Contactless Actuation of a Deformable Mirror by means of Radiative Heat Input

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Abstract

In the semiconductor industry the pursuit for ever decreasing feature size on integrated circuits demands for extreme performance specifications on optical components of photolithographic imaging systems. Moreover, light source wavelength used in such systems is brought down to the extreme ultraviolet (EUV) range to increase imaging resolution. The EUV light creates a thermal footprint on (reflective) optical components resulting in thermal deformation and consequently optical aberrations. A possible way of correcting optical aberrations is the active control of mirror surface shape. Current deformable mirror technologies are not suitable for application in EUV lithography systems. Mechanical noise due to physical connections to the environment is a major concern. Furthermore thin flexible structures that are used for deformable mirror faceplates are not suitable to handle the thermal load induced by the use of EUV light.

A novel idea is to deform the shape of mirror substrates by means of controlled thermal radiation on an absorptive coating accessible from the back side. This concept is unique because it is fully contactless and it allows for the controlled thermal deformation of a passive mirror substrate. Modeling work is done to identify substrate deformation properties and FEM analysis is used to find heat flux distributions for desired surface shapes. For the future design of a deformable mirror design parameters are identified and evaluated in an experiment setup. Experimental results confirmed the deformation model and showed system linearity, superposition and how material and geometry affect the actuated output. Furthermore, the mirror surface was deformed according to a set of Zernike polynomials. Lower order Zernikes (up to \( Z_3^3 \)) are actuated with amplitudes of \( \sim 50 \, \text{to} \, 70 \, \text{[nm]} \) and \( \sim 5\% \) RMS error.

The work described in this thesis is a first step towards the design of a deformable mirror by controlled radiative heat input. Essential features of the system are indicated and design parameters are evaluated. Although there are open ends for further research, experiments on the shaping of the surface according to the Zernike Polynomials proved that this concept for a deformable mirror is feasible.
Abstract

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Integrated Circuits (IC’s) have become an integral part of modern society. Composed of large numbers of transistors, IC’s have found their way in numerous applications in our daily lives: mobile phones, computer memory and household appliances just to name a few. IC’s are produced by photolithographic imaging systems. The theoretical resolution of any imaging system is bound by the diffraction limit which is proportional to the wavelength used. In the pursuit of Moore’s law to double the number of transistors on an integrated circuit every 18 months, deep ultraviolet (DUV) lithography systems have reached their limits. Due to the demand for ever decreasing feature size, shorter wavelength is needed. Current technological developments focus on the use of extreme ultraviolet (EUV) part of the electromagnetic spectrum, utilizing a wavelength of only 13.5 $[nm]$ instead of 193 $[nm]$ which is used in current DUV lithography systems [2].

![Reflective Mask, Vacuum Isolation, Illuminator Optics, EUV Generating Plasma, Collector, 6-Mirror Projection Optics, 300 mm Wafer](image)

Figure 1.1: Reflective optics in EUV Lithography. Figure modified from [3].
1.1 Problem statement

Problems arise with the use of EUV light mainly because virtually all materials absorb radiation in the EUV range. This puts some hard constraints on the use of EUV light in lithography systems. The entire imaging system must be situated in a vacuum environment to avoid absorption by a medium. Furthermore, the use of refractive optics is impossible because by nature it involves the travel of light through a transparent medium (lenses). This leaves reflective optics as the only option to comprise an EUV optical system. Nevertheless, for reflective optical components maximum achievable reflectivity of a typical mirror design is roughly around $R \approx 0.7^1$ by using two-component multilayer stacks of reflective coatings $^2$. A major problem of this low reflectivity is that the loss of optical power induces elevated temperature which can result in significant thermal deformations of the mirror. Unwanted mirror deformation results in optical aberrations and this affects imaging resolution.

1.2 Adaptive Optics Systems

The physical phenomenon of light can be described by a wave traveling through space. Taking a cross section of a bundle of light yields a spatial distribution of the phase of light. This spatial distribution of phase is the wavefront. Deviations in phase from a desired situation, phase aberration, implies deviation of the wavefront, so called wavefront error. In optical systems wavefront errors can be corrected by means of active control of one or more optical elements. Putting an active optical element in a control loop to correct for these wavefront errors goes by the name of Adaptive Optics (AO). This technology is best known for its application in astronomical observations. The resolution of a telescope is proportional its aperture as long as the system is diffraction limited. This makes it attractive to use a large diameter telescope for observing the night skies. However, above a certain diameter ($\varnothing \sim 1 \, [m]$) severe imaging limitations occur due to wavefront error induced by atmospheric turbulence. The immense practical and financial consequences of putting a large telescope into orbit can be avoided by the use of adaptive optics to increase imaging resolution.

The first to note that wavefront error can be corrected by active control of an optical element was H.W. Babcock. Babcock proposed a 'compensator' for astronomical seeing in 1953 $^5$. However, in 1953 the technology to successfully implement such a system was not yet at hands. In the 70’s adaptive optics was developed mainly for military applications and it became widely used in astronomical observations with large diameter optical telescopes from the 90’s on.

---

$^1 R$ is used to denote reflectivity. This dimensionless number between 0 and 1 indicates the portion of reflected light.
Nowadays there are numerous applications in which adaptive optics is used to reduce optical aberrations. Apart from astronomical observations, applications include (among others) laser communications, retinal imaging, microscopy and optical measurement systems.

Figure 1.2 show a schematic of a typical adaptive optics system showing all major components.

Figure 1.2: Schematic of an Adaptive Optics system showing major components: wavefront sensor, deformable mirror and control system.

In Figure 1.2 light with an aberrated wavefront is projected onto the deformable mirror. The deformable mirror is part of an imaging system. Using a beamsplitter in the optical path a part of the light is projected onto a wavefront sensor. The wavefront error, is used as error signal in the control loop. The controller actuates the deformable mirror to attenuate this error.

In an EUV lithography system it is possible that a wavefront error is introduced in the optical system itself by unwanted deformation due to the inherent heat input by the EUV light. Adaptive optics technology can be utilized to correct for these errors.
The deformable mirror is one of the key components of an adaptive optics system. There has been extensive research and development on deformable mirrors resulting in a variety of different concepts. Current deformable mirror designs are not suitable for the application of EUV lithography. There are concerns about mechanical noise of an actuator driven mirror affecting imaging resolution. In addition, deformable mirrors typically have a compliant face plate that is easy to deform because of limited thickness. This makes these structures unsuitable to deal with the thermal loading introduced by the EUV light.

In EUV lithography systems the amplitude of surface shapes that are to be actuated by a deformable mirror is up to tens of nanometers. Furthermore, the position tolerances of optical elements are such stringent that elements are suspended in space by electromechanic actuators. Any undesired vibration is suppressed by a control system. Using radiant (thermal) input, energy can be provided to a mirror substrate without physical contact. The provided energy manifests itself in the form of a temperature distribution in the substrate. Any mechanical stress is contained within the material of the mirror itself. An induced temperature distribution in the mirror substrate results in thermal deformation, shaping the substrate surface. This idea seems promising for application in EUV lithography systems.

1.3 Relevant research

There is some relevant research on contactless (thermal) adaptive optics systems, mainly for the application of detecting gravitational waves. Gravitational waves are minute distortions of spacetime, a concept in physics, and are predicted by Einstein’s relativity theory. To detect these distortions high-power laser interferometers are used to detect motion between two distant masses. In this application there is no room to budget errors introduced by mechanical disturbances due to actuation of optical components. To correct for laser wavefront error some solutions are proposed making use of thermal radiation.

A radiative corrector is proposed by Lawrence [6] to overcome errors due to variations of refractive index in lenses by absorption of laser beam power (so-called ‘thermal lensing’). The main idea is to compensate thermal lensing by thermal radiation onto the parts of the lens that absorb relative little power of the laser. Variations of the refractive index of the lens material can be counteracted to some extend using this concept. Correction towards a more homogeneous temperature of an optical element is a very useful concept when the errors induced by an optical element are due to an inhomogeneous temperature distribution. The idea of compensating an inhomogeneous refraction index by ensuring a homogeneous temperature distribution is easily extended to compensating an erroneous reflection surface.
Errors due to thermal lensing are predictable up to a certain extend because the power input distribution that results in the errors has a stationary shape. This allows the use of a corrector that provides thermal input that also has stationary shape. The objective of this project is to obtain a deformable mirror for a variety of shapes (the Zernike polynomials). Hence, a corrector must be found that can provide a (controlled) spatially varying thermal input instead of a stationary one.

Lück describes thermally adaptive optics [7] to control the surface profile of reflective components. A high power laser beam is used to control the thermal expansion of an optical element locally. Controlling the exposure time of a 5 by 5 grid of points on the mirror surface corrections were achieved for a displaced and tilted mirror.

The paper shows the ability to control an entire optical surface, by means of thermal radiative input. Although the study goes into surface deformation of one point only and the control of a surface, it does not go into the design of the optical element, the parameters and their relation to the deformation behavior of the element. The study of these design parameters is needed in order to be able to design a wavefront corrector for the use in EUV lithography systems.

1.4 Project Objectives

The goal of the project is to map out the possibilities of obtaining a deformable mirror, actuated solely by thermal radiative heat input. There is little relevant research done on this actuation concept (see: Section 1.3), so the focus will be at first to make a model of the behavior of such a system. It is key to investigate the concept properties and how they relate to parameters involved. A clear understanding of these relations is important for further development of a contactless thermally actuated mirror in an application setting. When a model of mirror deformation is at hand, it must be validated in a laboratory environment.

A deformable mirror must be able to deform into a certain set of surface shapes, the Zernike polynomials. This orthogonal set of shape functions is used to express optical aberrations. Actuation inputs for these surface shapes can serve as feed forward control signals. Deforming a mirror substrate according to the Zernike polynomials would be a proof of concept for a deformable mirror by thermal radiation. The range of amplitudes of a deformed shape that is of interest is up to tens of nanometers. To summarize, the project objectives are:

1. Obtain and validate clear model of mirror deformation by thermal input.

2. Find and evaluate deformable mirror design parameters and how they affect deformation behavior. The outcome of this can serve as input for deformable mirror design.

3. Obtain control inputs for a set of predefined surface shapes (Zernike polynomials) and actuate these surface shapes. This will be a proof of concept.
1.5 Thesis outline

In this thesis, Chapter 2 will start with discussing the thermo-mechanical behavior of a mirror substrate. The deformation of a substrate is discussed and divided into three different 'deformation modes'. This provides insight in the overall behavior of the element and how to actuate its shape. Chapter 3 introduces design parameters of a deformable mirror. The design parameters are related to expected substrate deformation behavior and this gives rise to a set of sample substrates for experimental validation. The practical side of this project is discussed in chapter 4. It will discuss the measurement setup and its properties and present the experiments that are performed on the sample substrates. The results of the experimental work are presented and discussed in Chapter 5. Chapter 6 will wind up this thesis with conclusions on the project and recommendations for future work.
When material is subjected to a thermal input, the temperature will rise and this will cause the material to expand. If this thermal input is simple, for example uniform throughout the material, modeling the deformation is trivial. Thermal input can also come in a more complex form, for example on a two-dimensional plane of a three-dimensional body. Possibly even with variation in amplitude across this input plane. Modeling of the thermal deformation due to such a thermal input is more complex.

This chapter will discuss the thermal deformation of the mirror substrate. The substrate can deform in several different ways that need to be identified and what temperature changes drive these deformations. A preliminary mirror configuration serves as the starting point of the deformation analysis. The deformation of this mirror is distinguished into three different modes of thermal deformation. Simple models are made to describe the amplitude and speed of the modes. A preliminary configuration is taken as a starting point for this analysis.

First, Section 2.1 will map out some boundary conditions of the system and list specifications of the preliminary mirror configuration. Section 2.2 will provide a short overview of thermo-mechanical theory that relates the mechanical behavior of the mirror to heat input. The three different deformation modes are discussed in Section 2.3.
2.1 Boundary conditions

This section discusses general boundary conditions as well as properties of the preliminary mirror configuration concerning material and heat transfer.

2.1.1 Heat transfer

The only way of heat transfer from a mirror in an EUV lithography system to its ambient is thermal radiation, which is a nonlinear phenomenon. In the case of an object of temperature $T_1$ that is completely surrounded by an object of temperature $T_2$ and the surface of the object is small compared to its surrounding, the heat transfer by radiation between these bodies is defined \[8\]:

$$q = \varepsilon_1 \sigma (T_1^4 - T_2^4)$$  \hspace{1cm} (2.1)

Where $q$ is the heat transfer in $[W/m^2]$, $\varepsilon_1$ the emissivity $[-]$ of the body with temperature $T_1$ in $[K]$ and $\sigma$ is the Stefan-Boltzmann constant $^1$. This non-linear relation can be linearized:

$$q \simeq \varepsilon_1 \sigma 4T_m^3 (T_1 - T_2)$$  \hspace{1cm} (2.2)

In which $T_m$ is the mean temperature of $T_1$ and $T_2$. At the cost of an acceptable error $^2$, this allows a much more easy model as Equation 2.2 reveals a radiative heat transfer coefficient:

$$h_r = \varepsilon_1 \sigma 4T_m^3$$  \hspace{1cm} (2.3)

This heat transfer coefficient in $[W/(m^2 \cdot K)]$ can be used for further analysis in Section 2.3, provided that care must be taken with using values for mean temperature $T_m$. To find this mean temperature in calculations an iterative process can be used to check whether this mean temperature has converged to the average of the found corresponding temperatures $T_1$ and $T_2$.

2.1.2 Preliminary configuration

The following conditions are basis of the preliminary configuration:

1. The mirror substrate will be situated in a vacuum enclosure as this will also be the case in an EUV applications.

---

$^1$Stefan-Boltzmann constant: $\sigma = 5.67 \cdot 10^{-8} [W/(m^2 \cdot K^4)]$

$^2$The error is 2% for $T_1 = 400 [K]$ and $T_2 = 300 [K]$
2. The top surface of the substrate has a reflective coating. This coating must be on top of the substrate as this is a requirement for any EUV mirror. Furthermore, thermal lensing is ruled out by placing the coating on top (this can possibly affect measurements). The reflective coating has an emissivity of $\epsilon_r = 0.04^3$.

3. The mirror must be able to absorb a thermal radiative input to control the temperature distribution and thus its shape. This is done by applying an absorptive coating which can be exposed to heat flux.

The preliminary substrate configuration that results from the listed boundary conditions is shown in Figure 2.1.

Figure 2.1: Schematic of the preliminary substrate configuration. The substrate receives a thermal input in the visible range through the transparent substrate material. The reflective coating is situated on top of the substrate. The substrate material mainly radiates in the infrared range (at room temperature).

The preliminary configuration consists of a transparent substrate with two coatings: one absorptive coating and a reflective coating on top. Most glass materials are transmissive for visible light. This means the absorptive coating can be subjected to a radiative thermal input in the visible range, through the substrate material. Also most glass materials do have substantial emissivity: they are radiative in the infra-red range$^4$. This means that the glass is able to transfer heat to the environment by means of thermal radiation.

The preliminary substrate is a square mirror of $50 \times 50 \text{ [mm]}$ reflective surface and has a thickness of $4 \text{ [mm]}$. The material used for the mirror is yet to be specified and can be chosen to suit the actuation principle best. To create a starting point for initial calculations and to get grip on the order of magnitude of effects that is likely to be expected, a material was selected for the preliminary substrate configuration. The selected material is BK7 glass which is known for its stable behavior in various thermal environments and optical quality. Relevant material

$^3$In $\epsilon_r$ the subscript ‘r’ denotes ‘reflective coating’.

$^4$According to Wien’s displacement law $^8$, peak thermal radiation takes place at the far-infrared ($\lambda \sim 10\mu m$) for objects at room temperature.
Figure 2.2: Schematic of the substrate showing coordinate system in space. These coordinates will be used throughout this report. The plane of the reflective coating is at $z = 0$.

properties for BK7 glass are listed in Table 2.1. Figure 2.2 shows a substrate together with a coordinate system. This definition of coordinates will be used throughout this report.

Table 2.1: Material properties for BK7 glass [9]. The table lists relevant properties for evaluating thermal deformation behavior of mirror substrates.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>$E$</td>
<td>81e9</td>
<td>$[N/m^2]$</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>0.3</td>
<td>$[-]$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>2510</td>
<td>$[kg/m^3]$</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>$\alpha$</td>
<td>7.1e-6</td>
<td>$[1/K]$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$k$</td>
<td>1.114</td>
<td>$[W/(m \cdot K)]$</td>
</tr>
<tr>
<td>Heat capacity at constant pressure</td>
<td>$C_p$</td>
<td>858</td>
<td>$[J/(kg \cdot K)]$</td>
</tr>
<tr>
<td>Emissivity</td>
<td>$\epsilon$</td>
<td>0.8</td>
<td>$[-]$</td>
</tr>
</tbody>
</table>

2.2 Thermo-mechanical behavior

This section will discuss the thermo-mechanical behavior of the mirror substrate according to theory. If an unconstrained body undergoes a uniform temperature change it will expand or contract homogeneously resulting in a stress-free material. A dimensionally constraint body undergoing a temperature change will also induce thermal stress. Providing a material with a non-uniform temperature change will result in a material that is partially deformed according to the temperature distribution, but also has induced thermal stress preventing it to fully expand. The thermal behavior of a solid is found from the energy balance [10] for an infinitesimal solid element.

\footnote{Note: summation convention of Einstein.}
In which \( e(T, \varepsilon_{ij}) \) denotes internal energy in \([J/m^3]\), \( t \) time in \([s]\), \( q_i \) the heat flux components in \([W/m^2]\), \( x_i \) spatial coordinates and \( i \) and \( j \) spatial directions, \( \sigma_{ij} \) mechanical stresses in \([N/m^2]\), \( \varepsilon_{ij} \) mechanical strains \([-\) and \( Q \) internal heat source in \([W/m^3]\). It is assumed the internal energy \( e \) is only a function of temperature.

\[
\frac{\partial e}{\partial t} + \frac{\partial q_i}{\partial x_i} - \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial t} = Q \tag{2.4}
\]

Where \( \rho \) is material density and \( C_p \) heat capacity. The heat flux component \( q_i \) in a solid in direction \( i \):

\[
q_i = -k \frac{\partial T}{\partial x_i} \tag{2.6}
\]

Where \( k \) is the thermal conductivity \([W/(m \cdot K)]\). By assuming the internal energy is only a function of temperature the mechanical work done by the strain rate \( \sigma \frac{\partial \varepsilon}{\partial t} \) can be left out. This assumption is discussed in Appendix A. The thermal behavior can now be described substituting Equations (2.5) and (2.6) in Equation (2.4). This results in Fourier’s heat equation:

\[
\rho C_p \frac{\partial T}{\partial t} - k \sum_{i=1}^{3} \frac{\partial^2 T}{\partial x_i^2} = Q \tag{2.7}
\]

From the above reasoning it follows:

1. Mechanical work does not affect the temperature response significantly.

2. The thermo-mechanical coupling is one-way: temperature profile affects mirror shape, vice-versa not.

The main important implication of this is that the surface shape response due to a heat input is a direct effect of the response of the temperature distribution in the substrate material. The temperature distribution in itself is again a direct effect of the thermal input. Because the thermo-mechanical coupling is one-way, the model stays simple as depicted in Figure 2.3.

