The Response of a Uniform Jig Bed in Terms of the Porosity Distribution
The Response of a Uniform Jig Bed in Terms of the Porosity Distribution

PROEFSCHRIFT

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Figure on cover:
Schematic view of the porosity distribution during a jigging stroke.

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To Jolanda
To Nienke
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Chapter 1

Introduction

1.1. History

Early in the history of mining it was found that heavy minerals could be separated from lighter ones by mixing with water in a tub and stirring or agitating the mixture until the heavier particles found their way to the bottom. It was soon found that if the tub had a porous bottom, and was moved up and down in water, the alternate upward and downward pulses produced a separation much more efficiently and quickly. This was jiggling in its primitive and fundamental form, and it was so effective and simple, both in equipment and operation, that it has remained an important concentration method up to the 20th century.\[1]\]

Throughout history, many theories have been developed to describe and explain the jiggling process. Due to its very complex character none of these theories can yet explain the jiggling process completely. This chapter will give a description of the jiggling process and a brief survey of the main current theories, leading to an introduction to the model described in this thesis.

1.2. The jig

In a jig a mixture of particles, supported on a perforated plate or screen in a layer or "bed" many times the thickness of the largest particle, is subjected to alternate rising and falling flows of fluid with the object of causing all the particles of high specific gravity to travel to the bottom of the bed while those of lower specific gravity collect at the top of the bed. In Figure 1.1 a schematic view of a jig is given.

During the upward and downward stroke of the fluid, parts of the jig bed will be fluidized and particles are able to move with respect to each other. During this stage stratification occurs, since it can generally be said that heavy particles will have a greater velocity downwards than will the light particles. After equilibrium is reached, layers with different densities will be present on the screen. The density will increase with decreasing distance from the screen.
The jigging process is, among others factors, influenced by:
- The jigging speed i.e. the number of strokes per second.
- The stroke length, i.e. twice the amplitude of the stroke.
- The stroke function. The fluid can be moved under a sinusoidal, a saw tooth, or an asymmetrical function.
- The bed thickness.
- The density of the different particle fractions.
- The diameter of the particles.
- The shape of the particles.

There are two main methods of jigging a mixture of particles:
1: On screen jigging
2: Through-screen jigging.

These two types differ from each other in the way in which the jig will discharge the heavy particles.
1.2.1. On screen jiggling

During the separation process both the heavy and light particles remain on the screen. The screen is of smaller aperture than the smallest particle in the mixture. The heavy particles will collect at the bottom of the bed and the light particles at the top of the bed. Removal of the lower stratified layer is accomplished by means of a gate of controllable height gate or of a variable-speed star valve that draws off a predetermined amount of concentrate.

1.2.2. Through-screen jiggling

During the separation process the heavy particles will fall through the screen. An intermediate layer, termed "ragging" (Magnetite, Haematite) is used, with a particle size larger than the screen aperture and a specific gravity between that of the light and heavy particles, to ensure that it remains at the bottom of the bed. During the jiggling process the light particles will collect at the top of the bed and the heavy particles will move through the ragging and fall through the screen. In Figure 1.2 a schematic view of the two types of jiggling is given.

![Figure 1.2: Schematic view of on-screen jiggling and through-screen jiggling.](image)

- ⊗ = Heavy particle
- o = Light particle
- ■ = Ragging
On-screen jiggling is generally employed for treating coarse material and through screen jiggling for fine material.

Throughout history the drive of the jig has been changed. Initially, only a mechanical drive was used; later on, hydraulic and pneumatic drives were developed to control the jiggling speed and jiggling function more accurately.

In this thesis, "jiggling" always refers to "on screen batch jiggling". This means that all the particles remain on the screen and that the discharge is left out of the considerations.

1.3. Practical use of Jigs

Jigging is today the ideal pre-concentration process, being relatively cheap in construction, operation, and maintenance, and relatively unaffected by the grade of feed. In some cases it will also produce a finished concentrate.

1.3.1. Metallic ore jiggling

In general, jiggling operations can be divided into two types:
1. Cobbing operations on coarse ore, in which the objective is to discard waste rock before final processing of the crude concentrate
2. Concentration operations, to recover finished concentrates from ores in which the mineral grains are liberated.

Jigs are in common use to recover cassiterite, scheelite, gold, manganese, lead-zinc, iron minerals, diamonds, barite and heavy minerals of all kinds that occur in decomposed rocks or that can be liberated by comminution without being reduced to slimes. The greater part of the world's tin is concentrated by jigs. Alluvial deposits of Southeast Asia, containing 0.13 kg of cassiterite per ton, are usually concentrated to 20% Sn with 98% recovery on dredges. In the concentration of fine heavy minerals from decomposed granite rocks in Africa, good recoveries of tantalite, columbite and cassiterite are obtained, down to 300 mesh.

The use of jigs to recover gold and other precious metals is widely practised since the
high specific gravity differential makes good recoveries possible with little operational attention. Jigs can tolerate large fluctuations in ore grade, tonnage rate, and dilution, which are characteristic of placer and dredging operations. Jigs used for pre-concentration of base metal ores can achieve high recoveries if the grade of concentrate is not important. This use of jigs can be very advantageous when carried out in conjunction with those separation processes which, although expensive on a tonnage basis, can produce high grade products with high recoveries. Examples of such processes are high-intensity magnetic concentration, selective flotation, and hydrometallurgy.

1.3.2. Coal jiggling

Jigging is an important process in the cleaning of coal: 65% of all the bituminous lignite and coal mined is mechanically cleaned, and almost 46% of this (approximately 100 million tons) is cleaned by jigs. Coal jiggling differs in several important aspects from metallic mineral jiggling:

- The coal is the light or low specific gravity part of the feed and the greater part by volume.
- The feed size range is large
- Ash, the impurity in the concentrate, is the criterion of quality.
- The coal has a specific gravity very close to that of the liquid.
- Coal particles, particularly coal fines, are difficult to wet.

1.3.3. Use in the secondary raw material industry, and some new applications:

- Jigging of car scrap: by means of a jig it is possible to concentrate several valuable metals from shredded car scrap.
- Cleaning of contaminated soil: by using jigs it is sometimes possible to clean up contaminated soil.

Generally a size range of about 7000 - 50 micrometers can be treated, with a capacity varying from 30 to 60 t/h/square meter of screen area for coal jiggling and 17 to 25 t/h/square meter for treating other material.[2]
1.4. Jigging theories

In the literature, several explanations of the jigging process may be found. Four main trends recognizable:

- A theoretical description of the jigging process, based on the movement of individual particles.
- A theoretical description of the jig bed as a whole, considering the energy changes in the bed.
- A theoretical description, based on a statistical approach to the movement of the particles.
- A more practical approach, based on segregation experiments.

1.4.1. The individual particle theories

The general mechanisms which have been postulated in these theories include differential acceleration, hindered settling, and interstitial trickling. These probably all occur, to a greater or lesser extent, in all jigs.[2]

Differential acceleration:

Differential acceleration of a particle is the initial acceleration at the start of particle movement. This acceleration depends only on the relative densities of the particle and fluid. Size of particle is not a factor. It follows that if the repetition of fall is frequent enough, and the duration of fall short enough, the distance travelled by the particles should be more influenced by their initial acceleration than by their terminal velocity. Under these conditions, stratification would take place on the basis of specific gravity alone.

Hindered settling:

After a slightly longer time, particles will have reached their terminal velocity. The terminal velocity of a single particle in a fluid can be calculated. As the concentration of particles in the fluid increases, the effect of particle crowding becomes apparent and the settling rate of the particles decreases. The system will behave as a heavy liquid
Figure 1.3 The three mechanisms of jiggling.

with the density of the slurry, not that of the fluid: this is known as "hindered settling". The terminal velocity depends on the particle's weight rather than on its specific gravity.

*Interstitial trickling:*

At the end of the downward stroke, as the bed grains begins to compact, the larger particles interlock, while the smaller particles move downwards through the interstices under the influence of gravity and of the fluid stream.

The movement of individual particles can be calculated by solving the equations of motion for the individual particles. Many authors have describe theories based on individual particle movement [3][4][5][6][7][8]. Generally, it may be concluded that these equations are very complicated; even if there is a mathematical or numerical solution it is not always possible to use it due to the fact that many of
the parameters necessary for solving these equations are not known, because they are
defined, in an unknown way, by time and by location in the jig bed. Such parameters
include the drag force on the individual particles and the density of the suspension.

Schubert[3] came to the following conclusion with respect to theories describing the
individual particle movement:
"Ein wesentlicher Mangel der Geschwindigkeits- bzw. Beschleunigungstheorien besteht
schliesslich darin, dass lediglich die Bewegung von Einzelkornern theoretisch verfolgt
wird. Es ist jedoch gerade die Eigenart des Setzprozesses, dass er nur mit einem
grössen Kollektiv von Körner überhaupt möglich ist. Diese Theorien können deshalb
nur die relative bewegung einzelner Körner erklären, nicht aber den Setzvorgang als
Ganzes beschreiben."

Of the theories based solely on the individual movement of particles the most
advanced is that developed recently by Beck and Holtham[9]. Once all particles have
been checked for collisions and the forces calculated and summed, the equations of
motion are integrated to determine the new velocity and position of each particle. Due
to the fact that the model simulates the movement of spherical particles in a two-
dimensional way, the moment at which the bed will start to fluidize is debatable. In
a two-dimensional bed consisting of spherical particles there are no channels between
the particles, so an increase of the superficial fluid velocity must result in an
instantaneous expansion of the bed.

1.4.2. Theories based on energy changes

Mayer[10][11][12][13] assumed that the action of the water pulse is purely
to open the jig bed, so that the denser particles are able to move down. In other words,
the bed is opened to permit the release of the potential energy of the mixed bed by
stratification and by lowering the apparent centre of gravity. King[14] has extended
the simple model of Mayer. Both models describe only the situation before and after
segregation; the dynamics associated with the attainment of the equilibrium are left out
of consideration. In Figure 1.4 a schematic view of the lowering of the centre of
gravity after stratification is shown.
Rong and Lyman\textsuperscript{[15]} made a general energy balance for a jig. The analysis is based on the energy and momentum equations of the fluid. Again, the way in which the particles will stratify is left out of consideration.

1.4.3. Theories based on a statistical approach

Siwiec\textsuperscript{[16]} assumed that the length of a single particle jump has a statistical gamma distribution. Based on this assumption, a mathematical model has been developed to describe the stratification. Again, it does not take into account the way in which the particles reach their new positions, and it is difficult to prove that the gamma function used is the right one.

Vetter\textsuperscript{[17],[18]} has developed a mathematical model based on a statistical dispersion approach. A description of bed pulsation and a quantitative treatment of this aspect of jigging are not directly included in the model.

1.4.4. Reflections on the jigging process based on practical experiments

Many authors carried out jigging experiments to investigate the influence of several jigging parameters, see references\textsuperscript{[19] -[30]}. They all based their work more on
the results of the experiments than on theory. These works are therefore of great interest with respect to the operation of jigs, but they generally do not make a fundamental contribution to jigging theory.

1.5. The porosity approach

From the literature, it is known that some authors recognize the importance of the porosity distribution during the jigging process. Hentzschel\textsuperscript{[31]} recognized the fact that particle movement is possible only if the jig bed is partially fluidized. He modelled the height of the fluidized parts as a function of time, but was unable to give the exact porosity distribution as a function of time and height in the jig bed. Uhlig\textsuperscript{[32]}, also, recognized the importance of the relation between stratification and the porosity distribution, but used only qualitative information, based on video images, regarding the porosity.

Rong et al.\textsuperscript{[33]}, in their article "Modelling jig bed stratification in a pilot scale Baum jig", conclude:

"Many researchers have focused on the loosening of the jig bed, or the bed porosity, and supposed that it is a significant parameter to control the degree of the bed stratification. However, the relationship between bed loosening and the operating jigging parameters has not yet been investigated in detail. Further, the measurement of bed loosening is sufficiently complex that no direct or indirect techniques have been developed to date."

It is clear that it is not possible to describe the jigging process in detail by using the theories of individual particle movement, or by theories using energy changes. Further, several authors have recognized that the loosening of the jig bed is an important parameter in describing the jigging process. Therefore, this thesis describes a study designed to predict the porosity distribution in a jig bed by using a mathematical model; to create a method for measuring the porosity in the jig bed; and to compare the results of the model with real conditions.

If the porosity distribution as a function of time and height in the jig bed is known it is possible to relate this knowledge to theories describing individual particle movement, giving more information about some parameters which are a function of
the time and height in the jig bed, such as drag force and relative density. This gives a greater insight into the jigging process and into the possibilities for optimizing the process in practice.

The theoretical model described in this thesis predicts the porosity distribution as a function of height and time in a uniform spherical jig bed. Uniform means that all the particles are of equal density and diameter. The model is based on the fact that zones with a certain porosity are generated at the bottom of the jig bed and will travel upwards with a time-dependent velocity.

A uniform jig bed was chosen as starting point to simplify the circumstances during one jigging stroke. The model given in this thesis will contribute to an improved insight into the behavior of particles in a jig bed, but it does not describe the entire jigging process with respect to stratification. To use the model to describe segregation it must to be extended to a multi-component system and equations of movement for the individual particles must be incorporated.

To investigate the phenomena in a jig, a laboratory jig was designed and built. The whole jig system was made transparent by using a particle - fluid system with the same refractive index. A method was developed for continually measuring the porosity in a uniform jig bed as a function of time and height in the bed. This method is based on the difference in light absorption between the fluid and the particles used in the jig bed. Using a laser at one and a photo diode at the other side of the jig it was possible to determine the transmission at a certain height above the screen.

1.6. References


[19] Batzer D.J., Brown D.J.; "Choice of jig stroke length and frequency in relation to concentrate flow";


[22] Aso K., Isayama Y.; "Studies on the stratification in Jig Concentration by the partition curve theory."; 1962


Chapter 2

The response of a liquid fluidized bed to sudden changes in the fluidizing velocity

2.1. Introduction

Slis et al. [1] have developed a mathematical model for the response of the level of a liquid fluidized bed to a sudden change in the fluidizing velocity. When the fluidizing velocity in a liquid fluidized bed of monosized solid particles of identical density is suddenly changed, a discontinuity in the porosity is introduced at the bottom of the bed. This discontinuity is propagated upwards through the bed. The boundary between the old and new porosity broadens or remains sharp depending on whether the porosity is increased or decreased. It was observed that, after a change in the fluidizing velocity, the new porosity of the bed (i.e. the equilibrium value for that velocity after the step-wise change) was created first at the bottom of the bed and then extended in an upward direction until the entire bed had obtained the new equilibrium porosity. The level change of the bed, after a change in the fluidizing velocity, is related to the way in which the porosity distribution travels upward.

2.2. Mathematical model for a fluidized bed.

If the fluidizing velocity is changed from \( U_0 \) to \( U_1 \), the porosity and the level of the bed change from the initial values \( \varepsilon_0 \) and \( h_0 \) to the final equilibrium values \( \varepsilon_1 \) and \( h_1 \). Because the total amount of solids in the bed remains constant, the mass balance is given by:

\[
(1-\varepsilon_0)h_0 = (1-\varepsilon_1)h_1
\]  

(2.1)

The relation between the fluidizing velocity and the porosity is given by the equation of Richardson and Zaki:
\[ U = V_T v^n \]  \hspace{1cm} (2.2)

where  
\begin{align*}
U & \quad \text{Superficial fluidizing fluid velocity} \quad \text{m/s} \\
V_T & \quad \text{Free falling velocity of one particle} \quad \text{m/s} \\
n & \quad \text{Richardson and Zaki constant}
\end{align*}

Eqs.(2.1) and (2.2) describe the situation before and at a sufficiently long time after the change in the fluidizing velocity. The mathematical model of Slis et al. describes the response of the level of a liquid fluidized bed to a sudden change in the fluidizing velocity, from the point at which this velocity is changed until the new equilibrium is reached. A discontinuity in the porosity is introduced at the bottom of the bed. This discontinuity is propagated upwards through the bed.

The equation of continuity for mass over a differential height, \( z \), is given by:

\[ \frac{\delta M}{\delta t} + \frac{\delta MV_p}{\delta z} = 0 \]  \hspace{1cm} (2.3)

where  
\begin{align*}
M & \quad \text{Mass density} \quad \text{kg/m}^3 \\
V_p & \quad \text{Average velocity of fluidized particles} \quad \text{m/s}
\end{align*}

Rewriting Eq.(2.3) gives:

\[ M = Cm = (1-\epsilon)\rho \]  \hspace{1cm} (2.4)

where  
\begin{align*}
C & \quad \text{Number of particles per unit of volume} \quad 1/\text{m}^3 \\
m & \quad \text{Mass of particles} \quad \text{kg} \\
\rho & \quad \text{Density of particles} \quad \text{kg/m}^3
\end{align*}

The total mass of the particles, \( m \), the total number of particles, \( C \), and the density remain constant during the whole process, so:

\[ \frac{\delta C}{\delta t} + \frac{\delta CV_p}{\delta z} = 0 \]  \hspace{1cm} (2.5)
\[ \frac{\delta (1-\epsilon)}{\delta t} + \frac{\delta (1-\epsilon)V_p}{\delta z} = 0 \]

so:
\[ \frac{\delta e}{\delta t} + (1-e) \frac{\delta V_p}{\delta z} + V_p \frac{\delta (1-e)}{\delta z} = 0 \] (2.6)

\[ \frac{\delta e}{\delta t} + (1-e) \frac{\delta V_p}{\delta z} - V_p \frac{\delta e}{\delta z} = 0 \]

This results in:

\[ \frac{\delta e}{\delta t} + V_p \frac{\delta e}{\delta z} = (1-e) \frac{\delta V_p}{\delta z} \] (2.7)

For the mean velocity, \( V_p \), of particles in a fluidized bed during the response time the following expression can be used (see also Appendix A).

\[ V_p(e) = U - U_\nu(e) \] (2.8)

Combining Eqs.(2.7) and (2.8) results in:

\[ \frac{\delta e}{\delta t} + [U - U_\nu(e)] \frac{\delta e}{\delta z} = (1-e) \left( \frac{\delta U_\nu(e)}{\delta e} \frac{\delta e}{\delta z} \right) \] (2.9)

where \( U_\nu(e) \): Free-falling velocity of a swarm particles m/s

Rewriting Eq.(2.9) gives:

\[ \frac{\delta e}{\delta t} + W(e) \frac{\delta e}{\delta z} = 0 \] (2.10)

in which:

\[ W(e) = U - U_\nu(e) \] (2.11)

where

\[ U_\nu(e) = U_\nu(e) - (1-e) \frac{\delta U_\nu(e)}{\delta e} \] (2.12)
W(ε) is the velocity of a zone with a given porosity ε. After a change in the fluidizing velocity from $U_0$ to $U_1$ a range of porosities, varying from $ε_0$ to $ε_1$, will move upwards with a velocity $W(ε)$, starting at the bottom of the fluidized bed.

For $U_γ(ε)$ the Richardson & Zaki equation for the hindered settling velocity can be used:

$$U_γ(ε) = V_T \cdot ε^n$$  \hspace{1cm} (2.13)

in which "n" is a function of the Reynolds number:

- $n = 4.65$ for $Re<0.2$
- $n = 4.35Re^{-0.03}$ for $0.2<Re<1.0$
- $n = 4.45Re^{-0.1}$ for $1.0<Re<500$
- $n = 2.39$ for $Re>500$  \hspace{1cm} (2.14)

Combining Eqs.(2.11), (2.12) and (2.13) gives:

$$W(ε) = U + V_T \cdot [nε^{(n-1)} - (n+1)ε^n]$$  \hspace{1cm} (2.15)

The mathematical model for the response of a fluidized bed to a sudden change in the fluidizing velocity is based on the assumption that $W(ε)$ decreases as $ε$ increases. This assumption is valid only if the function $W(ε)$ is monotonous declining, so the first derivative has to be negative.

$$\frac{δW(ε)}{δε} = V_T[ n(n-1) \cdot ε^{(n-2)} - n(n+1)ε^{(n-1)} ] = 0$$  \hspace{1cm} (2.16)

for

$$ε = \frac{n-1}{n+1}$$

Such is the case for $W(ε)$ for $ε > (n-1)/(n+1)$. Generally $n=2.39$ so $ε$ has to be greater than 0.41.
2.2.1. Increasing the velocity of the fluidizing medium:

At time \( t < t_0 \) the fluidized bed, with a porosity \( \varepsilon_0 \) and a height \( h_0 \), is in balance with the fluidizing velocity \( U_0 \). At \( t = t_0 \) the fluidizing velocity is increased from \( U_0 \) to \( U_1 \). The porosity of the new equilibrium is \( \varepsilon_1 \) and the corresponding height is \( h_1 \). At \( t = 0 \) the porosity at \( z = 0 \) will change immediately from \( \varepsilon_0 \) to \( \varepsilon_1 \), as shown in the left hand part of Figure 2.1, and a range of porosities varying from \( \varepsilon_0 \) to \( \varepsilon_1 \) will start moving upwards. In this case \( W(\varepsilon_0) > W(\varepsilon_1) \), so the boundary between the old and new porosity broadens.

![Figure 2.1](image)

**Figure 2.1** Porosity as a function of height during response time.

**I:** For \( t_0 \leq t < t^* \) the porosity is given by:

\[
\varepsilon(z) = \begin{cases} 
\varepsilon_1 & \text{for } 0 \leq z < b_1 \\
W^{-1} \left( \frac{z}{t} \right) & \text{for } b_1 \leq z \leq b_2 \\
\varepsilon_0 & \text{for } b_2 \leq z \leq h
\end{cases}
\]  

(2.17)
The transition points $b_1$ and $b_2$ in Figure 2.1 are given by:

$$b_1 = W(\varepsilon_1) \cdot t$$  \hspace{1cm} (2.18)

$$b_2 = W(\varepsilon_0) \cdot t$$  \hspace{1cm} (2.19)

$W^{-1}$ is the inverse function of $W(\varepsilon)$ and can be calculated from Eqs.(2.11) and (2.12). The porosity of the top of the bed remains $\varepsilon_0$ until the transition point $b_2$ reaches the top at $t = t^*$. 

II: For $t^* \leq t \leq t_1$ the porosity is given by:

$$\varepsilon(z) = \begin{cases} 
\varepsilon_1 & \text{for } 0 \leq z < b_1 \\
W^{-1}(\frac{z}{t}) & \text{for } b_1 \leq z \leq h 
\end{cases}$$  \hspace{1cm} (2.20)

III: At $t = t_1$ the new equilibrium is reached.

The change of height is determined by the velocity of the top particles in the fluidized bed. This velocity depends on the porosity, $\varepsilon_h$, at the top of the bed. The height of the bed is given by:

$$h(t) = h_0 + \int_0^t V_p(\varepsilon_h) \, dt$$  \hspace{1cm} (2.21)

I: For $0 < t < t^*$ the height of the bed is given by:

$$\varepsilon_h = \varepsilon_0$$

$$V_p(\varepsilon_h) = U_1 - U_v(\varepsilon_0) = U_1 - U_0$$  \hspace{1cm} (2.22)

so

$$h(t) = h_0 + (U_1 - U_0) t$$

$U_v(\varepsilon_0) = U_0$ because in a fluidized bed in equilibrium the hindered settling velocity of the particles is equal to the fluidizing velocity.
II: for \( t^* < t < t_1 \) the height of the bed is given by:

\[
h(t) = W(\varepsilon_h)t
\]

(2.23)

Calculation of time \( t_1 \):

Knowing \( U_0, h_0 \) and \( U_1 \) it is possible to calculate \( \varepsilon_0, \varepsilon_1 \) and \( h_1 \) by using Eqs.(2.1) and (2.2); it is now possible to calculate time \( t_1 \):

\[
t_1 = \frac{h_1}{W(\varepsilon_1)}
\]

(2.24)

\[
W(\varepsilon_1) = U_1 + V_T \left[ n \varepsilon_1^{(n-1)} - (n+1) \varepsilon_1^n \right]
\]

Calculation of time \( t^* \):

At \( t = t^* \): \( \varepsilon_h = \varepsilon_0 \) and \( U_\nu(\varepsilon_0) = U_0 \). Using Eqs.(2.8) and (2.21) \( h(t^*) \) is given by:

\[
V_p(\varepsilon_h) = U_1 - U_0
\]

\[
h(t^*) = h_0 + \int_0^{t^*} (U_1 - U_0) \, dt = h_0 + (U_1 - U_0) t^*
\]

(2.25)

Using Eqs.(2.13) and (2.23), \( h(t^*) \) is also defined by:

\[
h(t^*) = W(\varepsilon_h) t^* = \left[ U_1 + V_T (n \varepsilon_0^{(n-1)} - (n+1) \varepsilon_0^n) \right] t^*
\]

(B)

\[
V_T = \frac{U_\nu(\varepsilon_0)}{\varepsilon_0^n} = \frac{U_0}{\varepsilon_0^n}
\]

(2.26)
Combining Eqs.(2.25) and (2.26) gives for $t^*$:

$$t^* = \frac{h_0 \varepsilon_0}{U_0 \eta \left(1 - \varepsilon_0\right)}$$  \hspace{1cm} (2.27)

**Calculation of time $t$:**

$t$ is defined as the time at which $\varepsilon_h$ reaches the top of the fluidized bed and can be calculated by using the next derivation:

$$h(t) = h_0 + \int V_p(\varepsilon_h) \, dt$$

so:

$$\frac{\delta h(t)}{\delta t} = V_p(\varepsilon_h) = U_1 - V_T \varepsilon_h^n$$ \hspace{1cm} (A)

also:

$$h(t) = W(\varepsilon_h) t = (U_1 + V_T [n \varepsilon_h^{n+1} - (n+1) \varepsilon_h^n]) t$$ \hspace{1cm} (B)

combining A $\land$ B:

$$\frac{\delta t}{t} = \frac{n-1-(n+1) \varepsilon_h \delta \varepsilon_h}{\varepsilon_h (1-\varepsilon_h)}$$

integrating:

$$t = t^* - t = t \land \varepsilon_h = \varepsilon \Rightarrow \varepsilon_h = \varepsilon$$

$$\frac{t}{t^*} = \frac{\varepsilon_h^{(n-1)}(1-\varepsilon_0)^2}{\varepsilon_h^{(n-1)}(1-\varepsilon_h)^2} \hspace{1cm} t^* < t < t_1$$

2.2.2. Calculation of the height-time diagram:

Knowing $U_0$, $h_0$ and $U_1$ it is possible to calculate $\varepsilon_0$, $\varepsilon_1$ and $h_l$ by using Eqs.(2.1) and (2.2). Time $t_1$ can now be calculated by using Eq.(2.24). Time $t^*$ can be calculated by means of Eq.(2.27).
For $0 < t < t^*$: Eq.(2.22) can be used to calculate the height of the fluidized bed as a function of time. The position of transition points $b_1$ and $b_2$ can be calculated by using Eqs.(2.15), (2.18) and (2.19).

