Calibration and verification of a one-dimensional wave energy decay model

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CALIBRATION AND VERIFICATION OF A ONE-DIMENSIONAL WAVE ENERGY DECAY MODEL

1. Introduction

The present study was performed within the framework of the TOW Coastal Research Programme under responsibility of the task group Velocity Field in Waves. The breaking of waves is a main research topic of this task group. After initiating and completing an in-depth-study of the breaking wave phenomenon it was decided to extend the verification and thereby the operation of the energy decay formulation of Battjes and Janssen (1978). Wave propagation models incorporating this formulation are of great practical importance in nearshore engineering applications. This is confirmed by the close cooperation that developed already during the study with the study team concerned with the verification of the parabolic refraction-diffraction model CREDIZ (see Dingham, 1983).

Battjes and Janssen (1978) were the first to present an energy balance method in the modelling of the energy decay due to depth-limited breaking in random waves. The present extension of their concise laboratory verification concerns both laboratory and field measurements. The inclusion of field data made it necessary to construct a one-dimensional model with additional physical phenomena. This more general model is called ENDEC, which is an acronym for energy decay.

This report is built up as follows. Firstly, in Chapter 2 a calibration and verification of Battjes and Janssen's formulation is described based on laboratory data. Secondly, in Chapter 3 their formulation is extended so that an additional calibration and verification is possible based on field data. Thirdly, the overall model performance is discussed in Chapter 4. Finally, Chapters 5 and 6 give a discussion and the conclusions.
2. Calibration and verification based on laboratory data

2.1 Formulation of Battjes and Janssen's (1978) model

For convenience of the reader the formulation of the wave energy decay model as originally given by Battjes and Janssen (1978) - BJ in the following - is repeated here shortly.

Consider a two-dimensional situation of random waves normally incident on a beach with straight, parallel depth contours. The mean wave energy density $E$ is calculated from the wave energy balance in one spatial dimension:

$$\frac{dP}{dx} + D_b = 0,$$

where $x$ is the coordinate normal to the shore, where $P$ is the time-mean energy flux per unit width (approximated as $E_c g$) and where $D_b$ is the time-mean dissipated power per unit area due to breaking. The problem is to find an expression for $D_b$ taking into account the random wave character.

It is assumed that the non-broken waves obey a Rayleigh distribution with respect to the wave height $H$:

$$F(H) = P(H < h) = 1 - \exp\left[-\frac{1}{2}(H/\hat{H})^2\right] \quad \text{if} \quad 0 < H < H_m$$

$$= 1 \quad \text{if} \quad H > H_m$$

(2.2)

where $\hat{H}$ is some reference wave height. The maximum wave height is given by

$$H_m = \frac{0.88}{k} \tanh\left(\frac{\gamma}{0.88 k h}\right),$$

(2.3)

which expression is a variation of the one proposed by Miche. It is noted that in shallow water, $kh \ll 1$, one has $H_m \approx \gamma h$ and in deep water, $kh \gg 1$, $H_m \approx 0.88/k$, or $H_m/\lambda + 0.14$ where $\lambda$ is the wave length, $\lambda = 2\pi/k$. In fact we consider a whole spectrum of waves wherein the highest and steepest waves (according to $H_m$) are considered as being broken or breaking. The probability $Q_b$ that for some location $x$ a wave height is associated with a breaking or broken wave ($H > H_m$) is
\[ Q_b = P(H > H_m) = \exp(-\frac{1}{4}(H_m/\hat{H})^2). \quad (2.4) \]

The wave field is characterized by \( H_{\text{rms}} \). One has, by definition,

\[ H_{\text{rms}}^2 = \int_0^H H^2 dF(H) = \int_0^{H_m} H^2 dF(H) + H^2 \Delta F(H_m) = \int_0^{H_m} H^2 dF(h) + Q_b H_m^2. \]

so that

\[ H_{\text{rms}}^2 = 2\hat{H}^2 (1 - Q_b). \quad (2.5) \]

The not yet specified reference wave height \( \hat{H} \) can be eliminated from (2.4) and (2.5), yielding

\[ Q_b = \exp[-(1 - Q_b)/b^2] \quad , \quad b = H_{\text{rms}}/H_m. \quad (2.6) \]

For a bore of height \( H \) the dissipated power per unit width is approximately:

\[ D' \sim \frac{1}{4} \rho g H^3 (g/h) \frac{1}{4}, \quad (2.7) \]

where the approximation \( c = (gh)^{\frac{1}{4}} \) has been made. For periodic waves with frequency \( f \) BJ then obtain

\[ D_b = D'/\lambda = D'/c = \frac{fD'}{(gh)^{\frac{1}{4}}} \sim \frac{1}{4} \rho g f H^3/h, \quad (2.8) \]

where \( D_b \) is the dissipated power per unit area due to breaking.

Expression (2.8) is applied for waves with \( H > H_m \) in a random wave field. Because the probability of \( H > H_m \) is \( Q_b \), we obtain from (2.8), (with \( f \) a representative frequency of the spectrum,)

\[ D_b \sim \frac{1}{4} Q_b \rho g f H_m^3/h. \quad (2.9) \]

Since \( H_m/h = O(1) \) during breaking in shallow water, and since the estimation of \( D_b \) is based on order-relations, Battjes and Janssen took for \( D_b \) the expression

\[ D_b = \frac{a}{4} Q_b \rho g f H_m^2. \quad (2.10) \]
where \( \alpha \) is a proportionality constant which is \( O(1) \) when the estimate of \( D_b \) is correct.

The change in mean water level, \( \bar{n} \), due to the radiation stress effect is also accounted for in the BJ model. The momentum equation reads

\[
\frac{dS_{xx}}{dx} + \rho gh \frac{d\bar{n}}{dx} = 0 \quad , \quad h = d + \bar{n} \quad (2.11)
\]

with

\[
S_{xx} = \left( \frac{2c}{c} - i \right) E. \quad (2.12)
\]

Here \( d \) is the still-water depth* and \( c_g \) is obtained from the linear dispersion relation \( \omega^2 = gk \tanh kh \).

In summary, the BJ model for dissipation of wave energy due to breaking consists of the energy equation (2.1) and the momentum equation (2.11) where \( P = c_g E \), \( E = (1/8) \rho g H_{rms}^2 \). \( Q_b \) is solved iteratively from (2.6), \( D_b \) is given by (2.10) and the maximum wave height \( H_m \) is defined by (2.3). BJ gave the free parameter set \(( \alpha, \gamma )\) the plausible value \(( 1.0, 0.8 )\) a priori, which proved to be in satisfactory agreement with their measurements (see Figure 1 for a typical example of BJ's test cases). One of the main objectives of the present study is to verify or derive a generally applicable value for the parameter set \(( \alpha, \gamma )\).

2.2 The dissipation function

In this section the derivation of the dissipation function \( D_b \) is evaluated. Firstly, since - at first glance - unnecessary simplifications seem to have been made in BJ's derivation. Secondly, the bore approximation for steep waves needs further interpretation.

* Note that in the Figures the notations \( D \) and \( ETA \) correspond to \( d \) and \( \bar{n} \) but are not generally equal; the depth \( D \) is a reference waterdepth not necessarily equal to the still water depth. At any rate, however, \( D + ETA \) equals \( d + \bar{n} \) so that ETA contains the difference between the still water depth and the reference depth.
For a periodic bore we have (see, e.g., Stive (1984a)):

\[ D'_{\text{bore}} = \frac{\rho g c_{\text{bore}} h (d_2 - d_1)^3}{(4d_1 d_2)} \]  \hspace{1cm} (2.13)

and

\[ c_{\text{bore}} = \left[ \frac{g d_1 d_2}{(d_1 + d_2) h^2} \right]^{\frac{1}{2}}, \]  \hspace{1cm} (2.14)

where \( d_1 \) and \( d_2 \) are defined in the sketch below and \( h \) is the mean surface level. Note that for \( h = d_1 \) the classical bore relations are found.