![Figure 2.3: One-way thermo-mechanical coupling.](image-url)

---

6This assumption is discussed in Appendix A
2.3 Modes of thermal deformation

A basic idea of how the mirror substrate deforms when subjected to a thermal load must be developed in order to model the response. The mirror depicted in Figure 2.4 shows a mirror with a hotspot at the location of a heat input. The mirror is situated in an environment of temperature $T_0$. The colors depict profiles of equal temperature. A thermal input on a mirror will result in an elevated temperature of the mirror with respect to the environment. The elevated mirror temperature ensures heat transfer to the environment and rises to eventually reach a steady state temperature. However, if the heat input is local, as in Figure 2.4, heat transfer takes place from the location of the input to the rest of the body. This heat conduction can only occur due to a temperature gradient. This results in local temperature differences in the mirror material. From the temperature distribution in Figure 2.4 the following can be observed:

1. A global rise of the average material temperature with respect to ambient temperature: $\Delta T_{0-1}$. 

As a consequence, the mechanical response of a mirror substrate can be found by first solving the thermal response. The thermal response of a solid is governed by Fourier’s heat equation. Solving Fourier’s equation will result in an exact solution of the heat problem in the form of a series. For the understanding how the mirror deforms when it is subjected to a certain heat load, an exact solution of the temperature response is not a necessary prerequisite. It is important to obtain a simple model that relates mirror parameters to the deformation behavior. The base of this model is discussed in the next section and the extension to the design parameters follows in Chapter 3.
2. A temperature gradient across the thickness of the mirror: $\Delta T_{2-3}$.

3. A temperature gradient in the lateral direction of the mirror: $\Delta T_{4-1}$.

These three temperature differences give rise to three different deformation modes. These modes will be modeled separately to provide insight in their properties and the parameters that affect these properties.

### 2.3.1 Mode 1: Lumped Mass

Mode 1 is the deformation that occurs from the global rise of temperature, $\Delta T_{0-1}$ in Figure 2.4. When the internal thermal resistance of a body is small compared to the thermal resistance of the body to the environment, it can be assumed that the body temperature rises uniformly due to a thermal input in most engineering problems. This assumption of thermal energy distributed evenly throughout a body is often referred to as *lumped thermal analysis* [8].

The deformation that occurs due to a uniform temperature rise in a material is also uniform. The body expands equally in all directions as is seen in Figure 2.5. Because of the uniform expansion, the surface shape of a mirror cannot be actuated using this mode. The validity of using lumped thermal analysis is based on a dimensionless number, the *Biot number* [8]. This number can be seen as the ratio of the thermal resistance of the body to ambient over the body internal thermal resistance. The lower the Biot number, the lower temperature gradients in the body. For general engineering problems, lumped thermal analysis is used for thermal problems with $Bi < 0.1$.

$$Bi = \frac{h_l}{k}$$

Where $h$ is the generalized heat transfer coefficient in $[W/(m^2 \cdot K)]$. In this case of a mirror substrate, it is the average of the area-weighted radiative heat transfer coefficients $h_r$ of the reflective coating and the glass material. Length $l$ is an appropriate characteristic length of the object. The analysis of discrete thermal problems is analog to electric circuit analysis. Figure 2.6 shows the thermal circuit for the analysis mode 1. As with an RC circuit, the thermal circuit

---

7 This is only valid for isotropic materials.
Modes of thermo-mechanical behavior

![Equivalent RC circuit mode 1](image)

Figure 2.6: Equivalent RC circuit mode 1. The capacitance is the thermal capacitance of the substrate. The resistance is the thermal resistance to ambient.

has a time constant. This determines the speed of response of mode 1. It can be calculated when generalized heat transfer coefficient $\overline{h}$ is known.

$$\tau_1 = RC = \frac{\rho C_p V}{hA}$$

(2.9)

An iterative algorithm is used to solve the thermal radiation problem for an incident thermal radiation of 350 [W/m$^2$] on the absorptive layer of a 50 × 50 × 4 [mm] mirror. For the calculation of the Biot number a characteristic length of $l = 25$ [mm] is used to represent the maximum length of the conductive path from the center to the edges of the mirror in plane. The results are listed in Table 2.2.

Table 2.2: Model results: properties of mode 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biot number</td>
<td>Bi</td>
<td>0.077</td>
<td>[−]</td>
</tr>
<tr>
<td>Heat transfer coefficient</td>
<td>$\overline{h}$</td>
<td>3.37</td>
<td>[W/(m$^2$ · K)]</td>
</tr>
<tr>
<td>Time constant</td>
<td>$\tau_1$</td>
<td>1103</td>
<td>[s]</td>
</tr>
<tr>
<td>Temperature rise</td>
<td>$\Delta T_{0-1}$</td>
<td>44.8</td>
<td>[K]</td>
</tr>
<tr>
<td>Elongation</td>
<td>$\Delta x$</td>
<td>15.9</td>
<td>[µm]</td>
</tr>
<tr>
<td></td>
<td>$\Delta z$</td>
<td>1.27</td>
<td>[µm]</td>
</tr>
</tbody>
</table>

A transient response of the temperature rise $T_1$ is shown in Figure 2.7. It must be noted that this is the response of the model, using a linearized heat transfer coefficient for radiation based on the final temperature. This introduces error in a sense that the model is slightly slower than a real experiment would be. From the results in Table 2.2 can be concluded:

1. The Biot number, $Bi = 0.077$ shows that the internal temperature differences that are to be expected are up to at maximum 5 – 10% of the global temperature change $\Delta T_{0-1}$.

2. The time constant $\tau_1$ indicates the speed of response of the bulk temperature $T_1$. If experiments would be done on a mirror with the properties used in this analysis at constant temperature, a settling time of approximately 70 minutes must be considered.

---

8settling time: time to reach 98% of the final temperature change.
Modes of thermo-mechanical behavior

2.3.2 Mode 2: Mirror Curvature

Mode 2 is the deformation caused by the gradient across the thickness of the mirror, $\Delta T_{2-3}$ in Figure 2.4. The gradient is the result of heat flow through the material to the opposing side of the mirror. Due to the slope of the temperature distribution across the substrate thickness the material will expand more in lateral direction where temperature is high; in the vicinity of the absorptive coating. This causes the mirror to bend just like a bimetallic strip would bend when it undergoes temperature change. Actuating substrate curvature is also used as operational mode for bimorph piezoelectric deformable mirrors [11]. The thermal deformation of mode 2 is depicted in Figure 2.8. The response of the temperature gradient across the substrate thickness is modeled with an alternative two-element finite element approach. Instead of distributing the elements spatially (as is common in finite element analysis) the thermal energy content of the substrate is divided into two elements. One large element $E_1$ represents the bulk thermal energy of the substrate due to its temperature and one smaller element $E_2$ represents the relatively small amount of energy that goes into the additional temperature difference between both sides $\Delta T_{2-3}$. The energy capacity of both elements needs to be determined. To do this, a (one dimensional) temperature profile in final state is considered in Figure 2.9. The internal thermal

![Figure 2.7: Transient response of global substrate temperature $T_1$ to a 350 [W/m²] step input on an absorptive coating of 50 × 50 [mm].](image)

![Figure 2.8: Schematic deformation mode 2: curvature induced by unequal in plane expansion across the thickness of the substrate due to temperature gradient.](image)
Figure 2.9: Final temperature state of mode 2. The absorptive coating is subjected to a 350 W/m² heat flux. In the final state, a small portion of the input energy radiates from the reflective side. The majority of energy flows across the substrate to the opposing side to radiate to ambient from there due to the lower thermal resistance. Figure 2.9(b) shows both energy elements $E_1$ and $E_2$. 

Figure 2.9: Final temperature state of mode 2. The absorptive coating is subjected to a 350 W/m² heat flux. In the final state, a small portion of the input energy radiates from the reflective side. The majority of energy flows across the substrate to the opposing side to radiate to ambient from there due to the lower thermal resistance. Figure 2.9(b) shows both energy elements $E_1$ and $E_2$. 

resistance of the mirror has to be taken into account. Furthermore both radiative thermal resistances to ambient of the reflective surface and opposite site (BK7) must be considered. The figure shows a final state of the mirror subjected to a uniform heat flux $Q_{in}$. Figure 2.9(a) shows the heat input at the absorptive coating at $z = 0$. The reflective surface (emittance $\varepsilon = 0.04$) has a high radiative resistance compared to the internal resistance and the resistance of the opposing side (emittance $\varepsilon = 0.8$). This creates a heat flow across the substrate thickness and hence, a temperature gradient accompanying it. The gradient across the substrate will result in a spherical surface shape. In order to model the speed at which this spherical shape occurs, consider Figure 2.9(b). The temperature profile integrated over the substrate dimensions yields, regarding the concerning material properties, the amount of (thermal) energy build up in the material. The energy in the substrate is divided into two portions: $E_1$ represents the bulk energy and $E_2$ which represents the portion that makes up the difference between $T_2$ and $T_3$ as shown in the figure. To model the response of the temperature difference between both sides again a thermal circuit is used, shown in Figure 2.10. The circuit is an extension to that of mode one (Figure 2.6). It now consists of two thermal masses: again the same global mass as in mode 1 to allocate energy $E_1$ and an additional thermal mass to allocate thermal energy $E_2$. The capacitance that holds this energy $E_2$ is half the capacitance of the bulk material: due to the slope in the temperature profile, only 50% of the thermal capacitance of the material is used in the final state. Hence the ‘fill-factor’ of 0.5.

Using circuit analysis, a transient response to a step input of both sides of the substrate is plotted in Figure 2.11. The plot shows a difference in temperature between both sides of the mirror. This difference $\Delta T_{2-3}$ occurs faster than the global temperature rise. The plot also shows a generally higher temperature than the global temperature plot in Sub-
Modes of thermo-mechanical behavior

Figure 2.10: Equivalent RC circuit mode 2 showing two capacitances for allocating energy $E_1$ and $E_2$. Thermal resistances to ambient and internal resistance are incorporated.

$$Q_0 \left[ \begin{array}{c} \frac{1}{h_{z,0.04}} \frac{1}{h_{z,0.03}} \end{array} \right] \begin{array}{c} \frac{th}{k} \end{array} \begin{array}{c} 0.5 \rho C_p V \end{array} \begin{array}{c} \Delta T_{3,4} \end{array} \begin{array}{c} T_3 \end{array} \begin{array}{c} \rho C_p \epsilon \end{array} \begin{array}{c} \Delta T_{0,2} \end{array} \begin{array}{c} T_i \end{array} \begin{array}{c} T = T_0 \end{array}$$

Figure 2.11: Transient response $T_2$ and $T_3$ to a step input of 350 [W/m²], preliminary mirror configuration.

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Temperature [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>293.15</td>
</tr>
<tr>
<td>1800</td>
<td>300</td>
</tr>
<tr>
<td>3600</td>
<td>310</td>
</tr>
<tr>
<td>5400</td>
<td>320</td>
</tr>
<tr>
<td>7200</td>
<td>330</td>
</tr>
<tr>
<td>9000</td>
<td>340</td>
</tr>
</tbody>
</table>

Figure 2.11: Transient response mode 2: $T_2$ and $T_3$. The heat flux is applied to the absorptive coating of temperature $T_3$. The temperature on the opposing side is denoted $T_2$.

Section 2.3.1 on mode 1. This is due to the one dimensional model of this mode; the heat transfer from the substrate sides is omitted in this approach resulting in about one third less heat transfer from the body. In the response for the temperature difference $\Delta T_{2-3}$ two cases are identified:

- Case 1: Bulk temperature $T_1$ is subjected to change due to step in radiant net power on the substrate.
- Case 2: Bulk temperature $T_1$ is not changing due to stationary net radiant power on the substrate when an area of the substrate undergoes a step in heat flux (for example other when parts of the substrate are subjected to cooling).

This difference is of importance because in case 1, the thermal mass of the bulk capacity affects the response of mode 2 whereas in case 2 this does not occur. Figure 2.12 shows a transient responses of both cases. The plot shows the response of both cases. The red line depicts the thermal response of $\Delta T_{2-3}$ in case 1. This response shows a slight overshoot, denoted the
Figure 2.12: Transient response of temperature difference driving deformation mode 2: $\Delta T_{2-3}$. The red line shows the response for case 1: an overshoot is visible. The blue line depicts the response for case 2: it is the response of the individual first order system to the input of energy $E_2$. Note the different time scale of this figure compared to Figure 2.11.

The term ‘loading-effect’ throughout this report. This overshoot is due to the storage of thermal energy in the substrate while warming up: the temperature profile needs to accommodate the storage of thermal energy resulting in a larger temperature difference. When the body does not undergo an bulk temperature change when an area is subjected to a step in heat flux (for example when other areas of the substrate are subjected to cooling), the loading effect is not visible. This response is depicted by the blue line.

The temperature gradient across the thickness will result in a spherical deformation of the substrate with a certain radius. This deformation results in a curved substrate with 'pocket-depth', or *sagitta*\(^9\) that is the amplitude of the surface deformation. The properties mode 2 are determined using the model. They are listed in Table 2.3. The geometrical deformation properties are based on the calculated temperature difference $\Delta T_{2-3}$ over the entire substrate thickness.

Table 2.3: Model results: properties of mode 2

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time constant</td>
<td>$\tau_2$</td>
<td>15.5</td>
<td>[s]</td>
</tr>
<tr>
<td>Temperature rise</td>
<td>$\Delta T_{2-3}$</td>
<td>1.20</td>
<td>[K]</td>
</tr>
<tr>
<td>Radius of sphere</td>
<td>$R$</td>
<td>471.2</td>
<td>[m]</td>
</tr>
<tr>
<td>Curvature</td>
<td>$\kappa$</td>
<td>2.1e-4</td>
<td>[rad/m]</td>
</tr>
<tr>
<td>Sagitta on 50mm mirror</td>
<td>$S$</td>
<td>663</td>
<td>[nm]</td>
</tr>
</tbody>
</table>

\(^9\)This geometric distance known as *Sagitta* (or *Versine*) is the line segment drawn perpendicular to a chord, between the midpoint of that chord and the arc of the circle.
From the above reasoning and the results in Table 2.3 the following can be concluded:

1. The temperature gradient across the thickness occurs faster than the global temperature of the substrate when subjected to an homogeneous heat flux.

2. The temperature gradient across the mirror induces a deformation in the sense that the mirror surface becomes part of a sphere. This creates a pocket in the 50 [mm] wide mirror with a ‘depth’ of 663 [nm] for a 350 [W/m²] heat flux.

3. There is an overshoot effect due to the thermal gradients in the material to transport heat internally. This effect is denoted the ‘loading-effect’.

4. Due to the ‘bending’ nature of the deformation in mode 2, a local heat input that causes mode 2 deformation will affect the surface shape of the entire mirror.

2.3.3 Mode 3: Local Actuation

Deformation in mode 3 is the thermal expansion of the material in the $z-$direction, orthogonal to the mirror plane. This occurs due to a local higher temperatures of $T_4$ than $T_1$ (when heat input to the substrate is not spatially uniform). A local heat input will induce the body temperature to rise, but also an additional temperature rise at the location of the input. This temperature difference is denoted $\Delta T_{4-1}$ in Figure 2.4. Differences in thermal expansion across the mirror surface, orthogonal to the mirror surface, have direct effect on the surface shape of the mirror. Hence, this mode of thermal deformation is very useful in actuating the mirror surface. The thermal deformation of mode 3 is depicted in Figure 2.13. In order to model mode 3, the same approach is taken as was done for mode 2 previously. An element is defined of energy $E_3$ to allocate thermal energy that results in mode 3 deformation. Again the temperature distribution must be found to find the energy capacity of $E_3$. The temperature profile at final time can be found by the conservation of energy for an infinitesimal slice of material. consider the case that the mirror surface is subjected to a spatial periodic heat load. The heat input is a spatially periodic square distribution centered around zero: the thermal input is either $Q_{in}$ or $-Q_{in}$. This means there is no net energy input and no change of bulk temperature $T_1$. Figure 2.14(a) shows one half period, that is cutoff at the symmetry lines at $Q_{in}$ or $-Q_{in}$. As this
mode considers the average temperature across the mirror thickness, the radiative heat transfer coefficients to ambient of both mirror sides are averaged and denoted $\bar{h}$. The boundaries of the model in Figure 2.14(a) are chosen such that there is no heat flow across them (symmetry). Modeling the mode 3 temperature profile can be done by considering a slice of width $\Delta x$ in the mirror, shown in Figure 2.14(b). For this slice an energy balance is made. For the final steady state situation:

\[ th \cdot k \frac{\partial T}{\partial x} + \bar{h} (T - T_1) \Delta x = Q_{in} \Delta x + th \cdot k \left( \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} \Delta x \right) \quad (2.10) \]

In equation (2.10) the term $th \cdot k \frac{\partial T}{\partial x}$ appears on both sides and can thus be left out. Furthermore, dividing by $th \cdot k \cdot \Delta x$ yields the differential equation:

\[ \frac{\partial^2 T}{\partial x^2} - \alpha T = -\beta \quad (2.11) \]

where

\[ \alpha = \frac{T}{k \bar{h}} \quad \text{and} \quad \beta = \frac{(Q_{in} + \bar{h} T_1)}{k \bar{h}} \quad (2.12) \]

Which can be solved for $T$ when the two boundary conditions are applied (due to antisymmetry at $x = 0$ and symmetry at $x = L$):

\[ T|_{x=0} = T_1 \quad \text{and} \quad \frac{\partial T}{\partial x}|_{x=L} = 0 \quad (2.13) \]

In which $x = 0$ is at the antisymmetry axis and $x = L$ is at the symmetry axis in Figure 2.14(a), making length $L$ one quarter of the spatial heat flux period. Solving Equation (2.12) while regarding the boundary conditions stated above yields the final temperature profile:
\[ T(x) = A \cdot e^{-\sqrt{\alpha}x} + B \cdot e^{\sqrt{\alpha}x} + \frac{\beta}{\alpha} \]  

(2.14)

In which:

\[ B = \left( T_1 - \frac{\beta}{\alpha} \right) / \left( 1 + \exp\left( 2\sqrt{\alpha} \cdot L \right) \right) \quad \text{and} \quad A = B \cdot \exp\left( 2\sqrt{\alpha} \cdot L \right) \]  

(2.15)

The temperature profile is plotted in Figure 2.15.

\[
\begin{align*}
E_3 &= \rho C_p \cdot \int_0^L T(x) - T_1 \, dx \approx \rho C_p \cdot \frac{2}{3} (T_4 - T_1) \cdot L
\end{align*}
\]  

(2.16)

This shows that two-third of the thermal capacitance of the material considered is used.

