DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

Report LR-254

FOUR LECTURES ON FATIGUE CRACK GROWTH

by

J. Schijve

DELFT - THE NETHERLANDS

October 1977

DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

Report LR-254

FOUR LECTURES ON FATIGUE CRACK GROWTH

by

J. Schijve

DELFT - THE NETHERLANDS

October 1977

Summary

During the course year 1976/1977 the author presented a series of eight lectures on fatigue crack growth at the Department of Aerospace Engineering of the Delft University of Technology. Four of these lectures in a slightly modified version were presented in August 1977 as part of a Seminar on Fatigue, Fundamental and Applied Aspects, organized by the Linköping Institute of Technology (Professor T. Ericsson). These lectures are reproduced in this document. Titles and summaries are given below.

I. Fatigue crack growth and fracture mechanics

Summary: Aspects of the technical meaning of fatigue considerations in practice are indicated. The fatigue life is subdivided into a crack nucleation period and a crack propagation period. The significance of recognizing these periods for practical problems is illustrated by several examples. The similarity approach for correlating fatigue data is introduced. The meaning of the stress intensity factor for fatigue crack growth is discussed.

II. Fatigue cracks, plasticity effects and crack closure

Summary: Concepts introduced are residual plastic deformation, residual stress, reversed plastic deformation, plastic deformation in the wake of the crack and crack closure under tensile load. COD measurements as a method to determine the crack closure level are discussed. The significance of crack closure for fatigue crack growth is analysed and illustrated by several examples, including effects of yield stress, stress ratio and delayed crack growth after a peak load. Finally some attention is paid to three dimensional aspect following from thickness effects, shear lips and curved crack fronts.

III. Fatigue crack propagation, prediction and correlation

Summary: Two prediction techniques are introduced, (1) cycle-by-cycle prediction and (2) prediction by correlation. Attention is paid to the problem of describing variable-amplitude loading in terms of load cycles.

Aspects of fatigue damage are reviewed with reference to interaction effects and weaknesses in cycle-by-cycle prediction methods. The discussion on prediction by correlation is restricted to constant-amplitude loading. The validity of the similarity concept based on K-factors is reconsidered. Application of simple specimen data to complex structures is shown. Finally a variety of crack growth equations is reviewed, including aspects of curve fitting, a comparison between formulas of Walker and Elber and asymptotic values in the $da/dn-\Delta K$ relation.

IV. Fatigue crack growth under variable-amplitude loading

Summary: Stationary and non-stationary types of variable-amplitude loading are specified. Some attention is paid to the description of stationary random load. The stress intensity factor is then applied to crack growth under stationary variable-amplitude loading by defining first characteristic stresses and characteristic stress intensity factors. This is done for random loading, non-random loading and flight-simulation loading. It is discussed how and why this concept may break down if the stationarity is lost. Attention is paid to truncation of high loads in a flight-simulation test and the analogous problem of the crest factor under random loading. The significance of crack closure for understanding crack growth under variable-amplitude loading is emphasized.

Contents

	<u>page</u>
I Fatigue crack growth and fracture mechanics	1
1. Introduction	1
2. Fatigue crack initiation	4
3. Fatigue crack growth	10
4. The stress intensity factor K	12
5. K and cyclic loading	16
6. References	17
II Fatigue cracks, plasticity effects and crack closure	20
1. Introduction	20
2. Residual stress after local plastic deformation	20
3. Crack closure	24
4. How to measure crack closure	25
5. Crack closure and fatigue crack growth	28
6. Some consequences of crack closure	32
7. Crack closure and the effect of thickness	35
8. Crack closure and shear lips	38
9. Crack closure and crack front curvature	39
10. References	40
III Fatigue crack propagation, prediction and correlation	42
1. Introduction	42
2. Cycle-by-cycle prediction for variable-amplitude loading	42
3. Crack growth prediction by correlation	47
4. Crack growth equations for constant-amplitude loading	52
5. References	57
IV Fatigue crack growth under variable-amplitude loading	60
1. Introduction	60
2. Different types of variable-amplitude loading	60
3. How to describe random load	62

4. K-concept and random load	65
5. K-concept and stationary non-random load	66
6. Crack growth under flight-simulation loading	68
7. Truncation of high loads	72
8. Crack closure and non-stationary variable-amplitude loading	76
9. References	82

I. FATIGUE CRACK GROWTH AND FRACTURE MECHANICS

Summary

Aspects of the technical meaning of fatigue considerations in practice are indicated. The fatigue life is subdivided into a crack nucleation period and a crack propagation period. The significance of recognizing these periods for practical problems is illustrated by several examples. The similarity approach for correlating fatigue data is introduced. The meaning of the stress intensity factor for fatigue crack growth is discussed.

1. Introduction

As an introduction we will consider a kind of an overall picture about the relevance of fatigue. Fatigue is not an important problem because it is highly interesting to see a tip of a fatigue crack growing under a microscope at high magnification. That is why fatigue is an interesting phenomenon, but the essential arguments why we have to consider fatigue as a relevant problem are based on:

- economy (\$),
- safety (men)

Who should bother about fatigue? Probably the designer of a structure, but that answer is too specific and certainly not complete. At least two groups of people should be recognized, which are to be found in:

- the industry, the manufacturer of the structure,
- the operator, people using the structure in service.

Note that the first group involves more than the designer alone.

For instance the production shop is adding to the fatigue quality of a structure.

Now let us consider for a while why those two groups will have a different approach to the fatigue problem. The manufacturer will try to produce a structure, that will be free from fatigue as far as possible. In other words he will design and produce as to obtain a high fatigue resistance. Aspects of prime importance are:

- cyclic stresses,
- material properties,
- surface quality, etc.

The operator on the other hand should consider reasons why fatigue still might occur, the consequences it could have and how to prevent it. There are several reasons why fatigue still may occur, such as:

- a more severe utilization of the structure,
- fatigue damaging aspects overlooked by the designer,
- poor production standards,
- incidental damage,
- aggressive chemical environments, etc.

As a result the operator's fatigue problems are more associated with maintenance, inspections and non-destructive testing techniques. In terms of fatigue some questions of the operator are: at which locations will cracks start and how fast will they be growing?

At this point it should be said that the appreciation of the fatigue problem will be different depending of the type of structure to be considered. Let us consider three typical cases:

1. Motor car engine: Cracks should not occur, "infinite" life is required, crack growth is not of interest. Design and production against crack nucleation is of prime importance.
2. Nuclear pressure vessel: Initial flaws and defects in a welded steel structure have to be expected. Consequently crack nucleation is of little interest, but (very slow) crack growth should be considered.
3. Aircraft structure: A finite life has to be accepted. Hence both crack nucleation and propagation are significant.

In terms of crack nucleation and crack growth different situations are illustrated in Fig. 1, which shows schematic crack growth curves for both finite and infinite life. In the absence of initial defects some 90 percent of the fatigue life is spent in the micro-range. If we then ignore the remaining 10 percent the error made is small. In other words in such cases the fatigue damaging process is largely occurring in a very small

volume of the material and it will be highly depending on local conditions. This is significant for prediction problems. In other cases, where macro-crack growth has to be considered bulk properties of the material will also be involved.

Let us now limit the scope of the discussion by considering the prediction of fatigue properties of a technical structure. Predictions in the sense of Applied Mechanics imply prediction of the fatigue properties, usually starting from data of a more general and a more simple nature. Classical examples are:

- prediction of fatigue properties of a notched element from available data for the unnotched material (notch effect, size effect, stress distribution, K_t , stress gradient).
- prediction of fatigue life under complex load-time histories from

available S-N data (cumulative fatigue damage problem).

- prediction of fatigue behavior under biaxial loading from the behavior under uni-axial loading (fracture or yield criteria).

Solutions for these problems have the character of extrapolating available data to other conditions, assuming that some correlation exists between fatigue in the two cases, (1) one case for which data are known and (2) the second one for which predictions have to be made. Now a correlation between the two cases seems to be justified only if the same fatigue mechanism can and will occur in both cases. This question can be answered only if we know what is going on during fatigue of metals and which factors are having an influence on this phenomenon. In other words: application of fracture mechanics (extrapolation) requires some knowledge of the fracture mechanism (physical understanding). Note the almost similar writing, but the highly different meaning.

This lecture starts with some observations on fatigue of metals including crack nucleation and crack growth. At a later stage the stress intensity factor is defined in order to describe the "stress system in a crack tip area", with some comments on its significance.

2. Fatigue crack initiation

The fatigue life can be subdivided in some periods as shown in the figure below.

Fig. 2

The beginning and the end of each period is not easily defined, except for the last one. Final failure occurs in the very last cycle of the life and

usually this part of the failure is supposed to be quasi-static rather than fatigue.

Microscopical studies have shown that crack nucleation starts early in the fatigue life. We may ask when incipient cracking should be considered to be a microcrack, however, this question will be ignored since an answer is not of great interest for practical problems. The simplified picture then becomes:

$$\text{fatigue life} = (\text{nucleation period}) + (\text{crack growth period})$$

An obvious question now is: when does a microcrack become a macrocrack? The classical definition of a macrocrack is that the crack is large enough to be seen by the naked eye. This is not a very exact definition. An alternative definition is to consider a crack to be a macrocrack if it has sufficient depth (or length) to be sure that local conditions, responsible for crack nucleation do no longer affect crack growth. In other words crack growth will then depend on bulk properties. Physically this definition appears to be more reasonable, although not yet very precise. A third definition with some appeal to the present course might be: A crack is a macrocrack as soon as Fracture Mechanics are applicable. Note that this definition is depending on the definition of Fracture Mechanics. Perhaps the definition should be rephrased as: A crack is a macrocrack as soon as the stress intensity factor K has a real meaning for describing its growth.

Some observations will now be reviewed to illustrate the usefulness of considering crack nucleation and crack propagation as different phases. Fatigue cracks generally start at the surface of the material (macroscopic observation). A number of reasons may contribute:

- high stress level (K_t always > 1),
- surface roughness (inhomogeneous stress distribution on a small scale),
- environmental effects,
- lower restraint on plasticity.

These aspects will promote crack nucleation at the surface. Material structure at the surface (decarburizing) and residual stresses may also contribute, but they are not necessarily unfavourable for fatigue (e.g. shot peening).

The number of fatigue cracks occurring in one specimen has been observed in some laboratory studies.

They all show the same trend: The number is larger at a higher stress level (Fig. 3). However, at stress levels near the fatigue limit S_f

FIG. 3

it may well happen that only one crack has been nucleated. Apparently there is a typically weakest link in a material which may have as many as 1000 grains in 1 mm^2 . The weak link will be a compilation of unfavourable features, which will not be

discussed here. Anyhow, it is a highly local phenomenon in the nucleation period, while at higher stress levels several weak links are ready to produce a crack. Even then the fatigue phenomenon is still a local process for a long time, i.e. micro cracks are present in a rather small volume of the material only. It is highly worthwhile to have this observation in mind if the similarity between fatigue in different components or specimens of the same material is analysed. It has an essential meaning for the validity and the limitations of similarity concepts mentioned before.

The significance of surface roughness is shown in Fig. 4. Apparently the quality of the surface finish has a large effect on the nucleation period, whereas the effect on the crack growth period was negligible. Macro crack growth is depending on bulk properties of the material, since it is no longer a localized phenomenon.

Fig. 4. Effect of surface finish on the pre-crack life and the crack propagation life of unnotched rotating beam specimens of 0.2 % C steel (SAE 1020) (results of DeForest).

Another consequence is related to scatter. Crack nucleation being a highly localized phenomenon can be subject to considerable scatter, depending on local conditions. However, crack propagation, being dependent on bulk properties only, usually shows much less scatter.

Fig.5: Scatter bands for a strong Al-alloy

Figure 5 shows scatter bands of S-N data for an aluminium alloy (unnotched specimen). The bare material is highly depending on surface quality, especially at low stress levels, and scatter is large. At high stress levels more cracks will be nucleated which implies less scatter. In the same alloy with soft cladding layers of pure aluminium (corrosion protection) crack nucleation in the cladding is very easy at any stress level. This implies a well reproducible surface with a low fatigue resistance. Scatter is much lower then.

