An experimental study of reconstruction accuracy using a 12-Camera Tomo-PIV system

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ABSTRACT

A tomographic PIV system composed of a large number of cameras is used to experimentally investigate the relation between image particle density, number of cameras and the reconstruction quality. The large number of cameras allows to determine an asymptotic behavior for the object reconstruction over a wide range of values for the tracer particles concentration. In order to quantify the effect of additional cameras, the quality of the reconstruction is evaluated through the reconstruction signal-to-noise ratio and the normalized intensity variance.

The use of additional cameras even with small relative angular separation is shown to reduce the formation of ghost particles in the reconstructed volumes, resulting in a greater reconstruction signal-to-noise ratio and variance of the reconstructed object intensity. Object reconstructions are obtained for levels of the seeding density almost three times greater than a typical four-camera tomographic PIV system, based on evaluation of the reconstruction signal to noise ratio. This approach offers the perspective for future tomographic PIV measurements of higher resolution, particularly suited for the study of high-Reynolds number turbulent flows.

1. INTRODUCTION

The technique of tomographic particle image velocimetry has rapidly become widespread since its inception by Elsinga et al. (2006), in part due to its acute relevance to measurement needs in aerodynamics and turbulence research. This is highlighted by a recent review article (Scarano, 2013) outlining successful tomographic PIV experiments carried out on cylinder wakes, turbulent boundary layers, jets, and biological flows, to name a few. A well-known bottleneck of tomographic PIV is the maximum achievable particle density, which ultimately limits the spatial resolution or depth of the measurement. As particle density on the sensors increases, the quality of the reconstruction decreases due to the large number of active lines-of-sight contributing to the reconstruction (see e.g., Elsinga et al., 2006). This leads to the increased formation of ghost particles that contaminate the cross-correlation signal. This contamination can bias the velocity field (Elsinga et al., 2011) or render it immeasurable.

One measure of the particle density is the ratio of particles per pixel (ppp). By the use of synthetic and experimental tests, an early limit for an individual camera in a four-camera system was established at approximately 0.05 (Elsinga et al., 2006). However, this quantity does not take into account the effect of the particle image diameter and thus provides only a coarse estimate of how “full” the particle images are, and consequently, the number of active lines of sight in the reconstruction procedure. For example, considering two images at the same ppp, those with a larger particle diameter will render a larger proportion of total sensor pixels covered. Thus, fewer particles are needed before all lines of sight of the camera are active. A parameter for characterizing this effect is the source density $N_s$, which can be defined by following the approach of Novara et al. (2010) by multiplying the ppp by the particle image area or by working in terms of particles per voxel (ppv) and multiplying by the volume depth,

$$N_s = \text{ppv} \cdot \frac{\pi}{4} \cdot d_i^2 = \text{ppv} \cdot D \cdot \frac{\pi}{4} \cdot d_i^2$$

where $d_i^2$ is the particle image diameter in pixel units and $D$ is the volume depth in voxel units. From this relation it is clear that either increases in the particle concentration within the volume (ppv), increases in depth (D), or larger particle images (e.g., increased f-stop) lead to an increase in the source density. A range of experiments have suggested that for a four-camera system, the maximum attainable value of $N_s$ is approximately 0.3 (Scarano, 2013). The source density is critical in that it defines the number of active lines of sight that are capable of forming particles. Elsinga et al. (2006) explored this behavior by establishing a functional relation between the ratio of the number of real particles $N_p$ to ghost particles $N_g$ in a reconstructed volume as a function of the source density, the number of cameras, and the volume depth. This model has been updated in recent works such as Novara (2013),

$$\frac{N_p}{N_g} = \frac{\pi d_i^2}{4N_s N_c \Delta Z}$$

where $N_c$ is the number of cameras in the system and $\Delta Z$ is the length of the camera lines-of-sight (approximately the volume depth) in voxel units. This model predicts that adding cameras to a system will result in the reduction of the
number of ghost particles for a chosen value of the source density, in turn improving the quality of the resulting velocity measurement.