We consider now the two cases of shallow water and of deep water for the application of the dissipation function derived for a bore.

a) Shallow water

Let

\[ d_2 - d_1 = H \]

\[ d_1 d_2 = h^2 \]  \hspace{1cm} (2.15)

\[ c_{\text{bore}} = c_{\text{wave}} \]

With (2.15) one obtains from (2.13)

\[ D'_{\text{wave}} = \frac{\alpha}{4} \rho g c_{\text{wave}} H^3/h. \]

With \( D_{\text{wave}} = D'_{\text{wave}}/\lambda, \lambda \) wave length, one obtains, because \( c_{\text{wave}} = \omega/k, \omega = 2\pi f, \)

\[ D_{\text{wave}} = \frac{\alpha}{4} \rho g f H^3/h \]  \hspace{1cm} ; shallow water  \hspace{1cm} (2.16)
b) Deep water

In the case of deep water one may start with (2.13) only when the depth \( d_1 \) is not seen as the true water depth, but rather as a "penetration depth", denoting the depth of the layer in which the breaking wave characteristics may be compared with a bore. It seems reasonable to take the penetration depth to be about equal to the wave height \( H \) (the same is the case in shallow water). We then substitute in (2.13) the approximations:

\[
\begin{align*}
    d_2 - d_1 &= H', \\
    d_1 d_2 &= H^2, \\
    h &= H
\end{align*}
\]  

(2.17)

\[ c_{\text{wave}} \approx c_{\text{bore}} \]

We then obtain

\[
D_{\text{wave}} = \frac{a}{4} \rho g f H^2. 
\]

(2.18)

For periodic waves we thus found expressions (2.16) and (2.18) for \( D_{\text{wave}} \) in shallow water and deep water respectively. Battjes and Janssen (1978) took in (2.16) also \( H/h = O(1) \) which is a valid approximation in shallow water. With \( H/h \sim 1 \) we thus have expression (2.18) in both shallow and deep water.

For application to irregular waves the maximum wave height \( H_m \) is used and the fraction of breaking or broken waves, \( Q_b \). Then we obtain

\[
D_b = \frac{a}{4} \rho g Q_b f H_m^2, 
\]

(2.19)

the same expression as taken by Battjes and Janssen (1978), see also Eq. (2.10).
2.3 The model parameters

As described above BJ's energy decay model has two free parameters, \( \alpha \) and \( \gamma \). Both parameters may be used to adapt the amount of dissipated power \( D_b \) and thereby the rate of energy decay. For the above quoted test case of BJ Figures 2 and 3 show the sensitivity of the model for variations in \( \alpha \) and \( \gamma \) respectively. Inspection of these figures indicates that variations in \( \alpha \) and \( \gamma \) have qualitatively the same effect. This may be explained as follows.

From the elaboration in Appendix A it appears that for \( D_b \) to be constant and using an explicit approximation for \( Q_b \) of the form \( Q_b = 2.4 \left( \frac{H_{rms}}{H_m} \right)^7 \), the relation between \( \alpha \) and \( \gamma \) becomes in the case of shallow water (kh+o)

\[
\alpha \gamma^{-5} = \text{constant} \quad (2.20)
\]

This relation implies that variations in \( \alpha \) and \( \gamma \) result in qualitative similar effects. It is noted that relation (2.20) is not strictly valid. Firstly, an approximation for \( Q_b \) is used, which in itself is not very accurate and secondly, \( H_{rms} \) is assumed constant, so that (2.20) can only be applied locally. However, the qualitative applicability of (2.20) may be confirmed by the following.

As shown in Appendix A a more general relation between \( \alpha \) and \( \gamma \) is given as

\[
\alpha (\gamma + \varepsilon)^{-5} = \text{constant}, \quad (2.21)
\]

where \( \varepsilon \) is a function of \( kh \), such that \( \varepsilon \rightarrow 0 \) as \( kh \rightarrow 0 \). It is found that when the value of \( kh \) is taken at the location where the initial energy dissipation is relatively strongest, \( \varepsilon \) may be assumed nearly constant for the particular situation considered. From the sensitivity calculations it is derived that in the mentioned test case \( \varepsilon = 0.14 \). This allows us to calculate parameter sets generating identical results, e.g. for the parameter set \( (\alpha, \gamma) = (0.5, 0.68) \) the model should generate nearly the same results as for the set \( (\alpha, \gamma) = (1.0, 0.8) \). This is confirmed by the comparison in Figure 4. As expected only the parameter \( Q_b \) is affected, since the value of \( \gamma \) is changed of which \( Q_b \) is a complicated function. The latter aspect will be discussed furtheron.
2.4 Laboratory observations

Empirical laboratory data from various sources have been collected for calibration and verification of the model. The data are obtained for a variety of wave conditions and bottom profiles. The original data of BJ are included also.

All these data have been collected in wave flumes, using mechanically generated random waves. The bottom profiles include plane slopes and a schematized bar-profile in concrete, as well as concave and barred profiles in sand. The collected data are primarily based on small scale measurements with incident rms wave heights ranging from 0.10 to 0.21 m. One of the data series was obtained in a large scale wave facility (the Delta Flume) with an incident rms wave height of 1.00 m.

For each combination of incident waves and bottom profile a data series was obtained comprising the measurement of the bottom profiles, of the mean water level, the rms wave height and the peak frequency at an offshore reference point, and of the rms wave heights at various points in the profile. In some cases the variation of mean water level with distance onshore was measured also.

In all small scale measurements surface elevations were measured by means of parallel-wire conductivity wave gauges. Although aeration influences the response of the gauges, the air content in the breaking waves is estimated low enough to cause only negligible deviations. On the concrete beaches mean bottom pressures were measured by a piezometric system. This consisted of small taps mounted flush with the bottom, connected by tubes to stilling wells, of which the water level was read by point gauges.

In the large scale wave experiment, surface elevations were measured with a wave surface follower. The instrument consists of a vertical gauge with a conductivity sensor at the bottom tip. The gauge moves vertically so as to maintain a constant immersion depth of the sensor, thus registering the time variation of the surface. In order to increase the response in case of steep wave fronts a similar small horizontal gauge was fixed on the vertical gauge facing the incoming waves. The mean bottom pressures were measured by an alternative
piezometric system, consisting of narrow plastic tubes fixed to the flume walls with their open ends close to the sand bottom; the tubes were connected to transparent stilling wells of which the water level was read by eye.

As a standard procedure, the surface elevation signals were analyzed to estimate the variance, $\sigma^2$, and its spectral distribution. The measured variances were used to estimate $H_{rms}$ according to

$$H_{rms} = \sigma^2$$

### Table 1

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Table 1 Experimental and environmental parameters laboratory observations
A resume of the independent parameters is given in Table 1. Column 1 gives the serial number used herein. The columns 4, 5 and 6 list the values of \( h, H_{\text{rms}} \) and \( f_p \) in the offshore reference point (indicated with the subscript \( r \)). Values of a mean deep-water steepness \( s_o = H_{\text{rms}}/L_{\text{op}} \) are listed in column 7, based on \( L_{\text{op}} = \frac{g}{(2\pi f_p^2)} \) and a deep-water value of \( H_{\text{rms}} \) calculated from the values in the reference point using linear shoaling theory for periodic waves with frequency \( f_p \), i.e. \( H_{\text{rms_0}} = H_{\text{rms_r}} \left( \frac{c_g}{c_{g_0}} \right)^{\frac{4}{3}}. \) The parameter \( \gamma \) in column 8 will be explained below.

### 2.5 Model calibration

The energy balance (2.1) and the momentum balance (2.11) have been numerically integrated with respect to the onshore distance \( x \) for each case listed in Table 1, using the bottom profile, the reference values of \( H_{\text{rms}} \) and \( \bar{n} \), which were taken as initial values in the integration, the reference value of \( f_p \), and chosen values of \( (\alpha, \gamma) \). The peak frequency \( f_p \) and the coefficients \( (\alpha, \gamma) \) were kept constant with respect to \( x \). In the vicinity of the mean waterline on the beach, where \( h \approx 0 \) and \( Q \approx 1 \), the model reaches saturation; shoreward of the point where this situation is first encountered in the numerical integration, the \( H_{\text{rms}} \)-values were set equal to \( H_m \), i.e. their limiting value for \( Q_b = 1 \).

Each integration gives two functions \( \bar{n}(x) \) and \( H_{\text{rms}}(x) \) shoreward of the reference point, which can be compared to the measurements. Repeating the integration for several choices of \( (\alpha, \gamma) \), optimal values of these coefficients can be estimated, such that a maximum agreement is obtained between computed and measured values. Actually, this model calibration has been based on a comparison of \( H_{\text{rms}} \)-values only, since the mean water level measurements were available in only a few cases. The agreement between computed and measured values was maximized visually for each of the 16 cases, in which process most weight was given to \( H_{\text{rms}} \)-values which were about half of their initial value.

The coefficients \( \alpha \) and \( \gamma \) control the level of energy dissipation in a breaker, and the fraction of breaking waves, respectively. Formally, they can be varied independently of each other. However, in the calibration process just described there is a dependence between the two since in this process the model is forced to simulate a certain energy dissipation, which depends on \( \alpha \) and \( \gamma \) through its proportionality to the product \( \alpha Q_b H_m^2 \) (see Eq. 2.10). Therefore,
there is effectively only one degree of freedom in tuning the model to a measured wave height variation. The calibration was in fact carried out by estimating optimal values of $\gamma$ (denoted as $\hat{\gamma}$) under the constraint $a = 1$. The resulting values of $\hat{\gamma}$ are listed in column 8 of Table 1. They fall in the range from 0.60 to 0.83, which is physically realistic. A graphical impression of the computed and measured $H_{\text{rms}}$-variations is presented in a following section.

