DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

Report LR-272

FLUTTERANALYSIS OF A SMALL WINDTURBINE,
DESIGNED FOR MANUFACTURE AND USE IN
DEVELOPING COUNTRIES

by

P. C. Hensing

DELFT - THE NETHERLANDS

August 1978
Errata Report LR-272

Summary: 2nd line:
"de developing" should be "the developing"

page 4: 6th line:
"jhK₁" should be "jhK₁q"

equation (3):
"K" should be "K₂"

2nd line after equation (3):
"whereas the complex matrix Q is a function of the reduced frequency" should be "The vector Q can be written as:
Q = \sqrt{Q}q* where the complex matrix Q* is a function of
the reduced frequency"

page 5: 4th line, equation (9) and equation (10):
"Q" should be "Q*

page 6: second paragraph, 2nd line:
"is due" should be "is due to"

page 7: 5th line from below:
"whereas the torsional stiffness increases terms with a
factor r^3": "terms" should be omitted

page 10: 4th line from below:
"indepdendant on" should be "independent of"

page 12: ref. 3:
add: "J. Aircraft, November 1971, pp. 885-889"

figure 8: m = .05 kg in both cases.
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SUMMARY

The purpose of this report is the investigation of the flutter behaviour of a windturbine rotor designed for manufacture and use in developing countries. Possible improvements are discussed. The effect of scaling is considered. The addition of small tip masses turns out to have a curative influence on flutter sensitive rotordesigns.
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1. **INTRODUCTION**

In 1975 a national windenergy program has been started, coordinated by the Steering Committee Windenergy for Developing Countries, aiming at the investigation of the possibilities of windenergy utilisation in developing countries. Among others the universities of technology in Twente and Eindhoven contribute to this program by designing complete wind turbine installations, which can be manufactured with methods and materials usually applied in those countries. The stiffness of such designs is usually not extremely high, resulting in the occurrence of vibrations, which may become unstable by action of the aerodynamic forces. The occurrence of violent vibrations during windtunnel tests of a small windturbine developed by the Eindhoven University of Technology [4] has lead to the investigation of the aero-elastic behaviour of the concerning rotor.
2. THE DYNAMIC SYSTEM

The analyzed rotor has a diameter of 1.8 meter and consists of two aluminium blades clamped into an aluminium hub (see fig. 1). The blades are cambered plates which have relatively good aerodynamic properties. They can be simply made either by sawing them out of an aluminium tube with 300 mm diameter or sawing them out of aluminium sheet and rolling them afterwards. The blade twist is determined by the blade planform. The structure is rather simple, but the stiffness, especially the stiffness against torsion of the blades is poor.

For the computation of natural frequencies and damping factors a lumped parameter method is used as described in ref. 1. For that purpose a rotor-blade is divided into 5 elements. Figure 2 shows this schematization. Each element has three degrees of freedom, viz. displacements perpendicular to the blade elastic axis in and out of the plane of rotation and a rotation about this axis (see fig. 3a). Due to the lagwise bending the rotational speed of the rotor is not constant but slightly varies. This alteration is called the "rigid body"-motion of the rotor and completes the assumed degrees of freedom (see fig. 3b). The hub structure is assumed to be infinitely stiff and so is the supporting structure.

The elastic axis and the mass axis do not exactly coincide, as can be observed in fig. 2. The appearing static unbalance terms $S_{xy}$ and $S_{xz}$ are taken into account. These and other inertia quantities are given in table 1. The directions of the rotating reference frame $xyz$ differ from the directions of the principal axes of a blade section. So forces in $z$-direction cause displacements $v$ in $y$-direction, whereas forces in $y$-direction cause displacements $w$ in $z$-direction. This means that in addition to the "uncoupled" influence numbers $\epsilon_{ij}$, $\epsilon_{ij}$ and $\epsilon_{ij}$ also the "coupled" influence numbers $\epsilon_{w_{ij}}$ are taken into account (see tables 2 - 5).
3. THE EQUATIONS OF MOTION

The equations of motion are derived on the basis of Lagrange's equations. For this purpose the kinetic and potential energy are derived with respect to an inertial system of axes. A detailed description of the derivation of the equations of motion is given in ref. 1 for the case of a two-bladed rotor with tipvanes. The same computational method is used for this investigation, where the mass and moments of inertia of the tipvanes are set to zero. The equations of motion derived in ref. 1 are:

\[ M \ddot{\mathbf{q}} + 2 \Omega G \dot{\mathbf{q}} + \{K_1 - \Omega^2 (C - K_2)\} q = \mathbf{Q} \]  \hspace{1cm} (1)

where:

- \( M \) = structural mass matrix
- \( \Omega \) = rotor rotational frequency
- \( G \) = gyroscopic of Coriolismatrix \( \approx 0 \) for the case without tipvanes
- \( K_1 \) = structural stiffness matrix
- \( K_2 \) = geometric stiffness matrix
- \( C \) = centrifugal matrix
- \( \mathbf{Q} \) = aerodynamic force matrix
- \( q \) = displacement vector

The structural mass- and stiffness matrices are the matrices which would be also involved in the equations of motion of a non-rotating blade. The centrifugal matrix consists of elements representing the change of the centrifugal forces, caused by the displacements of the blade elements. The geometric stiffness matrix accounts for the extra spring effects caused by the centrifugal forces on a bended and twisted rotorblade.
The aerodynamic forces are derived from the unsteady strip theory as developed by Theodorsen.\textsuperscript{[2]} The flow around a strip is assumed to be a parallel flow, so the curvature of the streamlines is neglected. These aerodynamic forces are only valid for harmonic motions of the strips with respect to the flow. In order to effect pure sinusoidal motions a fictitious damping term $\phi$ $K_1$ is added. For the resultant harmonic solution can be written:

$$q = \bar{q} e^{j\omega t} \quad (2)$$

Substituting this in eq. (1) and differentiating yields:

$$\{ - \nu^2 M + K_1 (1 + j \phi) + \Omega^2 (K - C) \} q = Q \quad (3)$$

In this equation the matrices $M$, $K_1$, $K_2$ and $C$ contain constant elements, whereas the complex matrix $Q$ is a function of the reduced frequency

$$\omega = \frac{\nu \ell}{V_{res}} \quad (4)$$

where $V_{res}$ is the resultant airspeed at a blade section

$$V_{res} = \sqrt{\nu^2 + \Omega^2 \ell^2} \quad (5)$$

and $\ell$ is the half cord length of the concerning section.
4. THE COMPUTATIONAL METHOD

The form of eq. (3) invites to a solution with the so-called "k-method"[3]. Application of this method holds the solution of eq. (3) for a number of values of the reduced frequency \( \omega \). For each value of \( \omega \) the elements of the matrix \( Q \) can be determined. The rotor rotational frequency \( \Omega \) is after choice of a value of \( \omega \), not an independant variable any more. A value of \( \omega \) corresponds to a value of the dimensionless rotational frequency

\[
k = \frac{\Omega}{\nu}
\]  

(6)

for:

\[
\omega = \frac{\nu L}{v_{\text{res}}} = \frac{\nu}{\Omega} \frac{L}{R} \frac{\lambda}{\sqrt{1 + \lambda^2}} = \frac{1}{k} \frac{L}{R} \frac{\lambda}{\sqrt{1 + \lambda^2}}
\]  

(7)

where

\[
\lambda = \text{tipspeed ratio} = \frac{\Omega R}{\nu}
\]  

(8)

The eq. (3) can be rewritten in the form:

\[
K_1^{-1} \{ M + Q + k^2 (C - K_2) \} \quad q = \frac{1 + \frac{j}{h}}{v^2} q
\]  

(9)

Or with the use of the structural flexibility matrix \( E \) in stead of the structural stiffness matrix \( K_1 \):

\[
E \{ M + Q + k^2 (C - K_2) \} \quad q = \frac{1 + \frac{j}{h}}{v^2} q
\]  

(10)

For each value of \( \omega \) the form between \{ \} -brackets contains constant elements. This equation can easily be recognized as a complex eigenvalue problem, which is solved with the well-known LR-algorithm[5].
5. **DISCUSSION OF THE RESULTS**

5.1. **Comparison of theory and observation**

In fig. 4 the course of the natural frequencies and damping with the rotational frequency is given for the reference rotor, described in section 2. The first frequency branch represents mainly the fundamental flapwise bending mode (i.e. perpendicular to the plane of rotation), whereas the vibration mode of the second branch mainly consists of the fundamental torsional mode of the rotor blade. Because the other branches hardly affect the critical speed they are left out.

The increase of the bending frequency with increasing rotational frequency is due to the stiffening effect of the centrifugal forces, taken into account by the geometric stiffness matrix \( K_2 \). The influence of the geometric stiffness can be observed in fig. 6.