As an aside, it is worth noting that as an alternative to the hardware-based approach of adding cameras to the measurement, software-based approaches for increasing the achievable source density have been explored. In particular, the motion-tracking enhanced MART (MTE-MART) as proposed by Novara et al. (2010) is a suitable technique which takes into account the non-coherent motion of ghost particles within an image sequence; however, the required processing time is over an order of magnitude greater than standard tomographic PIV, rendering the technique suitable only in specialized circumstances. Other recent works include the simulacrum matching-based reconstruction enhancement (SMRE) technique proposed by de Silva et al. (2013) which aims to remove ghost particles a posteriori the reconstruction process. Results indicated notable improvements in reconstruction quality for source densities up to around 0.4. However, none of these works detail operation at source densities in excess of this amount, indicating that software-based approaches may face limitations in this regard.

Due to the continuing drop in prices of scientific interline CCD cameras, their use in large arrays is potentially attractive for overcoming the current seeding density limitations in tomographic PIV. A recent example of this is the work of Fukuchi et al. (2012) who used an 8-camera system for thick-volume measurements in the wake of a cylinder. Their work showed successful measurements at a high source density (a value of ppp = 0.5 has been reported) and a measurement volume on the order of 50 mm; however, a rigorous analysis of the reconstruction quality including a parametric study of increasing/decreasing the number of cameras is still missing.

This study investigates the fundamental relations between image source density, number of independent views and reconstruction quality making use of a 12-camera tomographic PIV system. The work aims at providing an experimental counterpart to the theoretical and numerical studies on this subject, which require scrutiny in terms of the validity of simplifying assumptions adopted in the simulations.

2. EXPERIMENTAL APPARATUS

A simple air flow setup is constructed that features a centrifugal fan channeled into a duct to produce a 2D jet. To enable a high particle concentration without diffuse reflections or fluid opacity interfering in the measurement, air is used as the working medium instead of water. The flow at the jet exit is obstructed by a small square-section prism that produces a typical bluff-body wake. This system is enclosed in a Plexiglas box of dimensions 60x45x45 cm. A stage-smoke generator is used to seed the flow with particles of 1 micron mean diameter.

The 12-cameras tomographic system is organized with three decks where four cameras are aligned, as shown in figure 1 (left). The cameras are LaVision Imager LX 2MP interline CCD (1628x1236 pixels, pixel pitch 4.4 μm). Each camera is equipped with a 75 mm lens attached to custom-manufactured Scheimpflug adapters and set to an aperture of f/# = 11. At this aperture, diffraction effects are the primary driver of particle image size; a calculated value of the particle image diameter is approximately 3.8 pixels. Note this is a large particle image diameter as compared to typical PIV studies, due primarily to the small pixel size of the cameras.

![Figure 1. Photos of the experimental setup including a view of the entire camera array and Plexiglas box (left) and close-up view of individual camera highlighting low-cost mounting brackets and Scheimpflug mounts.](image-url)

Illumination is provided by a Spectra-Physics PIV-400 laser (400 mJ/pulse). The output beam is conditioned using a series of optics and knife edges on all four sides to provide a collimated 20 mm thick illuminated volume. A mirror on the opposite side of the box is used to reflect the laser sheet back through the measurement volume to increase the laser intensity and equalize the light intensity for cameras in a backward-scattering configuration. The time separation...
between laser pulses was set to 40 μsec as previously determined by planar PIV to yield average displacements at the fan exit of approximately 12 pixels. A summary of these optical and laser parameters are given in table 1.

Table 1. Summary of physical parameters of the experimental setup.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel Size</td>
<td>4.4 μm</td>
</tr>
<tr>
<td>Sensor Size</td>
<td>1628 x 1236</td>
</tr>
<tr>
<td>Magnification</td>
<td>0.17</td>
</tr>
<tr>
<td>Digital resolution</td>
<td>40 vox/mm</td>
</tr>
<tr>
<td>Objective focal length</td>
<td>75 mm</td>
</tr>
<tr>
<td>F-number</td>
<td>11</td>
</tr>
<tr>
<td>Particle image diameter</td>
<td>3.8 px</td>
</tr>
<tr>
<td>Laser pulse separation</td>
<td>40 μsec</td>
</tr>
<tr>
<td>Laser energy</td>
<td>400 mJ/pulse</td>
</tr>
</tbody>
</table>