The temperature plotted in Figure 2.15 shows a profile with no lateral heat transfer at \( x = L \) due to symmetry. The slope at \( x = 0 \) corresponds with lateral heat transfer across the boundary due to antisymmetry. The ratio of thermal input that flows across the boundary \( x = 0 \) over the total thermal input is denoted \( \zeta \) in this report and is further discussed in Section 2.4. The amount of energy that radiates from the surface is the total thermal input factored by \( 1 - \zeta \). Integrating the temperature profile over \( x \) yields the amount of energy that is stored in the the 'capacitance' of mode 3:

\[
E_3 = \rho C_p \cdot \left( T_4 - T_1 \right) \cdot \frac{2}{3} L
\]  

(2.17)

which is found to be approximately equal to:

\[
\left. \frac{dE_m}{dt} \right|_{t=0} = \frac{E_3}{\tau_3} = Q_{in} L \quad \text{hence} \quad \tau_3 = \frac{E_3}{Q_{in} L}
\]  

(2.18)
\[
\tau_3 \simeq R_3 \cdot C_3 = \frac{L}{2k \cdot th} \cdot \frac{2\rho C_p th \cdot L}{3} = \frac{\rho C_p}{48k \cdot f_s^2}
\] (2.18)

In which the ‘path length’ of the thermal resistance \( R_3 \) was taken as \( L/2 \) as an average resistive path length to the antisymmetry axis at \( x = 0 \) and the capacitance has a ‘fill-factor’ of 2/3 as was found in Equation (2.16). Furthermore, \( f_s \) denotes the spatial frequency in \([1/m]\) which is \(1/4L\) as \( L \) is the length of one quarter spatial period. The found equivalent thermal resistance (using \( L/2 \) as resistive path length) can be used to express a relation for \( \Delta T_{4-1} \):

\[
\Delta T_{4-1} = \zeta \cdot Q_{in} L \cdot R_3 = \frac{\zeta \cdot Q_{in}}{32 \cdot f_s^2 \cdot k \cdot th}
\] (2.19)

The peak-valley amplitude of mode 3 deformation is proportional to the temperature difference \( \Delta T_{4-1} \) the substrate thickness and the coefficient of thermal expansion of the material: \( \Delta th = 2\Delta T_{4-1} \cdot \alpha \cdot th \) (the factor two to obtain peak valley amplitude). The relation in Equation (2.18) clearly shows the time constant \( \tau_3 \) is inversely proportional to the spatial frequency, squared: \( \tau_3 \propto f_s^{-2} \). Furthermore, Equation (2.19) shows the same relation and thus also the amplitude of mode 3 deformation. These relations are shown in Figure 2.16 for a number of spatial periods. The temperature difference and amplitude of deformation are based on heat flux \( Q_{in} = 350 \, [W/m^2] \).

The plots in Figure 2.16 show the behavior of the mode 3 parameters. The time constant is proportional to the square of the spacial frequency. Furthermore, the amplitude is also proportional to the square of the spatial frequency of the projected pattern onto the mirror. This is an important result of this analysis. It directly relates the amplitude of desired effects to the desired resolution of these effects over the mirror surface. The properties of mode 3 are also listed in Table 2.4. In this table the properties are dependent on the spatial frequency of the projected heat flux pattern: \( f_s \, [1/m] \). The ratio of incident power that flows across the boundary (instead of radiating to ambient) is denoted by \( \zeta \).
Table 2.4: Model results: properties of mode 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time constant:</td>
<td>$\tau_3 (f_s)$</td>
<td>$\frac{\rho C_p}{48k f_s}$</td>
<td>[s]</td>
</tr>
<tr>
<td>Temperature rise:</td>
<td>$\Delta T_{4-1} (f_s)$</td>
<td>$\frac{\zeta Q_{in}}{32 f_s^2 \cdot \theta \cdot k}$</td>
<td>[K]</td>
</tr>
<tr>
<td>Amplitude (peak-valley):</td>
<td>$\Delta th (f_s)$</td>
<td>$\frac{\zeta Q_{in} \cdot \alpha}{16 f_s \cdot k}$</td>
<td>[m]</td>
</tr>
</tbody>
</table>

From the model for mode 3 and the results as presented in Table 2.4, the following conclusions can be drawn:

1. The speed of response is inversely proportional to the square of the spatial frequency of the incident heat flux. This results in fast responses for higher order spatial patterns.

2. The magnitude of the temperature difference is inversely proportional to the square of the spatial frequency. Resulting in small deformations for higher order spatial frequencies. Also, the magnitude is proportional to the power flowing across the internal resistance, which is the incident power factored by the ratio of internal and external thermal resistance, $\zeta$. This will be further discussed in Section 2.4.

3. The deformations that are to be expected are proportional to the temperature differences. For incident heat flux of $Q_{in} = 350 \text{ [W/m}^2\text{]}$ this results in out of plane deformation of 13.8 [nm] (peak-valley) for 5 periods projected onto the mirror.

4. Although the model considers an average temperature across the substrate thickness, due to thermal diffusivity, temperature gradients across the substrate thickness do occur in mode 3. These gradients will result in curvature induced amplitude of the surface.

### 2.4 Actuating the mirror surface

This section will discuss a strategy on how the mirror surface is to be actuated. Furthermore, the relation between mode 2 and mode 3 is discussed as these modes both affect the substrate surface shape.

The mirror substrate can be provided by a homogeneous offset heat flux over its entire actuation area. A 'negative' heat flux can now be provided to the substrate by applying a lower heat flux intensity than offset level. As was discussed in Subsection 2.3.2, such a heat flux induces a spherical substrate shape. The shape induced by the offset heat flux imposes a manufacturing requirement on a deformable mirror. It must be manufactured in such way that the substrate surface is within specified tolerances when the offset heat flux is applied. This means that the substrate must be manufactured with an additional negative spherical shape to compensate the
offset heat flux. Due to the 'loading' of the thermal capacity of the substrate, the response of the surface shape can be affected as was discussed in Subsection 2.3.2. To minimize this effect the following strategy is proposed:

1. The substrate average temperature $T_1$ must be kept stable at an elevated temperature above ambient temperature. To do this, all actuation inputs must have the same net power on the substrate as the offset heat flux.

2. As was found in Subsection 2.3.3 lower order spatial heat flux patterns induce relatively large temperature difference across the mirror surface ($\Delta T_{4-1} \sim 5 [K]$ for one period on the substrate of 4 [mm] thickness). This possibly results in the loading-effect affecting the response. The temperature difference $\Delta T_{4-1}$ is inversely proportional to the substrate thickness $th$, indicating that the loading effect is smaller for thicker substrates actuated with equal spatial heat flux distribution.

As is concluded in the previous Section 2.3, two effects affect the mirror surface shape directly, the substrate bending in mode 2 and the effects of expansions orthogonal to the mirror surface in mode 3. In the analysis of mode 3 in Subsection 2.3.3 it was found that the power used in mode 3 is the fraction $\zeta$ of input power that flows across the internal resistance $R_3$ that is considered. This fraction $\zeta$ is the ratio of incident power on one quarter period (as discussed in Subsection 2.3.3) that flows across the internal resistance and is related to the ratio of the internal resistance and the total thermal resistance:

$$\zeta = \frac{R_3}{R_a + R_3} \quad (2.20)$$

Where $R_3$ is the internal resistance of the conducting path and $R_a$ is the radiative resistance to the ambient:

$$R_3 = \frac{1}{8 f_s k th} \quad \text{and} \quad R_a = \frac{1}{4 f_s h} \quad (2.21)$$

Note that $R_3$ was found in Subsection 2.3.3 as the internal resistance for one quarter period. The ratio $\zeta$ is plotted as function of spatial frequency, depicted in Figure 2.17. The number of periods on the 50mm mirror is shown on the x-axis (in stead of spatial frequency $f_s$) to make it more tangible. The plot shows that the ratio of power used for mode 3 approaches 1 when the number of spatial periods in the heat flux becomes large (or: the spatial frequency $f_s$ becomes high). When the ratio $\zeta$ approaches one, no power is used for mode 2. This does not exclude mirror bending; due to thermal diffusivity, temperature gradients do occur across the substrate thickness in mode 3. The result of this is that substrate curvature will occur in mode 3, effectively enlarging the mode 3 amplitude. The response of this curvature has time constant $\tau_3$. 

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Figure 2.17: Ratio of incident power used for mode 3.
CHAPTER 3

Mirror design considerations

The previous chapter discussed the thermal deformation of a substrate in a general sense. This chapter will discuss and evaluate design parameters of a deformable mirror by thermal radiation. Finally, a set of samples is proposed for experimental validation.

The Goal is to obtain mirror behavior as a function of these design parameters. Knowledge of these design parameters important when a deformable mirror is implemented in an application to fit mirror behavior to the desired specifications.

Design parameters of a thermally actuated deformable mirror are considered in Section 3.1. Section 3.2 proposes a set of sample substrates to use in experiments in order to verify the design parameters. These samples are obtained and used as part of an optical measurement setup to perform experiments.
3.1 Mirror design parameters

This section will discuss three design parameters: the substrate material, substrate dimensions and the location of the absorptive coating in the substrate.

3.1.1 Substrate material

The magnitude and speed of response of a substrate to a given heat input is a function of the properties of its material. Hence, the choice of substrate material is an important mirror design parameter. In this subsection the speed and magnitude of response, as found in the analysis of the modes of thermal deformation in Chapter 2, are related to the choice of material.

In the analysis on the modes of thermal deformation, the speed of the response to a heat input can be represented by the time constant. The time constant is inversely proportional to the speed (the larger \( \tau \) is, the slower the response is).

\[
\text{Speed} \propto \frac{1}{\tau} \propto \frac{k}{\rho C_p}
\]  

(3.1)

The material properties in Equation 3.1 combined are known as the thermal diffusivity of a material \([8]\):

\[
D = \frac{k}{\rho C_p}
\]  

(3.2)

The units of \( D \) is \([m^2/s]\). The higher the thermal diffusivity of a substrate material, the faster the substrate is able to converge to a new thermal equilibrium when boundary conditions change.

The magnitude of the response of a material is proportional to the coefficient of thermal expansion and the rise in temperature \( \Delta T \). The latter is proportional to the internal resistance and thus inversely proportional to the thermal conductivity of the material. Hence, a quantity can be defined that is an indication of the magnitude of response of a material:

\[
M = \frac{\alpha}{k}
\]  

(3.3)

The units of \( M \) is \([m/W]\). The larger \( M \) is, the larger the transfer of thermal input to mechanical output (deformation).

Note that the thermal conductivity \( k \) affects both the speed and magnitude of the response, although the effects are opposite for both stated quantities \( D \) and \( M \). As a consequence, the thermal conductivity of a material is a dominant property in the character of the response: materials with a relatively high conductivity have a relative fast but small response whereas a relatively low conductivity results in a slow but large response.

Selected materials are compared in their relative speed and magnitude of response to changing thermal boundary conditions. The materials chosen are transparent and widely used for optical elements and can be coated to absorb energy or reflect light. The compared materials are listed in Table 3.1 together with their material properties.
Table 3.1: Selected materials (common for optical elements) and their properties affecting thermal response.

<table>
<thead>
<tr>
<th>Material</th>
<th>k [W/(m K)]</th>
<th>α [-]</th>
<th>ρ [kg/m³]</th>
<th>C_p [J/(kg K)]</th>
<th>M [m/W]</th>
<th>D [m²/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B270</td>
<td>1</td>
<td>9e-6</td>
<td>2550</td>
<td>900</td>
<td>9e-6</td>
<td>4.4e-7</td>
</tr>
<tr>
<td>BK7 [9]</td>
<td>1.114</td>
<td>7.1e-6</td>
<td>2510</td>
<td>858</td>
<td>6.4e-6</td>
<td>5.2e-7</td>
</tr>
<tr>
<td>Borosilicate [9]</td>
<td>1.2</td>
<td>3.25e-6</td>
<td>2230</td>
<td>830</td>
<td>2.9e-6</td>
<td>6.0e-7</td>
</tr>
<tr>
<td>Fused Silica [13, 14]</td>
<td>1.3</td>
<td>5.5e-7</td>
<td>2200</td>
<td>725</td>
<td>4.2e-7</td>
<td>8.2e-7</td>
</tr>
<tr>
<td>Sapphire [13]</td>
<td>35</td>
<td>5e-6</td>
<td>3980</td>
<td>650</td>
<td>1.4e-7</td>
<td>1.4e-5</td>
</tr>
<tr>
<td>Zerodur [9]</td>
<td>1.64</td>
<td>5e-8</td>
<td>2530</td>
<td>821</td>
<td>3.0e-8</td>
<td>7.9e-7</td>
</tr>
</tbody>
</table>

For the materials listed in Table 3.1 the thermal diffusivity and relative magnitude are plotted in Figure 3.1. This figure can be seen as a magnitude-speed plane in which the materials have a location according to their properties. The figure shows Sapphire glass has an extraordinary fast (but small) response compared to the other materials. This is due to its superior thermal conductivity. Zerodur shows virtually zero magnitude in its response which is a prominent property of this material.

![Thermal response properties of selected materials](image)

(a) Thermal response properties of selected materials

![Close up of D](image)

(b) Close up of D.

Figure 3.1: Thermal response properties of glasses. Speed (thermal diffusivity), D on the horizontal axis and Magnitude, M on the vertical axis (a). A close up of the D axis is shown in (b), discarding Sapphire.
3.1.2 Coating placement

The absorptive coating ensures heat input to the substrate at the coating plane. The heat transfer from this coating plane to the rest of the substrate material results in temperature gradients with an obvious highest temperature at the location of the heat input. Hence, the largest thermal deformation will be in the vicinity of this absorptive coating. For the design of a deformable mirror, the deformation at the substrate surface is important. The goal of this section is to find a relation between the location of the absorptive coating and the deformation at the substrate surface.

The analysis on mode 3 in Subsection 2.3.3 regards the temperature profile along the reflective plane. The temperature distribution across the thickness is assumed to be uniform. This assumption allowed for making a simple model that provides an indication for the speed of response and average temperature across the thickness of the substrate. This model does not provide insight in temperature distribution across the thickness and the relation of this distribution to mechanical deformation. A FEM analysis is performed to provide insight in this relation. Consider the case the substrate is exposed to a spatial periodic (square wave) heat load as was also discussed in Subsection 2.3.3. Figure 3.2 shows part of the mirror on which one period of heat flux is applied. There is mechanical and thermal symmetry in the center of the period, but also mechanical and thermal antisymmetry one one- and three quarter period. Due to this, the model can be reduced to one quarter period.

Figure 3.2: Partial mirror substrate exposed to one period of heat flux. The model can be reduced to a quarter period to model thermal deformation.

Using FEM software a quarter period of an incident heat flux is modeled. Figure 3.3(a) schematically depicts one quarter period, showing all boundary conditions that are to be considered to reduce the model to a quarter period. Figure 3.3(b) shows the result for vertical displacement of a FEM analysis for $\eta = 0.25$, where $\eta = -z/th$. The figure clearly shows that on the reflective surface boundary, the vertical displacement ranges from 0 to more than 7 $\text{[nm]}$. On the opposing boundary the vertical displacement has a much smaller range, indicating less surface shape deformation. To find a relation between the location of the absorptive layer $\eta$, and the surface deformation on the reflective surface of the substrate, a parametric sweep is

$^1$Dimension $z$ is defined as in Figure 2.2
Figure 3.3: Schematic modeled quarter period (3.3(a)) as indicated in Figure 3.2. FEM results for vertical deflection (3.3(b)).

Figure 3.4: FEM results for normalized magnitude as a function of absorptive layer placement, $\eta [-]$. Boundary conditions applied as depicted in Figure 3.3(a).

The results of the parametric sweep in Figure 3.4 show that the deformation is largest when the absorptive coating is directly placed under the reflective coating. When the absorptive coating is on the opposing side of the substrate, the deformation is zero on the reflective side of the substrate. If the coating is placed halfway across the substrate thickness, the deformation is slightly smaller than 0.5. This being smaller than 0.5 is due to the fact that the thermal problem is not completely symmetric at half the thickness. Because of the different emissivity of the substrate material and the reflective coating, the heat transfer coefficients are different.
at these boundaries. Furthermore, Figure 3.4 clearly shows a non-linear relation between $\eta$ and surface deformation. Using multiple samples with different location of the absorptive coating, this behavior can be validated.

### 3.1.3 Substrate geometry

The geometry of a mirror substrate in an EUV lithography application is mainly dictated by the EUV optical design. However, the geometry of the substrates is a design parameter of which its influence on the actuation behavior must be known before a mirror can be designed for any application.

In Chapter 2 it is found that the thickness of the substrate does not affect the deformation amplitude, neither the speed of response. This is an interesting result which must be validated experimentally. For low spatial heat flux distributions there is a concern that the loading-effect affects the response for relatively thin substrates.

The surface size of the substrate is also of interest. When part of the substrate surface is outside the aperture for heat flux exposure, the area not affected by flux change will resist deformation and a mechanical equilibrium will occur. It is expected that this suppresses substrate curvature but not out of plane deformation. This must be validated experimentally. Furthermore, a larger area outside the aperture results in lower thermal resistance to ambient as there is more surface area for cooling (and the internal thermal resistance is relatively low). This results in lower average substrate temperature $T_1$ but will not affect the deformation properties of the substrate.

### 3.2 Sample set for experimental validation

For the validation of the model of substrate deformation and the design parameters, a set of substrates is manufactured. The substrates have varying design parameters to evaluate the influence of these parameters experimentally. The basic shape of the substrate has a reflective surface of $50 \times 50 \text{ [mm]}$ with a thickness of 4 $\text{[mm]}$. This surface area is approaching the size of EUV optics ($\sim 50 \times 50 \text{ [mm]}$ and larger) and it is the size of the optical components of the experimental setup to be used (beamsplitter, lenses etc.). Hence, it are the maximum possible dimensions to be used without acquiring larger optics (which would be costly). The chosen material is BK7 for its known stable and homogeneous thermal properties and availability. All substrates have a reflective aluminium (Al) coating on top of the substrate material. The surface deformation will be measured directly on this top surface. The default substrate configuration has a (reflective) Al top coating and an absorptive coating on the same plane as the reflective coating. This sample is equal to the example mirror used in Chapter 2 to derive the modes of thermal deformation. This sample is sample M3 and is used as a starting point for all analyses.
3.2.1 Sample for validating material as design parameter

Substrate M9 is made of different material: borosilicate is chosen as a comparison to BK7 (which is itself a modification of borosilicate). Because of availability, this substrate has thickness of 3.3 [mm] instead of 4 [mm]. Sample M9 is obtained to verify the effect of material properties on the speed and amplitude of response. Borosilicate was selected as material because of its availability and low cost (comparing to for example Fused Silica and Zerodur). It is expected the sample behaves according to its material parameters as was found in Subsection 3.1.1 and is depicted in Figure 3.1. The sample substrate is expected to have approximately 55% lower response magnitude and 15% higher response speed. Furthermore, it is expected that the substrates different thickness has no influence the temporal response as was found in Subsection 2.3.3.

3.2.2 Samples for validating the coating placement as design parameter

The response of sample M2, M4 and M5 is to be compared to the response of sample M3 to evaluate the coating placement design parameter. M2 has its absorptive coating on the back side of the substrate. That is the opposing side of the Al-coating. M5 has its absorptive coating halfway the substrate thickness: M5 actually consists of two substrate of half thickness on top of each other with an absorptive coating in between. M4 is constructed the same way as M5 but with its absorptive coating one quarter thickness from the Al-coating. The substrates are schematically shown in Figure 3.6.

It is expected that the difference in the amplitude of the response of the different substrates is according to Figure 3.4. The speed of response is expected to be similar for all samples because the considered thermal capacitance in mode 3 is equal.

3.2.3 Samples for validating geometry as design parameter

Samples M7 and M10 have varying geometric ratio compared to sample M3. Sample M7 has double thickness compared to M3 and was made by gluing two substrates with normal thickness together. M10 has equal thickness compared to M3 but has larger length of its sides: 75 ×
Figure 3.6: Cross-section of samples for layer placement study. Sample M2 (top left) has absorptive coating at \( \eta = 1 \). Sample M3 (top right) has absorptive coating at \( \eta = 0 \). Samples M4 and M5 (bottom left and right) have absorptive coating at \( \eta = 0.25 \) and \( \eta = 0.5 \).

Sample M7 can be used to prove that the thickness of the substrate has no direct effect on the response magnitude or speed. It is expected that both sample M7 shows similar behavior as sample M3.

Sample M10 is used to investigate the effect of its mechanical resistance to bending. It is expected to show lower amplitudes for mode 2 deformation. The different thermal resistance to ambient of this sample is expected not to have any influence on its behavior.

