 40 seconds while all signals were measured continuously.

At first it was tried to bring the sphere in contact with the substrate before the start of the experiment and move it back during the experiment. While doing this the sphere broke off from the cantilever and got stuck to the substrate. At least this confirmed that there are some attractive forces when the sphere is in contact with the substrate. With this in mind the sphere was brought very close to the substrate and was moved towards the substrate during the experiment. The following steps were performed for this experiment:

- The cantilever was brought into the focus of the laser spot.
- The piezo was extended to 10 µm, half of its full stroke.
- Using a flashlight to provide backlighting like in Figure 5.8 the substrate was moved manually towards the sphere to within a distance of \(\sim 5\) µm. This means that when the piezo is not extended it is \(\sim 15\) µm away from the sphere. When the piezo is fully extended the gap is -5 µm, so the point of contact is definitely within the stroke of the piezo actuator.
- The capacitive sensor was pressed against the substrate, this way it is certainly parallel to its surface. The screws on the capacitive sensor holder were tightened to secure the capacitive sensor and the motion stage was used to pull it back 10 µm. This way it is also certain that the point of contact is within the capacitive sensor’s measurement range.
- The positioning stage of the PSD holder was used to center the laser spot on the PSD.
- The extension of the piezo actuator was set back to zero.
- The measurement was started and the piezo was slowly extend to its full
range.

The raw data of one of the measurements where the sphere was brought into contact with the substrate for the first time is depicted in Figure 7.16.

![Graph](image)

**Figure 7.16**: Raw data of the \(x_{\text{pos}}\) signal for one of the experiments where the sphere was brought into contact with the surface (top). The sphere touches the surface at 25.3 seconds. Raw data of the capacitor from the same experiment (below).

Combining the graph of \(x_{\text{pos}}\) versus time and the graph of distance measured by the capacitor \(d_{\text{cap}}\) versus time together with the point of contact at \(d_{\text{cap}} = 4.016\) µm a plot of \(x_{\text{pos}}\) versus the real separation gap \(d\) is made for this experiment. This one and two similar plots are shown in Figure 7.17. For each of these three
7.2. Experiments in Air

measurements it was the first time that the sphere used in that experiment was brought into contact with the substrate.

There is a general trend here, the rotation of the cantilever changes when the sphere closes in on the substrate and this effect becomes stronger when the gap becomes smaller. The curve is not the same for each experiment. The curves were fitted to a power law function, a polynomial function and to a logarithmic function. The best fit for the first curve was the power law: $x_{\text{pos}} = 0.0087d^{0.2931} - 0.0615$. The second curve was fitted to the power law: $x_{\text{pos}} = 0.0043d^{0.6375} - 0.3674$. The best fit for the third curve was the polynomial function: $x_{\text{pos}} = 2.164 \times 10^{-5} \cdot d^3 - 6.4679 \times 10^{-4} \cdot d^2 + 0.0065d - 0.3914$. It is not clear why these relations are different, the difference implies that the experiment is not repeatable on different days. The resolution of the measured $x_{\text{pos}}$ voltage is 0.4 mV. The signal to noise ratio appears to be low in these experiments. Because of the continuous measurements and the many data points as a result of this the entire uncertainty area around the fitted line is flooded with data points making the graphs look very noisy.

In Figure 7.17 a series of measurements on the same day is shown. For all these experiments the same cantilever plus sphere was used. For the first few approaches the sphere was not brought into contact with the substrate. Then the piezo was extended to its full range to capture the moment where contact occurs. Then this full range experiment was repeated a few more times. After the sphere was brought into contact with the substrate for the first time the curve is not as steep any more, permanent damage of the cantilever might prevent the cantilever’s ability to change its rotation when the heat flow to the substrate increases. The contact point for these experiments was determined for each curve individually. The effect of the long term drift is obvious from the vertical translation of the curves with respect to each other. Except for the long term drift the experiments made on one day are reproducible, the slope of the curves remains consistent. Why the experiment is not reproducible on different days is not clear, this could be caused by changes in the measurement setup or changes in the pressure, temperature or humidity of the air in the AFM lab.
Figure 7.17: Three measurements on three different days where the sphere was brought into contact with the substrate for the first time that day. For each of these measurements a new cantilever plus sphere was used.
Figure 7.18: Comparison of results for a few approaches before the sphere was first brought into contact with the sphere (blue), for when the sphere touched the substrate for the first time (red) and for a few approaches after that (green). All data was fitted to power law functions successfully.
7.3 Vacuum Experiments

The bottom plate of the vacuum chamber had to be designed and ordered. It was the last part of the experimental setup and came in a few days before the end of this project. Because the temperature sensors in the air experiments yielded no useful information it was decided not to use them in the vacuum experiments, simplifying the system as much as possible in the remaining time available.