2.6 Coefficient parameterization

It is known that the process of wave breaking in shallow water is influenced by the incident wave steepness and by the bottom profile, in particular the slope. In some cases the influence of these two parameters is expressed sufficiently through a specific combination of wave steepness and bottom slope, the so-called surf similarity parameter (Battjes, 1974). It was therefore investigated whether the estimated $\hat{\gamma}$-values vary systematically with these parameters. No significant variation of $\hat{\gamma}$ with beach slope separately or with the surf similarity parameter could be found. However, there did appear to be a systematic dependence of $\hat{\gamma}$ on the deep-water steepness $s_0$, as can be seen in Figure 5. A tanh-function has been fitted to these data, with the result

$$\hat{\gamma} = 0.5 + 0.4 \tanh (33 s_0)$$

(2.23)

This relation, indicated in Figure 5 by the full line, can be used for purposes of prediction. Instead of using the deep water wave height in Eq. (2.23) it may be considered useful to use the wave height at the water depth four times the wave height in view of the discussion of Chapter 5. In this case the tanh-fit to the data becomes

$$\hat{\gamma} = 0.05 + 0.4 \tanh \left(35 \frac{H_{\text{rms}}}{L_{p,0}}\right)$$

(2.24)

where $H_{\text{rms}}$ is the rms wave height at the water depth of $4 H_{\text{rms}}$ and $L_{p,0}$ is the deep water wave length for the peak frequency.
3. Calibration and verification based on field data

3.1 Formulation of an extended model

In field situations where energy decay due to breaking is the primary process other phenomena—avoidable in the laboratory—are bound to be of secondary but unknown importance. Therefore an extension of Battjes and Janssen's one-dimensional model is constructed, which—in the parallel bottom contour topography—takes into account:

- refraction due to bottom variations
- refraction due to current variations
- three energy source terms, i.e. besides the main source term dissipation due to breaking, also dissipation due to bottom friction and gain due to a constant wind field.

As a starting point in the formulation we consider the wave action conservation equation for a stationary wave field in two horizontal dimensions:

$$\nabla \cdot \left[ \left( \frac{\dot{c}_g}{g} + \mathbf{\hat{U}} \right) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0$$  \hspace{1cm} (3.1)

where $E$ is the mean wave energy density, $D/\omega_r$ is the energy source term with $D$ the mean dissipated power, $\mathbf{\hat{U}}(x,y)$ a current field, $\frac{\dot{c}_g}{g} = \frac{\partial \omega_r}{\partial k}$ and $\omega_r$ the relative or intrinsic wave frequency with

$$\omega = \omega_r + k \cdot \mathbf{\hat{U}} \hspace{1cm} (3.2)$$

$$\omega_r^2 = gk \tanh(\kappa h), \quad k = |\kappa|$$

We now introduce a wave ray with a local coordinate $s$, along which the vector $\dot{c}_g$ is directed. Let the angle between the ray and the positive $x$-axis be $\Theta$, locally. Also, let the angle between the vector $\mathbf{\hat{U}}$ and the positive $x$-axis be $\nu$, locally. Then (3.1) can be written as

$$\frac{\partial}{\partial x} \left[ \left( c_g \cos \Theta + U \cos \nu \right) \frac{E}{\omega_r} \right] + \frac{\partial}{\partial y} \left[ \left( c_g \sin \Theta + U \sin \nu \right) \frac{E}{\omega_r} \right] + \frac{D}{\omega_r} = 0$$  \hspace{1cm} (3.3)

where $c_g = |\dot{c}_g|$ and $U = |\mathbf{\hat{U}}|$.
Finally, we simplify our situation to a bathymetry of parallel bottom contours, such that \( h = h(x) \) and thus the \( x \)-axis is normal to the bottom contours. In this case we have

\[
k_y = k \sin \theta = \text{constant} \quad \text{or} \quad \sin \theta / c = \text{constant (Snell's law).}
\] (3.4)

and it follows that (3.3) reduces to

\[
\frac{d}{dx} \left[ \left( c \cos \theta + U \cos \psi \right) \frac{E}{\omega} \right] + \frac{D}{\omega} = 0
\] (3.5)

To generalize our situation we consider the energy decay along the \( \xi \)-axis the still water depth \( d \) is given at distances \( \Delta \xi \). The situation is as sketched below.

Because \( d\xi = dx / \cos \psi \) Equation (3.5) may be written as

\[
\frac{d}{d\xi} \left[ \left( c \cos \theta + U \cos \psi \right) \frac{E}{\omega} \right] + \frac{D}{\omega} \cos \psi = 0
\] (3.6)

The equation is to be solved simultaneously with Eq. (3.4).

With respect to the momentum equation the effects of a variable current field and of bottom friction are neglected so far, considering that this concerns a third order effect on the energy decay only. Thus, the effect of radiation stress on the mean surface level is expressed by

\[
\frac{\partial S_{ij}}{\partial x_i} + \rho g h \frac{\partial h}{\partial x_j} = 0 \quad , \quad h = d + \tilde{n}
\] (3.7)
where the radiation stress tensor is given as

\[ S_{ij} = \frac{E}{2} \left( \frac{k_i k_j}{k^2} + \frac{2c}{c} \delta_{ij} \right) \left( \frac{\mathbf{g}}{c} - 1 \right) \]  \tag{3.8}

In our case, with \( h = h(x) \) and the wave direction being locally \( \theta \) with respect to the positive x-axis, we have with \( k_x = k \cos \theta, k_y = k \sin \theta \):

\[ \frac{dS_{xx}}{dx} + \rho gh \frac{dn}{dx} = 0, \]  \tag{3.9}

where

\[ S_{xx} = \left( \frac{c}{c} (1 + \cos^2 \theta) - \frac{1}{2} \right) E. \]  \tag{3.10}

Along the \( \xi \)-axis we have, from (3.9)

\[ \frac{dS_{xx}}{d\xi} + \rho gh \frac{dn}{d\xi} = 0. \]  \tag{3.11}

3.2 The energy source function

In the present case we have introduced the following terms in the energy source function \( D \):

\[ D = D_b + D_f - D_w \]  \tag{3.12}

where the terms on the right hand side are positively valued. In the situations under consideration the term \( D_b \), the power dissipated due to wave breaking, generally dominates over the term \( D_f \), the power dissipated due to bottom friction, and the term \( D_w \), the power gained due to a wind field, so that generally \( D \) is positive and a decay of wave energy results.

The expression for \( D_b \) is evaluated in Section 2.2. In the presence of a variable current field the earlier derived expression (2.19) needs to be slightly adapted by introducing the relative frequency in the substitution of the wave phase speed, i.e. now

\[ c_{\text{wave}} = \omega_r / k \]
so that the expression for \( D_b \) reads:

\[
D_b = \frac{a}{4} \rho g \frac{\omega_r}{2\pi} \frac{H_m^2}{H_m}.
\]

(3.13)

For a derivation of the dissipation term due to bottom friction we take the expression derived by Putnam and Johnson (1949) for regular waves as a starting point:

\[
D_f' = \frac{f_w}{6\pi} \left( \frac{\omega_r}{\sinh kh} \right)^3.
\]

(3.14)

where \( f_w \) is a friction factor. Here we consider unidirectional, random waves in the presence of currents where the wave heights are in principle Rayleigh distributed and the wave periods are characterized by one period. In this case the random counterpart of Putnam and Johnson's regular wave formulation reads:

\[
D_f = \frac{f_w}{6\pi} \left( \frac{\omega_r}{\sinh kh} \right)^3 \int_0^\infty H^3 \, dP(H)
\]

(3.15)

where \( P(H) \) is either the full or - in case of wave breaking - the clipped Rayleigh distribution:

\[
P(H) \equiv P(H < H) = 1 - \exp\left[ -\frac{1}{2}(H/H_m)^2 \right] ; \quad 0 < H < \infty
\]

(3.16)

or

\[
P(H) \equiv P(H < H) = 1 - \exp\left[ -\frac{1}{2}(H/H_m)^2 \right] ; \quad 0 < H < H_m
\]

\[
= 1 \quad ; \quad H > H_m,
\]

(3.17)

where the probability at breaking or broken waves \( Q_b \) is given by

\[
Q_b = \exp\left[ -\frac{1}{2}(H_m/H)^2 \right].
\]

The evaluation of expression (3.15) with \( P(H) \) given either by (3.16) or (3.17) is given in Appendix C. For the full Rayleigh distribution we obtain

\[
D_f = \frac{f_w}{8\pi} \left( \frac{\omega_r H_{rms}}{\sinh kh} \right)^3
\]

(3.18)
and for the clipped Rayleigh distribution we obtain:

\[ D_f = \rho \frac{\omega_r}{d \omega_r} \left( \frac{\omega_r}{\sinh(kh)} \right)^3 \left( \sqrt{2}H \right)^3 \gamma(a, z) \]  

(3.19)

where \( \gamma(a, z) \) is the incomplete gamma function with \( a = 1.5 \) and \( z = \frac{1}{4}(H_m/H)^2 \). Since it appears furtheron that where wave breaking occurs bottom friction is negligible it is decided to rely on the more simple expression (3.18) for the present study.

