National Codes in relation with
the CEB-FIP Model Code.

Ing. J.P. Straman
March 1981
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</table>
1. Introduction

In the treaty of Rome an article has been included to abolish commercial hindrances between the different countries of the E.C. This means among other things, to smooth out the differences between national concrete codes, so that contractors can build in every country of the E.C. and dealing in building products will be possible from one country to another without problems of national kind.

To get information about these differences, a comparison has been made of some West-European codes for structural concrete and of the CEB code, all as they were available in 1978.

The codes investigated are:
- CEB-Bulletin 124/125
- CP 110 England
- CCBA 68 France
- DIN 1045 Germany
- VB 1974 Netherlands.

To compare every subject of these codes is a tremendous work, so that this comparison has been limited to the design of the bearing structure of an office building, according to the above mentioned codes.

The required corresponding articles for the calculations have been investigated and some of them have been included in this report.

Because it was the intention to compare concrete codes - and not load prescriptions - equal loads have been taken for the calculation of the building. However to get information about the characteristic loads, prescribed in the different countries, these values have been investigated too.

So the report can be divided into:
- Load-prescriptions
- Comparison of corresponding articles of the different concrete codes
- Comparison of detailed analysis and design of reinforcement of the unbraced bearing structure of an office building.
- Conclusions

The following students have contributed to this report:
J. Laurens
R.B.G.H. Markslag
J. Vlagsma
W.H. Zwart
2. Symbols

\( A_c \)  
area of concrete

\( A_s \)  
area of reinforcement

\( E_{em} \)  
modulus of elasticity of concrete

\( E_s \)  
modulus of elasticity of steel

\( b \)  
width

\( d \)  
effective height of the section

\( e_o \)  
eccentricity of first order

\( e_c \)  
eccentricity due to creep

\( e_2 \)  
eccentricity of second order

\( e_{tot} \)  
total eccentricity

\( h \)  
total height of the section

\( i \)  
minimum radius of gyration

\( l_o \)  
effective buckling length

\( s \)  
standard deviation

\( z \)  
lever arm

\( f_c \)  
nominal compressive strength of concrete

\( f_{ck} \)  
characteristic strength of concrete

\( f_{cm} \)  
mean value compressive strength of concrete

\( f_{cd} \)  
concrete design strength

\( f_{yd} \)  
steel design strength

\( f_{yk} \)  
characteristic strength of steel

\( \varepsilon_c \)  
strain value of concrete

\( \varepsilon_s \)  
strain value of steel

\( \lambda \)  
slenderness

\( \gamma_C \)  
partial factor of safety of concrete

\( \gamma_S \)  
partial factor of safety of steel

\( \gamma_g \)  
partial safety-coefficient of permanent actions

\( \gamma_q \)  
"  "  "  variable actions

\( \gamma_w \)  
"  "  "  wind actions.
3 Load-prescriptions

3.1 Characteristic imposed loads (kN/m²)

<table>
<thead>
<tr>
<th>Use</th>
<th>Code/Country</th>
<th>floors</th>
<th>stairs/corridors</th>
<th>balconies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Houses</td>
<td>CEB</td>
<td>1,5</td>
<td>1,5</td>
<td>1,5</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td>1,5</td>
<td>4,0</td>
<td>1,5</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>1,75</td>
<td>2,5</td>
<td>3,5</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>2,0</td>
<td>3,5</td>
<td>3,5</td>
</tr>
<tr>
<td></td>
<td>Netherlands</td>
<td>1,5</td>
<td>2,0</td>
<td>2,5</td>
</tr>
<tr>
<td>Offices</td>
<td>CEB</td>
<td>2,5</td>
<td>2,5</td>
<td>2,5</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td>2,5</td>
<td>4,0</td>
<td>2,5</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>2,5</td>
<td>4,0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>2,0</td>
<td>3,5</td>
<td>3,5</td>
</tr>
<tr>
<td></td>
<td>Netherlands</td>
<td>2,0</td>
<td>2,5</td>
<td>2,5</td>
</tr>
<tr>
<td>Stores</td>
<td>CEB</td>
<td></td>
<td></td>
<td>depending on loaded area and type of goods</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td>4,0</td>
<td>4,0</td>
<td>4,0</td>
</tr>
<tr>
<td></td>
<td>France</td>
<td>4,0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>2,0</td>
<td>5,0</td>
<td>3,5</td>
</tr>
<tr>
<td></td>
<td>Netherlands</td>
<td>3,0</td>
<td>3,0</td>
<td>3,0</td>
</tr>
<tr>
<td>Warehouses</td>
<td>CEB</td>
<td></td>
<td></td>
<td>depending on loaded area and type of goods</td>
</tr>
<tr>
<td></td>
<td>England</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>France</td>
<td></td>
<td></td>
<td>2,4 metre</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>storage height</td>
</tr>
<tr>
<td></td>
<td>Germany</td>
<td>5,0</td>
<td>5,0</td>
<td>3,5</td>
</tr>
<tr>
<td></td>
<td>Netherlands</td>
<td>4,0</td>
<td>4,0</td>
<td>4,0</td>
</tr>
</tbody>
</table>
If the loads according to the CEB are assumed to be 100%, the following values can be found:

### Floor Loads (%)

<table>
<thead>
<tr>
<th></th>
<th>CEB</th>
<th>England</th>
<th>France</th>
<th>Germany</th>
<th>Netherlands</th>
</tr>
</thead>
<tbody>
<tr>
<td>houses</td>
<td>100</td>
<td>100</td>
<td>117</td>
<td>133</td>
<td>100</td>
</tr>
<tr>
<td>offices</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

### Loads Stairs/Corridors (%)

<table>
<thead>
<tr>
<th></th>
<th>CEB</th>
<th>England</th>
<th>France</th>
<th>Germany</th>
<th>Netherlands</th>
</tr>
</thead>
<tbody>
<tr>
<td>houses</td>
<td>100</td>
<td>267</td>
<td>167</td>
<td>234</td>
<td>133</td>
</tr>
<tr>
<td>offices</td>
<td>100</td>
<td>160</td>
<td>160</td>
<td>140</td>
<td>100</td>
</tr>
</tbody>
</table>
3.2 WINDLOADS

CCBA '68: the wind load is equally divided over the height and depends on the height of the building.
4 Comparison of corresponding articles

4.1 Grades of Concrete

The description of the quality of concrete often exists of a letter followed by a number, similar to the required characteristic compressive strength of the concrete of a certain age.

