Morphological effects of spatially varying grain size and bed roughness in rivers

M. Sc. Thesis

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Morphological effects of spatially varying grain size and bed roughness in rivers

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M. Sc. Thesis
Preface

The research described in this M. Sc. Thesis was carried out as a graduation project to complete my studies in Civil Engineering at the Delft University of Technology. The effects of a spatially varying grain size and a spatially varying bed roughness are investigated for various river sections.

The study was conducted for the Institute for Inland Water Management and Waste Water (RIZA) with support from WL | Delft Hydraulics. The basis for the research is formed by the linear analyses by Mosselman et al (1999), carried out to explain problems encountered during the calibration of a two-dimensional model for the bifurcation in the Rhine near Pannerden, The Netherlands.

Since I was a little boy, I have always been fascinated by water. This is why I decided to study Civil Engineering at the Christelijke Hogeschool Windesheim in Zwolle. After graduating with a specialisation of hydraulic engineering, I proceeded with the study of Civil Engineering at Delft University of Technology. During my studies I soon found the specialisation which fascinated me most, river dynamics. So when this research project emerged I seized the opportunity. Looking back at this graduation process this was a good choice. During the project I have learned a lot about hydraulic and morphological processes in river channels, as well as about working with numerical models.

I gratefully acknowledge the members of my graduation committee, prof. dr. ir. H.J. de Vriend, dr. ir. P.J. Visser, ir. F.J. Havinga, dr. ir. E. Mosselman. I also would like to thank dr. ir. J. Sieben (RIZA) and dr. ir. C.J. Slooff (WL | Delft Hydraulics) for their constructive comments and for supplying me with models of the Rhine branches. Special thanks go to ir. F.J. Havinga, who supported me with the use of the numerical models and their intricacies.

Finally I would like to thank all the graduate students who supported me and helped me tackle several little problems. Special thanks go to Gert-Jan Liek who helped me getting started with the numerical modelling using Delft2D-MOR. Furthermore, I would like to thank ing. S.P.A. Duinmeijer and ing. W.L.J. Peters for commenting on my report.

Alfons Smale
Delft, Oktober 2000
Abstract

Many morphological models that are in use today, make use of spatially constant grain sizes and bed roughness. Application of this spatially constant grain size and bed roughness is not a correct representation of reality. When making use of actually measured grain sizes, the prediction of the morphological changes does not necessarily improve.

Linear analyses of the application of spatially varying grain sizes show that the spatial variation of grain sizes can have much impact on the bed topography in rivers. Two different linear analyses have been conducted by Mosselman & Sloff (1998) and Sieben (2000). One shows a difference in the length and dampening of the bed perturbations due to free excitation. The other shows the existence of a response in the bed topography like a superimposed waveform, due to forced excitation.

Implementation of the spatially varying grain size resulted for Olesen's experiment (1985) in effects according to free and forced excitation. For the models of the Waal bend at Nijmegen and Pannerdene Kop, the effects were according to forced excitation.

Both linear analyses show morphological effects due to the spatially varying grain size as well as due to spatially varying bed roughness. The spatial variation of the bed roughness should, according to the analyses, have a counteracting morphological effect in regard to the morphological effects of the spatially variation of the grain size.

Application of spatially varying bed roughness resulted for Olesen's experiment in effects according to free and forced excitation. For the other two models the implementation of spatially varying bed roughness resulted in effects according to forced excitation.

Implementation of a grain size distribution which has reached an equilibrium state leads to a bed level response according to free excitation. While the implementation of a grain size distribution which has not reached an equilibrium, like in rivers with variable discharge, leads to a bed level response according to forced excitation.

Implementation of an alluvial bed roughness predictor led to no satisfactory results for Olesen's experiment and the Waal bend at Nijmegen. However, for the model of Pannerdene Kop the results of the computation matched the prototype better than the computation with uniform grain sizes and bed roughness. This was caused by two main effects. The first was a forced excitation due to the spatially varying bed roughness. The second was a change in the sediment distribution at the bifurcation.
Samenvatting

Veel van de huidige morfologische modellen maken gebruik van een ruimtelijk constant korrelveld en bodemruwheid. Toepassing van deze ruimtelijk constante korrelgroottes en bodemruw hedens is geen juiste weergave van de realiteit. Het toepassen van ruimtelijke variatie van korrelgrootte leidt echter niet altijd tot een verbetering van de voorspellingen van de bodemliggingen.

Lineaire analyses van de toepassing van ruimtelijke variatie van korrelgroottes laten zien dat toepassing van deze ruimtelijke variaties van korrelgroottes de bodemligging sterk kunnen beïnvloeden. Er zijn twee verschillende lineaire analyses uitgevoerd door Mosselman et al. (1998) en Sieben (2000). De eerste analyse voorspelt een verandering van de golflengte en demplingslengte van de verstoringen als gevolg van vrije excitatie. De tweede analyse voorspelt een bodemrespons in de vorm van een golf welke gesuperponeerd is op de bodemligging, als gevolg van geforceerde excitatie.

Implementatie van de ruimtelijke variatie van de korrelgrootte resulteerde voor Olesen's experiment (1985) in een vrije en een geforceerde excitatie. Voor de modellen van de Waalbocht bij Nijmegen en Pannerdense Kop, leidde dit tot geforceerde excitatie's.

Beide analyses laten tonen aan dat er morfologische effecten zijn als gevolg van ruimtelijke variatie van korrelgrootte en bodemruwheid. Volgens de analyses zijn de effecten van de korrelgrootte en de bodemruwheid elkaars tegengestelde.

Toepassing van de ruimtelijke variatie van de bodemruwheid resulteert voor Olesen's experiment in vrije en geforceerde excitatie. Voor de twee andere modellen resulteerde dit echter in enkel een geforceerde excitatie.

Het toepassen van een korrelgrootteverdeling welke een evenwicht heeft bereikt, leidt tot een bodem vernadering volgens de theorie van vrije excitatie. Terwijl het toepassen van een korrelgrootteverdeling welke nog niet in evenwicht is, zoals in rivieren met variabele afvoer, leidt tot een bodemverandering volgens de theorie van geforceerde excitatie.

Toepassing van een alluviale ruwheidsvoorspeller leidde niet tot een correcte voorspellingen van de bodemtopografie voor Olesen's experiment en de Waalbocht bij Nijmegen. Echter, voor het model van Pannerdense Kop leidde dit tot een resultaat dat dichter bij het prototype ligt dan de berekening met uniforme korrelgrootte en bodemruwheid. Dit werd veroorzaakt door twee verschillende effecten. Het eerste effect bestaat uit een geforceerde excitatie ten gevolge van de bodemruwheid. Het tweede effect werd veroorzaakt door een verandering van de sedimentverdeling bij het splitsingspunt.
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<th>Symbol</th>
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<th>Unit</th>
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<td>$a_{cal}$</td>
<td>coefficient for the transport rate</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_s$</td>
<td>coefficient on the bed load transport direction formula</td>
<td>[-]</td>
</tr>
<tr>
<td>$A$</td>
<td>coefficient for influence of secondary flow on direction of bed shear stress</td>
<td>[-]</td>
</tr>
<tr>
<td>$b$</td>
<td>degree of nonlinearity in $s(u)$</td>
<td>[-]</td>
</tr>
<tr>
<td>$b_{cal}$</td>
<td>exponent in sediment transport formula</td>
<td>[-]</td>
</tr>
<tr>
<td>$b_s$</td>
<td>coefficient in the bed load transport direction formula</td>
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</tr>
<tr>
<td>$B$</td>
<td>channel width</td>
<td>[m]</td>
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<tr>
<td>$c_{cal}$</td>
<td>exponent in sediment transport formula</td>
<td>[-]</td>
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<td>$C$</td>
<td>Chézy roughness coefficient</td>
<td>[m$^{1/2}$/s]</td>
</tr>
<tr>
<td>$C_f$</td>
<td>forced Chézy roughness</td>
<td>[m$^{1/2}$/s]</td>
</tr>
<tr>
<td>$D$</td>
<td>sediment grain size</td>
<td>[m]</td>
</tr>
<tr>
<td>$D_f$</td>
<td>forced grain size</td>
<td>[m]</td>
</tr>
<tr>
<td>$E_s$</td>
<td>correction factor for spiral flow</td>
<td>[-]</td>
</tr>
<tr>
<td>$f(\theta)$</td>
<td>function for transverse influence of gravity on sediment transport direction</td>
<td>[-]</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
<td>[m/s$^2$]</td>
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<tr>
<td>$h$</td>
<td>water depth</td>
<td>[m]</td>
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<td>$k$</td>
<td>complex wave number</td>
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<tr>
<td>$k_b$</td>
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<td>$k_{sn}$</td>
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</tr>
<tr>
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</tr>
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</tr>
<tr>
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<td>[-]</td>
</tr>
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<tr>
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<tr>
<td>$R_s$</td>
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<td>[m]</td>
</tr>
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<td>$s$</td>
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<td>[m$^2$/s]</td>
</tr>
<tr>
<td>$s_f$</td>
<td>volumetric sediment transport according to sediment transport formula</td>
<td>[m$^2$/s]</td>
</tr>
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<td>Description</td>
<td>Unit</td>
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<td>------------------------------------------------------</td>
<td>------</td>
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<td>( t )</td>
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<td>[s]</td>
</tr>
<tr>
<td>( u )</td>
<td>depth averaged flow velocity in x-direction</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( v )</td>
<td>depth averaged flow velocity in y-direction</td>
<td>[m/s]</td>
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<tr>
<td>( x )</td>
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<td>( y )</td>
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<td>[m]</td>
</tr>
<tr>
<td>( z_w )</td>
<td>water level</td>
<td>[m]</td>
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**Greek symbols**

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<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
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<td>degree of nonlinearity in ( D(u) )</td>
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</tr>
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<td>( \alpha_s )</td>
<td>direction of sediment transport</td>
<td>[°]</td>
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<td>( \alpha_t )</td>
<td>direction of bed shear stress</td>
<td>[°]</td>
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<td>( \alpha_{fy} )</td>
<td>correction of Shields parameter</td>
<td>[-]</td>
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<td>( \chi )</td>
<td>coefficient for influence of gravity on sediment transport rate</td>
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<tr>
<td>( \Delta )</td>
<td>relative density of sediment</td>
<td>[-]</td>
</tr>
<tr>
<td>( \Phi_D )</td>
<td>degree of nonlinearity in ( C(D) )</td>
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<td>( \Phi_u )</td>
<td>degree of nonlinearity in ( C(u) )</td>
<td>[-]</td>
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<td>( \kappa )</td>
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<td>( \lambda_D )</td>
<td>adaptation length of grain size</td>
<td>[m]</td>
</tr>
<tr>
<td>( \lambda_r )</td>
<td>adaptation length of streamline curvature</td>
<td>[m]</td>
</tr>
<tr>
<td>( \lambda_s )</td>
<td>adaptation length of bed topography</td>
<td>[m]</td>
</tr>
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<td>adaptation length of suspended transport</td>
<td>[m]</td>
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<td>( \lambda_w )</td>
<td>adaptation length of water motion</td>
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</tr>
<tr>
<td>( \mu )</td>
<td>ripple factor</td>
<td>[-]</td>
</tr>
<tr>
<td>( \xi )</td>
<td>coefficient for influence of gravity along longitudinal slopes on sediment transport</td>
<td>[-]</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Shields parameter</td>
<td>[-]</td>
</tr>
<tr>
<td>( \theta_c )</td>
<td>critical Shields parameter for initiation of sediment motion</td>
<td>[-]</td>
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<td>( \rho_s )</td>
<td>mass density of the sediment</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>mass density of water</td>
<td>[kg/m³]</td>
</tr>
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<td>( \psi )</td>
<td>angle between direction of sediment transport and x-axis</td>
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<td>( \Psi_C )</td>
<td>degree of nonlinearity in ( s(C) )</td>
<td>[-]</td>
</tr>
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<td>( \Psi_D )</td>
<td>degree of nonlinearity in ( s(D) )</td>
<td>[-]</td>
</tr>
<tr>
<td>( \Psi_u )</td>
<td>degree of nonlinearity in ( s(u) )</td>
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1 Introduction

1.1 Problem analyses
One of the successes in morphological modelling in the eighties was the successful reproduction of the Rhine bifurcation (the Netherlands) by a two-dimensional morphological numerical model. Application of this model resulted in a prediction of the bed topography which agreed better with the prototype than the bed topography obtained from a physical scale model.

Figure 1-1: Prototype and model bed topographies of Rhine bifurcation at Pannerden, Mosselman et al. (1999)
The model used spatially uniform sediment properties and a constant bed roughness. This, is not a correct representation of reality. Due to several processes, sorting of bed material occurs. In a recent computation a spatially varying grain size, according to measurements on site, was used. This computation, however, results in a bed topography which did not match the prototype at all (Struijsma, 1998).

Figure 1-2: Results from computations with uniform grain size, Sloff (2000)

Figure 1-3: Results from computation with spatial varying grain size, Sloff (2000)

As a result of this computation, several analyses have been carried out in order to identify the causes of this incorrect prediction of the bed topography. These analyses are based on two main assumptions. The first one is that the measured bed topography and spatial distribution of the grain size represents an equilibrium state. The morphological response is assumed to be due to variability of the grain size with the flow velocity, which is a free response. The second assumption, applied in another analyses, is that the horizontal grain size distribution leads to a forced morphological response. Both analyses show that the morphological effects of the spatially varying grain size might be counteracted by a spatially varying bed roughness. This assumption will be investigated in this research.
1.2 Basic hypotheses
The research is based on the following hypotheses:
- The basic model used for the calculations without spatially varying grain size predicts the bed topography reasonably well.
- The effects of spatially varying grain size are counteracted by spatially varying bed roughness.

1.3 Problem definition
Is it possible to counteract the morphological effects of spatially varying grain size with the use of spatially varying bed roughness, for example with a alluvial roughness predictor?

1.4 Objective of the research
The main objective of this study is to find answers to the following research questions:

1. To what extent does a spatially varying bed roughness counteract the effects of a spatially varying grain size?

2. Does the spatial variation of the bed roughness needed to reproduce the observed bed topography match the spatially variation of the bed roughness one can expect on the basis of alluvial roughness predictors?

3. To what extent can the idea of counteracting the morphological effects of a spatially varying grain size by a spatially varying bed roughness be applied to different situations?

4. To what extent can the problems with the calibration for the Pannerdense Kop be explained from the morphological effects of spatially varying grain size?
1.5 Outline of this report

This report contains the results from an M. Sc. Thesis at the Delft University of Technology, Faculty of Civil Engineering and Geosciences, Hydraulic and Offshore Engineering.

The first chapter consists of the problem definition, basic hypotheses and the resulting objective of the research.

The second chapter describes the two linear analyses which are the starting point of this study. First the linear analysis of Mosselman & Sloff (1998) is described. Second, the analysis of Sieben (2000) will be summarised. Both analyses describe the theoretical response of the bed level to the application of a spatially varying grain size.

The third chapter contains the link to the practical use of the results of the linear analyses. Assuming that the morphological effects of a spatially varying grain size are counteracted by a spatially varying bed roughness, the theoretical size and shape of the necessary spatially varying bed roughness are given.

Chapter four describes briefly the numerical mathematical model used for this investigation, Delft2D-MOR.

Chapters five to seven describe the validation of the theoretical analyses from chapter two and the theoretical compensation of chapter three. First a model of a flume experiment (Olesen, 1985), second a model of the Waal bend at Nijmegen and a model of the Pannerdene Kop will be used to validate the hypothesis.

Finally the conclusions and recommendations resulting from this research will be presented in chapter eight.
2 Theoretical analyses

2.1 Introduction

In order to explain the problems with the calibration of the numerical model for the Pannerdense Kop several analyses have been made. The table below gives an overview of the analyses, concerning the effects of spatial variation of grain size.

<table>
<thead>
<tr>
<th>Type of Analysis</th>
<th>Reference 1</th>
<th>Reference 2</th>
</tr>
</thead>
</table>

Table 2-1: Overview of conducted theoretical analyses

An axi-symmetrical analyses of the mathematical model (Mosselman & Sloff, 1998) showed that the equilibrium cross-sectional profile is rather insensitive to spatial variations in grain size. The effects observed by Struiksmo (1998) are therefore expected to be related to the dynamics along the river. This is investigated through the linear analysis of the mathematical model by Mosselman et al. (1999).