The energy gain term due to wind is included in a more approximate way. Use is made of growth curves as given in the wave prediction programme GONO, see Janssen et al. (1984). Because this method is essentially discrete, it is not possible to define the resulting dissipation function as \( D = D_b + D_f - D_w > 0 \) expressing the wave growth due to wind.

The growth curve has the general appearance

\[ \frac{gH_s^2}{W^2} = \beta \tanh F(gt/W), \]  

(3.20)

where \( W \) is the wind velocity component in the \( \xi \)-direction, measured at 10 m above mean sea level and \( F(.) \) is a function to be specified. Introducing the non-dimensional wave height \( z \) and the non-dimensional time \( \tau \) by

\[ z = \frac{gH_s}{W^2}, \quad \tau = gt/W, \]  

(3.21)

a growth curve has been chosen of the form

\[ z/\beta = \tanh \left[ pt^q \right] \]  

(3.22)

The numerical coefficients \( p \) and \( q \) as used in GONO are given as:

\[ p = c_2, \quad q = c_3 \quad \text{for} \quad \tau < 13.10^3 \]  

(3.23)

\[ p = c_4, \quad q = c_5 \quad \text{for} \quad \tau > 13.10^3 \]
and
\[ \beta = 0.22 \]
\[ c_2 = 4.62 \times 10^{-4} \]
\[ c_3 = 0.7786 \]
\[ c_4 = 1.91 \times 10^{-3} \]
\[ c_5 = 0.6286 \]
\[ c_6 = 0.62763, \]
where \( c_6 \) is the value obtained for \( z/\beta \) from \( z/\beta = \tanh \left[ c_2(13.103)^{c_3} \right] \).

Consider now the effect of wave growth due to wind over the distance \( \Delta \xi \) from \( \xi_i \) to \( \xi_{i+1} = \xi_i + \Delta \xi \). At the point \( \xi_i \) the energy \( E_i = (1/8) \rho g H_{\text{rms}i}^2 \) is known. Therefore, at \( \xi = \xi_i \), one has \( H_{\text{rms}i} = H_{\text{rms}i}/2 \). For the case of unsaturated waves (i.e., \( z/\beta < 1 \)) expression (3.22) is inverted in order to give the (non-dimensional) time \( \tau \) which would be needed to give waves of non-dimensional wave height \( z_i = g H_{\text{rms}i}/\beta^2 \). One obtains
\[ \tau_i = \left[ \frac{1}{p} \arctanh \left( \frac{z_i}{\beta} \right) \right]^{1/q}. \] (3.25)

For \( z_i/\beta < c_6 \) the coefficients \( c_2, c_3 \) are to be used for \( p, q \); for \( z_i/\beta > c_6 \) coefficients \( c_4, c_5 \) are used in (3.25).

The (dimensional) time-span \( \Delta t \) which is needed for the wave field to travel the distance \( \Delta \xi \) is given by
\[ \Delta t = \frac{\Delta \xi}{c g \cos(\theta - \psi)} \] (3.26)

With \( \Delta t = g \Delta t/W \), relations (3.22) and (3.23) are applied for \( \tau_{i+1} = \tau_i + \Delta t \) yielding \( z_{i+1} \) and thus \( H_{\text{rms}i+1} \). The wave growth due to wind over the distance \( \Delta \xi \) is then given by \( H_{\text{rms}i+1} - H_{\text{rms}i} \) and the amount \( (H_{\text{rms}i+1} - H_{\text{rms}i})/2 \) is simply added to the previously calculated value \( H_{\text{rms}i+1} \).

The integration of this approach in the numerical procedure is described in Appendix B.
3.3 Field observations

The field sites and conditions were selected so as to have more or less statistically uniform conditions alongshore and normal incidence (for the principal wave propagation direction).

Two sites on the Dutch coast were used, one on a beach near Egmond, the other on a shoal in the mouth of the Haringvliet estuary (see Figure 6). The beach has a typical double bar system. Here measurement series were collected both under moderate conditions with incident rms wave heights up to 1.3 m and under storm conditions with incident rms wave heights up to 2.8 m. The measurements in the Haringvliet estuary were conducted in a line across an elongated shoal, with more or less parallel depth contours over a distance of about 5000 m (see Figure 6). The minimum depth over the shoal is 0.1 m below Mean Sea Level, and about 1.5 m below Mean High Water Level. Here measurements were collected under a variety of conditions of which a swell series (incident rms wave heights of 0.9 m) and a storm series (incident rms wave height of 2.4 m) at near normal incidence were selected for the present study.

For each combination of incident waves and bottom profile a data series was obtained comprising the measurement of the bottom profile, of the mean water level, the rms wave height and the peak frequency at an offshore reference point, and of the rms wave heights at various points in the profile. In some cases the variation of mean water level with distance onshore was measured also. In these field cases a number of up to six measurement series differing only slightly in incident conditions were combined and averaged to reduce the influence of measurement errors and inaccuracies. The results are given in terms of an average value and a standard deviation.

In the measurements at Egmond beach surface elevations were measured by wave buoys offshore and by resistance wire gauges nearshore. The possibility of levelling of the gauges allowed the measurement of mean water level variations in some cases. In the measurements in the Haringvliet estuary wave buoys were used mainly. Both the buoys and the gauges can give problems in situations with considerable breaking activity, due to the occurrence of steep wave fronts. In such situations, the buoys occasionally capsize, and the gauge
response contains spurious peaks following a too rapid rise of surface elevation. Both phenomena are easily recognized in the registration. Those segments of the registration where capsizing occurred were eliminated, whereas the spurious response peaks of the gauges were removed by filtering.

As a standard procedure, the surface elevation signals were analyzed to estimate the variance, \(\sigma^2\), and its spectral distribution. The measured variances were used to estimate \(H_{\text{rms}}\) according to

\[
H_{\text{rms}} = 8\frac{1}{4} \sigma. \quad (3.27)
\]

A resume of the independent parameters is given in Table 2, in similar form as for the laboratory observations.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>source</th>
<th>profile</th>
<th>(h_R)</th>
<th>(H_{\text{rms}})</th>
<th>(f_{\text{PR}})</th>
<th>(s_0)</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>Derks and</td>
<td>bar-trough</td>
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<td>1.29</td>
<td>.157</td>
<td>.022</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>White (1984)</td>
<td>bar-trough</td>
<td>15.65</td>
<td>2.78</td>
<td>.115</td>
<td>.026</td>
<td>0.73</td>
</tr>
<tr>
<td>18</td>
<td>Dingemans (1983)</td>
<td>bar</td>
<td>16.40</td>
<td>.94</td>
<td>.143</td>
<td>.013</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>Dingemans (1983)</td>
<td>bar</td>
<td>11.10</td>
<td>2.43</td>
<td>.128</td>
<td>.028</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Table 2 Experimental and environmental parameters field observations

3.4 Model calibration

The energy balance of BJ's model contains no sources or sinks of energy other than that due to breaking nor does it incorporate current effects. The possible importance of dissipation due to bottom friction, of input of energy by wind and of current refraction was investigated by comparing the results obtained by BJ's model in these field cases with the results obtained by the extended model. For the Egmond case the effects of bottom friction were investigated (see Figure 7), while both the effects of wind energy input and current refraction were investigated for the Haringvliet case (see Figures 8 and 9). The
quantitative values describing these effects were chosen as realistically as possible. It appears that in all cases the effects of these phenomena are marginal compared to the effect of wave breaking as soon as the latter starts to play a role, say when the ratio of incident rms wave height over mean water depth reaches the value of 0.25. It is relevant to note that the other phenomena do each play a specific role in the non-breaking region (e.g. behind the Hinderplaat shoal, see Figure 8) which is not further investigated here.