### 3.2.4 Samples for thermography

Sample M1 only has an absorptive coating and no reflective coating, hence it is of interest for thermography. Because sample M1 is only of interest for thermography, it is not considered in this study. In addition, sample M6 is of interest of thermography but can also be used for topology measurements. Substrate M6 is equal to M3 but the sample has an SiO\(_2\) coating on top of the reflective coating. This is depicted in Figure 3.7. The SiO\(_2\) coating makes the sample radiative in the infra-red (IR) range on the front side. This makes the sample visible for an IR-camera. The coating also ensures the total radiative resistance to ambient is approximately half of that of sample M3. This will result in lower overall substrate temperature. It is expected that the higher heat transfer coefficient to ambient does not affect the response of mode 3 significantly. This is expected since the resistance to ambient will still be small compared to internal resistance as is discussed in Section 2.4. The Biot number will still be low for most considered spatial frequencies.
Mirror design considerations

Figure 3.7: Cross-section of sample M6. Compared to sample M3, sample M6 has an additional SiO$_2$ coating on top which results high emissivity on the top side.

The absence of sample M8 is because this substrate design was later removed from the proposed substrate set. This report has adopted the names of the samples as they are used in experiments. All samples and their main properties are listed in Table 3.2. Figure 3.8 shows a photograph of four samples out of the complete set.

Table 3.2: Set of samples for experimental validation. $\eta$ denotes the placement of the absorptive layer in the substrate $\eta = -z/th$ where $z$ is defined as in Figure 2.2.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Dimensions [mm]</th>
<th>Material</th>
<th>Coatings (top down)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>absorptive at $\eta = 1$</td>
</tr>
<tr>
<td>M2</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 1$</td>
</tr>
<tr>
<td>M3</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0$</td>
</tr>
<tr>
<td>M4</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0.25$</td>
</tr>
<tr>
<td>M5</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0.5$</td>
</tr>
<tr>
<td>M6</td>
<td>50 x 50 x 4</td>
<td>BK7</td>
<td>SiO$_2$, reflective, absorptive at $\eta = 1$</td>
</tr>
<tr>
<td>M7</td>
<td>50 x 50 x 8</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0$</td>
</tr>
<tr>
<td>M9</td>
<td>50 x 50 x 3.3</td>
<td>Borosilicate</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0$</td>
</tr>
<tr>
<td>M10</td>
<td>75 x 75 x 4</td>
<td>BK7</td>
<td>Al $\sim$ 90% reflective, absorptive at $\eta = 0$</td>
</tr>
</tbody>
</table>

3.3 Implementation in optical system

A mechanical clamp is devised to mount the substrates in an optical measurement system. An important requirement of the substrate mount is to minimize any heat transfer by conduction to the mount. Furthermore, the mechanical clamp must not introduce additional stress in the substrate material upon deformation.

In order to fulfill these requirements, the substrate must be mounted on low-stiffness flexures of a low conductive material with minimum contact surface area. The reflective surface of the mirror must be unobstructed in order to perform measurements. The opposing side must also be unobstructed to allow for applying the input heat flux.

A mount is devised out of three PVC plates. The center plate has three fixed supports and two leaf springs to ensure the lateral position of the sample. The front plate has three fixed
supports on the reflective side of the mirror. The back plate contains three leaf springs to fix the substrate onto the front plate supports. These rings are made by a rapid prototyping manufacturing process. This allowed for cheap and fast production of the mount.

Figure 3.9(a) shows a CAD rendering of the design and Figure 3.9(b) shows a photograph of the substrate mount with a substrate mounted in a 3-inch optical mount to implement it in an optical measurement setup. Table 3.3 shows the stiffness of the mounting springs to ensure proper mounting.

Table 3.3: Substrate mounting spring stiffness, deformations and applied forces. Due to the manufacturing tolerances of the prototyping process, notches where processed manually to ensure desired force on the substrate.

<table>
<thead>
<tr>
<th>Spring load w.r.t. coating plane</th>
<th>Stiffness $k$ [N/m]</th>
<th>Deflection [m]</th>
<th>Applied force $F$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>In plane (x, y)</td>
<td>6e3</td>
<td>0.5e-3</td>
<td>3</td>
</tr>
<tr>
<td>Orthogonal to plane (z)</td>
<td>12e3</td>
<td>0.5e-3</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 3.9: Substrate mounting: CAD rendering (a) of exploded view containing all components. Photograph (b) of the assembled substrate mount in a 3-inch optical mount that allows for easy alignment in an optical setup.
This chapter evaluates the measurement system, its physical principles and error sources of various nature. Furthermore, the preparation of experiments performed on the set of sample substrates is discussed.

Measuring nanometer topology on a surface with lengths in tens of millimeters requires adequate measuring principles. Because the measured quantity is very small it takes very little to introduce errors in a measurement. The reasoning behind the experiments performed is important to come to a set of experiments performed on the samples that is able to extract the information required to support the design parameter behavior.

An introduction to the physical principles involved in the measurement setup will be given in Section 4.1. Section 4.2 discusses the practical implementation of these principles by discussing the measurement setup hardware. The performance of the measurement setup is discussed in Section 4.3. Section 4.4 lists the experiments that are performed and their goals.
4.1 Spatial-carrier fringe interferometry for mirror topology measurements

The method used for measuring the topology and topology changes is spatial-carrier fringe-pattern analysis. Substantial contributions to the development of this method are done by M. Takeda [15],[16]. The used measurement tool is a Michelson interferometer using a large cross section bundle to cover the entire sample surface area. One of the mirrors is deliberately given a relative large tilt of \( \sim 10 \, [\mu m] \) deviation in x and y over the 50 \([mm]\) side lengths. The effect of this is that the returning wavefront from the tilted mirror has a slight angle. When the returning wavefronts of both optical paths meet again they interfere and a spatial carrier wave is generated. This is depicted in Figure 4.1.

Any unflatness of the mirrors, or topology, results in frequency modulation of the spatial-carrier wave. The topology information is extracted with the following algorithm. For simplicity only the \( x \)-coordinate is considered but the same principles apply for 2-dimensional analysis. The intensity captured by the CCD chip is expressed as follows:

\[
g(x) = a(x) + b(x) \cos[2\pi f_0 x + \phi(x)]
\] (4.1)
In which \( a(x) \) is the background intensity and \( b(x) \) the amplitude of the fringes. The distance from a set origin is denoted by \( x \). In the argument of the cosine, \( f_0 \) is the carrier fringe frequency and the phase addition \( \phi(x) \) is caused by the surface topology of the mirrors.

Equation (4.1) can be rewritten using Euler’s formula\(^1\):

\[
g(x) = a(x) + c(x)e^{2\pi if_0x} + c(x)^*e^{-2\pi if_0x}
\]

(4.2)

In which

\[
c(x) = \frac{1}{2}b(x)e^{i\phi(x)}
\]

(4.3)

In Equation (4.2), \( c(x)^* \) denotes the complex conjugate of \( c(x) \). The term \( c(x) \) being the amplitude and phase information of the fringes.

The multiplication of a signal by a complex exponential results in a shift in the frequency domain [17]:

\[
\mathcal{F}_x \left( c(x)e^{2\pi if_0x} \right) = C(f_x - f_0)
\]

(4.4)

Taking the Fourier transform of Equation (4.2) yields:

\[
G(f_x) = \mathcal{F}_x (g(x)) = A(f_x) + C(f_x - f_0) + C^*(-(f_x + f_0))
\]

(4.5)

If the frequency of the carrier wave is high enough with respect to the frequency content of the background intensity \( A(f_x) \) and fringe information \( C(f_x - f_0) \), the frequency shift is large enough to completely separate the three terms components in the frequency spectrum. This is depicted in Figure 4.2.

\[\text{Figure 4.2: Schematic frequency spectrum. The figure shows the frequency content of the background intensity, } A(f_x) \text{ centered around } f_x = 0 \text{ and the two 'lumps', } C(f_x - f_0), \text{ with frequency content of the fringe pattern. Figure modified from [16].}\]

The shifted information in \( C(f_x - f_0) \) can now be windowed by a filter function and shifted back to the origin. This effectively removes the carrier-fringe. Transforming the information \( C(f_x) \) back to the space-domain yields the desired information:

\[
\mathcal{F}^{-1} (C(f_x)) = \frac{1}{2}b(x)e^{i\phi(x)}
\]

(4.6)

\[\text{Euler’s formula: } e^{i\alpha} = \cos(\alpha) + i\sin(\alpha)\]

\[\text{39}\]
The amplitude of the fringes $b(x)$ is not of interest as the desired information is the phase $\phi$ (as the phase of the fringes is modulated by the topology of the mirror). This can easily be obtained by:

$$\phi(x) = \tan^{-1} \left\{ \frac{\text{Re}[c(x)]}{\text{Im}[c(x)]} \right\}$$

(4.7)

As this operation yields the phase between $-\pi$ and $\pi$, the phase needs to be unwrapped by an unwrapping algorithm that stitches the data together if sudden jumps close to $2\pi$ are present in the data. The phasemap is easily transformed into a topology map using the wavelength of the used laser.

### 4.2 Hardware setup

As discussed in Section 4.1, a Michelson interferometer is used to perform topology measurements on the samples. One of the two mirrors in the interferometer is the active mirror (i.e. one of the samples exposed to the heat flux) of which topology changes are of interest. Topology changes of the active mirror directly modulate the spatial-carrier frequency as projected onto a CCD chip.

The objective of the measurements is to measure topology changes caused by changing heat flux input. In order to do this, the initial topology is measured first. This initial topology is the shape difference between both substrate surfaces when the actuator provides only the offset heat input. The initial topology is subtracted from topology measurements recorded using actuation input. This isolated the topology caused by the heat flux input.

Table 4.1: List of key hardware components. Numbers correspond with numbers indicated in Figure 4.3.

<table>
<thead>
<tr>
<th>Component number</th>
<th>Component description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>He-Ne laser, wavelength: $\lambda = 632.8\ [nm]$</td>
</tr>
<tr>
<td>2</td>
<td>Polarizer</td>
</tr>
<tr>
<td>3</td>
<td>Beam expander with pinhole</td>
</tr>
<tr>
<td>4</td>
<td>Vacuum chamber: operating pressure 2e3 [Pa]</td>
</tr>
<tr>
<td>5</td>
<td>50/50 beamsplitter</td>
</tr>
<tr>
<td>6</td>
<td>$\lambda/20$ Zerodur reference mirror</td>
</tr>
<tr>
<td>7</td>
<td>50 x 50 [mm] active mirror in substrate mount assembly</td>
</tr>
<tr>
<td>8</td>
<td>Actuator: RGB projector (DLP technology): BENQ SP870, 5000 [ANSI lumen]</td>
</tr>
<tr>
<td>9</td>
<td>CCD camera: The Imagingsource DMK31, 1024 \times 768 [px]</td>
</tr>
</tbody>
</table>

Figure 4.3 shows a photograph of the measurement setup on an optical table. The numbers

---

$^2$As the interferogram results from the difference of both mirrors, neither of which is perfectly flat.
Figure 4.3: Photograph of the measurement setup. The numbers plotted represent the respective component as listed in Table 4.1.

indicate the key components of the measurement setup, which are listed in Table 4.1. A Helium-Neon laser (1) is used as monochromatic source of known wavelength. To avoid mode hopping of the laser affecting the measurements, a polarized (2) is used. A microscope objective (3) and a lens expand the beam to meet the geometric size of the optical components in the vacuum enclosure (4). This is to utilize as much of the sample (5) surface as measurement area. A 50/50 beamsplitter (6) splits the beam and directs one part to a reference mirror (7) and one part to the measurement sample. A 5000 [ANSI lumen] RGB video projector (8) is used to provide the heat flux pattern and actuate the substrate surface shape. The split paths meet again in the beamsplitter and interfere. A focusing lens and folding mirror are used to focus the interference pattern onto a CCD camera (9). The interference patterns is analyzed using a computer.

The offset heat flux provides the mirror with a spherical shape as was discussed in Section 2.3.2. This shape has a large amplitude compared to most topologies to be measured and results in relatively large alteration of the interferogram. Because it proved difficult to reconstruct topology from such an interferogram the interferometer was set up to compensate for this spherical shape. The beam expander (microscope objective with pinhole and an additional lens) was adjusted to create a slightly divergent beam. The divergent beam ensures a spherical wavefront with radius proportional to the optical path length. Two different path lengths for the interferometer branches result in a spherical interference pattern due to the interfering spherical wavefronts have different radii. The path length of the interferometer branches and the beam
Experimental Validation

divergence are chosen such that the spherical interference patterns of the mirror curvature induced by the offset heat flux and the different path lengths of the interferometer branches cancel out. This resulted in better topology reconstruction and a more robust measuring system.

4.2.1 Imaging area

Because it is practically impossible to use the full sample surface of 50 × 50 [mm] for actuation and measurements, the area exposed to the heat flux input is slightly smaller than the substrate surface. The area that is actuated has a size of 44 × 44 [mm]. This is due to the edges of the substrate are partly covered by the mirror clamp and care is taken to not expose the clamp to the heat flux (for geometric stability). Figure 4.4 schematically shows the mirror surface and the surface that is actuated by the heat flux pattern that consists of three periods in one dimension.

Figure 4.4: Input area and measurement area on mirror.

Figure 4.4 also shows the area effectively measured by the setup, indicated by the red lined square. This area has dimensions of 32 × 32 ± 1 [mm]. The interferogram is nearly the size of the mirror itself but ghost fringes distort the interferogram at the edges. Hence the interferogram sides are removed to get rid of the ghost fringes. The variance in measurement area (± 1 [mm]) is due to realignment that has to be done manually for all substrates. This results in a ratio of 16 ± 1 [px/mm] on a 512 px image.

4.2.2 Actuator calibration

The heat flux output of the video projector must be evaluated before any experiments are done. An 8-bit intensity level is provided as input to the video projector. The relation of the projector heat flux with the intensity level provided is to be found.
Preliminary measurements on the linearity of mode 2 showed non-linear mirror deflection as a result from providing linear grayscale values to the video projector. However, the deformation behavior is expected to be linear. A closer look is taken at the relation between grayscale values provided to the video projector and heat flux it produces.

An intensity meter is used to measure the optical output power of the video projector. The intensity meter has a sensitivity that is function of the wavelength of incident light. Because of this, the RGB projector is set to project 256 consecutive grayscale intensity levels. Grayscale levels are made of equal contributions of all RGB colors. This means that, although the intensity meter can possibly not measure the absolute intensity over all wavelengths, the measured irradiance is proportional to the projector intensity. Figure 4.5 shows measured irradiance (normalized) for an 8-bit grayscale intensity range on the actuator. Linear least squares is used to fit a polynomial to this data. The fitted polynomial is used to generate a lookup table for finding grayscale values for desired heat flux levels.

![Actuator Irradiance vs. Grayscale Intensity](image)

Figure 4.5: Actuator irradiance vs. grayscale intensity. The plot shows a nonlinear relation. The least squares fit is used to construct a lookup table to compensate for the actuator nonlinearity.

### 4.3 Performance of measurement system

The key in interpreting measurement data is knowledge of the performance of the measurement setup. There are three issues that need to be addressed to gain insight in how the measurement system output comes about:

1. Environmental influences disturbing the physical process.
2. Spatial sensitivity variance of the CCD pixel array.
3. Error due to processing of data (filtering of the signal).

4.3.1 Environmental influences

Environmental influences can severely disturb the measurement process. Two major disturbance sources were observed during measurements: mechanical vibrations and disturbance in the air.

Mechanical vibrations affect the position stability of the optical components of the interferometer. This can possibly result in optical aberrations. To suppress mechanical disturbances, the entire setup is built on an optical table that attenuates mechanical vibrations. The video projector (which contains a mechanical fan) is placed on a separate aluminium breadboard that is placed on foam cushions. In addition, posts holding optical components are connected together by steel rods to improve their mutual position stability. Furthermore, critical interferometer components are mounted on an aluminium breadboard that is supported by four rubber ball supports for additional mechanical in-plane isolation.

It was noted that disturbances of the atmosphere in the optical path can significantly affect the optical image on the CCD chip. The part of the optical path that is split into the two interferometer branches is especially delicate. Distortions in one of the branches directly distort the interference pattern itself. To suppress these distortions, both interferometer branches are completely situated in a vacuum enclosure. Disturbances in the rest of the optical path are suppressed as much as possible by shielding the entire optical path from the measurement room. Additionally, the air flow from the video projector cooling system is ducted away from the optical path using a shield.

To get an indication on the error incorporated by environmental influence, reproducibility experiments are performed. Two sets of 25 measurements are done on a stationary topology. The RMS error of the individual pixels is an indication of the reproducibility of the measurement system and also includes sensor noise.

\[ \epsilon_{RMS} \sim 0.1 \text{ [nm]} \]

As stated above, this number is a performance indicator of the measurement system. Transient environmental influences will affect the reproducibility error. The rms error of 0.1 [nm] is small compared to the topologies to be measured. It can be concluded that the precautions taken to attenuate the environmental influences are adequate.

\[ \text{3More detailed results (plots) are found in Appendix C.1.} \]
4.3.2 Spatial sensitivity variance of the CCD pixel array

An analysis is done to check the variance in the behavior of the individual CCD pixels. Variance in pixel independent sensitivity can result in changing information content when the interferogram is not stationary (i.e. when the interferogram changing because of actuated topology). From computer simulations it was found that sensor noise of the CCD camera can introduce significant error. As described in the part on environmental influences, reproducibility experiments are performed to get an indication of the performance of the measurement system. The rms error found ($\sim 0.1 \ [nm]$) on reproducibility also incorporates random sensor noise. However, this analysis is no indication for varying output of different pixels, as it is stationary.

To check reproducibility error of a non-stationary interferogram, a stationary interferogram of $512 \times 512 \ [px]$ is recorded on two different locations of the CCD array. Differences between the reconstructed topologies are evaluated.

On two locations of the CCD array, 25 recordings are made. The reproducibility of recording in these 25 frame sets is around 0.1 [nm] as was found previously.

When the average topologies of the two sets are compared there is a remarkable large difference. The average RMS error over the surface is much larger than the stationary reproducibility error:

$$e_{RMS} \sim 2.8 \ [nm]$$

This error can be caused by different reasons. Because the interferogram was shifted by hand, the interferogram on the CCD chip had to be selected manually as well. The difference between both topologies does not show a pattern to be expected from an alignment error $^3$. Although great care was taken to ensure the subtracted areas are the same areas on the mirror, this possibly affected the result.

Nevertheless, it is possible that the difference between the topology and the measured values is due to variance of properties between individual pixels. This indicates uncertainty in measurement sets with large differences in the individual interferograms which is the case if relatively large topologies are actuated.

4.3.3 Data Processing

The interference pattern projected onto the CCD chip is transformed to its frequency spectrum (as explained in Section 4.1) by a computer algorithm. Windowing of the utilized frequencies, or filtering, is done to extract the topology data from the interferogram.

There are some practical limitations of filtering the frequency spectrum and extracting the topology information. Consider Figure 4.6(a). It shows a typical interferogram as it is projected on the CCD chip $^4$.

$^4$The CCD chip has an array of 1024 × 768 pixels of which a square array of 512 × 512 pixels is used for measurements. This is to reduce computational time and it gives more freedom in aligning the measurement
Experimental Validation

Figure 4.6: Typical interferogram (a) as projected on (part of) the CCD-chip and its frequency spectrum (b). The frequency spectrum shows the background intensity information in the center and the two satellites containing the fringe information, denoted $A$ and $C$ in Section 4.1.