![Experimental Setup in Vacuum Chamber](image)

**Figure 7.19:** The experimental setup placed in the vacuum chamber. The capacitive sensor is removed. The two cables provide the actuation voltage of the piezo and the other carries the feedback signal from the strain gauge.

To prevent outgassing as much as possible all the sealing rings and KF adapters were ultrasonically cleaned. The other components that went into the vacuum chamber like the microscope objective and the piezo stage were cleaned with isopropanol. The base plate of the vacuum chamber was connected to the grounded
optical table, the substrate holder and the cantilever holder were both connected
to the base plate.

Aligning the laser path was a time consuming task. This had to be done step
by step using the alignment tool in Figure 7.20. First the stage of the collimator
needed to be rotated such that the laser beam is parallel to the rods of the 30
mm cage system. To make sure the 45 degree mirrors are indeed set to 45 degrees
a flat mirror was laid on the 45 degree mirror like in the middle of Figure 7.20.
The rotation of the 45 degree mirror was adjusted until the returning laser beam
was parallel to the laser beam from the collimator. The bottom plate of the
vacuum chamber needs to be positioned such that the laser beam passes through
the center of the optical viewport. Finally with the cantilever in place the laser
beam reflects off the cantilever and back through the optical viewport to the PSD.
The alignment tool was also used to test the angle of the 45 degree mirror inside
the vacuum chamber. The beam reflecting off the cantilever was also tested for
parallelism.

Figure 7.20: Laser path with the optical viewport (left). In this picture the optical
power meter is in the place where the PSD normally is. A mirror was laid on top
of the 45 degree mirror (middle). An alignment tool that allows accurate alignment
of the laser beam within the Thorlabs 30 mm cage system (right).

7.3.1 Vacuum Pumping

The pressure sensor is able to measure pressures from $10^{-5}$ mbar up to atmospheric
pressure. When the pump started it took a while before the pressure dropped below
the atmospheric pressure according to the pressure sensor. It was deemed unlikely that this was simply a 'startup' problem or a delay of the sensor. Several checks confirmed the connection from the analog output of the pressure sensor to the DAQ device was working properly. After that the settings of the sensor were checked using the serial port and it was discovered that the sensor was set to argon as the working gas instead of air. After that was corrected another problem that arose was that the capacitive sensor was not working in vacuum, this was explained in Section 5.3.9. The feedback from the strain gauge on the piezo actuator was used to measure the distance between the sphere and the substrate instead.

A simple way to test the lowest pressure the pump can achieve is to put the pressure sensor directly on the pump. However, this is not recommended because oil from the pump might contaminate the pressure sensor. It was chosen to connect the pressure sensor with the stainless steel hose to the pump. Turning the pump on revealed that the hose was leaking, it had to be replaced. With the hose replaced the pump was able to lower the pressure below 0.01 mbar in 15 minutes, see Figure 7.21. With an empty chamber connected with the hose to the pump was able to reach a pressure of 0.03 mbar in 15 minutes. The bell jar was placed directly on the bottom plate, no vacuum grease was used. With the whole measurement setup placed in the chamber pumping times became significantly longer. Pumping down to 0.1 mbar took about 1.5 hour, after that the pressure did not seem to change that much so it was chosen to use this as the working pressure. There are several reasons why pumping down became more complicated. The measurement setup consists of a lot of non-vacuum compatible components, outgassing can occur and air trapped inside the electric cables will escape slowly.

### 7.3.2 Cantilever Rotation versus Pressure in Vacuum

This experiment was meant to find the relationship between pressure and the cantilever rotation signal $x_{\text{pos}}$ of a free standing cantilever (no substrate nearby). At very low pressures $Q_{\text{air}}$ in Figure 6.1 should become negligible compared to the other heat losses. This occurs when the the $x_{\text{pos}}$ signal no longer changes while the pressure is lowered continuously. The value of $x_{\text{pos}}$ was monitored during pumping
7.3. Vacuum Experiments

![Graph showing pressure over time.](image)

**Figure 7.21:** *Pumping down the hose in 15 minutes, the pump is turned on 9 seconds after the measurement starts.*

and during the refilling of the chamber. Unfortunately during this experiment the $x_{sum}$ signal dropped considerably when the pressure was lowered, from 0.8 V at 1 bar to 0.58 V at 0.135 mbar. When the valve was opened and the chamber refilled with air the sum signal returned to 0.8 V. This is likely to be caused by the deformation of the chamber, causing misalignment of the laser spot. Because the intensity of the laser spot on the cantilever changes during this measurement the change in $x_{pos}$ is not only caused by the pressure drop and the results attained are not useful, the relationship between $x_{pos}$ and the pressure cannot be deduced from them.