The decrease of the torsional frequency is due to the aerodynamic forces as can be seen from a comparison of the reference case with and without aerodynamic forces (see fig. 5). Because the natural frequencies of the first and second branch come close together the vibration modes interact heavily. The possibility exists now that the aerodynamic forces caused by one mode, put energy in the other mode. According to the calculations this happens indeed at a rotational frequency of 90 rad/s. This results in an unstable vibration called flutter.

During windtunnel tests of this rotor violent vibrations were observed at a rotational frequency of 82 rad/s. No explanation was found for the difference between the calculated and the observed value. A part of it may be caused by differences between the calculated and present stiffness; an other part may be caused by the simplification that the supporting structure is infinitely stiff, which is actually not true.
The flutter calculations were carried out for a tipspeed ratio \( \lambda = 10 \) and \( \lambda = 5 \). The differences were very small as might be expected. The outer part of the rotor blade is the main part and at this part the resultant airspeed at a blade section is determined to a large extent by the velocity vector \( \Omega r \), so that the influence of \( V \) is not important.

5.2. Possibilities for improvement of the flutter properties

Because the minimum admissible flutter rotational frequency is about 160 rad/s attempts were made to improve the flutter properties. The following possibilities are considered:

- application of steel blades in stead of aluminium ones,
- increase of blade thickness,
- application of blade support,
- addition of small masses at the bladetips.

Other possibilities will rather directly affect the simplicity of the design.

Application of steel blades will increase the stiffness with a factor 3. However, also the inertia, centrifugal and geometric stiffness terms will increase with a factor 3. So the mechanical system has the same natural frequencies. The aerodynamic forces have the same magnitude, which means that relatively to the other terms the aerodynamic forces become smaller. This leads to the higher flutter rotational frequency of 127 rad/s, as can be seen in fig. 7.

Increase of blade thickness with a factor \( f \) will also increase the bending stiffness with a factor \( f \), whereas the torsional stiffness increases terms with a factor \( f^3 \). The inertia, centrifugal and geometric stiffness terms increase with a factor \( f \), while the aerodynamic terms are not affected. The result will be a higher flutterspeed. The question is if increase of the blade thickness
is a very realistic proposition, taking into consideration that a blade thickness of 3 mm is not sufficient for a 1.8 meter diameter rotor, so that scaling up to larger diameters will result in extreme blade thicknesses.

An other possibility to improve the flutter characteristics of the rotor is the application of a blade support, so that the blade structure is not of the cantilever kind any more.

Finally the possibility of adding small tipmasses will be discussed. The following considerations underlie this concept.

The mass centre of a blade can simply be shifted ahead when adding extra mass. From aircraft wing flutter theory it is known that the chordwise location of the mass centre of an aircraft wing has an important influence on the flutter behaviour. Shifting forward the mass centre the flutter speed (i.e. the speed above which unstable vibrations occur) will increase.

The same tendency proves to appear when the mass centre of a rotor-blade is shifted ahead by changing the chordwise situation of an additional tipmass (see fig.8).

The influence of the static unbalance of a tipmass with respect to the blade elastic axis mainly determines the favourable effect of the addition of tipmasses.

The influence of the mass itself, the static unbalance vanishing, is threefold. First both the bending frequency and the torsional frequency decrease, which has a lowering influence on the flutter speed. Secondly the geometric stiffness rises, with which the slope of the bending frequency branch in the $\nu$-$\Omega$ diagram is steeper and the interaction with the torsional branch takes place at a lower rotational frequency. This influence can be seen in fig. 6,
where results are shown of calculations with and without geometric stiffness. Finally because the added mass is concentrated at the bladetips the vibrational modes change in a sense that the displacements of the bladetips diminish relatively. Because the vibrational motions of the bladetip play the main part in the flutter behaviour, the described change of the vibrational modes has a stabilizing effect. Evidently this effect prevails the mentioned two unfavourable influences. So the influence of the mass itself without static unbalance is favourable, however, far less pronounced than the influence of its static unbalance. This can be observed by comparing the $S_{xz}=0$ characteristics of figure 8 ($\Omega_{\text{crit}} = 97 \text{ rad/s}$) and the characteristics of the reference case in fig. 4 ($\Omega_{\text{crit}} = 90 \text{ rad/s}$).