Figure 4.6(b) shows the spatial frequency transform of the interference pattern in Figure 4.6(a) processed with a FFT algorithm. It shows the same features as the 1D schematic in Figure 4.2. The background intensity is seen in the origin. Two satellites containing the fringe phase ($\phi(x,y)$) information are clearly shown. However, the two satellites are not completely separated (which would be ideal). The frequency content around the satellites is a mix of topology information and background intensity.

To obtain the desired topology information, the filter must be chosen carefully. Taking the bandwidth of the filter too wide will result in reconstructed topology affected by the frequency content of other phenomena like background intensity and sensor noise. On the other hand, taking the bandwidth of the filter too narrow can result in removing topology information. An analysis is done to gain insight in this problem.

The topology resulting from an interferogram is reconstructed using varying filter bandwidths. Figure 4.7 shows two reconstructed topologies from the interferogram depicted in Figure 4.6(a). Both topologies clearly show the same overall shape. Additionally, Figure 4.7(b) shows a lot of high frequency spikes compared to Figure 4.7(a) for which a more narrow filter was used.

Taking even higher filter bandwidths resulted in unreal topologies with even more small peaks at higher frequencies. This apparent interference of background frequencies can be filtered by taking the filter bandwidth carefully. An analysis is done on what spatial frequencies to expect in a topology, based on the spatial frequency of the heat flux pattern. The heat flux input that was provided for the interferogram in Figure 4.6(a) has the same spatial beam onto the CCD chip.

Note that opposing quadrants of the spectrum are shifted to get the origin of the spectrum in the center.
Figure 4.7: Reconstructed topology using two different filters. The topology in (a) is reconstructed using relatively narrow bandwidth filter, whilst for the topology in (b) a much wider filter was used.

period $f_p$ as the overall shape showing in both topologies in Figure 4.7. In order to determine the best filter bandwidth, a parametric sweep is performed on the bandwidth. The RMS amplitude is evaluated for all reconstructed topologies. This gives insight in the additional value of incorporating higher frequencies in the reconstruction. The results of the parametric sweep are shown in Figure 4.8. The figure clearly shows that adding higher spatial-frequencies than approximately 3 times the spatial-frequency does not affect the root-mean-square amplitude of the actuated profiles significant. It is therefore valid to not take these frequencies into the analyses of the behavior as the maximum possible error resulting from leaving them out is small.

### 4.4 Experimental validation of thermal model and design parameters

This section will discuss experiments performed on the samples. The aim of the experiments performed is twofold:

1. Verify system characteristics: modes, superposition of inputs and linearity.
2. Relate the design parameters of the substrates to specifications: resolution, amplitude and speed of response.

Experiments are devised to obtain results to fulfill both goals. Some boundary conditions must be kept in order to perform the experiments. In Section 2.4 it is described how the samples are to be actuated. The most important is that the net heat input on the substrate must be
stationary. There are two reasons to do this:

1. Overall mirror temperature $T_1$ is required to be in steady state to exclude most of the radiation nonlinearity.

2. When rising the overall substrate temperature the loading-effect can alter the temporal response, as was described in Subsection 2.3.2.

Practically, this is solved by providing a homogeneous 'offset heat flux' to the mirror. When a different heat flux distribution is provided for topology actuation, it must be balanced around the offset heat flux in order to keep the net heat input stationary. For the offset heat flux an intensity level of 50% of the maximum heat flux intensity is used. Next subsections will describe the experiments performed. A full list of all experiments performed can be found in Appendix B.

### 4.4.1 Linearity experiments

In Section 2.1 it was found that heat transfer by thermal radiation can be linearized without introducing significant error in temperature (due to the relatively small temperature range on the mirror compared to the difference between mirror temperature and ambient temperature). To verify this experimentally, linearity experiments are performed on both mode 2 and 3. This is done by consecutive topology measurements of the substrate with varying intensity of the heat flux pattern.

Mode 2 can be actuated isolated by providing the entire surface with a homogeneous heat flux. This is in contradiction with the set boundary condition that the net heat input must be sta-
tionary at all times: the mirror temperature might change significantly resulting in non-linear thermal behavior. A linearity experiment can be done for mode 2 only by taking a short actuation time in order to change the global temperature as little as possible. Due to this short actuation time, topology is recorded without the topology shape being completely converged to its final amplitude. This means that no conclusions can be drawn upon the quantitative results. The linearity of mode 3 can be validated by projecting a pattern of five square periods of heat flux intensity, as depicted in Figure 4.9(b). The heat flux is balanced around the offset intensity. The amplitude of the square wave is varied to validate linearity. The projected pattern varies only in one dimension. In the other dimension the heat flux is uniform over the substrate. This allows for averaging measurements over the uniform direction for a more clear representation of the results.

![Heatflux Input, mode 2](image1)

(a) Homogeneous input

![Heatflux Input, mode 3](image2)

(b) 5 periods square wave input

Figure 4.9: Inputs linearity experiment, heat flux patterns as projected onto the substrate absorptive coating.

For all experiments, the initial topology must be measured first. The topology upon actuation is measured and the initial topology is subtracted. This yields the actuated topology. For the linearity experiments, the heat flux is projected onto the absorptive coating after the initial topology is measured. The actuated topology is recorded after a specified actuation time.

### 4.4.2 Superposition experiment

Superposition is an important property of the actuation concept which allows for actuating a desired mirror shape by actuating multiple 'sub-shapes' that sum up to the desired shape. This is useful when a wavefront error is decomposed into several optical errors (see: Subsection 4.5). To show that the superpositions of thermal inputs will result in a superposition of mechanical outputs (deformed shapes), a superposition experiment is done. The superposition experiment can only be done in mode 3. Five exposure patterns are generated,
each consisting of one vertical slit exposed to maximum intensity $Q_{in}$ at a specific location. The remaining area is exposed to a negative heat flux input $-Q_{in}$ to keep the net input equal. When all five patterns are superimposed, this creates a five period square heat flux pattern as was also used in the linearity experiments for mode 3. This is illustrated in Figure 4.10.

![Individual heatflux components](image1)

![Superimposed heatflux](image2)

(a) Individual inputs  
(b) 5 periods input

Figure 4.10: Inputs superposition experiment, y axis uniform.

The five patterns sum up to a five period square wave pattern, but with relatively small amplitude. This is depicted in Figure 4.10(b). Because of the relatively small amplitude of the superimposed pattern, the amplitude of mechanical deformation is expected to be relatively small. Averaging measurements on the topologies over the y axis can possibly be useful for averaging out some error.

4.4.3 Verifying Design Parameters

To verify mirror design parameters as found in Chapter 3, transient measurements must be performed. The transient measurements will record the temporal properties of mode 3: amplitude and speed of response (time constant). Comparative studies are done for material, mirror geometry and the location (in z) of the absorptive coating. In these studies, the behavior of substrate M3 will serve as a benchmark to compare the other substrates to.

The mirror design parameters can be verified by relating the change in these temporal properties to the changing mirror design parameters in a comparative study. It is expected that the mirror design parameters will affect the substrate behavior as discussed in Chapter 3. Furthermore, it is expected that actuating different spatial frequencies will result in amplitude and time constant relations as where found in Subsection 2.3.3.

$^6$Negative meaning heat flux input lower than the offset heat flux
After measuring initial topology a heat flux pattern is projected onto the sample absorptive coating. The pattern is similar for the linearity experiment of mode 3 but now transient measurements are recorded. The spatial frequency of of the projected pattern is varied over several measurements. This is illustrated by Figure 4.11.

Figure 4.11: Inputs design parameters experiment, y axis uniform. Increasing heat flux spatial frequency.

Again the exposure pattern is a square wave, around the offset heat flux. The amplitude of the heat flux input is denoted by $Q_{in}$. Transient measurements are recorded with a set number of frames per second (fps). The number of frames per second is adjusted to expected temporal response. All experiment details are listed in Appendix B in Table B.1 An integer number of periods in the actuated topology is selected and the RMS amplitude is recorded in time. This will result in a step response of the amplitude of the actuated topology.

### 4.5 Input modeling for Zernike Polynomials

Wavefront abberations are often expressed in a series of Zernike polynomials \[18\]. The Zernike polynomials form an orthogonal set of polar polynomials, expressed in $\rho$ and $\theta$. Here, $\rho$ is the radial distance from the center and $\theta$ the azimuthal angle. The use of the Zernike polynomials (instead of a different set of orthogonal polynomials) is preferred because the lower Zernike polynomials correspond to the Seidel aberrations\[7\] which are commonly observed in optical systems. Knowing the inputs for the thermal actuated mirror to obtain the surface shape of the Zernike polynomials is useful. These inputs can serve as a feed forward signal in future work in the

\[7\]Most well known Seidel aberrations are Coma, Astigmatism and spherical aberration.
control of the mirror, to correct for aberrations. Furthermore, obtaining inputs for the Zernike polynomials would qualitatively proof that thermal actuation can be used to actuate a specific shape.

From the analysis in Section 2.3 it is clear that mode 3 is dominant in actuating the mirror surface. Nevertheless, in actuating mode 3 gradients occur across the substrate thickness, possibly affecting the surface profile by substrate curvature. There is not a clear inverse relation at hand for obtaining a heat input for a certain desired surface shape. Hence, FEM software is used to solve the inverse problem. An optimization procedure is used to optimize the thermal input for a desired surface deflection. The error of the surface deflection compared to the respective Zernike polynomial is minimized until within a specified tolerance.

The Zernike polynomials are polar polynomials for a circular aperture. As the samples are square, it is not possible to utilize the full surface of the samples. Furthermore, due to other constraints (ghost fringes) an aperture of 25 \( [\text{mm}] \) diameter is set in the center of the substrate. Inside the aperture, the target surface shape is described by the Zernike polynomials.

To cut computation time, only a quarter of the mirror is modeled in the FEM software. This can be done because for all Zernike polynomials symmetric or antisymmetric boundary conditions can be applied on the cut-off planes (when one quarter is modeled). These boundary conditions are listed in Table 4.2.

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Mechanical boundary condition</th>
<th>Thermal boundary condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetry</td>
<td>( \frac{\partial z}{\partial x} = 0 )</td>
<td>( \frac{\partial T}{\partial x} = 0 )</td>
</tr>
<tr>
<td>Zero slope</td>
<td></td>
<td>Thermal isolation</td>
</tr>
<tr>
<td>Antisymmetry</td>
<td>( \frac{\partial^2 z}{\partial x^2} = 0 ), ( z(t) = 0 )</td>
<td>( \frac{\partial^2 T}{\partial x^2} = 0 ), ( T(t) = T_1 )</td>
</tr>
<tr>
<td>Zero deformation ( (z) )</td>
<td>Fixed temperature</td>
<td></td>
</tr>
</tbody>
</table>

Apart from the boundary conditions in Table 4.2, the heat transfer boundary conditions must be taken into account. The model is programmed to optimize a heat flux pattern for minimizing the error between the surface shape and the desired surface shape.

The resulting heat flux for the quarter model must be extended to all quarters to make up the heat flux for the entire mirror. This means for some quarters the heat flux must be inverted. Figure 4.12 shows two surface plots. On the left \( 4.12(\text{a}) \) the target shape shows. The FEM software is set for optimizing heat flux for this shape. The resulting heat flux found the FEM software is depicted in Figure 4.12(\text{b}).
Figure 4.12: Input optimized for Zernike polynomial $Z_{3}^{1}$. Target surface shape inside aperture 4.12(a) and optimized heat flux distribution, 4.12(b).

All found heat flux patterns can be found in Appendix C.3 and the measured surface topologies actuated by the respective heat flux pattern. A more detailed description of the FEM modeling is found in [19].
CHAPTER 5

Results and discussion

This chapter will present an overview and a selection of the results from the experiments as described in Section 4.4. Experiment results on linearity are presented using the results of sample M3 because this was the preliminary design configuration used to describe and model the mirror behavior in Chapter 2. Substrate M4 is used for experiments on deforming the surface according to the Zernike polynomials because it was found actuating sample M4 yield the largest amplitudes (as will be described in Section 5.3). This makes sample M4 more suitable for actuating and measuring higher order shapes which are to be expected having relatively small amplitudes.

Section 5.1 will present and discuss the experiments on linearity and the modes of thermal deformation using the results of sample M3. Sample M3 is also used to present the results of the superposition experiment in Section 5.2. Transient measurements on substrate M3 are found in Section 5.3. An overview presenting the amplitude and speed of response (normalized to the amplitude and speed of sample M3) for all substrates is also found in this section to evaluate the mirror design parameters. Section 5.4 shows results for the Zernike polynomial surface shapes that where made using substrate M4. All detailed experimental results can be found in Appendix C.
5.1 Experiments on Linearity

This section reports on the linearity of mode 2 and mode 3. As stated in Subsection 4.4.1, a set of initial measurements is recorded and averaged. The heat flux input is changed from offset to the actuation heat flux and after a specified delay, another set of measurements is recorded. The delay is 10 [s] for both linearity of mode 2 and 3. Mode 2 is not converged at this instance but this short time is taken to avoid substrate temperature change (inducing thermal nonlinearity) as discussed in Subsection 4.4.1. According to the model described in Subsection 2.3.3 mode 3 is converges in this time for the actuated spatial frequency.

5.1.1 Linearity of mode 2

The experiment on linearity for a homogeneous heat flux pattern (shown in Figure 4.9, right) shows spherical surface deformation as was expected. These are depicted in Figure 5.1. The Figure shows two topologies for maximum negative heat flux input in 5.1(a) and maximum positive heat flux input in 5.1(b).

Figure 5.1: Reconstructed topologies of deformation in mode 2 of sample M3. The surface deformation has a spherical shape as was expected based on the discussing in Subsection 2.3.2.

The experiment on linearity is done by taking topology measurements whilst actuating the surface with a homogeneous heat flux in 11 linear steps from maximum negative to maximum positive irradiance. The amplitude is averaged over the y-axis and plotted for all irradiance steps in Figure 5.2(a).
Results and discussion

A spherical surface is fitted to the measured topologies using a least squares method. The curvature, $K$, of the fitted sphere is plotted vs. normalized irradiance of the source (video projector) in Figure 5.2(b). This fitted curvature is a magnitude indication of mode 2 for this sample. Mode 2 amplitudes of different measurements (and thus, samples) cannot be compared due to variance of measurement area and hence, differences in measured amplitude. Curvature values can be compared between samples. Figure 5.2(b) shows the curvature of all fitted spheres in the experiment. The plot shows a linear relation between curvature and source irradiance.

### 5.1.2 Linearity mode 3

The experiment on the linearity of mode 3 is performed using the five period heat flux input pattern as shown in Figure 4.9 right. The pattern is used with an irradiance magnitude of maximum negative up to maximum positive, centered around the offset heat flux. This means that the pattern for maximum positive input is the inverse of the pattern for maximum negative input. Figure 5.3 shows two topologies for both extreme cases. The topology plotted is a 24 × 24 [mm] cut out section on the sample surface.

Figure 5.2: Linearity experiment of sample M3 in mode 2. Figure 5.2(a) shows topology profiles obtained by averaging the actuated topology over the y-axis. Figure 5.2(b) shows fitted curvature to the spheres and a linear fit.

(a) Amplitude averaged over y

(b) RMS amplitude and linear fit
Figure 5.3: Reconstructed topologies of deformation in mode 3 of sample M3. The surface deformation of both plots show inverse features as was expected because of their inverse inputs.

The topologies show inverse features as expected based on their heat flux inputs. A set of measurements is done with irradiance amplitude of the heat flux pattern provided is changed from maximum negative to maximum positive. It must be noted that, as the heat flux pattern is a square wave around the offset heat flux, the net heat input is equal for all measurements. The amplitude of the surface shape is averaged over the y-axis and plotted in Figure 5.4(a) which shows profiles of the topology, averaged along the y-axis.

Figure 5.4: Linearity experiment of sample M3 in mode 3. Figure 5.4(a) shows topology profiles obtained by averaging the actuated topology over the y-axis. Figure 5.4(b) shows the RMS amplitudes of the actuated topologies and a linear fit.

The figure clearly shows similar topology shapes for all measurements with varying amplitude. The RMS amplitude is potted in Figure 5.4(b) in which also depicts a linear fit on the mea-
The magnitude of the actuated shapes in experiments for both mode 2 and 3 show to be linear with the irradiance of the heat flux pattern provided by the source. The linearity plot on mode 2 in Figure 5.2(b) shows more deviation than the plot on mode 3 in Figure 5.4(b). This is possible due to the fact that the data for mode 2 is the curvature from a spherical surface fitted to the actuated surface shape and the data for mode 3 is actual measurement data. In Subsection 2.1.1 the radiation phenomenon was linearized at cost of small error. Temperature differences on the substrate surface are much smaller, in the order of 2-3 Kelvin at maximum (as was found in Subsection 2.3.3). Based on the experimental results and the reasoning in Subsection 2.1.1 and Section 2.4 it is concluded that the system behavior can be considered linear around an operating temperature.

5.2 Superposition

This section presents the results of the superposition experiment as discussed in Subsection 4.4.2. Five individual heat flux patterns are projected onto the absorptive coating. The five individual patterns are summed up to the Sample M3 was used to perform the superposition experiment.

Five measurements where done, for each measurement using one of the individual periods as heat flux input pattern, shown in Figure 4.10 left. An additional measurement is done with all these individual heat flux patterns superimposed, shown in Figure 4.10 right. The individual measurements are super positioned. The resulting topologies are shown in Figure 5.5.

![Reference Topology](a) Reference topology  ![Superimposed Topology](b) Superimposed topology  ![Error](c) Error

Figure 5.5: Surface maps of the superposition experiment. The reference topology actuated by the superimposed heat flux pattern is shown in (a). Subplot (b) shows the superimposed topologies actuated by the individual heat flux patterns. The error is plotted in (c).

\(^1\)The error is 2% in case temperatures \(T_1 \sim 300 \, [K]\) and \(T_2 \sim 400 \, [K]\).
The left surface map shows the surface as was actuated by the full five period profile (superimposed inputs). It shows the same surface shape as the linearity experiments for sample M3 (Appendix C.2.2). The center surface map shows the sum of the five superimposed individual surface shapes (superimposed outputs), actuated by the individual heat flux patterns. It shows the same overall shape as the left surface. However, it also shows some defects on the surface, mostly at the left side of the plot. The right surface map, shows the error between both surfaces. The RMS amplitude over the entire error-surface is 2.0 [nm]. This is in the same range as the RMS amplitude error found in Section 4.3 for large deviations in the interferogram (which is the case for the individual actuated components of the superposition experiment) due to possible variance between pixel properties. The used heat flux input patterns are again only varying in one direction which allows for averaging in the other. This clarifies the results and is shown in Figure 5.6.

Figure 5.6: Superposition experiment: averaged over y-axis. Note the tilt is not subtracted in these measurements

There are some deviations in amplitude. The error is also depicted in the plot by the red line. The superposition experiment results are especially interesting when the reference profile (blue) and the superimposed profile (green) in Figure 5.6 are compared. Both plots show similar surface shape.

The error plotted in Figure 5.6 (right) has RMS amplitude of 2.0 [nm]. This is small when compared to the amplitudes of the individual profiles plotted in Figure 5.6 (left) and it is in line with previous found surface error for large changes in the interferogram, due to possible variance between pixel properties.