### 7.3.3 Cantilever Rotation versus Laser Power in Vacuum

It is interesting to see if the relationship between laser intensity and cantilever rotation is linear. This was predicted for vacuum and was experimentally proven in air. Just like in the experiment in air the laser power was slowly lowered from its maximum to zero over a 100 second period. For this experiment it had to be decided whether the pump had to stay on or off during this measurement. When the valve is closed and the pump is turned off when a pressure of 0.110 mbar is
reached the pressure rises to 0.143 mbar in 100 seconds. When the pump stays on when a pressure of 0.111 mbar is reached the pressure drops to 0.107 mbar in a 100 second time period, this change in pressure is much less compared to when the pump is turned off. To determine how large the influence of the pumps vibrations on the $x_{pos}$ signal is the laser was targeted on the cantilever and $x_{pos}$ was measured during a 100 second time period with a high sampling frequency (300 kHz). This was also done with the pump turned off. Both resulted in a peak to peak noise of 0.004 V, there was no measurable coupling between pump vibrations and $x_{pos}$. It was chosen to have the pump on during the experiment. The result is displayed in Figure 7.22. Remember that $x_{pos}$ is not reliable when $x_{sum}$ approaches zero. Like in the experiments in air the data points where $x_{sum} < 0.3$ V were not included in the linear fitting. The expected linear relation indeed exists. During these experiments there was no substrate in the chamber.

![Graph](image-url)  

**Figure 7.22:** The cantilever rotation signal $x_{pos}$ plotted against the sum signal $x_{sum}$ when the pressure in the surrounding air is 0.1 mbar.
7.3.4 Cantilever Rotation versus Distance in Vacuum

The steps performed to find the relation between $x_{\text{pos}}$ and the separation gap in vacuum are the same as in Section 7.2.5 only without the capacitive sensor. The pump was on throughout this experiment. The pressure dropped from 0.084 mbar at the start to 0.077 mbar at the end of the experiment. The result is shown in Figure 7.23. The $x_{\text{pos}}$ signal is given as a function of the elongation of the piezo measured by the strain gauge on it, $d_{\text{gauge}}$. When $d_{\text{gauge}}$ is zero the separation gap is the largest, when the piezo expands it decreases.

![Figure 7.23: Measure of $x_{\text{pos}}$ versus the strain gauge elongation $d_{\text{gauge}}$.](image)

What really happens near the point of contact remains unclear. There is no sharp signal change like in Figure 7.16, so it’s not certain where the sphere touches the substrate. Because the point of contact is not determined the real separation gap $d$ is unknown. It’s not clear if any part of the change in $x_{\text{pos}}$ is caused by near field thermal radiation or by mechanical contact. On the other hand it is clear what happens at the beginning and at the end of the curve. there is no change in signal
between 0 and 4.8 µm because there is no measurable gas conduction in this area. When the elongation of the piezo is larger than 7.5 µm the same steps in $x_{pos}$ are visible that can also be seen in some of the $x_{pos}$ versus distance curves in air, it is believed that these steps are caused by stick-slip motion of the sphere along the substrate.

After this final experiment the majority of the vacuum equipment had to be returned to their respective owners. For more definite answers on what happens in this experiment it has to be repeated more often.

7.4 Conclusion

The experiments in air proof that the setup works as predicted, it is an instrument sensitive to small changes in heat transfer. The gas conduction in air caused the cantilevers bending to change gradually over a gap of multiple micrometers whereas near field thermal radiation only becomes relevant below one micrometer. It was expected that the near field heat radiation causes the cantilever to bend sharply close to the substrate. It’s not clear whether this was observed or not because the point of contact in the vacuum experiments could not be determined. The experiment in vacuum where the relation between $x_{pos}$ and the separation gap was investigated needs to be repeated because the results are inconclusive so far.

The process of bringing the sphere into contact with the substrate damages the cantilever, for each time the relation between $x_{pos}$ and the separation gap is determined a new sphere had to be glued on the cantilever, a time consuming process.

A large drift in the $x_{pos}$ signal hindered long-lasting measurements, they had to be performed in short time periods. In advance the cantilever was expected to be very sensitive to temperature changes in the surrounding. The drift might be caused by small temperature changes. These were not recorded by the thermocouples, probably because they are not sensitive to small temperature changes or they were located too far away from the cantilever.

It was feared that the measurements were to be disturbed by electrostatic forces,
none were measured. The step response experiment revealed that the measurement setup needed 0.1 second to fully adapt to a change in laser intensity. It was confirmed that there is a linear relationship between laser power and $x_{pos}$ both in air and in vacuum.

The experiment wherein the influence of the heat losses to the surrounding air on the $x_{pos}$ signal were supposed to be determined by measuring $x_{pos}$ while lowering or raising the pressure was inconclusive.
Chapter 8

Closure

The results of this research project are summarized in this chapter and ideas for improvements and future directions are suggested.