Mosselman et al. (1999) used two alternative sub-models for spatial grain size variations. In the first model the local grain size $D$ is a function of flow velocity $u$. This is based on the idea that grain sorting leads to coarsening of the bed in areas of high flow velocities. The second model is based on the idea that the spatial variation of grain size is imposed as a function of the co-ordinates $x$ and $y$. Combination of the two sub-models gives a generalised form $D = D(x,y,u)$. Similarly, the sub-model of a Chézy coefficient, $C$, depending on local water depth, $h$, and grain size, $D$, is combined with a sub-model in which the spatial distribution of the Chézy coefficient is a function of the co-ordinates. This leads to the distribution $C = C(x,y,h,D)$.

The relations $D(x,y)$ and $C(x,y)$ correspond to the way spatial variations have been imposed in the numerical model. However, the relations $D(u)$ and $C(h,D)$ provide a plausible representation of the physical interactions occurring in nature.

In the following Paragraphs 2.2 and 2.3 these two sub-models are analysed separately. First the sub-model with the grain size variation as a function of the flow velocity $u$, $D(u)$, is taken into account. Second the variation of grain size imposed as a function of the co-ordinates is analysed. The models for the Chézy coefficient is analysed accordingly.
2.2 Linear analysis for free excitation

2.2.1 Introduction
For this linear analyses the river is schematised as a weakly curved channel of uniform width. The flow is quasi-steady. Because free alternate bars are known to be suppressed in the Rhine branches, the morphology is assumed steady as well. The model described in the following paragraph is set up by Mosselman & Slooff (1998).

2.2.2 Basic model
The depth-averaged quasi-stationary equation for downstream momentum component:

\[
          \frac{u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial z}{\partial x} + \frac{g \sqrt{u^2 + v^2}}{C^2 h} \cdot u - \frac{uv}{B} \frac{\partial B}{\partial x} + \frac{k_m u^2}{h} \frac{\partial}{\partial y} \left( \frac{1}{R_f} \right) = 0
\] (2.1)

All the symbols introduced are explained in the List of Symbols.

The model for the Chézy coefficient is defined as followed:

\[
C + \lambda_c \frac{\partial C}{\partial x} = C_f (x, y, h, D, u)
\] (2.2)

This model implies that the bed roughness will adapt to a equilibrium value which is a function of co-ordinates, local water depth and local grain size. In this equation \( \lambda_C \) represents the adaptation length of the hydraulic roughness. In this model the two submodels described in paragraph 2.1 are included.

The depth averaged quasi-stationary equation for cross-stream flow momentum is defined as:

\[
            \frac{u}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial z}{\partial y} + \frac{g \sqrt{u^2 + v^2}}{C^2 h} \cdot v - \frac{uv}{B} \frac{\partial B}{\partial x} - \frac{u^2}{R_f + y} = 0
\] (2.3)

The depth average quasi-stationary equation for preservation of water mass is:

\[
\frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} + \frac{hu}{B} \frac{\partial B}{\partial x} + \frac{hv}{R_f + y} = 0
\] (2.4)

The connection between water level, water depth and bottom level is given by:

\[ z_w = z_b + h \]

In which \( z_b \) is the bottom level in meters.
Due to the curvature of the flow streamlines a spiral flow will be induced. The streamline curvature can be seen as an indication of the spiral flow intensity, $I_\approx hu/R_s$ (De Vriend, 1976, 1981; Olesen, 1982, 1987). This leads to the following equation for the adaptation of the flow due to the streamline curvature:

$$
\frac{h}{R_f} + \lambda_r \frac{\partial}{\partial x} \left( \frac{h}{R_f} \right) = \frac{h}{R_c + y} - \frac{h}{u} \frac{\partial v}{\partial x} \tag{2.5}
$$

In this equation the coefficient $A$ represents the influence of the spiral flow on the bed shear stress, while $\lambda_r$ represents the adaptation length of the streamline curvature.

Theoretically the coefficient $A$ is a function of the Chézy coefficient, this dependency will be neglected because the spiral flow will adapt faster at the bottom. For this model the development of the bottom shear stress is relevant, for which the adaptation length is smaller than for the spiral flow itself.

The equation for conservation of sediment mass is given by:

$$
\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} + \frac{\partial (s \tan \psi)}{\partial y} + \frac{s}{B} \frac{\partial B}{\partial x} + \frac{s \tan \psi}{R_c + y} = 0 \tag{2.6}
$$

For the equation for conservation of mass, the assumption has been made that the sediment transport in $x$-direction approximates the total sediment transport. This assumption corresponds to the assumption that $\cos \psi \approx 1$. For this reason the sediment transport formula can be based on the flow velocity in $x$-direction.

Assuming that sediment transport consisting of bed load and suspended load needs some adaptation length to reach its equilibrium, the following model can be introduced with $\lambda_{sus}$ as the characteristic length scale for the adaptation length, Mosselman et al. (1998):

$$
S + \lambda_{sus} \frac{\partial s}{\partial x} = \left( 1 - \xi \frac{C_s}{g} \frac{\partial z_b}{\partial x} \right) s_f (u, D, C) \tag{2.7}
$$

The function $s_f$ stands for the volumetric sediment transport according to the sediment transport formula.

The direction of the sediment transport is influenced by the spiral flow and by the cross-slope. Mosselman & Sloff (1998) define the following equation for the direction of the sediment transport:

$$
\tan \psi = \frac{v}{u} \frac{Ah}{R_f} - \frac{1}{f(\theta)} \frac{\partial z_b}{\partial y} \tag{2.8}
$$

Finally the adaptation of the grain size to the equilibrium is defined as a function of co-ordinates and flow velocity, $\lambda_D$ is the adaptation length of the grain size:

$$
D + \lambda_D \frac{\partial D}{\partial x} = D_f (x, y, u) \tag{2.9}
$$
2.2.3 Linear equations

The described mathematical model will be linearised by writing every variable $x_i$ as a zero order solution $x_{i0}$ and a perturbation $x'_i$:

$$x_i = x_{i0} + x'_i,$$

Furthermore, all functions will be linearised by expanding the function to a Taylor series and neglecting all nonlinear terms of $x'_i$:

$$f(x_i) = f(x_{i0}) + \sum_i \left( \frac{df}{dx_i} \right)_{x_{i0}} x'_i,$$

Linearisation of the Equations (2.1) to (2.9) leads to the linearised set of Equations (2.10) to (2.18). The linearisation of the depth averaged quasi stationary equation for conservation of streamwise flow momentum results in:

$$u_0 \frac{\partial u'}{\partial x} + g \frac{\partial z'}{\partial x} + \frac{u_0^2}{2\lambda_w} \left( \frac{2}{u_0} \frac{u'}{h_0} - \frac{2}{C_0} \frac{C'}{C_0} \right) + \frac{u_0}{R_c} v' + \frac{k_m u_0^2}{h_0} \frac{\partial}{\partial y} \left( \frac{1}{R_c} \right)' = 0 \tag{2.10}$$

Because of the assumption that the river is only weakly curved, the influence of the curved streamlines is reduced to:

$$\frac{1}{R_c + y} \approx \frac{1}{R_c}$$

Also $\lambda_w$ is the adaptation length according to Struiiksma et al (1985), which is defined as:

$$\lambda_w = \frac{C_0^2 h_0}{2g}$$

The linearised equation for the Chézy coefficient is defined as:

$$C' + \lambda'_C \frac{\partial C'}{\partial x} = C'_f (x, y, h_0, D_0) + \Phi_h \frac{C_0}{h_0} h' + \Phi_D \frac{C_0}{D_0} D' + \Phi_u \frac{C_0}{u_0} u' \tag{2.11}$$

In the equation above the coefficient $\Phi_h$ represents the degree of non linearity of the function $C(h)$ and $\Phi_D$ the degree of non linearity of $C(D)$, as defined below:

$$\Phi_h = \frac{h_0}{C_0} \frac{dC}{dh}$$

$$\Phi_D = \frac{D_0}{C_0} \frac{dC}{dD}$$
Linearisation of the depth-averaged quasi-stationary equation for cross-stream flow momentum results in:

\[
\frac{u_0}{\partial x} + g \frac{\partial z'}{\partial y} + \frac{u_0^2}{2\lambda_w} \frac{\partial v'}{\partial x} + \frac{u_0}{B} \frac{\partial B}{\partial x} v' - \frac{2u_0}{R_c} u' = 0
\]  

(2.12)

The linear depth-averaged quasi-stationary equation for water mass preservation becomes:

\[
u_0 \frac{\partial h'}{\partial x} + h_0 \frac{\partial u'}{\partial x} + h_0 \frac{\partial v'}{\partial y} + \frac{h_0}{B} \frac{\partial B}{\partial x} u' + \frac{u_0}{B} \frac{\partial B}{\partial x} h' + \frac{h_0}{R_c} v' = 0 \]

(2.13)

For the adaptation of the secondary flow to the streamline curvature, the linearised model becomes:

\[
\left( \frac{1}{R_f} \right) + \lambda_c \frac{\partial}{\partial x} \left( \frac{1}{R_f} \right) = \frac{1}{R_c} - \frac{1}{u_0} \frac{\partial v'}{\partial x}
\]  

(2.14)

The assumption is made that the perturbation in the bottom level corresponds to the same perturbations in the water depth ("rigid-lid"-approach). This results in the following linear equation for sediment mass preservation:

\[
\frac{\partial s'}{\partial t} = \frac{\partial s'}{\partial x} + s_0 \frac{\partial v'}{\partial y} + \frac{1}{B} \frac{\partial B}{\partial x} s' + \frac{s_0}{R_c} \psi'
\]

(2.15)

The linear equation predicting the magnitude of the sediment transport:

\[
s' + \lambda_{av} \frac{\partial s'}{\partial x} = b \frac{s_0}{u_0} u' + \Psi_D \frac{s_0}{D_0} D' + \Psi_c \frac{s_0}{C_0} C' + s_0 \frac{C^2}{g} \frac{\partial h'}{\partial x}
\]

(2.16)

The degree of nonlinearity of the function \(s_t(u)\) is given by \(b\), \(\Psi_D\) is the degree of nonlinearity for \(s_t(D)\) and \(\Psi_c\) is the degree of nonlinearity of \(s_t(C)\). These coefficients are defined as followed:

\[
b = \frac{u_0}{s_0} \frac{ds_t}{dD}
\]

\[
\Psi_D = \frac{D_0}{s_0} \frac{ds_t}{dD}
\]

\[
\Psi_c = \frac{C_0}{s_0} \frac{ds_t}{dC}
\]
The linearised predictor for the direction of the sediment transport ("rigid-lid"-approach) is given by:

$$\psi' = \frac{v'}{u_o} - A h_0 \left( \frac{1}{R_f} \right)' + \frac{1}{f(\theta_0)} \frac{\partial h'}{\partial y}$$

(2.17)

The linearised equation for the adaptation of the grain size:

$$D' + \lambda_0 \frac{\partial D'}{\partial x} = D'_f(x, y, u_0) + \alpha \frac{D_0}{u_0} u'$$

(2.18)

In this equation $\alpha$ represents the degree of nonlinearity of the function $D(u)$.

$$\alpha = \frac{u_0}{D_0} \frac{dD}{du}$$
2.2.4 Substitution of solutions

The following solutions are substituted in the linearised model for the vector \( \mathbf{a} = (u', v', h', \vartheta_{w}, D', C', 1/R, s', \psi') \):

\[
\mathbf{a} = \hat{\mathbf{a}} \exp \left( ikx + \frac{m \pi y}{B} - \omega \right)
\]

(2.19)

In which \( \hat{\mathbf{a}} \) is the vector with complex amplitudes and \( k \) is the complex wave number and \( m \) the mode number of the linear analyses which is taken 1 for the following evaluation.

Substitution of the above solution in the linearised model gives the following set of equations, denoted in a matrix:

\[
\begin{bmatrix}
\frac{ik_u}{\lambda_u} + \frac{n_u}{\lambda_u} & \frac{n_v}{\lambda_u} & \frac{-1}{2 \lambda_u} & ik & 0 & \frac{n_v}{\lambda_v} & ik & \frac{n_s}{n_v} & 0 & 0 \\
\frac{-\Phi_{C_v}}{u_v} & 0 & \frac{-\Phi_{C_v}}{h_v} & 0 & - \frac{\Phi_{C_v}}{D_b} & 1 + ik \lambda_v & 0 & 0 & 0 & 0 \\
\frac{-2n_u}{R} & ik \lambda_u & \frac{n_u}{2 \lambda_u} & \frac{\partial \vartheta}{\partial x} & 0 & ik \lambda_u & 0 & 0 & 0 & 0 \\
\frac{ik \lambda_h}{\lambda_h} & \frac{\partial \vartheta}{\partial x} & \frac{n_v}{\lambda_v} & \frac{\partial \vartheta}{\partial x} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & ik & 0 & 0 & 0 & 0 & 1 + ik \lambda_v & 0 & 0 & 0 \\
0 & 0 & \frac{\partial \vartheta}{\partial x} & 0 & 0 & 0 & \frac{\partial \vartheta}{\partial x} & ik \lambda_u + \frac{n_u}{\lambda_u} & 0 & 0 \\
\frac{-\lambda_u}{n_u} & 0 & -ik \lambda_u & \frac{C_r^2}{g} & 0 & - \frac{\psi_{C_v}}{D_b} & \frac{\psi_{C_v}}{D_b} & 0 & 1 + ik \lambda_v & 0 \\
0 & \frac{1}{u_v} & \frac{ik}{f(u_v)} & 0 & 0 & 0 & Ah_v & 0 & 0 & 0 \\
\frac{-\alpha D_b}{u_v} & 0 & 0 & 0 & 0 & 1 + ik \lambda_v & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{h} \\
\dot{\vartheta} \\
\dot{\lambda_u} \\
\dot{\lambda_v} \\
\dot{\lambda_h} \\
\dot{\lambda_v} \\
\dot{\lambda_v} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
C_1(x,y) \\
0 \\
\lambda(x,y) \\
0 \\
0 \\
\psi \\
\psi_y \\
\psi_y \\
\end{bmatrix}
\]

(2.20)

This system of equations can be simplified (Mosselman & Slooff, 1998), by assuming no external forcing. This leads to elimination of the right hand vector and results is a homogeneous set of equations. Furthermore, Mosselman & Slooff assumed that there is no adaptation length for the bed roughness, spiral flow, sediment transport and grain size \( (\lambda_c = \lambda_r = \lambda_{sus} = \lambda_D = 0) \). Finally the river is assumed straight with a uniform width and the convection of the main flow caused by the spiral flow is also neglected.
The remaining system of equations has a trivial solution when all components in the variable vector are zero. The non-trivial solution of the system can be found when the determinant of the matrix is zero. This results in the following equation:

\[
\begin{pmatrix}
iku_n + \frac{u_n}{\lambda_n} & 0 & -\frac{u_n^2}{2\lambda_n h_n} & ikg & 0 & -\frac{u_n^2}{\lambda_n C_0} & 0 & 0 & 0 \\
\Phi_n C_0 & 0 & -\Phi_n C_0 & \Phi_n C_0 & 0 & -\Phi_n C_0 & 1 & 0 & 0 \\
0 &iku_n + \frac{u_n}{2\lambda_n} & 0 & ik \sigma g & 0 & 0 & 0 & 0 & 0 \\
ikh_n & ik \sigma h_n & iku_n & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{ik}{u_n} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & i\omega & 0 & 0 & 0 & 0 & ik & ik \sigma \tau_0 \\
-\frac{bs_n}{u_n} & 0 & -ik \sigma \gamma C_0^2 & 0 & -\Psi \omega s_0 & -\Psi \omega s_0 & C_0 & 0 & 1 \\
0 & -\frac{1}{u_n} & -ik \gamma \rho \frac{f(\theta_n)}{g} & 0 & 0 & 0 & Ah_n & 0 & 1 \\
-\alpha D_0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{pmatrix} = 0
\]
2.2.5 Second order polynomial for $k$

The determinant Equation (2.21) can be evaluated using elementary row and column operations. This finally results in a $3 \times 3$ system determinant. When assuming that the bars are stationary ($\omega=0$), $\Phi_h=0$, neglecting the spiral flow, the longitudinal influence of gravity on the sediment transport direction and neglecting the cross-current, the result is a $2 \times 2$ determinant equation.

$$\begin{vmatrix}
1 - \alpha \Phi_D + ik \lambda_w & \frac{1}{2} (1 + 2 \Phi_h) \\
(b - 1 + \alpha \Psi_D + \alpha \Psi_C \Phi_D) k \lambda_w & i \frac{m^2 \lambda_w}{\lambda_i(1)} - k \lambda_w (1 - \Psi_C \Phi_h)
\end{vmatrix} = 0 \quad (2.22)$$

According to Struiksma et al. (1985) the model of Equation (2.22) describes the morphological effects accurately within the relevant range of parameters. The range for this simplified model is shown in the figure below:

![Graph showing second and fourth order polynomials]

**Figure 2-1: Relative wave numbers**

Equation (2.22) leads to a polynomial of the following form:

$$(ik \lambda_w)^2 + \frac{2 \lambda_w}{L_D} (ik \lambda_w) + \left( \frac{2 \pi \lambda_w}{L_P} \right)^2 + \left( \frac{\lambda_w}{L_D} \right)^2 = 0 \quad (2.23)$$

In this equation the $L_D$ and the $L_P$ are, the damping length and the wavelength respectively. These are defined as:

$$L_D = \frac{1}{\text{Im}(k)}$$

$$L_P = \frac{2\pi}{\text{Re}(k)}$$
Rewriting Equation (2.22) in the form of Equation (2.23) leads to the following expressions for the damping length and the wave length:

\[
L_D = \frac{4(1 - \Psi_c \Phi_h) \lambda_w}{2m^2 \frac{\lambda_w}{\lambda_s(1)} + 2(1 - \alpha \Phi_D)(1 - \Psi_c \Phi_h)(1 + 2\Phi_s)\left(b - 1 + \alpha \Psi_D + \alpha \Psi_c \Phi_D\right)}
\]  

(2.24)

\[
L_P = \frac{2\pi \lambda_w}{\sqrt{(1 - \alpha \Phi_D)m^2 \frac{\lambda_w}{\lambda_s(1)} - \left(\frac{\lambda_w}{L_D}\right)^2}}
\]  

(2.25)

The expressions for \(L_D\) and \(L_P\) show that \(\alpha\), \(\Phi_D\) and \(\Phi_h\) influence the damping and wave length of the perturbations.