The cameras are connected to a single computer using a series of four-port gigabit Ethernet boards, and controlled simultaneously using Lavision Davis 8.1 software and a PTU9 triggering unit. The calibration of the cameras is performed using a dual-plane calibration target (Lavision Type 11) with dots positioned at 5 mm increments. The calibration target is lowered into the box and positioned at the center and both edges of the illuminated volume for a full volume calibration using a third-order polynomial fit. All cameras were checked for calibration accuracy and found to have calibration fitting errors no larger than 0.3 pixel. A further refinement of the calibration was performed using the volume self-calibration technique (Wieneke, 2008) using a set of 100 images at very low seeding density. Following two iterations of the self-calibration procedure, triangulation errors were reduced to the order of 0.01 pixel for all cameras. This procedure was performed before and after all data acquisition runs and cross-checked to ensure no degradation of the calibration accuracy during the acquisitions.

3. DATA PROCESSING

Image preprocessing consists of multiple steps: first, a subtraction of a full-set minimum calculated for each individual camera of the set. This has been suggested by Fukuchi (2012) as a superior technique for high source densities compared to local kernel methods such as sliding minimum subtraction. Following this, a normalization is applied by dividing the images by a smoothed average of the full set for each individual camera, and subtracting a constant noise floor that remains in the image as determined by visual inspection. No Gaussian smoothing is applied, in accordance with the already substantial particle image diameter of 3.8 px.

The selection of the cameras used as subsets of the full system are shown in figure 2. These are chosen in a manner to optimize the total aperture as additional cameras are added. Note that separate runs are not performed; to generate the subsets, the images are extracted from the full 12 camera measurement and then the reconstruction is performed on the reduced set.

The reconstruction is conducted using the fast MART procedure within the Lavision 8.1 software; the settings for the processing are outlined in table 2. A 25% buffer was added to the front and rear of the measurement volume to prevent edge effects of the MART reconstruction from interfering in the calculations. The reconstructed volume size is thus 20 x 20 x 40 mm.

Table 2. Summary of reconstruction parameters.
To vary the particle density during the run, the box is initially loaded with a large number of particles from a Safex fog generator. The top of the box contains a small opening which allows for mixing between the seeded air within the box and the unseeded ambient air. Thus, over the course of a run the particle density decays and the data collected from a single run can be used to sample the particle density rather than relying on multiple independent runs. However, this approach requires an in-situ method for determining the ppv and source density as the run progresses. As suggested by Fukuchi (2012), a removable slit can be used to temporarily thin the laser illumination to allow for an unambiguous determination of the particle density. In that work, a peak detection algorithm was used to determine the particle density of a 2-D slit image. The resulting pp from the slit image was then multiplied by the ratio of the full laser illumination depth to slit thickness to arrive at the actual particle density for the full measurement. A similar procedure is used in this work by acquiring data at 1 Hz, and manually inserting a 2 mm slit into the collimated portion of the laser beam once every 5 images. These images are extracted from the acquired sequence and processed separately, allowing for a sampling of the particle density during the run. An example of these images is provided in figure 3, showing the image with and without the slit. Clearly, individual particles are unable to be detected without the use of the slit.

For actual determination of pp and source density, a modification of the work of Fukuchi (2012) is proposed, whereby the reconstructed volumes from the slit images are used for a direct estimate of the ppv. Because the seeding density using the slit is approximately 10 times lower than the other images, and all 12 cameras are used for this reconstruction, the reconstructed slit volumes can be assumed to consist of only true particles (see e.g. equation 2 for low source density and large number of cameras). This was verified during data processing by watching the reconstructed volumes and clearly identifying the slit location and a complete lack of particles generated outside of the illuminated slit region. Within the reconstructed volume, a 3-D peak-finding algorithm is used to identify unique particles within the region of the volume containing the slit. By dividing by the total number of pixels searched, an estimate of the ppv is achieved. By multiplying by the full sheet thickness, the ppv is converted to the pp which would be imaged by a camera oriented normal to the laser volume.