8.1 Conclusions

The enhancement of radiative heat flow between objects with a separation between them smaller than the dominant wavelength of thermal radiation is caused by evanescent waves, surface phonon polaritons and surface plasmon polaritons. Other research groups have experimentally proven the existence of near field thermal radiation and its ability to surpass the black body radiation limit.

The slope of the near field enhancement as a function of separation gap depends on the shape of the two objects involved, if one of them is an infinite half space than this slope is predicted to be $d^{-3}$ if the other object is a dipole, it is $d^{-2}$ if the other object is also an infinite half space and $d^{-1}$ if the other object is a sphere much larger than the wavelength of thermal radiation.

The original premise of using thermal radiation as a sensing principle was that it applies to all materials because all materials emit thermal radiation. However the amount of heat transfer by near field thermal radiation strongly depends on optical
material properties, materials with surface polaritons in the thermal infrared are capable of transferring much more heat than materials without them. The heat transfer between a dielectric and a metal is much less than the black body limit in the nanometer and micrometer range. It is not possible to choose a sensor material that works equally well for all types of substrate materials.

A MATLAB script was written that calculates the near field thermal radiation between two infinite plates from the principles of fluctuational electrodynamics. It takes the material and temperature of the top plate, the material and temperature of the bottom plate and the separation gap between the plates as inputs. The script is supplied with 11 materials commonly used in the semiconductor industry. The script is also able to process magnetic materials. The proximity approximation can transform the results from this script into the heat radiation between curved objects.

To study the possibility to use near field thermal radiation as a sensing principle to measure the distance between two objects an experimental setup was built involving a 100 µm sphere on a silicon nitride cantilever, a substrate that was moved towards the sphere by a piezo actuator and a laser readout system that measures the rotation of the cantilever. This option was chosen because the tilt of the sphere does not have to be controlled and the used AFM cantilevers are very sensitive thermal sensors.

Of all studied types of cantilevers long silicon nitride cantilevers with a layer of gold on them will deflect the most when subjected to a certain temperature gradient. They have the highest sensitivity when used as thermal sensors.

A procedure was determined to successfully glue 20 µm and 100 µm spheres on the end of an AFM cantilever. The experiments in air revealed that the setup was indeed sensitive to the presence of the substrate, gas conduction between the sphere and substrate will cool down the sphere and the cantilever, lowering its thermally induced deflection.

It was predicted that the cantilever is very sensitive to temperature changes in the environment. It is believed that the long term drift in the cantilever rotation
signal is caused by its sensitivity to temperature changes. Because of the drift in the cantilever rotation signal the measurements were not reproducible and they have to be performed in short time periods, generally shorter than 100 s.

The rotation of the cantilever changes linearly with laser power both in air and in vacuum. The experiment that was supposed to find the relation between cantilever rotation signal and pressure failed because the intensity signal changes when the pressure in the chamber changes, it is believed that this is caused by deformations of the chamber induced by the pressure difference between the inside and outside of the chamber. When the pressure in the chamber was restored to atmospheric again the intensity signal returned back its regular value. No influence of electrostatic forces on the cantilever rotation signal was measured. The step response experiment revealed a total response time of the system of 0.1 s. The resolution of the cantilever rotation signal was 0.4 mV.

The results of the experiment to determine the relationship between the cantilever rotation signal and the separation gap in vacuum were unclear. No clear point of contact could be determined from the measured data and it is not clear if the changes in cantilever rotation signal are caused by near field thermal radiation or by mechanical contact between the sphere and the substrate. As a result of this it was not possible to determine a sensitivity or accuracy of the measurement principle wherein the cantilever rotation signal is a measure for the distance between the sphere and substrate.

8.2 Future Prospects

There are a few improvements possible in the measurement setup. The first one is to measure the temperature inside the vacuum chamber. The pump notably warms up during pumping, this heat can be conducted to the equipment inside the chamber. To minimize drift caused by temperature changes a temperature controlled chamber is required. To be sure that there is no heat conduction between the sphere and the substrate through air molecules the setup should be placed in a UHV environment. To increase the near field heat transfer between a glass sphere
and a glass substrate both should be made of pure SiO$_2$ and not of soda-lime glass or borosilicate glass. Increasing the laser power by using another higher power laser also increases the amount of near field heat flow to the substrate. Increasing the temperature of either the cantilever holder or the substrate holder by actively heating it also increases the near field thermal radiation between the sphere and substrate.

Because the cantilevers with a sphere on them seemed damaged after they were first brought into contact with the substrate this experiment needs a more gentle approach, where the piezo stops expanding when contact is made, this way the measurement can be repeated without having to replace the sphere and cantilever. Besides the sharp peak in the cantilever rotation signal the thermal noise peak should also shift to a higher frequency when contact is made, it changes the beam into a doubly clamped beam. This can also be used as an indication that contact is made.