For $t^* < t < t_1$: Eq.(2.28) can be used to calculate at what time, $t$, the porosities between $\varepsilon_0$ and $\varepsilon_1$ arrive at the top of the bed. By using Eqs.(2.15) and (2.23) it is possible to calculate the height of the bed at time $t$, knowing $\varepsilon_h$ at that time.

2.2.3. Decreasing the velocity of the fluidizing medium:

At time $t < t_0$ the fluidized bed, with a porosity $\varepsilon_0$ and a height of $h_0$, is in balance with the fluid velocity, $U_0$. At $t = t_0$ the fluid velocity is decreased from $U_0$ to $U_1$. The porosity at the new equilibrium is $\varepsilon_1$ and the height is $h_1$. At $t = t_0$ the porosity at $z = 0$ will change immediately from $\varepsilon_0$ to $\varepsilon_1$ (see right hand part of Figure 2.1), and a range of porosities, varying from $\varepsilon_0$ to $\varepsilon_1$, will start moving upwards. In this case $W(\varepsilon_0) < W(\varepsilon_1)$, so the boundary between the old and new porosity remains sharp. The new porosity tends to take over the original one.

For $t_0 \leq t < t_1$ the porosity is given by:

$$\varepsilon(z) = \begin{cases} 
\varepsilon_1 & \text{for } 0 \leq z < b \\
\varepsilon_0 & \text{for } b \leq z \leq h
\end{cases}$$

(2.29)

During the whole process $\varepsilon_h = \varepsilon_0$.

$$\varepsilon_h = \varepsilon_0$$

$$V_p(\varepsilon_h) = U_1 - U_0$$

$$h(t) = h_0 + \int_{t=0}^{t} V_p(\varepsilon_h) dt$$

$$h(t) = h_0 + (U_1 - U_0) t$$

(2.30)
The transition point, b, can be derived by calculating the surface of the diagram in the right hand part of Figure 2.1.

\[
\begin{align*}
(\varepsilon_0 - \varepsilon_i) b + h(1 - \varepsilon_0) &= h_0(1 - \varepsilon_0) \\
(\varepsilon_0 - \varepsilon_i) b + (h_0 + [U_1 - U_0] t)(1 - \varepsilon_0) &= h_0(1 - \varepsilon_0) \\
b(t) &= \frac{1 - \varepsilon_0}{\varepsilon_0 - \varepsilon_i} (U_0 - U_1) t
\end{align*}
\]

(2.31)

Calculation of \( t_1 \):

\[
\begin{align*}
\begin{aligned}
h_1 &= h_0 + (U_1 - U_0) t_1 \\
t_1 &= \frac{h_1 - h_0}{U_1 - U_0}
\end{aligned}
\end{align*}
\]

\[
\begin{align*}
\therefore (1 - \varepsilon_0) h_0 &= (1 - \varepsilon_1) h_1 \\
t_1 &= \frac{h_0 (\varepsilon_0 - \varepsilon_1)}{(1 - \varepsilon_1)(U_0 - U_1)} = \frac{h_1 (\varepsilon_0 - \varepsilon_1)}{(1 - \varepsilon_0)(U_0 - U_1)}
\end{align*}
\]

(2.32)

\[\text{Figure 2.2} \quad h(t) \text{ for fluidized bed during response time.}\]
In Figure 2.2 the height of the fluidized bed during the response time is given for increasing and decreasing fluidizing velocity. The curves are calculated for a system in which:

\[
\begin{array}{c|c|c}
\rho_s & 1185 \text{ kg/m}^3 & \rho_l & 888 \text{ kg/m}^3 \\
\eta_l & 0.001 \text{ kg/ms} & d_p & 0.005 \text{ m} \\
U_{\text{high}} & 0.11 \text{ m/s} & U_{\text{low}} & 0.04 \text{ m/s} \\
\varepsilon_{\text{high}} & 0.75 & \varepsilon_{\text{low}} & 0.5 \\
h_{\text{high}} & 0.2 \text{ m} & h_{\text{low}} & 0.1 \text{ m} \\
\end{array}
\]

There is a notable difference in the response time, \(t_1\), the time needed to reach the new equilibrium situation. If the superficial fluidizing velocity is increased this response time is 2.24 sec, in the case the velocity is decreased the response time is 1.44 sec.

2.3. References

Chapter 3

Mathematical model for the response of a uniform jig bed in terms of the porosity distribution

3.1. Introduction

The mathematical model, devised by Slis and described in the previous chapter, to describe the response of a liquid fluidized bed to a sudden change in the fluidizing velocity, can be translated into a model for the porosity distribution in a jig as a function of time and height in the bed. During the transition period in a fluidized bed, e.g. the time between the two equilibrium stages, all velocities mentioned in the mathematical model of Slis are time-independent, due to the fact that the superficial fluidizing velocity was changed in a step-wise mode. By contrast with a fluidized bed, the superficial fluid velocity in a jig is changed continuously. The Slis mathematical model can be translated into a quasi-static mathematical model for a spherical monosized, mono-density, jig bed by transforming the step wise change in the fluidizing velocity into a periodic function of time. Assuming that the porosity near the screen is in equilibrium with the corresponding instantaneous fluid velocity, according to the piston cycle of the jig, the porosity near the screen will change as a function of the time-dependent fluid velocity. These porosities will travel upwards with a time-dependent velocity according to the velocity described in the model for the fluidized bed.

3.2. Displacement and velocity functions in a jig.

As starting point for modelling the porosity distribution a basic sinusoidal jiggling function is chosen. The displacement of the plunger is defined by:

\[ S(t) = -A_{str} \cos \frac{2\pi t}{T} \]  \hspace{1cm} (3.1)

The superficial fluid velocity is defined by the differential of \( S(t) \) with respect to the time and is equal to:
\[ U(t) = \frac{2}{T} \pi A_{\text{str}} \sin \frac{2}{T} \pi t \] \hspace{1cm} (3.2)

in which:

- \( S(t) \): Displacement of the plunger \( (m) \)
- \( A_{\text{str}} \): Amplitude of the stroke \( (m) \)
- \( t \): Time \( (s) \)
- \( T \): Stroke time (1/frequency) \( (s) \)
- \( U(t) \): Superficial fluid velocity \( (m/s) \)

In Figure 3.1 a sinusoidal jiggling function is given. Positive values of the superficial fluid velocity mean that the plunger is moving upwards.

**Figure 3.1** Displacement and fluid velocity for a sinusoidal jiggling function.

### 3.3. Porosity distribution at the bottom of the jig bed.

Assuming that the bottom layer of particles of the jig bed is fixed on the screen, and that the porosity near the screen is in equilibrium with the corresponding instantaneous superficial fluid velocity, the porosity near the screen will change as a function of the
time-dependent fluid velocity of a given stroke.

Due to the fact that the particle velocity at \( z = 0 \) is zero, it is assumed the particles are fixed on the screen, the hindered settling velocity is equal to the superficial fluid velocity for a system in equilibrium (see Appendix A). The porosity distribution as a function of time at \( z = 0 \) can now be calculated by using the relation of Richardson and Zaki for the hindered settling velocity of particles:

\[
U(t) = U_v(\varepsilon(t)_{z=0}) = V_T \varepsilon(t)_{z=0}^n
\]

\[
\varepsilon(t)_{z=0} = \left( \frac{U(t)}{V_T} \right)^{1/n}
\]

(3.3)

Combining Eq. (3.3) and (3.2) gives:

\[
\varepsilon(t)_{z=0} = \left( \frac{2 \pi A_{mf}}{V_T T} \sin \frac{2 \pi t}{T} \right)^{1/n}
\]

(3.4)

The porosity distribution at \( z = 0 \) is now described by:

\[
\varepsilon(t)_{z=0} = \begin{cases} 
\varepsilon_p & \text{for } U(t) < U_{mf} \\
\left( \frac{U(t)}{V_T} \right)^{1/n} & \text{for } U_{mf} \leq U(t) \leq V_T \\
1 & \text{for } U(t) > V_T 
\end{cases}
\]

(3.5)

in which

- \( U_{mf} \): Minimum fluidization velocity
- \( \varepsilon_p \): Porosity of the packed bed
- \( V_T \): Free falling velocity of one particle.

The particles, at \( z = 0 \), are in a packed state until the superficial fluid velocity shall reach the minimum fluidization velocity. At this time the superficial fluid velocity equals the hindered settling velocity of a packed bed, therefore, the minimum fluidization velocity is defined by:
\[ U_{nf} = V_T \varepsilon_p^n \] (3.6)

**Figure 3.2** Porosity distribution as a function of time at \( z = 0 \).

In Figure 3.2 an example is given for the porosity distribution at \( z = 0 \) for the uniform standard jig bed, defined in chapter 4. Stroke length = 0.05 m, jigging frequency = 0.5 Hz, porosity packed bed = 0.376.

At time \( t < a \) the porosity is given by the porosity of the packed bed, \( \varepsilon_p \). At time \( t = a \) the superficial fluid velocity will reach the minimum fluidization velocity; from this moment on the porosity will increase, till the superficial fluid velocity, and the porosity, reach their maximum value at \( t = 1/4T \). At the point \( t = 1/2T-a \) the superficial fluid velocity will reach the minimum fluidization velocity again and the porosity will now decrease to \( \varepsilon_p \).

For the mathematical model it is important to know at what time, \( t \), a porosity of \( \varepsilon \) at \( z = 0 \) is generated, defined as time \( \tau(\varepsilon) \). Using the sinusoidal jigging function the superficial fluid velocity reaches a maximum value at \( t = 0.25T \). As can be seen from Figure 3.2, for \( t > 0.25T \) the same porosities are generated at the bottom as for
t < 0.25 T. The porosity zones moving through the jig bed can be split into two parts. By rewriting Eq.(3.4), \( \tau(\varepsilon) \) for the two different parts can be calculated by:

\[
\tau_1(\varepsilon) = \frac{T}{2\pi} \arcsin \left( \frac{U_v(\varepsilon)T}{2\pi A_{st}} \right) \quad \text{for } \tau < 0.25T
\]

\[
\tau_2(\varepsilon) = \frac{T}{2\pi} \left[ \pi - \arcsin \left( \frac{U_v(\varepsilon)T}{2\pi A_{st}} \right) \right] \quad \text{for } \tau > 0.25T
\]

(3.7)

3.4. Porosity velocity

The porosities generated at \( z = 0 \), will move through the jig bed with a porosity velocity \( W(\varepsilon, t) \). This velocity can be derived by using the equation of continuity of mass, similar to that described in Chapter 2 for the response of a fluidized bed.

The equation of continuity of mass is given by:

\[
\frac{\delta(1-\varepsilon)}{\delta t} + \frac{\delta(1-\varepsilon)V_p(\varepsilon,t)}{\delta z} = 0
\]

(3.8)

or:

\[
\frac{\delta \varepsilon}{\delta t} + V_z(\varepsilon,t) \frac{\delta \varepsilon}{\delta z} - (1-\varepsilon) \frac{\delta V_p(\varepsilon,t)}{\delta z} = 0
\]

(3.9)

As in the case of the mean particle velocity, \( V_p(\varepsilon) \) in the fluidized bed, the mean particle velocity \( V_p(\varepsilon,t) \) in the jig is given by the difference of the fluid velocity and the swarm velocity:

\[
V_p(\varepsilon,t) = U(t) - U_v(\varepsilon)
\]

(3.10)

The superficial fluid velocity, \( U(t) \), in Eq.(3.10) can have a negative or positive value depending on the direction of the plunger of the jig. For \( U_v(\varepsilon) \) the scalar value of the swarm velocity is used.
By using the equation for the mean particle velocity defined in Eq.(3.10) the model becomes quasistatic. The model neglects the kinetic energy. The energy contributed by the fluid to the particles is directly changed into potential energy, so the movement of the particles, necessary to change the potential energy is not considered by the model, so accelerative and decelerative forces are neglected. A mathematical explanation is given in Appendix D.

The equation of continuity of mass can be rewritten using Eq.(3.10):

$$\frac{\delta \epsilon}{\delta t} + \left( V_p(\epsilon,t) + (1-\epsilon) \frac{\delta U_s(\epsilon)}{\delta \epsilon} \right) \frac{\delta \epsilon}{\delta z} = 0$$

(3.11)

The porosity velocity, \( W(\epsilon,t) \), of a given porosity satisfies:

$$\frac{\delta \epsilon}{\delta t} + W(\epsilon,t) \frac{\delta \epsilon}{\delta z} = 0$$

(3.12)

Combining Eqs.(3.11) and (3.12) gives:

$$W(\epsilon,t) = V_p(\epsilon,t) + (1-\epsilon) \frac{\delta U_s(\epsilon)}{\delta \epsilon}$$

(3.13)

or:

$$W(\epsilon,t) = V_p(\epsilon,t) + V_w(\epsilon)$$

in which:

$$V_w(\epsilon) = (1-\epsilon) \frac{\delta U_s(\epsilon)}{\delta \epsilon}$$

(3.14)

The porosity velocity is equal to the mean particle velocity plus a term depending only on the porosity. Using the equation of Richardson and Zaki for \( U_s(\epsilon) \), and the jiggling function given in Eq.(3.2), the porosity velocity is given by:

$$W(\epsilon,t) = \frac{2\pi A_m}{T} \sin \frac{2\pi t}{T} + V_p(n \epsilon^{n-1} -(n+1) \epsilon^n)$$

(3.15)
In Figure 3.3 an example is given for the porosity velocity as a function of the porosity for the standard jig bed. \( V_T = 0.231 \text{ m/s} \), \( n = 2.39 \), Jigging frequency = 0.5 Hz and stroke length = 0.05 m.

![Graph of W(e, t) vs Porosity](image)

**Figure 3.3** Porosity velocity as a function of the porosity.

Similar to the porosity velocities in a fluidized bed, the curves in Figure 3.3 reach maximum value at \( \varepsilon_m = (n-1)/(n+1) \) due to the fact that:

\[
\frac{\delta W(e)}{\delta e} = V_T \left[ n(n-1) \varepsilon^{(n-2)} - n(n+1)\varepsilon^{(n-1)} \right] = 0
\]

for

\[
\varepsilon_m = \frac{n-1}{n+1}
\]  \hspace{1cm} (3.16)

If \( \varepsilon \) is smaller than this value \( W(e, t) \) will increase with increasing value of \( \varepsilon \). If \( \varepsilon \) is greater than this value, \( W(e, t) \) will decrease with increasing value of \( \varepsilon \). By contrast with the model for the response of a fluidized bed, in which it was assumed that the the porosities were always greater than \( \varepsilon_m \), the porosities in the jig bed do not satisfy
this assumption. The value of $\varepsilon_m$ is of great importance in the jig model, as will be described later.

3.5. Position of a given porosity $\varepsilon$ as a function of time

Knowing the porosity velocity it is possible to calculate the position of a given porosity at time $t$ by integrating the porosity velocity from $t = \tau(\varepsilon)$, the time at which the porosity was generated at $z = 0$, to $t = t$:

$$Z(\varepsilon, t) = \int_{\tau(\varepsilon)}^{t} W(\varepsilon, t) \, dt$$

(3.17)

Combining Eqs.(3.10), (3.14) and (3.17) gives:

$$Z(\varepsilon, t) = \int_{\tau(\varepsilon)}^{t} U(t) - U'(\varepsilon) + V_w(\varepsilon) \, dt$$

(3.18)

After integration, $z(\varepsilon, t)$ is defined by:

$$z(\varepsilon, t) = \prod (t) - \prod (\tau(\varepsilon)) - (U'(\varepsilon) - V_w(\varepsilon))(t - \tau(\varepsilon))$$

in which

$$\prod = \int U(t) \, dt$$

Using the Richardson and Zaki equation for $U'(\varepsilon)$ and the sinusoidal jigging function the position of given porosity is defined by:

$$Z(\varepsilon, t) = V_T \left\{ n \varepsilon^{n-1} -(n+1) \varepsilon^n \right\} (t - \tau(\varepsilon)) + A_{ai} \left\{ - \cos \left( \frac{2\pi t}{T} \right) + \cos \left( \frac{2\pi \tau(\varepsilon)}{T} \right) \right\}$$

(3.20)

By using Eq.(3.20) it is possible to calculate the porosity distribution in a uniform jig bed as a function of time and height, with the exception of places where discontinuities occur. These discontinuities are caused by the fact that mathematical porosities will pass each other. Theoretically it is impossible that more than one
porosity is present at a certain level in the jigbed.

In Figure 3.4 an example is given for three different situations of the porosity distribution in a jig bed, each with its own discontinuities.

![Discontinuities in the porosity distribution in a jig bed](image)

**Figure 3.4** Discontinuities in the porosity distribution in a jig bed.

**Situation 1:**
This situation occurs shortly after the jig bed, under the influence of the increasing superficial fluid velocity, $U(t)$, starts to move. The jig bed is in a packed stage for $z > b_1$ and fluidized for $z < b_1$. The porosity distribution at $z = b$ is not differentiable. It is also possible that the distribution at $z = b$ is discontinuous.

**Situation 2:**
The situation at $z = b_2$ occurs after the time at which the superficial fluid velocity, $U(t)$, has reached its maximum value. The porosities are now divided into two parts, as described by Eq.(3.7). One part is generated during increasing $U(t)$ and the other part during decreasing $U(t)$. The porosities at $z > b_2$ belong to the first or second part and the porosities at $z < b_2$ belong to the second part.

**Situation 3:**
In Situation 3 the superficial fluid velocity is decreased to a value at which the particles start packing again on the screen. The jig bed is in a packed stage for $z < b_3$
and fluidized for $z > b_3$.

As can be seen from Figure 3.4, more than one discontinuity may occur in the jigbed at a given time. To determine the places at which discontinuities occur in the jig bed the mass transport integrals are of great importance.

3.6. Mass Transport integrals

The equation of continuity of mass, Eq.(3.8), cannot be applied in places in which the porosity distribution is not a continuous function or is not differentiable. So Eq.(3.20) can be used only in places where the porosity distribution is a continuous function. The situation at places where the function is discontinuous, or is not differentiable, is determined by boundary conditions. These conditions can be derived by using the fact that the total mass of particles in the jig bed is constant.

A given porosity, $\varepsilon$, is generated at $z = 0$ at time $\tau(\varepsilon)$ and moves through the jigbed after that time. The particles situated under the $\varepsilon$ level at height $z(\varepsilon,t)$ at time $t > \tau(\varepsilon)$ have all passed the $\varepsilon$ level during the time interval $\tau(\varepsilon)$ to $t$. Due to this fact, the mass integral $M(\varepsilon,t)$ over the porosity distribution $(1 - \varepsilon)$ from the bottom of the jig bed to the $\varepsilon$ level at $z(\varepsilon,t)$ is equal to the integral over the mass transport, $T_m$, through the $\varepsilon$ level from $t = \tau(\varepsilon)$ to $t = t$, so:

$$M(\varepsilon,t) = \int_0^{z(\varepsilon,t)} (1 - \varepsilon) \, dz$$

(3.21)

is equal to

$$M(\varepsilon,t) = \int_{\tau(\varepsilon)}^{t} T_m \, dt$$

(3.22)

The density of the particles is taken as a constant value and is left out of the considerations.
The mass transport, $T_m$, is proportional to the difference between the porosity velocity, $W(\varepsilon,t)$, and the particle velocity, $V_p(\varepsilon,t)$, because this is the relative velocity at which the particles pass the $\varepsilon$ level. Combining this with Eq.(3.14) gives:

$$T_m = (1 - \varepsilon) \int W(\varepsilon,t) - V_p(\varepsilon,t) \, dt = (1 - \varepsilon) \int V_w(\varepsilon) \, dt$$  \hspace{1cm} (3.23)$$

Integrating $T_m$ from $t = \tau(\varepsilon)$ to $t = t$ gives:

$$M(\varepsilon,t) = (1 - \varepsilon) \int V_w(\varepsilon) \, [t - \tau(\varepsilon)]$$  \hspace{1cm} (3.24)$$

In Appendix E it is proven mathematically that Eq.(3.21) and Eq.(3.22) are identical.

At a discontinuity the porosity jumps from a value $\varepsilon_u$, directly under the discontinuity to a value $\varepsilon_a$, directly above the discontinuity. In this case, the calculated values of $Z(\varepsilon,t)$ for $\varepsilon_u$ and $\varepsilon_a$ have to be equal and the calculated values of the mass transport integrals for $\varepsilon_u$ and $\varepsilon_a$ have to be equal, so:

$$Z(\varepsilon_u,t) = Z(\varepsilon_a,t)$$

and

$$M(\varepsilon_u,t) = M(\varepsilon_a,t)$$  \hspace{1cm} (3.25)$$

The boundary conditions are defined by solving for the two porosities, $\varepsilon_u$ and $\varepsilon_a$, of the system of equations given in Eq.(3.25).

### 3.7. Computer simulation

Based on the theory described in paragraph 3.2 - 3.6 a computer program has been developed to simulate the response of a uniform jig bed, of spherical particles, in terms of the porosity distribution as a function of height and time in the jig bed. The program simulates for a given number of time steps the situation in the jigbed by calculating the boundary conditions and the position of each porosity zone. The computer program has been written in the computer language Turbo Pascal and runs on IBM compatible personal computers. In the next paragraphs the computer program will be described in the sequence of actions taken by the program.
3.7.1. Input data

For the calculation of the porosity distribution in the jig bed the computer program needs some input data, such as fluid and particle properties and jiggering conditions. As described in Appendix A there are several ways to calculate the terminal velocity of a particle and the constant "n" from the Richardson and Zaki equation. These equations are optional in the program and may be chosen arbitrarily during input. The program is able to give output files for further data treatment in other computer programs: file name and directory can be defined.

**Input:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of liquid, ( \rho_l )</td>
<td>(kg/m(^3))</td>
</tr>
<tr>
<td>Viscosity of liquid, ( \eta_l )</td>
<td>(kg/ms)</td>
</tr>
<tr>
<td>Density of particle, ( \rho_p )</td>
<td>(kg/m(^3))</td>
</tr>
<tr>
<td>Diameter of particle, ( d )</td>
<td>(m)</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>(-)</td>
</tr>
<tr>
<td>Height of packed jig bed, ( h_p )</td>
<td>(m)</td>
</tr>
<tr>
<td>Porosity of packed jig bed, ( \varepsilon_p )</td>
<td>(-)</td>
</tr>
<tr>
<td>Stroke length of jig, ( h_{str} )</td>
<td>(m)</td>
</tr>
<tr>
<td>Jigging frequency, ( F )</td>
<td>(Hz)</td>
</tr>
</tbody>
</table>

**Correlation terminal particle velocity:**
- Schiller and Naumann
- Stokes and Lapple
- Dallavalle
- Kahn and Richardson

**Correlation of "n":**
- Richardson and Zaki
- Garside and Al-Dibouni
- Rowe
- Kahn and Richardson
- Ergun

**Output files:** Directory and file name
3.7.2. Start values

The computer program starts with calculating some important variables:

- The Archimedes number.
- By using the Archimedes number and the chosen correlation for the terminal velocity (see Appendix A), the Reynolds number is calculated.
- From the Reynolds number the free falling terminal velocity, $V_T$, of one particle is calculated.
- The value of the constant "n" from the Richardson and Zaki equation.
- Duration of one jigging stroke, $T$ (s):

$$T = \frac{1}{F}$$  \hspace{1cm} (3.26)

- The maximum superficial fluid velocity derived from Eq.(3.2):

$$A_{vel} = \frac{2\pi A_{str}}{T}$$  \hspace{1cm} (3.27)

- The minimum fluidization velocity, $U_{mf}$:

$$U_{mf} = U_v(\epsilon_p) = V_T \frac{\epsilon_p}{T}$$  \hspace{1cm} (3.28)

- Time at which the jig bed starts expanding with increasing superficial fluid velocity, using Eq.(3.7), is:

$$t_{exp} = \frac{T}{2\pi} \arcsin \left( \frac{U_{mf} T}{2\pi A_{str}} \right)$$  \hspace{1cm} (3.29)

- Duration of each time step:

$$t_{step} = T \frac{T}{\text{number of time steps}}$$  \hspace{1cm} (3.30)
- Total mass in the jig bed, $M_{\text{tot}}$:

$$M_{\text{tot}} = h_p (1 - \varepsilon_p) \quad (3.31)$$

- Maximum value of the porosity reached during a jiggging stroke, $\varepsilon_{\text{max}}$:

$$U(t)_{\text{max}} = V_T \varepsilon_{\text{max}}$$

$$U(t)_{\text{max}} = A_{\text{vel}}$$

so:

$$\varepsilon_{\text{max}} = \left( \frac{A_{\text{vel}}}{V_T} \right)^{\frac{1}{n}} \quad (3.32)$$

- Time at which the bottom of the jig bed starts packing again, $t_{\text{pack}}$, is:

$$t_{\text{pack}} = \frac{1}{2} T - t_{\text{exp}} \quad (3.33)$$

The computer program now creates a certain number of arrays in which the calculated data can be stored. The arrays are time-dependent and each calculated phenomenon, described in the next paragraphs, is put in a separate array.