A comparison of the grades of concrete according to different codes is hardly possible because the way of testing is not the same.

Some differences are:
- the shape of test-pieces;
- the dimensions of the test-pieces;
- the way of storage;
- the speed of the tests.

According to CCBA 68 the conception "characteristic strength" is unknown. The concrete grade is represented by means of the formula:

\[ f_c = f_{cm} - 0.8s \]

where:
- \( f_c \) = nominal compressive strength,
- \( f_{cm} \) = mean compressive strength,
- \( s \) = standard deviation.

To make a comparison with regard to the concrete grades used in France, it is necessary to determine a frequency distribution curve and a standard deviation.
Correspondence between Concrete Grades assuming that compliance criteria are the same.

<table>
<thead>
<tr>
<th>Character</th>
<th>CEB</th>
<th>CP 110</th>
<th>CCBA 68</th>
<th>DIN 1045</th>
<th>VB 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder strength</td>
<td>cylinder</td>
<td>cube-grade strength</td>
<td>on = σ'28</td>
<td>cube-grade strength</td>
<td>cube-grade strength</td>
</tr>
<tr>
<td>10</td>
<td>C10</td>
<td>15 gr.15</td>
<td>s = 3</td>
<td>14.7 B15</td>
<td>12.5 B12.5</td>
</tr>
<tr>
<td>12</td>
<td>C12</td>
<td>20 gr.20</td>
<td>s = 4</td>
<td>17.5 B17.5</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td>s = 3</td>
<td>22.5 B22.5</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>C16</td>
<td>25 gr.25</td>
<td>s = 5</td>
<td>23.9 B25</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>C18</td>
<td></td>
<td>s = 5</td>
<td>30 B30</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>C20</td>
<td>30 gr.30</td>
<td>s = 5</td>
<td>34.8 B35</td>
<td>37.5 B37.5</td>
</tr>
<tr>
<td>24</td>
<td>C25</td>
<td>40 gr.40</td>
<td>s = 5</td>
<td>45 B45</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>52.5 B52.5</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>54.5 B55</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>C30</td>
<td>50 gr.50</td>
<td>s = 5</td>
<td>45 B45</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>52.5 B52.5</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>54.5 B55</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>C40</td>
<td>60 gr.60</td>
<td>s = 5</td>
<td>54.5 B55</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td></td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>C50</td>
<td></td>
<td>s = 5</td>
<td>60 B60</td>
<td></td>
</tr>
</tbody>
</table>

Note: σ' values are approximate and may vary slightly based on specific standards or guidelines.
4.2 Partial factors of safety

<table>
<thead>
<tr>
<th>Loads</th>
<th>$\gamma_F$</th>
<th>CEB</th>
<th>CP 110</th>
<th>CCBA 68</th>
<th>DIN 1045</th>
<th>VB 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>permanent</td>
<td>$\gamma_g$</td>
<td>1,35</td>
<td>1,2-1,4</td>
<td>1,0</td>
<td>1,75-2,1</td>
<td>1,7</td>
</tr>
<tr>
<td>prestress</td>
<td>$\gamma_p$</td>
<td>0,9-1,2</td>
<td>1,0</td>
<td>1,0</td>
<td>1,0</td>
<td>1,0</td>
</tr>
<tr>
<td>variable</td>
<td>$\gamma_q$</td>
<td>1,5</td>
<td>1,2-1,6</td>
<td>1,0-1,2</td>
<td>1,75-2,1</td>
<td>1,7</td>
</tr>
<tr>
<td>Material</td>
<td>$\gamma_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concrete</td>
<td>$\gamma_C$</td>
<td>1,5</td>
<td>1,5</td>
<td>1,4-2,8</td>
<td>1,0</td>
<td>1,0-1,33</td>
</tr>
<tr>
<td>steel</td>
<td>$\gamma_S$</td>
<td>1,15</td>
<td>1,15</td>
<td>1,5</td>
<td>1,0</td>
<td>1,0</td>
</tr>
</tbody>
</table>

1) depending on load combination
2) depending on kind of loads
3) for pressure $f_{yd} = \frac{f_{yk}}{\gamma_s + \frac{f_{yk}}{2000}}$

Global safety indications

<table>
<thead>
<tr>
<th></th>
<th>CEB</th>
<th>CP 110</th>
<th>CCBA 68</th>
<th>DIN 1045</th>
<th>VB 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0,85</td>
<td>0,84</td>
<td>-</td>
<td>0,76-0,86</td>
<td>0,75-1,0</td>
</tr>
<tr>
<td>$\gamma_F'\gamma_C'\frac{1}{\alpha}$</td>
<td>2,38-2,64</td>
<td>2,14-2,86</td>
<td>1,4-3,36</td>
<td>2,03-2,75</td>
<td>1,7-2,27</td>
</tr>
<tr>
<td>$\gamma_F'\gamma_S$</td>
<td>1,55-1,73</td>
<td>1,38-1,84</td>
<td>1,5-1,8</td>
<td>1,75</td>
<td>1,7</td>
</tr>
</tbody>
</table>
4.3 Moduli of elasticity of concrete

**CEB**

Modulus of longitudinal deformation:

\[ E_{cm} = 9.5 \left( f_{ck} + 8 \right)^{\frac{1}{3}} \]

\[ f_{ck} = \text{characteristic compressive strength for the age of 28 days.} \]

<table>
<thead>
<tr>
<th>( f_{ck} ) (N/mm(^2))</th>
<th>( E_{cm} ) (kN/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td>16</td>
<td>27.5</td>
</tr>
<tr>
<td>20</td>
<td>29.0</td>
</tr>
<tr>
<td>25</td>
<td>30.5</td>
</tr>
<tr>
<td>30</td>
<td>32</td>
</tr>
<tr>
<td>35</td>
<td>33.5</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>45</td>
<td>36</td>
</tr>
<tr>
<td>50</td>
<td>37</td>
</tr>
</tbody>
</table>

**CP 110**

Static modulus \( E_c = (1.25 E_{cq} - 19) \) kN/m\(^2\)

\( E_{cq} \) = dynamic modulus of elasticity.