During the step response experiment the step in laser intensity was not a perfect step function, making it hard to define a time constant of the system response. Exciting the laser sinusoidally results in the frequency response of the measurement system, this could reveal more information about the system response.

In future experiments the sphere and the substrate (be it a piece of glass or SiO$_2$ on a wafer) need their surface roughness to be sampled. what exactly is an acceptable roughness is not clear, this needs to be studied. the near field enhancement starts when the distance between objects is below 1 $\mu$m so if the roughness of one of the objects is comparable to this it is indeed too rough.

There are some aspects of near field thermal radiation that still need to be investigated experimentally. How large is the near field enhancement for materials with rough surfaces? Can the near field enhancement between two different materials be measured? So far it has only been measured between two similar materials.

In terms of technical readiness level the sphere-on-a-cantilever sensor is merely a proof of concept, it is not the final design of the sensor. But because these kind of cantilevers are very sensitive temperature sensors (their resolution is up to $10^4$ as
small as a regular thermocouple sensor) it will be a challenge to make a sensitive thermal sensor without an AFM cantilever.
### Appendix A: Optical Material Parameters

<table>
<thead>
<tr>
<th>material</th>
<th>parameters</th>
</tr>
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</table>
| Al [42]                         | $\omega_p = 22.415 \times 10^{15}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 12.532 \times 10^{13}\,\text{s}^{-1}$ |
| Au [42]                         | $\omega_p = 13.713 \times 10^{15}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 4.050 \times 10^{13}\,\text{s}^{-1}$ |
| cBN [40]                        | $\varepsilon_\infty = 4.46$          
  $\omega_{LO} = 1.985 \times 10^{14}\,\text{rad}\,\text{s}^{-1}$  
  $\omega_{TO} = 2.451 \times 10^{14}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 9.934 \times 10^{11}\,\text{s}^{-1}$ |
| Cu [42]                         | $\omega_p = 11.227 \times 10^{15}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 1.379 \times 10^{13}\,\text{s}^{-1}$ |
| doped Si (n-type $10^{20}\,\text{cm}^{-3}$) [45] | $\varepsilon_\infty = 11.7$        
  $\omega_p = 4.05 \times 10^{14}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 1.013 \times 10^{14}\,\text{s}^{-1}$ |
| GaAs [7]                        | $\varepsilon_\infty = 11$            
  $\omega_{LO} = 5.502 \times 10^{13}\,\text{rad}\,\text{s}^{-1}$  
  $\omega_{TO} = 5.061 \times 10^{13}\,\text{rad}\,\text{s}^{-1}$  
  $\gamma = 4.521 \times 10^{11}\,\text{s}^{-1}$ |
<table>
<thead>
<tr>
<th>Material</th>
<th>Optical Parameters</th>
</tr>
</thead>
</table>
| metallic wire/split ring resonator [55] | $\omega_p = 1 \times 10^{14} \text{ rad s}^{-1}$  
|                 | $\gamma_e = 1 \times 10^{12} \text{ s}^{-1}$  
|                 | $F = 0.5$  
|                 | $\omega_m = 4 \times 10^{13} \text{ rad s}^{-1}$  
|                 | $\gamma_m = 1 \times 10^{12} \text{ s}^{-1}$  |
| Pt [42]         | $\omega_p = 7.817 \times 10^{15} \text{ rad s}^{-1}$  
|                 | $\gamma = 10.511 \times 10^{13} \text{ s}^{-1}$  |
| SiC [40]        | $\varepsilon_\infty = 6.7$  
|                 | $\omega_{LO} = 1.925 \times 10^{14} \text{ rad s}^{-1}$  
|                 | $\omega_{TO} = 1.494 \times 10^{14} \text{ rad s}^{-1}$  
|                 | $\gamma = 8.966 \times 10^{11} \text{ s}^{-1}$  |
| SiO$_2$ [8]     | $\varepsilon_\infty = 2.001$  
|                 | $\kappa_1 = 4.477 \times 10^{27} \text{ s}^{-2}$  
|                 | $\kappa_2 = 2.358 \times 10^{28} \text{ s}^{-2}$  
|                 | $\omega_{r,1} = 8.673 \times 10^{13} \text{ rad s}^{-1}$  
|                 | $\omega_{r,2} = 2.022 \times 10^{14} \text{ rad s}^{-1}$  
|                 | $\gamma_1 = 3.303 \times 10^{12} \text{ s}^{-1}$  
|                 | $\gamma_2 = 8.398 \times 10^{12} \text{ s}^{-1}$  |
| Ti [42]         | $\omega_p = 3.824 \times 10^{15} \text{ rad s}^{-1}$  
|                 | $\gamma = 7.196 \times 10^{13} \text{ s}^{-1}$  |