3.7.3. Porosity at the bottom of the jig bed, at $z = 0$

The porosity at the bottom of the jigbed can be calculated by using Eq.(3.5):

$$\varepsilon(t)_{z=0} = \left( \frac{U(t)}{V_T} \right)^{\frac{1}{n}} \quad \text{for } U(t) > U_{\text{inf}} \quad \text{at } t_{\text{exp}} < t < t_{\text{pack}} \quad (3.34)$$

$$\varepsilon(t)_{z=0} = \varepsilon_p \quad \text{for } U(t) < U_{\text{inf}} \quad \text{at } 0 < t < t_{\text{exp}} \text{ or } t_{\text{pack}} < t < T$$

The time at which a given porosity, $\varepsilon$, is generated at $z = 0$ is given by Eq.(3.7). Two porosity parts exist. Depending on the part to which the porosity belongs, $\tau_1(\varepsilon)$ or $\tau_2(\varepsilon)$ has to be used for calculating the value of several equations.
3.7.4. Porosity movement through the jig bed

A zone with a given porosity, \( \varepsilon \), starts moving through the jig bed at time \( \tau(\varepsilon) \). The porosity velocity is defined by:

\[
W(\varepsilon,t) = U(t) = U_x(\varepsilon) + V_w(\varepsilon)
\]

in which

\[
V_w(\varepsilon) = (1 - \varepsilon) \frac{\delta U_x(\varepsilon)}{\delta \varepsilon} = (1 - \varepsilon) n V_t \varepsilon^{(n-1)}
\]

At time \( t = \tau(\varepsilon) \):

\[
U(\tau(\varepsilon)) = U_x(\varepsilon), \text{ see Eq.}(3.3), \text{ so:}
\]

\[
W(\varepsilon,\tau(\varepsilon)) = (1 - \varepsilon) n V_t \varepsilon^{(n-1)}
\]

From Eq.(3.36) it can be concluded that the porosity velocity at \( t = \tau(\varepsilon) \) is always positive, such that the porosity zone starts moving in an upward direction. The program calculates the porosity velocity for the whole range of porosities at every time step.

As described in paragraph 3.4 the porosity velocity as a function of the porosity reaches a maximum value at \( \varepsilon_m = (n-1)/(n+1) \). That means that porosities around the value of \( \varepsilon_m \) can pass each other. The program calculates the value of \( \varepsilon_m \). The importance of this value will become clear in some of the next paragraphs.

For the calculation of some boundary conditions it is important to know the first derivative of \( Z(\varepsilon,t) \) to \( \varepsilon \), derived in Appendix E:

\[
\frac{\delta Z(\varepsilon,t)}{\delta \varepsilon} = (t - \tau(\varepsilon)) \frac{\delta W(\varepsilon,t)}{\delta \varepsilon} - V_w(\varepsilon) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon}
\]

(3.37)

The program calculates the first derivative of \( Z(\varepsilon,t) \) to \( \varepsilon \) for the whole range of porosities at every time step.
3.7.5. Turn-over height of the porosity distribution

The superficial fluid velocity reaches its maximum value at \( t = 1/4T \). At this time, the porosity at the bottom of the jig bed is equal to \( \varepsilon_{\text{max}} \). The position of this porosity value constitutes the boundary between the porosities of the first part and of the second part (see paragraph 3.3). Due to this fact, the height position, \( h_{to} \), of \( \varepsilon_{\text{max}} \) is an important value in the porosity distribution. By using Eq.(3.20), \( h_{to} \) can be calculated:

\[
h_{to} = V_T \left\{ n \varepsilon_{\text{max}}^{n-1} - (n+1) \varepsilon_{\text{max}}^n \right\} (t - 1/4T) - A_{sc} \cos \left( \frac{2\pi t}{T} \right)
\]  
(3.38)

The maximum porosity, \( \varepsilon_{\text{max}} \), and the turn over height, \( h_{to} \), will disappear if:

- \( h_{to} \) passes a discontinuity.
- \( h_{to} \) reaches the top of the jig bed, no porosities of the first part are now present in the porosity distribution (see paragraph 3.7.6).
- \( h_{to} \) is overtaken by the upper boundary of the packed bed on the screen (see paragraph 3.7.8).
- \( h_{to} \) is overtaken by the "ridge", defined in paragraph 3.7.9.

At this moment the program also calculates the mass under the \( h_{to} \) level by using the mass transport integral, Eq.(3.24):

\[
M_{to} = M(\varepsilon_{\text{max}}, t) = (1 - \varepsilon_{\text{max}}) V_w(\varepsilon_{\text{max}}) (t - 1/4T)
\]  
(3.39)

3.7.6. Height position of the top of the jig bed

The jig bed starts expanding at \( t = t_{\text{exp}} \). The top of the jig bed remains, for the time being, in a packed stage. Similar to the situation described in the response of a fluidized bed, the height of the jig bed is determined by the velocity of the top particles, so that:

\[
h_{\text{top}} = h_p + \int_{t_{\text{exp}}}^{t} V_p(\varepsilon_p, t) \, dt
\]  
(3.40)
Combining with Eq.(3.10) gives:

$$h_{\text{top}} = h_p + \int_{t_{\text{top}}}^{t} U(t) - U_v(e_p) \, dt$$  \hspace{1cm} (3.41)

resulting in:

$$h_{\text{top}} = h_p + A_{st} \left\{ \cos \left( \frac{2 \pi t_{\text{exp}}}{T} \right) - \cos \left( \frac{2 \pi t}{T} \right) \right\} - U_v(e_p)(t - t_{\text{exp}})$$  \hspace{1cm} (3.42)

The porosity at the top of the jig bed is, at this stage, the porosity of a packed bed, $e_p$. The porosities generated at $z = 0$ at $t > t_{\text{exp}}$ create a fluidized zone, which moves upwards in the jig bed. The upper side, $h_{\text{open}}$, of this fluidized zone moves faster in an upward direction than $h_{\text{top}}$ from Eq.(3.42). At a given time $h_{\text{open}}$ will pass $h_{\text{top}}$ and the porosity at the top of the jig bed will change from $e_p$ to a value $e_{\text{top}}$, which is time-dependent.

The mass transport integral defined in Eq.(3.24) can be used to determine the value of $e_{\text{top}}$, due to the fact that, in this case, the mass transport integral is equal to the total mass in the jigbed, $M_{\text{tot}}$, given by Eq.(3.31):

$$(1 - e_{\text{top}}) V_{\text{w}}(e_{\text{top}})(t - \tau(e_{\text{top}})) = M_{\text{tot}}$$  \hspace{1cm} (3.43)

The height of the jig bed is now given by:

$$h_{\text{top}} = Z(e_{\text{top}} t)$$  \hspace{1cm} (3.44)

The first porosities reaching the top of the jig bed are of the first part. After a given time, porosities of the second part are also able to reach the top of the jig bed. This will be the case at the moment at which the mass under the turn-over height, calculated from Eq.(3.39), becomes greater than the total mass, $M_{\text{tot}}$. Depending on which porosities occur at the top of the jig bed, $\tau_1(\varepsilon)$ or $\tau_2(\varepsilon)$ should be used to solve Eqs.(3.43) and (3.44).

3.7.7. Upper side fluidized part of the jig bed

The porosity, $e_{\text{open}}$, at the upper side, $h_{\text{open}}$, of the fluidized part of the jig bed, directly after the time the jig bed starts expanding, $t_{\text{exp}}$, is equal to the porosity of the
packed bed, $\epsilon_p$. The upper side of the fluidized part can now be calculated by:

$$h_{\text{open}} = Z(\epsilon_p, t)$$

(3.45)

The porosity of a uniform packed bed of spherical particles is about 0.36. This porosity is always smaller than the porosity, $\epsilon_m$, having the maximum porosity velocity, defined in paragraph 3.7.4. Due to this fact, the porosity, $\epsilon_p$, at the upper side of the fluidized part of the jig bed can be passed by other porosities, lying between $\epsilon_p$ and $\epsilon_m$. After the time that $\epsilon_p$ has been passed by other porosities, $\epsilon_{\text{open}}$ and $h_{\text{open}}$ can be calculated by solving the next set of equations, based on the mass transport integrals:

$$(1 - \epsilon_{\text{open}}) V_w (\epsilon_{\text{open}}) (t - \tau(\epsilon_{\text{open}})) = (1 - \epsilon_p) (h_p - h_{\text{top}}(t) + h_{\text{open}}(t))$$

and

$$h_{\text{open}}(t) = Z(\epsilon_{\text{open}}, t)$$

(3.46)

In Figure 3.5 the situation as described by Eq.(3.46) is visualized.

---

![Figure 3.5](image_url)  
**Figure 3.5** Mass distribution in a jig bed.

The time at which the porosity at the upper side of the fluidized part of the jig bed will change from $\epsilon_p$ to $\epsilon_{\text{open}}$ can be determined by considering the first derivative of $Z(\epsilon, t)$ to $\epsilon$. If, at a given time, $t$, the first derivative is negative, the maximum value
of \( Z(\varepsilon,t) \) lies at a porosity smaller than \( \varepsilon_p \), so \( \varepsilon_p \) has not been passed. If the first derivative is positive the maximum of \( Z(\varepsilon,t) \) lies at a porosity greater than \( \varepsilon_p \), so \( \varepsilon_p \) has been passed by another porosity. In Figure 3.6 the two situations described above are depicted.

![Figure 3.6: \( Z(\varepsilon,t) \) as a function of the porosity.](image)

Depending on the first derivation of \( Z(\varepsilon,t) \) to the porosity, the program calculates \( h_{\text{open}} \) by using Eq.(3.45) or by using Eq.(3.46).

The upper side of the fluidized part will leave the porosity distribution at the moment at which this height reaches the top of the jig bed. At this moment, \( \varepsilon_{\text{open}} \) is changed into \( \varepsilon_{\text{top}} \).

### 3.7.8. Packing at the bottom of the jig bed

After time \( t_{\text{pack}} \), the superficial fluid velocity \( U(t) \) is smaller than the minimum fluidization velocity. From this moment on, the particles start settling and a packed bed builds up on the screen. The height, \( h_{\text{pack}} \), of the packed phase will increase progressively from \( z = 0 \) to \( z = h_p \), the original situation of the jig bed.

The mass of the packed particles under the \( h_{\text{pack}} \) level has to be equal to the mass transport integral of porosity \( \varepsilon_{\text{pack}} \) just above the height \( h_{\text{pack}} \). By solving the next set of equations, \( \varepsilon_{\text{pack}} \) and \( h_{\text{pack}} \) can be determined.
\[(1 - e_{\text{pack}})V_w(e_{\text{pack}}) \left( t - \tau(e_{\text{pack}}) \right) = (1 - e_p)h_{\text{pack}}(t)\]

and

\[h_{\text{pack}}(t) = Z(e_{\text{pack},t})\] (3.47)

The upper side of the packed stage first passes porosities of the second part, after some time, porosities of the first part can also be passed. At the transition point from the second to the first part of porosities, \(h_{\text{pack}}\) will pass the turn-over height, \(h_{\text{to}}\), described in paragraph 3.7.5. To determine which part of the porosities is passed a comparison is necessary between the mass under the turn over height, \(M_{\text{to}}\), calculated by Eq.(3.39) and the mass under the turn over-height, assuming that the jig bed is in a packed stage under this height:

\[(1 - e_p)h_{\text{to}}(t) > M_{\text{to}} \quad e_{\text{pack}} \quad \text{second part} \] (3.48)

\[(1 - e_p)h_{\text{to}}(t) < M_{\text{to}} \quad e_{\text{pack}} \quad \text{first part} \]

Figure 3.7  Schematic view of the packed phase at the bottom of the jig bed.
As can be seen from the left hand part of Figure 3.7, the mass calculated by means of \((1 - \varepsilon_p)h_{to}\) is greater than the mass \(M_{to}\) calculated by using Eq.(3.39), due to the fact that a part of the jig bed is not in a packed stage. In this case, the porosities passed by the upper side of the packed phase belong to the second part, because all porosities at heights smaller than \(h_{to}\) belong to the second part.

The computer programme first determines to which part the porosities belong by using Eq.(3.48), and thus establishes which \(\tau(\varepsilon)\) has to be taken for solving Eq.(3.47).

### 3.7.9. Ridge in the porosity distribution

Porosities of the second part greater than \(\varepsilon_m\) (see paragraph 3.4), can be passed by other porosities, due to the fact that the porosity velocity \(W(\varepsilon, t)\) reaches a maximum value at \(\varepsilon_m\) as a function of the porosity. If porosities of the second part pass each other a discontinuity occurs, e.g. a "ridge" in the porosity distribution. So long as the porosities of the second part did not pass each other, the height position \(Z(\varepsilon, t)\) of these porosities is a monotonous increasing function. As soon as the porosities pass each other the first derivative of \(Z(\varepsilon, t)\) to \(\varepsilon\) does not have positive values for all the porosities of the second part. The computer program determines the values of the first derivative of \(Z(\varepsilon, t)\) to \(\varepsilon\) for the porosities of the second part; if one or more porosities has a negative value the above described ridge will occur.

The height position of this ridge, \(h_r\) and the porosities, \(\varepsilon_{ra}\) just above the ridge and \(\varepsilon_{ru}\) just below the ridge, can be determined by solving the next set of equations, based on the mass transport integrals:

\[
(1 - \varepsilon_{ru})V_w(\varepsilon_{ru})\{t - \tau(\varepsilon_{ru})\} = (1 - \varepsilon_{ra})V_w(\varepsilon_{ra})\{t - \tau(\varepsilon_{ra})\}
\]

and

\[h_r = Z(\varepsilon_{ra}, t) = Z(\varepsilon_{ru}, t)\] (3.49)

The porosity \(\varepsilon_{ru}\) always belongs to the second part of the porosities; \(\varepsilon_{ra}\) can belong to both parts.

A ridge in the porosity distribution does not necessarily occur in every jig simulation. A ridge occurs if \(\varepsilon_{max}\) has a relatively large value and if the turn-over height, \(h_{to}\), by
virtue of the decreasing superficial fluid velocity, moves downwards. The particles under the $h_i$ level create a layer of almost packed particles. After some time, the height of the ridge will be passed by the height of the packed bed building up on the screen.

3.7.10 Jig bed simulation

The computer program simulates the porosity distribution in a uniform jigbed by calculating the position of each porosity as a function of the height in the jig bed for a given number of time steps. For each time step the program moves through the procedures described in paragraph 3.7.3. - 3.7.10. The data are stored in time-dependent arrays:

<table>
<thead>
<tr>
<th>Array 1:</th>
<th>Time steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array 2:</td>
<td>Porosity distribution at $z = 0$</td>
</tr>
<tr>
<td>Array 3:</td>
<td>Turn-over height</td>
</tr>
<tr>
<td>Array 4:</td>
<td>Height of the top of the jig bed</td>
</tr>
<tr>
<td>Array 5:</td>
<td>Porosity at the top of the jig bed</td>
</tr>
<tr>
<td>Array 6:</td>
<td>Height of the upper side of the opened part of the jig bed</td>
</tr>
<tr>
<td>Array 7:</td>
<td>Porosity at the upper side of the opened part</td>
</tr>
<tr>
<td>Array 8:</td>
<td>Height of the packed phase at the bottom of the bed</td>
</tr>
<tr>
<td>Array 9:</td>
<td>Porosity at just above the packed phase</td>
</tr>
<tr>
<td>Array 10:</td>
<td>Height of the ridge</td>
</tr>
<tr>
<td>Array 11:</td>
<td>Porosity just below the ridge</td>
</tr>
<tr>
<td>Array 12:</td>
<td>Porosity just above the ridge</td>
</tr>
</tbody>
</table>

The program determines the packed parts of the jig bed and the fluidized parts of the jig bed as a function of the height. Combining this with the discontinuities and with the fact that the position of a given porosity between the discontinuities can be calculated by using Eq.(3.20), a porosity distribution can be created for each time step.

3.7.11. Output computer program

The computer program is able to give several output graphs. All data can be written to files for further treatment by other computer programs, such as spreadsheet
programs or specific presentation programmes. The following output graphs are available:

- Height - porosity diagrams at a given time.
- Porosity - time diagrams at a given height.
- Porosity - time diagram at $z = 0$.
- Turn-over height as a function of time.
- Height of jig bed as a function of time.
- Porosity at the top of the jig bed as a function of time.
- Height of the upper side of the fluidized part as a function of time.
- Porosity at the top of the fluidized part as a function of time.
- Height of the packed phase at the bottom of the bed as a function of time.
- Porosity just above the packed phase as a function of time.
- Height of the ridge as a function of time.
- Porosity just above the ridge as a function of time.
- Porosity just below the ridge as a function of time.

3.7.12. Example of computer simulation

In this paragraph an example is given of the computer simulation of the porosity distribution in a uniform jig bed.

*Input parameters*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>liquid viscosity</td>
<td>1.00000E-03 kg/ms</td>
</tr>
<tr>
<td>liquid density</td>
<td>8.61000E+02 kg/m$^3$</td>
</tr>
<tr>
<td>particle density</td>
<td>1.18500E+03 kg/m$^3$</td>
</tr>
<tr>
<td>height of the packed bed</td>
<td>1.00000E-01 m</td>
</tr>
<tr>
<td>porosity of the packed bed</td>
<td>3.60000E-01</td>
</tr>
<tr>
<td>particle diameter</td>
<td>4.76200E-03 m</td>
</tr>
<tr>
<td>stroke length</td>
<td>5.00000E-02 m</td>
</tr>
<tr>
<td>jigging frequency</td>
<td>5.00000E-01 Hz</td>
</tr>
<tr>
<td>number of time steps:</td>
<td>100</td>
</tr>
</tbody>
</table>

Re calculated by: Schiller and Naumann
n calculated by: Khan and Richardson
Output:  Height - Porosity diagrams
Figure 3.8 - 3.19. Different stages of the jig bed during one jigging stroke at $t = 0$, 0.09, 0.027, 0.47, 0.67, 0.87, 0.95, 1.13, 1.23, 1.29, 1.37 and 1.43 sec. At $t = 1.45$ s the entire jigbed is packed again.
Output: porosity / Height - Time diagrams

In Figure 3.20 the Height - Time diagrams for the position of some discontinuities are given.

![Graph of Height vs. Time](image)

**Figure 3.20** Simulated heights as a function of time of some discontinuities during one jigging stroke.

In Figure 3.21 The Porosity - Time diagrams for some discontinuities are given

![Graph of Porosity vs. Time](image)

**Figure 3.21** Simulated porosities as a function of time during one jigging stroke.

In Figure 3.22 the porosity as a function of time at a fixed height above the screen is
given for five different heights.

![Porosity diagram](image)

**Figure 3.22** Porosity as a function of time at a given height.

As has been shown in this paragraph, the computer simulation is able to show the porosity distribution in a uniform jig bed in two ways:

- Porosity as a function of height at a given time
- Porosity as a function of time at a given height.

### 3.8. Assumptions and limitations of the model

In the mathematical model some assumptions have been made which restrict the applicability of the model:

- The model is quasistatic: accelerative and decelerative forces are not considered. The model assumes that the particles are instantaneously in equilibrium with the new superficial fluid velocity. In reality, the particles need time to reach the equilibrium velocity relating to a given superficial fluid velocity.
- The bottom particles are fixed on the screen. In reality, it is possible that the particles are lifted by the superficial fluid velocity.
- The inter-particle forces are also not considered; the result is that the minimum fluidization velocity calculated by the model will be lower than in reality.
- The model neglects the sedimentation behavior, or close-packing phase of a particle system. Also the transition between a packed phase and a fluidized phase is simplified by using only the hindered settling velocity of a packed bed as the minimum fluidization velocity.

Limitations:

- The model does not describe segregation but gives only the porosity distribution in a uniform jig bed consisting of spherical particles, all with the same density and diameter.
Chapter 4

Experimental set-up

4.1. Introduction

To investigate the phenomena in a jig a laboratory jig was designed and built. The whole jig system was made transparent by using a particle-fluid system with the same refractive index. A method has been developed for continuous measurement of the porosity in a uniform jig bed as a function of time and height in the bed. This method is based on the difference in light absorption between the fluid and the particles used in the jig bed. Using a laser at one side and a photo diode at the other side of the jig it is possible to determine the transmission at a certain height above the screen. For the alignment of the laser with the photo diodes an optical bench was constructed around the jig. To calibrate the output signal of the diode in terms of the porosity a fluidized bed was built, based on the same principles as the jig. In the first part of this chapter a description is given of the equipment used during the experiments, in the second part an explanation is given of why the porosity was measured by using the difference in light transmission.

PART ONE: 4.2. Delft University Laboratory Jig.\[1\]

A laboratory jig was designed and built for the observation of phenomena in a jig and for comparing these with the mathematical model. The jig contained a vertically-moving box, filled with a fluid, around a fixed screen (see Figure 4.1). Due to this construction there was no difference between the velocity of the box and of the fluid; thus, no wall effects existed between the two. By changing the refractive index of the fluid used in the jig it was possible to make the bed grains visible or invisible. The uniform jig bed consisted of transparent polymethylmethacrylate, PMMA, spheres and the fluid was a mixture of two Shellsols, AB and H. The composition of the mixture determines the refractive index of the fluid. The walls of the jig were also made from PMMA, making the whole system transparent.

The experimental jig was equipped with an hydraulic drive. The speed and stroke length, following a certain wave form, of the drive were set by a function generator and an amplifier. The jiggling process was managed by a process control system. The
position of the jig was measured by a displacement transmitter. This signal was fed back to the process control system and compared with the original signal from the generator. In Table 4.1 some relevant technical data of the Delft University Laboratory Jig are summarised.

![Diagram of the Delft University Laboratory Jig]

**Figure 4.1** Delft University Laboratory Jig

**Table 4.1** Technical data Laboratory Jig

<table>
<thead>
<tr>
<th>Density</th>
<th>kg/m$^3$</th>
<th>Screen</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;PMMA&quot;</td>
<td>1185</td>
<td>Dimensions</td>
</tr>
<tr>
<td>&quot;Shellsol AB/H&quot;</td>
<td>861</td>
<td>Open area</td>
</tr>
<tr>
<td>Viscosity Shellsol</td>
<td>$1.01 \times 10^{-3}$ kg/ms</td>
<td>Diameter holes</td>
</tr>
<tr>
<td>Diameter sphere</td>
<td>4.762 mm</td>
<td>4 mm</td>
</tr>
</tbody>
</table>

| Stroke jig       | Variable | PMMA            |
| Frequency jig    | Variable | Shellsol AB     |
| Jigging function | Variable | Shellsol H      |

Refraction index

<table>
<thead>
<tr>
<th>Density</th>
<th>kg/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;PMMA&quot;</td>
<td>1.4900</td>
</tr>
<tr>
<td>&quot;Shellsol AB/H&quot;</td>
<td>1.5119</td>
</tr>
<tr>
<td>Shellsol H</td>
<td>1.4405</td>
</tr>
</tbody>
</table>

Due to the fact that the whole system was transparent it was possible to use the laboratory jig for a wide range of applications, such as:
- Investigation of the response of a uniform monosized jigbed by means of light absorption techniques.
- Investigation of particle trajectories, with a refractive index different from that of the fluid. These trajectories can be followed visually in the middle of the jigbed. For the observation of the particle trajectories during the jigging process a high speed camera could be used, which took 50 synchronised images per second, with an exposure time of 1/1000 sec.. It was possible to construct the particle trajectories by observing the video, image by image, after the experiments.[2]
- Investigation of the behavior of irregularly shaped particles during the jigging process, in order to find a relation between the density and shape of particles associated with segregation during the jigging process.

4.3. Delft University Laboratory Fluidized Bed.

Figure 4.2  Delft University Laboratory Fluidized Bed
To calibrate the output signal of the diode (a voltage output signal linear to the amount of light detected by the diode) to the porosity, a fluidized bed was built. The particles and fluid used were the same as in the laboratory jig. The length over which the transmission was measured was equal to the length in the jig. In Figure 4.2 a schematic view of the fluidized bed is given.

Two flow meters were used, one for high and low flows, respectively. By mean of a by-pass the flow through the fluidized bed could be regulated.

4.4. Optical Bench

To measure the transmission phenomena in the jig and in the fluidized bed an optical bench was designed and built (see Figure 4.3). The light beam was split into two equal light beams by using a 50% transparent mirror at an angle of 45°. One light beam was received by a reference diode to check the stability of the light source, while the other beam passed through the jig or fluidized bed and was received by the second photo diode, monitoring the phenomena in the jig.

Figure 4.3 Optical bench.
The optical bench could be translated in the vertical direction. The output signals from the diodes were collected by a computer data acquisition system, using a Labmaster DMA interface system and Labtech Notebook software. The output of both diodes was a voltage signal. The computer system was able to measure at a sampling rate of up to 1000 Hz, depending on the number of channels used. The displacement of the jig was also measured by the computer system, so the phenomena in the jig could be correlated to the period, stroke and frequency of the jig. During the experiments both the jig and the fluidized bed were placed in a dark chamber to prevent interference from other light sources.

Technical data for the components of the optical bench:

Laser type: NEC HeNe laser, 5mW, linear polarized, wavelength 632.8 nm.

Diode types:
Reference diode: Silicon Diffused Pin Photodiode. EG en G Electro Optics, active area 20 mm$^2$

Two types of measurement diode were used:

Measurement diode I: Silicon Diffused Pin Photodiode. EG en G Electro Optics, active area 20 mm$^2$, small area diode.

Measurement diode II: Planar Diffused Silicon Photodiode. UDT Sensors Inc., active area 613 mm$^2$, wide area diode.