There are two sources likely affecting the amplitude deviation. One is the measurement system accuracy: the superimposed surface plot in Figure 5.5(b) shows the surface has some irregularities, likely due to the filtering of the measurement data. A second possible error source is
Results and discussion

The heat flux input by the video projector. As was explained in Subsection 4.2.2, the heat flux provided by the video projector is calibrated to behave linearly in 256 discrete steps. There are some concerns that the projector properties vary in time due to source temperature changes, controlled by its internal cooling system. Furthermore, aging of the projector lamp (and changing properties due to this) is a concern. The used lookup table based on actuator calibration (Subsection 4.2.2) might be subjected to error due to these effects. This possibly results in intensity values calculated in order to provide equal net heat input for every heat flux pattern might be off. This can explain the slight spherical trend that is seen in the error between both surfaces in Figure 5.6 by the red line.

The reference profile and the superimposed profile show similar surface shape. The error can be attributed to possible sources. Taking this into account it can be concluded that if the super positioning of heat flux inputs lead to a super positioning of their respective outputs in terms of surface shape.

5.3 Validating mirror design parameters

This section presents the results for the experiments validating the mirror design parameters. First the loading-effect is evaluated and the properties of the transient response of sample M3. Transient measurements are performed on all samples with increasing spatial frequency of the input heat flux pattern. On the actuated topology, the RMS amplitude of a selected number of spatial periods is recorded in time. This yields the amplitude and speed of response of the sample to the actuated spatial frequency. Samples are compared on amplitude and speed of response to evaluate the mirror design parameters.

5.3.1 The loading-effect

Before all other samples, the special case of sample M2 must be considered. As sample M2 has its absorptive coating on the opposite side of the substrate with respect to the reflective coating it is expected that it shows no actuation of mode 3, as was found in Subsection 3.1.2. However, mirror curvature can be seen on the reflective surface. This makes sample M2 useful to evaluate the ‘loading-effect’ as was discussed in Subsection 2.3.2 for spatial input patterns that are about equal to the substrate thickness. Figure 5.7 shows the transient response of sample M2 to a step input with the respective heat flux spatial frequency. The RMS amplitude of the surface is plotted in time.
Results and discussion

Measured data: RMS amplitude [nm]

Substrate: 2
Experiment: transient response, P3
Amplitude: RMS

<table>
<thead>
<tr>
<th>time [s]</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS amplitude [nm]</td>
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<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

(a) 3 periods on mirror

Measured data: RMS amplitude [nm]

Substrate: 2
Experiment: transient response, P5
Amplitude: RMS

<table>
<thead>
<tr>
<th>time [s]</th>
<th>0</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS amplitude [nm]</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

(b) 5 periods on mirror

Figure 5.7: The loading effect in the step response of sample M2. Projecting 3 periods on the substrate shows more than 100% overshoot. Projecting 5 periods shows less than 50% overshoot. No overshoot is seen with 7 or more periods on the 4 [mm] thick substrate (Figure C.11(a)).

The figures clearly show the overshoot due to the loading effect. The loading effect is more distinct for the 3 period pattern on the mirror. This is due to the geometric ratio of the actuated spatial period and the substrate thickness. When the thickness of the substrate is less than one quarter of the spatial period, the loading effect occurs. This is due to the heat flux inducing a significant temperature rise on both sides of the substrate (thermal loading).

It can be concluded that the loading-effect can affect the step response of the actuated amplitude when the thickness of the substrate is smaller than one quarter period of the applied input heat flux pattern. This can obscure the step response for lower spatial frequencies by apparent faster response than expected.

5.3.2 Sample speed and magnitude of response

In the following the temporal properties of sample M3 are considered. The results for all sample substrates are then compared and evaluated in the next subsections. Figure 5.8 shows a transient response of a measurement on sample M3. The heat flux pattern used to actuate the sample for this measurement has five periods as shown in Figure 4.11(left). The right Figure 5.8(b) shows a cut out section of just two periods on the actuated mirror surface. The RMS amplitude in time of this surface is plotted in Figure 5.8(a). Nonlinear least squares is used to fit an exponential function to the data with amplitude and time constant as depicted in the figure. The amplitude and time constant of the fit are used to compare all other samples to sample M3 to evaluate mirror design parameters.

Multiple recordings are done per substrate with increasing spatial frequency of heat flux pattern, all incorporated in Appendix C.2. For substrate M3, values for fitted amplitude and time
Results and discussion

Nonlinear least squares fit

Measured data: RMS amplitude [nm]

Nonlinear fit:

\[ A \cdot (1 - e^{-t/\tau}) \]

Amplitude: 14.0 [nm]

Time constant: 2.45 [s]

Substrate: 3

Experiment: transient response, P5

Amplitude: RMS and linear fit

RMS amplitude [nm] vs. time [s]

(a) Transient response

(b) Reconstructed topology (cropped to 2 periods)

Figure 5.8: Transient response the RMS amplitude of sample M3 in Figure 5.8(a). Nonlinear least squares is used to fit a response to the data. Figure 5.8(b) shows the reconstructed topology actuated by the 5-period heat flux pattern at final measurement time.

Figure 5.9: Amplitude of response vs. spatial heat flux input pattern for substrate M3.

The fitted relations in Figure 5.9 relating time constant and amplitude to the heat flux input pattern are made for all sample substrates. Figure 5.10 shows the magnitude of the fitted relations for the samples with magnitude normalized to the magnitude of sample M3. This
data can be used to evaluate the mirror design parameters as described in Chapter 3.

Figure 5.10: Bar graphs indicating the fitted magnitude for time constant and amplitude as depicted for sample M3 in Figure 5.9. Sample M2 is discarded from the comparison because of its deviant behavior.

### 5.3.3 Substrate material as design parameter

Sample M3 (default design) and sample M9 (3.3 [mm] borosilicate) are compared for evaluating substrate material as a design parameter (Subsection 3.1.1). The following is observed:

1. The Amplitude of M9 is 43% of the amplitude of sample M3.
2. The time constant of sample M9 is approximately 78% of the M3 time constant. This indicates that is about one quarter faster.

The amplitude difference of the two samples made of different materials is very close to the expected difference (see: Subsection 3.1.1). A 42% amplitude of the borosilicate sample was expected based upon the assumed material parameters found in Table 3.1. Also the difference in speed of response is close to the expected difference. Based on the properties in Table 3.1 a time constant of sample M9 of 80% of the M3 time constant was expected.

Both differences are close to expected and this validates the use of thermal diffusivity $D$ and magnitude parameter $M$ as described in Subsection 3.1.1. The slight observed deviations 1% (amplitude) and 2% (speed) can be due to error in the fitting of the exponential to the RMS amplitude. It is also possible that the material parameters of the used substrates deviate from the parameters in Table 3.1 as the values in the table are found in the stated references and
where not measured or observed from the actual samples.

5.3.4 Coating placement as design parameter

Sample M3 (default design) is compared to samples M2, M4, and M5 (coating placement according to Figure 3.6) for evaluating coating placement as a design parameter (Subsection 3.1.2). The following is observed:

1. Sample M4 has 140% amplitude compared to sample M3 and 126% time constant.
2. Sample M5 has 67% of the M3 amplitude and 176% time constant.
3. The behavior of sample M2 deviates from all other samples. It strongly shows the 'loading effect' as described in Subsection 2.3.2.

The experimental results contradict the expected amplitudes, based on the FEM analysis found in Subsection 3.1.2. The FEM model assumes homogeneous material behavior and heat input at a plane on location $x$ in Figure 3.3(a). The samples used consist of two samples, each grinded down to the specified thickness, glued together with the absorptive coating in between. The glue used is BK7-glue that has equal material properties. The effect of the presence of this layer is possibly not to be neglected.

It can be observed that the time constant that is found is lower for samples having the absorptive coating further away from the reflective coating. This suggests a possible increase of thermal capacitance for the third mode due to more material near the coating location (material on both sides of the coating instead of only on one side). The fact that the coating location is at the layer of glue between two substrates possibly causes an increase in thermal resistance and hence a slower response. It is recommended to further investigate the temporal actuation behavior in relation to the coating location in further research.

5.3.5 Substrate geometry as design parameter

Sample M3 (default design) is compared to sample M7 (double thickness) and sample M10 (1.5 times side lengths) for evaluating the substrate dimensions as a design parameter (Subsection 3.1.3). The following is observed:

1. Sample M7 has equal amplitude as M3 but 133% of its time constant.
2. Sample M10 has 110% amplitude of M3 and equal time constant.
The identical amplitude for mirrors M3 and M7 was to be expected, based on the reasoning in Subsection 2.3.3. The difference in speed of response was not expected. A closer inspection of the all measurement data (Appendix C.2) shows this difference is mainly due to a large difference for the lowest spatial heat flux distribution (3 periods on sample). The measurement data shows approximately equally fast response for M3, M6 and M10 (∼ 4.9 [s]) compared to M7 (∼ 6.8 [s]) for the lowest spatial heat flux distribution. Based on the measured speed of response to higher order spatial heat flux distributions (that are approximately equal for all mirrors) and the expected relation (∝ fₙ⁻²), the speed of response, it can be said that the speed of M7 is as expected and M3, M6 and M10 are unexpectedly fast. An explanation of this behavior is the 'loading-effect' as was described in Subsection 2.3.2. The thickness of samples M3, M4 and M6 is 4 [mm] and the width of one half period heat flux input (one 'stripe') projected is ∼ 7.5 [mm]. It is likely that the temperature profile across the substrate thickness at the 'hot' (or 'cold') locations is significantly higher (or lower) than the average sample temperature. This means these areas are 'loaded' in the first part of exposure. The loading effect induces additional mirror curvature during heat up, speeding up the amplitude response.

The amplitude of M10 is 10% larger when compared to the amplitude of M3, which was not expected. The speed of response of M10 is equal to the speed of response of M3. Therefore it is likely that the unexpected amplitude difference has its cause in some external factor. A possibility is the variance of video projector intensity.

5.4 Input modeling for Zernike Polynomials

This section presents the results for the experiments concerning the Zernike surface shapes. Inputs are generated by the FEM software as described in Section 4.5. Transient measurements are done on the surface shape and the final surface shape is decomposed into Zernike terms. The amplitude of the fitted objective Zernike polynomial is recorded as well as the RMS amplitudes of the actuated shape and the error resulting from the difference in objective Zernike polynomial and the actuated shape. This is an indication of how well the substrate surface is shaped into the desired shape. The sample surface is formed according to a total of 21 Zernike polynomials using thermal radiation only. All results are found in Appendix C.3. This section will show two selected examples: one lower order Zernike (Astigmatism) and one higher order Zernike (5th order Coma).

Figure 5.11 shows the results for Zernike Z₂² (denoted as astigmatism). On the left Zernike Z₂² is shown with the amplitude as fitted to the surface shape. In the center the actual measured surface shape is shown. The error between both shapes is plotted on the right. The RMS Amplitudes are incorporated in the plot titles. The figure shows similar surface shape of the measured topology and the objective surface shape. This is also illustrated by the Zernike decomposition of the surface shape, depicted in Figure 5.12.
Results and discussion

Error [nm]
RMS 3.1 [nm]

y [mm]
x [mm]

Target Shape: \( Z_2^2 \)
Fit amplitude 71.5 [nm]

Measured topology [nm]
RMS amplitude 29.3 [nm]

y [mm]
x [mm]

Error [nm]
RMS 3.1 [nm]

Figure 5.11: Topologies \( Z_2^2 \). The center plot shows the measured topology of the sample. The left plot shows the target shape with amplitude fitted to the amplitude of the recorded shape. The residual error is plotted on the right.

Figure 5.12: Decomposition measurement \( Z_2^2 \). The bar graph shows the contribution of the first 36 Zernike Polynomials\(^4\) in the measured topology.

The decomposition shows \( Z_2^2 \) being dominant in the measured topology. In the residuals \( Z_4^2 \) and \( Z_6^2 \) show up over other Zernike terms. These are higher order astigmatisms and are an indication of the error in the modeling of the heat flux input. The found magnitudes of other Zernike shapes are small and can result of modeling error but also from error introduced by the measurement system.

The results for the higher order Zernike \( Z_5^{-1} \) are shown in Figure 5.13\(^5\). The plot again shows similar surface shapes but now with lower amplitudes. Because of this the percentage error is larger due to error introduced by the measurement system.

---

\(^4\) Zernike Polynomial

\(^5\) Figure 5.13
Results and discussion

Figure 5.13: Topologies $Z_5^{-1}$. The center plot shows the measured topology of the sample. The left plot shows the target shape with amplitude fitted to the amplitude of the recorded shape. The residual error is plotted on the right.

Figure 5.14: Decomposition measurement $Z_5^{-1}$. The bar graph shows the contribution of the first 36 Zernike Polynomials in the measured topology.

Figure 5.14 shows $Z_5^{-1}$ being dominant and shows significant contribution of $Z_9^{-1}$ which is an higher order aberration of the same type indication modeling error. The contribution of the other Zernike terms is in the same range as error introduced in shaping of $Z_2^2$ in Figure 5.12 but relatively large due to the smaller amplitude of $Z_5^{-1}$. Some Zernike shapes where constructed on the substrate surface with larger deviation (possibly due to modeling error) than the two described above. All results can be found in Appendix C.3. All fitted amplitudes and errors are found in Table 5.1.
Table 5.1: Results for all Zernike polynomial surface shapes in the experiment set showing the amplitude of the fitted Zernike polynomial and the RMS error with this desired surface shape. All topologies are found in Appendix C.3.

<table>
<thead>
<tr>
<th>Zernike</th>
<th>Common name</th>
<th>Fit amplitude [nm]</th>
<th>RMS error [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_2^2)</td>
<td>Power (paraxial focus)</td>
<td>76.3</td>
<td>5.11</td>
</tr>
<tr>
<td>(Z_2^2)</td>
<td>3rd Order Astigmatism</td>
<td>71.5</td>
<td>3.05</td>
</tr>
<tr>
<td>(Z_{-2}^2)</td>
<td>3rd Order 45° Astigmatism</td>
<td>66.4</td>
<td>3.10</td>
</tr>
<tr>
<td>(Z_3^3)</td>
<td>3rd Order x-coma</td>
<td>60.1</td>
<td>3.31</td>
</tr>
<tr>
<td>(Z_{-1}^3)</td>
<td>3rd Order y-coma</td>
<td>58.0</td>
<td>3.39</td>
</tr>
<tr>
<td>(Z_1^0)</td>
<td>3rd Order Spherical</td>
<td>41.6</td>
<td>5.62</td>
</tr>
<tr>
<td>(Z_2^3)</td>
<td>Trefoil</td>
<td>67.4</td>
<td>3.08</td>
</tr>
<tr>
<td>(Z_{-3}^3)</td>
<td>45° Trefoil</td>
<td>67.0</td>
<td>2.96</td>
</tr>
<tr>
<td>(Z_2^4)</td>
<td>5th Order Astigmatism</td>
<td>46.1</td>
<td>13.09</td>
</tr>
<tr>
<td>(Z_{-2}^4)</td>
<td>5th Order 45° Astigmatism</td>
<td>40.7</td>
<td>15.16</td>
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<tr>
<td>(Z_3^5)</td>
<td>5th Order x-coma</td>
<td>22.2</td>
<td>1.85</td>
</tr>
<tr>
<td>(Z_{-1}^5)</td>
<td>5th Order y-coma</td>
<td>21.3</td>
<td>1.63</td>
</tr>
<tr>
<td>(Z_2^6)</td>
<td>5th Order Spherical</td>
<td>18.9</td>
<td>3.79</td>
</tr>
<tr>
<td>(Z_{-3}^6)</td>
<td>Quadrafoil</td>
<td>45.6</td>
<td>2.60</td>
</tr>
<tr>
<td>(Z_4^4)</td>
<td>45° Quadrafoil</td>
<td>46.3</td>
<td>2.20</td>
</tr>
<tr>
<td>(Z_2^5)</td>
<td>5th Order Trefoil</td>
<td>25.1</td>
<td>2.41</td>
</tr>
<tr>
<td>(Z_{-2}^5)</td>
<td>5th Order 45° Trefoil</td>
<td>25.3</td>
<td>2.22</td>
</tr>
<tr>
<td>(Z_3^6)</td>
<td>7th Order Astigmatism</td>
<td>19.8</td>
<td>6.18</td>
</tr>
<tr>
<td>(Z_{-1}^6)</td>
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<td>19.4</td>
<td>7.82</td>
</tr>
<tr>
<td>(Z_2^7)</td>
<td>Pentafoil</td>
<td>32.1</td>
<td>1.63</td>
</tr>
<tr>
<td>(Z_{-3}^7)</td>
<td>45° Pentafoil</td>
<td>32.7</td>
<td>1.77</td>
</tr>
</tbody>
</table>

The table shows the lower order Zernike polynomials (up to \(Z_3^3\)) are being constructed making up with \(\sim 5\%\) RMS error of their total amplitude. Higher order zernike polynomials where constructed with relatively large error. Zernikes \(Z_2^2\) and \(Z_2^2\) are two examples that show larger deviation due to modeling error.

It was found that for these (and other higher order Zernikes, not taken up in this report) surface shapes, it was difficult to obtain a FEM model that was able to run on a normal computer as this modeling required fine meshing of 3D structure in an elaborate iterative optimization procedure. It can be concluded that the modeling of the heat flux inputs by using FEM software to optimize the heat flux pattern is well feasible for lower order Zernike polynomials. For higher order polynomials it proved difficult to obtain a proper result without introducing relatively large error in the measured surface.

This conclusion leads to the following recommendation: In further research it is recommended that work is done on finding an inverse relation for finding heat flux input patterns to actuate the surface into a desired shape.
Contactless surface shape actuation of a mirror substrate solely by thermal radiative input is a new concept for a deformable mirror. The actuation of the surface shape of a sample substrate according to a set of Zernike polynomials is an unmistakable proof of this concept. An effective model was made that relates mirror design parameters and heat flux input properties to the behavior of the substrate response. This model and mirror design parameters are evaluated with an experimental setup.

Based upon the modeling and experimental work done during the course of this project, the following conclusions can be drawn:

1. The amplitude of any actuated surface deformation is linearly dependent on the amplitude of the heat flux pattern as projected onto the mirror surface. The nonlinear character of a heat transfer by radiation does not have any significant influence at the (small) temperature differences on the substrate. This was found by modeling in Subsection 2.1.1 and shown by the results of the linearity experiments as found in Section 5.1.

2. Superpositioning of heat flux inputs lead to a superpositioning of their respective outputs in terms of surface shape as described in Section 5.2. This is an additional proof of the linearity of the system behavior.

3. Both amplitude (A) and time constant (τ) of the response of an actuated surface shape are inversely proportional to the spatial frequency of the applied heat flux distribution, squared: \[ [A, \tau] \propto f_s^{-2} \]. The time constants measured are in agreement with values modeled in Subsection 2.3.3.
Conclusions and recommendations

4. The thickness of the mirror has no direct influence on the achievable final amplitude of the actuation mechanism. However if the thickness of the mirror is larger than half of the spatial period of the actuated shape, the ‘loading-effect’ affects the temporal response. This can make the response initially faster due to bending of the substrate and it can result in overshoot as was observed in experiments on sample M2 (Appendix C.2.1).

5. When a material is selected for a deformable mirror substrate, the speed of response is proportional to the thermal diffusivity $D$. The Amplitude of the response is proportional to the magnitude parameter $M$ as described in Subsection 3.1.1. This needs to be used to compare different materials considered for a design.

6. It is possible to use FEM software to generate feed forward heat flux distributions to generate Zernike polynomial surface shapes. This is effective for most lower order shapes: Zernike shapes up to $Z_{33}$ are actuated with amplitudes of $\sim 50 - 70 \text{ nm}$ and $\sim 5\%$ RMS error. Inputs for higher order Zernike Polynomials proved difficult to obtain and showed larger errors in experiments.