### 2.2.6 Fourth order polynomial for \(k\)

As stated before, the second order polynomial is only valid within a certain range of the interaction parameter, IP [-]. For the computations of Olesen's Experiment (1985) the interaction parameter is not within that range. In order to be able to use Olesen's experiment for this research the fourth order polynomial has to be used.

The fourth order polynomial can be obtained in the same way as the second order polynomial has been obtained. Now however, the cross-current will not be neglected. This results in the following equation:

\[
\begin{vmatrix}
1 - \alpha \Phi_D + ik \lambda_w & -\left(\frac{1}{2} + ik \lambda_s\right) \left(\frac{k}{k_s}\right)^2 & -\frac{1}{2}(1 + 2\Phi_s) \\
1 & 1 & 1 \\
(b - 1 + \alpha \Psi_D + \alpha \Psi_c \Phi_D) \frac{k}{k_s} & 0 & -\frac{k}{k_s}(1 - \Psi_c \Phi_h) + i \frac{k_s h_s}{f(\theta_s)}
\end{vmatrix} = 0
\]

(2.26)

This equation can be rewritten as a fourth order polynomial. Assuming no spatial variation of the grain size, no variation of the bed roughness, neglecting the spiral flow and the longitudinal influence of gravity on the sediment transport direction this equation equals the equation found by Struiksma et al. (1985).

It is not possible to obtain the analytical solution of this equation. Only numerical solutions are available.
2.3 Linear analysis of forced excitation

2.3.1 Introduction

The model which is described in this chapter, is used to investigate the morphological effects of a forcing by a spatially varying grain size, Sieben (2000). The analysis coincides best with the numerical model, because this is the way the grain sizes are implemented in the model. The mathematical model used for this analysis is only valid for relatively small perturbations to the initial condition. The analysis aims at investigating the forced solution of the system, instead of the homogeneous solution investigated by Mosselman & Sloff (1998). Because the imposed perturbation is stationary, only the equilibrium state will be considered.

The hypothesis of this analyses is based on the assumption that spatial variation of bed roughness and grain size causes a gradient in the sediment transport, which will lead to a bed level change. The interaction between bed level change, grain size and bed roughness will be neglected. This implies that there are no time-dependent processes, such as free bars.

2.3.2 Basic linear model

The most important assumptions are:

- steady, non-erodible river banks
- hydrostatic pressure (no vertical velocities)
- sediment motion determined by local conditions
- quasi-stationary hydraulic conditions
- 'rigid-lid'-approach

The linearised model is equal to the model presented in Equation (2.20). Sieben (2000) is interested in the inhomogeneous solution of the model. After several simplifications, the model results in two linearised partial differential equations.

\[
\frac{\partial^3 u'}{\partial x^3} + \frac{\partial^4 u'}{\partial x \partial y^2} + \frac{1}{2\lambda_w} \frac{\partial^3 u'}{\partial x^2} + \frac{1}{\lambda_w} \frac{\partial^3 h'}{\partial y^2} + \frac{1}{\lambda_w} \frac{\partial^3 h'}{\partial x^2} - \frac{1}{2\lambda_w} \frac{\partial^3 h'}{\partial y^2} = \frac{1}{\lambda_w} \frac{\partial^2 C'_f}{\partial y^2} \tag{2.27}
\]

\[
\frac{\partial h'}{\partial x} \left( \chi - \frac{Ag}{C_o^2} \right) 2\lambda_w \frac{\partial^2 h'}{\partial x^2} - \frac{a}{\theta^2} \frac{\partial^3 h'}{\partial y^2} - (b - 1) \frac{\partial u'}{\partial x} - \frac{2}{\lambda_w} \frac{Ag \partial^3 u'}{\partial x^2} = \Psi_c \frac{\partial C'_f}{\partial x} + \Psi_v \frac{\partial D'_f}{\partial x} \tag{2.28}
\]

The first Equation, (2.27), represents the first order perturbation to the water motion. The second Equation, (2.28), represents the combined equations for conservation of water and sediment mass and the equations for the sediment transport.
2.3.3 Amplitude equations

The following solutions are substituted into the linearised differential equations:

\[ \begin{pmatrix} \Delta h/h_0 \\ \Delta u/u_0 \\ \Delta D/D_0 \\ \Delta C/C_0 \end{pmatrix} = \begin{pmatrix} h'(x, y) \\ u'(x, y) \\ D'(x, y) \\ C'(x, y) \end{pmatrix} \exp \left( ikx + \frac{\pi y}{B} \right) \quad (2.29) \]

In which $\Delta h/h_0$ and $\Delta u/u_0$ are complex dimensionless amplitudes, in order to obtain information about possible phase shifts. The dimensionless amplitudes $\Delta D/D_0$ and $\Delta C/C_0$ are purely real.

![Figure 2-2: Principle excitation and bed level response](image)

Inserting the solution (2.29) into Equations (2.27) and (2.28), gives the following amplitude equations, which will be used to analyse the first order perturbation as a result of the imposed grain size and bed roughness amplitudes:

\[ \left[ ik \lambda_s^2 \left( \frac{k \lambda_x}{k_s \lambda_s} \right)^2 + 1 \right] + \frac{1}{2} \left( \frac{k \lambda_x}{k_s \lambda_s} \right)^2 \frac{\Delta u}{u_0} + \left[ ik \lambda_s^2 \left( \frac{\lambda_x}{\lambda_s} + 1 \right) - \frac{1}{2} \right] \frac{\Delta h}{h_0} = \frac{\Delta C}{C_0} \quad (2.30) \]

\[ \left[ 1 - \Psi_u - ik \lambda_s^2 \frac{2Ag}{C_0^2} \right] \frac{\Delta u}{u_0} + \left[ 1 - ik \lambda_s \frac{\lambda_x}{\lambda_s} \left( \frac{Ag}{C_0^2} \right) - \frac{i}{k \lambda_s} \right] \frac{\Delta h}{h_0} = \Psi_c \frac{\Delta C}{C_0} + \Psi_d \frac{\Delta D}{D_0} \quad (2.31) \]

The solution of the above set of equations consists of two complex amplitudes for depth and flow velocity as a function of the excitation. These amplitudes can be derived analytically, which results in the expressions given in Appendix A.
The expression for the complex amplitude of the depth can be plotted as a function of $k\lambda_s (k=2\pi/L_p)$, in which $L_p$ is the length of the perturbation. The first plot shows the modulus of the complex amplitude, which represents the size of the amplitude. The second plot shows the argument of the complex amplitude, which represents the phase shift of the depth response compared to the excitation. Governing parameters from Olesen’s experiment (1985) are used. ($R_D = (\Delta h/h_0)/(\Delta D/D_0)$)

$$R_D = \frac{\Delta h}{h_0} \frac{h_0}{\Delta D}$$ $$R_C = \frac{\Delta h}{h_0} \frac{h_0}{\Delta C}$$

Figure 2-3: Amplitude of forced excitation

Figure 2-4: Phase shift of forced excitation

Figure 2-3 shows a peak in the depth response to C and D. This indicates that there might be some form of resonance in the system.
3 Compensation hypotheses

3.1 Objective
According to the analyses in Chapter 2, the spatial variation of grain size can have two effects. First, a change in the characteristics of the perturbation may occur. Second a bed response can appear superimposed on the point bar. According to the analyses these effects can be counteracted by applying spatial variation of the bed roughness. The following chapter describes the conditions for complete compensation of the effects of spatial variation of grain size.

3.2 Compensation for free excitation

3.2.1 Introduction
According to the analyses in Chapter 2, the spatial variation of grain size and bed roughness influence the length and dampening of the perturbations. Both analyses (Mosselman & Sloff (1998) and Sieben (2000)) show that it is possible to counteract the effects of spatially varying grain size with a spatially varying bed roughness, by means of $\Phi_D$ and $\Phi_h$. For the second order polynomial for $k$ it is possible to give an analytical expression for complete compensation. A numerical investigation for compensation according to the fourth order polynomial is described.

3.2.2 Compensation in case of the second-order polynomial
The compensation hypothesis for the second-order polynomial refers to the expressions for the dampening length, (2.24), and the wave length, (2.25). Stipulating that the model without spatial variation of grain size gives a correct approximation of reality, the following can be defined:

$$r_D = \frac{L_D(\alpha, \Phi_h, \Phi_D)}{L_D(0,0,0)}$$  \hspace{1cm} (3.1)

$$r_p = \frac{L_p(\alpha, \Phi_h, \Phi_D)}{L_p(0,0,0)}$$  \hspace{1cm} (3.2)

The effects of $\alpha$ are exactly compensated by those of $\Phi_h$ and $\Phi_D$ when $r_D = r_p = 1$. This results in two Equations (3.1) and (3.2) with three unknowns. This allows $\Phi_h$ and $\Phi_D$ to be expressed in values of $\alpha$, in the case of compensation. The resulting expressions are:

$$\Phi_h = \frac{\alpha \Psi_D}{\left(-\Psi_c - 2b + 2 - 2\alpha \Psi_D + 2 \Psi_c \frac{\lambda_c}{\lambda_y} - \Psi_c b - \Psi_c^2\right)}$$  \hspace{1cm} (3.3)
\[ \Phi_D = \frac{\Psi_C \Psi_D}{\left(-\Psi_C - 2b + 2 - 2\alpha \Psi_D + 2\Psi_C \frac{\lambda_s}{\lambda_s} - \Psi_C b - \Psi_C^2 \right)} \] (3.4)

When applying spatial variation of grain size, the effects of this implementation can be compensated by applying spatial variation of bed roughness. This compensation can be expressed a degree of non-linearity for the bed roughness, for C(D) and C(h). The optimal values of \( \Phi_h \) and \( \Phi_D \) can be obtained from Eq. (3.3) and (3.4).

Plotting \( \Phi_h \) and \( \Phi_D \) as a function of \( \alpha \), with parameters of Olesen's experiment (Olesen, 1985), results in the figures below.

![Figure 3-1: Compensation for Olesen's experiment](image1)

![Figure 3-2: Compensation for river Waal](image2)

The figures above show that \( \Phi_D \) should have a negative value. This is coincides with the conclusion of van Rijn (1993), who states that for certain ranges the Chézy coefficient should increase as the grain size decreases. Furthermore, \( \Phi_h \) should have a positive value, which indicates that an increase in water depth results in an increase in the Chézy coefficient.
3.2.3 Compensation in case of the fourth-order polynomial

As explained in Section 2.2.6, it is not possible to determine the analytical solution for the fourth order polynomial of $k$. In order to predict the magnitude of compensation the values found by the second-order compensation hypothesis will be inserted in the fourth order polynomial, according to Equation (2.26). When all parameters are known except $k$, the polynomial can be solved numerically.

The figures below show the wavelength as a function of $\Phi_h$ and $\Phi_D$, for various values of $\alpha$. Values have been obtained by numerical computations of $k$ with governing parameters from Olesen's experiment.

![Figure 3-3: $L_P(\Phi_h)$](image1)

![Figure 3-4: $L_P(\Phi_D)$](image2)

The numerical results of the fourth order polynomial for $k$, show that the system is very sensitive for changes in $\Phi_h$. Small degrees of nonlinearity of $C(h)$ lead to large differences in the wave length of the system. However the system seems relatively insensitive to the degree of nonlinearity of $C(D)$, in the range shown. Changes in $\Phi_D$ seem to have smaller effects on the wave length then changes in $\Phi_h$. 

Morphological effects of spatially varying grain size and bed roughness in rivers
3.3 Compensation for forced excitation

Complete compensation according to the theory of forced excitation, should occur when the complex amplitude of the depth response is zero. This means that Equation (A.4) in Appendix A, should be set to zero. Solving the resulting equation for the excitation of $\Delta C/C_0$ results in an expression for $\Delta C/C_0$ as a function of $\Delta D/D_0$.

$$\frac{\Delta C}{C_0} \left( \frac{\Delta D}{D_0} \right) = \frac{-\left( -ik \frac{k^2}{k_b} \lambda_w + ik \lambda_w + \frac{1}{2} \frac{k^2}{k_b^2} + 1 \right) \Psi_D}{\left( -\frac{1}{\Psi_C} + \frac{b}{\Psi_C} + \frac{2gA}{C_0^2 \Psi_C} - ik \lambda_w + ik \frac{k^2}{k_b^2} + ik \lambda_w + \frac{1}{2} \frac{k^2}{k_b^2} + 1 \right) \Psi_D} \frac{\Delta D}{D_0}$$  \hspace{1cm} (3.5)

Here the result is a complex amplitude for $\Delta C/C_0$. The modulus or real part of the complex amplitude for $\Delta C/C_0$, represents the size of the necessary amplitude for compensating. The argument of the complex amplitude for $\Delta C/C_0$ represents the necessary phase shift for complete compensation, compared to the distortion $\Delta D/D_0$.

The figures below show the necessary size and phase shift for complete compensation in which $k = 2\pi/L_P$ (Parameters from Olesen’s experiment are used).

Figure 3-5: Amplitude for $\Delta C/C_0$

Figure 3-6: Phase shift for $\Delta C/C_0$
4 Delft2D-MOR

4.1 Introduction
Reliable information on water quantity, sediment transport and morphology can be obtained by the use of numerical models. In the present study we use Delft2D-MOR as the two-dimensional numerical modelling tool. This tool is applied to the generation of bed topography with spatially constant and spatially varying grain sizes.

Delft2D-MOR, an integrated flow and transporting modelling system, is a product of WL | delft hydraulics. The flow module of this system, called FLOW, provides the hydrodynamic basis for sediment transport and morphological computations. The sediment transport module (called TRSTOT for total load and TRSSUS for suspended load) and the morphological module (called BOTTOM for bed level updates) use the flow field generated by the flow module. The dynamic interaction between the separate modules is controlled by the MAIN module.

![Diagram](image)

Figure 4-1: Demonstration of the dynamic interaction of the MAIN module
4.2 Flow module
Delft3D-FLOW (WL | delft hydraulics, 1999) solves the Navier Stokes equations for an incompressible fluid, under the shallow-water and Boussinesq approximations. In the vertical momentum equation, the vertical accelerations are neglected, which leads to a hydrostatic pressure equation. The vertical velocities are computed from the continuity equation.

Delft3D-FLOW is a numerical model based on a finite difference approximation. To discretise the 3D shallow water equations in space, the model area is covered by a rectangular, curvilinear or spherical grid. It is assumed that the grid is orthogonal and well structured. In the discretisation, the variables are arranged in a special way on the grid. The water level points are defined in the centre of the continuity cell. The velocity components are defined perpendicular to the grid cell faces at the grid cell faces. This type of discretisation is called the staggered grid. For details about Delft3D-FLOW, see WL | delft hydraulics (1999).