Aperture: Diameter: 0.49mm

4.5. Sphere holder

For measuring the transmission through a row of spheres, a sphere holder was built, see Figure 4.4. The holder could be placed in the jig as well as in the fluidized bed. Using this holder, the centres of the spheres were at exactly the same height. This holder was used to carry out some calibration experiments, and for determining the appropriate refractive index of the fluid (see next paragraph).
4.6. Refractive index system

For all the experiments it was very important that the fluid have the same refractive index as the PMMA spheres. A small difference in the refractive index causes absorption, as well as refraction and reflection of the light, to take place on the surface of the spheres. Reflection and refraction disturb the measurements and have to be minimized by creating a fluid with a refractive index as close as possible to that of the spheres.

The refractive index of the PMMA spheres is about 1.4900. By mixing the two types of Shellsols (AB with a refractive index of 1.5119 and H with an index of 1.4405) it is possible to create a fluid with the same refractive index as the spheres. A refractometer was used to measure the refractive index of the fluid. To check if the refractive index of the fluid, determined by the refractometer as 1.490, was equal to that of the PMMA spheres the following experiment was carried out.

A total of 32 spheres of PMMA, diameter 5mm, was placed in the jig on the sphere holder shown in Figure 4.4. The laser beam was placed 1.5 mm above the centre line.
Figure 4.5 Refraction of light through a row of spheres as a function of the refractive index of the fluid.

of the spheres. It could clearly be seen that the horizontal collapsed laser beam deviated from the horizontal axis during its passage through the row of spheres. This can be explained by the fact that the light was refracted, owing to a difference between the refractive index of the fluid and that of the particles.

By using Snellius's law of refraction the light trajectory through a row of spheres can be calculated as a function of the refractive index of the fluid (see appendix B). In Figure 4.5 the light trajectory, calculated by means of Snellius's law, through a row of 32 PMMA spheres in Shellsol is shown. It can be seen that a difference of 0.0005 between the two refractive indices will cause a significant deviation from the horizontal axis. From the experiment described above it could be concluded that the accuracy of three digits of the refractometer used is too low to determine the appropriate refractive index of the fluid. The most accurate method seemed to be visual observation, using the experimental set-up described. Depending on the way the laser beam travels through the row of spheres, Shellsol H (low refractive index) or Shellsol AB (high refractive index) have to be added in small quantities until the laser beam shows no deviation from the horizontal axis.
PART TWO: 4.7. Measuring the porosity by means of the transmission

For measuring the porosity in the jig bed as a function of the height in the bed and of the time, it is important that the method be able to measure porosity over a very small height, the specific measuring height $\Delta h$ (see Figure 4.6).

In the laboratory jig it was possible to use a maximum bed thickness, $h$, of about 10 cm. The smaller the specific measuring height the higher the resolution of the measurements as a function of the height.

For measuring the porosity in the Delft University Laboratory Jig it was possible to use several possible physical phenomena:

I Electric fields
II Röntgen (X-rays)
III Visible light

Figure 4.6 The specific measuring height, $\Delta h$, for measuring the porosity.

I Electric fields

The conductivity and the di-electric constant are both functions of the concentration of solids in a fluid particle system, so they are a function of the porosity. Looking for methods for measuring the conductivity or the di-electric constant it seemed that $\Delta h$ is large in comparison with $h$, i.e. several times the diameter of the spheres used in the jig (see dotted lines in Figure 4.6).

II Röntgen (X-rays)

Using Röntgen rays or radioactive tracers it would be possible to measure the porosity. Neither method is usable in the laboratory used for the experiments, on account of safety regulations and to their high investment costs.
III Visible light

From a video tape of the transparent jigbed it could clearly be seen that the intensity of light transmitted through the jigbed changed during a stroke length of the jig. Based on these observations the idea of measuring the porosity by means of light transmission techniques was considered. The advantage of using these light transmission techniques in a particle fluid system with the same refractive index is that the specific measuring height can be made very small. For measuring transmission it is necessary that the light source have a parallel light beam and a high output stability. In order to determine any difference in absorption, or transmission, between PMMA and Shellsol some experiments were carried out in a spectrophotometer, Type SP6-300 Pye Unicam. In Figure 4.7 the transmission curve, as a function of the wavelength of the light, is given for pure PMMA and for a mixture of Shellsol AB and H, with the same refractive index as PMMA. It can be seen that there is a significant difference between the transmission of the fluid and of the PMMA, over a wide range of wavelengths of light.

![Diagram](image)

**Figure 4.7** Transmission as a function of the wavelength of light
Based on these experiments a cuvette was built for the spectrophotometer, in which it was possible to measure the transmission of PMMA spheres in Shellsol AB/H as a function of the concentration of spheres in the light beam, similar as shown in Figure 4.4. The light beam was placed through the centre line of the spheres. In Figure 4.8 the transmission as a function of the wavelength and the number of spheres is given. It can be seen that the intensity of light detected by the photo diode in the spectrophotometer is a measure of the concentration of spheres in the fluid, and is thus proportional to the porosity.

\[ \text{Transmission (\%)} \]

\[ \begin{array}{c}
\text{Wavelength light (nm)} \\
300 & 400 & 500 & 600 & 700 & 800 & 900 \\
\end{array} \]

**Figure 4.8** Transmission as a function of the wavelength for different numbers of spheres.

One of the inherent problems when measuring with light arises from the fact that the spheres used have to be perfectly homogeneous in any direction (isotropic). In Figure 4.8 a dip in the curve for two spheres can be distinguished at about 430 nm; this phenomenon can be attributed to the use of an imperfect sphere. To minimize this effect, perfectly polished and securely-made spheres were used during the experiments.

Based on these experiments the experimental set-up described in the first part of this chapter was built.
Light source

For all the experiments it was important to use polarized light to minimize problems with reflection and transmission (see Appendix C).

First a xenon light source was used to measure the transmission. Xenon light has an wide spectral distribution (300 - 800 nm). To create a parallel light beam a set of lenses and apertures is needed, and the adjustment of the optical system is very critical. Small changes in the distance between the lenses create a non parallel light beam, resulting in inaccurate measurements. Because of the stroke length and frequency of the jigging operation it is important to use a high sampling rate (100 measurements/second) during experiments to measure transmission in the jig. For example, jigging with a frequency of 2 Hz means that one full jigging stroke takes only 0.5 sec. To collect enough data to create a porosity time diagram high sampling rates are necessary. Measuring the output stability of the xenon light source at this sampling rate it seemed that the xenon light source twinkled with the electrical frequency (50 Hz), so the output continuity of this light source was too low to use in for the experiments measuring the transmission.

Finally, a polarized helium-xenon laser was chosen as light source for the experiments in the jig. Laser light has a cylindrical, parallel beam shape, of small diameter, with a high output stability. The wavelength of a helium-xenon laser is 632.8 nm. As can be seen from Figure 4.8 there is a significant difference in transmission as a function of the number of spheres at this wavelength.

4.8. The law of Lambert and Beer\(^3\)

Theoretically, the transmission and the porosity can be correlated by using the Law of Lambert and Beer. If \(I_0\) is the intensity (flux) of the incidence light and \(I\) is the intensity of the light passing through a medium with concentration \(C\) of the absorbent species and of length \(l\), the Lambert-Beer law is:

\[
I = I_0 e^{-\alpha C l}
\]  \hspace{1cm} (4.1)

in which \(\alpha\) is the specific absorption coefficient, determined by measuring standard samples. The transmission, \(T\), is defined by:
\[ T = \frac{I}{I_0} = e^{-\alpha Cl} \]  

(4.2)

For the transmission experiments carried out in the jig the concentration C can be replaced by:

\[ C = 1 - e \]  

(4.3)

Thus, theoretically the transmission is defined as:

\[ T = \frac{I}{I_0} = e^{-\alpha(1 - e)t} \]  

(4.4)

The intensity of the laser light is reversed with respect to the voltage output signal by the photo diode. The output signal of the diode is linear to the intensity of the light received by the photo diode. Therefore, the transmission can be defined as:

\[ T = \frac{I}{I_0} = \frac{V}{V_0} \]  

(4.5)

in which V is the output signal of the photo diode measured through a row of spheres, and \( V_0 \) is the output signal of the photo diode measured in the jig without spheres in the laser beam.

In practice, the Lambert-Beer law, described above, is not always valid in practice due to the facts that:

- the light is not fully monochromatic and the wavelength \( \alpha \) for the different spectral components differs, and
- in addition to transmission, reflection and refraction also occur.
4.9. References


Chapter 5

Calibration of the porosity

5.1. Introduction.

As described in Chapter 4, the porosity distribution in the jig bed was measured by means of light transmission through the jig bed. The intensity of the laser light was converted to a voltage output signal by the photo diode. The output signal of the diode was linear to the intensity of the light received by the diode, so was linear with the transmission intensity. For measuring the porosity in the jig bed, transmission has to be related to porosity.

First, some experiments using a row of spheres in a sphere holder are described, to calibrate the porosity. The laser light was moved from the centre line of the spheres in an upward direction. By using this device a whole range of porosities could be achieved. Measuring the output signal of the diode, and knowing the position of the laser with respect to the centre line of the spheres, it was possible to create a porosity-transmission curve.

The next stage in the calibration research was measuring the transmission in a fluidized bed. In a fluidized bed it is possible to achieve any desired porosity by changing the superficial fluid velocity. The particles in a fluidized bed move randomly through the bed, by contrast with the particles on the sphere holder; this situation was more realistic in relation to the situation in a jig bed.

5.2. Calibration of the porosity using the sphere holder.

To calibrate the output signal of the photo diode in relation to the porosity, some experiments were carried out using the sphere holder described in Chapter 4. By moving the laser beam in an upward direction from the centre line of the row of spheres, a range of porosities occurred (see Figure 5.1). In Appendix B, the porosity as a function of the concentration of spheres and the height above the centre line is derived. This porosity is given by:
\[ \varepsilon(h) = \frac{L - nL_{mi}}{L} = \frac{L - 2nR_s(1 - \varepsilon_1(h)_{\text{one sphere}})}{L} \]  

(5.1)

To cover the whole range of porosities the sphere holder was placed in the jig. The screen was removed and the holder was fixed in the moving box of the jig. The optical bench was situated on the horizontal centre line of the spheres. By moving the jig with a saw tooth function with a very low frequency of 0.05 Hz and a stroke length of twice the radius of the spheres the whole porosity range could be measured continuously. The signals of the diodes and the position of the jig were collected by the computer system, making it possible to determine the transmission at every required height above the centre line of the spheres.

![Diagram](image)

**Figure 5.1** Porosity as a function of the height.

The intensity of the laser light is reversed to a voltage output signal by the photo diode. The output signal of the diode is linear to the intensity of the light received by the photo diode. Therefore the transmission can be defined as:

\[ T = \frac{I}{I_0} = \frac{V}{V_0} \]  

(5.2)
in which \( V \) is the output signal of the photo diode measured through a row of spheres, and \( V_0 \) is the output signal of the photo diode measured in the jig without spheres in the laser beam.

In Figure 5.2 the transmission curves as a function of the distance to the centre line of the spheres are given for 7 - 22 spheres on the sphere holder, corresponding to a porosity of 0.79 - 0.34 on the centre line of the spheres. The laser light was vertically polarized. For measuring the transmission the wide area diode was used (see Chapter 4). The sampling rate of the computer programme was 100 samples/sec. The numbers in the graph correspond to the amount of spheres in the row. The measurements were repeated several times, giving exactly the same results.

![Transmission Graph](image)

**Figure 5.2** Transmission as a function of the distance to the centre line of the spheres.

In Figure 5.3 the transmission as a function of the porosity is shown. The distance from the centre line can be translated to a porosity by using the equations derived in Appendix B. Three curves, representing three heights, are shown:
I Transmission through the centre line of the spheres.
II Transmission at 1 mm above the centre line.
III Transmission at 2 mm above the centre line.

![Transmission graph](image)

Figure 5.3 Transmission as a function of the porosity through a row of spheres. The numbers in the graph correspond to the number of spheres in the row.

Theoretically, the three curves have to coincide because for one porosity only one transmission can exist, due to the fact that there is the same amount of acrylic plastic in the laser beam.

Measuring on the centre line of the spheres the angle of incidence of the laser light with the spheres was about 0°; hence, almost no refraction or reflection of the light occurred even if there is a small difference in refractive index between the fluid and the particles.

The curve based on the experiments through the centre line of the spheres therefore gives the optimum curve. Based on the curve measured through the centre line of the spheres it can be concluded that a part of the light was lost or was not detected by the photo diode during its passage through the row of spheres at different heights. This is possible only if a part of the light is reflected, or if the light is refracted out of the
detection area of the diode. The reflected light can move in any direction, depending on the reflection angle with the different surfaces; therefore, a part of this light will never reach the photo diode. The reflection and refraction can be explained by the fact that there was always a small difference between the refractive index of the fluid and the particles, in spite of the procedure followed to determine the appropriate refractive index as described in Chapter 4. A second reason for reflection and refraction is the fact that the PMMA spheres used are not fully isotropic in light, causing disturbances in the passage of the light through the spheres.

It is obvious that the deviation from the curve measured through the centre line of the spheres increased with increasing distance from the centre line because the angle of incidence of the laser beam with the spheres increased with increasing height. Looking at Snellius's law (see appendix C), it can be seen that refraction and reflection will be stronger with increasing angle of incidence.

The transmission during these experiments was measured with the wide-area diode, (radius of active area: 13.97 mm). This diode was chosen because initial experiments, using the small area diode with a radius of active area of 2.52 mm, showed acceptable transmission curves only on the centre line of the spheres. The output of the small-area diode, when measuring the transmission above the centre line of the spheres, decreased too rapidly to get reasonable curves. The radius of the small area diode was about the same as the radius of the spheres used during the experiments. Therefore, given the fact that there was always some reflection and refraction, it is obvious that the output signal of the small area diode would decrease rapidly if the transmission were measured at increasing height above the centre line of the spheres. The wide-area diode will detect light that has some deviation from the vertical axis of the laser beam, so a part of the reflected or refracted light will be collected by this diode, resulting in a higher output signal at increasing heights from the centre line of the spheres.

To demonstrate the fact that a part of the light was refracted or reflected, a third diode was placed on the optical bench. This diode was placed under an angle of 45° (see Figure 5.4). By using this experimental set-up a qualitative experiment could be carried out to prove that there is more reflection or refraction near the top of the spheres, corresponding to the results given in Figure 5.3.
Figure 5.4 Optical bench for measuring the reflection.

In Figure 5.5 two curves are shown, determined with the experimental set-up given in Figure 5.4. The curves are based on a row of seven spheres.

I A transmission curve measured by the transmission diode
II A reflection and refraction curve measured by the reflection diode.

The output values of both the diodes are given in a voltage signal due to the fact that not only transmission was measured, but also reflection. For the reflection diode the wide-area diode was used, for the transmission diode a small-area diode was used. Due to the fact that the diodes have different amplifications and to the fact that the reflection was measured only in one direction above the row of spheres, this experiment gives only qualitative information.

From Figure 5.5 it can be seen that transmission decreased with increasing distance from the centre line of the spheres until the upper part of the laser beam reached the top of the spheres. It must be remembered that the laser beam is not a line beam but a cylindrical beam, with a diameter of 0.49 mm. From this point, a part of the beam was travelling only through the fluid, resulting in increased transmission, and a part is travelling through the spheres. The transmission would now increase now until the entire laser beam was travelling through the fluid, corresponding to the dotted line. Theoretically, the transmission curve must increase with increasing distance from the centre line, due to the fact that the porosity increased with increasing distance from the centre line.

As can be seen, a decrease in the transmission resulted in an increase of reflection.
The top of the reflection curve and the dip in the transmission curve do not coincide. This can be explained by the fact that the transmission will increased from the moment that a part of the laser beam travelled only through the fluid, but reflection reached a maximum value when the angle of incidence of the laser beam and the spheres reached a maximum value. This took place when a part of the laser beam travelled through the fluid. In this case, the laser beam was built up of an infinite number of infinitely small beams, all with a certain angle of incidence. In Figure 5.5 the position of the laser beam with respect to the sphere is given for the situations described.

![Diagram](image)

**Figure 5.5** Transmission and reflection through a row of 7 spheres.

Therefore, in spite of the fact that the refractive index of the particles is very close to that of the fluid it seemed that reflection and refraction occurred.
5.3. Calibration of the porosity using a fluidized bed.

The next stage in the calibration research was measuring the transmission in a fluidized bed. In a fluidized bed it is possible to achieve any desired porosity by changing the superficial fluid velocity. The particles in a fluidized bed move randomly through the bed, by contrast with the particles on the sphere holder used in the calibration experiments described earlier. This situation is more realistic in relation to the situation in a jig.

Knowing the number of spheres used in the fluidized bed, the volume of particles is known. By measuring the height of the fluidized bed it is possible to calculate the mean porosity in the fluidized bed, assuming that the porosity in the bed is constant. This porosity is given by:

\[
\varepsilon = \frac{Ah - n \pi r^3}{3Ah} \tag{5.3}
\]

where:

\( A \): Surface of the fluidized bed  
\( h \): Bed height  
\( n \): Number of spheres  
\( r \): Radius of spheres

In Chapter 4 the fluidized bed used for these experiments was described. 17835 spheres were used, corresponding to a bed thickness of 9.34 cm in a packed state. The optical bench was situated in such a way that the laser beam intersected the bed at a height of 7.5 cm. By changing the superficial fluid velocity every desired porosity could be achieved. Before measuring the transmission through the fluidized bed after each change of the fluidizing velocity, a 15 minutes stabilization time was maintained. The sampling rate of the computer programme was 100 samples/sec and the sampling time was 30 sec, so 3000 measurements were performed at each porosity. The transmission was measured at porosities of: 0.39, 0.41, 0.44, 0.47, 0.51, 0.54, 0.57, 0.60, 0.64, 0.67, 0.70 and 0.73.
Transmission as a function of the porosity measured in the fluidized bed. Mean = mean values of 3000 samples, Max = maximum value measured, Min = minimum value measured.

In Figure 5.6 the transmission is given as a function of the porosity. The mean measured value and the maximum and minimum value measured are given for each porosity.

The mean values measured for the different porosities in a fluidized bed are far below the values measured on the centre line of the spheres, given in Figure 5.3. It can be concluded that in a fluidized bed the reflection and refraction of the light are of greater importance than during the experiments on a row of spheres on the sphere holder.

It can also be seen that there is a significant spread between the maximum and minimum values measured at each porosity. In Figure 5.7 and Figure 5.8 the frequency distribution is given for each porosity. The frequencies are normalized to the value 1 by using the following equation:
Frequency = \frac{\text{Number of measurements with transmission value } X}{\text{total number of measurements}} \tag{5.4}

It can be seen the mean value of the transmission increases with increasing porosity and that the spread in the frequency distribution also increases with increasing porosity. Looking at the distributions in detail it can be seen that they are not fully symmetrical. The left hand part of the distributions is steeper than the right hand part. To explain these phenomena, the porosity in a fluidized bed is described in more detail in the next paragraph.

5.3.1. Porosity in a fluidized bed.

The porosity in a fluidized bed can be described in analogy of Eq.(5.1) by:

\[ \varepsilon = \frac{L - L_{\text{through spheres}}}{L} \tag{5.5} \]

In Figure 5.9 a schematic view is given for three situations in a fluidized bed, all with the same porosity. The diameter of the spheres is large in comparison to the dimensions of the fluidized bed. Measuring the porosity by means of a laser beam with a diameter several times smaller than the diameter of the spheres, the porosity is influenced by the number of spheres the beam intersects and the place in which the beam intersects the spheres.

Mean distance travelled

If the diameter of the spheres is very small in comparison with the dimensions of the fluidized bed, the mean distance travelled by the laser beam through the spheres is given by the length for which surface 1 = surface 2 (see right hand corner of Figure 5.9). This will be true for every porosity. The standard deviation will be very small.
Figure 5.7  Frequency distribution for the transmission for different porosities measured in a fluidized bed.
Figure 5.8  Frequency distribution for the transmission for different porosities measured in a fluidized bed.
In cases the diameter of the spheres cannot be neglected in comparison to the dimensions of the fluidized bed, the spread around the mean distance travelled will be greater, due to the fact that there is a relatively small number of particles in the laser beam; statistically, it can be said that the population is very small. This is depicted schematically by the three two-dimensional situations shown in Figure 5.9.; in reality, the situation is three-dimensional. The spread around the mean distance travelled will increase with increasing porosity, due to the fact that at high porosities there are fewer particles in the laser beam than at low porosities.

\[ L \text{ } 1t/m3 = L \text{ } 4t/m8 = L \text{ } 9t/m12 \]

Figure 5.9  Three situations in a fluidized bed all with the same porosity.

In the fluidized bed used for the experiments the ratio of the diameter of the spheres to the length through the bed is not infinitely small, so there will be a great spread around the mean distance travelled through the spheres.
Relation between transmission and the distance travelled

From the experiments measuring the transmission through a row of spheres on a sphere holder it is known that the height at which the laser intersects the spheres is of great importance with respect to the measured transmission. For a given porosity the transmission will decrease with increasing height to the centre line of the spheres, corresponding to a decreasing intersection length. Reflection and refraction are of more importance at smaller intersection lengths, owing to the fact that the angle of incidence increases.

Therefore, a spread around the mean distance travelled will cause a spread in the measured transmission. This results, for the three situations given in Figure 5.9, in the fact that:

Transmission situation 1  >  Transmission 3  >  Transmission 2

in spite of the fact that the porosity of the three situations is the same.

Due to the fact that the spread around the mean travelled distance will increase with increasing porosity, the spread in the transmission will also increase with increasing porosity.

Relation between the porosity in a fluidized bed and the values measured

The spread in the frequency distributions measured in the fluidized bed, shown in Figure 5.7 and Figure 5.8, can be explained by the mechanisms described above.

Another reason for the measured spread arises from the probability that not all of the spheres are perfectly isotropic and that during fluidization some spheres will be damaged, resulting in non-isotropic spheres.

If one or more imperfect spheres are in the laser beam, the measured transmission will be disturbed by these spheres. It is impossible to quantify the effect of this phenomenon on the measured transmission. The transmission can be totally blocked or be influenced only slightly.

If we assume that 1% of the spheres is imperfect the probability that one or more
imperfect spheres are in the laser beam can be calculated as a function of the porosity, due to the fact that the total number of spheres used during the experiments is known.

Table 5.1  Probability of one or more imperfect spheres in the laser beam as a function of the porosity

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Number of spheres in the laser beam to obtain the porosity</th>
<th>Probability that one or more imperfect spheres are in the beam (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.39</td>
<td>21</td>
<td>19.0</td>
</tr>
<tr>
<td>0.41</td>
<td>20</td>
<td>18.2</td>
</tr>
<tr>
<td>0.44</td>
<td>19</td>
<td>17.4</td>
</tr>
<tr>
<td>0.47</td>
<td>18</td>
<td>16.6</td>
</tr>
<tr>
<td>0.51</td>
<td>17</td>
<td>15.7</td>
</tr>
<tr>
<td>0.54</td>
<td>16</td>
<td>14.9</td>
</tr>
<tr>
<td>0.57</td>
<td>15</td>
<td>14.0</td>
</tr>
<tr>
<td>0.60</td>
<td>14</td>
<td>13.1</td>
</tr>
<tr>
<td>0.64</td>
<td>12</td>
<td>12.3</td>
</tr>
<tr>
<td>0.67</td>
<td>11</td>
<td>11.4</td>
</tr>
<tr>
<td>0.70</td>
<td>10</td>
<td>10.5</td>
</tr>
<tr>
<td>0.73</td>
<td>9</td>
<td>8.7</td>
</tr>
</tbody>
</table>

In Table 5.1, the number of spheres in the laser beam to obtain a given porosity is rounded off to a whole number of spheres, assuming that the laser beam travels through the centre line of the spheres.

From Table 5.1, it can be seen that the probability is a function of the number of spheres in the laser beam, and so is a function of the porosity. The higher the porosity the smaller the number of spheres in the row and the lower the probability that one or more imperfect spheres are in the laser beam.

The asymmetrical shape of the frequency distributions can be explained by the fact that, in a fluidized bed, local porosity fluctuations always exist. From the literature[1]
it is known that a local increase in porosity is more common than a local decrease. An increase in porosity will increasing the measured transmission. Therefore, relatively more high-transmission measurements will occur at one given porosity due to these fluctuations, resulting in a asymmetrical shape of the frequency distributions. The fluctuations will increase with increasing porosity, so the asymmetry of the distributions will also increase with increasing porosities.

5.3.2. The standard deviation.

The standard deviation of a set of data can be reduced by doing many more measurements. If $\sigma_1$ is the standard deviation for $M_1$ measurements than the standard deviation, $\sigma_2$, for $M_2$ measurements is given by:

$$\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{M_2}{M_1}}$$  \hspace{1cm} (5.6)

The frequency distributions for the fluidized bed are based on 3000 transmission measurements for each porosity. To reduce the spread to a reasonable level many more measurements are needed. In the jig bed, the same number of measurements for each porosity has to be carried out to reach the same statistical stability, due to the fact that, in a jig bed, the porosity is described in the same way as in the fluidized bed.

In the jig bed the porosity changes as a function of time, so a range of porosities will occur during one full jigging stroke. In other words, each point of time during the stroke corresponds to a certain porosity. To reach the same statistical stability as in the fluidized bed the number of stroke lengths that have to be measured is equal to the amount of measurements in the fluidized bed.

Example: Sampling one stroke length of the jig at a jigging speed of 0.5 Hz, with a sampling rate of the computer programme of 100 samples/sec, gives 200 samples. To reach the same stability as described above 3000 stroke lengths have to be measured resulting in $3000 \times 200 = 600000$ data points.
Calibration of the porosity

To check the computer model a range of jiggling speeds and stroke lengths has to be evaluated, resulting in an enormous number of data points.

The computer equipment used in this research is not able to handle many more data than those collected during the described experiments in the fluidized bed. Measuring the transmission in the jig bed with the same statistical stability will give many more data than can be handled.