The above leads to the conclusion that we may extend the calibration of the model of BJ with the present field results in a similar way as has been done for the laboratory results. Again the calibration was carried out by estimating optimal values \( \hat{\gamma} \) under the constraint \( \alpha = 1 \). The resulting values of \( \hat{\gamma} \) are listed in column 8 of Table 2. These values fall in the same range as found for the laboratory cases. Moreover, it appears that the values \( \hat{\gamma} \) also systematically depend on the deep-water steepness \( s_o \), as can be seen in Figure 5. It is concluded that the tanh-function fitted to the laboratory data holds equally well for the field data.
4. Overall model performance

In order to gain an impression of the overall performance of the model, we have applied it again to each of the 20 cases considered, this time using the parameterization described above. All results are shown in the Figures 10 through 29. It can be seen that the model is quite realistic in the simulation of the observed rms wave heights. A comparison of normalized computed and measured $H_{rms}$ values in the zone of wave shoaling and breaking covering all 20 cases is given in Figure 30. The correlation coefficient is 0.98; the model bias based on the best-fitting proportional relationship is 0.01, and the rms relative error, normalized with the mean value of all measured values of $H_{rms}/H_{rms0}$ shown in Figure 30, is 0.06. These numbers confirm in a quantitative sense the high degree of realism possessed by the model for the prediction of the wave height variation in areas of wave breaking.

In those cases where the variation of the mean water level has been measured this variation (set up) also is well predicted, but in areas of large set-up gradient the predicted rise is systematically too far seaward. This phenomenon has been noted previously by Battjes (1972) and by BJ. It suggests that the decrease in momentum flux lags behind the decrease in wave energy as measured through the variance of the surface elevation. A possible explanation for this phenomenon would be a relative surplus of kinetic energy in the area of intensive wave energy dissipation. This might consist partly of a surplus of kinetic wave energy (coherent with the surface elevation) and partly of turbulence energy. The latter possibility can be investigated by adding turbulent Reynolds stresses to the radiation stresses, e.g. on the basis of the model for turbulence in the surf zone presented by Battjes (1975). However, this matter has not been pursued in the present study.

In order to obtain more insight in the working of the model, a very limited comparison is also made with an internal parameter, i.e. for $Q_b$ to be interpreted as the fraction of waves in the process of breaking at a certain water depth. For the three laboratory cases 9, 10 and 11 (figures 18, 19 and 20) the calculated $Q_b$ is compared to the visually observed fraction of waves in the process of breaking. It appears that $Q_b$ for $\gamma = 0.8$ underestimates the breaking wave fraction by a factor of about 5. As shown in Paragraph 2.2 different parameter combinations can yield identical results for the wave
height decay and the mean water level variation while the value of $Q_b$ changes. For instance in the BJ tests case considered (Figures 1 and 4) $Q_b$ more than doubled when applying $\gamma = 0.68$ instead of $\gamma = 0.8$. This may imply that a parameter combination with $(\alpha, \gamma) < (1.0, 0.8)$ is more internally consistent with laboratory results. However, if we are only interested in the correct prediction of the end results of the model, i.e. the wave height decay and the mean water level variation it is unnecessary and perhaps confusing to change the parameter combination. In that case $Q_b$ must only be considered as a qualitative measure of the fraction of breaking waves.
5. Discussion

The model calibration described above is based on situations of one-dimensional wave propagation. However, the key element in the model is the estimate of the mean energy dissipation in random waves due to breaking, which is believed to apply equally well in case of two-dimensional wave propagation.

In the situations used in the calibration described herein, there were no significant sources or sinks of energy between deep water and the surf zone. In these situations, a parameterization of the model coefficient(s) in terms of a characteristic deep water wave steepness is meaningful, as in Eq. 2.23. In applications, more complicated situations may arise, in which processes other than pure shoaling play a non-negligible role between deep water and the surf zone. Examples are wave refraction, and energy dissipation in the near-bottom boundary layer. For such situations, we use the following procedure.

We distinguish areas of negligible breaking and areas of significant breaking. (In barred profiles, more than one area of significant breaking may occur.) Somewhat arbitrarily, we define an area of significant breaking as the region of space (in the horizontal plane) where $H_{\text{rms}}/h > 0.25$. This corresponds roughly with $H_{\text{rms}}/H_m > 0.35$, approximately the limit above which the fraction of breaking waves is no longer negligible.

In a process of numerical integration along a wave ray in water of decreasing depth, in which refraction and various energy sources and sinks can be taken into account, the dissipation due to breaking is neglected as long as $H_{\text{rms}} < 0.25 \ h$. At the point where $H_{\text{rms}} = 0.25 \ h$, $\gamma$ is determined according to Eq. 2.24. Alternatively, the local value of $H_{\text{rms}}$ may be converted to an equivalent deep water value using linear shoaling only (i.e. $H_{\text{rms}} = (c_g/c_0)^4 H_{\text{rms}}$), and $\gamma$ is determined according to Eq. 2.23. The value of $\gamma$ so obtained is used in the integration of the energy balance (including the dissipation due to breaking according to Eq. 2.10 or in the case of currents Eq. 3.13) across the area of non-negligible breaking downwave of the point of transition (i.e., as long as $H_{\text{rms}}/h > 0.25$).
6. **Conclusions**

The present report mainly deals with the calibration and verification of the method presented by Battjes and Janssen (1978) to model the energy decay due to breaking in random waves. Although some internal aspects of the model are discussed, the main emphasis is on the empirical investigation of the optimal value or functional dependence of the free parameter set $(a, \gamma)$. These free parameters both adapt the amount of dissipated power and thereby the rate of energy decay.

In Chapter 2 the original formulation of BJ's method is evaluated and their concise laboratory calibration is extended on the basis of small and large scale $(H_{rms}$ up to 1 m) flume data. It is shown that the theoretical model has effectively one adjustable parameter. Based on a comparison of calculated and measured variations of rms wave heights in areas of wave breaking, optimal values of this coefficient have been determined.

In Chapter 3 extensions have been made to the original formulation. These relate to the generalization to oblique wave propagation on a coast of parallel bottom contours, to the inclusion of current refraction and to the incorporation of bottom friction dissipation and wind force gain in the energy source term. With the aid of this model the calibration has been extended on the basis of two field data sets. It is found that the influences of the additional phenomena, as bottom dissipation, wind gain, bottom and current refraction are marginal compared to wave breaking. So, identical to the laboratory calibration optimal values of the single adjustable coefficient have been determined which purely relate to the breaking wave formulation.

For both the laboratory and the field experiments alike it is found that the optimal values of the single adjustable coefficient vary slightly in a physically realistic range with the incident wave steepness. A parameterization of this dependence is presented. Using this parameterization, the overall performance of the model has been evaluated. The coefficient of correlation between predicted and observed $H_{rms}$ values is 0.98; the model bias is not significantly different from zero, and the rms relative error is 0.06. The variation of the mean water level (set-up) also is well predicted, although the predicted interval of steepest rise of the mean water level is systematically
too far downwave. It is hypothesized that this is due to an underestimation of
the total kinetic energy and momentum flux in areas of intense breaking, for a
given potential energy of the waves.

In summary, the theoretical model presented by Battjes and Janssen (1978) for
the prediction of the variation of wave energy and the associated radiation
stresses in areas of wave breaking is found to perform very well for a wide
range of conditions. It can be used with confidence for purposes of predic-
tion, using a parameterization of its single adjustable coefficient in terms
of wave steepness.
Acknowledgement

Prof. J.A. Battjes, chairman of the task group "Velocity field in waves", is thanked for his valuable guidance. Also we thank H. Derks, J.P.F.M. Janssen and J. van Overeem, who supplied additional information on the measurement results obtained by them in their respective studies.
APPENDIX A: THE RELATION BETWEEN THE MODEL PARAMETERS $\alpha$ AND $\gamma$

This Appendix concerns the relation between the free parameters $\alpha$ and $\gamma$. This relation is derived such that for different combinations of $(\alpha, \gamma)$ the dissipation function $D_b$ remains nearly unchanged.

We have

$$ H_m = \frac{0.88}{k} \tanh\left[\frac{\gamma}{0.88} kh\right] \quad (A.1) $$

$$ Q_b = \exp[-(1-Q_b)/b^2] \quad , \quad b = H_{rms}/H_m \quad (A.2) $$

$$ D_b = \frac{\alpha}{4} \rho g Q_b^2 H_m^2 \quad (A.3) $$

The standard values are $(\alpha, \gamma) = (1.0, 0.8)$. These standard values are denoted by $\alpha_n, \gamma_n$ in this Appendix.