Multiple advantages exist by using this approach versus the use of the raw camera images: first, the elimination of a variation in pp due to the geometry of the camera configuration. Briefly, a camera oriented at a large angle to the measurement volume will image a slightly larger particle density due to the increased length of the lines of sight intersecting the illuminated volume. Second, the slit thickness can be directly measured, providing an accurate scaling factor for conversion between the slit illumination and the full illumination. Finally, the sparsity of the reconstructed volumes is well-suited for 3-D peak finding and removes the problem particle overlap which is encountered when analyzing the particle density in 2-D images.

The results of this algorithm on the acquisition run is shown in figure 4. Due to the measurement being performed in air, the stability of the particle density is more difficult to achieve compared to water tunnel testing, and inadequate mixing of the air leads to masses of highly seeded fluid entering/exiting the measurement volume during the run. However, an overall decaying trend is observed which is consistent with the operation of the experimental apparatus. This curve is used to associate each image of the run with a specific particle/source density.
3. ANALYSIS OF RECONSTRUCTION QUALITY

The quality of the reconstruction in an experiment cannot be explicitly determined due to the fact that the actual reconstructed object is unknown. However, indirect methods can be used to analyze the quality and characteristics of the reconstructed object. This study relies on the representation of the laser sheet within the volume, and a second is based on the probability density function of the intensity distribution within the reconstructed object.

Laser Sheet Representation

The MART procedure is an approximate solution method for a highly underdetermined system of equations, thus multiple incorrect solutions manifest themselves in the true solution as a collection of ghost particles distributed within the reconstruction. Within the illuminated region of the volume, both real particles and ghost particles will be formed. Outside of this illuminated volume no scattering of light occurs, so the residual intensity reconstructed in these regions is only due to the formation of ghost particles. A well-performed measurement should allow the laser sheet to be unambiguously identified within the reconstructed volume. As more cameras are added to the system, the number of uniquely intersecting lines of sight decreases, which reduces the formation of ghost particles and redistributes their intensity towards the real particles (see e.g., Elsinga et al., 2006). This has a double effect of increasing the signal level within the laser sheet and decreasing the signal outside of the laser sheet.

To provide further context for the discussion and show this process visually, consider figure 5 which provides a profile of the laser sheet intensity along the depth direction for the case of 4, 8, and 12 camera reconstruction from a single measurement snapshot at an identical source density of approximately 0.3. As more cameras are added, the intensity outside of the laser sheet is reduced, and in general, the intensity within the laser sheet is increased.

To quantify the effect more clearly, a signal-to-noise ratio can be established for the tomographic reconstruction, which represents the ratio of ghost signal to actual signal. Since ghost particles are generated within the illuminated region with an equal probability as ghost particles generated outside, an SNR calculated as the ratio of illuminated region intensity versus nonilluminated region intensity provides a global estimate of the reconstruction quality. Thus the SNR is defined simply as,

\[
\text{SNR} = \frac{E_t}{E_o}
\]  

(6)

where \(E_t\) and \(E_o\) represent the intensities inside and outside of the laser sheet, respectively, as indicated by the flat bold lines in each of the reconstructions presented in figure 6. These levels are calculated by averaging over the profile within regions of the volume known to contain the laser sheet as determined by reconstructions at a low seeding density. Note that there are two sets of levels shown in each plot, corresponding to the first and second frames of the reconstruction, respectively. The differences in these two reconstructions highlight the fact that while the geometrical arrangement of cameras and number of particles is approximately constant between frames, variations in illumination can result in large differences in the reconstruction behavior.

![Figure 5. Laser sheet intensity profiles and reconstruction SNR values for 4, 8, and 12 cameras.](image)

The above figure considers the case of a single source density; however, by varying the source density as an independent variable it is possible to evaluate the robustness of tomographic systems with varying numbers of cameras by considering the SNR as the dependent variable. In addition to the experimentally determined SNR values, a theoretical estimate of the reconstruction SNR can be established based on the ratio of ghost to real particles generated, as per the expression given in the introduction with equation 2. Both analyses are given in figure 6. Note that the theoretical values have a value of 1 added to correspond to the definition of equation 6 (i.e., within the volume both real
and ghost particles are formed). As additional cameras are added to the system, the range of source densities that result in SNR values above 2 is enlarged. For example, the threshold of 2 for the SNR is crossed at a source density of approximately 0.25 for a 4-camera system, 0.4 for a 8-camera system, and 0.5 for a 12-camera system. This is consistent with the best practices established by Scarano (2013) which suggested a useful maximum source density of 0.3 for a 4-camera system. Figure 7 suggests that the use of 12 cameras can provide a doubling of the maximum source density achievable for the reconstruction.