5.4. Conclusion

Based on the measurements of the transmission through a row of spheres and through a fluidized bed it can be concluded that it is impossible to create one single calibration curve, relating transmission to porosity. The transmission seemed to be a function not only of the porosity but also of the intersection length of the laser beam with the individual spheres, or angle of incidence, due to the fact that reflection and refraction occur. The transmission is also influenced by the number of imperfect spheres in the laser beam.

The experiments carried out on the sphere holder demonstrate that transmission, for a given porosity, decreases with decreasing intersection length of the laser beam and the spheres. The results of the calibration experiments carried out in the fluidized bed show a great spread in the transmission values for a given porosity, and generally lower transmission values than those measured on the sphere holder.

The calibration results obtained from the experiments carried out in the fluidized bed are more realistic than those obtained from the sphere holder, due to the fact that the particles in a fluidized bed and in a jig move randomly through the bed.

The numbers of measurements needed to reach statistical stability is far beyond the number that the computer systems can handle.

Generally, it can still be concluded that transmission will increase with increasing porosity. In spite of the fact that it is impossible to relate absolute values of transmission to absolute values of porosity, the transmission measurements can still be used to give relative information about porosity.
Using continuous measurements of transmission in the jig bed it will be possible to
give information about increases or decreases of porosity as a function of the time.
Due to the fact that it is possible to measure transmission continuously in the jig bed
it is possible to compare these transmission values with the mathematical model,
providing qualitative information about the validity of the model.

5.5. References

[1] Haughey D.P., Beveridge G.S.G.; "Local voidage variation in a randomly packed bed
Chapter 6

Light transmission measurements through a jig bed
in relation to the mathematical model

6.1. Introduction

In Chapter 3 a mathematical model for the response of a uniform jig bed in terms of the porosity distribution was described. To check the model, measurements have been carried out in a jig. The jig bed consisted of uniform PMMA spheres with a refractive index equal to that of the fluid used, making the jig bed invisible. By means of light transmission techniques the porosity could be qualified. The experimental set-up for the experiments is been described in detail in Chapter 4. In Chapter 5 it was been proven that it is impossible to give a unique relation between the transmission and the porosity, due to the fact that a fraction of the light is always refracted and reflected, in spite of the procedure followed to determine the correct refractive index of the fluid.

Generally, it can still be concluded that transmission will increase with increasing porosity. In spite of the fact that it is impossible to relate absolute values of transmission to absolute values of porosity, transmission measurements will give relative information about the porosity in the jig bed. Using transmission measurements made continuously in the jig bed it is possible to obtain information about any increase or decrease of the porosity in the jig bed as a function of time. The transmission values can be compared with the porosity values calculated by the mathematical model and give qualitative information about the validity of the mathematical model.

6.2. Experimental procedure

The optical bench was constructed around the jig. The laser was vertically polarized and the transmission was measured with the wide-area diode. Before measuring the transmission at a certain height above the screen as a function of time the jig was adjusted to operate at a certain stroke length and jiggling speed.
During the experiments the following parameters were registered by the computer system, with a sampling rate of 100 samples a second over a period of 30 seconds:

- Time
- Position of the jig
- Output reference diode
- Output measuring diode

By measurement of these four parameters it was possible to create transmission diagrams as a function of time, relative to the position of the jig.

The transmission was measured as a function of the height in the jig bed, jigging speed, stroke length, and bed thickness. For the experiments the following values were used:

**I**

Bed thickness: 5 cm

Measuring height above the screen: 5, 15, 30, 45, 55 mm
At every height a jigging speed of: 0.25, 0.5, 1.0, 1.5, 2.0 Hz
At every jigging speed a stroke length of: 8, 17, 26, 35 mm

**II**

Bed thickness: 7 cm

Measuring height above the screen: 15, 30, 45 mm
At every height a jigging speed of: 1.0 Hz
At a stroke length of: 17, 26 mm

**III**

Bed thickness: 3 cm

Measuring height above the screen: 5, 15, 25 mm
At every height a jigging speed of: 1.0 Hz
At a stroke length of: 17, 26 mm

Depending on the jigging speed, between 7 and 60 strokes could be measured, due to the fact that a fixed sampling time of 30 seconds was used. So for every defined set of experimental values several duplicate measurements were made.
The transmission is defined by:

$$T = \frac{V}{V_0} \times 100\%$$  \hspace{1cm} (6.1) $$

where $V$ is the voltage output signal of the photo diode measured through the jig bed at a given time and height, and $V_0$ is the voltage output signal of the diode in the absence of spheres in the jig, i.e. the 100% transmission value.

6.2.1. Procedure to determine the 100% transmission value

Due to the fact that the entire jiggling box is moved upwards and downwards around a fixed screen, the laser does not intersect the wall of the jig at a fixed position. To eliminate the influence of any small damage to the wall of the jig, the 100% transmission value was measured as a function of the position of the box.

At every measuring height a transmission experiment was carried out without spheres in the jig to determine the 100% transmission values. These measurements were carried out at the greatest stroke length and lowest jiggling speed used for a given set of experiments, resulting in the determination of the 100% transmission value for each intersection point of the laser with the wall.

Measuring the light intensity through the jig with spheres, the position of the wall was also recorded by the computer system. Every transmission value measured could thus be joined to its specific 100% value measured in the empty jig.

During all the experiments the output value of the reference diode was very stable, and thus the intensity of the laser as a function of time was also stable. Because of this it was not necessary to correct the values of the measuring diode for fluctuations in the output of the laser.

6.3. General remarks with respect to the data processing of the transmission measurements

From all the duplicate measurements it appeared that there was a large spread in the
transmission curves under identical jiggling conditions. This can be explained by the same theory used to describe the spread in the results of the experiments carried out in the fluidized bed to calibrate the porosity (see Chapter 5). The transmission is a function not only of the porosity but also of the intersection length of the laser beam with the individual spheres. The more spheres intersected at a given porosity, the higher the reflection and refraction and the lower the transmission. The particles in the jig bed move randomly, so that a given porosity can be obtained by a different number of particles in the laser beam (see Figure 5.9), resulting in different transmission values for one porosity. It is reasonable to assume that the particle orientation at a given height in the jig bed is not the same for every measured stroke at a given time point under the same jiggling conditions. This will result in different transmission values but does not necessarily imply that the porosity at that height was not the same in all cases.

Another phenomenon explaining the spread in the measured transmission curves is the fact that not all the spheres are perfectly isotropic (see also Chapter 5). Non-isotropic spheres will cause a spread in the transmission measured at a given porosity. It is impossible to quantify this effect due to the facts that transmission can either be blocked or be influenced only slightly by non-isotropic spheres, and that a given porosity can be obtained by a different number of spheres in the laser beam, depending on the intersection length of the different spheres. The probability that one or more imperfect spheres are in the laser beam is a function of the total number of spheres in the beam and the ratio of non-isotropic spheres to the total number of spheres used in the system.

So, at a given porosity the transmission value can be disturbed by:

- Reflection and refraction, a function of the intersection length, and
- Non-isotropic spheres in the laser beam.

Each phenomenon is a function of the number of spheres in the laser beam and each has its own statistical probability of occurring at a given number of spheres, or porosity.

Due to the two mechanisms described above it is not significant to use an average transmission curve obtained from several measurements under the same jiggling conditions, due to the fact that the two phenomena cannot be quantified. To compare the transmission values with the porosity values calculated by the mathematical model, the first stroke measured with an amplitude equal to the mean amplitude (plus or
minus 5%) of all measurements was used.
In Figure 6.1 an example is given for the transmission, as a function of time, at a specific measuring height and under specified jiggling conditions. The arrow in the graph indicates the stroke used for comparison with the mathematical model.

6.4. Experimental results.

The transmission curves measured and the porosity curves calculated by the model are visualized in the following way:

- On the X axis the time of one stroke of the jig is represented, e.g. one period. At period = 0 the jig is in its lowest position and will start moving upwards. At period = 0.5 the jig reaches its highest position and will start moving downwards until period = 1 when the jig again reaches its lowest position.
- On the left Y axis the measured transmission is represented.
- On the right Y axis the porosity calculated by the model is given.
For a given bed height, measuring height, and jiggling speed the transmission values and porosities are given for several stroke lengths in one graph. This results in 4 transmission and 4 porosity lines per graph for a bed height of 5 cm, due to the fact that four different stroke lengths were measured. The numbers in the graphs correspond to a specific stroke length:

1: Stroke length: 8 mm
2: Stroke length: 17 mm
3: Stroke length: 26 mm
4: Stroke length: 35 mm

If there was no significant difference between the transmission curves the four numbers were placed together above the curves. A straight line for the porosity curve means that the porosity remains 0.376 or 1, depending on the measuring height.
6.4.1 Bed height 5 cm, measuring height 5 mm

**Bed height 5 cm, measuring height 5 mm, jiggling speed 0.25 Hz**

Transmission   | Porosity
---|---

**Bed height 5 cm, measuring height 5 mm, jiggling speed 0.5 Hz**

Transmission   | Porosity
---|---
Bed height 5 cm, measuring height 5 mm, jigging speed 1.0 Hz

Bed height 5 cm, measuring height 5 mm, jigging speed 1.5 Hz
Bed height 5 cm, measuring height 5 mm, jigging speed 2.0 Hz

![Graph showing Transmission vs. Porosity against Period]

**Figure 6.2 - 6.6** Transmission / Porosity - Period curves. Bed height 5 cm, measuring height 5 mm.

**Differences between transmission and porosity curves:**

Not all the predicted porosity curves for a given stroke length can be reconciled with the transmission curves. For example, in the first graph the model predicts a porosity increase for a stroke length of 35 mm but this increase cannot be reconciled with the transmission curves. From the graphs it can be seen that this phenomenon occurs in almost all cases.

**Explanation:**

From the graphs it can be seen that an increase of the porosity which cannot be found in the transmission curves occurs at porosities with values below about 0.5. This implies that the resolution of the transmission measurements is too low to detect the difference between the porosities of a packed bed and about 0.5. It can be concluded that the transmission will increase significantly at porosity values, calculated by the model, of above 0.5.
The phenomenon described above cannot be attributed to the fact that the inter-particle forces are neglected by the model, resulting in a higher minimum fluidization velocity than that used in the model. This can be explained by the following consideration. For the particle system used during the experiments, the minimum fluidization velocity calculated by the model (see Eq.3.38) is 20.98 mm/s. The maximum superficial fluid velocity as a function of the jiggling speed and stroke length, calculated by using Eq.3.27, is given in Table 6.1.

Table 6.1 Maximum superficial fluid velocity as a function of the jiggling speed and stroke length, in mm/s.

<table>
<thead>
<tr>
<th></th>
<th>Str. 8 mm</th>
<th>Str. 17 mm</th>
<th>Str. 26 mm</th>
<th>Str. 35 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.S. 0.50 Hz</td>
<td>12.57[1][2]</td>
<td>26.70[2]</td>
<td>40.84</td>
<td>54.98</td>
</tr>
<tr>
<td>J.S. 1.0 Hz</td>
<td>25.13[2]</td>
<td>53.40</td>
<td>81.68</td>
<td>109.96</td>
</tr>
<tr>
<td>J.S. 1.5 Hz</td>
<td>37.70[2]</td>
<td>80.11</td>
<td>122.52</td>
<td>164.93</td>
</tr>
<tr>
<td>J.S. 2.0 Hz</td>
<td>50.27</td>
<td>106.80</td>
<td>163.36</td>
<td>219.91</td>
</tr>
</tbody>
</table>

[1] The maximum superficial fluid velocity is below the minimum fluidization velocity used by the computer model; the porosity remains 0.376 during the whole stroke, as can be seen from the graphs.


The minimum fluidization velocity used by the model is based on the assumption that the porosity will increase at the moment that the porosity in the Richardson and Zaki equation exceeds the porosity of a packed bed, 0.376, due to an increase of the superficial fluid velocity. From the literature (see Appendix A) it is known that there is a transition zone between a packed phase and a fluidized phase, due to the inter-particle forces. It is generally accepted that fluidization occurs at porosities above 0.41. Therefore, at below this porosity the validity of the Richardson and Zaki equation is
 Transmission jig bed

 debatable; above this value the use of the equation is accepted. The corresponding superficial fluid velocity calculated by the Richardson and Zaki equation to obtain a porosity of 0.41 is given by:

\[ U = V_T (\varepsilon)^n = 0.23 (0.41)^{2.44} = 25.88 \times 10^{-3} \text{ m/s} \quad (6.2) \]

(Values are based on the particle system used during the experiments)

Therefore, even if there is a difference between the minimum fluidization velocity used by the model and the minimum fluidization velocity in reality, the porosity has to increase at superficial fluid velocities greater than 25.88 mm/s at a measuring height of 5 mm, just above the screen.

For example, the maximum superficial fluid velocity at a stroke length of 8 mm and a jiggling speed of 1.5 Hz is 37.70 mm/s; this value will always exceed the minimum fluidization velocity, resulting in an increase in porosity. This increase has not been found in the transmission curve for these jiggling conditions.

II The increase of transmission, at a given stroke length, starts at a later point of time with respect to the increase of porosity.

Explanation 1:

As described above, the resolution of the transmission measurements is too low to detect porosities below about 0.5. All porosity curves start increasing from the porosity of a packed bed, 0.376. Until the value of the porosity exceeds a value of about 0.5 the transmission measurements show no response. This results in a later starting point of these curves with respect to the porosity curves.

Explanation 2:

As described above, the minimum fluidization velocity used in the model is too low, resulting in the fact that the time at which the superficial fluid velocity reaches the minimum fluidization velocity is later in reality than that predicted by the model. Therefore, the time at which the jig bed starts expanding is a later point than that
predicted by the model. In Figure 6.7 a schematic view of this situation is given for both cases. It should be remembered that the measuring height is 5 mm, i.e. just above the screen, so the porosity will follow the superficial fluid velocity immediately.

**Figure 6.7** Schematic view of the superficial fluid velocity in relation with the minimum fluidization velocity.

Explanation 3:

Due to the fact that the model is quasistatic the acceleration and deceleration of the particles are neglected, i.e. it is assumed that the particles are instantaneously in equilibrium with the corresponding superficial fluid velocity. In reality, the particles need time to reach their new positions by acceleration, deceleration and movement (see Appendix D).

The phenomenon described above is the result of a combination of the three explanations given. The influence of each single explanation on the curves is difficult to determine.
III The point of time at which the transmission curves again reach their starting values, i.e. those values corresponding to a packed bed, is in almost all cases later than the time at which the porosity curves again reach the values of a packed bed. Also, the decrease in the porosity curves is steeper than the decrease in the transmission curves.

Explanation:

The origin of this phenomenon lies in the fact that the model is quasistatic. In reality, acceleration and deceleration of the particles cause a delay in time with respect to the model. In other words, the particles need more time to reach their original position than the model predicts.

The porosity curves show a very steep decrease in porosity. This means that the particle concentration is increasing very fast between two time points. A decrease of porosity at a certain height and time in the jig bed is obtained by particles moving downwards from a level above the height considered. Due to the quasistatic character of the model it is possible that the model predicts a high increase of the particle concentration between two time points but that in reality it is physically impossible to obtain such a high particle concentration in such a small time interval. The time interval is too short to move the particles from a higher level to the level considered.

The mechanisms described above will result in a delay of the packing and a smoother decrease of the transmission curves with respect to the porosity curves.

This delay is in contrast with the fact the minimum fluidization velocity used by the model is too low. As can be seen from Figure 6.7, an increase in the minimum fluidization velocity will cause packing to occur earlier. The quasistatic character of the model dominates this phenomenon.

IV The positive slope of the transmission curves is sometimes steeper than that of the porosity curves. At higher jiggling speeds the transmission curves shows almost the same values for different high values of the stroke lengths.
Explanation 1:

The fact that the transmission curves sometimes increase faster can in some cases be explained by the fact that the resolution of the transmission measurements is too low to detect low porosities. Due to this fact, the increase of the transmission curves starts at the moment when the porosity exceeds a value of about 0.5, reached at a later point of time. This results in a steeper start to the transmission curves.

Looking, for example, to the transmission curves at a jigging speed of 1.0 Hz it can be seen that the slope of the transmission curves is steeper with increasing stroke length. This cannot be explained by the interpretation given above.

Explanation 2:

The mathematical model assumes that the bottom particles are fixed on the screen. This assumption is necessary for the use of the Richardson and Zaki equation for predicting the porosity distribution at $z = 0$ (see Chapter 3). In reality, it can be seen that the jig bed is sometimes lifted entirely to a certain height. The higher the jigging speed the lower the stroke length necessary to produce this phenomenon. Two situations are now possible:

- At a given moment the particles start settling from the bottom of the lifted bed.
- The jig bed remains packed and will move as a plunger.

At low measuring heights, just above the screen, the porosity will increase from 0.376 to a value of 1, i.e. fluid only, resulting in steeply increasing transmission curves with almost 100 % end values, i.e. values measured in fluid only. At a given jigging speed this can occur for more than one stroke length. The settling of the particles from the bottom of the lifted jig bed will also result in a smooth decrease of the transmission curves.

Similarity between transmission and porosity curves:

Under the restriction of the differences described above the following similarities can be seen:
I  At a given jiggling speed an increase of the stroke length results in an increase of porosity and also in an increase of the measured transmission values.

II  At a given stroke length an increase in jiggling speed results in an increase in porosity and also in an increase in the transmission values measured.

III The time for which the jig bed is fluidized is limited to a restricted time of the stroke. An increase of the stroke length at a given jiggling speed and an increase of the jiggling speed at a given stroke length results both in an increase in the time for which the jig bed is fluidized.

IV  The porosity curves and the transmission curves both show that the jig bed is opened during the time the fluid is moving upwards as well as that for which the fluid is moving downwards, turning point of the fluid is period = 0.5. But the main point of the opening of the jig bed at this measuring height lies during the upward part of the stroke.
6.4.2. Bed height 5 cm, measuring height 15 mm

Bed height 5 cm, measuring height 15 mm, jiggling speed 0.25 Hz

Bed height 5 cm, measuring height 15 mm, jiggling speed 0.5 Hz
Bed height 5 cm, measuring height 15 mm, jigging speed 1.0 Hz

Bed height 5 cm, measuring height 15 mm, jigging speed 1.5 Hz
Bed height 5 cm, measuring height 15 mm, jiggling speed 2.0 Hz

Figure 6.8 - 6.12 Transmission / Porosity - Period curves. Bed height 5 cm, measuring height 15 mm.

Remarkable differences with the curves measured at a height of 5 mm:

Both the porosity and the transmission curves are moved to the right, resulting in the fact that both curves start increasing at a later point of time and also reach their original value at a later point of time.

Explanation:

The porosities generated at z = 0 need time to reach a certain height in the jig bed, resulting in the fact that both curves start increasing at a later point of time. Particles first start packing at the screen, building up a packed bed. The higher the measuring height the later the point of time at which the original packed bed values are again reached, provided that the jig bed has been opened at this measuring height during one stroke. In some cases, the upper part of the jig bed remains packed during one stroke,
and the fluidized part will not reach the measuring height.

II The number of transmission curves which reach the 100 % transmission value is reduced with respect to a measuring height of 5 mm.

Explanation:

For some jigging strokes and speeds the jig bed is lifted entirely to a certain height. The higher the jigging speed at a given stroke, or the higher the jigging stroke at a given jigging speed, the higher the jig bed will be lifted. Therefore, at a measuring height of 15 mm fewer curves will reach the 100 % value than at a measuring height of 5 mm.

III Both the porosity curves and the transmission curves at a given jigging speed and stroke length show lower values with respect to the values measured at a measuring height of 5 mm.

Explanation:

High porosities are generated at late points of time at $z = 0$ and the porosity velocity $W(\varepsilon, t)$ decreases for increasing porosities, if these porosities $> \varepsilon_m$, defined in Eq.3.16. For the particle system used $\varepsilon_m$ is 0.419. Combining these two facts results in an decrease of the porosity as a function of height for a given stroke length and jigging speed.
6.4.3. Bed height 5 cm, measuring height 30 mm

Bed height 5 cm, measuring height 30 mm, jiggling speed 0.25 Hz

Bed height 5 cm, measuring height 30 mm, jiggling speed 0.5 Hz
Bed height 5 cm, measuring height 30 mm, jigging speed 1.0 Hz

Bed height 5 cm, measuring height 30 mm, jigging speed 1.5 Hz
Bed height 5 cm, measuring height 30 mm, jiggling speed 2.0 Hz

Figure 6.13 - 6.17 Transmission / Porosity - Period curves. Bed height 5cm, measuring height 30 mm.

Remarkable differences with the curves measured at a height of 5 mm and 15 mm:

I Both the porosity curves and the transmission curves are moved further to the right.

II The transmission curves generally reach lower values at a given jigging speed and stroke length than could be expected from the porosity curves, based on comparison of the porosity - transmission values at a measuring height of 15 mm. In other words, the model predicts porosity values which are too high at this measuring height.

Explanation:

Due to the fact that the model is quasistatic and acceleration and deceleration are
neglected, the model predicts, at greater measuring heights, porosities with higher values than occur in reality. The particles do not have enough time to reach the porosity distribution described by the model.

At a given point of time, the porosity velocity becomes negative and the zones start moving downwards, depending on the superficial fluid velocity. In Figure 6.18 the porosity velocity as a function of the period is given.

**Figure 6.18** Porosity velocity as a function of the period.

Combining these two considerations results in the fact that the porosity values predicted by the model are too high at increasing measuring heights. From the transmission curves it can be concluded that the porosities do not reach such high values in reality.
6.4.4. Bed height 5 cm, measuring height 45 mm

Bed height 5 cm, measuring height 45 mm, jiggling speed 0.25 Hz

Bed height 5 cm, measuring height 45 mm, jiggling speed 0.5 Hz
Bed height 5 cm, measuring height 45 mm, jigging speed 1.0 Hz

Bed height 5 cm, measuring height 45 mm, jigging speed 1.5 Hz
Bed height 5 cm, measuring height 45 mm, jigging speed 2.0 Hz

Figure 6.19 - 6.23  Transmission / Porosity -Period curves. Bed height 5 cm, measuring height 45 mm.

Remarkable differences with the curves measured at a height of 5 mm, 15 mm and 30 mm:

1. The transmission curves do not show a response for all jigging speeds and stroke lengths used.

Explanation 1:

The observation that there is no response at a measuring height of 45 mm results from the facts that the porosity values predicted by the model are too high at increasing height and that the resolution of the transmission measurements is too low to detect porosities below 0.5.

Explanation 2:

As described above, it is possible that the jig bed is lifted entirely, resulting in the fact that it remains packed at this measuring height.
6.4.5. Bed height 5 cm, measuring height 55 mm

Bed height 5 cm, measuring height 55 mm, jiggling speed 0.25 Hz

Bed height 5 cm, measuring height 55 mm, jiggling speed 0.5 Hz
Bed height 5 cm, measuring height 55 mm, jiggling speed 1.0 Hz

Transmission

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Porosity

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Period

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

Bed height 5 cm, measuring height 55 mm, jiggling speed 1.5 Hz

Transmission

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Porosity

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Period

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Bed height 5 cm, measuring height 55 mm, jiggling speed 2.0 Hz

Figure 6.24 - 6.28  Transmission / Porosity - Period curves. Bed height 5cm, measuring height 55 mm.

Explanation of the porosity curves:

Due to the fact that the bed height in a packed stage is 50 mm, the porosity at a measuring height of 55 mm remains 1, i.e. fluid only, until the top of jig bed passes the 55 mm level. Several options are possible, depending on the stroke length and jiggling speed:

- The top of the jig bed will not reach a height of 55 mm, i.e. the porosity remains 1 for the whole period. For example, this is the situation given for a jiggling speed of 0.25 Hz, for all stroke lengths.

- The top of the jig bed will pass a height of 55 mm and the part of the jig bed lifted above the 55 mm level remains in a packed stage. For the first part of the period the porosity has a value of 1; at the moment when the top of the jig bed passes the 55 mm level the porosity will decrease to a value of 0.376 and
remains constant until the top again passes the 55 mm level. The porosity now increases to a value of 1. For example, this is the situation given for a jiggling speed of 2.0 Hz, for a stroke length of 17, 26 and 35 mm.

The top of the jig bed will pass a height of 55 mm and the part of the jig bed lifted above the 55 mm level is partially fluidized after a given time. The first part of the period the porosity has a value of 1, at the moment the top of the jig bed will pass the 55 mm level the porosity will decrease to a value of 0.376, at the moment the fluidized part of the jig bed will pass the 55 mm level the porosity will obtain the corresponding value. Now two situations are possible:

- The jig bed remains in a fluidized stage at the 55 mm level until the top of the jig bed again passes this level. At this moment the porosity will change from a specific value to the value of 1. For example, this is the situation given for a jiggling speed of 1.0 Hz for stroke lengths of 26 and 35 mm.

- After a given time the jig bed will again reach the packed stage at the 55 mm level and the porosity will now decrease to a value of 0.376. At the moment the top of the jig bed again passes the 55 mm level the porosity will change to the value of 1. For example, this is the situation given at a jiggling speed of 1.5 Hz, for a stroke length of 35 mm.

Differences between transmission and porosity curves:

1. The porosity predicted at a jiggling speed of 1.5 Hz and a stroke length of 35 mm cannot be found in the corresponding transmission curve.

Explanation 1:

This phenomenon can be explained by the fact that the resolution of the transmission measurements is too low to detect these porosities.

Explanation 2:

The model predicts porosity values which are too high with increasing measuring
height.

II  The transmission curves start decreasing at an earlier point of time with respect to the porosity curves.

Explanation:

The mathematical model assumes that the bottom particles are fixed on the screen. In reality it can be seen that the jig bed is lifted entirely to a certain height sometimes. The higher the jigging speed the lower the stroke length necessary to obtain this phenomenon. Due to this fact the top of the jig bed will reach the 55 mm level earlier than predicted by the model. Comparing the point of time the transmission curves start increasing at a measuring height of 5 mm, just above the screen, for stroke lengths at which the jig bed is lifted entirely, with the point of time at which the transmission curves start decreasing at a measuring height of 55 mm, it can be seen that these points of time roughly coincide.