**Table A1:** Optical parameters of various materials.
Appendix B: MATLAB code

% This matlab code calculates the radiative heat flow between two materials % as a function of separation gap

% Temperatures
T1 = 295;
T2 = 275;

%-------------------------------------------------------------------
% Physical constants
hbar = 6.62606957e-34/2/pi;
c = 299792458;
kB = 1.381e-23;
sigma = 5.67e-8;

%-------------------------------------------------------------------
% Optical constants in the dielectric functions

% SiC
eps_inf_SiC = 6.7;
w_LO_SiC = 1.825e14;
w_TO_SiC = 1.494e14;
Gamma_SiC = 8.966e11;

% GaAs
eps_inf_GaAs = 11;
w_LO_GaAs = 5.5021e+13;
w_TO_GaAs = 5.0614e+13;
Gamma_GaAs = 4.5208e+11;
% SiO2
eps_inf_SiO2 = 2.0014;
sigma_1 = 4.4767e27;
sigma_2 = 2.3584e28;
omega_01 = 8.6732e13;
omega_02 = 2.20219e14;
gamma_1 = 3.3026e12;
gamma_2 = 8.3983e12;

% cBN
eps_inf_cBN = 4.46;
w_LO_cBN = 1.985e14;
w_TO_cBN = 2.451e14;
Gamma_cBN = 9.934e11;

% Au
omega_p_Au = 13.713e15;
gamma_Au = 4.0499e13;

% Al
omega_p_Al = 22.415e15;
gamma_Al = 12.432e13;

% Cu
omega_p_Cu = 11.227e15;
gamma_Cu = 1.3788e13;

% Pt
omega_p_Pt = 7.8172e15;
gamma_Pt = 10.511e13;

% Ti
omega_p_Ti = 3.8238e15;
gamma_Ti = 7.1955e13;

% doped Si
epsinf=11.7;
wp=4.05e14;
te=9.87e-15;
% metamaterial

omega_p_met = 1e14;
gamma_e_met = 1e12;
F_met = 0.5;
omega_0_met = 4e13;
gamma_m_met = 1e12;

%--------------------------------------------------------------------------------------------
% Boundaries and steps in distance, frequency and surface wavevector

% distances
dis_min = -9;   % minimum distance is 10^-9
dis_max = -4;   % maximum distance is 10^-4
dis_steps = 20; % number of distances between 10^-9 and 10^-4 to calculate the heat flow for
dis = logspace(dis_min,dis_max,dis_steps);

Q = zeros(length(dis_steps),1); % this is the vector that will contain the
% heat flow for each distance

Q_prop_s = zeros(length(dis_steps),1);
Q_prop_p = zeros(length(dis_steps),1);
Q_evan_s = zeros(length(dis_steps),1);
Q_evan_p = zeros(length(dis_steps),1);

% frequencies
w_min = 11;
w_max = 18;
w_steps = 1000;
w_Hz = logspace(w_min,w_max,w_steps);
w = w_Hz*2*pi;       % convert to rad/s
q_spec = zeros(length(w),1);  % spectral heat flow container

% wavevector beta (this is the wavevector along the surface)
min_beta = -9;
max_beta = 100; %normal = 100, cutoff = log10(2*pi/(22e-9))
beta_steps = 10000;
% space over which beta is integrated
beta = logspace(min_beta,max_beta,beta_steps);

di=0;
% di is a counter, di = 1 is the first distance, di = 2 is the 2nd etc.

for d = dis
    di = di + 1;
    di

    Q_int = 0;

    Q_prop_s_int = 0;
    Q_prop_p_int = 0;
    Q_evan_s_int = 0;
    Q_evan_p_int = 0;

    steps_omega = diff(w);

    for n = 2: length(w)
        omega = w(n);
        deltaomega = steps_omega(n-1);
        % choose your materials here
        % epsilon1 = eps_inf_SiC*(omega^2-w_LO_SiC^2+1i*Gamma_SiC*omega) ...
        %/(omega^2-w_TO_SiC^2+1i*Gamma_SiC*omega); % SiC
        % epsilon1 = eps_inf_GaAs*(omega^2-w_LO_GaAs^2+1i*Gamma_GaAs ...
        %*omega)/(omega^2-w_TO_GaAs^2+1i*Gamma_GaAs*omega); % GaAs
        % epsilon1 = eps_inf_cBN*(omega^2-w_LO_cBN^2+1i*Gamma_cBN*omega) ...
        %/(omega^2-w_TO_cBN^2+1i*Gamma_cBN*omega); % cBN
        % epsilon1 = eps_inf_SiO2 + sigma_1/(omega_01^2 - omega^2 - ...
        % 1i*omega*gamma_1) + sigma_2/(omega_02^2 - omega^2 - ...
        % 1i*omega*gamma_2); % SiO2
        % epsilon1 = 1-omega_p_Au^2/(omega*(omega+1i*gamma_Au)); % Au
        % epsilon1 = 1-omega_p_Cu^2/(omega*(omega+1i*gamma_Cu)); % Cu
        % epsilon1 = 1-omega_p_Al^2/(omega*(omega+1i*gamma_Al)); % Al
        % epsilon1 = 1-omega_p_Pt^2/(omega*(omega+1i*gamma_Pt)); % Pt
        % epsilon1 = 1-omega_p_Ti^2/(omega*(omega+1i*gamma_Ti)); % Ti