In fig. 9 the results are shown of calculations of the rotor with 25 gram and 50 gram tipmasses. The location of the masses is in both cases the same, viz. .05 meter in front of the elastic axis. As can be observed, the addition of 50 gram tipmasses stabilizes the rotor in the area of possibly occurring rotational frequencies up to 148 rad/s. At that rotational frequency an other unstable aeroelastic phenomenon takes place, viz. divergence. Divergence is a static aeroelastic instability (so $\nu = 0$), appearing when the increase of the aerodynamic moment at a blade section, caused by a change of the angle of attack of this blade section, is bigger than the increase of the elastic restoring moment. This results in extreme torsional and bending deformations.

Divergence is a well-known phenomenon in aircraft engineering, occurring at airfoils, specially when the airfoil structure is lacking in torsional stiffness.

From fig. 9 it is obvious that the addition of small tipmasses makes the rotor flutterfree, but this intervention does not affect the divergence rotational frequency. From flutter-point-of-view the blade thickness can be reduced when applying tipmasses (the thickness of 3 mm was chosen on behalf of the vibrational (mis)-behaviour), but divergence will then appear at lower
rotational frequencies because of the further reduction of the torsional stiffness. This can be observed in fig. 10, where results are shown of calculations of a rotor with 2 mm thick blades. With the help of tipmasses the rotor can be made flutterfree. However, divergence occurs at a rotational frequency $\Omega_{\text{div}} = 82$ rad/s.

5.3. Larger scale applications

An interesting question is what happens to the flutterspeed when all dimensions of the rotor are enlarged with a factor $f$. In eq. (10) all terms between the $\{\}$ - brackets will increase for a certain value of the reduced frequency $\hat{\omega}$, (which means also a certain value of $k$) with a factor $f^3$. The elements of the flexibility matrix $E$ decrease with a factor $f$. Consequently the eigenvalues are a factor $f^2$ higher, so the calculated natural frequencies at a certain $\omega$ are a factor $f$ smaller, whereas the damping factors remain unchanged.

Solutions were obtained at a certain value of $\omega = \frac{u_k}{V}$, so when the natural frequencies decrease with a factor $f$, the velocities do not change. The resultant flutterspeed is not affected by the scaling factor $f$, also the critical tip rotational speed $\Omega R$ does not change. This means that flutter occurs at a value of $\Omega$, which is a factor $f$ smaller.

The operational limit of $\Omega$-values goes down with a factor $f$, because the design tipspeed ratio $\lambda = \frac{\Omega R}{V}$ is independant on the rotorsize.

The conclusion is that the enlargement of the rotordimensions does not affect the "flutterpain".
6. CONCLUSIONS

1. The application of materials with high mass density for the manufacture of rotorblades, the \( \rho/E \) ratio remaining unchanged, has a favourable influence on the flutterspeed.

2. The addition of tipmasses situated on the blade elastic axis or in front of it, has a favourable influence on the flutterspeed.

3. The longer the distance about which these masses are situated in front of the blade elastic axis, the better the improvement of the flutterbehaviour.

4. When all dimensions of a rotor, with blades of the massive cantilever kind, are enlarged with a factor \( f \), the operational rotational frequencies as well as the flutter rotational frequencies are reduced with a factor \( f \).
7. REFERENCES


Table 1: Inertia properties of a rotor blade.

<table>
<thead>
<tr>
<th>element number</th>
<th>s [mm]$^1$</th>
<th>m [kg]</th>
<th>$\alpha$</th>
<th>$S_{xy}$ [kgm]</th>
<th>$S_{xz}$ [kgm]</th>
<th>$I_{x}$ [kgm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>145.4</td>
<td>0.294</td>
<td>11.9$^\circ$</td>
<td>$3.42 \times 10^{-3}$</td>
<td>$-0.717 \times 10^{-3}$</td>
<td>$5.440 \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>132.2</td>
<td>0.268</td>
<td>9.5$^\circ$</td>
<td>$2.56 \times 10^{-3}$</td>
<td>$-0.433 \times 10^{-3}$</td>
<td>$4.056 \times 10^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>132.2</td>
<td>0.100</td>
<td>7.8$^\circ$</td>
<td>$0.840 \times 10^{-3}$</td>
<td>$-0.115 \times 10^{-3}$</td>
<td>$1.306 \times 10^{-4}$</td>
</tr>
<tr>
<td>4</td>
<td>117.9</td>
<td>0.095</td>
<td>6.7$^\circ$</td>
<td>$0.731 \times 10^{-3}$</td>
<td>$-0.086 \times 10^{-3}$</td>
<td>$1.142 \times 10^{-4}$</td>
</tr>
<tr>
<td>5</td>
<td>112.7</td>
<td>0.091</td>
<td>5.7$^\circ$</td>
<td>$0.643 \times 10^{-3}$</td>
<td>$-0.064 \times 10^{-3}$</td>
<td>$0.994 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

$^1$s defined in fig. 2.