III  The transmission curves start increasing at a later point of time with respect to the porosity curves.

Explanation:

The origin of this phenomenon lies in the fact that the model is quasistatic. In reality, acceleration and deceleration of the particles cause a delay in time with respect to the model. In other words, the particles need more time to reach their original position than the model predicts.

Similarity between transmission and porosity curves:

I  For jigging conditions at which the model predicts a response in the porosity, the transmission also shows a response. The shape of the porosity curves is approximately comparable with the shape of the transmission curves.

II  A response of both curves at a measuring height of 55 mm goes together with a response of both curves at a measuring height of 5 mm, i.e. just above the screen.
Explanation:

A response at a measuring height of 55 mm implies that the jig bed has been expanded. This can happen only if there is a response at the bottom of the jig bed, due to the fact that expansion will start at the bottom of the jigbed.

**General remarks with respect to all measuring heights:**

From both the transmission curves and porosity curves it can be seen that the time for which the jig bed has been fluidized, during one period, decreases with increasing measuring height.
6.4.6. Bed height 7 cm, jiggling speed 1.0 Hz

Bed height 7 cm, measuring height 15 mm, jiggling speed 1.0 Hz

Bed height 7 cm, measuring height 30 mm, jiggling speed 1.0 Hz

Transmission

Porosity
Bed height 7 cm, measuring height 45 mm, jigging speed 1.0 Hz

Figure 6.29 - 6.31 Transmission / Porosity curves. Bed height 3 cm, jigging speed 1.0 Hz.

Comparing the results with a bed height of 5 cm:

The porosity curves predicted by the model are independent of the bed height for the measuring heights considered, due to the fact that the porosity velocity $W(\varepsilon, T)$ is independent of bed-height and the porosities start moving from the bottom of the jig bed.

Comparing the transmission curves measured at a bed height of 7 cm with the corresponding transmission curves at a bed height of 5 cm, measured under the same jigging conditions, it can been seen that the curves show agreement with respect to amplitude and time position.
6.4.7. Bed height 3 cm, jiggling speed 1.0 Hz

**Bed height 3 cm, measuring height 5 mm, jiggling speed 1.0 Hz**

![Graph showing transmission and porosity data for bed height 3 cm and measuring height 5 mm.](Image)

**Bed height 3 cm, measuring height 15 mm, jiggling speed 1.0 Hz**

![Graph showing transmission and porosity data for bed height 3 cm and measuring height 15 mm.](Image)
Figure 6.32 - 6.34 Transmission / Porosity - Period curves. Bed height 3 cm, jigging speed 1.0 Hz.

Comparing the results with a bed height of 5 cm:

The porosity curves predicted by the model are independent of the bed height for the measuring heights considered, due to the fact that the porosity velocity $W(e,T)$ is independent of bed-height and the porosities start moving from the bottom of the jig bed.

Comparing the transmission curves measured at a bed height of 3 cm with the corresponding transmission curves at a bed height of 5 cm, measured under the same jigging conditions, it can be seen that the curves show agreement with respect to amplitude and time position.
6.5. Conclusions

From the experiments described it can be concluded that transmission measurements may be used to derive qualitative information about the porosity distribution in a uniform jig bed as a function of time and height in the bed.

Comparing the measured transmission values and the porosity values calculated by the model, it can be seen that there is a similarity in amplitude change and time position as a function of jigging speed, stroke length and measuring height between these two values.

Differences can be explained by the facts that:

- The resolution of the transmission measurements is too low to detect the difference between the porosity of a packed bed and about 0.5.
- The model neglects the transition zone between a packed phase and a fluidized phase.
- Accelerative and decelerative forces are neglected by the model
- The mathematical model assumes that the bottom particles are fixed on the screen.

Based on the similarity between the transmission values measured and the porosity values calculated it can be concluded that the porosity distribution in a uniform jig bed calculated by the mathematical model is comparable in a qualitative way with the porosity distribution in reality.

Due to the assumptions made in the mathematical model it is clear that the porosity distribution calculated by the model cannot be in full agreement with reality.

In spite of the great spread in the duplicate measurements and of the fact that the transmission curves used for comparison with the porosity curves were based not on an average transmission curve but only on one curve with a mean amplitude, it can be concluded that the transmission curves used for the comparison give a general picture of the situation. This can be concluded because all the trends expected under different jigging conditions and measuring heights can be found in the transmission curves used.
The mathematical model described in this thesis has given greater insight into the behaviour of particles in a uniform jig bed, in terms of the porosity distribution, but is not intended to describe the whole jiggling process with respect to stratification. To use the model for the description of segregation it must be extended to a multi-component system and the equations of movement of the individual particles have to be incorporated.

Based on the porosity distribution calculated by the model in a uniform jig bed some general remarks can be made regarding the segregation of particles in a multi-component jiggling system. The porosity distribution in such a system cannot be predicted completely from the porosity distribution in a uniform system, due to differences in diameter, density and shape of the particles.

Under the assumption that the greatest part of the segregation will take place at those moments at which the jig bed is fluidized it can be concluded that:

- During both upward and downward stroke, parts of the jig bed are fluidized. From the curves it can be seen that during the upward stroke the lower parts of the jig bed are fluidized and during the downward stroke the upper parts of the bed are fluidized. Therefore, segregation will take place during both parts of the stroke.

- The time during which particles can move relative to one another is restricted to limited parts of one period, due to the fact that the fluidized parts are time-limited. The higher the position in the jig bed the shorter the time for which the bed is fluidized. It is not claimed that the higher the porosity the faster the segregation because if a heavier particle moves 1 mm downwards through surroundings with high porosity it will pass relative fewer lighter particles than when moving 1 mm in surroundings with low porosity.

- The situation where the jig bed moves as a plunger must be avoided, since the jig bed will not then fluidize and segregation will then be difficult. The higher the jiggling speed the lower the stroke necessary to obtain this phenomenon.
Appendix A

Velocities in particle - fluid systems

A.1. Introduction

In a vertical flow of a mixture consisting of a continuous and a discontinuous component, many velocities exist, e.g. the superficial and interstitial fluid velocity, the particle velocity, the swarm velocity, and the slip velocity. Three main types of two-component vertical flow systems can be distinguished:

I  Fluidization
II  Sedimentation
III Combination of I and II

Each has its own velocity. In this Appendix the particle and fluid velocities of each system will be defined.

For the free-falling terminal velocity of a particle and for the velocity of a swarm of uniform particles several correlation are known; some of them were used in the computer model and will be described in this Appendix.

In fluid - solid systems the following velocities can be distinguished:

$V_p$: Particle velocity with respect to the wall of the system.
$U$: Superficial fluid velocity with respect to the wall of the system.
i.e. the fluid velocity based upon the total cross section of the empty tube.
$U_i$: Interstitial fluid velocity with respect to the wall of the system.
i.e. the fluid velocity in the pores between the particles.
$U_{s}(e)$: Velocity of a swarm of particles in the absence of a superficial fluid velocity.
$V_T$: Free-falling terminal velocity of one particle.
$U_s$: Slip velocity. i.e. the relative velocity between particle and fluid.

Velocities are vector quantities, so they have a length and a direction. The direction is given by a minus or plus sign, depending on the definition of the positive direction. It is possible that the scalar component of two velocities is equal but the orientation is different.
A.2. Fluidization.

Richardson and Zaki\textsuperscript{[1]} have given the following definition for fluidization: "When a fluid is passed slowly through a bed of granular solids, the bed remains static. If the velocity is increased, a stage is reached when the particles reorientate themselves and present a greater cross-sectional area to the flow of fluid; this readjustment continues until the loosest stable arrangement is attained. With further increase, the particles are individually supported by the fluid, and the bed becomes fluidized. At velocities greater than the minimum required to produce fluidization, the bed continues to expand and the particles remain uniformly dispersed in the liquid."

In a fluidized bed, see Figure A.1, the particle swarm is in equilibrium with the corresponding superficial fluid velocity, so the superficial fluid velocity $U$ is equal to the value of the swarm velocity $U_s(e)$ but in an opposite direction. The individual particle velocity $V_p = 0$. The interstitial fluid velocity $U_i$ is equal to the superficial fluid velocity $U$ divided by the open area between the particles, the porosity, $e$. 

**Figure A.1** Velocities in the three systems.
For a fluidized system the following equations can be defined:

\[ V_p = U_y(\varepsilon) = 0 \]

\[ U \neq 0 \] \hspace{1cm} (A.1)

\[ U_i = \frac{U}{\varepsilon} \] \hspace{1cm} (A.2)

\[ U_s = V_p - U_i \] \hspace{1cm} (A.3)

Combining Eq. (A.1), (A.2) and (A.3) gives:

\[ U_s = 0 - \frac{U}{\varepsilon} = -\frac{U}{\varepsilon} \] \hspace{1cm} (A.4)

**A.3. Sedimentation.**

Sedimentation can be defined as the falling of a suspension of particles at terminal velocity in a stationary fluid (see Figure A.1). The swarm particles have a velocity \( U_y(\varepsilon) \) and all the particles in the swarm have the same velocity equal to the swarm velocity. The interstitial fluid velocity is equal to the amount of fluid displaced by the particles as a function of time, divided by the open area between the particles.

Now, for a system during sedimentation the following equations can be defined:

\[ U = 0 \]

\[ U_y(\varepsilon) = V_p \neq 0 \] \hspace{1cm} (A.5)

\[ U_i = \frac{(1-\varepsilon)V_p}{\varepsilon} \] \hspace{1cm} (A.6)

\[ U_s = V_p - U_i \] \hspace{1cm} (A.7)
\[ U_s = V_p + \frac{(1-\varepsilon)V_p}{\varepsilon} = \frac{V_p}{\varepsilon} \quad (A.8) \]

A.4. Similarity between Fluidization and Sedimentation.

\( U_s(\varepsilon) \) is defined as the velocity of a swarm of particles, with porosity \( \varepsilon \), in the absence of a superficial fluid velocity during sedimentation. \( U \) is defined as the superficial fluid velocity in order to obtain a porosity \( \varepsilon \) in a fluidized bed. The drag on a particle in each case is equal to its weight in the liquid, and since the slip velocity, \( U_s \), is a unique function of the physical properties of the liquid, the particle diameter, the distance between the particles and the drag force, \( U_s \) must be the same in both cases to obtain a porosity \( \varepsilon \).[1]

**Fluidization:**

\[ U_{s1} = -\frac{U}{\varepsilon} \]

**Sedimentation:**

\[ U_{s2} = \frac{V_p}{\varepsilon} = \frac{U_s(\varepsilon)}{\varepsilon} \quad (A.9) \]

\[ U_{s1} = U_{s2} \text{ so:} \]

\[ U_s(\varepsilon) = -U \]

So the sedimentation velocity of a swarm particles with porosity \( \varepsilon \), \( U_s(\varepsilon) \), is equal to the superficial fluid velocity, \( U \), necessary to obtain a porosity \( \varepsilon \) in a fluidized bed.

A.5. Combination of Fluidization and Sedimentation.

In a system in which the superficial fluid velocity does not equal the swarm velocity particles will move in an up- or downwards direction depending on the value of the superficial velocity (see Figure A.1). The particle velocity is the sum of the superficial
fluid velocity and the swarm velocity, and the interstitial fluid velocity is a combination of the displaced fluid by the moving particles and the influence of the superficial fluid velocity. For a combined system the following velocities can be defined:

\[ U_s(e) + V_p = U = 0 \]  \hspace{1cm} (A.10)

\[ V_p = U + U_s(e) \]  \hspace{1cm} (A.11)

\[ U_i = \frac{(1-e)V_p}{e} + \frac{U}{e} \]  \hspace{1cm} (A.12)

\[ U_s = V_p - U_i \]  \hspace{1cm} (A.13)

\[ U_s = \frac{V_p}{e} - \frac{U}{e} = \frac{U_s(e)}{e} \]  \hspace{1cm} (A.14)

From these equations of the combined system the equations of a sedimentation or fluidization system can be obtained by using \( U = 0 \) or \( V_p = 0 \).

A.6. Volumetric flux in a combined system.\(^2\)

The symbol \( J \) is used to represent volumetric flux, in meters per second, or volumetric flow rate per unit area. The flux is related to the local component concentration and velocities.

The volume of particles leaving the unit area per second is given by:

\[ J_p = V_p (1-e) \]  \hspace{1cm} (A.15)

The volume of fluid leaving the unit area per second is given by:

\[ J_f = e U_i \]  \hspace{1cm} (A.16)
In the literature\cite{3\cite{4}\cite{5}} these fluxes are sometimes defined as the superficial fluid or particle velocities, but $J_f$ is not always the same as the superficial fluid velocity "$U$" defined in the previous paragraph. In a fluidized system $J_f$ is equal to $U$ because $U_i = U/\varepsilon$, but in a combined system $V_i$ is given by Eq. (A.12) so:

$$J_f = -(1-\varepsilon)V_p + U$$ \hspace{1cm} (A.17)

The total local flux is given by:

$$J = J_f + J_p$$ \hspace{1cm} (A.18)

$$J = V_p(1-\varepsilon) + \varepsilon U_i$$

Combining Eqs.(A.13), (A.15) and (A.16) the slip velocity can be defined as:

$$U_s = \frac{J_p}{1-\varepsilon} - \frac{J_f}{\varepsilon}$$ \hspace{1cm} (A.19)
A.7. Free-falling terminal velocity

The behavior of a single particle moving through a stationary fluid can be described by\cite{6}:

$$\sum \mathbf{F} = M \frac{\mathbf{dv}}{dt}$$  \hspace{1cm} (A.20)

in which:

\(\sum \mathbf{F}\) : Resultant of the forces acting on a particle. (N)

M: Mass particle (kg)

dv/dt: Acceleration (m/s\(^2\))

If the particle is moving under an external accelerative force, \(F_x\), it will have opposing forces of \(F_d\), the drag force, and \(F_b\), the buoyancy force, so:

$$F_x - F_d - F_b = M \frac{dv}{dt}$$  \hspace{1cm} (A.21)

For a gravitational acceleration force Eq.(A.21) can be replaced by:

$$Mg - \frac{1}{2} f_d \rho_f v^2 A - \frac{M \rho_g g}{\rho_s} = M \frac{dv}{dt}$$  \hspace{1cm} (A.22)

in which:

\(\rho_f\): Density of fluid (kg/m\(^3\))

\(\rho_s\): Density of particle (kg/m\(^3\))

g: Gravitational acceleration (9.81 m/s\(^2\))

A: Surface area of particles perpendicular to the direction of movement (m\(^2\))

\(f_d\): Drag coefficient

The terminal velocity of the particle is reached at the moment that dv/dt = 0, so that:

$$M \frac{\rho_s - \rho_f}{\rho_s} = \frac{1}{2} f_d \rho_f V_T^2 A$$  \hspace{1cm} (A.23)

in which:

\(V_T\): Free falling terminal velocity (m/s)
For a spherical particle A and M are defined by Equ. A.24:

\[ A = \frac{\pi d^2}{4} \quad \text{and} \quad M = \frac{\pi d^3 \rho_s}{6} \]  

(A.24)

in which:

\( d \): Diameter of sphere (m)

Combining Eqs. (A.23) and (A.24) gives for the terminal velocity of a spherical particle:

\[ V_T = \sqrt{\frac{4 gd \rho_s - \rho_f}{3 f_d \rho_f}} \]  

(A.25)

Eq. (A.25) can be rewritten by using the dimensionless groups of Archimedes, Ar, and Reynolds, Re:

\[ Ar = \frac{3}{4} f_d Re^2 \]  

(A.26)

in which:

\[ Ar = \frac{gd^3 \rho_f (\rho_s - \rho_f)}{\eta^2} \]  

(A.27)

\[ Re = \frac{\rho_f V_T d}{\eta} \]  

(A.28)

in which:

\( \eta \): Viscosity of fluid (kg/ms)

The Ar number is only a function of the system parameters and is independent of the velocity of the particle. Writing the equation of the terminal velocity in terms of the Ar and Re number makes it easy to determine the correct value of the drag coefficient; this will be described in the next paragraph.
A.7.1. The drag coefficient $f_d$

The drag coefficient is a function of the Reynolds number. There are many correlations available between $f_d$ and Re, four of which will be described and are optional in the computer program.

*Correlation of Schiller and Naumann:*\(^{17}\)

\[
f_d = \frac{24}{Re} (1 + 0.15 Re^{0.687}) \quad \text{for } Re < 988
\]

\[
f_d = 0.44 \quad \text{for } Re > 988
\]

(A.29)

Combining with Eq.(A.26) gives:

\[
Ar = 18Re + 2.7Re^{1.687} \quad \text{for } Ar < 322236
\]

\[
Ar = \frac{Re^2}{3} \quad \text{for } Ar > 322236
\]

(A.30)

Knowing the Archimedes number, which is independent of the velocity of the particle, it is possible to calculate the Reynolds number of the particle by solving Eq.(A.30). By using Eq.(A.28) it is possible to calculate the terminal velocity of the particle.

*Correlation of Stokes and Lapple:*\(^{8}\)\(^{9}\)

\[
f_d = \frac{24}{Re} \quad \text{for } Re < 0.2
\]

\[
f_d = 18.5Re^{-0.6} \quad \text{for } 0.2 < Re < 500
\]

\[
f_d = 0.44 \quad \text{for } Re > 500
\]

(A.31)

Combining with Eq.(A.26) gives:
\[ Ar = 18 \text{Re} \quad \text{for } Ar < 3.6 \]
\[ Ar = \frac{55.5 \text{Re}^{1.4}}{4} \quad \text{for } 3.6 < Ar < 83330 \]  
(A.32)
\[ Ar = \frac{\text{Re}^2}{3} \quad \text{for } Ar > 83330 \]

*Correlation of Dallavalle:*\(^{[10]}\)

\[ f_d = \left(0.63 + \frac{4.8}{\sqrt{\text{Re}}}\right)^2 \]  
(A.33)

Combining with Eq.(A.26) gives:

\[ Ar = (0.63 \text{Re} + 4.8 \text{Re}^{0.5})^2 \]

*Correlation of Richardson and Kahn:*\(^{[11]}\)

\[ f_d = (2.25 \text{Re}^{-0.51} + 0.36 \text{Re}^{0.06})^{3.45} \]  
(A.35)

Combining with Eq.(A.26) gives:

\[ Ar = (2.07 \text{Re}^{0.27} + 0.33 \text{Re}^{0.64})^{3.45} \]  
(A.36)

**A.8. Terminal velocity of a swarm of uniform particles.**

The falling velocity of a swarm of particles is lower than that of a single particle. This phenomenon is the result of a combination of factors. In a multi-particle system, there is a significant upflow of displaced fluid, changed buoyancy effects, and steeper velocity gradients, at a given particle velocity relative to the fluid. In addition to the particle diameter and density a third variable is introduced, i.e. the volumetric fraction occupied by the particles or the inverse term, the porosity.
A literature study was carried out to find relationships between the swarm velocity and the porosity.\textsuperscript{[12]} From this study it can be concluded that the Richardson and Zaki equation is the most suitable to use for the mathematical jigging model. By comparison with other equations\textsuperscript{[13] [14] [15] [16] [17]} the Richardson and Zaki equation\textsuperscript{[1]} gives mean values for the velocity as a function of the porosity and, due to its simple mathematical form, it is easy to integrate and differentiate the equation. The equation of Richardson and Zaki is given by:

\[ U_s(\varepsilon) = V_T \varepsilon^n \]  \hspace{1cm} \text{(A.37)}

in which:
\[ n: \text{ Constant} \]

\textbf{A.8.1. The constant, } n \text{, from the Richardson and Zaki equation}

The constant \( n \) in Eq.(A.37) is a function of the Reynolds number. For \( n \) several relationships are available, five of which will be described and are optional in the computer program.

\textit{Correlation of Richardson and Zaki.\textsuperscript{[1]}}

\[ n = 4.65 \quad \text{for } Re < 0.2 \]
\[ n = 4.35 Re^{-0.03} \quad \text{for } 0.2 < Re < 1 \]  \hspace{1cm} \text{(A.38)}
\[ n = 4.45 Re^{-0.1} \quad \text{for } 1 < Re < 500 \]
\[ n = 2.39 \quad \text{for } Re > 500 \]

\textit{Correlation of Garside and Al-Dibount\textsuperscript{[14]}}

\[ \frac{5.09 - n}{n - 2.73} = 0.104 Re^{0.877} \]  \hspace{1cm} \text{(A.39)}
Correlation of Rowe.\textsuperscript{[18]}

\[
\frac{4.70 - n}{n - 2.35} = 0.175 \text{Re}^{0.75} \tag{A.40}
\]

Correlation of Khan and Richardson.\textsuperscript{[19]}

\[
\frac{4.8 - n}{n - 2.4} = 0.043 \text{Ar}^{0.57} \tag{A.41}
\]

Correlation derived from the equation of Ergun.

The pressure drop over a packed bed is given by the equation of Ergun\textsuperscript{[20]}:

\[
\frac{\Delta P}{L} = 150 \frac{(1 - \varepsilon)^2 \eta U}{\varepsilon^3 d^2} + 1.75 \frac{1 - \varepsilon}{\varepsilon^3} \frac{\rho_f U^2}{d} \tag{A.42}
\]

At minimum fluidization the pressure drop is given by:

\[
\frac{\Delta P}{L} = g(\rho_s - \rho_f)(1 - \varepsilon) \tag{A.43}
\]

Combining Eqs. (A.42) and (A.43) gives for minimum fluidization:

\[
A - g(\rho_s - \rho_f) = 0 \tag{A.44}
\]

in which:

\[
A = 150 \frac{1 - \varepsilon_{mf}}{\varepsilon_{mf}^3} \frac{\eta U_{mf}}{d^2} + 1.75 \frac{\rho_f U_{mf}^2}{\varepsilon_{mf}^3 d} \tag{A.45}
\]

in which:

- $U_{mf}$: Minimum fluidization velocity.
- $\varepsilon_{mf}$: Minimum fluidization porosity.
Given \( \varepsilon_{mf} \) it is possible to calculate \( U_{mf} \) by using Eqs. (A.44) and (A.45). From the literature it is known that: \( 0.40 < \varepsilon_{mf} < 0.42 \). By using Eq. (A.37) it is possible to calculate the value of the constant \( n \):

\[
 n = \frac{\log U_{mf} - \log V_T}{\log \varepsilon_{mf}} \tag{A.46}
\]

**A.9. References**


[8] Stokes G.G.; "Mathematical and Physical papers."., 1901


[12] Bunnik J.J.; "Review of hindered settling and sedimentation theories in relation with a jig bed theory". Thesis Delft University of Technology, faculty of Mining and Petroleum Engineering, Department of Mineral resources Technology, September


Appendix B

Porosity distribution through a row of spheres

B.1. Introduction

In this Appendix the mathematical derivation is given for the calculation of the porosity through a row of spheres as a function of the height at which the laser beam intersects the sphere.

The laser used in the experiments was not a line beam, with an infinitely small diameter, but a cylindrical beam with a radius of 0.49 mm. Therefore, the distance travelled by the beam through a sphere must be corrected for this effect. If the intersection volume from the cylindrical beam and the sphere is known, it is possible to calculate the mean distance travelled through the sphere by dividing this volume by the surface area of the beam. It is also possible that the laser beam partially intersects the sphere; this phenomenon also has a great effect on the calculated porosity. The porosity for a line beam is described first in this Appendix; based on this derivation, the porosity for a cylindrical light beam is described.

B.2. The porosity as a function of the height through a row of spheres for a line light beam

Generally, for a fluid-solid system the porosity is defined as:

\[
\varepsilon = \frac{\text{Total volume} - \text{Volume solids}}{\text{Total volume}}
\]  

(B.1)

In Figure B.1 the intersection of a line beam with one sphere is given as a function of the height the light intersects the sphere. A line beam is a beam with an infinitely small radius. For the porosity through one sphere a system is defined in which the total length of the system is equal to the diameter of one sphere. It can be seen that, with increasing h, the distance for

![Figure B.1](image-url)
which the light travelled through the sphere decreases, so the porosity will increase. In this case the porosity can be defined by:

\[
\varepsilon = \frac{\text{Total length travelled by the light} - \text{Length travelled through the sphere}}{\text{Total length travelled}}
\]

Rewriting Eq.(B.2) gives the porosity as a function of height:

\[
\varepsilon(h)_{\text{one sphere}} = \frac{2R_s - L_s}{2R_s} = \frac{2R_s - 2\sqrt{R_s^2 - h^2}}{2R_s}
\]  \hspace{1cm} (B.3)

In which:
- \(\varepsilon\) = Porosity
- \(R_s\) = Radius of sphere
- \(L_s\) = Distance travelled by the light through the sphere.
- \(h\) = Height

For a system with total length \(L\) and \(n\) spheres in the line beam the porosity is given by:

\[
\varepsilon(h) = \frac{L - nL_s}{L} = \frac{L - 2nR_s(1 - \varepsilon(h)_{\text{one sphere}})}{L}
\]  \hspace{1cm} (B.4)

B.3. The porosity as a function of the height through a row of spheres for a cylindrical light beam

The laser used in the experiments was not a line beam but a cylindrical beam with a diameter of 0.49 mm. In this case, the distance \(L_s\) has to be corrected. \(L_s\) can be replaced by the mean distance travelled, \(L_m\). This is defined as the intersection volume of the beam and the sphere, divided by the surface area of the beam.
In Figure B.2 a schematic view is given of the intersection of the cylindrical laser beam and one sphere. The front view is defined in the direction of the beam.

In Figure B.3 the front view is given in more detail. The volume of intersection is calculated by integrating the side view intersection surface area in the x direction from $x=0$ to $x=r_1$, in which $r_1$ is the radius of the laser beam. To calculate this volume the side view intersection surface area has to be derived.