<table>
<thead>
<tr>
<th>cube strength ( N/mm^2 )</th>
<th>modulus of elasticity ( kN/mm^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>30</td>
<td>28</td>
</tr>
<tr>
<td>40</td>
<td>31</td>
</tr>
<tr>
<td>50</td>
<td>34</td>
</tr>
<tr>
<td>60</td>
<td>36</td>
</tr>
</tbody>
</table>

**CCBA 68**

Short-term: \( E_i = 21000 \sqrt{\sigma_j} \) (bars) (< 24 hours)

\( \sigma_j \) denotes compression strength for the age of \( j \) days.

Long term: \( E_v = 7000 \sqrt{\sigma_j} \) (bars).

If only \( \sigma_{28}' \) is known:

\[ E_i = 23000 \sqrt{\sigma_{28}'} \] (bars)

\[ E_v = 7670 \sqrt{\sigma_{28}'} \] (bars)
DIN 1045  Short-term elastic modulus, relevant to the serviceability limit states:

<table>
<thead>
<tr>
<th>Cube strength (N/mm²)</th>
<th>E_b (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>26</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>35</td>
<td>34</td>
</tr>
<tr>
<td>45</td>
<td>37</td>
</tr>
<tr>
<td>55</td>
<td>39</td>
</tr>
</tbody>
</table>

VB 1974  Short-term elastic modulus:

\[ E'_b = (C_{1m}^{3/2} - C_2 f'_{cmj}) \sqrt{10 f'_{cmj}} \quad E'_b = 0.9 E'_b. \]

- \( f'_{cmj} \) = mean value cube strength for an age of \( j \) days
- \( C_{1m} \) = 1800 (normal weight concrete)
- \( C_2 \) = 4 (normal weight concrete)

<table>
<thead>
<tr>
<th>Cube strength (N/mm²)</th>
<th>E'_b (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12,5</td>
<td>21,5</td>
</tr>
<tr>
<td>17,5</td>
<td>25,0</td>
</tr>
<tr>
<td>22,5</td>
<td>28,0</td>
</tr>
<tr>
<td>30</td>
<td>30,5</td>
</tr>
<tr>
<td>37,5</td>
<td>32,5</td>
</tr>
<tr>
<td>45</td>
<td>34,0</td>
</tr>
<tr>
<td>52,5</td>
<td>35,5</td>
</tr>
<tr>
<td>60</td>
<td>37,0</td>
</tr>
</tbody>
</table>

The figure on page 13 shows the moduli of elasticity of concrete in relation with the concrete grade:

a. without influence of creep
b. with creep influence (\( t = \infty \)) in relation with a relative humidity (R.H.) of 40 to 50%.

The differences are very small.
4.4 Shrinkage

CEB, DIN 1045 and VB 1974 prescribe extensive formulae to determine the influence of shrinkage.
CP 110 refers to CEB-FIP recommendations.
CCBA prescribes an equally divided stress $E \times \varepsilon$ depending on the location of the building.

The differences are very small as shown on page 15. Only according to VB 1974 shrinkage depends on the concrete grade. Also according to that code the influence of the reinforcement can be taken into account.

4.5 Creep

CEB, DIN 1045 and VB 1974 prescribe extensive formulae to determine the influence of creep.
CP 110 refers to CEB-FIP recommendations.
CCBA prescribes that the long term elastic modulus is 1/3 of the short elastic modulus, so $E(t_{\infty}) = 2$.

The figure on page 16 shows small differences. Only according to VB 1974 creep depends on the concrete grade.
4.5 CREEP

- Only according to VB 1974: creep depends on concrete grade
- CCBA 68: long term elastic modulus is $\frac{1}{3}$ of short term $\varepsilon(t) = 2$
4.6 Stress-strain diagrams

4.6.1 Steel

The stress-strain diagrams of steel grade S 400 are shown on the next page. According to CEB and CP 110 the yield stress has to be reduced by the material-factor $\gamma_S$.

4.6.2 Concrete

The stress-strain curves of concrete grade C20 are shown on page 19. VB 1974 is the only code which describes the possibility of variation of the modulus of elasticity in the curve.
STRESS STRAIN DIAGRAM REINFORCEMENT
Design diagram S 400

CEB
S 400 $f_{yk} = 400 \text{ N/mm}^2$
tension and compression:
$$f_{yd} = f_{yk} = \frac{400}{1,15} = 347 \text{ N/mm}^2$$

CP 110
Hot rolled high yield $f_{yk} = 410 \text{ N/mm}^2$
tension:
$$f_{yd} = f_{yk} = \frac{410}{1,15} = 357 \text{ N/mm}^2$$
$$0,8 \frac{f_{yk}}{\gamma_s} = 0,8 \cdot 357 = 285 \text{ N/mm}^2$$
compression:
$$f_{yd} = \frac{f_{yk}}{\gamma_s + \frac{2000}{\gamma_s f}} = 303 \text{ N/mm}^2$$

CCBA 68
FeE 40
$\phi < 20 \text{ mm}$ yield stress $412 \text{ N/mm}^2$
$\phi > 20 \text{ mm}$ yield stress $392 \text{ N/mm}^2$
Permitted: $2/3$ of the yield stress
$E_s = 200 \text{ kN/mm}^2$