\begin{verbatim}
epsilon1 = 1 - \omega_p_{\text{met}}^2/(\omega_0^2+1i*\gamma_{\text{m}}_{\text{met}}); \% metamaterial from petersen 2013
\%
epsilon1 = 11.7; \% intrinsic silicon
\%
epsilon1 = \epsilon_{\text{inf}} - \omega_p^2/(\omega_0^2+1i*\omega/\tau_e); \% doped silicon
\%
\mu1 = 1; \% choose this for nonmagnetic materials
\mu1 = 1 - F_{\text{met}}\omega_0^2/(\omega_0^2-\omega_0_{\text{met}}^2+1i*\gamma_{\text{m}}_{\text{met}}\omega); \% metamaterial from petersen 2013
\%
\epsilon2 = \epsilon_{\text{inf}}_{\text{SiC}}(\omega_0^2-\omega_{\text{LO}}_{\text{SiC}}^2+1i*\Gamma_{\text{SiC}}\omega) /
/(\omega_0^2-\omega_{\text{TO}}_{\text{SiC}}^2+1i*\Gamma_{\text{SiC}}\omega); \% SiC
\%
\epsilon2 = \epsilon_{\text{inf}}_{\text{GaAs}}(\omega_0^2-\omega_{\text{LO}}_{\text{GaAs}}^2+1i*\Gamma_{\text{GaAs}}\omega) /
/(\omega_0^2-\omega_{\text{TO}}_{\text{GaAs}}^2+1i*\Gamma_{\text{GaAs}}\omega); \% GaAs
\%
\epsilon2 = \epsilon_{\text{inf}}_{\text{cBN}}(\omega_0^2-\omega_{\text{LO}}_{\text{cBN}}^2+1i*\Gamma_{\text{cBN}}\omega) /
/(\omega_0^2-\omega_{\text{TO}}_{\text{cBN}}^2+1i*\Gamma_{\text{cBN}}\omega); \% cBN
\%
\epsilon2 = \epsilon_{\text{inf}}_{\text{SiO2}} + \sigma_1/(\omega_0^2 + 1i*\omega*\gamma_1) + \sigma_2/(\omega_0^2 + 1i*\omega*\gamma_2); \% SiO2
\%
\epsilon2 = 1 - \omega_p_{\text{Au}}^2/(\omega_0^2+1i*\gamma_{\text{Au}}); \% Au
\%
\epsilon2 = 1 - \omega_p_{\text{Cu}}^2/(\omega_0^2+1i*\gamma_{\text{Cu}}); \% Cu
\%
\epsilon2 = 1 - \omega_p_{\text{Al}}^2/(\omega_0^2+1i*\gamma_{\text{Al}}); \% Al
\%
\epsilon2 = 1 - \omega_p_{\text{Pt}}^2/(\omega_0^2+1i*\gamma_{\text{Pt}}); \% Pt
\%
\epsilon2 = 1 - \omega_p_{\text{Ti}}^2/(\omega_0^2+1i*\gamma_{\text{Ti}}); \% Ti
\%
\epsilon2 = 1 - \omega_p_{\text{met}}^2/(\omega_0^2+1i*\gamma_{\text{m}}_{\text{met}}); \% metamaterial
\%
\epsilon2 = 11.7 +0.0001*1i; \% intrinsic silicon
\%
\epsilon2 = \epsilon_{\text{inf}} - \omega_p^2/(\omega_0^2+1i*\omega/\tau_e); \% doped silicon
\%
\mu2 = 1; \% choose this for nonmagnetic materials
\mu2 = 1 - F_{\text{met}}\omega_0^2/(\omega_0^2-\omega_0_{\text{met}}^2+1i*\gamma_{\text{m}}_{\text{met}}\omega); \% metamaterial from petersen 2013
\%
s_int = 0;
s_prop_s_int = 0;
s_prop_p_int = 0;
s_evan_s_int = 0;
s_evan_p_int = 0;
steps_beta = \text{diff}(\beta);
\end{verbatim}
for a = 2: length(beta)

    b = beta(a); % current value of beta
    deltab = steps_beta(a-1); % Delta_beta to multiply with