With

$$ \mu = (\gamma - \gamma_n)/\gamma_n \quad (A.4) $$

expression (A.1) for $H_m$ can be written as

$$ H_m = \frac{0.88}{k} \tanh\left[(1+\mu) \frac{\gamma_n}{0.88} kh\right] \quad (A.5) $$

The maximum wave height for $\gamma = \gamma_n$ (thus, $\mu = 0$) is denoted by $H_n$. Using the expansion

$$ \tanh[(1+\mu)z] = \tanh z + \mu z[\tanh^2 z] + $$

$$ -\frac{1}{4}(\mu z)^2 (\tanh z)[1-\tanh^2 z] + \ldots, $$

$H_m$ can be expressed in terms of $H_n$ as

$$ \tilde{H}_m = H_n + \mu \gamma_n h\left[1 - \left(\frac{kH_n}{0.88}\right)^2\right] + \frac{\mu k h \gamma_n}{0.88} \left[1 - \left(\frac{kH_n}{0.88}\right)^2\right] H_n \quad (A.6) $$

Taking $\gamma = 0.6$ ($\mu = -0.25$) a relative error of 1.67% in $\tilde{H}_m$ corresponding to the true $H_m$ is obtained for $kh = 1$. When only first-order terms in $\mu$ are retained, the relative error is 3.17% for $kh = 1$. In the case $kh = 0.5$ these errors are 0.63% and 1.31% respectively.
It is clear that the first approximation in (A.6) (only linear in $\mu$) suffices for our purposes here:

$$\tilde{H}_m = H_n + \mu \gamma_n h \left[ 1 - \left( \frac{kH_n}{\gamma_n} \right)^2 \right].$$ \hspace{1cm} (A.7)

Expression (A.7) is now used in the expression for $D_b$. Because also $Q_b$ depends on $H_m$, it is desirable that an expression for $Q_b$ is used which is explicit in terms of $H_{rms}/H_m$. We take

$$\bar{Q}_b = 2.4 (H_{rms}/H_m)^7.$$ \hspace{1cm} (A.8)

This approximation is compared with the exact expression for $Q_b$ (Eq. (2.6)) for some values in the Table below.

<table>
<thead>
<tr>
<th>$H_{rms}/H_m$</th>
<th>$Q_b$</th>
<th>$\bar{Q}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.3</td>
<td>$1.49 \times 10^{-5}$</td>
<td>$0.52 \times 10^{-5}$</td>
</tr>
<tr>
<td>.4</td>
<td>$0.195 \times 10^{-2}$</td>
<td>$0.393 \times 10^{-2}$</td>
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</tr>
<tr>
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<td>$7.65 \times 10^{-2}$</td>
<td>$6.72 \times 10^{-2}$</td>
</tr>
<tr>
<td>.7</td>
<td>$19.2 \times 10^{-2}$</td>
<td>$19.8 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table A.1

It is seen from Table A.1 that especially for the lower values of $H_{rms}/H_m$ $\bar{Q}_b$ is not a very good approximation to $Q_b$. Inserting expression (A.8) in (A.3) one obtains

$$\bar{D}_b = 0.6 \alpha \rho g \frac{H_{rms}}{H_m}. \hspace{1cm} (A.9)$$

Inserting approximation (A.7) for $H_m$ into this expression then yields

$$\bar{D}_b = 0.6 \alpha \rho g \frac{H_{rms}}{H_m} \left[ H_n + \mu \gamma_n h \left[ 1 - \left( \frac{kH_n}{\gamma_n} \right)^2 \right] \right]^{-5}. \hspace{1cm} (A.10)$$
For the expression (A.10) for the dissipation function $\tilde{D}_b$, a relation between $\alpha$ and $\gamma$ can be obtained under the condition of a constant value of $\tilde{D}_b$. We take now also a constant value for $H_{\text{rms}}$ although in practice $H_{\text{rms}}$ will be different for a certain location along the ray, which location may be characterized by $kh$. That $H_{\text{rms}}$ is different may be seen as follows. For the incipience of breaking to be resulting in measurable loss of energy over a short distance, one should have at least $H_{\text{rms}}/H_m > 0.30$; besides, for $H_{\text{rms}}/H_m < 0.30$ there is simply defined $Q_b = 0$ in the algorithm by which $Q_b$ is evaluated. When $\gamma$ is decreased from its standard value $\gamma_n = 0.80$ there will be obtained a value $Q_b \neq 0$ at a location further off-shore than is the case for $\gamma = \gamma_n$; say that $H_{\text{rms}}/H_m > 0.30$ at location $(kh)_1$ for the first time, when $\gamma = \gamma_n$ is taken. Off-shore from this location an increase in $\alpha$ has no effect because $D_b$ remains zero. However, when decreasing $\gamma$ the value 0.30 is reached at a point off-shore from $(kh)_1$ so that the breaking region begins earlier. An increase in $\alpha$ becomes effective at a point further in-shore than a decrease in $\gamma$.

To simplify matters we take $H_{\text{rms}}$ to be constant at some location. Then one obtains from (A.10)

$$a[kH_n + \mu \gamma_n kh[1 - \left(\frac{kH_n}{.88}\right)^2]^5 = \text{constant}$$  \hspace{1cm} (A.11)

This functional relation depends still on $kh$. Therefore the relation (A.11) is worked out for a few $kh$ values. At first it is noted that relation (A.11) can be rewritten as

$$a[\gamma + \varepsilon]^{-5} = \text{const}$$  \hspace{1cm} (A.12)

where

$$\varepsilon = \frac{kH_n}{kh} \left[1 - \left(\frac{kH_n}{.88}\right)^2\right]^{-1} - \gamma_n$$  \hspace{1cm} (A.13)

Substituting the expression for $H_n$ in (A.13), one obtains

$$\varepsilon = (kh)^{-1} 0.88 \tanh(\gamma_n kh/.88)[1 - \tanh^2(\gamma_n kh/.88)]^{-1} - \gamma_n$$  \hspace{1cm} (A.14)

The value of $\varepsilon$ is now computed for several values $kh$, where $\gamma_n = 0.80$ is used. We obtain the values shown in the next Table:
It is clear from Table A.2 that for shallow water, \( kh \ll 1 \), the simple relationship
\[
\alpha \gamma^{-5} = \text{constant} \cdot \ kh \ll 1
\]  
(A.15)
is obtained. This relation could easily be derived directly by noting that 
\( H_m + \gamma h \) for \( kh + 0 \); substitution of \( H_m = \gamma h \) in the dissipation function \( \tilde{U}_b \)
given in (A.9) leads then to (A.15).

A few examples of corresponding pairs \((\alpha_n, \gamma)\), \((\alpha, \gamma_n)\) are given below; relation 
(A.12) is used with \( \varepsilon \)-values from (A.14) for both \( kh = 0 \) (\( \varepsilon = 0 \)) and \( kh = 0.4 \) 
(\( \varepsilon = 0.0724 \)). These pairs are obtained from 
\[
\alpha (\gamma + \varepsilon)^{-5} = \alpha_n (\gamma + \varepsilon_n)^{-5}.
\]

<table>
<thead>
<tr>
<th>( kh = 0 )</th>
<th>( kh = 0.4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_n = 1 )</td>
<td>( \gamma_n = .80 )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>.80</td>
<td>1</td>
</tr>
<tr>
<td>.75</td>
<td>1.38</td>
</tr>
<tr>
<td>.70</td>
<td>1.95</td>
</tr>
<tr>
<td>.696</td>
<td>2</td>
</tr>
<tr>
<td>.65</td>
<td>2.82</td>
</tr>
<tr>
<td>.606</td>
<td>4</td>
</tr>
<tr>
<td>.60</td>
<td>4.214</td>
</tr>
<tr>
<td>.589</td>
<td>4.632</td>
</tr>
</tbody>
</table>

Table A.3 Pairs of \((\alpha_n, \gamma)\) and \((\alpha, \gamma_n)\) satisfying 
(A.12) for the cases \( kh = 0 \) and \( kh = 0.40 \)
APPENDIX B: NUMERICAL PROCEDURE EXTENDED MODEL ENDEC