It is a good question whether crack nucleation is always followed by crack propagation. Under certain conditions it does not and then non-propagating cracks are present. Such cracks are found under compressive mean stresses and this explains the asymmetric shape of fatigue diagrams. In area A (Fig. 6) microcracks are initiated early in the life, but these cracks do not grow, due to compressive stresses. In area B microcracks are just not initiated, while above the S_f -line cracks are initiated, which then will grow further.

Fig. 6

Non-propagating cracks have also been observed at positive mean stresses, especially for specimens with sharp notches. Frost (Ref. 2) observed the trend as shown in figure 7. Apparently for moderate K_t -values the fatigue limit is defined as the minimum stress for crack initiation. However, for high K_t -values the fatigue limit is the minimum stress for crack growth. Apparently there are two essentially different

Fig. 7

definitions of the fatigue limit.

It still requires an explanation why a microcrack, once being initiated, should not grow any further. For a homogeneous material the more obvious argument is the increased restraint on (cyclic) plasticity going from A (free surface) to B (triaxial state of stress).

Fig. 8

Difficulties in the growth of a small microcrack may also affect the shape of the S-N curve. Two typical shapes are shown below.

Fig. 9

In the first case (sharpknee) nucleation of a crack is the problem to obtain a finite life. Crack growth will follow anyhow. In the second case crack initiation at a very low stress amplitude is possible, for instance from an inclusion (in high strength steel), or by fretting corrosion, or from a cladding layer. However, in view of the low stress level crack growth initially occurs extremely slowly and complete failure may require $10^7 - 10^8$ cycles.

3. Fatigue crack growth

For technical materials fatigue crack propagation has been shown to be crack extension in every load cycle (striations, fractographic observations by the electron microscope). Since crack extension implies decohesion in the material a fracture mechanism is operating. Details of the mechanism on an atomic level actually are unknown, but on a larger scale several observations have been made, e.g.:

Microscopic level:

- Crack propagation in several metals follows a transcrystalline path.
- Crack propagation occurs along slipplanes in some materials only (Forsyth, Stage I), while in general it will not do so (Stage II). Slip on more than one crystallographic plane is operative then (in the latter case Stage I may still occur in the nucleation period and also later on at the free surface, due to low restraint on slip deformations and crack path).

Macroscopic level:

- Cracks usually grow in a macroscopic plane perpendicular to the main principal stress, at least as long as the crack rate is low (tensile mode fracture). At faster crack rates the growth direction remains perpendicular to the maximum principal stress, but the plane of the fatigue fracture (for several materials) will be under an angle of 45° with that stress (shear mode fracture). In sheet materials this leads to shear lips as shown in figure 10.
- Crack propagation is strongly affected by the environment, the crack rate being slower in inert environments and faster in aggressive environments. Humid air for several materials should be considered to be aggressive

as compared to inert environments. Obviously salt water is still more aggressive. The environment may also affect the fracture mode. For Al-alloys an aggressive environment is postponing the transition from the tensile mode to the shear mode.

Fig. 10

The crack growth rate, usually denoted as da/dn (= slope of crack growth curve), should be considered to be the crack extension Δa of a crack (length a) occurring in one cycle.

$$da/dn = \Delta a$$

From the previous arguments it will be clear that Δa will depend on:

- the cyclic stress on the crack tip area,
- the (cyclic) elasto-plastic response of the material in the same area,
- the environment,
- some fracture criterion.

The plastic deformations around a tip of a fatigue crack are depending on the (cyclic) strain-hardening behavior of the material. The distribution

of stress and strain in the crack tip area will be highly inhomogeneous (large gradients) and this makes exact calculations for a cyclic loading difficult and expensive. Even if this information would be available a fracture criterion, including environmental effects, is not available, although some speculations have been published. It should be concluded that predicting Δa from first principles is a tremendous problem as yet unsolved.

To overcome this problem a practical approach is to correlate crack growth rates under similar conditions. The similarity approach implies:

Similar conditions imply the same loading on the crack tip area and the same environment surrounding the crack tip. The loading on the crack tip area can be described by the stress intensity factor K , to be discussed later.

The similarity approach is physically sound, but it should be carefully examined whether the required similarity is satisfied. It may be recalled that the similarity approach is also adopted for predicting the fatigue limit of a notched component by employing $K_f = K_t$. It is based on the fact that similar cyclic peak stresses in a notched and an unnotched component should cause (or should not cause) crack nucleation in both components. Actually the similarity approach is the basis for many predictions in Applied Mechanics. A fundamental understanding of the mechanism going on in the material is not required for the applicability of the similarity approach. However, understanding of the phenomenon is essential to see the limitations of the validity.

4. The stress intensity factor "K"

With the theory of elasticity, assuming linearly elastic behavior (Hooke's

law), the stress distribution in a cracked element can be calculated. The most simple problem is an infinite sheet loaded by a tensile stress S . The solutions for the stresses in the sheet are still fairly complex, and cannot be given as explicit functions.

Fig.11

However, if we consider stresses in the neighbourhood of the crack tip only (i.e. $r \ll a$) the formulas for the stress component become relatively simple (Ref. 3):

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (1)$$

$$\sigma_Y = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \quad (2)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (3)$$

or:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) \quad (4)$$

with the stress intensity factor:

$$K = S \sqrt{\pi a} \quad (5)$$

If the sheet has finite dimensions, but the load and the geometry are symmetric with respect to the X-axis (crack opening mode I), equations (1) - (4) still apply, while a geometry correction factor C (dimensionless) has to be added to Eq. (5). This factor accounts for the shape of the component.

$$K = C S \sqrt{\pi a} \quad (6)$$

Equations (1) - (6) imply that stress distributions around crack tips are all geometrically similar, while the stress intensity factor fully determines all stress components. The similarity of stress distributions around different cracks is illustrated in the figure below.

Fig. 12. Photo-elastic picture of a beam with three different cracks (Ref. 4).

The equations show:

$$\sigma_{ij} \div S \quad (\text{Hooke's law})$$

$$\sigma_{ij} \div r^{-1/2} \quad (\text{square root stress singularity})$$

The singularity implies that $\sigma_{ij} \rightarrow \infty$ if $r \rightarrow 0$. Infinite elastic stresses are impossible and ductile materials will always show some plasticity at the tip of a crack. Two limitations of K to describe the stress field around a crack tip are apparent now:

- (1) Equations (1) - (4) are valid only if $r \ll a$ (asymptotic solutions for the near field stresses).
- (2) Equations (1) - (4) cannot be valid if $r \rightarrow 0$ (plasticity).

The first limitation need not bother us too much because we are interested in the crack tip area only. However, the second one should be a matter of concern.

In Fig. 13 an elastic zone (radius r_1) is considered, which is sufficiently small ($r_1 \ll a$) for Eqs. (1) - (6) to be valid. If no plasticity occurs the

Fig.13

stresses acting on the periphery of this zone are determined by K. If a small plastic zone is formed some redistribution of stress and strain in the elastic zone (r_1) will occur, but as long as $p \ll r_1$ the redistribution will have largely faded at the periphery of the elastic zone (r_1). Consequently it may well be expected that the same K-value will always produce the same plastic zone

as long as the plastic zone is small (small-scale yielding). In other words the similarity approach predicts: the same K-values will produce (in the same material) the same elastic-plastic deformation in the crack tip zone.

As a result the same amount of crack extension Δa has to be expected.

5. K and cyclic loading

From the definition: $K = C S \sqrt{\pi a}$ it follows that a cyclic variation of S will cause a similarly cyclic variation of K . The stress intensity in the crack tip area will thus be characterized by $K_{\max}$ and $K_{\min}$.

The similarity approach thus predicts:

$$\left. \begin{aligned} \frac{da}{dn} &= f(K_{\min}, K_{\max}) \\ \text{where:} \\ K_{\max} &= \frac{\Delta K}{1 - R} \end{aligned} \right\} \begin{aligned} \frac{da}{dn} &= f(\Delta K, R) \\ \text{or: } \frac{da}{dn} &= f_R(\Delta K) \end{aligned} \quad (7)$$

where:

$$K_{\max} = \frac{\Delta K}{1 - R}$$

and:

$$K_{\min} = \frac{R \Delta K}{1 - R}$$

A formula proposed by Forman (Ref. 5) is:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1 - R) (K_c - K_{max})} \quad (9)$$

Figure 15 gives an illustration of the R-effect for an Al-alloy. Different S_a and S_m values produced the same $da/dn - \Delta K$ relation, provided the same R value applied.

Another illustrative example of the applicability of K is the comparison between crack growth results from specimens with end loading and specimens with crack edge loading, see figure 16 (Ref. 6). For crack edge loading the K-factor is decreasing for increasing crack length! There is a full agreement between the crack growth rates for the two types of loading if plotted as a function of ΔK . In other words the results of one specimen can be predicted if the results for the other one are known. For this prediction the functional relationship in equations (7) and (8) need not be known analytically. The similarity approach leads to a correlation between the two cases, but not to analytical relations for crack growth. Such relations can be assumed by employing empirical crack growth data (curve fitting), but unfortunately they cannot be derived from physical arguments so far.

6. References

1. A.V. de Forest - The rate of growth of fatigue cracks. J. Appl. Mech. 3, 1936, p. A-23.
2. N.E. Frost, K.J. Marsh and L.P. Pook - Metal fatigue. Clarendon Press, Oxford 1974.
3. P.C. Paris and G.C. Sih - Stress analysis of cracks. ASTM STP 381, 1965, p. 30.
4. J. Schijve - Fatigue crack propagation and the stress intensity factor. Delft Un. of Technology, Dept. of Aerospace Eng., Report M-191, 1973.
5. R.G. Forman, V.E. Kearney and R.M. Engle - Numerical analysis of crack propagation in cyclic-loaded structures. J. Basic Engrg., Trans. ASME D 89, 1967, p. 459.
6. I.G. Figue and J.C. Newman - Fatigue crack propagation in structures with simulated rivet forces. ASTM STP 415, 1967, p. 71.

da/dn ($\mu\text{m}/\text{cycle}$)

Fig. 15. Correlation between da/dn and K_a . Effect of stress ratio. Sheet specimens of an Al-alloy (2024-T3) (Ref. 4).

Fig. 16

Results in one scatter band (Ref. 6)

II. FATIGUE CRACKS, PLASTICITY EFFECTS AND CRACK CLOSURE

Summary

Concepts introduced are residual plastic deformation, residual stress, reversed plastic deformation, plastic deformation in the wake of the crack and crack closure under tensile load. COD measurements as a method to determine the crack closure level are discussed. The significance of crack closure for fatigue crack growth is analysed and illustrated by several examples, including effects of yield stress, stress ratio and delayed crack growth after a peak load. Finally some attention is paid to three dimensional aspects following from thickness effects, shear lips and curved crack fronts.

1. Introduction

Plastic deformation occurring around the tip of a fatigue crack is usually considered to be "small scale yielding" in technical materials with a limited ductility, having a relatively high yield stress. However, small plastic zones are still sufficient to leave residual deformations and stresses and to cause crack closure. This is the main topic of the present lecture.

2. Residual stress after local plastic deformation

Let us consider a strip with a central hole loaded in tension (Fig. 1). If the load is sufficiently high plastic deformation will occur in small zones at the edge of the hole and this implies permanent deformation. If the strip is unloaded ($\Delta P \rightarrow 0$) the plastic zones do no longer fit in the elastic surroundings as a result of the plastic deformation. This misfit causes residual stresses. Assuming that unloading will cause elastic spring back only, a residual stress distribution as shown in Fig. 2 will occur along the X-axis.

It should be noted that there is a two-dimensional residual stress field which obviously will extend outside the plastic zones.

Let us now consider a strip with a much sharper notch, say a narrow

Fig. 1

Fig. 2

elliptical hole, which is loaded and unloaded (Fig. 3). During uploading a plastic zone will be formed (along the X-axis approximately between A and C). Full elastic unloading will cause residual compressive stresses at A exceeding the compressive yield limit. As a result reversed plastic deformation will occur between A and B. Clearly enough AB will be considerably smaller than AC.

A similar picture applies if we consider a crack during uploading followed

by unloading (Fig. 4). The

monotonic plastic zone is significantly larger than the reversed plastic zone.