DIN 1045
BSt 42/50 $f_{yk} = 420 \text{ N/mm}^2$
tension and compression:
$$f_{yd} = f_{yk} = 420 \text{ N/mm}^2$$

VB 1974
FeB 400 $f_{yk} = 400 \text{ N/mm}^2$
tension and compression:
$$f_{yd} = f_{yk} = 400 \text{ N/mm}^2$$
STRESS STRAIN DIAGRAM CONCRETE

Design diagram C 20

CEB
C 20 (cylinder-strength)

\[ f_{cd} = 0.85 \frac{f_{ck}}{Y_c} = 0.85 \cdot \frac{20}{1.5} = 11.3 \text{ N/mm}^2 \]

CP 110
Grade 25 (cube-strength)

\[ f_{cd} = \frac{0.67 f_{cu}}{Y_c} = \frac{0.67 \times 25}{1.5} = 11.2 \text{ N/mm}^2 \]

CCBA 68
\[ \sigma_n = \sigma_{28} = 23.4 \text{ N/mm}^2 \]
Permitted: \( 0.6 \times 23.4 = 14.0 \text{ N/mm}^2 \)

DIN 1045
B 25 (cube-strength)

\[ f_{cd} = 0.7 f_{cu} = 0.7 \times 25 = 17.5 \text{ N/mm}^2 \]

VB 1974
B 25 (cube-strength)

\[ f_{cd} = 0.6 f_{cu} = 0.6 \times 25 = 15 \text{ N/mm}^2 \]
4.7 Relation moment-normalforce

- The design charts on page 21 are based on the characteristic values of M en N.

To determine this values the following load-factors have been used:

- CEB : \( \frac{1.35 + 1.50}{2} = 1.425 \)
- CP 110 : \( \frac{1.4 + 1.6}{2} = 1.5 \)
- DIN 1045 : \( = 1.75 \) - \( = 2.1 \)
- VB 1974 : \( = 1.7 \)

- According to CP 110 the maximum value of \( \frac{N}{A_C} \) corresponds to a minimum design eccentricity given by \( \frac{M}{N} \geq 0.05 \) h so that \( \frac{N}{A_C} \geq 20 \frac{M}{A_C \cdot h} \)

- DIN 1045: If the value \( \frac{N}{A_C} \) increases, the loadfactor increases from 1.75 to 2.1.
RELATION $e_{tot}$ vs. $h$ of uniaxially loaded column

$\xi$ depends on ratio normal force to long term total normal force

$e_o = h$

$e_o = 0.5h$

$e_o = 0.25h$
4.8 Bending

Example

Materials: C18 S400

Actions: deadweight $g_{fk} = 14.4 \text{kN/m}^2$
live load $q_{fk} = 14.4 \text{kN/m}^2$

<table>
<thead>
<tr>
<th>Codes</th>
<th>load-coefficient</th>
<th>$M_{sd}$ (kNm)</th>
<th>Stress distribution comp. zone</th>
<th>Design strength</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_g$</td>
<td>$\gamma_p$</td>
<td>$f_{cd}$ (N/mm²)</td>
<td>$f_{yd}$ (N/mm²)</td>
</tr>
<tr>
<td>CEB</td>
<td>1.35</td>
<td>1.5</td>
<td>128.25</td>
<td>$0.85f_{ck}$</td>
</tr>
<tr>
<td>CP 110</td>
<td>1.40</td>
<td>1.60</td>
<td>135.0</td>
<td>$0.45f_{cube}$</td>
</tr>
<tr>
<td>CCBA 68</td>
<td>1.0</td>
<td>1.2</td>
<td>99.0</td>
<td>$f_{cube}$</td>
</tr>
<tr>
<td>DIN 1045</td>
<td>1.75</td>
<td>1.75</td>
<td>157.5</td>
<td>$0.7f_{cube}$</td>
</tr>
<tr>
<td>VB 1974</td>
<td>1.7</td>
<td>1.7</td>
<td>153.0</td>
<td>$0.8f_{cube}$</td>
</tr>
</tbody>
</table>
4.9 Shear

Calculations of shear stresses and shear reinforcement in beams are based on comparing the average shear stress on a section with a nominal value of ultimate shear stress. When the average shear stress is greater than the nominal stress, shear reinforcement is provided in proportions, calculated under the assumption that the reinforcement forms the tension members of one or more series of pin-jointed trusses (truss analogy). The figure on page 30 show the required shear reinforcement by using vertical stirrups and the maximum shear stresses.

With regard to punching shear two values has been defined $\tau_1$ and $\tau_2$ (page 35). The lowest value $\tau_1$ indicates the ultimate value above which shear reinforcement should be provided. In no case, even with shear reinforcement, shear stress should exceed the highest value $\tau_2$. 
Shear

CEB

There are two methods:
- Standard method
- Accurate method.

Standard method

a) \( \tau_1 = 2.5 \frac{T_d}{R_d w} \) no shear reinforcement should be provided.

\[
\begin{array}{cccccccccc}
\text{f}_{ck} & 12 & 16 & 20 & 25 & 30 & 35 & 40 & 45 & 50 \\
\hline
\tau_{Rd} & 0.18 & 0.22 & 0.26 & 0.30 & 0.34 & 0.38 & 0.42 & 0.46 & 0.50 \\
\end{array}
\]

b) \( \tau_2 = 0.3 \frac{f_{cd}}{\tan \alpha} \)

where inclined stirrups are used \((45^\circ < \alpha < 90^\circ)\)

\( \tau_2 = 0.3 \frac{f_{cd}}{\sin \alpha} \times 0.45 \frac{f_{cd}}{\sin \alpha} \)

\[
\begin{array}{cccccccccc}
\text{f}_{ck} & 12 & 16 & 20 & 25 & 30 & 35 & 40 & 45 & 50 \\
\hline
\text{f}_{cd} & 8 & 10.7 & 13.3 & 16.6 & 20.0 & 23.3 & 26.7 & 30 & 33.3 \\
\end{array}
\]

c) Design shear stress:

\[
\tau = \frac{V_{sd}}{b_d w} \left( 1 + \cot \alpha \right) \sin \alpha = \frac{V_{sd}}{b_d w} \cdot \tau_1 \cdot \beta_1
\]