4.3 Sediment transport module
The total sediment transport module TRSTOT determines the transport components in a 2-D horizontal plane, using the time dependent flow and wave fields and a fixed bed level available from the previous time step. The sediment transport magnitude is computed by the selected sediment transport relation. The computed transport magnitude can be corrected for a physical bed slope effect, non-erodible layers as well as for numerical stability/accuracy. The direction of the sediment transport is corrected by including the effects of transverse bed level gradients and spiral motion. For details about the transport module, see WL | delft hydraulics (1996).

4.4 Bottom module
The module BOTTOM computes the bed level variations induced by the sediment transports. Next, these changes are superimposed on the original bottom, which gives the new bottom.

4.5 Main module
In Delft2D-MOR, the execution of the process simulation is controlled by the module called MAIN. The control function of the module MAIN concerns the process starting, the splitting up of the total simulation time into time intervals, the splitting up of the process into subprocesses, the activation and deactivation of the process modules and the process stopping.

The output file of the MAIN module, the communication file, allows exchange of data between the modules FLOW, TRSTOT and BOTTOM, Figure 4-1. For details about the transport module, see WL | delft hydraulics (1996).
5 Olesen's experiment

5.1 Description of the experiment
In order to investigate the morphological effects of the spatial variation of grain size, computations have been made reproducing the results of Olesens' experiment (Olesen, 1985). This experiment consists of several movable bed experiments in the DHL curved flume. The layout of the DHL curved flume is shown in the figure below.

Figure 5-1: Layout of the curved flume experiment
The report on experimental investigation describes three experiments. In the first experiment, T4, a sediment with a steep sieve curve (uniform sediment) was used, sand II. The second, T5, and third experiment, T6, were conducted with a sediment which had the same mean diameter as the sediment in the first experiment, but with a less steep sieve curve, 2/3 sand I and 1/3 sand III. Table 5-1 and Figure 5-2 show the characteristics of the sediment used.

<table>
<thead>
<tr>
<th>sand</th>
<th>D_{10} [mm]</th>
<th>D_{50} [mm]</th>
<th>D_{90} [mm]</th>
<th>D_{m} [mm]</th>
<th>\sigma_{d} [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.38</td>
<td>0.45</td>
<td>0.605</td>
<td>0.46</td>
<td>1.21</td>
</tr>
<tr>
<td>II</td>
<td>0.725</td>
<td>0.795</td>
<td>0.855</td>
<td>0.80</td>
<td>1.09</td>
</tr>
<tr>
<td>III</td>
<td>1.37</td>
<td>1.60</td>
<td>1.78</td>
<td>1.61</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Table 5-1: Sediment characteristics

![Sieve curves](image)

Figure 5-2: Sieve curves for sand I, II and III

The difference between experiment T5 and T6 is the sediment distribution at the entrance of the flume. The sediment was uniformly distributed in experiment T5, while an asymmetrical distribution was used for experiment T6.

This difference in the sediment distribution led to the choice to use experiment T5 for the numerical computations. Experiment T6 would lead to more complicated boundary conditions and possibly could result in side effects.

The table below shows the governing parameters for experiment T5.

<table>
<thead>
<tr>
<th>Q [m³/s]</th>
<th>B [m]</th>
<th>h [m]</th>
<th>u [m/s]</th>
<th>i *10^{-3}</th>
<th>C [m^3/s]</th>
<th>D_{m} [mm]</th>
<th>s [m^3/s]*10^{-6}</th>
<th>B/h [-]</th>
<th>\lambda_{w} [m]</th>
<th>\lambda_{s} [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>2</td>
<td>0.135</td>
<td>0.46</td>
<td>1.49</td>
<td>32.7</td>
<td>0.84</td>
<td>8.36</td>
<td>14.8</td>
<td>7.4</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 5-2: Governing parameters of experiment T5

For more detailed information about the model, reference is made to Olesen (1985).
5.2 Model set-up
The numerical model is set up according to the parameters given in the previous chapter. The flume is schematised with 115 by 10 grid cells. These cells have a length of approximately 0.40 meters and a width of approximately 0.20 meters.

The upstream boundary has been defined as a discharge of 0.125 m³/s. Furthermore, the bottom level at the upstream boundary is fixed. At the downstream boundary the water level is fixed. Also secondary flow has been selected, which makes use of the depth averaged equation to determine the spiral flow intensity. The secondary flow will not instantaneously reach its equilibrium state, but needs an adaptation length.

The sediment transport has been computed with the option "General Formula" from Delft2D-MOR. This option is based on the following equation:

\[
\frac{s}{\sqrt{g\Delta D_{50}}} = a_{col} \theta^{\gamma_{cr}} (\mu \theta - \theta_{cr})^{\gamma_{cr}}
\]  \hspace{1cm} (5.1)

5.3 Reference computation
In order to investigate the morphological effects of the spatial variation of grain size, a good reference computation has to be made. This reference model was calibrated with the following steps:

Water movement
The first step in the calibration was calibrating the water movement in the model, to fit with the measurements from Olesen (1985). Calibrating the water flow started with settings using the governing parameters from Olesen. These settings resulted in a water flow which corresponds well with the experiment.

Because the model is relatively small, tangential shear stress from vertical walls should not be neglected. Some computations have been made to investigate the effects of this phenomenon. These computations did not show any improvement of the results. So in the later computations this option was not used.
**Sediment transport**

After calibrating the water flow, the sediment transport has to be calibrated. First of all the correct sediment transport has to be established. The first computations showed too little sediment transport. This transport can be adjusted by changing $a_{cal}$ in the sediment transport equation:

$$ s = a_{cal} D (\Delta g D)^{0.5} \theta^{b_{cal}} \left( \mu \theta - \theta_{cr} \right)^{c_{cal}} $$

(5.2)

where

$$ \theta = \left( \frac{u}{C} \right)^2 \frac{1}{\Delta D} $$

$$ \Delta = \frac{(\rho - \rho_w)}{\rho_w} $$

in which after calibration:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{cal}$</td>
<td>2.7</td>
<td>[-]</td>
</tr>
<tr>
<td>$b_{cal}$</td>
<td>0.0</td>
<td>[-]</td>
</tr>
<tr>
<td>$c_{cal}$</td>
<td>1.5</td>
<td>[-]</td>
</tr>
<tr>
<td>$D$</td>
<td>0.00084</td>
<td>[m]</td>
</tr>
<tr>
<td>$C$</td>
<td>32.7</td>
<td>[m^2/s]</td>
</tr>
<tr>
<td>$\theta_{cr}$</td>
<td>0.047</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Table 5-3: Calibration parameters for Olesen's experiment

**Bed topography**

The last step in creating a correct reproduction of the experiment is adjusting the shape of the bed topography to fit the experiment. Adjusting the coefficients $a_y$, $b_y$ and $E_y$ results in variation in the bed topography. These coefficients influence the sediment transport direction:

$$ \tan(\alpha_y) = \frac{\sin(\alpha_y) + \frac{1}{a_y, \theta^{b_y}} \frac{\partial a_y}{\partial y}}{\cos(\alpha_y) + \frac{1}{a_y, \theta^{b_y}} \frac{\partial a_y}{\partial x}} $$

(5.3)

where

$$ \theta = \alpha_y \frac{u^2}{g \Delta d_{s0}} $$

(5.4)
\[ \tan (\alpha_i) = \frac{u}{q} - \frac{I_s}{u + \frac{v}{q}} \]  

(5.5)

\[ \alpha_i = \frac{2}{\kappa^3} E_s \left( 1.0 - 0.5 \frac{\sqrt{g}}{\kappa C} \right) \]  

(5.6)

In which after calibration:

| $\alpha_f$ | correction Shields number | 1.00 | [-] |
| $a_k$     | correction slope effect   | 1.70 | [-] |
| $b_k$     | correction slope effect   | 0.50 | [-] |
| $E_s$     | correction spiral flow    | 0.95 | [-] |

Table 5-4: Calibration parameters bed topography

The final computations of the reference model approximate the point bar reasonably well. However, the erosion in the outer bend is overestimated, as shown in the figure below. The crossings are estimated quite well.

Figure 5-3: Results from the reference computation
5.4 Spatially varying grain size

Once the reference computation is complete, the spatially varying grain size can be implemented. Computations with the spatially varying grain size require a horizontal grain size distribution. First computations have been carried out with a horizontal grain size distribution measured by Olesen (1985), without any smoothing. This field has a distribution as shown in the figure below.

![Figure 5-4: Horizontal grain size distribution](image)

Figure 5-4: Horizontal grain size distribution

The results from the initial computation with spatially varying grain size show a lot of noise in the bed level. This noise might be caused by the noise in the spatial grain size distribution. In order to focus on the essence, the grain distribution has been smoothened. This horizontal grain size distribution has been obtained by averaging the measured horizontal grain size distribution according to Olesen. The resulting grain size distribution is shown in the figures below.

![Figure 5-5: Computation with D(x,y)](image)

Figure 5-5: Computation with D(x,y)

![Figure 5-6: Grain size distribution (right bank)](image)

Figure 5-6: Grain size distribution (right bank)

![Figure 5-7: Grain size distribution (left bank)](image)

Figure 5-7: Grain size distribution (left bank)
Computations with the smoothened horizontal grain size distribution show no more noise in the bed topography:

![Graph 1](image1)

![Graph 2](image2)

**Figure 5-8: Implementation of spatially varying grain size**

Comparing the computation with the prototype, one can clearly see that the overshoot phenomenon is predicted much better. Furthermore, the results from the computation with the smoothened spatially grain size distribution show two almost separate point bars in comparison with the computations with spatially constant sediment.

There are two separate ways of interpretation of the results. When assuming that the computed bed topography shown in Figure 5-8 is the result of free excitation of the bed topography, the figures below can be plotted. In these figures, the analytical equilibrium has been obtained by using the expression for the axi-symmetrical solution found by Mosselman & Sloff (1998).

![Graph 3](image3)

![Graph 4](image4)

**Figure 5-9: Water depth at right bank**

**Figure 5-10: Water depth at left bank**
Figure 5-9 and Figure 5-10 clearly show a shortening of the perturbation length $L_p$, as well as an increase in the dampening length $L_D$, when applying spatially varying grain size. Despite the fact that the grain size distribution is given as $D(x,y)$, the combination of this grain size distribution and the flow distribution results in $\alpha \neq 0$. An indication of the size of $\alpha$ during the computations can be estimated according to the relation below:

$$\alpha \approx \frac{1}{n} \sum_{i=1}^{n} \left( \frac{u_0}{D(x,y)} \left( \frac{D_0 - D(x,y)}{u_0 - u(x,y)} \right) \right)$$

(5.7)

When applying this on the numerical results, gives an approximation of $\alpha \approx 0.1$. Numerical results from Equation (2.26) in combination with the results from the numerical model are presented in Table 2-1. The wave length $L_p$ can be found by determining the characteristics of the numerical solution with regard to the axi-symmetrical solution.

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>Theoretical $L_p$ [m]</th>
<th>Numerical $L_p$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = \text{constant}$</td>
<td>0</td>
<td>16.8</td>
</tr>
<tr>
<td>$D(x,y)$</td>
<td>0.1</td>
<td>16.5</td>
</tr>
<tr>
<td>Difference</td>
<td>-2%</td>
<td>-7%</td>
</tr>
</tbody>
</table>

Table 5-5: Change of $L_p$

The dampening length of the numerical model with constant grain size can not be exactly determined. The wave dampens out after half a wave length. The dampening length of the numerical model with $D(x,y)$ has increased, the wave dampens out less. The theoretical results, however, predicts a decrease in the dampening length.

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = \text{constant}$</td>
<td>0</td>
</tr>
<tr>
<td>$D(x,y)$</td>
<td>0.1</td>
</tr>
<tr>
<td>Difference</td>
<td>-3%</td>
</tr>
</tbody>
</table>

Table 5-6: Change of $L_D$

The quality of the change in $L_p$ is correct. However, the quantity of the change is not according to the theory of free excitation. The change of $L_D$ is not corresponding to the theory of free excitations at all. The model shows a large increase in the dampening length, while the theory in Chapter 2 predicts a decrease of the dampening length.

Deviations from the theory might be caused by the extremely low $\lambda_d/\lambda_w$ [-]. This low value makes the theoretical analysis not completely valid. This was also mentioned in Chapter 2.
It must be denoted that the assumption of free excitation is not entirely correct. The assumption of free excitation is only valid when the grain size distribution is defined as D(u). However, when assuming that grain sorting occurs in nature, than the grain size distribution measured by Olesen (1985) is some sort of D(u). This makes the assumption of free excitation less incorrect.

Another way of investigating the phenomenon of two separate point bars is to assume that the response of the bed topography is a forced excitation. The relative difference in water depth between the computation with uniform and spatially varying grain size has been calculated. The longitudinal profiles of the relative difference in the total water depth between both computations are plotted in the figure below.

**Figure 5-11:** Effects of D(x,y) (right bank)

**Figure 5-12:** Effects of D(x,y) (left bank)

Figures 5.9 and 5.10 show some difference between the effects on the right bank and on the left side. The effects on the right and left side show some kind of waveform. The wave form at the left bank shows some chaos. The numerical model might become unstable as the water depth decreases or when $\theta$ approaches $\theta_{cr}$, resulting in the observed chaos.

The effect of the spatially varying grain size plotted in the figure above, shows some kind of waveform. The amplitude of this waveform is approximately 30% of the total water depth. The shape and size of the effects of this resulting wave form agree with the theory of forced excitation. The resulting wave form due to the effects of spatially varying grain size has the following characteristics:

<table>
<thead>
<tr>
<th>$k \lambda_s = (2\pi/L_p) \lambda_s$</th>
<th>$R_D = (\Delta h/h)/(\Delta D/D)$</th>
<th>Phase = $x_f/L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sieben (2000)</td>
<td>0.25</td>
<td>0.93</td>
</tr>
<tr>
<td>Model</td>
<td>0.25</td>
<td>1.00</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>7%</td>
</tr>
</tbody>
</table>

**Table 5-7:** Waveform characteristics of effect grain size variation

The characteristics of the model are obtained by investigation of the local maxima of Figure 5-11 and Figure 5-12.
Plotting these characteristics in the response graphs according to Sieben (2000) gives:

Figure 5-13: Amplitude response according to forced excitation

Figure 5-14: Phase shift response according to forced excitation

Results of the computations with spatial varying grain size indicate that the analysis of Sieben (2000) gives an accurate description of amplitude of the bed level response, overestimated with 7%. The phase shift of the bed response is underestimated with 5%.
5.5 Morphological effects of C(D)

According to the hypotheses mentioned in Chapter 3 the effects of the spatial variation of grain size can be counteracted with spatial variation of the bed roughness. In order to investigate these hypotheses, computations have been made adjusting the bed roughness to the grain size distribution. The adjustments of the bed roughness to the grain size distribution are made according to the following equation:

\[ C(D, h, u) = C_0 \Phi_D \frac{C_0}{D_0} (D(x, y) - D_0) + \Phi_h \frac{C_0}{h_0} (h(x, y) - h_0) + \Phi_u \frac{C_0}{u_0} (u(x, y) - u_0) \] (5.8)

Several computation have been made using this function for deriving the bed roughness. Variations have been made with \( \Phi_D \), ranging from 0 to -1.0 with steps of 0.05. In order to investigate the effects of \( \Phi_D \), \( \Phi_h = \Phi_u = 0 \). During one computation, the value of \( \Phi_D \) was kept constant. No iteration took place.

Computations with Delft2DMOR resulted in the following longitudinal profiles for the relative bed level, with \( \Phi_D = -0.55 \) [-]:

![Graphs showing relative water depth and distance along outer bend for 0.3 m from right bank, 0.3 m from left bank, and prototype.]

**Figure 5-15: Implementation of C(D)**

The results show a prediction of the bed level which fits better with the prototype and the reference computation than the computation without C(D).
Here are also two separate ways of interpretation of the results. When assuming that the grain size distribution is the result of grain size sorting $D(u)$, one might conclude that the application of $C(D)$ could result in effects described in the analysis for free excitations. When assuming free excitations, the following figures can be plotted with the axi-symmetrical solution found by Mosselman et al. (1998).

![Figure 5-16: Water depth at right bank](image1)

![Figure 5-17: Water depth at left bank](image2)

Figure 5-16 and Figure 5-17 show an increase in the perturbation length $L_P$, as well as a shortening of the dampening length $L_D$ with regard to the computation with only $D(x,y)$.