As a first approximation the size of a plastic zone

(r_p) is inversely proportional

to the square of the yield stress. During unloading the stress increment to cause yielding in the reversed direction may be assumed to be twice the yield stress during uploading (Fig. 5, Bauschinger effect) and as a consequence (Ref. 1):

$$\left. \begin{aligned} (r_p)_{\text{uploading}} &\div \frac{1}{(\sigma_{\text{yield}})^2} \\ (r_p)_{\text{unloading}} &\div \frac{1}{(2\sigma_{\text{yield}})^2} \end{aligned} \right\} \quad \text{and:} \quad \left(\begin{array}{c} \text{reversed} \\ \text{plastic} \\ \text{zone size} \end{array} \right) = \frac{1}{4} \left(\begin{array}{c} \text{monotonic} \\ \text{plastic} \\ \text{zone size} \end{array} \right)$$

3. Crack closure

Let us now consider a growing fatigue crack (Fig. 7). During its growth

Fig. 7.

the plastic zone is moving with the tip of the crack. It is also increasing in size and as a first approximation (central crack)

$$r_p \div K^2 \div a$$

That means that the plastic

zone size is proportional to the crack length (a). The same will be true for the reversed plastic zone. Since the monotonic plastic zone is considerably larger than the reversed plastic zone, the consequence of the growing fatigue crack is that monotonic plastic deformation has been left in the wake of the crack. This deformation involves elongation in the y -direction. As a result of this elongation the crack will close (at least partly) during unloading, and after full unloading ($P = 0$) compressive residual stresses will be present in the wake of the crack. It means that residual compressive stresses are transmitted through the crack, because the fracture surfaces are pressed together by the plastic deformation left in the wake of the crack.

The phenomena that the upper and lower fracture surface of a fatigue crack come together before complete unloading (i.e. at $P > 0$, tensile load) implies that the crack is no longer fully open. This phenomena in the literature is referred to as "crack closure". It was first observed by Elber (Ref. 3) and it is sometimes referred to as the Elber mechanism. The proof of the occurrence of crack closure can be obtained from direct evidence, such as stiffness measurements (COD measurements) to be discussed later. It also may come from indirect evidence, which is the effect on fatigue crack growth.

The picture in Fig. 8 shows two fatigue cracks started from a small notch (hole with two saw cuts) as viewed through a rectangular window. The plastic zone in the wake of the crack is visible due to a slight lateral contraction of the material and a simple illumination technique.

Fig. 8

From the picture the wedge opening angle α can be measured: $\tan \alpha \sim 0.11$. As a first approximation:

$$r_p = \frac{1}{\beta\pi} \left(\frac{K_{\max}}{\sigma_{\text{yield}}} \right)^2 = \frac{1}{\beta} \left(\frac{C S_{\max}}{\sigma_{\text{yield}}} \right)^2 \cdot a \rightarrow \tan \alpha = \frac{r_p}{a} = \frac{1}{\beta} \left(\frac{C S_{\max}}{\sigma_{\text{yield}}} \right)^2$$

With $a \sim 21$ mm, $2w = 160$ mm, $S_{\max} = 161$ N/mm² and $\sigma_{\text{yield}} \sim 450$ N/mm² the result is $\beta = 1.39$. This order of magnitude agrees with suggested values for plane stress.

4. How to measure crack closure

The most wellknown method to indicate crack closure is by COD measurements (COD = crack opening displacement). For crack opening between two points A and B, close to the edges of the crack and in the center of the panel, the relation for an infinite sheet is:

$$\text{COD} = \frac{4a\sigma}{E}$$

Fig. 9

Fig. 10

It means that COD is linearly proportional to σ (Hooke's law) and to the crack length a . For a finite sheet a geometry correction factor has to be added. Measurements on panels with fine saw cuts (see Fig. 10) have confirmed the linear relationship with excellent agreement between measured slopes (σ/COD) and theoretical values. If a similar test is carried out on a panel with a fatigue crack the σ -COD record shows a non-linear part. Above point A the record is linear and the slope is in agreement with the crack length, which implies that the crack is fully open. Below point A the slope (tangent to σ -COD record) is larger, which implies that the panel behaves as a panel with a shorter crack. This indicates that the crack is partly closed. The stress corresponding to point A is called the crack closure stress (σ_{cl}) or the crack opening stress (σ_{op}).

Fig. 11

A full loop of a COD record (Fig. 11) usually shows a slight hysteresis, but there is no doubt about the occurrence of full crack opening at A' the onset of crack closure at A''. The existence of a non-linear part followed by a linear part is easily observed, but the problem is to determine accurately the point A where the transition occurs.

Measurements suggest that A' and A'' coincide, however, experience shows that the unloading branch (A'') gives a slightly

better reproduction and a more unambiguous determination of the closure stress. Nevertheless it cannot be denied that it is difficult to achieve a high accuracy.

In order to improve the accuracy Paris (Ref. 4) suggested a compensation method, illustrated by Fig. 12. Instead of recording the COD signal it is compensated by a signal that would have been obtained under full linear behaviour (i.e. no crack closure). This leads to a vertical line as long as the crack is fully open. The compensated COD signal now allows a much larger amplification which will bring out the transition point A more clearly. If the amplification is selected too high the linear part (vertical line) may become erratic.

Fig.12

Another possibility to improve the sensitivity of the crack closure measurement is to locate the COD meter more closely to the crack tip. The effect on the σ -COD record is shown in Fig. 13.

The non-linear part (A'' - A') of the record is small and the transition is more easily observed. This method can be used for a particular crack length, but if the crack is growing the COD meter has to be moved also. If the COD-meter is too close to the crack tip plasticity effects may obscure the measurements.

Fig. 13.

Several other methods to measure crack closure have been proposed in the literature (for a survey see Ref. 5). Some of them may be mentioned here.

- (1) Electrical potential method. This method was developed for automatic crack length measurements. It is possible to use this method, but difficulties may arise due to electrical short circuits over the crack, e.g. at the shear lips or some other minor asperities. A mechanically open crack is not the same as an electrically open crack. The risk of problems is smaller (but not absent) if the material forms an oxide layer, which acts as an insulator on the fracture surface (which will not happen in an inert environment, e.g. in vacuum).
- (2) Strain gages on the surface near the crack. Indications of non-linear behaviour are obtained, but interpretation problems may occur.
- (3) Ultrasonic transmission of surface waves through the crack line. Transmission is possible if the crack is closed. However, acoustic short circuiting is possible. A mechanically open crack is not the same as an "acoustically" open crack.

5. Crack closure and fatigue crack growth

In 1970 W. Błazewicz carried out fatigue tests in Delft (unpublished results). He made ball impressions on 2024-T3 sheet specimens before the crack growth test was started, see Fig. 14. As a result there is a zone between the impressions with residual compressive stresses. This

Fig. 14

caused a delay in the crack growth, but (surprisingly enough at that time) the delay was small during the growth through the zone between the impressions, whereas it was significant at a later stage. The explanation is that the deformations of the ball impressions were the cause of crack closure after the crack had grown through the affected zone.

Elber (Ref. 3) suggested the following relation between crack closure and crack growth. During a stress cycle a fatigue crack will be partly or fully closed as long as $\sigma < \sigma_{cl}$ (Fig. 15). He then suggested that the stress variation will contribute to crack extension only if

$$\sigma < \sigma_{cl}.$$

which leads to the definition of an effective stress range:

$$\Delta\sigma_{eff.} = \sigma_{max} - \sigma_{cl}.$$

Fig.15

and an effective stress intensity factor:

$$\Delta K_{\text{eff.}} = C \Delta \sigma_{\text{eff.}} \sqrt{\pi a}$$

The crack rate was supposed to be dependent on $\Delta K_{\text{eff.}}$ only

$$\frac{da}{dn} = f(\Delta K_{\text{eff.}})$$

This relation includes the effect of the stress ratio R because crack closure (and thus $\Delta K_{\text{eff.}}$) will depend on R . For 2024-T3 material Elber found that $\sigma_{\text{cl.}}$ was approximately constant during a fatigue test, implying that $\sigma_{\text{cl.}}$ was independent of the crack length a . This is an empirical result. He defined the ratio:

$$U = \frac{\Delta K_{\text{eff.}}}{\Delta K} \left(= \frac{\Delta \sigma_{\text{eff.}}}{\Delta \sigma} \right)$$

and the test results indicated the relation:

$$U = 0.5 + 0.4 R$$

This is again an empirical result. Combining the above equations leads to:

$$\log \Delta K_{\text{eff.}} = \log \Delta K + \log (0.5 + 0.4 R)$$

The difference between $\log \Delta K_{\text{eff.}}$ and $\log \Delta K$ is constant for constant R . The assumption $da/dn = f(\Delta K_{\text{eff.}})$ then implies that plots of da/dn as a function of $\log \Delta K$ should give parallel curves for different R -values. Results from Ref. 6 in the left graph of Fig. 16 confirm this trend. In these tests crack closure measurements were not made, however, adopting Elber's formula for $\Delta K_{\text{eff.}}$ the right hand graph of Fig. 16 shows that all three curves practically coincide.

Some comments on crack closure and Elber's proposals are appropriate now.

(1) Although it is not easy to measure accurately the stress level at

da/dn ($\mu\text{m}/\text{cycle}$)

Fig. 16

which crack closure does occur, there is abundant evidence that it occurs in several materials under tensile load.

- (2) The relation between ΔK_{eff} and R will depend on the type of material. The above relation ($U = 0.5 + 0.4 R$) was found for an Al-alloy (2024-T3) as an empirical result. It may be expected that such relations can be calculated by elasto-plastic finite element techniques.

The same applies to the effect of crack length and type of specimen. Elber found ΔK_{eff} to be independent of crack length for a center cracked specimen.

- (3) The relation $da/dn = f(\Delta K_{eff})$ assumes that cyclic stress is significant only if the crack is fully open, whereas it is insignificant as long as the crack is partly closed. The basic idea is that there is no stress singularity at the real tip of the crack unless it is fully open. However, if the crack tip is almost open it should be expected that some preliminary plastic deformation will occur. As a first approximation it is ignored in Elber's proposals.

6. Some consequences of crack closure

Crack closure is caused by plastic deformations left in the wake of the crack. It should be expected that material with a lower ductility will form smaller plastic zones and thus show less crack closure. This was recently explored by comparing 2024-T3 (as received) with the same material after 3 percent plastic prestraining (Ref. 7). The prestraining raised the yield stress from 428 to 480 MN/m². The material was then

Fig. 17.

cycled between 13.8 and 138 MN/m². As a result of the prestraining the crack closure level was systematically lower (Fig. 17) in agreement with the higher yield stress. The crack growth rate was about twice as high as for the unprestrained material as shown in the left hand graph of Fig. 18. The

same results plotted as a function of ΔK_{eff} (calculated from the measured closure stresses) are shown in the right hand graph. The two curves come closer together, but they do not coincide. Obviously, crack closure can

Fig.18

• pre-strained material

○ not pre-strained

explain half of the effect of the higher yield stress, but it cannot explain the full difference. It should be said that plastic prestraining also implies that the material itself is changed by strain-hardening. However, without knowing $\Delta\sigma_{\text{eff}}$, an explanation anyhow will be incomplete.

An intermediate peak load applied in a constant-amplitude test causes the wellknown crack growth delay. An example from recent tests is shown in Fig. 19 (Ref. 8). When a crack length (a) of 15 mm was reached a peak load was applied causing a delay of 33000 cycles as compared to the original crack propagation curve for constant-amplitude loading only. Crack closure measurements until the peak load occurred were measured on other specimens. The closure level was lower than the minimum stress of the cycle ($\sigma_{\text{cl.}} < \sigma_{\text{min}}$). As a result of the peak load it decreased even further, because the plasticity ahead of the crack induced by the peak load opens the wake of the crack. However, shortly afterwards when the crack penetrated into the plastic zone of the peak load the closure stress raised significantly up to a level beyond σ_{min} . Then $\Delta\sigma_{\text{eff}}$ is reduced and a delaying effect should be expected. Later on $\sigma_{\text{cl.}}$ decreased and when it dropped below σ_{min} no further delay was observed. As Fig. 19 shows the delay occurred during a crack length increment $\Delta a = 5$ mm which is significantly larger than the plastic zone size associated with the peak load. It

Fig. 19

confirms that crack closure can still occur after the crack has fully grown through the plastic zone of the peak load. A similar indication was obtained in the tests of Błazewicz as mentioned before.