\( V_{sd} = \) shear force due to ultimate limit loads
\( b_d w = \) width of the section
\( d = \) effective depth.

d) Shear reinforcement

\[
A_{SW} = 0.9 d \times \frac{f_{yk}}{\gamma} \left( 1 + \cot \alpha \right) \sin \alpha = \frac{V_{sd}}{b_d w} \cdot \tau_1 \cdot \beta_1
\]

\( A_{SW} = \) cross-sectional area of the two legs of a stirrup
\( \beta_1 = \) coefficient by which the value \( \tau_1 \) may be increased for members subjected to longitudinal compression (included prestress)

\( 1.0 < \beta_1 < 2.0. \)
C30 S400 vertical stirrups \( \theta = 1 \)  

\[
\frac{A_{sw}}{s} \cdot \frac{f_{yk}}{b_w} = \frac{1,15 \times 6}{0,9} - \frac{1,15 \times 0,85}{0,9} = 6,6 \text{ N/mm}^2
\]

Accurate method  
This method is provided for special cases in particular where both shear and torsion are involved. It allows a choice for the inclination of web compression \((3/5 < \cot \theta < 5/3)\). Reduction of the shear reinforcement is only possible in a limited transitional range. 

\[
\frac{A_{sw}}{s} \cdot 0,9 \cdot d \cdot \frac{f_{yk}}{Y} (\cot \theta + \cota) \sin \alpha = \frac{V}{b_w d \sin \theta} \quad (\leq 0,3 f_{cd})
\]

C30 S400 vertical stirrups \( \theta = 45^\circ \)  

\[
\frac{A_{sw}}{s} \cdot \frac{f_{yk}}{b_w} = \frac{1,15 \times 6}{0,9} = 7,7 \text{ N/mm}^2 \quad \tau = \frac{6}{1,425} = 4,2 \text{ N/mm}^2
\]

CP 110  

a) \( \tau_1 = \nu_c \) depends on longitudinal tension reinforcement \( A_s \)

| 100 \( A_s \) | concrete grade |
|---|---|---|---|---|---|
| bd | 20 | 25 | 30 | 40 or more |
| 0,25 | 0,35 | 0,35 | 0,35 | 0,35 |
| 0,50 | 0,45 | 0,50 | 0,55 | 0,55 |
| 1,00 | 0,60 | 0,65 | 0,70 | 0,75 |
| 2,00 | 0,80 | 0,85 | 0,90 | 0,95 |
| 3,00 | 0,85 | 0,90 | 0,95 | 1,00 |

b) \( \tau_2 \)

| concrete grade |
|---|---|---|---|---|
| 20 | 25 | 30 | 40 or more |
| \( \tau_2 \) | 3,35 | 3,75 | 4,10 | 4,75 |

c) Design shear stress:  
\[
\nu = \frac{V}{b_d} \quad V = \text{shear force due to ultimate loads} \\
b = \text{width of the section} \\
d = \text{effective depth}.
d) Shear reinforcement

\[ A_{sv} > \frac{b(v-v_y)}{0,87 f_{yv}} \]

C30 = grade 37,5 \( f_{yv} = 400 \text{ N/mm}^2 \)

\[ A_{sw yk} = \frac{1,15 \times 3,6}{0,9} - \frac{1,15 \times 0,35}{0,9} = 5,4 \text{ N/mm}^2 \]

\[ \frac{1,15 \times 3,6}{0,9} - \frac{1,15 \times 1,0}{0,9} = 4,6 \text{ N/mm}^2 \]

CCBA 68

a) Shear reinforcement must always be provided

b) \( \tau_2 \leq 3,5 \sigma_b \) if \( \sigma_b \) < \( \sigma_{bo} \)

\[ \tau_2 \leq (4,5 - \frac{\sigma_b}{\sigma_{bo}}) \sigma_b \text{ if } \sigma_b < \sigma_{bo} < 2 \sigma_{bo}. \]

\( \sigma_b \) = admissible tensile strength of concrete
\( \sigma_{bo} \) = concrete compression strength in the same section
\( \sigma_{bo} \) = admissible compression strength with normal force
\( \tau_2 \) = 5 \( \sigma_b \) if inclined reinforcement is used.

<table>
<thead>
<tr>
<th>C10</th>
<th>C14</th>
<th>C18</th>
<th>C24</th>
<th>C30</th>
<th>C36</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_b )</td>
<td>0,44</td>
<td>0,51</td>
<td>0,59</td>
<td>0,72</td>
<td>0,83</td>
</tr>
<tr>
<td>( \sigma_{bo} )</td>
<td>3,76</td>
<td>4,96</td>
<td>6,41</td>
<td>8,46</td>
<td>10,26</td>
</tr>
</tbody>
</table>

c) Design shear stress:

\[ \tau = \frac{T}{b z} \]

\( T \) = shear force due to serviceability loads
\( b \) = width of the section
\( z \) = lever arm \( \left( \frac{7}{8} d \right) \)

d) Shear reinforcement

\[ A_{sw yk} = \frac{b(v-v_y)}{s} \]

C30 \( \sigma_{en} = 34,2 \text{ N/mm}^2 \) Steel: \( \sigma_{at} = 2/3 f_{yk} \)

\[ A_{sw yk} = \frac{3}{2} \times 3,5 \times 0,83 = 4,4 \text{ N/mm}^2 \]

\[ \tau = \frac{7}{8} \times 3,5 \times 0,83 = 2,5 \text{ N/mm}^2 \]

* Admissible steel stress \( \sigma_{at} = \frac{\tau}{\rho \sigma_b} \)

\[ \rho = 1 - \frac{\tau}{\sigma_{bo}} \]