**Figure B.2** Front and side views of the intersection of the laser and a sphere,

**Figure B.3** Front view of the intersection at plane $x=x$. 
B.3.1. Derivation of the side view intersection surface area

The side view intersection surface area is shown in the right hand part of Figure B.2. The length \( L \) and \( h \) can be given as a function of the angle \( \alpha \):

\[
L = 2R \cos \alpha \\
h = R \sin \alpha \quad \text{or} \quad \alpha = \arcsin \frac{h}{R} \tag{B.5}
\]

The side view intersection surface area, \( S \), can be calculated by integrating \( L \).

\[
dS = L \, dh \\
dS = 2R \cos \alpha \, dh \tag{B.6}
\]

\[
S = \int_{h-h-r}^{h+h-r} 2R \cos \alpha \, dh \\
= \int_{h-h-r}^{h-h+r} 2R \cos \alpha \, dh
\]

Changing from \( h \) coordinates to \( \alpha \) coordinates gives:

\[
S = \int_{a_1}^{a_2} 2R \cos \alpha \, dR \sin \alpha \\
= \int_{a_1}^{a_2} 2R^2 \cos^2 \alpha \, d\alpha \tag{B.7}
\]

\[
S = 2R^2 (0.5 \alpha + 0.25 \sin 2\alpha) \bigg|_{a_1}^{a_2} \\
S = 2R^2 [0.5 (\alpha_2 - \alpha_1) + 0.25 (\sin 2\alpha_2 - \sin 2\alpha_1)]
\]

\( \alpha_1 \) and \( \alpha_2 \) are given by:

\[
\alpha_1 = \arcsin \left( \frac{h-r}{R} \right) \tag{B.8} \\
\alpha_2 = \arcsin \left( \frac{h+r}{R} \right)
\]
The parameters $R$ and $r$ used in Eq.(B.5) to Eq.(B.8) are functions of $x$, the distance from the centre of the laser beam (see Figure B.3). These functions are given by:

$$R(x) = \sqrt{R^2 - x^2}$$
$$r(x) = \sqrt{r_1^2 - x^2}$$  \hspace{1cm} (B.9)

So:

$$\alpha_1(x) = \arcsin \left( \frac{h-r(x)}{R(x)} \right)$$
$$\alpha_2(x) = \arcsin \left( \frac{h+r(x)}{R(x)} \right)$$  \hspace{1cm} (B.10)

The side view intersection surface area $S$ as a function of $x$ is now given by:

$$S_1(x) = 2R(x)^2 \left[ 0.5 \left\{ \alpha_2(x) - \alpha_1(x) \right\} + 0.25 \left\{ \sin 2\alpha_2(x) - \sin 2\alpha_1(x) \right\} \right]$$  \hspace{1cm} (B.11)

**B.3.2. Derivation of the intersection volume**

Knowing the side view intersection surface area it is possible to calculate the intersection volume by integrating this surface area in the $x$ direction.

Two situations can be distinguished:

I. $h < R_x - r_1$: The laser intersects the sphere totally

II. $h > R_x - r_1$: The laser intersects the sphere partially

I. $h < R_x - r_1$:

As long as $h < R_x - r_1$ the intersection volume $V$ can be calculated by integrating the side view intersection surface area $S(x)$ from $x = 0$ to $x = r_1$. 


\[ V_i = 2 \int_{x=0}^{x=r_i} S_i(x) \, dx \quad \text{(B.12)} \]

II. \( h > R_i - r_i \):

In Figure B.4 the situation is given for partial intersection.

The intersection coordinates \( x_i \) and \( y_i \) can be calculated by:

\[ y_i = \frac{R_s^2 - r_i^2 + h^2}{2h} \quad \text{(B.13)} \]
\[ x_i = \sqrt{R_s^2 - y_i^2} \]

As can be seen in Figure B.4, three situations can be distinguished:

1: \( h < h_c \)

2: \( h = h_c \)

3: \( h > h_c \)

The critical height \( h_c \) is given by:

\[ h_c = \sqrt{R_s^2 - r_i^2} \quad \text{(B.14)} \]

These three situations are important because they have an influence on the value of \( \alpha_2 \) defined in Eq.(B.10) and on the way in which the side view intersection surface area must be integrated.

**situation 1: \( h < h_c \)**

\( \alpha_1 \): is given by Eq.(B.10)

\( \alpha_2 \): \( \pi/2 \) for \( 0 < x < x_i \)

is given by Eq.(B.10) for \( x_i < x < r_i \)
Porosity through a row of spheres

The intersection volume is given by:

\[
V_2 = 2 \left[ \int_{x=0}^{x=x_i} S_3(x) \, dx + \int_{x=x_i}^{x=x_i'} S_2(x) \, dx \right]
\]  
(B.15)

in which:

\[
S_2(x) = 2R(x)^2 \left\{ 0.5 \left[ \alpha_2(x) - \alpha_1(x) \right] + 0.25 \left[ \sin 2 \alpha_2(x) - \sin 2 \alpha_1(x) \right] \right\}
\]  

\[
S_3(x) = 2R(x)^2 \left\{ 0.5 \left[ \frac{\pi}{2} - \alpha_1(x) \right] + 0.25 \left[ \sin \left(2 \frac{\pi}{2}\right) - \sin \left(2 \alpha_1(x)\right) \right] \right\}
\]  
(B.16)

**Situation 2 and 3: \( h > h_c \)**

\( \alpha_1 \): is given by Eq.(B.10)

\( \alpha_2 \): \( \pi/2 \) for \( 0 < x < x_i \)

For this situation the side view intersection surface area has to be integrated only from \( x=0 \) to \( x=x_i \). The intersection volume is given by:

\[
V_3 = 2 \int_{x=0}^{x=x_i} S_3(x) \, dx
\]  
(B.17)

By using Eqs.(B.12),(B.15) and (B.17) it is possible to calculate the intersection volume for each situation.

B.3.3. Front view surface for partial intersection

For the calculation of the mean distance travelled through the sphere it is necessary to calculate the front view intersection surface area. For total intersection this surface area is equal to the surface area of the laser beam.
\[ F_1 = \pi r_i^2 \] \hspace{1cm} (B.18)

For partial intersection two situations can be distinguished:

1. \( h < h_c \)
2. \( h > h_c \)

![Diagram showing intersection surface for different values of \( h \)]

**Figure B.5**  Front view intersection surface

*Situation 1: \( h < h_c \):*

Front view intersection surface area = \( \pi r_i^2 \) - Surface area of outside sphere

Surface area of outside sphere BDCEB = Surface area BDCB - Surface area BECB
Surface area of BDCB = Surface area FBDCF - Surface area FBCF
Surface area of BECB = Surface area ABEC - Surface area ABCA

The front view intersection surface area \( F_2 \) is given by:
\[ F_2 = \pi r_i^2 - \left\{ \frac{1}{2} \beta_2 r_i^2 - x_i (y_i - h) \right\} - \left\{ \frac{1}{2} \beta_1 R_s^2 - x_i y_i \right\} \]  
(B.19)

In which:

\[ \beta_1 = 2 \arctan \left( \frac{x_i}{y_i} \right) \quad \text{(rad)} \]  
(B.20)

\[ \beta_2 = 2 \arctan \left( \frac{x_i}{y_i - h} \right) \quad \text{(rad)} \]

For the definition of \( x_i \) and \( y_i \) see Figure B.4.

**Situation 2: \( h > h_c \):**

Surface area outside sphere BDCEB = Surface area BDCGB - Surface area BCGB - Surface area BECB

The front view intersection surface area \( F_3 \) is given by:

\[ F_3 = \pi r_i^2 - \left\{ \pi r_i^2 - \frac{1}{2} \beta_2 r_i^2 - x_i (h - y_i) \right\} - \left\{ \frac{1}{2} \beta_1 R_s^2 - x_i y_i \right\} \]  
(B.21)

in which:

\[ \beta_1 = 2 \arctan \left( \frac{x_i}{y_i} \right) \quad \text{(rad)} \]  
(B.22)

\[ \beta_2 = 2 \arctan \left( \frac{x_i}{h - y_i} \right) \quad \text{(rad)} \]
B.3.4. Mean distance travelled

Knowing the intersection volume and the front side intersection surface area it is possible to calculate the mean distance travelled by the laser beam through a sphere as a function of the height of the laser.

**Situation 1:** \( h < R_s - r_l \)

For total intersection the mean distance travelled is given by dividing the intersection volume by the laser surface area.

\[
L_{m1} = \frac{V_1}{F_1} \quad (B.23)
\]

**Situation 2:** \( R_s - r_l < h < h_c \)

For partial intersection the mean distance travelled is given by dividing the intersection volume divided by the front side intersection surface area.

\[
L_{m2} = \frac{V_2}{L_2} \quad (B.24)
\]

**Situation 3:** \( h > h_c \)

\[
L_{m3} = \frac{V_3}{L_3} \quad (B.25)
\]

\( V_i \) and \( F_i \) are defined in the previous sections.
B.3.5. Porosity as a function of the height

For the porosity through one sphere a system is defined in which the total length of the system is equal to the diameter of one sphere (see Figure B.1). The porosity through one sphere for a cylindrical laser beam is given by:

\[ \varepsilon_1(h)_{\text{one sphere}} = \frac{2R_s - h}{2R_s} \]  \hspace{1cm} (B.26)

**Situation 2:** \( R_s - r_1 < h < h_c \):

For partial intersection between the laser beam and the sphere the ratio between the front view intersection surface area and the total surface area of the laser beam is needed:

\[ Q_i = \frac{\text{Front side intersection surface}}{\text{Surface laser beam}} \]  \hspace{1cm} (B.27)

\[ Q_i = \frac{F_i}{\pi r_1^2} \]

The fraction \( Q_i \) of the laser beam travels through the sphere with a specific mean travelled distance and porosity; the fraction \( 1-Q_i \) does not travel through the sphere, so the porosity of this fraction is 1.

\[ \varepsilon_2(h)_{\text{one sphere}} = (1 - Q_2)1 + Q_2 \left[ \frac{2R_s - h}{2R_s} \right] \]  \hspace{1cm} (B.28)

**Situation 3:** \( h_c < h \):

\[ \varepsilon_3(h)_{\text{one sphere}} = (1 - Q_3)1 + Q_3 \left[ \frac{2R_s - h}{2R_s} \right] \]  \hspace{1cm} (B.29)
Generally, for a system with total length $L$ and $n$ spheres in the line beam, the porosity is given by:

$$
\epsilon(h) = \frac{L - nL_{\text{ini}}}{L} = \frac{L - 2nR_s(1 - \epsilon_i(h))_{\text{one sphere}}}{L} \quad (B.30)
$$

Analytical expressions for the integrals used in this appendix are difficult to give. A computer program has been written within the software program "Mathcad", solving the equations numerically. For a given radius of sphere and laser beam the program calculates the porosity through one sphere as a function of the height.

The equations derived above are valid only in cases such that the porosity is evaluated at the vertical centre line of the spheres. In other words, the centre of the laser has to be on the line $x = 0$ (see Figure B.3).

![Graph showing porosity as a function of height](image)

**Figure B.6** Porosity through one sphere as a function of the height. $R_s = 2.381$ mm, $r_1 = 0.245$ mm.
In Figure B.6 the porosity through one sphere as a function of the height is given for a line beam and a cylindrical beam. The diameter of the spheres used in the experiments was 4.762 mm, and the diameter of the laser beam was 0.49 mm. It can be seen that the line beam reached a zone with a porosity of 1 after 2.381 mm, the radius of the sphere. Due to its diameter, the cylindrical beam reached this porosity zone after 2.626 mm, corresponding with: $R_s + r_l$. The radius of the laser beam can be controlled with a diaphragm. The smaller the diameter of the laser beam the smaller the difference between it and a line beam, but the lower the light output of the laser. Some experiments were carried out to find the smallest usable diaphragm with respect to a reasonable light output.
Appendix C

The laws of geometrical optics

C.1. Introduction

The three laws of geometrical optics are usually stated in terms of light rays.\textsuperscript{[1]}

I \hspace{1em} \textit{The law of rectilinear propagation.} Light rays in homogeneous media propagate in straight lines.

II \hspace{1em} \textit{The law of reflection.} At an interface between two different homogeneous, isotropic optical media, an incident disturbance is (partially) reflected, and the reflected ray is in the plane of incidence (the plane determined by the incident ray and the normal to the surface). The angle it makes with the normal (the angle of reflection) equals the angle made by the incident ray with the normal (angle of incidence).

III \hspace{1em} \textit{The law of refraction.} At an interface between dielectric media, there is also a refracted ray in the second medium, lying in the plane of incidence, making an angle \( r \) with the normal, and obeying Snellius’s law.

C.2. Snellius’s law\textsuperscript{[1][2]}

The law of refraction is given by:

\[
\frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1} = \frac{v_1}{v_2} \tag{C.1}
\]

where \( v_1 \) and \( v_2 \) are the velocities of propagation in the two media and \( n_1 = c/v_1 \), \( n_2 = c/v_2 \) are the indices of refraction. Here, \( c \) is the velocity of light in a vacuum, a universal constant: \( c = 3.0 \times 10^8 \) m/s. In Figure C.1 the refraction of light is given for \( n_1 > n_2 \) and \( n_1 < n_2 \).

if \( n_2 > n_1 \): \hspace{1em} and light change from medium 1 to medium 2 than \( r < i \).
and light change from medium 2 to medium 1 than \( r > i \).

if \( n_2 < n_1 \): \hspace{1em} and light change from medium 1 to medium 2 than \( r > i \).
and light change from medium 2 to medium 1 than \( r < i \).
The refractive index, $n$, is a function of the wavelength of the light. This function is given by Cauchy’s Dispersion Formula:

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$  \hspace{1cm} (C.2)

in which:

- $n$: Refractive index
- $A, B, C$: Material constants
- $\lambda$: Wavelength

If a light beam passes from a medium with a high index of refraction to one with a lower index of refraction it is possible that the angle of refraction becomes $90^\circ$. The angle of incidence at which this phenomenon occurs is called the Critical Angle. The critical angle is given by:

$$i_{\text{critical}} = \arcsin \left( \frac{n_2}{n_1} \right) \quad n_1 > n_2$$  \hspace{1cm} (C.3)

If the angle of incidence becomes greater than the critical angle, total reflection will occur.

Using Snellius’s law it is possible to calculate a light trajectory through a row of spheres in a fluid as a function of the height the light intersects the row and the refractive indices of the fluid and the spheres.
C.3. Calculation of the light trajectory through a row of spheres.

In Figure C.2 the light trajectory through the first sphere is given, in this case $n_{\text{sphere}} > n_{\text{fluid}}$.

For a horizontal light beam the angle of incidence, $i_1$, is equal to the angle $\alpha$, the angle of the radius with the horizontal axis. $h_1$, the input height, may be chosen arbitrarily for the first sphere in the row.

**Figure C.2**  Light trajectory through one sphere, $n_{\text{sphere}} > n_{\text{fluid}}$

Calculation of the parameters given in Figure C.2:

\[ i_1 = \alpha = \sin \left( \frac{h_1}{R} \right) \]
\[ i_2 = r_1 \]
\[ B = \left( \frac{h_1}{\tan \alpha} \right) \]
\[ A = R - B \]
\[ \gamma = 180 - 2r_1 \]
\[ \beta = 180 - \alpha - \gamma = 2r_1 - \alpha \]
\[ r_1 = \sin \left( \frac{\sin i_1}{n_2} \right) \]
\[ r_2 = \sin \left( \frac{\sin i_2}{n_t} \right) \]
\[ h_2 = R \sin \beta \]
\[ C = \frac{h_2}{\tan \beta} \]
\[ D = R - C \]

(C.4)
In Figure C.3 the light trajectory between two spheres is given. Knowing the parameters from the first sphere in the row it is possible to calculate the new angle of incidence for the second sphere and the height the light intersects the second sphere.

\[ A + B = R \]
\[ h_1^2 + B^2 = R^2 \]

Combining gives:
\[ h_1 = \sqrt{2RA - A^2} \quad (C.5) \]

\( A_{\text{new}} \) can be calculated by using the known parameters from the first sphere by using the following derivation:
\[
\begin{align*}
\tan(\tau_2 - \beta) &= \left( \frac{h_2 - h_1}{D + A} \right) \\
\tan(\tau_2 - \beta) &= \frac{h_2 - \sqrt{2RA - A^2}}{D + A} \\
\text{suppose: } \tan(\tau_2 - \beta) &= Z \quad \text{then} \\
(Z^2 + 1)A^2 + (2Z^2D - 2h_2Z - 2R)A + (h_2^2 + z^2D^2 - 2h_2ZD) &= 0 \\
\text{or} \\
\Pi A^2 + \Pi A + \text{III} &= 0 \quad \text{so:} \\
A_{\text{new}} &= \frac{-\Pi \pm \sqrt{\Pi^2 - 4\Pi(\text{III})}}{2\Pi} \\
\text{Knowing } A_{\text{new}} \text{ it is possible to calculate } h_{1\text{new}}: \\
h_{1\text{new}} &= \sqrt{2RA_{\text{new}} - A_{\text{new}}^2} \\
\text{and} \\
B_{\text{new}} &= R - A_{\text{new}} \\
\alpha_{\text{new}} &= \arctan \left( \frac{h_{1\text{new}}}{B_{\text{new}}} \right) \\
i_{\text{new}} &= \alpha_{\text{new}} - (\tau_{\text{old}} - \beta_{\text{old}}) \\
\end{align*}
\]

Using Eqs. (C.3) - (C.7) it is possible to calculate the light trajectory through a row of spherical particles.

**C.4. Energy reflection coefficients**

The reflection coefficients describe the reflection at an interface between two media having different optical properties.

Let the interface lie in the XY plane and let the plane of incidence be the ZY plane as shown in Figure C.4. The electric field \( E \) and the magnetic field \( H \) of the light may
be resolved into their components in the plane of incidence and their X components normal to the plane of incidence. Two independent cases then arise:

I  σ light: The electric field is vibrating perpendicular to the plane of incidence.
II  π light: The electric field is vibrating parallel to the plane of incidence.

For the σ light the reflection coefficient is given by:

$$\rho_\sigma = \frac{n_1 \cos i - n_2 \cos r}{n_1 \cos i + n_2 \cos r}$$  \hspace{1cm} (C.8)

For the π light the reflection coefficient is given by:

$$\rho_\pi = \frac{n_2 \cos i - n_1 \cos r}{n_2 \cos i + n_1 \cos r}$$  \hspace{1cm} (C.9)

For both cases the reflectivity $R$ is defined as:
\[ R = \rho^2 \]  \hspace{2cm} \text{(C.10)}

For the transmissivity we can write: \( T = 1 - R \).

In Figure C.5 and C.6 the reflectivity is given for \( n_1/n_2 = 1.5 \) and for \( n_2/n_1 = 1.5 \). It can be seen that reflectivity increases with increasing angle of incidence.

**Figure C.5** Reflectivity for \( n_1/n_2=1.5 \), the critical angle is 41°.

**Figure C.6** Reflectivity for \( n_2/n_1=1.5 \)
C.4.1. Polarization by reflection and transmission.

Light reflected from a smooth interface becomes partially polarized if the original angle of incidence is not zero. This effect becomes particularly strong at a dielectric interface when the angle of incidence is at, or near, the so-called Brewster's angle. When the reflected and transmitted rays meet an angle of 90°, it is impossible for light to be reflected if its electric field lies in the plane of incidence. This π light will only be transmitted. σ light will be reflected and transmitted. If the light incident at Brewster's angle is unpolarized, the reflected light is pure σ light, and the transmitted light is partially polarized with more π light than σ light.

Therefore, at Brewster's angle of incidence $i_b$ we have:

$$i_b + r_b = 90$$  \hspace{1cm} (C.11)

For all the experiments it is important to use polarized light to provide the two kinds of reflection and transmission. Laser light is polarized. It has to be analyzed if the polarized light is of the σ type or π type, to determine the amount of reflected or transmitted light at an interface.

C.5. References.

[1] Miles V.Klein; "Optics"

L.C.G.Malmberg. 's-Hertogenbosch 1971
Appendix D

Quasistatic jig model

D.1. Introduction

The model used to describe the porosity distribution in a jig bed as a function of time and height is a quasistatic model. In this Appendix a mathematical explanation is given of this fact by considering the equations of continuity.

D.2. Equations of continuity

Equation of movement:

For a particle in a jig bed the equation of movement can be written as:

\[
\frac{m}{dt} = G + F_w \tag{D.1}
\]

in which:
- \(m\): Mass of particle
- \(V\): Velocity of particle
- \(G\): Gravity force minus buoyancy force
- \(F_w\): Drag forces

The computer model ignores the interactive forces between the particles. Due to this fact, a term containing the interaction forces is absent from the right hand part of Eq.(D.1)

Equation of continuity of impulse:

The equation of continuity of impulse for the jig bed can be derived from the equation of movement.

\(V_p\) is the mean particle velocity as a function of the distance \(z\) from the screen, so:
\[ V_p = \langle V_z \rangle_z \]  \hspace{1cm} (D.2)

and

\[ \frac{dV_p}{dt} = \frac{\delta V}{\delta t} + V_p \frac{\delta V}{\delta z} \]  \hspace{1cm} (D.3)

By combining Eqs.(D.1) and (D.3) the equation of impulse is now given by:

\[ \frac{\delta V_p}{\delta t} + V_p \frac{\delta V_p}{\delta z} = f_w + g^* \]  \hspace{1cm} (D.4)

in which:

\[ g^* = \frac{G}{m} = g \left( 1 - \frac{\rho_t}{\rho_s} \right) \]  \hspace{1cm} (D.5)

\[ f_w = \frac{1}{m} \langle F_w \rangle \]

*Equation of continuity of mass:*

The equation of continuity of mass over a differential height is given by:

\[ \frac{\delta M}{\delta t} + \frac{\delta MV \rho}{\delta z} = 0 \]  \hspace{1cm} (D.6)

M: Mass density \hspace{1cm} \text{kg/m}^3

Rewriting Eq.(D.6) gives:

\[ M = Cm = (1-\varepsilon)\rho \]  \hspace{1cm} (D.7)

C: Particles per unit of volume \hspace{1cm} \text{l/m}^3
\nm: Mass of particles \hspace{1cm} \text{kg}
\n\rho: Density of particles \hspace{1cm} \text{kg/m}^3
The total mass of the particles, \( m \), the total number of particles, \( c \), and the density remain constant during the whole process, so:

\[
\frac{\delta c}{\delta t} + \frac{\delta c V_p}{\delta z} = 0
\]

(\( D.8 \))

\[
\frac{\delta (1-\varepsilon)}{\delta t} + \frac{\delta (1-\varepsilon)V_p}{\delta z} = 0
\]

so:

\[
-\frac{\delta \varepsilon}{\delta t} + (1-\varepsilon) \frac{\delta V_p}{\delta z} - V_p \frac{\delta (1-\varepsilon)}{\delta z} = 0
\]

(\( D.9 \))

\[
-\frac{\delta \varepsilon}{\delta t} + (1-\varepsilon) \frac{\delta V_p}{\delta z} - V_p \frac{\delta \varepsilon}{\delta z} = 0
\]

resulting in:

\[
\frac{\delta \varepsilon}{\delta t} + V_p \frac{\delta \varepsilon}{\delta z} = (1-\varepsilon) \frac{\delta V_p}{\delta z}
\]

(\( D.10 \))

**Equation of continuity of energy:**

Neglecting the interactive forces between particles the energy density, \( \varepsilon \), can be described by the summation of the potential energy, \( c_p \) and the kinetic energy, \( c_k \):

\[
e = e_p + e_k
\]

(\( D.11 \))

\[
e = \rho_s (1-\varepsilon) g z + \frac{1}{2} \rho_s (1-\varepsilon) V_p^2
\]

Combining Eqs.(\( D.4 \)), (\( D.10 \)) and (\( D.11 \)) gives for the equation of continuity of energy:

\[
\frac{\delta \varepsilon}{\delta t} + \frac{\delta e V_p}{\delta z} = \rho_s (1-\varepsilon) f_w V_p
\]

(\( D.12 \))
D.3. Quasistatic model

In the mathematical model the particle velocity, \( V_p \), is described by:

\[
V_p(t,x) = U(t) - U_0(x)
\]  

(D.13)

Using Eq.(D.13) implies that the forces acting on the particle are zero, because no accelerative term is present, resulting in the fact that the equation of continuity of impulse has no value. In this case \( f_w = g^* \). To explain the influence of this fact the equation of continuity of energy is considered.

The equation of continuity of the potential energy, \( e_p \), is given by:

\[
\frac{\delta e_p}{\delta t} + \frac{\delta e_p V_p}{\delta z} = \rho_s (1 - \varepsilon) g^* V_p
\]  

(D.14)

The equation of continuity for the kinetic energy, \( e_k \), is given by:

\[
\frac{\delta e_k}{\delta t} + \frac{\delta e_k V_p}{\delta z} = \rho_s (1 - \varepsilon) f_w (g^*) V_p
\]  

(D.15)

Due to the fact that \( f_w = g^* \), the kinetic energy term is constant. This does not agree with Eq.(D.13) in which the particle velocity, thus also the kinetic energy, follows the change of superficial fluid velocity directly. The mathematical model neglects the kinetic energy. The energy, contributed by the water to the particles, is changed directly into potential energy. The movement of the particles, necessary to change the potential energy, is ignored in the model, making the mathematical model a quasistatic one.
Appendix E

Derivation of some equations used in Chapter 3

E.1. Introduction

In this Appendix it is proven mathematically that the two mass transport integrals used in Chapter 3 are equal. To prove this several mathematical expressions are necessary, derived in the first paragraphs of this Appendix.