Crack closure was also observed in more complex variable-amplitude tests, such as a flight-simulation tests. This will be discussed in another lecture.

Crack closure measurements were also made on two Al-alloys (2024-T3 and 7075-T6) tested in three environments, viz. vacuum, humid air and salt water (Ref. 9). Although the crack growth rates in the three environments were highly different, the crack closure stress was the same in all environments. The size of plastic zones should not be expected to be dependent on the environment and consequently the same amount of plasticity in the wake of the crack will occur, which implies similar σ_{cl} values. Environmental aspects should therefore be explained on the basis of other arguments (Ref. 10).

7. Crack closure and the effect of thickness

In the preceding sections fatigue crack growth in a sheet or plate has been considered as a two-dimensional problem with crack growth in X-direction and loading in the Y-direction. The thickness of the material is adding the third dimension (Z-direction) and it should be expected that this will complicate the problem. The most simple case is a crack with a straight crack front perpendicular to the surface of the material, as depicted in Fig. 20. The reason why the thickness may be important is the difference between the state of stress at the free surface (plane stress, $\sigma_z = 0$) and the state of stress along the crack front away from the free surface (approximately plane strain, $\epsilon_z = 0$). Frequently quoted formula's for the plastic zone size for the two states of stress are:

$$\text{Plane stress: } r_p = \frac{1}{2\pi} \left(\frac{K}{\sigma_{\text{yield}}} \right)^2$$

$$\text{Plane strain: } r_p = \frac{1}{6\pi} \left(\frac{K}{\sigma_{\text{yield}}} \right)^2$$

Fig 20

The formulas suggest that the plastic zone size is three times larger in the plane stress condition. Larger plastic zones imply more deformation left in the wake of the growing crack and hence more crack closure. Increasing the thickness will lead to a relatively smaller part of the crack front in plane stress, see Fig. 21, and as a consequence less effective crack closure. This was confirmed in some recent tests (Ref. 8) on an Al-alloy as

shown in Fig. 22. Whether this should fully account for faster crack growth in thicker material is doubtful, because a thickness effect on crack rate is also observed if $\sigma_{cl.} < \sigma_{min.}$

Fig. 21

The above picture about the relation between thickness and crack closure suggests that crack closure should predominantly occur near the free surface and to a much lesser extent at the interior of the material.

This was checked in fatigue

Fig. 22

tests (Ref. 11) on material with an original thickness of 10 mm which gave a crack closure level of 33 MN/mm^2 , see Fig. 23. The thickness of the specimen was then reduced from both sides by removing the surface material. After some steps σ_{cl} was measured again and the results indicate that the major part of crack closure was associated with larger plastic zones at the free surface.

Fig. 23

Another elegant confirmation that crack closure predominantly occurs at the free surface was presented by McEvily (Ref. 12). He found in tests on the Al-alloy 6061 (Al Si Mg) that a peak load introduced a significant crack growth delay. He then reduced the thickness of the specimen immediately after the peak load and a much smaller delay occurred.

The real elasto-plastic three-dimensional problem is more complicated than discussed before. Along the crack front there is a transition from plane stress at the surface to approximately plane strain at the interior. However, the r_p -formulas given before apply to either pure plane stress (extremely thin material) or pure plane strain (very thick material). The formulas then suggest that the plastic zone is three times larger in plane stress. Probably the factor will be less for intermediate thicknesses. Consider a line AB in Fig. 20 from one free surface to the other one ($r = \text{constant}$, $\theta = \text{constant}$). A plane stress zone at the surface and a plane strain zone at the interior cannot deform independently. Both zones have to be coherent and as a first approximation it may be assumed that the line AB will remain a straight line after deformation. This is equivalent to assuming that ϵ_x and ϵ_y are independent of z . Different zone sizes will still be obtained because at the free surface due to the plane stress situation a lower effective yield limit will apply (von Mises criterion, $\sigma_z = 0$). As a consequence the plastic zone will still be larger at the free surface.

In the literature it is sometimes suggested that a plane stress situation through the full thickness will be obtained, if the estimated plastic zone size (r_p) calculated with the plane stress formula is equal or larger than half the material thickness, i.e.:

$$r_p \geq \frac{1}{2} t$$

The paradox may be that adopting the plane strain formula will indicate $r_p < \frac{1}{2} t$. Furthermore considering cyclic plasticity at the crack tip, which is causing crack growth, the reversed plastic zone size estimated with the plane strain formula is:

$$r_p = \frac{1}{6\pi} \left(\frac{\Delta K}{2\sigma_{\text{yield}}} \right)^2$$

Considering $\Delta\sigma$ values in the order of $0.5 \sigma_{\text{yield}}$ or smaller we obtain:

$$r_p \leq \frac{1}{6\pi} \left(\frac{0.5 \sqrt{\pi a}}{2} \right)^2$$

or:

$$\frac{r_p}{a} \leq \sim 0.01$$

It then may be expected that many fatigue cracks will be predominantly in plane strain during the larger part of crack growth. However, further study of this issue is necessary.

8. Crack closure and shear lips

In general the fracture surface of a fatigue crack is perpendicular to the main principal stress. However, at a free surface the growing fatigue crack is forming shear lips (see previous lecture) which become wider as the crack is growing faster. In the past shear lips were associated with plane stress conditions at the free surface. If this were true shear lips might be important for the occurrence of crack closure. However, since we know more about environmental effects on fatigue fracture behaviour, the correlation between shear lips and plane stress conditions seems to be less positive. In aggressive environments shear lips are much smaller than in inert environments under the same cyclic loading (Ref. 10). In other words shear lip formation is also depending on the fatigue fracture mechanism and since the environment will not affect the state of stress the width of shear lips cannot be an unambiguous indication of the plastic zone size.

In spite of the above complexity it still should be expected that the free surface edges of shear lips were part of larger plane stress plastic zones. As a consequence crack closure during unloading will start at the shear lip edges (Ref. 13). If contact between the upper and lower fracture surface start in the slant shear lips some rubbing should be expected. As a matter of fact black debris is observed on the shear lips of fatigue

fractures in aluminium alloy material, whereas it is absent on the tensile mode part of the fracture.

9. Crack closure and crack front curvature

In the previous sections it was assumed that the crack front in a plate was a straight line perpendicular to the material surface. In reality a tendency to slightly curved crack fronts is frequently observed. Under certain conditions large curvatures may occur (tongues) but they will not be discussed here.

It is not fully correct that a curved crack front implies a slower crack growth at the free surface. If the amount of curvature of a growing crack does not change the crack rate is the same all along the crack front. However, the curvature anyhow implies that crack growth is lagging behind at the surface. It similarly could be said that crack growth at the interior remains ahead of the growth at the surface. It is a good question to ask why this can occur. In terms of crack closure a straightforward explanation seems to be possible, see Fig. 24. More crack closure at the surface and less crack closure at the interior implies a higher $\Delta K_{eff.}$ at the interior. Figure 24 suggests that the crack tip will be fully closed at σ_{min} , but it is possible that it will not fully close at the interior. Plastic infiltrations of cracks, now carried out by Bowles (Ref. 14) may answer this question.

In addition to the above arguments it should not be overlooked that the crack can lag behind at the surface because plastic deformation is more easy there (lower restraint) which may delay the cracking mechanism.

Elastic deformations are also more easy at the surface. In a recent publication (Ref. 15) Newman analysed the elastic stress intensity along a crack with a straight crack front. Three-dimensional FEM calculations showed a drop of the K-value near the free surfaces of the plate.

Fig. 24

10. References

1. J.R. Rice - The mechanics of crack tip deformation and extension by fatigue. ASTM STP 415, 1967, p. 247.
2. A.U. de Koning (to be published).
3. W. Elber - Fatigue crack closure under cyclic tension. Eng. Fracture Mechanics, 1970, Vol. 2, p. 37, and: The significance of fatigue crack closure. ASTM STP 486, 1971, p. 230.
4. P.C. Paris (private communication).
5. V. Bachmann and D. Munz - Crack closure in fatigue crack propagation. Proc. S.E.E. Conference: Fatigue Testing and Design, London, April 1976.
6. J. Schijve - Fatigue crack propagation and the stress intensity factor. Delft Un. of Technology, Dept. of Aerospace Eng., Memorandum M-191, Feb. 1973.

7. J. Schijve - The effect of pre-strain on fatigue crack growth and crack closure. Eng. Fracture Mechanics, Vol. 8, p. 575.
8. W.J. Arkema, results quoted in J. Schijve - Observations on the prediction of fatigue crack growth propagation under variable-amplitude loading. ASTM STP 595, 1976, p. 3.
9. J. Schijve and W. Arkema - Crack closure and the environmental effect on fatigue crack growth. Delft Un. of Technology, Dept. of Aerospace Eng., Report VTH-217, 1976.
10. L.B. Vogelesang - Some factors influencing the transition from tensile mode to shear mode under cyclic loading. Delft Un. of Technology, Dept. of Aerospace Eng., Report LR-222, 1976.
11. R.Th. Furnee - Thesis (in Dutch), Department of Metallurgy, Delft Un. of Technology, March 1977.
12. A.J. McEvily - Current aspects of fatigue. Appendix: Overload experiments. Fatigue 1977 Conference, Un. of Cambridge, March 28-30, 1977.
13. T.C. Lindley and C.E. Richards - The relevance of crack closure to fatigue crack propagation. Materials Science and Engineering, Vol. 14, 1974, p. 281.
14. C.Q. Bowles - An experimental technique for vacuum infiltration of cracks with plastic and subsequent study in the scanning electron microscope. Delft Un. of Technology, Dept. of Aerospace Eng., Report LR-249, June 1977.
15. I.S. Raju and J.C. Newman - Three-dimensional finite-element analysis of finite-thickness fracture specimens. NASA TN D-8414, 1977.

III. FATIGUE CRACK PROPAGATION, PREDICTION AND CORRELATION

Summary

Two prediction techniques are introduced, (1) cycle-by-cycle prediction and (2) prediction by correlation. Attention is paid to the problem of describing variable-amplitude loading in terms of load cycles. Aspects of fatigue damage are reviewed with reference to interaction effects and weaknesses in cycle-by-cycle prediction methods. The discussion on prediction by correlation is restricted to constant-amplitude loading. The validity of the similarity concept based on K-factors is reconsidered. Application of simple specimen data to complex structures is shown. Finally a variety of crack growth equations is reviewed, including aspects of curve fitting, a comparison between formulas of Walker and Elber and asymptotic values in the $da/dn-\Delta K$ relation.

1. Introduction

Basically there are two different prediction techniques:

1. Cycle-by-cycle prediction
2. Prediction by correlation, employing characteristic K-values.

In the literature the first technique has been adopted by several authors for predicting crack growth under variable-amplitude loading (VA-loading). All methods of this type include some correlation with crack growth data obtained under constant-amplitude loading (CA-loading).

The second technique is well-known for CA-loading, but application to VA-loading received limited attention.

In this lecture problems involved in the first technique are summarized, while the discussion on the second one is restricted to CA-loading. In a fourth lecture more attention is paid to crack growth under VA-loading.

2. Cycle-by-cycle prediction for variable-amplitude loading

Before we can discuss the cycle-by-cycle prediction methods it is necessary to specify in some detail variable-amplitude loading (VA-loading) as a load-time history. Two samples of VA-loading, as they may occur on a structure in service, are shown in Fig. 1. Although many more types of

a. Narrow-band random load.

Fig. 1

b. Broad-load random load.

VA-loading are possible (and will be defined in the fourth lecture) these samples are shown here, because in the first one a definition of load cycles seems relatively easy, whereas in the second one it is more complex.

Whatever the type of a cyclic load is, the significant events for the fatigue behaviour of the material are the maxima and the minima, i.e. the moments at which the loading direction is reversed. That is what the material feels. After passing a maximum or a minimum of the load the reversion of stress and strain will start immediately. For a crack tip it implies in view of the stress and strain singularity, that reversed plasticity will also start immediately as soon as the loading direction is reversed.