7. In the design and manufacturing of a thermally actuated deformable mirror as described in this report, the shape of the substrate surface induced by the offset heat flux (provided to allow for ‘negative’ input) must be taken into account. The negative of this shape must be made in the substrate surface in addition to the surface shape as specified to acquire the target optical specifications for the element.

Recommendations for future work

The following is recommended for future work:

1. The behavior of samples M4 and M5 is not fully understood. The cause of this unexpected behavior is to be identified as it has implications on future design of contactless deformable mirrors. A future step can be the modeling the effects of possible properties of the glue layer between two substrates. Furthermore, for experiments, samples can be obtained that are compromised of glued substrates with the absorptive coating on yet a different location. This can possibly isolate behavior of the layer between the substrates.

2. In this project specific surface shapes where made (Zernike polynomials) by generating the heat flux input pattern using FEM software. This method is elaborate and proved to be inconvenient for higher order polynomials. It would be highly useful if a clear input-output relation would be defined. In such a relation the surface deformation would be a function of heat inputs on any point on the surface. This relation can be used for the design of a closed-loop control system.
3. A lookup table was made to cope with the non-linear behavior of the actuator (the RGB projector) as described in Subsection 4.2.2. An initial lookup table was discarded in a later stage of the project because deviation was apparent in actuated amplitudes. A new lookup table was made and differences were found with the initial one. At the end of the project it again seemed that the used lookup table provides awkward inputs to the projector. A plausible explanation of this is the aging of the projector lamp during this project and the apparent changing of its radiative properties. It seems sensible to device some kind of auto calibration tool for the projector intensity for further experiments and ruling out any changing properties. The main requirement on this calibration tool is that the incident heat flux on the mirror substrate is always proportional to the specified intensity to the projector.

4. The substrate mounting device as described in Section 3.3 is not fully thermally isolated from the sample. Changing temperature in the material of this clamp resulted in significant position changes of the mirror surface (and thus changes in the interferogram). Because of this the optical mount needs to be re-adjusted after warming up (with offset heat flux applied). This can only be done manually and requires breech of the vacuum which again affects the temperature due to convection. Hence, this results in a cumbersome iterative adjusting procedure after the warming up of the substrate. Having the possibility to remotely control the adjustments of the optical mount that holds the sample assembly would greatly improve the ease of use of the experimental setup.
Bibliography


Thermo-mechanical modeling

The internal energy of a solid is generally a function of its temperature and strain:

\[ e(T, \varepsilon) = \rho C_p T + \frac{1}{2} E \varepsilon^2 \]  (A.1)

If a material is initially stress-free and unconstraint in expansion, the material will remain stress free (no strain energy). If all material expansion would be constraint, thermal strain energy is induced in the material apart from just thermal energy.

\[ e = \rho C_p \Delta T + \frac{1}{2} E \alpha^2 \Delta T^2 \]  (A.2)

Although all internal energy is now a function of temperature, it still distinguishes a term for internal (thermal) energy and thermally induced strain energy. The ratio of these terms is found easily:

\[ \frac{\text{thermal strain energy}}{\text{thermal internal energy}} = \frac{E \alpha^2 \Delta T}{2 \rho C_p} \]  (A.3)

This ratio remains small (using material parameters as found in Table 2.1) unless very high temperatures are reached which is not intended in this project.
Table of experiments on all samples

Table B.1: Experiments performed on substrates. In the superposition experiment, OL denotes outer left, CL center left, C center, CR center right and OR outer right.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Experiment</th>
<th>Analysis</th>
<th>Input</th>
<th>Intensity</th>
<th>Output time</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN2_00</td>
<td>linearity mode 2</td>
<td>stationary</td>
<td>homogeneous</td>
<td>00%</td>
<td>t = 10 s.</td>
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<td>5 periods</td>
<td>20%</td>
<td>t = 10 s.</td>
<td>30 fps</td>
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<td>30 fps</td>
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<td>90%</td>
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Continued on next page ...
Table B.1 – continued from previous page

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<th>Measurement</th>
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<th>Analysis</th>
<th>Input</th>
<th>Intensity</th>
<th>Output time</th>
<th>note</th>
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<td>superposition</td>
<td>transient</td>
<td>1 period OL</td>
<td>100%</td>
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<td>1 period CL</td>
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<td>transient</td>
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<td>transient</td>
<td>7 periods</td>
<td>100%</td>
<td>t = 0-10 s.</td>
<td>7.5 fps</td>
</tr>
<tr>
<td>DPR_05</td>
<td>substrate parameters</td>
<td>transient</td>
<td>11 periods</td>
<td>100%</td>
<td>t = 0-5 s.</td>
<td>15 fps</td>
</tr>
<tr>
<td>DPR_06</td>
<td>substrate parameters</td>
<td>transient</td>
<td>15 periods</td>
<td>100%</td>
<td>t = 0-2 s.</td>
<td>30 fps</td>
</tr>
<tr>
<td>DPR_07</td>
<td>substrate parameters</td>
<td>transient</td>
<td>21 periods</td>
<td>100%</td>
<td>t = 0-2 s.</td>
<td>30 fps</td>
</tr>
<tr>
<td>DPR_08</td>
<td>substrate parameters</td>
<td>transient</td>
<td>31 periods</td>
<td>100%</td>
<td>t = 0-2 s.</td>
<td>30 fps</td>
</tr>
</tbody>
</table>
C.1 Measurement system

Figure C.1: Average measured topology and RMS error on 25 individual measurements with the interferogram on the far left side of the CCD array as shown in figure C.3.
Figure C.2: Average measured topology and RMS error on 25 individual measurements with the interferogram on the far right side of the CCD array as shown in figure C.3.

Figure C.3: Shifting interferograms. Both interferograms were made with the same mirror topologies: a folding mirror was used to shift the interferogram itself over the CCD array without any other adjustments to the optical setup components.
Measurement results

Differences between topology on interferogram locations [nm]

RMS over surface: 2.81 [nm]

Figure C.4: Topology difference between interferograms.
C.2 Individual Sample Measurements

This Appendix will present all individual measurements on the set of sample substrates. These results include:

- Results for the experiments on linearity of mode 2 for all sample substrates.
- Results for the experiments on linearity of mode 3 for all sample substrates.
- Results for the transient measurements on all sample substrates.

Note: In the results for the transient measurements, not all experiments on the samples as presented in table [B.1] are found. Measurements of topology actuated by higher order spatial frequencies are discarded as it was found they induce topology amplitudes that could not be measured using the current measurement tool.
C.2.1 Sample M2

Linearity mode 2 sample M2

Heat flux projected on mirror substrate: homogeneous

Substrate: 2
Experiment: Linearity mode 2
Reconstructed topology: $I = -50\%$

Substrate: 2
Experiment: Linearity mode 2
Reconstructed topology: $I = 50\%$

Figure C.5: Reconstructed topologies sample M2, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Substrate: 2
Experiment: Linearity mode 2
Deflection: averaged over y

Substrate: 2
Experiment: Linearity mode 2
Spherical curvature: amplitude (fit) and linearity

Figure C.6: Linearity sample M2, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M2

Heat flux projected on mirror substrate: 5 periods

![Reconstructed topologies sample M2, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).](image)

Figure C.7: Reconstructed topologies sample M2, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

![Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).](image)

Figure C.8: Linearity sample M2, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M2

Periods projected on mirror substrate: 3
Measurement spatial resolution: 2.1 [mm]

![Graph 1](image1.png)

Figure C.9: Transient response sample M2 (a) showing RMS amplitude in time and fitted response. Reconstructed (partial) topology for 3-period-pattern.

Periods projected on mirror substrate: 5
Measurement spatial resolution: 2.1 [mm]

![Graph 2](image2.png)

Figure C.10: Transient response sample M2 (a) showing RMS amplitude in time and fitted response. Reconstructed (partial) topology for 5-period-pattern.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 1.8 [mm]

Substrate: 2
Experiment: transient response, P7
Amplitude: RMS

(a) Transient response

Figure C.11: Transient response sample M2 (a) showing RMS amplitude in time and fitted response. Reconstructed (partial) topology for 7-period-pattern.
C.2.2 Sample M3

Linearity mode 2 sample M3

Heat flux projected on mirror substrate: homogeneous
Measurement spatial resolution: x [mm]

Substrate: 3
Experiment: Linearity mode 2
Reconstructed topology: I = −50%

Substrate: 3
Experiment: Linearity mode 2
Reconstructed topology: I = 50%

(a) Minimum flux
(b) Maximum flux

Figure C.12: Reconstructed topologies sample M3, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Substrate: 3
Experiment: Linearity mode 2
Deflection: averaged over y

Substrate: 3
Experiment: Linearity mode 2
Spherical curvature: Fit and linearity

(a) Amplitude averaged over y
(b) RMS amplitude and linear fit

Figure C.13: Linearity sample M3, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M3

Heat flux projected on mirror substrate: 5 periods

Figure C.14: Reconstructed topologies sample M3, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.15: Linearity sample M3, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M3

Periods projected on mirror substrate: 3

Substrate: 3  
Experiment: transient response, P3  
Amplitude: RMS and linear fit

Nonlinear least squares fit

Measured data: RMS amplitude [nm]
Nonlinear fit: \[ A \cdot \left(1 - e^{-t/\tau}\right) \]
Amplitude: 40.8 [nm]  
Time constant: 4.94 [s]

(a) Transient response

(b) Reconstructed topology

Figure C.16: Transient response sample M3 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5

Measurement spatial resolution: 2.8 [mm]

Substrate: 3  
Experiment: transient response, P5  
Amplitude: RMS and linear fit

Nonlinear least squares fit

Nonlinear fit: \[ A \cdot \left(1 - e^{-t/\tau}\right) \]
Amplitude: 14.0 [nm]  
Time constant: 2.45 [s]

(a) Transient response

(b) Reconstructed topology (cropped to 2 periods)

Figure C.17: Transient response sample M3 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.1 [mm]

Substrate: 3
Experiment: transient response, P7
Amplitude: RMS and linear fit

Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 7.0 [nm]
Time constant: 1.41 [s]

(a) Transient response
(b) Reconstructed topology (cropped to 3 periods)

Figure C.18: Transient response sample M3 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.

Periods projected on mirror substrate: 11
Measurement spatial resolution: 2.1 [mm]

Substrate: 3
Experiment: transient response, P11
Amplitude: RMS and linear fit

Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 1.8 [nm]
Time constant: 0.68 [s]

(a) Transient response
(b) Reconstructed topology (cropped to 4 periods)

Figure C.19: Transient response sample M3 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 11-period-pattern: four periods plotted.
C.2.3 Sample M4

Linearity mode 2 sample M4

Heat flux projected on mirror substrate: homogeneous

Figure C.20: Reconstructed topologies sample M4, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.21: Linearity sample M4, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M4

Heat flux projected on mirror substrate: 5 periods

Figure C.22: Reconstructed topologies sample M4, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.23: Linearity sample M4, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M4

Periods projected on mirror substrate: 3
Measurement spatial resolution: 2.1 [mm]

Nonlinear least squares fit

Measured data: RMS amplitude [nm]
Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 56.1 [nm]
Time constant: 6.24 [s]

Substrate: 4
Experiment: transient response, P3
Amplitude: RMS and linear fit

Substrate: 4
Experiment: transient response, P3
Reconstructed topology: $t = 30.5$ [s]

(a) Transient response
(b) Reconstructed topology (cropped to 1 period)

Figure C.24: Transient response sample M4 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: 2.1 [mm]

Nonlinear least squares fit

Measured data: RMS amplitude [nm]
Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 21.9 [nm]
Time constant: 2.85 [s]

Substrate: 4
Experiment: transient response, P5
Amplitude: RMS and linear fit

Substrate: 4
Experiment: transient response, P5
Reconstructed topology: $t = 15.0$ [s]

(a) Transient response
(b) Reconstructed topology (cropped to 2 periods)

Figure C.25: Transient response sample M4 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.1 [mm]

Substrate: 4
Experiment: transient response, P7
Amplitude: RMS and linear fit

<table>
<thead>
<tr>
<th>Measured data: RMS amplitude [nm]</th>
<th>Nonlinear least squares fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear fit: ( A \cdot (1 - e^{-t/\tau}) )</td>
<td>Amplitude: 10.8 [nm]</td>
</tr>
<tr>
<td>Time constant: 1.60 [s]</td>
<td></td>
</tr>
</tbody>
</table>

(a) Transient response

(b) Reconstructed topology (cropped to 3 periods)

Figure C.26: Transient response sample M4 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.

Periods projected on mirror substrate: 11
Measurement spatial resolution: 2.1 [mm]

Substrate: 4
Experiment: transient response, P11
Amplitude: RMS and linear fit

<table>
<thead>
<tr>
<th>Measured data: RMS amplitude [nm]</th>
<th>Nonlinear least squares fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear fit: ( A \cdot (1 - e^{-t/\tau}) )</td>
<td>Amplitude: 3.0 [nm]</td>
</tr>
<tr>
<td>Time constant: 0.90 [s]</td>
<td></td>
</tr>
</tbody>
</table>

(a) Transient response

(b) Reconstructed topology (cropped to 5 periods)

Figure C.27: Transient response sample M4 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 11-period-pattern: four periods plotted.
C.2.4 Sample M5

Linearity mode 2 sample M5

Heat flux projected on mirror substrate: homogeneous

![Graph](image1)

Figure C.28: Reconstructed topologies sample M5, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

![Graph](image2)

Figure C.29: Linearity sample M5, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M5

Heat flux projected on mirror substrate: 5 periods

Substrate: 5
Experiment: Linearity mode 3
Reconstructed topology: $I = -50\%$

Substrate: 5
Experiment: Linearity mode 3
Reconstructed topology: $I = +50\%$

Figure C.30: Reconstructed topologies sample M5, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Substrate: 5
Experiment: Linearity mode 3
Deflection: averaged over y

(a) Amplitude averaged over y

Substrate: 5
Experiment: Linearity mode 3
Amplitude: RMS and linear fit

(b) RMS amplitude and linear fit

Figure C.31: Linearity sample M5, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M5

Periods projected on mirror substrate: 3
Measurement spatial resolution: $2.8 \, [mm]$

- Substrate: 5
- Experiment: transient response, P3
- Amplitude: RMS and linear fit

<table>
<thead>
<tr>
<th>Measured data: RMS amplitude [nm]</th>
<th>Nonlinear least squares fit</th>
</tr>
</thead>
</table>
| Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$  
Amplitude: 27.2 [nm]  
Time constant: 8.89 [s] |

(a) Transient response

(b) Reconstructed topology (cropped to 1 period)

Figure C.32: Transient response sample M5 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: $2.8 \, [mm]$

- Substrate: 5
- Experiment: transient response, P5
- Amplitude: RMS and linear fit

<table>
<thead>
<tr>
<th>Measured data: RMS amplitude [nm]</th>
<th>Nonlinear least squares fit</th>
</tr>
</thead>
</table>
| Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$  
Amplitude: 9.7 [nm]  
Time constant: 3.69 [s] |

(a) Transient response

(b) Reconstructed topology (cropped to 2 periods)

Figure C.33: Transient response sample M5 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Measurement results

Periods projected on mirror substrate: \( 7 \)
Measurement spatial resolution: \( 2.1 \, \text{[mm]} \)

Substrate: \( 5 \)
Experiment: transient response, P7
Amplitude: RMS and linear fit

Nonlinear least squares fit

Measured data: RMS amplitude [nm]
Nonlinear fit: \( A \cdot (1 - e^{-t/\tau}) \)
Amplitude: 3.8 [nm]
Time constant: 2.13 [s]

Figure C.34: Transient response sample M5 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.
C.2.5 Sample M6

Linearity mode 2 sample M6

Heat flux projected on mirror substrate: homogeneous

Figure C.35: Reconstructed topologies sample M6, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.36: Linearity sample M6, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M6

Heat flux projected on mirror susbtrate: 5 periods

Figure C.37: Reconstructed topologies sample M6, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.38: Linearity sample M6, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M6

Periods projected on mirror substrate: 3
Measurement spatial resolution: 2.8 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
Amplitude: 40.0 [nm]
Time constant: 4.88 [s]

Substrate: 6
Experiment: transient response, P3
Amplitude: RMS and linear fit

RMS amplitude [nm] vs. time [s]
(a) Transient response

Reconstructed topology: \( t = 30.4 \) [s]

Figure C.39: Transient response sample M6 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: 2.1 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
Amplitude: 15.2 [nm]
Time constant: 2.46 [s]

Substrate: 6
Experiment: transient response, P5
Amplitude: RMS and linear fit

RMS amplitude [nm] vs. time [s]
(a) Transient response

Reconstructed topology: \( t = 15.1 \) [s]

Figure C.40: Transient response sample M6 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.1 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 7.7 [nm]
Time constant: 1.45 [s]

Figure C.41: Transient response sample M6 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.

Periods projected on mirror substrate: 11
Measurement spatial resolution: 1.7 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: $A \cdot (1 - e^{-t/\tau})$
Amplitude: 3.0 [nm]
Time constant: 0.82 [s]

Figure C.42: Transient response sample M6 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 11-period-pattern: four periods plotted.
C.2.6 Sample M7

Linearity mode 2 sample M7

Heat flux projected on mirror substrate: homogeneous

![Graphs showing reconstructed topologies sample M7, mode 2.](image)

(a) Minimum flux  
(b) Maximum flux

Figure C.43: Reconstructed topologies sample M7, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

![Graphs showing topology averaged over y.](image)

(a) Amplitude averaged over y  
(b) RMS amplitude and linear fit

Figure C.44: Linearity sample M7, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M7

Heat flux projected on mirror substrate: 5 periods

Figure C.45: Reconstructed topologies sample M7, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.46: Linearity sample M7, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M7

Periods projected on mirror substrate: 3
Measurement spatial resolution: 2.8 [mm]

- Nonlinear least squares fit
- Measured data: RMS amplitude [nm]
- Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
  - Amplitude: 40.7 [nm]
  - Time constant: 6.81 [s]

Substrate: 7
Experiment: transient response, P3
Amplitude: RMS and linear fit

(a) Transient response

Reconstructed topology: \( t = 30.4 \) [s]

(b) Reconstructed topology (cropped to 1 period)

Figure C.47: Transient response sample M7 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: 2.1 [mm]

- Nonlinear least squares fit
- Measured data: RMS amplitude [nm]
- Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
  - Amplitude: 13.6 [nm]
  - Time constant: 2.67 [s]

Substrate: 7
Experiment: transient response, P5
Amplitude: RMS and linear fit

(a) Transient response

Reconstructed topology: \( t = 15.0 \) [s]

(b) Reconstructed topology (cropped to 2 periods)

Figure C.48: Transient response sample M7 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.1 [mm]

Substrate: 7
Experiment: transient response, P7
Amplitude: RMS and linear fit

Nonlinear fit: \( A \cdot \left( 1 - e^{-t/\tau} \right) \)
Amplitude: 7.8 [nm]
Time constant: 1.49 [s]

(a) Transient response
(b) Reconstructed topology (cropped to 3 periods)

Figure C.49: Transient response sample M7 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.