E.2. Porosity velocity $W(\varepsilon, t)$

Combining the equation of continuity of mass:

$$\frac{\delta \varepsilon}{\delta t} - \frac{\delta (1 - \varepsilon) V_p(\varepsilon, t)}{\delta z} = 0$$  \hspace{1cm} (E.1)

and the fact that the porosity velocity satisfies:

$$\frac{\delta \varepsilon}{\delta t} + W(\varepsilon, t) \frac{\delta \varepsilon}{\delta z} = 0$$  \hspace{1cm} (E.2)

results in the fact that the porosity velocity $W(\varepsilon, t)$ can be written as:

$$W(\varepsilon, t) = -\frac{\delta (1 - \varepsilon) V_p(\varepsilon, t)}{\delta \varepsilon}$$  \hspace{1cm} (E.3)

$W(\varepsilon, t)$ is also defined in Chapter 3 by:

$$W(\varepsilon, t) = V_p(\varepsilon, t) + V_w(\varepsilon)$$  \hspace{1cm} (E.4)

Combining Eqs.(E.3) and (E.4) gives:

$$(1 - \varepsilon) \frac{\delta W(\varepsilon, t)}{\delta \varepsilon} = \frac{\delta (1 - \varepsilon) V_w(\varepsilon)}{\delta \varepsilon}$$  \hspace{1cm} (E.5)
E.3. First derivative of \( z(\epsilon,t) \) to \( \epsilon \)

\( z(\epsilon,t) \) is defined in Chapter 3 by:

\[
z(\epsilon,t) = \prod (t) - \prod (\tau(\epsilon)) - (U_\epsilon(\epsilon) - V_\epsilon(\epsilon)) t - \tau(\epsilon)
\]

in which

\[
\prod = \int U(t) dt
\]

(E.6)

Partial differentiation with respect to \( \epsilon \) gives:

\[
\frac{\delta z(\epsilon,t)}{\delta \epsilon} = - \frac{\delta \prod (\tau(\epsilon))}{\delta \epsilon} - (t - \tau(\epsilon)) \frac{\delta U_\epsilon(\epsilon) - V_\epsilon(\epsilon)}{\delta \epsilon} - \frac{(U_\epsilon(\epsilon) - V_\epsilon(\epsilon)) \delta (t - \tau(\epsilon))}{\delta \epsilon}
\]

(E.7)

The porosity velocity is defined in Chapter 3 by:

\[
W(\epsilon,t) = U(t) - U_\epsilon(\epsilon) + V_\epsilon(\epsilon)
\]

(E.8)

Partial differentiation with respect to \( \epsilon \) gives:

\[
\frac{\delta W(\epsilon,t)}{\delta \epsilon} = \frac{\delta V_\epsilon(\epsilon) - U_\epsilon(\epsilon)}{\delta \epsilon}
\]

(E.9)

Combining Eqs.(E.7) and (E.9) gives:

\[
\frac{\delta z(\epsilon,t)}{\delta \epsilon} = (t - \tau(\epsilon)) \frac{\delta W(\epsilon,t)}{\delta \epsilon} - V_\epsilon(\epsilon) \frac{\delta \tau(\epsilon)}{\delta \epsilon} + U_\epsilon(\epsilon) \frac{\delta \tau(\epsilon)}{\delta \epsilon} - \frac{\delta \prod (\tau(\epsilon))}{\delta \epsilon}
\]

(E.10)

This is equal to:

\[
\frac{\delta z(\epsilon,t)}{\delta \epsilon} = (t - \tau(\epsilon)) \frac{\delta W(\epsilon,t)}{\delta \epsilon} - V_\epsilon(\epsilon) \frac{\delta \tau(\epsilon)}{\delta \epsilon}
\]

(E.11)

due to the fact that:
\[ U_0(\varepsilon) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon} = \frac{\delta \prod (\tau(\varepsilon))}{\delta \varepsilon} \]  

(E.12)

because:

At time \( \tau(\varepsilon) \), the time that porosity \( \varepsilon \) arises at \( z = 0 \), the hindered settling velocity \( U_0(\varepsilon) \) is equal to the superficial fluid velocity, \( U(\tau(\varepsilon)) \), so the left hand part can be written as:

\[ U_0(\varepsilon) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon} = U(\tau(\varepsilon)) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon} \]  

(E.13)

The right hand part can be written as:

\[ \frac{\delta \prod (\tau(\varepsilon))}{\delta \varepsilon} = U(\tau(\varepsilon)) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon} \]

so:

\[ \frac{\delta \prod (\tau(\varepsilon))}{\delta \varepsilon} = U(\tau(\varepsilon)) \frac{\delta \tau(\varepsilon)}{\delta \varepsilon} \]  

(E.14)

The two terms are therefore equal.

Combining Eqs.(E.5) and (E.11) gives:

\[ (1 - \varepsilon) \frac{\delta Z(\varepsilon,t)}{\delta \varepsilon} = \frac{\delta (1 - \varepsilon) V_0(\varepsilon) (t - \tau(\varepsilon))}{\delta \varepsilon} \]  

(E.15)

E.4. Mass transport integrals

The two mass transport integrals used in Chapter 3 are given by:

\[ M(\varepsilon,t) = \int_0^{x(\varepsilon,t)} (1 - \varepsilon) dz \]  

(E.16)

and
\[ M(\varepsilon,t) = \int_{\tau(\varepsilon)}^{t} T_m \, dt \quad (E.17) \]

As has been proven in Chapter 3, Eq. (E.17) can be rewritten as:

\[ M(\varepsilon,t) = (1 - \varepsilon) \, V_w(\varepsilon) \, (t - \tau(\varepsilon)) \quad (E.18) \]

Combining Eqs. (E.18) and (E.15) gives:

\[ \frac{\delta M(\varepsilon,t)}{\delta \varepsilon} = (1 - \varepsilon) \, \frac{\delta Z(\varepsilon,t)}{\delta \varepsilon} \quad (E.19) \]

or:

\[ \delta M(\varepsilon,t) = (1 - \varepsilon) \, \delta Z(\varepsilon,t) \quad (E.20) \]

Eq. (E.20) is equal to Eq. (E.16) so the mass transport integrals can be derived from each other and are equal.
Appendix F

List of symbols

Chapter 2

*b*; (m) Transition point

C: (1/m³) Particles per unit volume.

h: (m) Height of fluidized bed

h₀; (m) Height of fluidized bed before change in fluid velocity.

h₁; (m) Height of fluidized bed after change in fluid velocity.

M: (kg/m³) Mass density.

m: (kg) Mass of particles.

n: Constant in the Richardson and Zaki equation

Re: Reynolds number

t: (s) Time

t₀; (s) Time at which superficial fluid velocity is changed

t₁; (s) Time at which fluidized bed reaches new equilibrium

t*: (s) Time at which transition point reaches the top of the fluidized bed

U: (m/s) Superficial fluid velocity.

U₀; (m/s) Superficial fluid velocity before change

U₁; (m/s) Superficial velocity after change

Uᵥ(ε): (m/s) Velocity of a swarm of particles in the absence of a fluid velocity

Vᵥ(ε): (m/s) Average velocity of fluidized particles.

Vₜ: (m/s) Free falling terminal velocity of one particle.

W(ε): (m/s) Porosity velocity

z: (m) Height

ε₀; Porosity before change in superficial fluid velocity.

ε₁; Porosity at new equilibrium of the fluidized bed.

ε₇; Porosity at the top of the fluidized bed

ρ: (kg/m³) Density of particles
Chapter 3

$A_{str}$: (m) Amplitude of the jiggling stroke
$b_i$: (m) Height in the jiggbed at which a discontinuity occurs
$F$: (Hz) Jiggling frequency
$h_{open}$: (m) Height of the upper side of the fluidized part of the jig bed
$h_p$: (m) Height of the packed jig bed
$h_{pack}$: (m) Height of the packed phase at the bottom of the jig bed
$h_r$: (m0) Height of the ridge in porosity distribution
$h_{to}$: (m) Turn-over height
$h_{top}$: (m) Height of the top of the jig bed
$M(e,t)$: Mass transport integral
$M_{to}$: Mass under turn-over height
$M_{tot}$: Total mass in jig bed
$n$: Constant in the Richardson and Zaki equation
$S(t)$: (m) Displacement
$T$: (s) Stroke time
$T_m$: Mass transport
$t$: (s) Time
$t_{exp}$: (s) Time at which the jig bed starts expanding
$t_{pack}$: (s) Time at which the bottom of the jigbed starts packing
$t_{step}$: (s) Duration of one time step
$U(t)$: (m/s) Superficial fluid velocity
$U_{mf}$: (m/s) Minimum fluidization velocity
$U_v(e)$: (m/s) Velocity of a swarm of particles in the absence of a fluid velocity, hindered settling velocity.
$V_T$: (m/s) Free falling terminal velocity of one particle.
$V_p(e,t)$: (m/s) Average velocity of fluidized particles.
$W(e,t)$: (m/s) Porosity velocity
$Z(e,t)$: (m) Height of porosity zone $e$ at $t = t$
$z$: (m) Height above screen
$e$: Porosity
$e_m$: Porosity at which $W(e,t)$ reaches a maximum value
$e_{max}$: Maximum value of the porosity during a jiggling stroke
$e_{open}$: Porosity at the upper side of the fluidized part of the jig
Symbols

$\varepsilon_p$: Porosity of the packed bed

$\varepsilon_{\text{pack}}$: Porosity just above packed phase.

$\varepsilon_{\text{ra}}$: Porosity just above ridge

$\varepsilon_{\text{ru}}$: Porosity just under ridge

$\varepsilon_{\text{top}}$: Porosity at the top of the jig bed

$\varepsilon_{\text{u}}$: Porosity directly under discontinuity

$\varepsilon_d$: Porosity directly above discontinuity

$\tau(\varepsilon)$: (s) Time at which porosity $\varepsilon$ is generated at $z = 0$

Chapter 4

C: Concentration

I: Intensity of light

$I_0$: Intensity of the incident light

l: (m) Length

T: Transmission

V: (volt) Voltage output signal photo diode

$\alpha$: Specific absorption coefficient

$\varepsilon$: Porosity

Chapter 5

A: $(m^2)$ Surface area of the fluidized bed

h: (m) Height above the centre line of spheres

or:

Fluidized bed height

I: Intensity of light

$I_0$: Intensity of the incident light

L: (m) Total length

$L_m$: (m) Mean distance travelled through one sphere

n: Number of spheres in a row

or:

number of spheres in fluidized bed

M Number of measurements

$R_s$: (m) Radius of sphere
Appendix A

A: \( (m^2) \) Surface area perpendicular to the direction of movement

Ar: Archimedes number

d: \( (m) \) Diameter of sphere

F: \( (N) \) Force

F_b: \( (N) \) Buoyancy force

F_d: \( (N) \) Drag force

F_x: \( (N) \) Accelerative force

f_d: Drag coefficient

g: \( (m/s^2) \) Gravitational acceleration

J_p: \( (m/s) \) Volumetric flux of particles

J_f: \( (m/s) \) Volumetric flux of fluid.

M: \( (kg) \) Mass

n: Constant in the Richardson and Zaki equation

Re: Reynolds number

ΔP: \( (N/m^2) \) Pressure drop

t: \( (s) \) Time

U: \( (m/s) \) Superficial fluid velocity with respect to the wall of the system

U_i: \( (m/s) \) Interstitial fluid velocity with respect of the wall

U_{mf}: \( (m/s) \) Minimum fluidization fluid velocity

U_v(ε): \( (m/s) \) Velocity of a swarm of particles in the absence of a superficial fluid velocity.

U_s: \( (m/s) \) Slip velocity

V_T: \( (m/s) \) Free falling terminal velocity.

V_p: \( (m/s) \) Particle velocity with respect to the wall of the system

v: \( (m/s) \) Velocity

ε: Porosity

ε_{mf}: Minimum fluidization porosity
Appendix B

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_f )</td>
<td>(kg/m(^3))</td>
<td>Density of fluid</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>(kg/m(^3))</td>
<td>Density of solid</td>
</tr>
<tr>
<td>( \eta )</td>
<td>(kg/ms)</td>
<td>Viscosity of fluid</td>
</tr>
<tr>
<td>h</td>
<td>(m)</td>
<td>Height above centre line of sphere</td>
</tr>
<tr>
<td>L</td>
<td>(m)</td>
<td>Total length</td>
</tr>
<tr>
<td>( L_s )</td>
<td>(m)</td>
<td>Length travelled by the light through one sphere</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>Number of spheres in row</td>
</tr>
<tr>
<td>( R_s )</td>
<td>(m)</td>
<td>Radius of sphere</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td></td>
<td>Porosity</td>
</tr>
<tr>
<td>r</td>
<td>(m)</td>
<td>Radius of laser beam</td>
</tr>
<tr>
<td>S,F</td>
<td>(m(^2))</td>
<td>Surface area</td>
</tr>
<tr>
<td>x,y</td>
<td>(m)</td>
<td>Coordinates</td>
</tr>
<tr>
<td>V</td>
<td>(m(^3))</td>
<td>Volume</td>
</tr>
</tbody>
</table>

Appendix C

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>(m/s)</td>
<td>Velocity of light in vacuum</td>
</tr>
<tr>
<td>i</td>
<td></td>
<td>Angle of incidence</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>Refractive index</td>
</tr>
<tr>
<td>R</td>
<td></td>
<td>Reflectivity</td>
</tr>
<tr>
<td>r</td>
<td></td>
<td>Angle of refraction</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>Transmissivity</td>
</tr>
<tr>
<td>v</td>
<td>(m/s)</td>
<td>Light velocity</td>
</tr>
<tr>
<td>( \rho )</td>
<td></td>
<td>Reflection coefficient</td>
</tr>
</tbody>
</table>

Appendix D

See chapter 3

Appendix E

See chapter 3
Summary of the thesis

In the course of time a number of theories have been developed to describe and explain the jiggling process. Due to the very complex character of the process, none of the existing theories has been successful in comprehensively explaining its nature.

Since it has not been possible to describe the jiggling process in detail by using theories based on individual particle movement or by using theories involving energy changes of particles in the jigbed, other means had to be found. The fact that loosening of the jigbed is an important parameter for describing the jiggling process has been recognized by several authors. The investigation, described in this thesis, was performed with the objective to be able to predict and measure the porosity distribution in a jig bed as a function of time and location in the bed during operation.

If the porosity distribution as a function of time and height in the jig bed could be measured, it would then be possible to relate this information to the theories describing individual particle behaviour involving a density different from that of the particles in the jig bed. This would generate additional information concerning some parameters which are a function of the time and height in the jig bed, such as drag force and relative density. Combining all the results would give improved insight into the jiggling process, and possibilities could be identified to optimize the separation process in practice.

A mathematical model has been developed by Slis et al. for the response of the level of a liquid fluidized bed to a sudden change in the fluidizing velocity. When the fluidizing velocity in a liquid fluidized bed of monosized solid particles, all with the same density, is suddenly changed, a discontinuity in the porosity is introduced at the bottom of the bed which will propagate itself upwards through the bed. This mathematical model pertaining to a liquid fluidized bed can be adapted into a model for the porosity distribution in a jig as a function of the time and height in the bed. During the transition period in a fluidized bed, all velocities defined in the mathematical model of Slis are time independent, due to the fact that the superficial fluidizing velocity changes in a single step. This is in contrast with the simulation in a jig bed, where the superficial fluid velocity is subject to continuous change.

The mathematical model derived by Slis can be transformed into a quasistatic
mathematical model for a spherical monosized, mono density jig bed by transforming the step change in the fluidizing velocity into a periodic function of time. Assuming that the porosity near the screen is always in equilibrium with the locally prevailing fluid velocity, the porosity near the screen will change as a function of the time dependent fluid velocity according to the piston cycle of the jig. These porosities will travel upwards with a time dependent velocity corresponding to the velocity described in the model for a fluidized bed. By using mass balances it is then possible to describe the porosity in the jig bed as a function of time and height.

In order to test the validity of the mathematical model, a laboratory jig was designed and constructed. In the jig the bed has been made transparent by using clear particles and a clear fluid, both with the same refractive index. For continuous measurement of the porosity in a uniform jig bed as a function of time and height in the bed, special instrumentation has been developed. The measurements are based on the difference in light absorption between the fluid and the particles used in the jig bed. The measurements were made by placing a laser source at one side of the jig and a photo diode at the other side of the jig on an optical bench. In this way it was possible to determine the changes in transmission with times at different heights above the screen.

For calibration of the output signal of the diode in terms of the porosity at a given height, a fluidized bed was built with the same dimensions as the jig.

Based on the calibration experiments in the fluidized bed it was concluded that it is impossible to create a single calibration curve which can directly relate the light transmission to the porosity. The obtained transmission curves turned out to be a function of not only the porosity, but also of the intersection length and the angle of incidence of the laser beam in combination with the reflection and refraction.

Generally, it can still be concluded that the transmission increases with increasing porosity. Despite the fact that it was impossible to relate absolute values of the transmission to absolute values of the porosity, the transmission measurements could still be used as an indicator, to give relative information regarding the porosity.

By using the continuous transmission measurements in the jig bed it was possible to obtain information regarding the increase or decrease of the porosity as a function of time. Due to the fact that it was possible to continuously measure the light transmission
in the jig bed, it was also possible to compare the trends in the transmission values with the mathematical model, resulting in qualitative information indicating the validity of the model in terms of trends and relative porosities.

The light transmission was measured during operation as a function of the height in the jig bed, the jiggling speed, the stroke length and the bed thickness, respectively.

By comparing the light transmission values measured and the porosity values calculated by the model, it is apparent that there is correspondence with regard to amplitude changes and time position as a function of jiggling speed and measuring height.

Differences between data derived from the model and the measured results can be explained as follows:

- The resolution of the transmission measurements is too low to detect the difference between the porosity of a packed bed, $\epsilon = 0.37$ and $\epsilon = 0.5$.
- The model has been designed to neglect the transition zone between a packed phase and a fluidized phase.
- Acceleration and deceleration forces on the particles are neglected by the model.
- The mathematical model assumes that the first layer of particles on the bottom are fixed on the screen, which is not observed to be the case in reality.
- The model does not take 3-dimensional effects into account since the measurements are only made in a 2-dimensional plane.

Based on the similarity between the trends in the light transmission values measured and the porosity values calculated it can be concluded that the porosity distribution in a uniform jig bed calculated by the mathematical model is comparable in a qualitative way with the porosity distribution in reality.

The mathematical model described in this thesis has given more insight in the behaviour of particles in a uniform jig bed, in terms of the porosity distribution, but the description of the entire jiggling process with respect to stratification of particles to be separated was not possible. To use the model for the description of the segregation process in the jig it has to be extended to a multi component system and the equations of movement of the individual particles have to be added to the mathematical equation of the model.
Samenvatting dissertatie

In de loop van de tijd zijn een groot aantal theorieën ontwikkeld om het jigproces te beschrijven en te verklaren. Gezien het complexe karakter van het jigproces is tot op heden echter geen van de bestaande theorieën er in geslaagd het jigproces in zijn geheel te doorgronden.

Aangezien het niet mogelijk is gebleken het jigproces in detail te beschrijven op basis van het individuele deeltjesgedrag of door middel van theorieën die gebaseerd zijn op energie veranderingen in het jigbed, is gezocht naar andere parameters die kunnen leiden tot modellvorming van het jigproces. In de literatuur wordt onderkend dat de opening van het jigbed, met andere woorden de porositeit, een belangrijke parameter is in het jigproces. Het onderzoek, beschreven in dit proefschrift, had als doel de porositeitsverdeling in een jigbed te voorspellen als functie van tijd en plaats en tevens deze porositeitsverdeling in de praktijk te meten door gebruik te maken van een proefopstelling.

Als de porositeitsverdeling in het jigbed bekend is, is het naar alle waarschijnlijkheid mogelijk deze kennis te combineren met theorieën die het individuele gedrag beschrijven van deeltjes met een onderling afwijkende dichtheid. Tevens zal dit meer inzicht geven in andere parameters die voor het jigproces en de modellering van belang zijn, zoals o.a. wrijvingskrachten. Combinatie van alle resultaten zal uiteindelijk meer inzicht geven in het jigproces en biedt op termijn de mogelijkheid het scheidingsproces in de praktijk te verbeteren.

Vanuit de literatuur is bekend dat Slis e.a. een mathematisch model hebben ontwikkeld om de respons van een fluide bed, als gevolg van een veranderende fluidisatie snelheid, te beschrijven. Als deze snelheid in een fluide bed, bestaande uit uniforme vaste deeltjes, allen met dezelfde dichtheid, plotseling wordt veranderd ontstaat een discontinuïteit in de porositeit. Deze discontinuïteit ontstaat als eerste op de bodem van het bed en zal zich in opwaartse richting door het bed bewegen.

Dit model voor een fluide bed kan worden getransformeerd in een model dat de porositeitsverdeling in een jigbed beschrijft als functie van tijd en hoogte. Gedurende de overgangsfase, tussen de oude en de nieuwe evenwichtssituatie, zijn alle snelheden gedefinieerd in het model van Slis tijdsonafhankelijk als gevolg van het feit dat de vloechtofsnelheid tijdsonafhankelijk is. Dit in tegenstelling tot de situatie in een jigbed.
waar de vloeistofsnellheid een tijdsafhankelijke functie is.

Het model van Slis is omgezet in een quasistatisch wiskundig model voor een uit bolvormige deeltjes opgebouwd uniform jigsaw, waarin tevens alle deeltjes dezelfde dichtheid hebben. Hiertoe is de stapsgewijze verandering van de vloeistofsnellheid in het fluïde bed model omgezet naar een snelheidsverandering die tijdsafhankelijk is. Als wordt aangenomen dat de porositeit in de buurt van de zee van de jig altijd in evenwicht is met de op dat moment heersende vloeistofsnellheid, welke een gevolg is van de beweging van de plunjer van de jig, zal deze porositeit een tijdsafhankelijke verandering te zien geven. De verschillende porositeiten die gedurende een jigcyclus op de zeef ontstaan zullen zich met een tijdsafhankelijke snelheid door de rest van het jigsaw omhoog bewegen, analoog aan het fluïde bed model, met dien verstande dat hier de porositeitsnelheid tijdsonafhankelijk is. Door mede gebruik te maken van massabalansen is het mogelijk gebleken hiermee de porositeitsverdeling in een uniform jigsaw te beschrijven als functie van plaats en tijd.

Om het ontwikkelde model te toetsen aan de praktijk is een laboratorium jig ontwikkeld. De vloeistof die in de jig gebruik wordt heeft dezelfde brekingsindex als de uniforme bolvormige perspex deeltjes, zodat het jigsaw als zodanig niet te zien is. Om de porositeit in het jigsaw continu te kunnen meten als een functie van plaats en tijd is een meetopstelling ontwikkeld. De meetmethode is gebaseerd op het verschil in lichtabsorptie tussen de gebruikte vloeistof en die van de deeltjes. De metingen werden verricht met behulp van een optische bank waarop een laser aan de ene kant van jig was geplaatst en een fotodiode aan de andere zijde. Met behulp van deze opstelling is het mogelijk de veranderingen in de lichttransmissie als functie van de tijd en hoogte in het jigsaw te meten.

Om het meet systeem te kalibreren is gebruik gemaakt van een fluïde bed met dezelfde dimensies als de jig.

Naar aanleiding van de kalibratie experimenten is gebleken dat het niet mogelijk is een eenduidige kalibratie curve te creëren die de lichttransmissie correleert aan de porositeit. De oorzaak hiervan is gelegen in het feit dat de transmissie niet alleen een functie is van de porositeit, maar ook wordt beïnvloed door de afgelegde weg door de bolletjes en de hoek van inval die de laserstraal maakt met de verschillende bollen, met andere woorden reflectie en refractie van het licht spelen een rol.
Toch kan algemeen geconcludeerd worden dat de transmissie toeneemt met toenemende porositeit. Ondanks het feit dat er geen kwantitatieve relatie aanwezig is tussen de gemeten transmissie en de porositeit is het mogelijk gebleken de transmissiemetingen te gebruiken als indicatie voor de porositeitsveranderingen.

Door gebruik te maken van continue transmissiemetingen in het jigbed is het mogelijk informatie te verkrijgen over de toename en afname van de porositeit als functie van de tijd en hoogte in het jigbed. Door de trends in de gemeten transmissiewaarden te vergelijken met de door het wiskundig model berekende porositeiten is gebleken dat hiermee de geldigheid van het model aan te tonen is.

Om het wiskundig model te vergelijken met het gedrag van deeltjes in de jig, zijn transmissiemetingen uitgevoerd gedurende verschillende instellingen van de jig, zoals frequentie, slaglengte en bedhoogte.

Door vergelijking van de gemeten transmissiewaarden en de berekende porositeiten blijkt dat er een grote mate van overeenkomst bestaat tussen amplitude veranderingen en tijdsfasering van beiden. Verschillen worden ondermeer veroorzaakt door het feit dat:

- De resolutie van de transmissiemetingen te laag is om verschillen te meten in porositeiten tussen 0.37 en 0.5
- Het wiskundig model geen rekening houdt met de overgangsfase tussen een fluide toestand en een gepakte bed toestand.
- Versnellings- en vertragingskrachten op deeltjes door het model niet worden meegenomen.
- Het model er vanuit gaat dat de eerste laag deeltjes gefixeerd is op de zeef, dit in tegenstelling tot de praktijk waar dit niet in alle gevallen zo behoeft te zijn.

Op basis van de uitgevoerde licht transmissiemetingen kan geconcludeerd worden dat de berekende porositeitsverdeling kwalitatief overeenkomt met de porositeitsverdeling zoals deze optreedt in de praktijk.

Het wiskundig model, zoals dat in dit proefschrift beschreven wordt, heeft geleid tot meer inzicht in het gedrag van deeltjes in een uniform jigbed, en wel met betrekking tot de optredende porositeitsverdelingen. Om echter het jigproces in zijn geheel te beschrijven, de ontmenging van deeltjes naar dichtheid, zal het ontwikkelde model
uitgebreid moeten worden naar een multi-component systeem en zullen ondermeer vergelijkingen met betrekking tot individuele deeltjessnelheden ingebouwd moeten worden.
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