Regarding the theoretical estimates, the overall trend is valid: adding additional cameras suffers from the problem of diminishing returns to the exponential behavior of equation 2. However, the correspondence between the experimental and theoretical values is tenuous. This is conjectured to result from two factors: first, the geometrical configuration of the camera system is not modeled using the simple expression of equation 2; second, the theoretical model only considers the ratio of the number of ghost particles generated rather than the intensity. The experimental estimates here are a result of an intensity maximum operation and thus are not a suitable estimate of the number ratio of particles.

![Figure 6](image)

**Figure 6.** Reconstruction SNR for varying numbers of cameras and varying source densities. Dashed lines indicate theoretical values as predicted from equation 7.

**Volumetric Intensity Variance**

A second estimate of reconstruction quality is found by considering the contrast of the reconstructed particles within the volume as suggested by Novara et al. (2010). The contrast can be statistically defined as the variance of the signal; a high variance indicates a sparse reconstructed field with high-amplitude peaks representative of particles; conversely, a low value is associated with low-amplitude, dense peaks which are trademark features of ghost particles. To generalize the metric, the variance can be normalized by the mean value of the volume, yielding the normalized intensity variance $\sigma_E^2$.

$$\sigma_E^2 = \frac{\sum_i (E_i - \bar{E})^2}{\bar{E}}$$

where $i$ is a voxel index and $\bar{E}$ is a scalar representing the average of all voxels. This approach has the advantage of not requiring a reference volume, allowing the possibility of establishing a simple criterion for reconstruction quality that does not require large reconstructions and averaging such as the reconstruction SNR. Figure 7 shows the value of $\sigma_E^2$ for the cases of 4 and 12 cameras. For high source densities the 12 camera system returns a variance approximately two to three times higher than the 4 camera case, indicating a possible scaling similar to the reconstruction SNR. However, at lower seeding densities, the values appear comparable to each other, since the two systems are reconstructing approximately an identical volume with very few ghost particles present.
CONCLUSIONS

A preliminary study into the effect of a large number of cameras on tomographic reconstruction quality has been performed, to serve as an analogue to synthetic studies performed by various other researchers. Measurements were performed in air and the particle density was varied in a decaying fashion by allowing particles to escape from a transparent box. The technique of slit illumination was used to provide an estimate of the particle density, and a new method for calculating this particle density was proposed based on the determination of the ppv from reconstructed volumes. This method was shown to be more robust compared to previous 2-D particle density determination efforts, and also free from ambiguities involving the geometric configuration of the cameras.

The reconstructions were analyzed using two metrics, the first being based on the reconstruction SNR and the second being based on the intensity variance within the volume. The former indicated drastic increases in system performance as additional cameras were added: based on an SNR threshold of 2, a 4-camera system was able to perform successful reconstructions at a maximum source density of approximately 0.12, and a 12-camera system increases this to approximately 0.32 (almost 3 times higher). This suggests that the asymptotic behavior of adding cameras seen in synthetic studies may begin at a greater number of cameras when experimental data is considered. Additionally, the theoretical estimates of ghost-to-real particle ratio seem to roughly fit the data for small numbers of cameras, but deviate significantly when large numbers of cameras are used. An open question remains whether the theoretical estimates overpredict the performance of a tomographic PIV system, or if the ghost-to-real particle ratio is a suitable metric for comparison to reconstruction SNR.

A final analysis was performed by investigating the variance of the intensity within the volume to determine if a metric could be identified for reconstruction quality that does not depend on large numbers of statistics or a reference volume. Initial results showed a scaling of the variance approximately equal to the increase in reconstruction SNR performance, however the values need additional analysis to identify a threshold of intensity contrast that corresponds to a successful reconstruction.

REFERENCES