\( \rho_{min} = 2/3 \)
DIN 1045

a) \( \tau_1 = \tau_{012} \)

if \( \tau_{012} < \tau_o < \tau_{02} \) it is allowed to reduce \( \tau_o \rightarrow \tau = \frac{\tau_o^2}{\tau_{02}} > 0,4 \tau_o \)

\( \tau_o \) = design shear stress.

b) \( \tau_2 = \tau_{03} \)

\( B15 \rightarrow \tau_2 = 2,0 \text{ N/mm}^2 \)

\( B55 \rightarrow \tau_2 = 5,0 \text{ N/mm}^2 \)

c) Design shear stress

\( \tau_o = \frac{Q_s}{b_o \cdot z} \)

\( Q_s \) = shear force due to serviceability loads

\( b_o \) = width of the section

\( z \) = lever arm

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
& \text{B15} & \text{B25} & \text{B35} & \text{B45} & \text{B55} \\
\hline
\tau_{012} & 0,50 & 0,75 & 1,00 & 1,10 & 1,25 \\
\tau_{02} & 1,20 & 1,80 & 2,40 & 2,70 & 3,00 \\
\tau_{03} & 2,00 & 3,00 & 4,00 & 4,50 & 5,00 \\
\hline
\end{array}
\]

d) Shear reinforcement

\( C30 \rightarrow B35 \quad f_{yk} = 400 \text{ N/mm}^2 \), vertical stirrups

\( \tau_{03} = 1,75 \times 4 \times 0,9 = 6,3 \text{ N/mm}^2 \) \((z = 0,9d)\)

\[
\frac{\tau_{03}}{1,75} = 3,6 \text{ N/mm}^2
\]

\[
\frac{A_{sw} \cdot f_{yk}}{s \cdot b} = \frac{1405,6 \cdot 3}{0,9} = 7,4 \text{ N/mm}^2
\]
VB 1974

a) \( \tau_1 = 0,5 f_b \) \( f_b \) = design tensile strength of concrete

<table>
<thead>
<tr>
<th></th>
<th>B12.5</th>
<th>B17.5</th>
<th>B22.5</th>
<th>B30</th>
<th>B37.5</th>
<th>B45</th>
<th>B52.5</th>
<th>B60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>0,5</td>
<td>0,55</td>
<td>0,65</td>
<td>0,75</td>
<td>0,90</td>
<td>1,00</td>
<td>1,10</td>
<td>1,25</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>2,5</td>
<td>3,5</td>
<td>4,5</td>
<td>6,0</td>
<td>7,5</td>
<td>9,0</td>
<td>9,0</td>
<td>9,0</td>
</tr>
</tbody>
</table>

b) \( \tau_2 = 0,25 f_{bk} \) \( f_{bk} \leq 9 \text{ N/mm}^2 \) \( f_{bk} \) = characteristic compression strength of concrete.

c) Design shear stress:
\[ \tau_d = \frac{T_d}{bh} \]
\( T_d \) = shear force due to ultimate loads
\( b \) = width of the section
\( h \) = effective depth

d) Shear reinforcement
\( C30 = B37.5 \) \( f_{yk} = 400 \text{ N/mm}^2 \), vertical stirrups

\[ \frac{A_{sw} f_{yk}}{s \cdot b} = \frac{7,5}{0,9} - \frac{0,9}{0,9} = 7,3 \text{ N/mm}^2 \]
\[ \tau = \frac{7,3}{1,7} = 4,3 \text{ N/mm}^2 \]
4.10 Punching shear

CEB

a. $\tau_1$ depends on: - concrete grade C
- percentage longitudinal reinforcement
  in both directions $\rho_x$ and $\rho_y$
- effective height $d$.

$$\tau_1 = 1,6 \cdot \tau_{Rd} \cdot \kappa \cdot (1 + 50 \rho_1)$$

$$\kappa = 1,6 - d \text{ (m)} + 1,0$$

$$\rho_1 = \sqrt{\rho_{lx} \cdot \rho_{ly}} + 0,008$$

<table>
<thead>
<tr>
<th>$f_{ck}$</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{Rd}$</td>
<td>0,18</td>
<td>0,22</td>
<td>0,26</td>
<td>0,30</td>
<td>0,34</td>
<td>0,38</td>
<td>0,42</td>
<td>0,46</td>
<td>0,50</td>
</tr>
</tbody>
</table>

$C12 + \tau_1 = 0,29 - 0,56 \text{ N/mm}^2$

$C50 + \tau_1 = 0,8 - 1,6 \text{ N/mm}^2$

b. $\tau_2 = 1,6 \tau_1$

$C12 + \tau_2 = 0,47 - 0,9 \text{ N/mm}^2$

$C50 + \tau_2 = 1,3 - 2,5 \text{ N/mm}^2$

c. Design shear stress

$$\tau_d = \frac{\tau_{d_1}}{\eta \cdot \frac{e}{W}}$$

e denotes the eccentricity of the load with respect to
the centroid of the critical section.

W denotes the modulus of inertia of the critical section
\eta is a factor depending on the shape of the periphery
The critical section should be taken on a perimeter 0,5d
from the boundary of the loaded area.