Analysing the results from the numerical computations results in the following data:

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_D$ [-]</th>
<th>Theoretical $L_P$ [m]</th>
<th>Numerical $L_P$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(x,y)$</td>
<td>0.1</td>
<td>16.5</td>
<td>17.6</td>
</tr>
<tr>
<td>$D(x,y)$ and $C(D)$</td>
<td>0.1</td>
<td>-0.55</td>
<td>16.9</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>3%</td>
<td>19%</td>
</tr>
</tbody>
</table>

Table 5-8: Change of $L_P$

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_D$ [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(x,y)$</td>
<td>0.1</td>
<td>2.56</td>
</tr>
<tr>
<td>$D(x,y)$ and $C(D)$</td>
<td>0.1</td>
<td>-0.55</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>7%</td>
</tr>
</tbody>
</table>

Table 5-9: Change of $L_D$

The quality of the change in $L_P$ is correct. However, the quantity of the change is not according to the theory of free excitation. The change of $L_D$ is not corresponding to the theory of free excitations at all. The model shows a large decrease in the dampening length, while the theory indicates an increase of the dampening length. Deviations are probably due to the small IP [-].
The other interpretation of the computational results is based on the assumption of forced excitation. From the longitudinal profiles the effect caused by the spatially varying bed roughness can be obtained. The effect of the spatially varying grain size and spatially varying grain size are plotted in the figures below.

Figure 5-18: Implementing C(D) (right bank)  
Figure 5-19: Implementing C(D) (left bank)

On the right bank, the effects of the spatially varying bed roughness are of the same size and shape as the effects of the spatially varying grain size. The amplitude of the effect of C(D) is also approximately 30%. However, a phase shift between the two separate effects can be observed, which results in a net effect of both spatially varying grain size and bed roughness, as plotted in the figures above.

The observed phase shift of the bed roughness can be explained by the theory of forced excitation. The difference in the phase shift of the effects of the spatially varying grain size and bed roughness coincide with the phase shift needed for optimal counteraction, according to the theory of forced excitation. Computations with the bed roughness shifted with respect to the variations of the grain size should lead to better results. Deviations from theory are probably caused by nonlinear effects or curvature.
5.6 Counteracting with C(D) and phase shift

The results presented in the previous paragraph indicated that the hypothesis of forced excitation might be correct. The counteraction with C(D) led to effects which were shifted in phase. This is confirmed by the hypothesis of forced excitation, the magnitude of the shift coincides with the theory. In order to confirm the validity of the hypothesis, computations have been made with a shifted C(D). According to the theory the necessary phase shift for optimal compensation should be -0.05*L_p metres (Paragraph 3.3). This leads to a phase shift of -1.5 metres for the wave length, L_p, of 20 metres.

![Figure 5-20: Implementing C(D) with phase shift](image)

![Figure 5-21: Effects of C(D) without phase shift](image)

![Figure 5-22: Effects of C(D) with phase shift](image)

The figures 5-21 and 5-22 show the effect of the counteraction with C(D) implemented with a phase shift. The net effect of D(x,y) and C(D) is now reduced to less than 5% in the first part of the flume. Application of the phase shift leads to an improvement of the net effect of D(x,y) and C(D) of approximately 66%, this justifies the application of such a phase shift.
At a distance of 20 meters, the excitation of grain size ends and the bed topography goes to an equilibrium state. Due to different effects of the grain size and bed roughness on the adaptation lengths of the bed topography, no compensation is established from this point on. In order to obtain compensation in this area, further investigation on the effects of grain size and bed roughness on the adaptation lengths are required.

The physical process which might cause a phase shift between the morphological effects of spatially varying grain size and bed roughness could be a difference in adaptation lengths for flow and sediment transport. The found phase shift implies that the adaptation lengths for the case of spatially varying bed roughness is smaller than the adaptation length of the spatially varying grain size.

5.7 Morphological effects of C(h)

Similar to the counteracting with C(D), computations have been carried out with use of C(h). These computations also made use of equation (5.8). The results of the computations are shown in the figure below:

Figure 5.23: Counteracting with C(h)

Here are also two separate ways of interpretation of the results. When assuming that the grain size distribution is the result of grain size sorting D(u), one can conclude that the application of C(h) could result in effects described in the analysis for free excitations. Assuming free excitations, the following figures can be plotted with the axi-symmetrical solution found by Mosselman & Sloff (1998).
Comparing the change in characteristic wave lengths with the predictions according to the theory of free excitations leads to the following tables:

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_h$ [-]</th>
<th>Theoretical $L_P$ [m]</th>
<th>Numerical $L_P$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(x,y)</td>
<td>0.1</td>
<td>16.5</td>
<td>17.6</td>
</tr>
<tr>
<td>D(x,y) and C(D)</td>
<td>0.1</td>
<td>25.3</td>
<td>18.7</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>35%</td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 5-10: Change of $L_P$

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_h$ [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(x,y)</td>
<td>0.1</td>
<td>2.56</td>
</tr>
<tr>
<td>D(x,y) and C(D)</td>
<td>0.1</td>
<td>2.75</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>7%</td>
</tr>
</tbody>
</table>

Table 5-11: Change of $L_D$

The increase in $L_P$ is overestimated by the theoretical model. The decrease in dampening length is not confirmed by numerical computations.

Like the computations with C(D), the results of the computations with C(h) also show an imposed wave on the point bar. This wave is not as pronounced as the wave resulting from computations with C(D). By determining the difference between computation with spatially varying grain size and the computation with C(h), the effects of C(h) on the bed level can be visualised. The effects of D(x,y), C(h) and the combination of both effects are plotted in the figures 5-26 and 5-27.
The effect of the application of C(h) results in a wave which is not as strong as the wave caused by using C(D). The amplitude of the effect of C(h) is much smaller than the amplitude of the effect of C(D). Also the shape of the amplitude does not coincide with the shape of the amplitude caused by D(x,y). The observed phase shift is opposite to the phase shift observed in computations with C(D).

### 5.8 Morphological effects of C(u)

Finally computations have been carried out with the use of C(u). Results show a weakly or rather no effect of C(u) on the bed topography.

The effects of the implementation of C(u), are very small. Large $\Phi_u$ lead to effects smaller than 5%. The shape of the effects implies that there is no connection with the counteraction for D(x,y). Combinations with other counteracting measurements could lead to better results. However, this will be a shot in the dark because there is no theoretical basis for such an assumption. For this reason this possibility of counteraction will be neglected in further investigations.
5.9 Counteracting with alluvial bed roughness predictor

After confirming the counteracting effects of spatially varying grain size, now the link with alluvial roughness predictors has to be made. This link is important in order to find practical use for the relation found between spatially varying grain size and bed roughness.

Perhaps it is possible to choose the counteracting effects so that the computation will result in a more accurate prediction of the bed topography. The numerical computations have shown that the best counteracting measure was achieved by applying C(D) and C(h). These findings implicate that an alluvial bed roughness predictor with dependencies on grain size and water depth could result in more accurate predictions of the bed topography.

For this study the alluvial bed roughness predictor of van Rijn (1993) has been tested. The predictor was verified for a large amount of field data by van Rijn and proved to be accurate for the river sections and experiment used in this study. Julien and Klaassen (1995) determined approximation for the dune length and height as a function of grain size and water depth (Appendix B), to be used in the alluvial bed roughness predictor. The generation of the resulting bed roughness field is explained in Appendix C. The resulting bed levels after implementation of van Rijn (1993) are plotted in the following figures:

![Figure 5-30: Implementation of alluvial bed roughness predictor](image)

The figures show an increase in the cross-sectional bed slope. The sensitivity of the alluvial bed roughness predictor to grain size and water depth variations was not according to the optimal counteraction, found during the computations.
For Olesen's experiment $\Phi_D = -0.55$ was found to result in the best counteracting for application of $C(D)$. Furthermore, $\Phi_h = 0.06$ was found to result in the best counteracting for the application of $C(h)$.

A comparison has been made with several general alluvial bed roughness predictors, Appendix B. These alluvial bed roughness predictors also require information about the bed-forms. Olesen (1985) found the following characteristics during the experiment:

<table>
<thead>
<tr>
<th></th>
<th>h [m]</th>
<th>H [m]</th>
<th>L [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>0.105</td>
<td>0.034</td>
<td>0.2</td>
</tr>
<tr>
<td>Right</td>
<td>0.165</td>
<td>0.078</td>
<td>1.1</td>
</tr>
<tr>
<td>Average</td>
<td>0.135</td>
<td>0.056</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 5-12: Bed-form characteristics for Olesen's experiment

Given the bed-form characteristics found during the experiment, the bed roughness can be predicted by several bed roughness predictors.

<table>
<thead>
<tr>
<th></th>
<th>$C$ [m$^3$/s]</th>
<th>$\Phi_D$ [-]</th>
<th>$\Phi_h$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation</td>
<td>32.7</td>
<td>-0.55</td>
<td>0.06</td>
</tr>
<tr>
<td>Engelund (1977)</td>
<td>60</td>
<td>-0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>van Rijn (1993)</td>
<td>27</td>
<td>-0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Vanoni and Hwang (1967)</td>
<td>60</td>
<td>-0.22</td>
<td>0.36</td>
</tr>
<tr>
<td>Yalin (1977)</td>
<td>22</td>
<td>-0.01</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 5-13: Comparison of compensation found with alluvial bed roughness predictors

Plotting these alluvial bed roughness predictors with the necessary counteraction leads to the following figures:

Figure 5-31: $\Phi_D$ as a function of grain size
Figure 5.32: $\Phi_h$ as a function of water depth

The table and figures show that the alluvial bed roughness predictor of van Rijn is not the correct approach for determining the bed roughness for this model. The general bed roughness predicted by van Rijn is still the best estimate.
5.10 Conclusions

Computations with Delft2D-MOR with a spatially constant grain size predict the bed topography reasonably well. The erosion in the outer right bank is overestimated, but the location of the crossings of the point bar are predicted well.

Imposing a spatial variation of the grain size resulted in a bed topography which showed two almost separate point bars. There are two separate ways of interpretation of the results. The first is to assume that the effects are caused by free excitation of the bed topography. The second assumes a forced excitation of the bed topography.

When investigating the free excitation, the changes is the characteristic perturbation lengths are coincide with the theory. Deviations from the predicted theoretical effects are smaller than 10%.

Assuming forced excitation the difference in bed topography shows a superimposed wave form on the bed topography. This wave corresponded with the forced bed level response due to grain size variations predicted by Sieben (2000).

Counteracting the effects of the spatially varying grain size with a spatially varying bed roughness shows that counteracting might be possible for both theories. In the case of free excitation the bed perturbation characteristics change to the quantities for the computations with spatially constant grain size. When assuming forced excitation the superimposed wave is counteracted by another superimposed wave caused by the spatially varying bed roughness.

For forced excitation, the resulting wave due to the spatial variation of bed roughness seemed to be shifted in phase, compared to the response due to spatial variation of grain size. According to the hypothesis a phase shift in the bed roughness was introduced, which resulted in a better prediction of the bed topography. This phase shift might be the result of adaptation length of the bed roughness to grain size and water depth variations.

Implementation of the alluvial bed roughness predictor according to van Rijn (1993) did not lead to better prediction of the bed topography. For morphological models with characteristics as Olesen's experiment the application of the alluvial bed roughness predictor of van Rijn (1993) is not recommended.

Computations with spatially varying bed roughness indicate that there is a combination of spatially varying grain size with spatially varying bed roughness which gives the same results as the computation with constant grain size and bed roughness. Since the computation with constant bed roughness gives a good prediction of the bed topography, one might conclude that there is a similar process in nature.
6 Waal bend at Nijmegen

6.1 Introduction
Olesen's experiment is conducted under controlled circumstances. Due to the scale of the experiment it might be possible that certain physical phenomena are overestimated or underestimated, when the experiment is seen as a scale model of a real river. In order to investigate the effect of spatial variation of grain size with the assumed correct representation of a natural river, computations have been made for the river Waal.

Several bends of the river Waal have been taken into consideration. Most of them were excluded by lack of bed level and grain size measurements or availability of previous numerical computations. The bend at Hulhuizen was the first option, but the upstream boundary condition could not be successfully modelled. Finally the bend at Nijmegen remained the best option, despite the natural fixed layer in the river bed.

The model used represents the bend at Nijmegen, section km 882-887, as shown in the figure below. Computations have been made according to the situation before the construction of an artificial fixed layer in 1987.

![Figure 6-1: Location of numerical grid](image)

The table below shows the governing parameters for the Waal bend at Nijmegen.

<table>
<thead>
<tr>
<th>Q  [m³/s]</th>
<th>B  [m]</th>
<th>h  [m]</th>
<th>u  [m/s]</th>
<th>i [-]*10^-4</th>
<th>C  [m/²/s]</th>
<th>Dm  [mm]</th>
<th>s  [m²/s]*10^-5</th>
<th>B/h [-]</th>
<th>λw  [m]</th>
<th>λa  [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1620</td>
<td>240</td>
<td>5.3</td>
<td>1.13</td>
<td>1.0</td>
<td>45</td>
<td>1.5</td>
<td>3.8</td>
<td>45</td>
<td>536</td>
<td>356</td>
</tr>
</tbody>
</table>

Table 6-1: Governing parameters of the Waal bend at Nijmegen
6.2 Model setup

The numerical model is set up according to a previous case study by Wolters (1998). The model uses a grid as shown in Figure 6-1. The size of the grid cells is approximately 50 * 20 meters.

The initial bathymetry for this model consists of a bed level under a longitudinal slope of $1 \times 10^{-4}$. Except at the upstream boundary the initial cross-sectional slope is taken zero. In order to get an accurate prediction of the equilibrium bed level, the bed level at the upstream boundary has to be specified. The bed level at this boundary is obtained by taking the averaged measured bed level at this location over the period 1982-1986. This is visualised in the figure below.

![Figure 6-2: Upstream bed level boundary](image)

The upstream hydraulic boundary condition is given by a discharge. This discharge is also averaged over the years 1982-1986. This leads to a discharge of 1620 m³/s over the total cross-sectional area. Since there is no record of significant high water effects at this location the assumption of a mean discharge is justified. The total discharge is distributed over the upstream boundary with a Chézy formulation. This formulation is based on the assumption that the discharge is a function of the water depth to the power 1.5.

The downstream hydraulic boundary condition is given by a water elevation based on the rating curve of the river Waal. According to this curve the downstream water elevation belonging to a discharge of 1620 m³/s is 7.6 m + NAP.

The upstream morphological boundary condition is given by a fixed bed level. This bed level is equal to the initial bed level given with the bathymetry, Figure 6-2. The sediment transport is predicted by the "General formula" from Delft3D-MOR.

As mentioned before, a natural occurring fixed layer due to sedimentation during ice age is present underneath the river basin, here called the glacial layer. Due to this glacial layer the erosion of the bed level in the left outer bank is dampened.
6.3 Reference computation

In order to investigate the morphological effects of the spatial variation of grain size, a good reference computation has to be made. This reference computation was calibrated according to the following steps:

**Water movement**

The first step was calibration of the water movement in the model. The initial boundary conditions resulted in a satisfactory representation of the water movement. This water movement could not be compared to actual measured data, due to lack of this data. The criteria therefore, are the theoretical approximations and the one-dimensional model SOBEK, van der Veen et al. (1997).

**Sediment transport**

After calibration of the water movement, the sediment transport has to be calibrated. First of all the correct sediment transport rate has to be established. According to Equation (5.2) the correct sediment transport rate can be obtained by adjusting the calibration parameters. The calibration resulted in the following sediment transport parameters.

<table>
<thead>
<tr>
<th>a&lt;sub&gt;cal&lt;/sub&gt;</th>
<th>correction transport rate</th>
<th>2.0</th>
<th>[-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>Ripple factor</td>
<td>0.7</td>
<td>[-]</td>
</tr>
<tr>
<td>b&lt;sub&gt;cal&lt;/sub&gt;</td>
<td>correction factor</td>
<td>0.0</td>
<td>[-]</td>
</tr>
<tr>
<td>c&lt;sub&gt;cal&lt;/sub&gt;</td>
<td>correction factor</td>
<td>1.5</td>
<td>[-]</td>
</tr>
<tr>
<td>D</td>
<td>D&lt;sub&gt;50&lt;/sub&gt;</td>
<td>0.0015</td>
<td>[m]</td>
</tr>
<tr>
<td>C</td>
<td>Chezy coefficient</td>
<td>45</td>
<td>[m&lt;sup&gt;0.5&lt;/sup&gt;/s]</td>
</tr>
<tr>
<td>θ&lt;sub&gt;cr&lt;/sub&gt;</td>
<td>Critical Shields parameter</td>
<td>0.037</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Table 6-2: Calibration parameters sediment transport

**Bed topography**

The last step in creating a correct reproduction of the prototype is adjusting the shape of the bed topography to fit the averaged measured bed topography (1982-1986). By adjusting the calibration parameters the correct representation can be obtained.