The basic equations of the extended energy decay model ENDEC are the energy and momentum conservation equations. The extensions with respect to Battjes and Janssen's (1978) original model relate to the generalization to oblique wave propagation on a coast of parallel bottom contours, to the inclusion of current refraction and to the incorporation of the effects of bottom friction and wind force on the energy source term in the energy equation. The basic equations form a system of two ordinary, first order differential equations, from which the variation of wave energy, \( H_{\text{rms}} \), and mean water level, \( \bar{n} \), may be derived. As shown in Paragraph 3.2 they read:

\[
\frac{d}{dx} \left( \frac{E}{\omega_r} \left( c_g \cos \Theta + U_x \right) \right) + \frac{D}{\omega_r} = 0 \tag{B.1}
\]

\[
\frac{d}{dx} S_{xx} + \rho g (d + \bar{n}) \frac{d\bar{n}}{dx} = 0 \tag{B.2}
\]

where

\[
E = \frac{1}{8} \rho g H_{\text{rms}}^2
\]

\[
\omega_r = \omega - k_x U_x - k_y U_y
\]

with

\[
\omega = 2\pi f
\]

\[
k = \sqrt{k_x^2 + k_y^2}
\]

\[
U_x = \left| \mathbf{U} \right| \cos \nu
\]

\[
U_y = \left| \mathbf{U} \right| \sin \nu
\]

\[
\omega_r = \left[ gk \tanh(kh) \right]^{1/4}
\]

\[
c_g = \left| \mathbf{c}_g \right| = \omega_r / \partial k
\]

\[
D = D_b + D_f
\]

with

\[
D_b = \frac{a}{4} \rho g \frac{\omega_r}{2\pi} H_{\text{rms}}^2 q_b
\]
\[ D_f = \rho_f c_w \left( \frac{\omega}{\sinh \, kh} \right)^3 \]
\[ S_{xx} = \left( \frac{c_g}{c} \right) (1 + \cos^2 \Theta) - \tfrac{1}{4} E. \]

It is noted that the effects of a current field on the momentum equation are neglected so far, considering that this concerns a third order effect on the energy decay only. In addition to the equations above we use Snell's law:

\[ k_y = k \sin \Theta = \text{constant} \quad (B.3) \]

For each step in the direction normal to the depth contours, i.e. the \( x \)-direction, the differential equations are solved alternately in an iteration procedure. The energy equation generalized to the wave action equation is tackled first.

The reduced wave action equation is solved by a fourth order Runge-Kutta method. In this method first approximations are needed for \( c_g, c, k, H_m \) and \( \Theta \) on the forward step being functions of \( h = d + \eta \). So as an approximation on the forward step \( i+1 \) we use

\[ h_{i+1} = d_{i+1} + \bar{\eta}_1 \quad (B.4) \]

which is accurate in general since the variations in \( \eta \) are small compared to the variations in \( d \). The reduced action equation reads:

\[ \frac{d}{dx} \left\{ E' \left( c_g \cos \Theta + U_x \right) \right\} + \frac{D_b'}{\omega_r} + \frac{D_f'}{\omega_r} = 0 \quad (B.5) \]

where

\[ E' = H_{\text{rms}}^2 = E/(\rho g/\delta) \]
\[ D_b' = \frac{a}{\pi} \omega_r H_m^2 Q_b \]
\[ D_f' = \frac{f_w}{\gamma} \left( \frac{H_{\text{rms}}}{\sinh \, kh} \right)^3 \]

Now let \( y = \frac{H_{\text{rms}}^2}{\omega_r} \left( c_g \cos \Theta + U_x \right) \quad (B.6) \)
then \( \frac{dy}{dx} = f(x,y) = - \left( \frac{D'_b}{\omega_r} + \frac{D'_r}{\omega_r} \right) \) (B.7)

and the fourth order Runge-Kutta method yields

\[
\Delta y = y_{i+1} - y_i = \frac{1}{6} \left( k_1 + 2k_2 + 2k_3 + k_4 \right)
\]

where

\[
\begin{align*}
k_1 &= \Delta x.f(x_i, y_i) \\
k_2 &= \Delta x.f(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}k_1) \\
k_3 &= \Delta x.f(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}k_2) \\
k_4 &= \Delta x.f(x_i + \Delta x, y_i + k_3)
\end{align*}
\]

so \( k_1 = k_{1b} + k_{1f} = \alpha u^{-1} Q_{b_1} H_{1i}^{2} + f \omega g^{-1} \pi \frac{1}{r_i} H_{3i}^{3} \sinh^{-3} k_i h_i \)

etc.

In order to solve \( k_2 \), \( k_3 \) and \( k_4 \) we need first approximations to \( H_{rms,i+\frac{1}{2}} \) and \( H_{rms,i+1} \). Here we may use for \( k_2 \):

\[
H_{rms,i+\frac{1}{2}}^2 = \left( y_{i+\frac{1}{2}} + k_1 \right) \frac{\omega}{r_{i+\frac{1}{2}}} \frac{c}{g_{i+\frac{1}{2}}} \frac{\cos \theta_{i+\frac{1}{2}} + U_{x_{i+\frac{1}{2}}}}{x_{i+\frac{1}{2}}}
\]

(B.9)

and similarly for \( k_3 \) and \( k_4 \).

Here \( y = H_{rms,i}^2 \left( c \cos \theta + U_{x_i} \right) / \omega \).

The final solution for \( H_{rms,i+1} \) gives:

\[
H_{rms,i+1}^2 = y_{i+1} \frac{\omega}{r_{i+1} g_{i+1}} \frac{c}{g_{i+1}} \frac{\cos \theta_{i+1} + U_{x_{i+1}}}{x_{i+1}}
\]

(B.10)

where \( y_{i+1} = \Delta y + y_i \).

At this point in the procedure the wave growth due to wind is calculated according to the method of Paragraph 3.3. The growth in terms of \( H_{rms} \) is simply added to the calculated \( H_{rms,i+1} \).

The second step in the iteration procedure is now to solve the momentum equation. In an explicit differential form for \( \bar{\eta} \) this equation may be written as
\[ \tilde{\eta}_{i+1} = \tilde{\eta}_i + \frac{\frac{1}{8} H^2 \text{rms} \left[ \frac{c_{\text{rms}}}{c_i} \left( 1 + \cos^2 \theta_{i+1} \right) \right] - \frac{1}{8} H^2 \text{rms} \left[ \frac{c_{\text{rms}}}{c_{i+1}} \left( 1 + \cos^2 \theta_{i+1} \right) \right]}{\frac{1}{2} (h_{i+1} + h_i)} \] (B.11)

A first approximation to all the variables on step i+1 is now obtained. The procedure is repeated at least once more and thereafter until the variation of one of the variables is below a required level or when a limit number of iterations is reached. This concludes the procedure for one step, so the programme proceeds to the next step.

Finally, it is noted that in the evaluation of the parameters \(k\) and \(Q_b\) - which are given by transcendental functions - iteration procedures are involved. In the first case the standard Newton-Raphson method is used. In the second case, however, it was decided to use a more efficient algorithm for \(Q_b\) since the Newton-Raphson method converges rather slowly in this case. This algorithm was developed in the context of the study of Dingemans (1983), to which the reader is referred.
APPENDIX C: THE BOTTOM FRICTION DISSIPATION TERM

In this Appendix the function $D_f$ is evaluated which is given as

$$D_f = \frac{1}{6\pi} \rho \omega w s_o^3 \int_0^\infty H^3 F(H), \quad (C.1)$$

where $s_o = \omega / \sinh kh$ and $F(H)$ is given either by the full or the clipped Rayleigh distribution:

$$F(H) \equiv P(H < H) = 1 - \exp \left[-\frac{1}{4} \left(\frac{H}{\hat{H}}\right)^2\right] \quad ; \quad 0 < H < \infty \quad (C.2)$$

or

$$F(H) \equiv P(H < H) = 1 - \exp \left[-\frac{1}{4} \left(\frac{H}{\hat{H}}\right)^2\right] \quad ; \quad 0 < H < H_m$$

$$= 1 \quad ; \quad H > H_m \quad (C.3)$$

The integral in Equation (C.1) given as

$$I = \int_0^\infty H^3 dF(h)$$

is to be evaluated.

The full Rayleigh distribution

With $F(H)$ given by (C.2) one obtains

$$I = - \int_0^\infty H^3 d \exp \left[-\frac{1}{4} \left(\frac{H}{\hat{H}}\right)^2\right] \quad (C.4)$$

and a simple calculation, using $\int_0^\infty \exp(-z^2/2) \, dz = \sqrt{\pi}/2$, yields

$$I = 3\hat{H}^3 (\pi/2)^{3/2}. \quad (C.5)$$

Because $H^2_{rms} = \int_0^\infty H^2 \, dF(H) = 2\hat{H}^2$ we have then

$$I = \frac{3}{4} \sqrt{\pi} H^3_{rms}, \quad (C.6)$$
and thus, from (C.1),

\[
D_f = \frac{1}{8\pi^\frac{3}{2}} \rho f_w s_0^3 H_m^3 \rho \]

(C.7)

This formulation (C.7) is used for the dissipation function in the one-dimensional wave model.