CP 110

a. $\tau_1$ depends on: - concrete grade (grade)
- percentage of reinforcement in both directions
- overall depth (h)

$$\tau_1 = \xi_s \cdot \tau_c$$

<table>
<thead>
<tr>
<th>$\frac{100 \ A_s}{b.d}$</th>
<th>grade 20 N/mm²</th>
<th>grade 25 N/mm²</th>
<th>grade 30 N/mm²</th>
<th>grade 40 or more</th>
<th>Overall slab depth (mm)</th>
<th>$\xi_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,25</td>
<td>0,35</td>
<td>0,35</td>
<td>0,35</td>
<td>0,35</td>
<td>250 or more</td>
<td>1,00</td>
</tr>
<tr>
<td>0,50</td>
<td>0,45</td>
<td>0,50</td>
<td>0,55</td>
<td>0,55</td>
<td>225</td>
<td>1,05</td>
</tr>
<tr>
<td>1,00</td>
<td>0,60</td>
<td>0,65</td>
<td>0,70</td>
<td>0,75</td>
<td>200</td>
<td>1,10</td>
</tr>
<tr>
<td>2,00</td>
<td>0,80</td>
<td>0,85</td>
<td>0,90</td>
<td>0,95</td>
<td>175</td>
<td>1,15</td>
</tr>
<tr>
<td>3,00</td>
<td>0,85</td>
<td>0,90</td>
<td>0,95</td>
<td>1,00</td>
<td>150 or less</td>
<td>1,20</td>
</tr>
</tbody>
</table>
grade 20 \( \rightarrow \tau_1 = 0,35 - 1,02 \text{ N/mm}^2 \)
grade 40 or more \( \rightarrow \tau_1 = 0,35 - 1,20 \text{ N/mm}^2 \)

b. \( \tau_2 \) depends on concrete grade and should not exceed half the appropriate value given in the table below


<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>N/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>grade 20</td>
<td>3,35</td>
</tr>
<tr>
<td>grade 25</td>
<td>3,75</td>
</tr>
<tr>
<td>grade 30</td>
<td>4,10</td>
</tr>
<tr>
<td>grade 40 or more</td>
<td>4,75</td>
</tr>
</tbody>
</table>

grade 20 \( \rightarrow \tau_2 = 1,68 \text{ N/mm}^2 \)
grade 40 or more \( \rightarrow \tau_2 = 2,38 \text{ N/mm}^2 \)

c. Design shear stress \( v = \frac{V}{U_{\text{crit}} \cdot d} \) \( U_{\text{crit}} \) = length of critical parameter

There are no data about the determination of eccentricity.
The critical section should be taken on a perimeter 1,5 \( h \) from the boundary of the loaded area and if \( h > 200 \text{ mm} \) a similar amount on a parallel perimeter at a distance of 0,75 \( h \) inside it.

CCBA 68

a. \( \tau_1 < 1,2 \bar{\sigma}_b \)
\( \tau_d = \frac{1,5 P}{P_c \cdot h_o} \)
\( P \) is concentrated load
\( P_c \) is perimeter
\( h_o \) is overall depth (\( h \))
\( \bar{\sigma}_b \) is tensile strength of concrete


<table>
<thead>
<tr>
<th>Concrete Grade</th>
<th>C10</th>
<th>C14</th>
<th>C18</th>
<th>C24</th>
<th>C30</th>
<th>C36</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{\sigma}_b ) (N/mm²)</td>
<td>0,44</td>
<td>0,51</td>
<td>0,59</td>
<td>0,72</td>
<td>0,83</td>
<td>0,93</td>
</tr>
</tbody>
</table>

C10 \( \rightarrow \tau_1 = 0,53 \text{ N/mm}^2 \)
C36 \( \rightarrow \tau_1 = 1,11 \text{ N/mm}^2 \)

b. The critical section should be taken on a perimeter 0,5 \( h \) from the boundary of the loaded area.
DIN 1045  
a. $\tau_1$ depends on:  
- concrete grade (B)  
- way of anchorage of tension reinforcement  
- percentage of reinforcement in both directions  
- quality of the steel  

$$\tau_1 = x_1 \cdot \tau_{011}$$  
$x_1 = 1,3a \sqrt{U_g}$, $1,5 > U_g \geq 0,5$  
$U_g$ = mean percentage of reinforcement

<table>
<thead>
<tr>
<th>B St 22/34</th>
<th>B St 42/50</th>
<th>B St 50/55</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1,0</td>
<td>1,3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>B15</th>
<th>B25</th>
<th>B35</th>
<th>B45</th>
<th>B55</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>$\tau_{011}$</td>
<td>0,25</td>
<td>0,35</td>
<td>0,40</td>
<td>0,50</td>
</tr>
<tr>
<td>1b</td>
<td>$\tau_{011}$</td>
<td>0,35</td>
<td>0,50</td>
<td>0,60</td>
<td>0,70</td>
</tr>
<tr>
<td>2</td>
<td>$\tau_{02}$</td>
<td>1,20</td>
<td>1,80</td>
<td>2,40</td>
<td>2,70</td>
</tr>
</tbody>
</table>

* Values if reinforcement has been anchored in tensile area.

B15 $\tau_1 = 0,23 - 0,78 \text{ N/mm}^2$, serviceability state  
B55 $\tau_1 = 0,50 - 1,80 \text{ N/mm}^2$, serviceability state

b. $\tau_2$ depends on:  
- concrete grade  
- percentage of reinforcement  
- quality of the steel  

$$\tau_2 = x_2 \cdot \tau_{02}$$  
$x_2 = 0,45a \sqrt{U_g}$

B15 $\tau_2 = 0,38 - 0,93 \text{ N/mm}^2$, serviceability state  
B55 $\tau_2 = 0,96 - 2,31 \text{ N/mm}^2$, serviceability state

c. Design shear stress  

$$\tau_r = \frac{Q_{r\max}}{u \cdot h_m} \quad u = \text{perimeter}$$  
$h_m = \text{effective depth (d)}$

At edge and corner columns $\tau_r$ has to be increased with 40%.  
The critical section should be taken on a perimeter $0,5 \text{ d}$  
from the loaded area. At edge and corner columns the  
perimeter is 60% and 30% of the perimeter of an inner column.
VB 1974

a. $\tau_1$ depends on the concrete grade

$$\tau_1 = f_b'$$

$f_b' = \text{design tensile strength of concrete}$

<table>
<thead>
<tr>
<th>Concrete grade</th>
<th>$f_b'$ N/mm$^2$</th>
<th>$f_{bk}'$ N/mm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B12.5</td>
<td>1.0</td>
<td>10</td>
</tr>
<tr>
<td>B17.5</td>
<td>1.1</td>
<td>14</td>
</tr>
<tr>
<td>B22.5</td>
<td>1.3</td>
<td>18</td>
</tr>
<tr>
<td>B30</td>
<td>1.5</td>
<td>24</td>
</tr>
<tr>
<td>B37.5</td>
<td>1.8</td>
<td>30</td>
</tr>
<tr>
<td>B45</td>
<td>1.9</td>
<td>36</td>
</tr>
<tr>
<td>B52.5</td>
<td>2.2</td>
<td>42</td>
</tr>
<tr>
<td>B60</td>
<td>2.5</td>
<td>48</td>
</tr>
</tbody>
</table>