    k0 = omega/c;
    k1 = sqrt(epsilon1*mu1)*omega/c;
    k2 = sqrt(epsilon2*mu2)*omega/c;
    kz0 = sqrt(k0^2-b^2);
    kz1 = sqrt(k1^2-b^2);
    kz2 = sqrt(k2^2-b^2);

    rs01 = (mu1*kz0-kz1)/(mu1*kz0+kz1);
    rs02 = (mu2*kz0-kz2)/(mu2*kz0+kz2);
    rp01 = (epsilon1*kz0-kz1)/(epsilon1*kz0+kz1);
    rp02 = (epsilon2*kz0-kz2)/(epsilon2*kz0+kz2);

if b < k0

    sprops = b*(1-abs(rs01)^2)*(1-abs(rs02)^2)/(4*abs(1- rs01*rs02*exp(2*i*im(kz0+d))^2));
    spropp = b*(1-abs(rp01)^2)*(1-abs(rp02)^2)/(4*abs(1- rp01*rp02*exp(2*i*im(kz0+d))^2));
    s = sprops + spropp;
    s_prop_s_int = s_prop_s_int + sprops*deltab;
    s_prop_p_int = s_prop_p_int + spropp*deltab;
else

    sevans = imag(rs01)*imagin(rs02)*b*exp(-2*d*im(kz0))/ ... 
            abs(1-rs01*rs02*exp(-2*d*im(kz0)))^2;
    sevanp = imag(rp01)*imagin(rp02)*b*exp(-2*d*im(kz0))/ ... 
            abs(1-rp01*rp02*exp(-2*d*im(kz0)))^2;
    s = sevans + sevanp;
    s_evan_s_int = s_evan_s_int + sevans*deltab;
    s_evan_p_int = s_evan_p_int + sevanp*deltab;
end

    s_int = s_int + s*deltab;
end
% multiply the integrated value of s with the difference in planck energies
Theta1 = hbar*omega/(exp(hbar*omega/kB/T1)-1);
Theta2 = hbar*omega/(exp(hbar*omega/kB/T2)-1);

Q_int = (Theta1-Theta2)*s_int*deltaomega + Q_int;
% vector containing the spectral heat flow
q_spec(n) = (Theta1-Theta2)*s_int*deltaomega;

Q_prop_s_int = (Theta1-Theta2)*s_prop_s_int*deltaomega + Q_prop_s_int;
Q_prop_p_int = (Theta1-Theta2)*s_prop_p_int*deltaomega + Q_prop_p_int;
Q_evan_s_int = (Theta1-Theta2)*s_evan_s_int*deltaomega + Q_evan_s_int;
Q_evan_p_int = (Theta1-Theta2)*s_evan_p_int*deltaomega + Q_evan_p_int;

end

Q(di) = Q_int/pi^2
Q_prop_s(di) = Q_prop_s_int/pi^2;
Q_prop_p(di) = Q_prop_p_int/pi^2;
Q_evan_s(di) = Q_evan_s_int/pi^2;
Q_evan_p(di) = Q_evan_p_int/pi^2;

end
Qbb = zeros(length(dis),1)+sigma*(T1^4-T2^4);

%-----------------------------------------------------------------------------------------------
% plotting spectral heat flow
figure
semilogx(w,q_spec, 'r', 'linewidth', 2)
xlim([1e13 1e15])
ylabel('q_{spec} (W s/rad m^2)')
xlabel('frequency (rad/s)')
% legend('10 nm', '100 nm', '1 um')
set(gca,'Fontsize', 20)
Matlab code

set(findall(gcf, 'type', 'text'), 'Fontsize', 20)

% Plotting
figure
loglog(dis,Q, 'r', 'linewidth', 2)
hold on
loglog(dis,Qbb, 'k', 'linewidth', 2)
% title('Heat flux between two plates')
%ylim([1e-1 1e7])
ylabel('q (W/m^2)')
xlabel('gap size (m)')
legend('total heat flow', 'black body limit')
set(gca,'Fontsize', 20)
set(findall(gcf, 'type', 'text'), 'Fontsize', 20)

% Plotting
figure
loglog(dis, Q_prop_s, '--', 'color',[0 0.7 0], 'linewidth', 2)
hold on
loglog(dis, Q_prop_p, 'color',[0 0.7 0], 'linewidth', 2)
hold on
loglog(dis, Q_evan_s, '--b', 'linewidth', 2)
hold on
loglog(dis, Q_evan_p, '-b', 'linewidth', 2)

% title('Heat flux divided into s and p and evanescent and propagating')
%ylim([1e-1 1e7])
ylabel('q (W/m^2)')
xlabel('gap size (m)')
legend('propagating s', 'propagating p', 'evanescent s', ...
'evanescent p', 'Location','SouthWest')
set(gca,'Fontsize', 20)
set(findall(gcf, 'type', 'text'), 'Fontsize', 20)
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140


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