In addition to the minima and the maxima the load-time history is characterized by a loading rate, which can vary in some systematic way (e.g. sinusoidal loading) or in a random way. If time dependent effects on fatigue are involved (corrosion fatigue e.g.) the loading rate has to be considered (effects of wave shape, and cyclic frequency). If time dependent effects are not involved the load-time history is described in sufficient detail by a series of successive minima and maxima.

The load on a structure in service can be measured for a long period in

order to be representative. As a result the series of maxima and minima will become a very large one and for practical reasons it is then condensed to statistical distribution functions of peak values (maxima and minima) or ranges (differences between successive maxima and minima) or both. This can be done by counting how many times events of a certain magnitude have occurred in a certain period, preferably a long period. A variety of counting methods have been developed (Refs. 1-3) but a discussion here is outside the scope. It should be pointed out, however, that a distribution function of counted events does not give information on the sequence in which the events have occurred. The counted results will be informative only if information on the sequence is available also. For a stationary random load (e.g. gust loads on an aircraft wing during constant weather conditions) and for systematically planned service loads (e.g. pressure cycles on a pressure vessel) this information can be provided. For unstationary random loading and unscheduled manoeuvre loadings assumptions on the sequence can sometimes be made.

For simplicity let us assume that load cycles occurring under VA-loading are fully described by a minimum and an associated maximum, from which related stress values ($\sigma_{\min}$ and $\sigma_{\max}$) and K-values ($K_{\max}$ and $K_{\min}$, or ΔK and R) can be derived. The crack length after N cycles according to the non-interaction cycle-by-cycle prediction technique can be obtained by calculating:

$$a_N = a_0 + \sum_{i=1}^N \Delta a_i \quad (1)$$

with:

$$\Delta a_i = \frac{da}{dn} \text{ for } \Delta K \text{ and } R \text{ in cycle } i \text{ as obtained under CA-loading} \quad (2)$$

In this relation a_0 is the initial value (a for $N = 0$). Each cycle will contribute a crack length increment (Δa_i) which is assumed to be equal to the increment obtained at the same ΔK and R value in a constant-

amplitude test. This increment is equal to the crack rate observed under CA-loading. The approach assumes that Δa_i in cycle i is independent of the history of the preceding crack growth. Unfortunately in view of so-called interaction effects this is not correct. A well-known example of interaction effects is delayed crack growth after a high peak load (Ref. 4). In general terms the crack length increment Δa in a cycle will be a function of:

1. the crack geometry being present before the cycle started,
2. the condition of the crack tip material, and
3. the magnitude of the load cycle.

Only the magnitude of the cycle and one dimension of the crack (a) is accounted for by ΔK and R . However, significant aspects resulting from

CRACK GEOMETRY	Additional aspects
<p>a. Amount of cracking</p> <ul style="list-style-type: none"> - crack length - shape of crack front, straight or curved
<p>b. Crack front orientation</p> <ul style="list-style-type: none"> - tensile mode - shear mode, single shear or double shear - mixed mode
<p>c. Crack blunting (and sharpening)</p> <ul style="list-style-type: none"> - shape of crack tip
<p>d. Crack closure</p> <ul style="list-style-type: none"> - plastic deformation in the wake of the crack
MATERIAL CONDITION AT TIP OF CRACK
<p>e. (cyclic) strain-hardening</p> <ul style="list-style-type: none"> - distribution in crack tip zone
<p>f. Residual stress</p> <ul style="list-style-type: none"> - plastic deformation in crack tip zone

Fig. 2. Aspects of fatigue damage

the preceding fatigue cycles are disregarded by ΔK and R . This is illustrated in Fig. 2.

Various interaction effects between the "fatigue damage" as left from preceding cycles and the increments of damage in future cycles are possible, depending on changes of the cyclic load (i.e. in VA-loading). Several interaction effects, surveyed in Ref. 4, can be understood qualitatively by considering crack closure (see fourth lecture). However, such interaction effects are ignored in Eqs. (1) and (2) and as a result predictions obtained are generally conservative and sometimes highly conservative (see e.g. Ref. 5).

The conservatism of the non-interaction prediction has prompted some models for crack growth under VA-loading with simple assumptions for the effect of high load cycles on delayed crack growth in subsequent cycles of a lower magnitude. Two models (Refs. 6, 7) receiving more attention in the literature are referred to as: - Willenborg-model and the
- Wheeler-model

The main aim of the models was to account for delayed growth and the assumptions made by Wheeler (Ref. 7) are certainly more reasonable than for the other model.

He replaced Eq. (2) by:

$$\Delta a_i = \left(\frac{da}{dn} \right)_{VA, a=a_i} = \beta \left(\frac{da}{dn} \right)_{CA, a=a_i} \quad (3)$$

with a delay factor β (≤ 1) depending on plastic zone sizes. Assumptions are made about this dependence including the introduction of one empirical material constant. In view of this constant there is a possibility to adjust its value in order to fit empirical crack growth rates obtained under VA-loading. This may well offer a possibility to correlate crack growth data obtained under specific types of VA-loading occurring in service (Ref. 8) and thus offer some promise for crack growth predictions in practical problems. At the same time we should realize that such a model has to be physically incorrect because it is fundamentally incapable

to account for physical observations, such as: (1) accelerated crack growth under certain load variations ($\beta > 1$), (2) delayed retardation after a high peak load (Ref. 9) and (3) a sustained delay after a high peak load even when the crack has already fully penetrated through the plastic zone created by the high peak load (see previous lecture). Qualitatively these observations can be explained by crack closure as will be shown in the fourth lecture.

In closing it should be noted that Eq. (3) implies that it is still tried to correlate crack extension under VA-loading with crack extension under CA-loading! In view of the complexity of fatigue crack extension and possible interaction mechanisms, one cannot be very optimistic about deriving a reliable prediction technique with some general validity based on equations like Eq. (3).

3. Crack growth prediction by correlation

Predictions made by correlation are based on the similarity approach discussed in the first lecture. Briefly: Similar conditions on the same system will cause similar consequences. For crack growth it implied: the same ΔK and R in a load cycle will produce the same crack growth increment $\Delta a = da/dn$. This approach seems reasonable if applied to CA-loading. However, it will be clear that the similarity is no longer satisfied if a single cycle of VA-loading is compared to a single cycle of CA-loading, in spite of the same ΔK and R being applicable. The reason is that we do not consider the same system, because that includes:

- the same material in the plastic zone
- the same geometry at the crack tip

Under VA-loading this cannot be guaranteed since all aspects mentioned in Fig. 2 will depend on previous history. In the fourth lecture it will be shown that the stress intensity factor can still be used to correlate crack growth rates as obtained under similar VA-loadings, but it has to be concluded here that the K -concept is unsuitable to correlate crack growth between dissimilar types of cyclic loading.

Restricting ourselves to CA-loading the requirement of having the same material in the plastic zone should be given some more attention. This material has seen monotonic plasticity and considerable cyclic plasticity. Similar material implies similar (cyclic) strain-hardening. In Fig. 3

Fig. 3

two cracks are compared, obtained under a low and a high CA-loading respectively. For the crack length shown in the figure they have the same ΔK (and R) and hence the same plastic zone size.

However, the cyclic strain-hardening in these zones (and also other damage aspects) will be similar only if the crack growth in the preceding cycles occurred with the same growth rate in both cases. That requires equal da/dn values in previous cycles, implying that the derivative

$$\frac{d \left(\frac{da}{dn} \right)}{dn}$$

should be equal. In the first lecture we arrived at

$$\frac{da}{dn} = f_R (\Delta K) \quad (4)$$

and thus

$$\frac{d \left(\frac{da}{dn} \right)}{dn} = f' (\Delta K) \cdot \frac{d\Delta K}{da} \quad (5)$$

Consequently similar cyclic strain-hardening requires that dK/da should be similar also (in addition to similar ΔK and R values). However, the requirements of both equal K -values and equal dK/da -values are not compatible in general, as illustrated in Fig. 4.

Fig. 4

As a result we should rewrite Eq. (4) more generally as:

$$\frac{da}{dn} = f_R (\Delta K, dK/da) \quad (6)$$

Results obtained in tests with CA-loading do not reveal any noticeable effect of dK/da . It should be realized that this is an empirical observation, although it is a rather fortunate finding. One extreme example of a negligible dK/da influence was given in the first lecture (Fig. 16) by comparing the $da/dn - \Delta K$ results of a specimen with end loading and a specimen with crack edge loading. For the first specimen dK/da is positive and for the second one it is negative. Nevertheless $da/dn = f_R (\Delta K)$ was apparently applicable.

If dK/da has little effect on crack growth we may feel more confident that crack growth results of CA-tests on simple specimens will be applicable to CA-loading on structures with a more complex relation between K and a . This will be illustrated by considering crack growth in a stiffened panel, see Fig. 5. The crack growth data obtained in tests on a simple sheet specimen are used as a calibration curve, i.e. the relation $da/dn = f_R (\Delta K)$. For the stiffened panel (a skin with a number of stringers) the relation between ΔK and the crack length has to be calculated. For simple stiffened panels analytical solutions are available, but otherwise finite-element methods have to be adopted. The prediction of crack growth in the stiffened panel is then a simple

Fig. 5. Prediction of crack growth in a stiffened panel

I. Material data

Fatigue tests on unstiffened panel produce $\frac{da}{dn} = f(\Delta K)$ (calibration curve)

II. Stress analysis

ΔK is calculated as a function of crack length "a"

III. Prediction

by integration

- Steps: (1) Select a-value
 (2) Derive ΔK -value from stress analysis
 (3) Read da/dn at ΔK in calibration curve

Fig. 6. Crack growth rate in stiffened panel, comparison between prediction and test results (Ref. 10).

conversion of the ΔK - a relation of the panel into a da/dn - a relation by application of the calibration curve (Fig. 5). An example of predicted an observed growth rates in the skin of a stiffened panel is shown in Fig. 6. The prediction is that da/dn will initially increase, but when approaching a stringer the stress intensity factor of the crack in the skin is reduced by the stringer. As a result the growth rate is decreasing. The agreement between prediction and observation is considered to be promising.

In the literature K -values for a large variety of geometries have been compiled in handbooks (Refs. 11-13) and the number of available solutions is steadily increasing. Nevertheless for several practical cases K -values cannot be found in the literature, but sometimes clever estimates can be derived from known solutions. If this is not sufficiently accurate, calculations (FEM) have to be made, while experimental techniques have also been developed.

4. Crack growth equations for constant-amplitude loading

In the literature many crack growth equations can be found. Older equations recognizing the effects of crack length, σ_a and σ_m were of the type:

$$\frac{da}{dn} = C \sigma_a^\alpha \sigma_m^\beta a^\gamma \quad (7)$$

where α , β and γ had to be determined from empirical data. A significant step was the recognition that K is a good parameter to indicate the severity of the stress field at the tip of a crack (Irwin) and to relate da/dn to ΔK (Paris):

$$\frac{da}{dn} = f(\Delta K) \quad (8)$$

An example frequently referred to as the Paris relation is:

$$\frac{da}{dn} = C \Delta K^m \quad (9)$$

with C and m being material constants (Ref. 14). The function implies a linear relation in a double-log plot. When more data became available it was obvious that the linearity was not generally confirmed, see Fig. 15 of the first lecture. Moreover Eq. (9) does not include an effect of mean stress or stress ratio. It was suggested by Walker (Ref. 15) that this effect could be accounted for by assuming:

$$\frac{da}{dn} = f (\Delta K^\alpha \cdot K_{\max}^\beta) \quad (10)$$

which he also wrote as:

$$\frac{da}{dn} = f (\overline{\Delta K}) \quad (11)$$

with:

$$\overline{\Delta K} = \Delta K^{\frac{\alpha}{\alpha+\beta}} \cdot K_{\max}^{\frac{\beta}{\alpha+\beta}} = \Delta K^m \cdot K_{\max}^{1-m} \quad (12)$$

or:

$$\overline{\Delta K} = C \Delta \sigma^m \cdot \sigma_{\max}^{1-m} \sqrt{\pi a} = C \overline{\Delta \sigma} \sqrt{\pi a} \quad (13)$$

with:

$$\overline{\Delta \sigma} = \Delta \sigma^m \cdot \sigma_{\max}^{1-m} = \frac{\Delta \sigma^m}{(1 - R)^{1-m}} \quad (14)$$

Walker referred to $\overline{\Delta \sigma}$ as an "effective stress".