<table>
<thead>
<tr>
<th>α&lt;sub&gt;f&lt;/sub&gt;v</th>
<th>correction Shields number</th>
<th>1.00</th>
<th>[-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a&lt;sub&gt;s&lt;/sub&gt;</td>
<td>correction slope effect</td>
<td>0.70</td>
<td>[-]</td>
</tr>
<tr>
<td>b&lt;sub&gt;s&lt;/sub&gt;</td>
<td>correction slope effect</td>
<td>0.50</td>
<td>[-]</td>
</tr>
<tr>
<td>E&lt;sub&gt;s&lt;/sub&gt;</td>
<td>correction spiral flow</td>
<td>1.00</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Table 6-3: Calibration parameters of bed topography
After calibration the resulting bed topography approximates the averaged bed topography reasonably well.

![Graph showing bed levels over distance](image)

**Figure 6-3: Reference computation**

The result of the computation without the glacial layer clearly shows the effect of this layer. In that case, the outer left bank the depth is clearly overestimated. Furthermore, measurements of the inner right bank show an almost horizontal bed level. This can be explained from dredging activities just before the measurements. This discrepancy will therefore not be taken into account.

### 6.4 Spatially varying grain size

Computations with the spatially varying grain size require a horizontal grain size distribution. This horizontal grain size distribution is obtained from various measurements conducted by RIZA. These measurements have been analysed by Wolters (1998), which resulted in an approximation of the horizontal grain size distribution as shown in Figure 6-4.

![Graph showing grain size over distance](image)

**Figure 6-4: Horizontal grain size distribution**
This grain size distribution results in the following bed levels:

![Graphs showing grain size distribution](image)

**Figure 6-5: Implementation of spatially varying grain size**

As can be seen in the figures above, that the effect of spatially varying grain size is not as strong as with Olesen’s experiment. The deviations from the reference computation are not more than 1.0 meters.

As explained in the previous chapter, there are two ways of interpretation of the computational results. When assuming free excitation, comparison has to be made with the analytical equilibrium as obtained using the expression for the axi-symmetrical solution found by Mosselman & Sloff (1998). Due to the complexity of the used model the analytical solution has been simplified. Only the large changes in curvature have been taken into account. The result is shown in the figures below.

![Graphs showing water depth](image)

**Figure 6-6: Water depth right bank**

Figure 6-6 clearly shows a shortening of the perturbation length in section 883.5-885 km, as well as in section 885.5-887 km. This effect is not so obvious in Figure 6-7, which is probably due to the natural fixed layer.
According to Equation (5.7) an approximation of $\alpha$ can be made. Application of this relation results in $\alpha \approx 1.45$ [-] for the section 883.5-885 km for the right bank. Applying this value leads to the following theoretical perturbation length:

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$ [-]</th>
<th>Theoretical $L_P$ [m]</th>
<th>Numerical $L_P$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D = constant</td>
<td>0</td>
<td>2866</td>
<td>2444</td>
</tr>
<tr>
<td>D(x,y)</td>
<td>1.45</td>
<td>3222</td>
<td>1920</td>
</tr>
<tr>
<td>Difference</td>
<td>12%</td>
<td></td>
<td>-21%</td>
</tr>
</tbody>
</table>

Table 6-4: Change of $L_P$

The dampening length of the numerical model with constant grain size and spatially varying grain size can not be exactly determined. The length of the perturbation is too short to determine the exact dampening length. The theoretical results, however, predict a decrease in the dampening length.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$ [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D = constant</td>
<td>0</td>
<td>1519</td>
</tr>
<tr>
<td>D(x,y)</td>
<td>1.45</td>
<td>834</td>
</tr>
<tr>
<td>Difference</td>
<td>45%</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-5: Change of $L_D$

The change of $L_P$ is not corresponding to the theory of free excitations at all. The model shows a decrease in the perturbation length, the theory in Chapter 2 also predicts an decrease of the dampening length. For this model prediction of $L_D$ is qualitative correct. The actual size of the decrease could not be determined.
When assuming forced excitation, the effect on the bed level can be obtained by determining the difference between the bed level computed with and without spatial variation of grain size. The differences are plotted in the figures below.

Figure 6-8: Effect of $D(x,y)$ without glacial layer
Figure 6-9: Effect of $D(x,y)$ with glacial layer

The computation with the glacial layer shows no difference in bed level for the section km 883-884. This coincides with the location of the glacial layer, which causes this phenomenon. Also quite a lot of disturbances can be observed in the effects of the spatially varying grain size compared to the theoretical analysis. These disturbances might be caused by higher order terms. Especially the theoretical simplification to a straight channel might contribute to these disturbances. Higher order terms resulting from the curvature might be responsible for the effects. Due to the strong variability of the curvature in the model, the effects of higher order terms of the grain size variation could be amplified. The differences between Figure 6-8 and Figure 6-9 are caused by the glacial layer.

The results of the computations with spatial variation of grain size show generally a similar effect as with Olesen's experiment. The spatial variation of grain size results in some kind of wave form superimposed on the bed topography. This wave form has approximately the following characteristics:

<table>
<thead>
<tr>
<th></th>
<th>$k\lambda_4/(2\pi/L_p)\lambda_4$</th>
<th>$R_D = (\Delta h/\Delta D)/(\Delta D/D)$</th>
<th>Phase $= \chi_g/L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sieben (2000)</td>
<td>0.28</td>
<td>0.24</td>
<td>-0.23</td>
</tr>
<tr>
<td>Model</td>
<td>0.28</td>
<td>0.16</td>
<td>-0.25</td>
</tr>
<tr>
<td>Difference</td>
<td>33%</td>
<td>8%</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-6: Waveform characteristics of effect of grain size variation
When plotting these wave characteristics with the theoretical response function according to the theory of forced excitation, the characteristics of the response in the computation nicely match with the theory.

**Figure 6-10: Amplitude response according to forced excitation**

**Figure 6-11: Phase shift response according to forced excitation**
6.5 Morphological effects of C(D)

The effects of the spatial variation of grain size might be counteracted with spatial variation of the bed roughness, according to the analyses of Mosselman & Sloff (1998) and Sieben (2000). This counteraction is implemented according to the following relation between bed roughness, grain size, water depth, flow velocity.

\[ C(D, h, u) = C_0 + \Phi_D \frac{C_D}{D_0} (D(x, y) - D_0) + \Phi_h \frac{C_h}{h_0} (h(x, y) - h_0) + \Phi_u \frac{C_u}{u_0} (u(x, y) - u_0) \] (6.1)

Counteracting with C(D) only, implies that \( \Phi_h = \Phi_u = 0 \). During computations with Delft2D-MOR, the parameter \( \Phi_D \) has been varied between \(-0.4 < \Phi_D < 0\), with steps of 0.1. The best results from the counteraction were obtained with \( \Phi_D = -0.1 \). The results from this computation are plotted below. During one computation, \( \Phi_D \) was kept constant, no iteration took place.

![Figure 6-12: Implementation of C(D)](image)

The results of the computation with C(D) show a bed topography which matches the prototype and the computation with spatially constant sediment better.
When assuming that the grain size distribution is the result of grain sorting $D(u)$, one might conclude that the application of $C(D)$ could result in effects described in the analysis for free excitations. When assuming free excitations, the following figures can be plotted with the axi-symmetrical solution found by Mosselman & Sloff (1998).

![Figure 6-13: Water depth at right bank](image)

![Figure 6-14: Water depth at left bank](image)

Figure 6-13 and Figure 6-14 show no substantial change in the perturbation length. The dampening length of the perturbation seems to have increased. This is concluded from the fact that the amplitude of the perturbation has increased.

Analysing the results from the numerical computations results in the following data:

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_D$ [-]</th>
<th>Theoretical $L_P$ [m]</th>
<th>Numerical $L_P$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(x,y)$</td>
<td>1.45</td>
<td>3222</td>
<td>1920</td>
</tr>
<tr>
<td>$D(x,y)$ and $C(D)$</td>
<td>1.45</td>
<td>-0.1</td>
<td>2823</td>
</tr>
<tr>
<td>Difference</td>
<td>-12%</td>
<td>16%</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6-7: Change of $L_P$**

The dampening length of the numerical model with constant grain size and spatially varying grain size can not be exactly determined. The length of the perturbation is too short to determine the exact dampening length. The theoretical results, however, predict an increase in the dampening length.

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_D$ [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(x,y)$</td>
<td>1.45</td>
<td>834</td>
</tr>
<tr>
<td>$D(x,y)$ and $C(D)$</td>
<td>1.45</td>
<td>-0.1</td>
</tr>
<tr>
<td>Difference</td>
<td>17%</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6-8: Change of $L_D$**

The change of $L_P$ is not corresponding to the theory of free excitations at all. The model shows an increase in the perturbation length, while the theory in Chapter 2 predicts a decrease of the perturbation length. The actual size of the increase could not be determined.
When assuming that the effects of the implementation of C(D) are due to a forced excitation, the difference between the computations with and without the counteraction by C(D) have to be determined.

![Figure 6-15: Effect of C(D) (right bank)](image1)

![Figure 6-16: Effect of C(D) (left bank)](image2)

The figures show a response due to C(D), which looks quite irregular. Here also the higher-order terms can be responsible. However, when taking into account these effects, the figure shows some kind of wave form. This wave form looks like the wave form which is predicted by the hypothesis of Sieben (2000).

<table>
<thead>
<tr>
<th>kλ<em>s = (2π/Lp)λ</em>s</th>
<th>R_C = (Δh/h)/(ΔC/C)</th>
<th>Phase = x_p/L_p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sieben (2000)</td>
<td>0.28</td>
<td>0.64</td>
</tr>
<tr>
<td>Model</td>
<td>0.28</td>
<td>1.5</td>
</tr>
<tr>
<td>Difference</td>
<td>135%</td>
<td>16%</td>
</tr>
</tbody>
</table>

Table 6-9: Wave characteristics

Table 6-9 shows that the amplitude of the wave form is not completely according to the theory. The amplitude is mismatched by approximately 135%, while the phase shift is overestimated by 16%. The deviations from the theory might be explained by the fact that the model is strongly curved. Due to this curvature the theoretical analyses are not valid for this model.
6.6 Counteracting with C(D) and phase shift

According to the compensation hypothesis, the C(D) should be shifted in phase compared to the D(x,y). The necessary phase shift for optimal compensation is given by Equation (3.5) in Section 3.3. This equation can be visualised with the following graph:

![Graph showing phase shift](image)

**Figure 6-17: Necessary phase shift for optimum compensation**

From the analytical solution the phase shift is obtained, \( x_f = -0.1 \cdot L_p = -800 \) [m], \( (k\lambda_a=0.28 \ldots) \). Implementing this phase shift in the model leads to the bed level as shown in Figure 6-18 and Figure 6-19.

![Graph showing bed level](image)

**Figure 6-18: Compensating C(D) with phase shift (right bank)**

![Graph showing bed level](image)

**Figure 6-19: Compensating C(D) with phase shift (left bank)**

Results from the computation with a phase shift show some improvement in the prediction of the bed level. The strong curvature and width/depth-ratio probably causes deviations from the theory.

The physical process which might cause a phase shift between the morphological effects of spatially varying grain size and bed roughness could be a difference in adaptation lengths for flow and sediment transport. The found phase shift implies that the adaptation lengths for the case of spatially varying bed roughness is smaller than the adaptation length of the spatially varying grain size.
6.7 Morphological effects of C(h)

Similar to the counteracting with C(D), computations have been carried out with use of C(h). These computations also make use of Equation (5.8). The results of the computations are shown below:

Figure 6-20: Counteracting with C(h)

When assuming free excitation, comparison has got to be made with the analytical axi-symmetrical solution according to Mosselman & Sloff (1998). This results in the following figures:

Figure 6-21: Water depth at right bank

Comparison of the results with show no significant difference with the computation with spatially varying grain size.

Because there is no significant difference between the reference computation and the computation with D(x,y) and C(h), there also is no significant forced excitation.
6.8 Counteracting with alluvial bed roughness predictors

After confirming the possibility of counteracting the effects of spatially varying grain size, now the link with alluvial roughness predictors has to be made. This link is important in order to find practical use for the relation found between spatially varying grain size and bed roughness.

Result from the computations with C(D) and C(h) indicate that the application of an alluvial bed roughness predictor with dependencies on grain size and water depth could result in a better prediction of the bed topography when implementing spatial variation of grain size. In order to investigate this, an alluvial bed roughness predictor has been implemented according to Appendix C. For this study the alluvial bed roughness predictor of van Rijn (1993) and Julien and Klaassen (1995) has been tested. The predictor was verified by van Rijn (1993) for a large amount of field data, including the river Waal. The resulting bed levels are plotted in the figures below:

![Figure 6-23: Implementation of alluvial bed roughness predictor](image)

The figures show a slight increase in the cross-sectional slope. The bed topography, however, was not predicted better with this alluvial bed roughness predictor. The sensitivity of the alluvial bed roughness predictor to grain size and water depth variations was not according to the optimal counteraction, found during the computations.
For the Waal bend at Nijmegen $\Phi_D = -0.1$ [-] was found to result in the best counteracting. A comparison with several general alluvial bed roughness predictors has been made (Appendix B). These alluvial bed roughness predictors also require information about the bed-forms. Wilbers (1998) found the following characteristics for a discharge of approximately 1800 m$^3$/s:

<table>
<thead>
<tr>
<th>h [m]</th>
<th>H [m]</th>
<th>L [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>5.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 6-10: Bed-form characteristics for Waal bend at Nijmegen

Given the bed-form characteristics, the bed roughness can be predicted by several bed roughness predictors. The $\Phi_h$ has not been taken into account, because computations showed no large effects due to C(h).

<table>
<thead>
<tr>
<th>C [m$^3$/s]</th>
<th>$\Phi_D$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counteraction</td>
<td>45</td>
</tr>
<tr>
<td>Engelund (1977)</td>
<td>90</td>
</tr>
<tr>
<td>van Rijn (1993)</td>
<td>40</td>
</tr>
<tr>
<td>Vanoni and Hwang (1967)</td>
<td>96</td>
</tr>
<tr>
<td>Yalin (1977)</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 6-11: Comparison found counteraction with alluvial bed roughness predictors

Plotting the table above leads to the figure below:

Figure 6-24: $\Phi_D$ as a function of grain size

The table and figure confirm the results from the computation with the alluvial bed roughness predictor. The table and figure show that van Rijn (1993) and Julien & Klaassen (1995) is not the correct approach for determining the bed roughness. However, the alluvial bed roughness predictor has the best results with regard to the other alluvial bed roughness predictors.
6.9 Conclusions

Computations with Delft2D-MOR with spatially constant grain sizes, predict the bed topography reasonably well. The effects of the glacial layer are very obvious. Implementing spatial variation of grain size resulted in a bed topography which did not differ very much from the reference computation. The differences where not larger than 1 meter.

According to the theory of free excitation, the perturbation length should increase when implementing spatial variation of grain size. Numerical computations showed a decrease in the perturbation length. The change in the dampening length was predicted right, a decrease of the dampening length occurred.

When assuming forced excitation the wave corresponded to the bed level response due to grain size variations predicted by Sieben (2000). The size of the bed level response was overestimated by 33%, while the occurring phase shift was also overestimated but with 8%.

Counteracting the effects of the spatially varying grain size with spatially varying bed roughness shows that counteracting might be possible for forced excitation. For free excitation the change in perturbation length was not according to theory. However, the superimposed wave on the bed level, resulting from forced excitation, was of the correct shape. The amplitude and phase shift of this wave form were underestimated by the theory.

For forced excitation, the resulting wave due to the spatial variation of bed roughness seemed to be shifted in phase, compared to the response due to spatial variation of grain size. According to the hypothesis a phase shift in the bed roughness was introduced, which resulted in a better prediction of the bed topography.

Implementation of the alluvial bed roughness predictor of van Rijn did not result in a satisfactory prediction of the bed topography. Theoretical comparison of several alluvial bed roughness predictors showed that van Rijn was not the best alluvial bed roughness predictor for this river section.

Computations with spatially varying bed roughness indicate that there is a combination of spatially varying grain size with spatially varying bed roughness which gives the same results as the computation with constant grain size and bed roughness. Since the computation with constant bed roughness gives a good prediction of the bed topography, one might conclude that there is a similar process in nature.
7 Pannerdense Kop

7.1 Introduction

After confirming the compensation hypothesis in the previous cases, the hypothesis was tested for a more complex case, viz. the bifurcation at Pannerden. As stated in the problem definition, the question remains whether the bed level response due to spatial variation of grain size is causing the problems with modelling the Pannerdense Kop. Furthermore, the compensation principle should also be tested here.