The clipped Rayleigh distribution

With \( F(H) \) given by (C.3) one has

\[
I = \int _0 ^ H \frac{H^m}{H} dF(H) = \int _0 ^ H \frac{H^3}{H} dF(H) + Q_b H_m^3 = 
\]

\[
= H^3 \int _0 ^ H \frac{(H/H)^3}{H} dF(H) + Q_b H_m^3 = H^3 I + Q_b H_m^3 
\]

(C.8)

The integral \( J \) is to be evaluated. With the substitution

\[
z = H/H, \quad z_m = H_m/H
\]

one obtains

\[
J = -z_m^3 Q_b + \int _0 ^ {z_m} 3z^2 \exp(-z^2/2)dz,
\]

and thus,

\[
I = H^3 \int _0 ^ {z_m} 3z^2 \exp(-z^2/2)dz = H^3 K
\]

(C.9)

The integral \( K \) is now to be evaluated. It is possible to express \( K \) in terms of the incomplete Gamma function \( \gamma(a,x) \) or in terms of the error function \( \text{erf}(x) \). One has

\[
\gamma(a,x) = \int _0 ^ x t^{a-1} e^{-t} dt; \quad \text{(C.10)}
\]

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int _0 ^ x e^{-t^2} dt. \quad \text{(C.11)}
\]

At first the expression in terms if the incomplete Gamma function is given.

Using the substitution \( t = z^2/2 \) one finds for \( K \):
\[ K = 3^{1/2} \gamma(\frac{3}{2}, \frac{z_m^2}{2}), \]

and thus \( I \) is given as

\[ I = 3^{1/2} \hat{H}^3 \gamma[\frac{3}{2}, \frac{1}{4}(\frac{H_m}{\hat{H}})^2], \quad \text{(C.12)} \]

where \( H_{rms}^2 = 2(1-Q_b)\hat{H}^2. \)

A further elaboration is given in Dingemans(1983). His numerical evaluation leads to the conclusion that the dissipation function \( D_f \) for bottom friction is not very sensitive for the accounting of the occurrence of wave breaking as reflected in the use of the clipped Rayleigh distribution for the wave height. This is especially true in the range \( 0.4 < H_{rms}/H_m < 0.6 \) for which most of the dissipation due to wave breaking takes place. For \( H_{rms}/H_m = 0.6 \) one has with clipped distribution a \( D_b \) which is 94% in value of the \( D_b \) for the unclipped Rayleigh distribution. In view of the fact that the choice of the friction factor \( f_w \) is rather arbitrary, there is not enough reason to choose expression (C.12) in favour of (C.7), although the use of (C.12) is more consistent when dissipation due to breaking is included than (C.7) is.
REFERENCES


Dingemans, M.W., Verification of numerical wave propagation models with field measurements; Crediz verification Haringvliet, Report W 488, Delft Hydraulics Laboratory, 1983.


Stive, M.J.F., A scale comparison of waves breaking on a beach, Accepted for publication to J. Coast. Eng., 1984b.

TYPICAL EXAMPLE OF BJ's TESTCASES
THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSEN (1978) RUN 15
DELFT HYDRAULICS LABORATORY  M1882  FIG. 1
MODEL SENSIVITY FOR A VARIATION IN $\alpha$

DELFT HYDRAULICS LABORATORY

RUN 155
DELTAX=1.0
GAMMA=0.8

M1882 FIG. 2
MODEL SENSIVITY FOR A VARIATION IN $\gamma$

RUN 15S
DELTA$\alpha$=1.
ALFA=1.0

DELFIT HYDRAULICS LABORATORY

M1882 FIG. 3
MODEL COMPARISON FOR DIFFERENT (α,γ) COMBINATIONS

DELTAX = 1

DELT HYDRAULICS LABORATORY

M1882 FIG. 4
ESTIMATED VALUES OF BREAKERHEIGHT COEFFICIENT $\gamma$ VS. DEEP WATER WAVE STEEPNESS $S_0$

DELFt HYDRAULICS LABORATORY
THE EFFECT OF DISSIPATION DUE TO BOTTOM PRICITION
THEORY (LINES)
EQUID EXP. (SYMBOLS) OF 5.6.81 (EG03, EG04, EG05)
DELFT HYDRAULICS LABORATORY
M1882 FIG. 7
THE EFFECT OF GAIN DUE TO WIND
THEORY (LINES)
RWS MEAS. HARINGVLIET/SELECTION 171182, SEA.WL+NAP+0.2M
DELFT HYDRAULICS LABORATORY

RUN 1711A  DELTAX=50
ALFA=1.0/FW=0.0/WS=VAR

M1882  FIG. 8
RESULTS FOR CASE 1
THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSSEN (1980) RUN HJ02

DELTAX=.25
ALPHA GAMMA STANDARDFORM.

DELFIT HYDRAULICS LABORATORY
M1882 FIG. 10
RESULTS FOR CASE 2
THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSEN (1980) RUN HJ03
ALFA&GAMMA STANDARDFORM.
DELTAX = .25
M1882 FIG. 11
RESULTS FOR CASE 3

THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSSEN (1980) RUN HJ04

ALFA&GAMMA STANDARDFORM.

DELF HydRULICS LABORATORY

M1882 FIG. 12
RESULTS FOR CASE 4

THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSEN (1980) RUN HJ11

DELFT HYDRAULICS LABORATORY

RUN HJ11  DELTAX=0.25
ALPHA & GAMMA STANDARD FORM

M1882  FIG. 13
RESULTS FOR CASE 5
THEORY (LINES) EXPERIMENTS (SYMBOLS) OF JANSSSEN (1980) RUN HJ12
DELFT HYDRAULICS LABORATORY

DELTA HYDRAULICS LABORATORY

M1882 FIG. 14
RESULTS FOR CASE 6

THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF JANSSEN (1980) RUN HJ13

DELFT HYDRAULICS LABORATORY

RUN HJ13 DELTAX=.20 ALFA&GAMMA STANDARDFORM.

M1882 FIG. 15
RESULTS FOR CASE 7
THEORY (LINES) EXPERIMENTS (SYMBOLS) OF JANSSEN (1980) RUN HJ14
ALFA&GAMMA STANDARDFORM.
DELFT HYDRAULICS LABORATORY M1882 FIG. 16
RESULTS FOR CASE 8

THEORY (LINES) EXPERIMENTS (SYMBOLS) OF JANSSEN (1980) RUN HJ15

ALFA&GAMMA STANDARD FORM.

DELFt HYDRAULICS LABORATORY

M1882 FIG. 17
RESULTS FOR CASE 9
THEORY (LINES)
EXPERIMENTS (SYMBOLS) OF STIVE (1977) RUN10

DELFT HYDRAULICS LABORATORY

RUN MS10 DELTAX=0.5
ALFA&GAMMA STANDARDFORM.
RESULTS FOR CASE 11
THEORY (LINES)
DELTAGOOT EXPERIMENTS (SYMBOLS) OF STIVE (1983) RUN DG 01
ALFA&GAMMA STANDARDFORM.

DELFHYDRAULICS LABORATORY
RESULTS FOR CASE 12
THEORY (LINES)
LIDO DI DANTE EXP. (SYMBOLS), TEST P180 AND P186

DELFT HYDRAULICS LABORATORY

RUN P1B   DELTA=0.5
ALFA&GAMMA STANDARD FORM.
RESULTS FOR CASE 13
THEORY (LINES)
LIDO DI DANTE EXP. (SYMBOLS), TEST PICO AND PICO6
ALFA&GAMMA STANDARDFORM.
DELFT HYDRAULICS LABORATORY

M1882  FIG. 22
RESULTS FOR CASE 15

RUN P400  DELTAX=.5
ALFA&GAMMA STANDARDFORM

DELT HYDRAULICS LABORATORY  M1882  FIG. 24
RESULTS FOR CASE 17

THEORY (LINES)
EGMUND EXP. (SYMBOLS) OF S6.81 (T5=ENG1, ENG2, ENG3, ENG4, ENG5)
ALFA & GAMMA STANDARDFORM.

DELFT HYDRAULICS LABORATORY

RUN EG/T5/81 DELTAX=10
M1882 FIG. 26
RESULTS FOR CASE 19
THEORY (LINES)
RWS MEAS. HARINGVLIET/MT7 220982&161082 SWELL WL:NAP+1.43M

DELFT HYDRAULICS LABORATORY M1882 FIG. 28
RESULTS FOR CASE 20
THEORY (LINES)
RWS MEAS. HARINGVLIET/SELECTION 141182,SEA.WL:NAP+0.2M

DELFT HYDRAULICS LABORATORY
M1882 FIG. 29
BEST PROPORTIONAL FIT:
HRMS/HRMS0,COMP = 1.01*HRMS/HRMS0,MEAS
CORRELATION COEFFICIENT R=.98
RELATIVE RMS ERROR=.06

COMPARISON OF MEASUREMENTS AND COMPUTATIONS
WITH STANDARD FORMULATION FOR ALFA AND GAMMA

DELFt HYDRAULICS LABORATORY M1882 FIG.30