Table 2: Influence numbers $e_{vij}$ representing the bending displacement of blade element $i$ in the direction of the y-axis, caused by a unit force in the direction of the y-axis applied on blade element $j$.

\[
\begin{array}{cccccc}
0.013 & 0.087 & 0.140 & 0.173 & 0.200 \\
0.880 & 1.547 & 1.913 & 2.287 \\
*10^{-6} \text{ m/N}
\end{array}
\]
Table 3: Influence numbers $e_{w_{ij}}$ representing the bending displacement of blade element $i$ in the direction of the $z$-axis, caused by a unit force in the direction of the $z$-axis applied on blade element $j$.

<table>
<thead>
<tr>
<th></th>
<th>.240</th>
<th>1.387</th>
<th>2.187</th>
<th>2.647</th>
<th>3.107</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14.57</td>
<td>25.86</td>
<td>32.11</td>
<td>38.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>46.07</td>
<td>60.59</td>
<td>75.13</td>
<td></td>
<td>* $10^{-6}$ m/N</td>
</tr>
<tr>
<td></td>
<td>81.33</td>
<td>103.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>133.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Influence numbers $e_{y_{v_{ij}}}$ representing the bending displacement of blade element $i$ in the direction of the $y$-axis, caused by a unit force in the direction of the $z$-axis applied on blade element $j$.

<table>
<thead>
<tr>
<th></th>
<th>-.007</th>
<th>-.307</th>
<th>-.487</th>
<th>-.593</th>
<th>-.693</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.07</td>
<td>-5.433</td>
<td>-6.727</td>
<td>-8.053</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-9.153</td>
<td>-11.90</td>
<td>-14.64</td>
<td></td>
<td>* $10^{-6}$ m/N</td>
</tr>
<tr>
<td></td>
<td>-15.71</td>
<td>-19.67</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-24.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Influence numbers $e^{ij}_{t}$ representing the torsion angle of blade element $i$ about the $x$-axis, caused by a unit torque applied at blade element $j$.

\begin{tabular}{|c|c|c|c|c|}
\hline
.333 & .333 & .333 & .333 & .333 \\
\hline
1.049 & 1.049 & 1.049 & 1.049 & \\
\hline
1.594 & 1.594 & 1.594 & \\
\hline
1.923 & 1.923 & \\
\hline
2.268 & \\
\hline
\end{tabular}

$\ast 10^{-2}$ rad/Nm
Figure 2: Schematization of the rotor
Figure 3a: Degrees of freedom of a blade element

Figure 3b: Rigid body motion of the rotor
Figure 4: Frequency and damping characteristics of the analyzed rotor
(l = 5, aluminium blades, thickness 3 mm)
Figure 5: Frequency characteristics of the reference case

- without aerodynamic forces
- with aerodynamic forces
Figure 6: Frequency and damping characteristics of the rotor with and without geometric stiffness terms.

- Without geometric stiffness terms, $K_2 = 0$
- With geometric stiffness
Figure 7: Frequency and damping characteristics of a rotor with steel blades.
Figure 8: Frequency and damping characteristics of the reference rotor (λ=5, aluminium blades, thickness = 3 mm) with lip masses; influence of the static unbalance

- $m=0.05$ kg $S_{x_{2}}=0.0$ kgm $I_{x_{x}}=0$ kg m$^2$
- $m=0.05$ kg $S_{x_{2}}=0.025$ kgm $I_{x_{x}}=0.00125$ kg m$^2$
Figure 9: Frequency and damping characteristics of the rotor with and without tip masses
$L = 9$, aluminium blades, thickness = 3 mm

- without tip masses
- with tip masses $m = .025$ kg $S_{xz} = .00125$ kg m
  $I_x = .0000625$ kg m$^2$
- with tip masses $m = .050$ kg $S_{xz} = .0025$ kg m
  $I_x = .000125$ kg m$^2$
Figure 10: Frequency and damping characteristics of the rotor with 2 mm blades (2×5, aluminium blades)

- without tipmasses
- with tipmasses m = 0.5 kg, \( S_{2Z} = 0.025 \text{kgm}, J_{zf} = 0.00125 \text{kg m}^2 \)