The model showed two different problems during calibration, Struiksma (1998). The first consisted of an instability of the bifurcation. The river Waal continued to erode, whereas the Pannerdens Kanaal accreted. The second problem led to the present study: computations without spatial variation of grain size gave better results than with the observed spatial variation of grain size.

The model used is equal to the model of Struiksma (1998). The figure below gives the modelling area. The model represents the Rhine branches from km 862.750 on the Bovenrijn, to km 870.500 of the river Waal and 871.250 on Pannerdens Kanaal.

![Map of Pannerdense Kop](image)

Figure 7-1: Location of numerical grid (Pannerdense Kop)

The table below shows the governing parameters for Pannerdense Kop.

<table>
<thead>
<tr>
<th>Q [m³/s]</th>
<th>B [m]</th>
<th>h [m]</th>
<th>u [m/s]</th>
<th>i [-] x 10⁻⁴</th>
<th>C [m²/s]</th>
<th>Dₘ [mm]</th>
<th>s [m²/s] x 10⁻⁵</th>
<th>B/h [-]</th>
<th>λᵥ [m]</th>
<th>λₛ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2400</td>
<td>330</td>
<td>6.3</td>
<td>1.13</td>
<td>1.0</td>
<td>45</td>
<td>2.6</td>
<td>4.9</td>
<td>52</td>
<td>637</td>
<td>447</td>
</tr>
</tbody>
</table>

Table 7-1: Governing parameters of Pannerdense Kop

Morphological effects of spatially varying grain size and bed roughness in rivers
7.2 Model set-up

As mentioned in the introduction, the numerical model is set up according to the previous study by Struiksma (1998). The size of the grid is approximately 100 * 20 metres.

The initial bathymetry for this model consists of a bed level which is obtained by averaging measurements of the bed topography during the period 1988-1991. This bathymetry therefore is also the prototype, with which the reference computation will be calibrated.

The upstream hydraulic boundary is given by a discharge. This discharge is 2400 m³/s over the total cross-sectional area, which is the average discharge over the period 1988-1991. The total discharge is distributed over the upstream boundary with a Chézy formulation. This formulation is based on the assumption that the discharge is a function of the water depth raised to the power 1.5.

The downstream hydraulic boundary conditions are given by a water elevation. The water elevation is based on the rating curves of the Rhine branches. According to these curves the downstream water elevations belonging to a discharge of 2400 m³/s are 9.56 m + NAP for the river Waal and 9.54 m + NAP for the Pannerdens Kanaal.

The upstream morphological boundary condition is given by a fixed bed level. This bed level is equal to the initial bed level given by the initial bathymetry. The sediment transport is predicted by the "General formula" in Delft2D-MOR.

7.3 Reference computation

The reference computation for Pannerdense Kop has not been obtained by the standard calibration cycle. Since Struiksma (1998) already tried to calibrate the model, parameters from that study have been used. These parameters, according to Equation (5.2), are given in the table below.

<table>
<thead>
<tr>
<th>a_cal</th>
<th>correction transport rate</th>
<th>4.4</th>
<th>[-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>μ</td>
<td>ripple factor</td>
<td>1.0</td>
<td>[-]</td>
</tr>
<tr>
<td>b_cal</td>
<td>correction factor</td>
<td>2.3</td>
<td>[-]</td>
</tr>
<tr>
<td>c_cal</td>
<td>correction factor</td>
<td>0.0</td>
<td>[-]</td>
</tr>
<tr>
<td>D</td>
<td>Dₜ₀</td>
<td>0.0026</td>
<td>[m]</td>
</tr>
<tr>
<td>C</td>
<td>Chézy coefficient</td>
<td>45.0</td>
<td>[m³/s]</td>
</tr>
<tr>
<td>θ_cr</td>
<td>critical Shields parameter</td>
<td>0.0</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Table 7-2: Calibration parameters of sediment transport

The sediment transport relation is, with use of the parameters above, adapted to some kind of Engelund-Hansen transport relation.
Table 7-3: Calibration parameters for bed topography

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_S$</td>
<td>correction Shields number</td>
<td>1.00</td>
<td>[-]</td>
</tr>
<tr>
<td>$a_S$</td>
<td>correction slope effect</td>
<td>0.85</td>
<td>[-]</td>
</tr>
<tr>
<td>$b_S$</td>
<td>correction slope effect</td>
<td>0.50</td>
<td>[-]</td>
</tr>
<tr>
<td>$E_S$</td>
<td>correction spiral flow</td>
<td>1.00</td>
<td>[-]</td>
</tr>
</tbody>
</table>

The resulting bed topography approximates the average bed topography reasonably well, as shown in the figure below.

![Figure 7-2: Reference computation for Pannerdense Kop](image)

As can be observed in the figure above, the total bed level for the river Waal has dropped about 1 meter. This can be due to the problems with the bifurcation, as mentioned by Struiksma (1998). However, the crossings between the bends are predicted quite well. In order to investigate the effects of spatial variation of grain size, this computation can serve as a reference.

### 7.4 Spatially varying grain size

Computations with spatially varying grain size require a horizontal grain size distribution. This horizontal grain size distribution is obtained from various measurements conducted by RIZA. Analysis of these measurements resulted in a horizontal grain size distribution, plotted in the figure below (Struiksma, 1998).

![Figure 7-3: Horizontal grain size distribution](image)
Implementing the horizontal grain size distribution results in the following bed levels:

Figure 7-4: Implementation of spatial variation of grain size

The results from the computation with spatial variation of grain size shows a shift of the crossing between the bends and a less strongly pronounced point bar. This suggests that the hypothesis of free excitation is correct. When assuming free excitation the following figures can be plotted:

Figure 7-5: Water depth at right bank
Figure 7-6: Water depth at left bank

Figure 7-5 and Figure 7-6 show two different effects of the implementation spatial variation of grain size. On the right bank the perturbation length seems to have increased, while on the left bank the perturbation length decreased.

Combining the numerical and theoretical results leads to the table below:

<table>
<thead>
<tr>
<th>α [-]</th>
<th>Theoretical L_P [m]</th>
<th>Numerical L_P (right) [m]</th>
<th>Numerical L_P (left) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D = constant</td>
<td>0</td>
<td>3474</td>
<td>3140</td>
</tr>
<tr>
<td>D(x,y)</td>
<td>-0.3</td>
<td>3430</td>
<td>3880</td>
</tr>
<tr>
<td>Difference</td>
<td>-1%</td>
<td>24%</td>
<td>-53%</td>
</tr>
</tbody>
</table>

Table 7-4: Change of L_P
The dampening length of the numerical model with constant grain size and spatially varying grain size can not be exactly determined. The model is too complex to determine the exact dampening length.

<table>
<thead>
<tr>
<th>α [-]</th>
<th>Theoretical $L_D$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = \text{constant}$</td>
<td>0</td>
</tr>
<tr>
<td>$D(x,y)$</td>
<td>-0.3</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
</tr>
</tbody>
</table>

Table 7-5: Change of $L_D$

The results from the model do not match with the theory of free excitation.

Assuming forced excitation the effects on the bed level can be obtained by determining the difference between the bed level computed with and without spatial variation of grain size. The differences are plotted in the figures below:

![Figure 7-7: Effect D(x,y) (right bank)](image)

![Figure 7-8: Effect D(x,y) (left bank)](image)

The results of the computation with spatial variation of grain size show a very distinguished wave form on the left bank, according to the theory of forced excitation.

The spatial variation of grain size results in some kind of wave form superimposed on the bed topography. This wave form has the following characteristics:

<table>
<thead>
<tr>
<th>$k = (2\pi/L_P)\lambda_s$ [-]</th>
<th>$R_0 = (\Delta h/\lambda)/(\Delta D/D)$ [-]</th>
<th>Phase $= x_L/L_P$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sieben (2000) 0.55</td>
<td>0.72</td>
<td>-0.25</td>
</tr>
<tr>
<td>Model 0.55</td>
<td>0.75</td>
<td>-0.25</td>
</tr>
<tr>
<td>Difference</td>
<td>4%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Figure 7-9: Wave form characteristics of effect grain size variation
Plotting these wave characteristics with the theoretical response function according to the theory of forced excitation, the characteristics of the response in the computation fit the theory nicely. The figures show clearly that the theory of forced excitation is the correct approach for the problem with the calibration found by Struijsma (1998).

![Amplitude response according to forced excitation](image)

**Figure 7-10: Amplitude response according to forced excitation**

![Phase shift according to forced excitation](image)

**Figure 7-11: Phase shift according to forced excitation**

The results from the computation with spatial variation of grain size indicate that the effects of the spatial variation of grain size can be counteracted with spatial variation of bed roughness.
7.5 Morphological effects of C(D)

The effects of the spatial variation of grain size can be counteracted with spatial variation of the bed roughness, according to the analysis of forced excitation. This counteracting is implemented according to the following relation between bed roughness, grain size, water depth and flow velocity.

\[ C(D, h, u) = C_0 + \Phi_D \frac{C_0}{D_0} (D(x, y) - D_0) + \Phi_h \frac{C_0}{h_0} (h(x, y) - h_0) + \Phi_u \frac{C_0}{u_0} (u(x, y) - u_0) \] (7.1)

Counteracting with C(D) only, implies that \( \Phi_h = \Phi_u = 0 \).

Computations show the best counteraction with \( \Delta C/C_0 = 0.11 \). The best results were obtained using \( \Phi_D = -0.25 \). The resulting bed levels are plotted below:

**Figure 7-12: Bed levels without C(D)**

**Figure 7-13: Bed levels with C(D)**

The computations with counteracting bed roughness show significant improvement of the prediction of the bed topography.
When assuming free excitation comparison has to be made with the axi-symmetrical solution, according to Mosselman & Sloff (1998). This results below:

Figure 7-14: Bed level at right bank

Figure 7-15: Bed level at left bank

Figure 7-14 and Figure 7-15 show a decrease in the perturbation length on both sides. In the table below the change in characteristic lengths of the model is compared with the theoretical change. For the table the characteristic lengths of the right bank are compared.

<table>
<thead>
<tr>
<th>α [-]</th>
<th>ΦD [-]</th>
<th>Theoretical Lp [m]</th>
<th>Numerical Lp (right) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(x,y)</td>
<td>-0.3</td>
<td>3430</td>
<td>3880</td>
</tr>
<tr>
<td>D(x,y) and C(D)</td>
<td>-0.3</td>
<td>3654</td>
<td>2940</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>7%</td>
<td>-24%</td>
</tr>
</tbody>
</table>

Table 7-6: Change of Lp

The dampening length of the numerical model with constant grain size and spatially varying grain size can not be exactly determined. The model is too complex to determine the exact dampening length.

<table>
<thead>
<tr>
<th>α [-]</th>
<th>ΦD [-]</th>
<th>Theoretical Ld [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(x,y)</td>
<td>-0.3</td>
<td>2522</td>
</tr>
<tr>
<td>D(x,y) and C(D)</td>
<td>-0.3</td>
<td>2048</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>-19%</td>
</tr>
</tbody>
</table>

Table 7-7: Change of Ld

According to the theory of free excitation the perturbation length should increase. The model shows a decrease in the perturbation length. The dampening length should decrease according to the theory. This seems to be the case in the model, the amplitude of the excitation is more dampened.
When assuming forced excitation the effects of $D(x,y)$ and $C(D)$ can be obtained by determining the difference between the computations. The found differences are plotted in the figures below:

![Figure 7-16: Effects of C(D) (right bank)](image)

![Figure 7-17: Effects of C(D) (left bank)](image)

Some disturbance in the effects of $C(D)$ can be seen at km 867. This disturbance is caused by the bifurcation, which is located at that position. The bifurcation causes non-uniform flow patterns, which on their turn cause bed changes.

The characteristics of the wave form as a result of applying $C(D)$ has the following characteristics, derived from local maxima:

<table>
<thead>
<tr>
<th></th>
<th>$k\lambda_n/(2\pi/L_p)\lambda_n$</th>
<th>$R_C = (\Delta h/h)/(\Delta C/C)$</th>
<th>$\text{Phase} = \chi x/L_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sieben (2000)</td>
<td>0.55</td>
<td>2.9</td>
<td>-0.15</td>
</tr>
<tr>
<td>Model</td>
<td>0.55</td>
<td>2.9</td>
<td>-0.14</td>
</tr>
<tr>
<td>Difference</td>
<td>0%</td>
<td>7%</td>
<td></td>
</tr>
</tbody>
</table>

Table 7-8: Wave form characteristics
Plotting the characteristics found with the theoretical response function results in the following figures:

**Figure 7-18: Amplitude response according to forced excitation**

![Amplitude response graph]

**Figure 7-19: Phase shift according to forced excitation**

Also the response of the bed level due to spatial variation of the bed roughness coincides nicely with the theory of forced excitation. However, the results of the computations indicate that here also a phase shift of the bed roughness compared to the grain size should be implemented. This will be further investigated in the following paragraph.
7.6 Counteracting with C(D) and phase shift

As mentioned in the previous paragraph, according to the compensation hypothesis, the C(D) should be shifted in phase compared to the D(x,y). The necessary phase shift for the optimal compensation is given by equation (3.5) in Section 3.3. This equation can be visualised with the following graph:

![Graph showing necessary phase shift for optimum compensation](image)

**Figure 7-20:** Necessary phase shift for optimum compensation

From the analytical solution the phase shift is obtained, $\chi_f = -0.1 \times L_p = 500$ m ($k \lambda_s = 0.55 [-]$). Implementing this phase shift in the model leads to the bed level as shown below.

![Graph showing compensating C(D) without phase shift](image)

**Figure 7-21:** Compensating C(D) without phase shift

![Graph showing compensating C(D) with phase shift](image)

**Figure 7-22:** Compensating C(D) with phase shift

Morphological effects of spatially varying grain size and bed roughness in rivers
The resulting bed levels show an almost complete compensation, except for the section km 867-868. There the bifurcation, located at km 867, causes some reduction in the effect of C(D) at the left bank, as can be seen in the following figures.

Figure 7-23: Effect compensating with C(D) and phase shift (right bank)

Figure 7-24: Effect compensating with C(D) and phase shift (left bank)

Figure 7-24 clearly shows that at km 867 the bifurcation causes a distortion in the effect of the spatially varying grain size. This phenomenon will not be investigated further. Overall a good compensation has been established. For example, the effects of the spatially varying grain size on the right bank have been reduced from an amplitude of 2 meters, to a amplitude of 0.5 meters, this means 75 % reduction.
7.7 Morphological effects of C(h)

Similar to the counteracting with C(D), computations have been carried out with the use of C(h). These computations also made use of Equation (5.8). The results of the computations are shown in the figure below:

**Figure 7-25: Counteracting with C(h)**

The results from the computation with C(h) shows no effects on the bed topography due to the implementation of C(h). When assuming free excitation the results from the computation should be compared to the axi-symmetrical solution (Mosselman & Sloff, 1998). This results in the figures below:

**Figure 7-26: Water depth at right bank**

**Figure 7-27: Water dept at left bank**

The figures show no significant change in the characteristic perturbation lengths.

Because there is no significant difference between the reference computation and the computation with D(x,y) and C(h), there also is no significant forced excitation.
7.8 Counteracting with alluvial bed roughness predictors

After confirming the possibility of compensating the effects of spatially varying grain size, now the link with alluvial roughness predictors have to be made. This link is important in order to find practical use for the found relation between spatially varying grain size and bed roughness.

Perhaps it is possible to choose the counteracting effects so that the computation will result in a more accurate prediction of the bed topography. The numerical computations have shown that the best counteracting measure was achieved by applying C(D) and C(h). These findings implicate that an alluvial bed roughness predictor with dependencies on grain size and water depth could result in more accurate predictions of the bed topography.

For this study the alluvial bed roughness predictor of van Rijn (1993) has been tested. The predictor was verified for a large amount of field data by van Rijn and proved to be accurate for the river sections and experiment used in this study. Julien and Klaassen (1995) determined an approximation for the dune length and height as a function of the grain size and water depth, to be used in the alluvial bed roughness predictor. The generation of the resulting bed roughness field is explained in Appendix C. The resulting bed levels after implementation of van Rijn (1993) are plotted in the following figures:

Figure 7-28: Implementation of the alluvial bed roughness predictor

The figures show a general increase of the bed level. This increase of the bed level is caused by a change in the sediment distribution at the bifurcation. The ratio Swael/Spannedens kanaal as decreased from 3.1 to 2.6, while according to Struiksma (1998) the distribution ratio should be about 2.3. This implies that the implementation of the alluvial bed roughness predictor has a positive effect on the sediment distribution. It should be denoted that the total sediment transport has increased with 16%, this is caused by a general decrease of the Chézy-coefficient. Furthermore, the crossings are shifted towards their measured location according to the prototype. Finally the cross-sectional slope has increased to the slope according to the prototype.
When assuming free excitation the implementation of the alluvial bed roughness predictor leads to an increase in the perturbation length. This increase is shown in the figures below:

![Figure 7-29: Water depth right bank](image1)

![Figure 7-30: Water depth left bank](image2)

The figures above show a decrease in the perturbation length, while the theory predicts an increase in the perturbation length. The differences in perturbation length are summarised in the table below, where $\Phi_D$ and $\Phi_h$ are determined from the alluvial bed roughness predictor.