A relation proposed in Ref. 16 for 2024-T3 material was:

$$\frac{da}{dn} = C \Delta K \cdot K_{\max}^2 \quad (15)$$

or in terms of Walker's formula:

$$\frac{da}{dn} = C (\overline{\Delta K})^3 \quad \text{with} \quad \overline{\Delta K} = \Delta K^{1/3} \cdot K_{\max}^{2/3} \quad \text{or} \quad m = 1/3$$

However, Walker found a better fit for this alloy with $m = 0.5$, while he found $m = 0.425$ for the 7075-T6 alloy.

With $K_{\max} = \Delta K / (1 - R)$ equation (10) becomes:

$$\frac{da}{dn} = f \left[\frac{\Delta K^m}{(1 - R)^{1-m}} \right] \quad (16)$$

Some similarity with Forman's equation (Ref. 17) is apparent:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1 - R) K_c - \Delta K} \quad (17)$$

Both formulas account for effects of ΔK and R and both fit test data quite well, but it should be realized that the formulas are purely empirical, i.e. no physical arguments were involved in the derivation.

Comparison between the formulas of Elber and Walker

It should be noted from Eq. (14) that Walker's effective stress $\overline{\Delta \sigma} \geq \Delta \sigma$ for $R \geq 0$, contrary to Elbers effective stress range $\Delta \sigma_{\text{eff.}} \leq \Delta \sigma$ since crack closure can only reduce $\Delta \sigma$.

Elber started from physical arguments (crack closure $\rightarrow \Delta K_{\text{eff.}}$, see previous lecture) to account for the R -effect. For 2024-T3 material he then arrived at the empirical relation:

$$U = \frac{\Delta S_{\text{eff.}}}{\Delta S} = \frac{\Delta K_{\text{eff.}}}{\Delta K} = 0.5 + 0.4 R$$

Walker's result for the same material is ($m = 0.5$):

$\overline{\Delta K} = \Delta K^{1/2} \cdot K_{\max}^{1/2}$ and with $K_{\max} = \Delta K / (1 - R)$ we obtain:

$$\frac{\overline{\Delta K}}{\Delta K} = \frac{1}{\sqrt{1 - R}} \quad (19)$$

The empirical relations from Elber and Walker are evidently different, but it is interesting to see the quantitative differences for R-ratios between 0 and 0.6 covering most available test data.

R:	0	0.1	0.2	0.3	0.4	0.5	0.6
Elber : $\Delta K_{\text{eff.}}/\Delta K$	0.50	0.54	0.58	0.62	0.66	0.70	0.74
Walker: $\overline{\Delta K}/\Delta K$	1	1.05	1.12	1.20	1.29	1.41	1.58
Ratio : $\Delta K_{\text{eff.}}/\overline{\Delta K}$	0.50	0.51	0.52	0.52	0.51	0.49	0.47

The last line indicates that there is an almost constant ratio between $\Delta K_{\text{eff.}}$ and $\overline{\Delta K}$. Consequently a satisfactory correlation between data for different R-values obtained with $\Delta K_{\text{eff.}}$ will automatically imply that a satisfactory correlation will also be obtained with $\overline{\Delta K}$. Different relations for the R-effect are apparently fitting the data equally well and hence the data are not a good instrument to discriminate between different formulas. It may be stressed once again that both R-effect formulas are the result of curve fitting (regression analysis). Some people prefer $\overline{\Delta K}$ and $\overline{\Delta S}$ because it does not have the pretension or the necessity to know something about crack closure, while the formulas are relatively simple.

Asymptotic K-values

Considering the relation between da/dn and ΔK there are two obvious limitations to ΔK . If ΔK becomes too large a static failure will follow immediately, which implies that K_{max} exceeded the fracture toughness K_{IC} , which is equivalent to ΔK exceeding $(1 - R) K_{\text{IC}}$. The other limitation comes from the fact that cracks will not grow if ΔK is too low. There is a threshold value ΔK_0 implying that crack growth requires that $\Delta K > \Delta K_0$. As a result of the two limitations the trend is a sigmoidal relation on a double log plot with two vertical asymptotes, see Fig. 7. For an analytical relation the two limitations imply:

$$\frac{da}{dn} \rightarrow 0 \text{ if } \Delta K - \Delta K_0 \rightarrow 0 \quad (20)$$

$$\frac{da}{dn} \rightarrow \infty \text{ if } K_c - K_{\max} \rightarrow 0 \quad (21)$$

In analytical relations these limitations are easily satisfied by having $\Delta K - \Delta K_0$ in the numerator and $K_c - K_{\max}$ in the denominator. Equation (17) of Forman can also be written as:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1-R)(K_c - K_{\max})} \quad (22)$$

The second limitation has been included although it could be done in a variety of ways, e.g.:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(K_c - K_{\max})^\beta} \cdot f(R) \quad (23)$$

Apparently Forman selected $\beta = 1$ and $f(R) = 1/(1-R)$, which fitted the data reasonably well. In the Forman equation a lower asymptote is missing. Such an asymptote is not clearly evident from data for Al-alloys, but it seems applicable to carbon steels. The variety of formulas suggested in the literature, including both asymptotes, will be illustrated by two examples. The first one is proposed by Priddle (Ref. 18)

$$\frac{da}{dn} = C \left[\frac{\Delta K - \Delta K_0}{K_c - K_{\max}} \right]^m \quad (24)$$

and a somewhat more complex one by Branco, Radon and Culver (Ref. 19)

$$\frac{da}{dn} = C \left[\frac{K_m \cdot \Delta K (\Delta K - \Delta K_0)}{K_c^2 - K_{\max}^2} \right]^m \quad (25)$$

Both formulas clearly satisfy the asymptotic requirements of Eqs. (20)

and (21). The formulas were mainly checked for various types of steel and it was shown that the threshold ΔK_0 was depending on R (i.e. on mean stress). A relation proposed is:

$$\Delta K_0 = A (1 - R)^\gamma \quad (26)$$

with $\gamma = 0.5 - 1.0$ (Refs. 18-20) depending on the type of steel.

None of the formulas discussed before can claim a physical background. The formulas are adjusted to agree with observed trends, while constants in the formulas are determined by regression analysis. It sometimes require K_{IC} -values different from measured fracture toughness values. The usefulness of the formulas is that they show the effect of the variables involved as they are observed under CA-loading. Secondly they represent a large amount of data in a condensed analytical form, which can be useful in certain computerized life calculation techniques.

5. References

1. J. Schijve - The analysis of random load-time histories with relation to fatigue tests and life calculations. Fatigue of Aircraft Structures. Pergamon Press, 1963, p. 115.
2. O. Buxbaum - Statistische Zählverfahren als Bindeglied zwischen Beanspruchungsmessungen und Betriebsfestigkeitsversuch. Lab. für Betriebsfestigkeit TR No. TB-64, Darmstadt 1966.
3. G.M. van Dijk - Statistical load data processing. Advanced Approaches to Fatigue Evaluation. NASA SP-309, 1972, p. 565.
4. J. Schijve - Observations on the prediction of fatigue crack propagation under variable-amplitude loading. Fatigue Crack Growth Under Spectrum Loads, ASTM-STP 595, 1976, p. 3.
5. J. Schijve, F.A. Jacobs and P.J. Tromp - Fatigue crack growth in aluminium alloy sheet material under flight-simulation loading. Effects of design stress level and loading frequency. NLR-TR 72018, 1972.
6. J.D. Willenborg, R.M. Engle and H.A. Wood - A crack growth retardation

- model using an effective stress concept. AFFDL-TM-FBR-71-1, Air Force Flight Dynamics Lab., 1971.
7. O.E. Wheeler - Spectrum loading and crack growth. J. of Basic Eng., Trans. ASME, 1972, p. 181.
 8. D. Broek and S.H. Smith - Spectrum loading fatigue-crack-growth predictions and safety-factor analysis. Naval Air Dev. Center NADC-76383-30, Sep. 1976.
 9. V.W. Trebules, R. Roberts and R.W. Hertzberg - Effect of multiple overloads on fatigue crack propagation in 2024-T3 aluminium alloy. Progress in Flaw Growth and Fracture Toughness Testing, ASTM-STP 536, 1973, p. 115.
 10. C.C. Poe - Fatigue crack propagation in stiffened panels. Damage Tolerance in Aircraft Structures, ASTM STP 486, 1971, p. 79.
 11. H. Tada, P. Paris and G. Irwin - The stress analysis of cracks handbook. Del Research Corp., Hellertown, Pa. 1973.
 12. G.C. Sih - Handbook of stress intensity factors. Lehigh Un., 1973.
 13. D.P. Rooke and D.J. Cartwright - Stress intensity factors. London, Her Majesty's Stationary Office, 1976.
 14. P. Paris and F. Erdogan - A critical analysis of crack propagation laws. Trans ASME, Series D, Vol. 85, 1963, p. 528.
 15. E.K. Walker - The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7075-T6 aluminium. Effects of Environments and Complex Load History on Fatigue Life. ASTM-STP 462, 1970, p. 1.
 16. D. Broek and J. Schijve - The influence of the mean stress on the propagation of fatigue cracks in aluminium alloy sheets. NLR TR M2111, 1963.
 17. R.G. Forman, V.E. Kearney and R.M. Engle - Numerical analysis of crack propagation in cyclic-loaded structures. J. Basic Engrg, trans. ASME-D, Vol. 89, 1967, p. 459.
 18. E.K. Priddle - High cycle fatigue crack propagation under random and constant amplitude loadings. Int. J. Pres. Ves. & Piping. Vol. 4, 1976, p. 89.
 19. C.M. Branco, J.C. Radon and L.E. Culver - Growth of fatigue cracks

in steel. Metal Science, Vol. 10, 1976, p. 149.

20. A. Ohta and E. Sasaki - Influence of stress ratio on the threshold load for fatigue crack propagation in high strength steels. Eng. Fract, Mech., Vol. 9, 1977, p. 307.

IV. FATIGUE CRACK GROWTH UNDER VARIABLE-AMPLITUDE LOADING

Summary

Stationary and non-stationary types of variable-amplitude loading are specified. Some attention is paid to the description of stationary random load. The stress intensity factor is then applied to crack growth under stationary variable-amplitude loading by defining first characteristic stresses and characteristic stress intensity factors. This is done for random loading, non-random loading and flight-simulation loading. It is discussed how and why this concept may break down if the stationarity is lost. Attention is paid to truncation of high loads in a flight-simulation test and the analogous problem of the crest factor under random loading. The significance of crack closure for understanding crack growth under variable-amplitude loading is emphasized.

1. Introduction

In the previous lecture we have seen that the K-concept was quite successful in correlating crack growth data obtained under CA-loading (constant-amplitude loading). In the present lecture it will be explored if the K-concept can be useful for VA-loading (variable-amplitude loading) if this type of loading has a stationary character. For this purpose several aspects of VA-loading have to be specified first. Characteristic K-values for both random loading and flight-simulation loading will then be defined and their usefulness will be discussed. If the meaning of characteristic K-values breaks down the significance of interaction effects has to be reconsidered. This leads to a discussion on load spectrum truncation effects and the significance of crack closure during VA-loading.

2. Different types of variable-amplitude loading

For the purpose of the present discussion VA-loading should be defined in two categories:

1. Stationary VA-loading
2. Non-stationary VA-loading

Simple examples of both categories are shown in Fig. 1 on the next page.

Fig. 1

From these illustrations it appears that stationary VA-loading is defined by the following requirement: After a sufficiently small period (return period, recurrence period) the same sequence of load cycles is repeated.

Mathematically in terms of Fourier series the definition is that stationary VA-loading is a periodic type of loading, which can be developed into a Fourier-series with a finite number of terms.

However, a stationary VA-loading has a broader meaning if we consider random loading caused by some random process. Examples from the field of aeronautics are: air turbulence causing gusts loads on a wing, and random noise from jet engines, causing sheet bending in stiffened skin panels. From other fields we can refer to loads on ships and off-shore structures, caused by sea waves, and loads on transport vehicles riding on a variety of roads with random roughness. A random process is defined as a stationary random process if its statistical properties are constant, i.e. independent of time.