Periods projected on mirror substrate: 11
Measurement spatial resolution: 1.7 [mm]

Substrate: 7
Experiment: transient response, P11
Amplitude: RMS and linear fit

Nonlinear fit: \( A \cdot \left( 1 - e^{-t/\tau} \right) \)
Amplitude: 2.9 [nm]
Time constant: 0.74 [s]

(a) Transient response
(b) Reconstructed topology (cropped to 5 periods)

Figure C.50: Transient response sample M7 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 11-period-pattern: four periods plotted.
C.2.7 Sample M9

Linearity mode 2 sample M9

Heat flux projected on mirror substrate: homogeneous

Substrate: 9  
Experiment: Linearity mode 2  
Reconstructed topology: \( I = -50\% \)

![Minimum flux](image)

Substrate: 9  
Experiment: Linearity mode 2  
Reconstructed topology: \( I = 50\% \)

![Maximum flux](image)

Figure C.51: Reconstructed topologies sample M9, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

![Amplitude averaged over y](image)

Substrate: 9  
Experiment: Linearity mode 2  
Deflection: averaged over y

![RMS amplitude and linear fit](image)

Substrate: 9  
Experiment: Linearity mode 2  
Spherical curvature: amplitude (fit) and linearity

Figure C.52: Linearity sample M9, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Linearity mode 3 sample M9

Heat flux projected on mirror substrate: 5 periods

Figure C.53: Reconstructed topologies sample M9, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.54: Linearity sample M9, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M9

Periods projected on mirror substrate: 3
Measurement spatial resolution: 2.8 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \( A \cdot (1 - e^{-t/\tau}) \)
Amplitude: 17.4 [nm]
Time constant: 3.76 [s]

Figure C.55: Transient response sample M9 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: 2.8 [mm]

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \( A \cdot (1 - e^{-t/\tau}) \)
Amplitude: 6.3 [nm]
Time constant: 1.99 [s]

Figure C.56: Transient response sample M9 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Measurement results

Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.8 \( [\text{mm}] \)

Nonlinear least squares fit

Measured data: RMS amplitude \( [\text{nm}] \)
Nonlinear fit:
\[ A \cdot (1 - e^{-t/\tau}) \]
Amplitude: 3.0 \( [\text{nm}] \)
Time constant: 1.19 \( [\text{s}] \)

Substrate: 9
Experiment: transient response, P7
Amplitude: RMS and linear fit

Figure C.57: Transient response sample M9 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.
C.2.8 Sample M10

Linearity mode 2 sample M10

Heat flux projected on mirror substrate: homogeneous

Figure C.58: Reconstructed topologies sample M10, mode 2. Maximum positive homogeneous heat flux (a) and maximum positive homogeneous heat flux(b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.59: Linearity sample M10, mode 2. Topology profiles averaged over y (a) for all heat flux levels. Induced spherical shape curvature for all heat flux levels and linear fit (b).
Measurement results

Linearity mode 3 sample M10

Heat flux projected on mirror substrate: 5 periods

Figure C.60: Reconstructed topologies sample M10, mode 3. Maximum negative heat flux pattern (a) and maximum negative heat flux pattern (b).

Topology averaged over y. Topology profiles and RMS amplitude linearity:

Figure C.61: Linearity sample M10, mode 3. Topology profiles averaged over y (a) for all heat flux levels. RMS amplitude for all heat flux levels and linear fit (b).
Transient parameters sample M10

Periods projected on mirror substrate: 3
Measurement spatial resolution: $3.8 \text{ [mm]}$

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit:
$$A \cdot (1 - e^{-t/\tau})$$
Amplitude: $44.2 \text{ [nm]}$
Time constant: $4.88 \text{ [s]}$

Substrate: 10
Experiment: transient response, P3
Amplitude: RMS and linear fit

RMS amplitude [nm]

(a) Transient response

Figure C.62: Transient response sample M10 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 3-period-pattern: one period plotted.

Periods projected on mirror substrate: 5
Measurement spatial resolution: $2.8 \text{ [mm]}$

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit:
$$A \cdot (1 - e^{-t/\tau})$$
Amplitude: $16.7 \text{ [nm]}$
Time constant: $2.50 \text{ [s]}$

Substrate: 10
Experiment: transient response, P5
Amplitude: RMS and linear fit

RMS amplitude [nm]

(a) Transient response

Figure C.63: Transient response sample M10 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 5-period-pattern: two periods plotted.
Periods projected on mirror substrate: 7
Measurement spatial resolution: 2.8 [mm]

Substrate: 10
Experiment: transient response, P7
Amplitude: RMS and linear fit

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
Amplitude: 8.6 [nm]
Time constant: 1.47 [s]

Figure C.64: Transient response sample M10 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 7-period-pattern: three periods plotted.

Periods projected on mirror substrate: 11
Measurement spatial resolution: 2.8 [mm]

Substrate: 10
Experiment: transient response, P11
Amplitude: RMS and linear fit

Nonlinear least squares fit
Measured data: RMS amplitude [nm]
Nonlinear fit: \[ A \cdot (1 - e^{-t/\tau}) \]
Amplitude: 3.3 [nm]
Time constant: 0.78 [s]

Figure C.65: Transient response sample M10 (a) showing RMS amplitude in time and fitted response. Reconstructed topology for 11-period-pattern: four periods plotted.
C.3 Input modeling for Zernike Polynomials

Table C.1: Results for all Zernike polynomial surface shapes in the experiment set showing the amplitude of the fitted Zernike polynomial and the RMS error with this desired surface shape. The most right column shows the spatial resolution of the measurement (limited by filtering in processing data).

<table>
<thead>
<tr>
<th>Zernike</th>
<th>Common name</th>
<th>Fit amplitude [nm]</th>
<th>RMS error [nm]</th>
<th>Spatial res. [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0^2$</td>
<td>Power (paraxial focus)</td>
<td>76.3</td>
<td>5.11</td>
<td>2.5</td>
</tr>
<tr>
<td>$Z_2^3$</td>
<td>3rd Order Astigmatism</td>
<td>71.5</td>
<td>3.05</td>
<td>2.5</td>
</tr>
<tr>
<td>$Z_2^{-2}$</td>
<td>3rd Order 45° Astigmatism</td>
<td>66.4</td>
<td>3.10</td>
<td>2.5</td>
</tr>
<tr>
<td>$Z_3^3$</td>
<td>3rd Order x-coma</td>
<td>60.1</td>
<td>3.31</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_3^{-1}$</td>
<td>3rd Order y-coma</td>
<td>58.0</td>
<td>3.39</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_4^0$</td>
<td>3rd Order Spherical</td>
<td>41.6</td>
<td>5.62</td>
<td>1.2</td>
</tr>
<tr>
<td>$Z_3^1$</td>
<td>Trefoil</td>
<td>67.4</td>
<td>3.08</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_3^{-3}$</td>
<td>45° Trefoil</td>
<td>67.0</td>
<td>2.96</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_4^1$</td>
<td>5th Order Astigmatism</td>
<td>46.1</td>
<td>13.09</td>
<td>1.2</td>
</tr>
<tr>
<td>$Z_4^{-2}$</td>
<td>5th Order 45° Astigmatism</td>
<td>40.7</td>
<td>15.16</td>
<td>1.2</td>
</tr>
<tr>
<td>$Z_5^1$</td>
<td>5th Order x-coma</td>
<td>22.2</td>
<td>1.85</td>
<td>1.2</td>
</tr>
<tr>
<td>$Z_5^{-1}$</td>
<td>5th Order y-coma</td>
<td>21.3</td>
<td>1.63</td>
<td>1.2</td>
</tr>
<tr>
<td>$Z_6^0$</td>
<td>5th Order Spherical</td>
<td>18.9</td>
<td>3.79</td>
<td>2.5</td>
</tr>
<tr>
<td>$Z_4^1$</td>
<td>Quadrafoil</td>
<td>45.6</td>
<td>2.60</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_4^{-4}$</td>
<td>45° Quadrafoil</td>
<td>46.3</td>
<td>2.20</td>
<td>1.5</td>
</tr>
<tr>
<td>$Z_5^3$</td>
<td>5th Order Trefoil</td>
<td>25.1</td>
<td>2.41</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z_5^{-3}$</td>
<td>5th Order 45° Trefoil</td>
<td>25.3</td>
<td>2.22</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z_6^2$</td>
<td>7th Order Astigmatism</td>
<td>19.8</td>
<td>6.18</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z_6^{-2}$</td>
<td>7th Order 45° Astigmatism</td>
<td>19.4</td>
<td>7.82</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z_5^5$</td>
<td>Pentaofoil</td>
<td>32.1</td>
<td>1.63</td>
<td>2.0</td>
</tr>
<tr>
<td>$Z_5^{-5}$</td>
<td>45° Pentafoil</td>
<td>32.7</td>
<td>1.77</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Measurement results
Zernike Polynomial $Z^0_2$

Common name: Power (paraxial focus)
Measurement spatial resolution: 2.5 [mm]

![Figure C.66: Topologies for actuation of $Z^0_2$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

![Figure C.67: Decomposition measurement of actuation of $Z^0_2$. The first 36 Zernike Polynomials are included in this decomposition.](image)

![Figure C.68: Input for actuating $Z^0_2$ as was found using FEM software.](image)
Zernike Polynomial $Z_2^2$

Common name: 3rd Order Astigmatism
Measurement spatial resolution: 2.5 [mm]

Figure C.69: Topologies for actuation of $Z_2^2$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.70: Decomposition measurement of actuation of $Z_2^2$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.71: Input for actuating $Z_2^2$ as was found using FEM software.
Zernike Polynomial $Z_2^{-2}$

Common name: 3rd Order 45° Astigmatism
Measurement spatial resolution: 2.5 [mm]

![Figure C.72: Topologies for actuation of $Z_2^{-2}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

Zernike decomposition of measured topology, $Z_2^{-2}$

![Figure C.73: Decomposition measurement of actuation of $Z_2^{-2}$. The first 36 Zernike Polynomials are included in this decomposition.](image)

Input heat flux distribution for $Z_2^{-2}$

![Figure C.74: Input for actuating $Z_2^{-2}$ as was found using FEM software.](image)
Zernike Polynomial $Z^1_3$

Common name: 3rd Order x-cosa
Measurement spatial resolution: 1.5 [mm]

Target Shape: $Z^1_3$
Fit amplitude 60.1 [nm]

Measured topology [nm]
RMS amplitude 21.5 [nm]

Error [nm]
RMS 3.3 [nm]

Figure C.75: Topologies for actuation of $Z^1_3$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Zernike decomposition of measured topology, $Z^1_3$

Figure C.76: Decomposition measurement of actuation of $Z^1_3$. The first 36 Zernike Polynomials are included in this decomposition.

Input heat flux distribution for $Z^1_3$

Figure C.77: Input for actuating $Z^1_3$ as was found using FEM software.
Zernike Polynomial $Z_{3}^{-1}$

Common name: 3rd Order y-coma
Measurement spatial resolution: 1.5 [mm]

Figure C.78: Topologies for actuation of $Z_{3}^{-1}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Zernike decomposition of measured topology, $Z_{3}^{-1}$

Figure C.79: Decomposition measurement of actuation of $Z_{3}^{-1}$. The first 36 Zernike Polynomials are included in this decomposition.

Input heat flux distribution for $Z_{3}^{-1}$

Figure C.80: Input for actuating $Z_{3}^{-1}$ as was found using FEM software.
Zernike Polynomial $Z_4^0$

Common name: 3rd Order Spherical
Measurement spatial resolution: 1.2 $[mm]$

![Graph](image)

Figure C.81: Topologies for actuation of $Z_4^0$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Graph](image)

Figure C.82: Decomposition measurement of actuation of $Z_4^0$. The first 36 Zernike Polynomials are included in this decomposition.

![Graph](image)

Figure C.83: Input for actuating $Z_4^0$ as was found using FEM software.
Zernike Polynomial $Z_3^3$

Common name: Trefoil
Measurement spatial resolution: 1.5 [mm]

![Figure C.84: Topologies for actuation of $Z_3^3$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

Figure C.84: Topologies for actuation of $Z_3^3$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Figure C.85: Decomposition measurement of actuation of $Z_3^3$. The first 36 Zernike Polynomials are included in this decomposition.](image)

Figure C.85: Decomposition measurement of actuation of $Z_3^3$. The first 36 Zernike Polynomials are included in this decomposition.

![Figure C.86: Input for actuating $Z_3^3$ as was found using FEM software.](image)

Figure C.86: Input for actuating $Z_3^3$ as was found using FEM software.
Zernike Polynomial $Z_3^{-3}$

Common name: 45° Trefoil
Measurement spatial resolution: 1.5 [mm]

<table>
<thead>
<tr>
<th>Target Shape: $Z_3^{-3}$</th>
<th>Measured topology [nm]</th>
<th>Error [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit amplitude 67.0 [nm]</td>
<td>RMS amplitude 23.9 [nm]</td>
<td>RMS 3.0 [nm]</td>
</tr>
</tbody>
</table>

Figure C.87: Topologies for actuation of $Z_3^{-3}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Zernike decomposition of measured topology, $Z_3^{-3}$

Figure C.88: Decomposition measurement of actuation of $Z_3^{-3}$. The first 36 Zernike Polynomials are included in this decomposition.

Input heat flux distribution for $Z_3^{-3}$

Figure C.89: Input for actuating $Z_3^{-3}$ as was found using FEM software.
Zernike Polynomial $Z_4^2$

Common name: 5th Order Astigmatism
Measurement spatial resolution: 1.2 [mm]

Target Shape: $Z_4^2$
Fit amplitude 46.1 [nm]

Measured topology [nm]
RMS amplitude 19.6 [nm]

Error [nm]
RMS 13.1 [nm]

Figure C.90: Topologies for actuation of $Z_4^2$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Zernike decomposition of measured topology, $Z_4^2$

Figure C.91: Decomposition measurement of actuation of $Z_4^2$. The first 36 Zernike Polynomials are included in this decomposition.

Input heat flux distribution for $Z_4^2$

Figure C.92: Input for actuating $Z_4^2$ as was found using FEM software.
Zernike Polynomial $Z_4^{-2}$

Common name: 5th Order 45° Astigmatism
Measurement spatial resolution: 1.2 [mm]

Target Shape: $Z_4^{-2}$
Fit amplitude 40.7 [nm]

Measured topology [nm]
RMS amplitude 19.9 [nm]

Error [nm]
RMS 15.2 [nm]

Figure C.93: Topologies for actuation of $Z_4^{-2}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Zernike decomposition of measured topology, $Z_4^{-2}$

Figure C.94: Decomposition measurement of actuation of $Z_4^{-2}$. The first 36 Zernike Polynomials are included in this decomposition.

Input heat flux distribution for $Z_4^{-2}$

Figure C.95: Input for actuating $Z_4^{-2}$ as was found using FEM software.
Zernike Polynomial $Z^1_5$

Common name: 5th Order x-coma
Measurement spatial resolution: 1.2 \( [\text{mm}] \)

![Figure C.96](image1.jpg)

Figure C.96: Topologies for actuation of $Z^1_5$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Figure C.97](image2.jpg)

Figure C.97: Decomposition measurement of actuation of $Z^1_5$. The first 36 Zernike Polynomials are included in this decomposition.

![Figure C.98](image3.jpg)

Figure C.98: Input for actuating $Z^1_5$ as was found using FEM software.
Zernike Polynomial $Z_5^{-1}$

Common name: 5th Order y-coma
Measurement spatial resolution: 1.2 [mm]

Figure C.99: Topologies for actuation of $Z_5^{-1}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.100: Decomposition measurement of actuation of $Z_5^{-1}$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.101: Input for actuating $Z_5^{-1}$ as was found using FEM software.
Zernike Polynomial $Z_6^0$

Common name: 5th Order Spherical  
Measurement spatial resolution: 2.5 [mm]

![Figure C.102: Topologies for actuation of $Z_6^0$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

Figure C.102: Topologies for actuation of $Z_6^0$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Figure C.103: Decomposition measurement of actuation of $Z_6^0$. The first 36 Zernike Polynomials are included in this decomposition.](image)

Figure C.103: Decomposition measurement of actuation of $Z_6^0$. The first 36 Zernike Polynomials are included in this decomposition.

![Figure C.104: Input for actuating $Z_6^0$ as was found using FEM software.](image)

Figure C.104: Input for actuating $Z_6^0$ as was found using FEM software.
Zernike Polynomial $Z_4^4$

Common name: Quadrafoil
Measurement spatial resolution: 1.2 [mm]

Figure C.105: Topologies for actuation of $Z_4^4$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.106: Decomposition measurement of actuation of $Z_4^4$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.107: Input for actuating $Z_4^4$ as was found using FEM software.
Zernike Polynomial $Z_{4}^{-4}$

Common name: 45° Quadrafoil
Measurement spatial resolution: 1.5 [mm]

Figure C.108: Topologies for actuation of $Z_{4}^{-4}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.109: Decomposition measurement of actuation of $Z_{4}^{-4}$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.110: Input for actuating $Z_{4}^{-4}$ as was found using FEM software.
Zernike Polynomial $Z_5^3$

Common name: 5th Order Trefoil
Measurement spatial resolution: 2.0 [mm]

![Measurement results](image)

Figure C.111: Topologies for actuation of $Z_5^3$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Zernike decomposition of measured topology, $Z_5^3$](image)

Figure C.112: Decomposition measurement of actuation of $Z_5^3$. The first 36 Zernike Polynomials are included in this decomposition.

![Input heat flux distribution for $Z_5^3$](image)

Figure C.113: Input for actuating $Z_5^3$ as was found using FEM software.
Zernike Polynomial $Z_5^{-3}$

Common name: 5th Order 45° Trefoil
Measurement spatial resolution: 2.0 [mm]

Figure C.114: Topologies for actuation of $Z_5^{-3}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.115: Decomposition measurement of actuation of $Z_5^{-3}$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.116: Input for actuating $Z_5^{-3}$ as was found using FEM software.
Zernike Polynomial $Z_6^2$

Common name: 7th Order Astigmatism
Measurement spatial resolution: 2.0 [mm]

Figure C.117: Topologies for actuation of $Z_6^2$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.118: Decomposition measurement of actuation of $Z_6^2$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.119: Input for actuating $Z_6^2$ as was found using FEM software.
Zernike Polynomial $Z_{6}^{-2}$

Common name: 7th Order 45° Astigmatism
Measurement spatial resolution: 2.0 [mm]

![Figure C.120: Topologies for actuation of $Z_{6}^{-2}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

![Figure C.121: Decomposition measurement of actuation of $Z_{6}^{-2}$. The first 36 Zernike Polynomials are included in this decomposition.](image)

![Figure C.122: Input for actuating $Z_{6}^{-2}$ as was found using FEM software.](image)
Zernike Polynomial $Z_5^5$

Common name: Pentafoil
Measurement spatial resolution: 2.0 [mm]

Figure C.123: Topologies for actuation of $Z_5^5$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

Figure C.124: Decomposition measurement of actuation of $Z_5^5$. The first 36 Zernike Polynomials are included in this decomposition.

Figure C.125: Input for actuating $Z_5^5$ as was found using FEM software.
Zernike Polynomial $Z_5^{-5}$

Common name: 45° Pentafoil
Measurement spatial resolution: 1.2 [mm]

![Figure C.126: Topologies for actuation of $Z_5^{-5}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).](image)

Figure C.126: Topologies for actuation of $Z_5^{-5}$. Target shape with fitted amplitude (left) and measured topology (center) and error between these shapes (right).

![Figure C.127: Decomposition measurement of actuation of $Z_5^{-5}$. The first 36 Zernike Polynomials are included in this decomposition.](image)

Figure C.127: Decomposition measurement of actuation of $Z_5^{-5}$. The first 36 Zernike Polynomials are included in this decomposition.

![Figure C.128: Input for actuating $Z_5^{-5}$ as was found using FEM software.](image)

Figure C.128: Input for actuating $Z_5^{-5}$ as was found using FEM software.