<table>
<thead>
<tr>
<th>$\alpha$ [-]</th>
<th>$\Phi_D$ [-]</th>
<th>$\Phi_h$ [-]</th>
<th>Theoretical $L_P$ [m]</th>
<th>Numerical $L_P$ (right) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(x,y)$</td>
<td>-0.3</td>
<td>0</td>
<td>3430</td>
<td>3880</td>
</tr>
<tr>
<td>$D(x,y)$ and $C(D)$</td>
<td>-0.3</td>
<td>-0.1</td>
<td>4230</td>
<td>3280</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>0.1</td>
<td>23%</td>
<td>-15%</td>
</tr>
</tbody>
</table>

Table 7-9: Change of $L_P$

When assuming forced excitation of the bed topography the following figures can be plotted:

![Figure 7-31: Effects on the right bank](image3)

![Figure 7-32: Effects on the left bank](image4)
Figure 7-31 and Figure 7-32 show the effects of C(D) compared to the effects of the application of van Rijn (1993) and Julien & Klaassen (1995). The computation with C(D) consists of a $\Phi_D = -0.3$ and a phase shift of -500 meters. When taking into account that from km 866 to km 871 the bed level at the right bank has dropped approximately 1 meter due to a change in the sediment distribution at the bifurcation, the alluvial bed roughness predictor of van Rijn (1993) and Julien & Klaassen (1995) has approximately the same effect as the computation with C(D), as mentioned in Paragraph 7.6. The opposite applies to the left bank.

For Pannerdense Kop, $\Phi_D = -0.3$ was found to be the optimal compensation. Comparison with several general alluvial bed roughness predictors has been made, Appendix B. These alluvial bed roughness predictors also require information about the bed-forms. Wilbers (1998) found the following characteristics, for a discharge of approximately 2600$m^3$/s:

<table>
<thead>
<tr>
<th>$h$ [m]</th>
<th>$H$ [m]</th>
<th>$L$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>6.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 7-10: Bed-form characteristics for Pannerdense Kop

Given the bed-form characteristics found, the bed roughness can be predicted by several bed roughness predictors. As shown before the influence of $\Phi_h$ is not significant, therefore it is not taken into account.

<table>
<thead>
<tr>
<th>Compensation</th>
<th>$C$ [m$^3$/s]</th>
<th>$\Phi_D$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engelund (1977)</td>
<td>100</td>
<td>-0.14</td>
</tr>
<tr>
<td>van Rijn (1993)</td>
<td>40</td>
<td>-0.09</td>
</tr>
<tr>
<td>Vanoni and Hwang (1967)</td>
<td>95</td>
<td>-0.15</td>
</tr>
<tr>
<td>Yalin (1977)</td>
<td>50</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Table 7-11: Comparison found compensation with alluvial bed roughness predictors

Plotting the table leads to the figure below:

The table and figure show that implementation of van Rijn and Julien & Klaassen theoretically can not lead to satisfactory results.
7.9 Conclusions

Computations with Delft2D-MOR with spatially constant grain sizes, predict the bed topography reasonably well. The reference computation shows some accretion of Pannerdens Kanaal.

Implementing spatial variation of grain size resulted in a bed topography which showed a shift of the crossings between the bends. Also a decrease in the transverse bed slope is observed. The effects of the spatial variation of grain size have been investigated for free and forced excitation.

When comparing the effects if spatial variation of grain size with the theoretical effects for free excitation, the results show that there is no free excitation of the bed level. The wave, resulting from forced excitation, corresponded to the bed level response due to grain size variations predicted for forced excitation.

Continuation with the hypothesis of forced excitation led to the application of spatial variation of bed roughness as a function of the grain size. The result of this application was another wave in the bed level, opposite to the wave caused by the spatial variation of grain size.

The resulting wave due to the spatial variation of bed roughness seemed to be shifted in phase, compared to the response due to spatial variation of grain size. According to the hypothesis a phase shift in the bed roughness was introduced, which resulted the best counteraction.

Implementation of the alluvial bed roughness predictor of van Rijn and Julien & Klaassen, resulted in a different sediment distribution at the bifurcation. Furthermore, when taking into account this change in the sediment distribution, the alluvial bed roughness predictor has approximately the same characteristics as needed according to the forced excitation.

Computations with spatially varying bed roughness indicate that there is a combination of spatially varying grain size with spatially varying bed roughness which gives the same results as the computation with constant grain size and bed roughness. Since the computation with constant bed roughness gives a good prediction of the bed topography, one might conclude that there is a similar process in nature.
8 Conclusions and recommendations

8.1 Conclusions
The purpose of the present investigation is to gain insight into the effects of spatial variation of grain size on the bed topography. Below the most important conclusions of this investigation are summarised.

The computations with Delft2D-MOR show that both free and forced excitation may occur. In the case of Olesen's experiment the grain size distribution has reached its equilibrium. Due to the constant discharge no variations in time disturb the solution. For this reason the hypotheses of free and forced excitation can be applied to Olesen's experiment. Both hypotheses do not deviate more than 10% from the model results.

Counteracting the effects of spatial variation of grain size with spatial variation of bed roughness in Olesen's experiment confirmed both hypotheses. In the case of free excitation the change in characteristic lengths were according to theory. For forced excitation the resulting superimposed wave coincided with the theory.

In the case of the river Waal and Pannerdense Kop the discharge is not constant. Due to this variation in time, the occurring grain size field can not reach its equilibrium state. When the grain size distribution has not reached its equilibrium, the hypothesis of free excitation is no longer valid, which is confirmed by the computations. The resulting effects of the spatial variation of grain size are therefore caused by the forced excitation.

Counteracting with spatial variation of bed roughness resulted in a superimposed wave according to the theory of force excitation. Due to the strong curvature of the bend at Nijmegen, the theory did not coincide with the model completely.

For the model of the bifurcation in the Rhine branches at Pannerden the computations showed that a forced excitation caused the shift in the crossings. When applying a spatial variation of bed roughness according to the theory of forced excitation the effects of the spatial variation of grain size can be counteracted. Applying the alluvial bed roughness predictor of van Rijn and Julien & Klaassen, resulted in a forced excitation which approximated the necessary counteraction.

Computations with spatially varying bed roughness indicate that there is a combination of spatially varying grain size with spatially varying bed roughness which gives the same results as the computation with constant grain size and bed roughness. Since the computation with constant bed roughness gives a good prediction of the bed topography, one might conclude that there is a similar process in nature.
8.2 Recommendations

First of all, more detailed information regarding the grain size distribution should be obtained from the study areas. At present the known density of the grain sizes is far too rough. Detailed information regarding the grain size distribution could show other effects in the model.

The current model uses spatially varying grain size only. The current insight in the processes in alluvial channels indicate that other parameters which relate to grain size should also vary spatially. For example, the critical Shields parameter should also vary if the grain size varies. Investigations of the influence of this phenomena is recommended.

The spatially varying grain size, investigated in this research, is caused by natural occurring grain size sorting. This grain size sorting is partly caused by hiding and exposure of individual grains. Investigations have started on modelling this hiding and exposure. The combination of results from both researches should also be investigated.

The computations with Delft2D-MOR have shown that free excitation can only occur when the grain size distribution has reached its equilibrium. For variable discharges a model should be constructed with grain sorting. Results from this model should coincide with the theory of free excitation. Further research is recommended to obtain the characteristics of the grain sorting and the effect on the bed topography in regard to the investigated theory.

When looking at the river Waal further investigation is recommended in order to obtain insight into the variability of the grain size distribution due to variable discharges. This should gain insight whether the grain size distribution always results in forced excitation or if the distribution has time to go to an equilibrium state which leads to free excitation.

The implementation of an alluvial bed roughness predictor in the model for Pannerdene Kop, showed that an alluvial bed roughness predictor might counteract the effects due to spatial variation of grain size. This implies that such a process occurs also in nature. Further investigation is needed to determine the best alluvial bed roughness predictor and the best way of implementing such a predictor.

Computations with the alluvial bed roughness predictor also showed a change in the sediment distribution at the bifurcation. This implies that the bifurcation is sensitive to the occurring grain size and bed roughness distributions. Further investigation is needed to obtain insight in the effects of those distributions on the stability of the bifurcation.
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Appendix A: Analytical solution of Sieben (2000)
The resulting simplified model of Sieben (2000) is given by the following system of equations:

\[
\frac{\Delta u}{u_0} = \left[ \frac{\frac{k s_d}{k_s}}{k_s} \right] + \left( \frac{\frac{k s_d}{k_s}}{k_s} \right)^2 + \frac{1}{2} \left( \frac{\frac{k s_d}{k_s}}{k_s} \right)^3 - \frac{1}{2} \left( \frac{\frac{k s_d}{k_s}}{k_s} \right)^4 + \frac{\Delta h}{h_0} = \frac{\Delta c_s}{c_s} \tag{A.1}
\]

\[
\left[ 1 - Y_s - \frac{2}{C_s} \frac{\Delta f}{f_0} \right] \frac{\Delta u}{u_0} = \left[ 1 - \frac{2}{C_s} \right] \frac{\Delta h}{h_0} = \frac{\Delta c_s}{c_s} + \frac{\Delta d_s}{d_s} \tag{A.2}
\]

Solving this set of equations leads to the following expressions:

\[
\frac{\Delta u}{u_0} = \left[ \frac{\left( \frac{k s_d}{k_s} \right)^2 + 1}{\frac{k s_d}{k_s} + \frac{1}{2}} \right] + \left( \frac{\frac{k s_d}{k_s}}{k_s} \right) - \frac{1}{2} \left( \frac{\frac{k s_d}{k_s}}{k_s} \right)^2 - \frac{1}{2} \left( \frac{\frac{k s_d}{k_s}}{k_s} \right)^3 + \frac{\Delta h}{h_0} = \frac{\Delta c_s}{c_s} + \frac{\Delta d_s}{d_s} \tag{A.3}
\]

\[
\left[ 1 - Y_s - \frac{2}{C_s} \frac{\Delta f}{f_0} \right] \frac{\Delta u}{u_0} = \left[ 1 - \frac{2}{C_s} \right] \frac{\Delta h}{h_0} = \frac{\Delta c_s}{c_s} + \frac{\Delta d_s}{d_s} \tag{A.4}
\]

Morphological effects of spatially varying grain size and bed roughness in rivers
Appendix B: Alluvial bed roughness predictors
The following alluvial roughness predictors have been compared during the research:

Engelund (1977):

\[
C(h, D, H, L) = \sqrt{\left(18 \cdot \log \left(\frac{11 \cdot h}{2 \cdot D}\right)\right)^2 + \frac{8 \cdot g}{10 \cdot \frac{H^2}{h \cdot L} e^{-2.5 \frac{H}{h}}}}
\]

van Rijn (1993):

\[
C(h, D, H, L) = 18 \cdot \log \left(\frac{12 \cdot h}{3 \cdot D + 1.1H \left(1 - e^{-\frac{25H}{L}}\right)}\right)
\]

Vanoni and Hwang (1967):

\[
C(h, D, H, L) = \sqrt{\left(18 \cdot \log \left(\frac{12 \cdot h}{D}\right)\right)^2 + 8 \cdot g \left(3.3 \cdot \log \left(\frac{h \cdot L}{H^2}\right) - 2.3\right)^2}
\]

Yalin (1977):

\[
C(h, D, H, L) = \frac{8 \cdot g}{\sqrt{2.5 \cdot g \cdot \ln \left(\frac{11 \cdot h}{D}\right)^2 \left(1 - 1.6 \frac{H}{L}\right) + 4 \left(\frac{H^2}{h \cdot L}\right)}}
\]

Julien & Klaassen (1995):

\[
L \equiv 6.5 \cdot h
\]

\[
H \equiv 2.5 \cdot h^{0.7} \cdot D_{50}^{0.3}
\]
Appendix C: Implementation of alluvial bed roughness predictor
The implementation of the alluvial bed roughness predictor can be specified as follows:

- 10 x computation with uniform bed roughness
- reading water depth and grain size from output files
- adjustment of bed roughness with application of van Rijn and Julien & Klaassen according to computed water depth
- writing of new bed roughness input file
- 10 x computations with new bed roughness
- and so on

The following program was constructed in order to compute the new bed roughness:

```
Program for_calculating_new_C_using_waterdepth_field;

Const
    p=12;
    q=117;
Var
    d50,h,C,delta,k :array [1..p,1..q] of real;
    bottom :array [1..p,1..q] of real;
    d, l :real;
    i,j :integer;
    inf, inf1, inf2, ouf1, ouf2, inf3 :text;
    f, f1, f2, f3, f4, f5 :string;

begin
    writeln ('read the file with grain size field');
    f1 := 'dd50.txt';
    writeln ('read the file with waterdepth field');
    f2 := 'wdepth.txt';
    writeln ('write output file name for new d50');
    f3 := 'bottom.rgh';
    writeln ('read input file name for new bottom');
    f4 := 'bottom.txt';
    writeln ('write output file for new bottom');
    f5 := 'olesgoot.dep';
    assign(inf1, f1);
    reset(inf1);
    assign(inf2, f2);
    reset(inf2);
    assign(ouf1, f3);
    rewrite(ouf1);
    assign(inf3, f4);
    reset(inf3);
    assign(ouf2, f5);
    rewrite(ouf2);
    {readln(inf1);} 
    {readln(inf1);}
```

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{ reading the grain size field starts here }

readln(inf1);
readln(inf1);
readln(inf1);
readln(inf1);
for i := 1 to p do 
  begin
    for j := 1 to q do 
    begin
      read(inf1, d50[i,j]);
      {if i=17 then readln(inf1);}
    end;
    writeln(oufl);
  close(inf1);

{ reading grain size field ends here }

{ reading the waterdepth field starts here }

readln(inf2);
readln(inf2);
readln(inf2);
readln(inf2);
for i := 1 to p do 
  begin
    for j := 1 to q do 
    begin
      read(inf2, hj[i,j]);
      {if i=q then readln(inf2);}
    end;
    writeln(oufl);
  close(inf2);

{ reading the waterdepth field ends here }

{ reading the bottom field starts here }

readln(inf3);
readln(inf3);
readln(inf3);
readln(inf3);
for i := 1 to p do 
  begin
    for j := 1 to q do 
    begin
      read(inf3, bottom[i,j]);
      {if i=q then readln(inf3);}
    end;
    writeln(oufl);
  close(inf3);
{ reading the bottom field ends here }

{ calculating new C(i,j) starts here }
for i:=1 to p do
  begin
    for j:=1 to q do
      begin
        lambda[i,j]:=abs(6.5 * h[i,j])+0.005;
        t[i,j]:=abs(h[i,j])+0.001;
        delta[i,j]:=abs(2.5*exp(0.7*ln(abs(h[i,j]+0.001)))*exp(0.3*ln(abs(d50[i,j])))))+0.001;
        k[i,j]:=abs(3*(d50[i,j]+0.001)+1.1*0.7*delta[i,j]*
                    (1-exp(-25*delta[i,j]*(abs(6.5*h[i,j]+0.005)))));
        C[i,j]:=18*ln(12*abs(h[i,j]+0.001))/(abs(3*(d50[i,j]+0.00123)+1.1*0.7*
                    delta[i,j]*(1-exp(-25*delta[i,j]*(abs(6.5*h[i,j]+0.00123))))))/ln(10);
        end;
      end;
  end;
{ calculatoin ends here }

{write new C(i,j) matrix begins here}
for i:=1 to p do
  begin
    for j:=1 to q do
      begin
        write(out1, C[i,j]:12:4);
      end;
  end;
for i:=1 to p do
  begin
    for j:=1 to q do
      begin
        write(out1, C[i,j]:12:4);
      end;
    writeln(out1);
  end;
close(out1);

{write new bottom matrix begins here}
for i:=1 to p do
  begin
    for j:=1 to q do
      begin
        write(out2, bottom[i,j]:12:4);
      end;
    writeln(out2);
  end;
close(out2);
end.