In various practical cases service loads are a mixture of deterministic

loads and random loads. As an example a transport aircraft will see deterministic ground-air-ground transitions (one cycle per flight) and random gust loads (many cycles per flight), see Fig. 12 later on. As a more general definition of stationary VA-loading we now require that the description of the loads, including both deterministic and random loads, should not vary as a function of time.

3. How to describe random load

Several random processes are supposed to behave as stationary Gaussian random noise, and if it is "Gaussian" interesting input-output calculations for linear systems can be made (e.g. Ref. 1), but this is outside the scope of this lecture. However, some comments on the power spectral density function (PSD) should be made. For Gaussian random noise this function fully describes its statistical properties. It should be

Fig. 2

Fig. 3

considered as a density distribution function of the energy in the frequency domain (Fig. 2). To evaluate this concept a bit further, we will consider a Fourier series with an infinite number of terms with infinitesimally small differences between frequencies of successive terms, and coefficients A being a function of ω , see Figure 3. In an oscillation the energy is proportional to the square of the amplitude, which implies:

$$\Phi(\omega) \div [A(\omega)]^2$$

The sum of the infinite Fourier series is random noise. Some examples of random noise and corresponding power spectral density functions are

shown in Figure 4, taken from Ref. 2.

Fig.4

In Fig. 4a the energy is concentrated in a narrow frequency band and as a result the load-time history is somewhat similar to an amplitude modulated signal, in this case with a random modulation. This narrow band random loading is typical for resonance systems, which predominantly respond at one single resonance frequency if activated by some external random process over a broader frequency range.

In Fig. 4c the spectral density function of the random signal covers a much wider frequency band and the corresponding broad band random loading shows a higher degree of irregularity. White noise is referring to $\phi(\omega) = \text{constant}$, i.e. the same energy at all frequencies.

Since $\phi(\omega)$ fully characterizes random load some characteristic properties

can be derived from $\Phi(\omega)$ (Ref. 1). If we consider a stress $\sigma(t)$ varying randomly around a (constant) mean stress σ_m , the root-mean-square of $(\sigma - \sigma_m)$ follows from:

$$\sigma_{\text{rms}} \equiv \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\sigma - \sigma_m)^2 dt \right]^{1/2} = \left[\int_0^\infty \Phi(\omega) d\omega \right]^{1/2} \quad (1)$$

O = mean crossing
• = peak values

Fig. 5

Characteristic events of a random signal are the peaks and the mean-crossings, see Fig. 5. It will be clear that the number of peaks of a random signal will always be larger than the number of mean-crossings, and this will be more so for more irregular signals. This

has led to defining an irregularity factor k as the ratio between the number of peaks and the number of mean-crossings with $k \geq 1$. The relations to $\Phi(\omega)$ are given below.

$$\text{Number of mean crossings per sec} = N_O = \frac{1}{\pi} \left[\frac{\int_0^\infty \omega^2 \Phi(\omega) d\omega}{\int_0^\infty \Phi(\omega) d\omega} \right]^{1/2} \quad (2)$$

(↗ and ↘)

$$\text{Number of peaks per sec} = N_1 = \frac{1}{\pi} \left[\frac{\int_0^\infty \omega^4 \Phi(\omega) d\omega}{\int_0^\infty \omega^2 \Phi(\omega) d\omega} \right]^{1/2} \quad (3)$$

(maxima and minima)

$$\text{Irregularity factor } k = \frac{\text{number of peaks}}{\text{number of mean-crossings}} = \left[\frac{\int_0^\infty \Phi(\omega) d\omega \int_0^\infty \omega^4 \Phi(\omega) d\omega}{\left\{ \int_0^\infty \omega^2 \Phi(\omega) d\omega \right\}^2} \right]^{1/2} \quad (4)$$

The distribution density function of the peak values can also be derived from $\Phi(\omega)$ but it is a fairly complex equation. For narrow band random loading ($k \sim 1$) it reduces to a simple relation, which is known as the Rayleigh distribution.

With a "normalized" stress $\alpha = \frac{\sigma - \sigma_m}{\sigma_{rms}}$ $f(\alpha) = \alpha \cdot e^{-\alpha^2/2}$

Figure 6 shows the distribution density function (for maxima only). It confirms for narrow band random loading ($k \sim 1$) that all maxima occur above the mean (σ_{max} always $> \sigma_m$), whereas for more irregularly random loading ($k = 1.4$) σ_{max} -values below σ_m do occur.

Perhaps it should be pointed out that the irregularity factor k gives a good indication of the irregularity of random loading, but it does not fully describe random loading. This can only be done by $\Phi(\omega)$ and theoretically different $\Phi(\omega)$ functions can still give the same k -value. (Note: In the literature some authors define the irregularity factor as $1/k$.)

4. K-concept and random load

If a fatigue crack is growing under a random load, $\sigma(t)$, the stress intensity in the crack tip area will vary in the same random way. We then can define a root-mean-square value of the variation of the stress

intensity factor K:

$$K_{rms} = C \sigma_{rms} \sqrt{\pi a} \quad (6)$$

with σ_{rms} as defined in Eq. (1) and C and a as defined in the first lecture. For a certain spectral density function we may then expect that K_{rms} will fully describe the random variation of the stress intensity at the tip of the crack.

It should then be expected that the crack rate will be a function of K_{rms} , or:

$$\frac{da}{dn} = f(K_{rms}) \quad (7)$$

The relation will depend on the mean stress σ_m . A stress ratio R as defined for CA-loading ($R = \sigma_{min}/\sigma_{max}$) cannot be defined in the same way for random loading, but the relative severity of the mean stress is given by:

$$\gamma = \frac{\sigma_m}{\sigma_{rms}} \quad (8)$$

Similarity then implies that random stress variations will be similar if both K_{rms} and γ are equal, and consequently Eq. (7) should read:

$$\frac{da}{dn} = f(K_{rms}, \gamma) \quad \text{or} \quad \frac{da}{dn} = f_{\gamma}(K_{rms}) \quad (9)$$

Limited evidence in the literature has confirmed the applicability of this approach (Refs. 3-5). The function f_{γ} might depend on the PSD function, although some experiments have suggested that this effect will be small (Refs. 3,4).

5. K-concept and stationary non-random loading

A simple example as shown in Fig. 7 can be considered as a periodic phenomenon, for which a period and a characteristic stress are easily

Fig. 7

defined. Similarly to Eqs. (6) - (9) we now define a characteristic stress intensity factor:

$$K_{char} = C \sigma_{char} \sqrt{\pi a} \quad (10)$$

and it may then be expected that:

$$\Delta a / \text{period} = f(K_{char}) \quad (11)$$

Since there is always the same number of cycles in one period Eq. (11) can be written as:

$$\frac{da}{dn} = f(K_{char}) \quad (12)$$

The larger cycles will delay crack growth during the smaller cycles, and this will affect the function f . However, if crack growth, including delay effects, can still be considered to be a stationary process the similarity approach should still be expected to be valid. Of course the two sequences shown in Fig. 8 will not have the same $f(K_{char})$ since the "wave shapes" are different. It implies that Eq. 12 should be more generally written as:

$$\frac{da}{dn} = f(K_{char}, \text{wave shape}) \quad (13)$$

Fig. 8

Let us now consider an example with a much longer period as shown below (Fig. 9). This sequence is almost CA-loading with periodically

inserted peak loads, which subsequently cause noticeable crack growth delay (Fig. 10).

Fig. 9

Fig.10

Fig.11

The period is supposed to be sufficiently long (i.e. to continue a sufficiently large number of cycles) in such a way that the delay is over before the next peak occurs. That implies that crack growth on a macro-scale apparently is no longer a stationary process. The size of the discontinuity in the crack growth curve is of a similar magnitude as the plastic zone (r_p) caused by the peak load. It should be expected that Eq. (13) is no longer applicable.

If the period is shorter the delays still occur, but the delays are less obvious from the crack growth curve, see Fig. 11 (except perhaps for the first one). The crack growth curve suggests an almost quasi-stationary process, while in reality a continuous superposition of delay effects is causing a most effective growth retardation. The applicability of Eq. (13) might be questionable.

Since high peak loads have a large effect on crack growth under VA-loading, the return period of the high loads has a significant meaning for the stationarity of VA-loading. Large return periods will upset the stationarity of VA-loading. This can apply to flight-simulation loading and random loading, as discussed later (truncation of high loads, crest factor).

6. Crack growth under flight-simulation loading

A flight-simulation test as applied nowadays to full-scale aircraft structures, components and specimens, consists of a sequence of loads as they occur in service. The ground-air-ground transition is a deterministic

load cycle, occurring once per flight. Statistically variable loads will be added in flight (gusts, manoeuvres) or in the ground phase (taxiing loads). The example shown in Fig. 12 (Ref. 6) applies to a civil transport aircraft for which gust loads in flight are predominant, while taxiing loads were omitted since they occurred in compression. Ten different types of flight (A - K) were applied in a random sequence and the gusts in each flight were also applied in a random sequence.

Fig. 12

The question now is whether such a flight-simulation loading could still be a stationary VA-loading. Let us consider four possibilities of aircraft operation in service.

- (a) Always the same flight under the same weather conditions (apparently not in Fig. 12).
- (b) Always the same flight under various weather conditions, but the variation of the weather being a stationary random process (Fig. 12).
- (c) As (b) but different flights.
- (d) Good weather conditions during summer and poor weather conditions during winter time.

In spite of the mixture of deterministic and random loading as it occurs in flight-simulation loading, it still may be expected to be stationary VA-loading, if the statistical properties of all types of loads are independent of time. We then should expect that cases (a) and (b) are examples of stationary VA-loading, while the same could apply to case (c), provided that the variation of the flights has a stationary character. However, case (d) appears to have a systematic return period of one year,

which could be too long for crack growth and thus upset the stationarity. Another restriction may be caused by rarely occurring very high loads, as will be discussed later.

We now have to define a characteristic stress for flight-simulation loading. The most obvious choice from a design point of view is the ultimate design stress level. However, any other typical stress related to this level is equally suitable. It is common by now to adopt the mean stress in flight (1g-condition) as a characteristic stress for flight-simulation loadings. Denoting this stress as σ_{mf} a characteristic stress intensity factor for flight-simulation loading can be defined as:

$$K_{FS} = C \sigma_{mf} \sqrt{\pi a} \quad (14)$$

Both σ_{mf} and K_{FS} can be meaningful only if all other stresses are linearly related to σ_{mf} , and we may ask whether that is a practical situation.

Fortunately it is, because:

(1) A change of a local stress level in a structure by a structural modification (effect on nominal stress level, effect on stress concentration) will affect all induced stresses by gusts, manoeuvres, ground-air-ground transitions, etc., in a proportional way.

All stresses will change with the same percentage as σ_{mf} .

(2) If we compare stress levels at two different locations in the same structure all induced stresses will occur in the same ratio. Consequently the ratio of σ_{mf} values will be characteristic.

The effect of changing σ_{mf} on the load spectrum is illustrated by Fig. 13.

It now might be hoped that the crack growth rates for different σ_{mf} -values can be correlated by a single function:

$$\frac{da}{dn} = f(K_{FS}) \quad (15)$$

Fig.14

This was checked in crack growth tests on 2024-T3 and 7075-T6 sheet material (Ref. 6). The results of the former alloy are plotted in Fig. 14. If Eq. (15) were applicable the five curves in the right hand graph should have merged into a single scatter band. Unfortunately they do not, and

this is unfortunate indeed, because otherwise Eq. (15) would allow to extrapolate results obtained for a certain design stress level to another stress level. This would be most helpful in a design office in studies on the effect of design stress levels.

If $da/dn = f(K_{rms})$ (Eq. 7) is still valid to random load, why does Eq. (15) not apply to flight-simulation loading? It should be expected that an insufficient stationary character of flight-simulation loading will be significant here. This will be discussed in the following section.

7. Truncation of high loads

A "load spectrum" as it occurs in service is frequently presented as an exceedance curve, i.e. the number of times that load levels are exceeded in a specified time interval. As a qualitative example Fig. 15 shows how many times stress amplitude levels are exceeded in a wing structure during 5000 flights. Such a curve is closely related to a distribution

Fig. 15

function. It requires a good deal of effort to arrive as such a curve. First the anticipated utilisation of the aircraft has to be analysed. Second, gust load statistics for all relevant flying conditions have to be compiled. Finally the gust loads have to be converted into stress

levels in the structure.

A weak point in the gust load statistics is the upper end of the exceedance curve, associated with the very severe gust loads (stormy weather), having a very low probability of occurrence. In this area gust load statistics are less reliable. Secondly, we cannot be sure whether all aircraft will meet the same severe storms. It is even more

likely that they will not, which implies that the upper end of the load spectrum will vary from aircraft to aircraft. Unfortunately the very high loads can have a predominant effect on fatigue life and crack growth.

Fatigue critical components of an aircraft are carrying a positive mean stress in flight. It implies that the extreme stress levels induced by a symmetric gust load spectrum will be larger in tension than in compression (see Fig. 16), or:

$$(\sigma_{\text{max,upwards}}) > |\sigma_{\text{max, downwards}}| \quad (16)$$

Very high loads will cause local yielding at notches and in view of Eq.(16) the probability of having compressive residual stresses (which are favourable) are much better than for tensile residual stresses (which are unfavourable). It implies that load spectrum I in Fig. 15 (higher maximum loads) will give a larger life than load spectrum II (lower maximum load). The difference can even be larger for crack growth, since negative loads will close the crack and the crack will not be a stress raiser under high downward loads (see also the 2nd lecture).

As a consequence flight-simulation tests will produce more conservative results (i.e. lower fatigue lives, faster crack growth) if spectrum II is adopted, although spectrum I may be the best estimate of the "average" load spectrum. Spectrum II implies that the rarely occurring high load amplitudes of spectrum I are truncated (not omitted!) to a lower level. Comparative fatigue tests with flight-simulation loading have amply confirmed the predominant effect of truncating high loads on fatigue life of notched specimens (e.g. Refs. 7,8,9) and on crack propagation (e.g. Refs. 7,10). As an illustration Fig. 17 shows results for three truncation levels. It will be noted that a higher truncation level implies longer crack initiation periods and longer crack growth periods.

Fig.17 (Ref.7)

Fig. 18. (Ref. 11)

In another test program (Ref. 11) with a somewhat more severe gust spectrum the truncation level was selected at a level occurring in 5 out of 4000 flights only (flights of type A, B and C). An example of average crack growth curves obtained is presented in Fig. 18. All three curves show growth delays caused by the severe flights. Obviously crack growth was not a stationary process in all three environments, although it almost appears to be so in dry air at a later stage (compare Fig. 18 with Figs. 10 and 11). The more severe flights upset the stationary character of crack growth and this may well explain why Eq. (15) was unable to correlate data obtained at different σ_{mf} -values (see Fig. 14). Actually the invalidity of Eq. (15) can also be interpreted to be a result of a dK/da -effect discussed in the 3rd lecture. This effect was expected as a result of the preceding K-history. Fortunately it was absent under CA-loading. However, crack growth delays as observed under flight simulation loading are clear manifestations of history effects.

8. Crack closure and non-stationary variable-amplitude loading

From the previous section we have learned that apparently stationary flight-simulation loading was insufficiently stationary due to rarely occurring high loads. A more simple example of unstationary VA-loading was discussed before with reference to figures 10 and 11. Under such conditions a prediction of crack growth by correlation, employing a characteristic K-value breaks down. We then have to return to the equation discussed in the third lecture:

$$a_N = a_0 + \sum_{i=1}^N \Delta a_i \quad (17)$$

It was pointed out in that lecture that crack growth could not be such a simple additive process as assumed in Eq. (17), because Δa_i will depend on the fatigue damage caused by the preceding load history (size of crack, mode of crack, crack closure, tip blunting, etc.). It implies that Δa_i will differ from the expected value (da/dn) based on the magnitude of the cycle and test data obtained under CA-loading. Effects causing such deviations are referred to as interaction effects, which can be either

positive or negative. For a positive interaction effect Δa is smaller than expected (retardation) and for a negative interaction effect Δa is larger than expected (accelerated growth). For certain load sequences retardations are easily observed as crack growth delays. Acceleration effects are not easily observed, but they have been indicated from striation observations.

Various mechanisms can contribute to interaction effects and they probably will do so. However, it seems that crack closure could be a predominant one. This was already proposed by Elber (Ref. 12) for a test with step loading. The principle how it could introduce an interaction effect is illustrated qualitatively in Fig. 19 by indicating how the crack closure level (σ_{cl}) changes from one stable level to another one. During the transition period ΔK_{eff} will differ from the expected value observed under CA-loading, and as a result crack growth will be slower (Hi-Lo case) or faster (Lo-Hi case).

Fig. 19

A similar transient behavior can occur after a peak load, see Fig. 20. The peak load itself will reduce σ_{cl} , but when the crack growth into the plastic zone created by the peak load it will increase beyond the original level, while it will return to the original stable level at a later time. As a consequence the peak load will be followed by an acceleration first,

but later on the retardation will take over. This was already discussed in the second lecture with reference to Fig. 19 of that lecture. If the peak load is relatively high $\sigma_{cl.}$ can increase beyond σ_{max} and crack growth will stop (lower part of Fig. 20). It was shown that a Limit Load applied to a wing structure successfully stopped the growth of fatigue cracks (Ref. 7).

Fig. 20

Some empirical evidence of the transitional behavior of $\sigma_{cl.}$ and the associated effects on crack growth was discussed in the 2nd lecture. Further, crack closure and the transitional behavior during step loading (Fig. 19) also follow from elasto-plastic calculations employing finite-element techniques (Refs. 14,15). Such calculations are fairly expensive, but it is highly instructive to know that crack closure considerations can be checked by calculations also. Beyond any doubt a physically sound theory on crack growth under VA-loading should include crack closure. However, it should be kept in mind that interaction effects may stem from other sources as well. This should be clear from the discussion in the two previous lectures. The significance of other sources can be explored in experiments, provided that crack closure is measured also. It certainly will remain a difficult issue to separate the

contribution of all possible mechanisms. Fortunately crack closure is a mechanism that can be observed experimentally, and it would be stimulating to see that crack closure has a predominant effect during VA-loading with service load-time histories, which might justify to ignore other interaction mechanisms.

Some exploratory COD-measurements during flight-simulation loading have clearly confirmed the occurrence of crack closure (Ref. 8). An example of measurements during a severe flight is shown in Fig. 21. After each severe upward gust cycle the position of the recorder pen was given a small horizontal shift to avoid coinciding σ - COD records. The non-linear behavior can be observed easily and the arrows indicate the crack closure stress.

After the first maximum load σ_{cl} was reduced to a lower level, in agreement with the results of more simple tests (e.g. Fig. 19 of the 2nd lecture). After some crack growth in subsequent flights the crack closure level increased again as should be expected. More systematic measurements are now carried out at the University in Delft, including the effect of truncation level.

Fig. 21.

Crack closure measurements during random load test were recently carried out by Elber (Ref. 16). The variation of σ_{cl} during his tests on 7075-T6 sheet specimens was fairly small (see Fig. 22) and he therefore proposed to adopt the average value of a test to define ΔK_{eff} -values for all individual cycles. Employing these values and a linear damage summation with $a = a_o + \sum \Delta a$ (Eq. 17) and $\Delta a = da/dn = f(\Delta K_{eff})$ he could make a comparison between prediction and test results. A reasonable agreement was found, but the number of tests was still fairly low. Moreover the return period (~ 1500 cycles) was small. Anyhow, it seems worthwhile to study this average ΔK_{eff} -approach in more detail in the future.

Fig.22 (Ref.16)

One aspect that should be studied is the significance of crack closure in explaining the truncation effect mentioned in the previous section. Higher truncation levels will give larger plastic zones and possibly more crack closure in flight-simulation tests. The same problem can arise under pure random loading. For the larger load amplitudes the distribution density function is approximately given by (Ref. 17):

$$f(\alpha) = \frac{\alpha}{k} e^{-\alpha^2/2}$$

(compare to Eq. 5) and the distribution function by:

$$F(\alpha) = \int_{\alpha}^{\infty} f(\alpha) d\alpha = \frac{1}{k} e^{-\alpha^2/2} \quad (19)$$

or

$$\ln F(\alpha) = -\ln k - \frac{\alpha^2}{2} \quad (20)$$

Fig. 23

Plotting α^2 as a function of $\ln F(\alpha)$ a linear relationship should be obtained. In practice deviations are found for larger α -values, which implies that the load spectrum is truncated (see Fig. 23). The deviations are due to physical restraints, such as damping and non-linear behavior at high α -values. Equation (19) would theoretically allow an infinite amplitude, which is obviously impossible. It is usual

to specify the physical restraint by the so-called crest factor (or clipping ratio), which is defined as:

$$\text{crest factor} = \frac{\text{maximum amplitude}}{\text{rms-value}}$$

For the normalized stress amplitude α the crest factor would simply be $\alpha_{\max}$. The magnitude of the crest factor will depend on the dynamic response of the vibrating system. Since the crest factor is representative for the truncation of random load, more should be known about its effect on fatigue. However, this problem hardly received any attention in the literature. Several investigators reporting results of random load fatigue tests even do not specify the value of the crest factor in their tests. An exploratory test program seems to be worthwhile.

9. References

1. J.S. Bendat - Principles and applications of random noise theory. John Wiley Sons, New York, 1958.
2. J. Kowalewski - On the relation between fatigue lives under random loading and under corresponding program loading. Full-Scale Fatigue Testing of Aircraft Structures. Pergamon 1961, p. 60.
3. P.C. Paris - The fracture mechanics approach to fatigue. Fatigue, an Interdisciplinary Approach, Syracuse Un. Press, 1964, p. 107.
4. S.H. Smith - Fatigue crack growth under axial narrow and broad band random loading. Acoustical Fatigue in Aerospace Structures, Syracuse Un. Press, 1965, p. 331.
5. S.R. Swanson, F. Cicci and W. Hoppe - Crack propagation in Clad 7079-T6 aluminium alloy sheet under constant and random amplitude fatigue loading. ASM STP 415, 1967, p. 312.
6. J. Schijve, F.A. Jacobs and P.J. Tromp - Fatigue crack growth in aluminium alloy sheet material under flight-simulation loading. Effects of design stress level and loading frequency. NLR TR 72018, Amsterdam 1972.
7. J. Schijve - The accumulation of fatigue damage in aircraft materials and structures. AGARDograph No. 157, 1972.
8. J. Schijve, F.A. Jacobs and P.J. Tromp - Flight simulation tests on notched elements. NLR TR74033, Amsterdam 1974.
9. D. Schütz and H. Lowak - Application of design rules for fatigue life estimates based on service simulation test results (in German). LBF Bericht FB-109, Darmstadt 1976.
10. W. Schütz and H. Zenner - Crack growth in titanium alloy sheets under flight-simulation loading (in German). IABG Report TF-224, Ottobrunn 1972.
11. J. Schijve, F.A. Jacobs and P.J. Tromp - Environmental effects on crack growth in flight-simulation tests on 2024-T3 and 7075-T6 material. NLR TR 76104, Amsterdam 1976.
12. W. Elber - The significance of crack closure. ASTM STP 486, 1971, p. 230.
13. J. Schijve - Observations on the prediction of fatigue crack propagat-

- ion under variable-amplitude loading. ASTM STP 595, 1976, p. 3.
14. K. Ohji, K. Ogura and Y. Ohkuba - Cyclic analysis of a propagating crack and its correlation with fatigue crack growth. Eng. Fract. Mech., Vol. 7, 1975, p. 457.
 15. J.C. Newman - A finite-element analysis of fatigue crack closure. ASTM STP 590, 1976, p. 281.
 16. W. Elber - Equivalent constant-amplitude concept for crack growth under spectrum loading. ASTM STP 595, 1976, p. 236.
 17. J. Schijve - Some notes on random load and fatigue (in Dutch) NLR TM M2071, 1960.

Rapport 254

60141070355