$B12.5 \rightarrow \tau_1 = 1.0 \text{ N/mm}^2$

$B60.0 \rightarrow \tau_1 = 1.9 \text{ N/mm}^2$

b. $\tau_2$ depends on the concrete grade

$$\tau_2 = 0.25 f_{bk}' (< 9 \text{ N/mm}^2)$$

$f_{bk}' = \text{characteristic compression strengths of concrete}$

$B12.5 \rightarrow \tau_2 = 2.5 \text{ N/mm}^2$

$B45$ or more $\rightarrow \tau_2 = 9.0 \text{ N/mm}^2$

c. Design shear stress $\tau_d = \frac{T_d}{ph} + \frac{T_d}{ph} \cdot \frac{2e}{h+d}$ (inner column)

$h = \text{effective depth (d)}$

$p = \text{perimeter}$

$d = 2 \left( \frac{a_x + a_y}{\pi} \right)$

The critical section should be taken on a perimeter $0.5h$ from the loaded area.

Punching shear reinforcement

CEB : The vertical component of the force resisted by this reinforcement should be not less than 75% of the load.

CP 110 : Only if the slab is at least 200 mm thick is it allowed to apply shear reinforcement.

CCBA 68 : No articles about punching shear reinforcement

DIN 1045 : like CEB

VB 1974 : No articles about punching shear reinforcement.
PUNCHING SHEAR
serviceability state

\[ \tau'_1 (\text{N/mm}^2) \]

\[ \tau'_2 (\text{N/mm}^2) \]

---

<table>
<thead>
<tr>
<th>Code</th>
<th>( f_{ck} ) (N/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEB</td>
<td>1.425</td>
</tr>
<tr>
<td>CP 110</td>
<td>1.5</td>
</tr>
<tr>
<td>CCBA 68</td>
<td>1.1</td>
</tr>
<tr>
<td>DIN 1045</td>
<td>1.75</td>
</tr>
<tr>
<td>VB 1974</td>
<td>1.7</td>
</tr>
</tbody>
</table>
4.11 Staggering rule

The envelope line of the tensile forces is obtained by a horizontal displacement $a_1$ of the envelope line $F_t = \frac{M}{z}$

---

![Envelope of tensile forces](image-url)

---

CEB : $a_1 = \frac{V_{sd,s}}{2A_{sw,fywd,sin\phi}} - d \cotga$ (if shear reinforcement is used)

- stirrups $\phi 6 \ s = 100 \ A_{sw} = 57 \ mm^2 \ f_{ywd} = 400 \ N/mm^2$
- $V_{sd} = 162 \ kN \ \alpha = 90^\circ \quad a_1 = 410 \ mm$

CCBA 68 : $a_1 = \frac{V_{sd,s}}{2} \ if \ z = 0,9 \ d \quad a_1 = 200 \ mm$

DIN 1045:
- $a_1 = 0,25 \ d$ inclined stirrups
- $a_1 = 0,50 \ d$ bent-up bars + vert. stirrups
- $a_1 = 0,75 \ d$ vertical stirrups $\quad a_1 = 340 \ mm$

- By reducing the shear stress the values $a_1$ have to be increased by 0,25 $d$.
- If no shear reinforcement is used $a_1 = 0,75 \ d$.

VB 1974 : $a_1 = d \quad a_1 = 450 \ mm$.

CP 110 : no prescriptions
4.12 EFFECTIVE WIDTH FLANGED BEAMS

- One-span beam
  - \( h_l \) = 2.0

- Continuous beam
  - \( h_l / h \) = 0.1, 0.2, 0.3
  - \( l_1 \) = 0.1, 0.2, 0.3

- Symbols:
  - \( h \): Beam depth
  - \( l \): Span length
  - \( a \): Effective width factor

- Standards:
  - CEB
  - CP 110
  - CCBA 68
  - DIN 1045
  - VB 1974
Cases for which checking of the deflections may be omitted

\( l = \text{span} \)
\( d = \text{effective height} \)
\( \sigma_s = \text{steel stress} \)
slabs

spanning in two directions
simply supported

\[
\frac{l_x}{d} \quad l_x [m]
\]

\[
\begin{array}{c}
35 \quad 298 \quad 289 \quad 20 \\
\end{array}
\]

\[
\begin{array}{c}
10 \quad 20 \\
\end{array}
\]

\[
\begin{array}{c}
I_x = 143 \\
= 235 \\

I_y = 145 \\
= 240 \\
\end{array}
\]

restrained or continuous

\[
\frac{l_x}{d} \quad l_x [m]
\]

\[
\begin{array}{c}
50 \quad 467 \quad 397 \quad 374 \\
\end{array}
\]

\[
\begin{array}{c}
10 \quad 20 \\
\end{array}
\]

\[
\begin{array}{c}
I_x = 143 \\
= 235 \\

I_y = 145 \\
= 240 \\
\end{array}
\]
### 4.14 Minimum Cover (mm)

<table>
<thead>
<tr>
<th>Code</th>
<th>Grade of concrete</th>
<th>Conditions of exposure</th>
<th>Additions</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C12, C16, C20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>general case</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>slabs, shells</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C25, C30, C35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>general case</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>slabs, shells</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C40, C45, C50</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>general case</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>slabs, shells</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CP 110</td>
<td></td>
<td>dry</td>
<td>subject to salt used for de-icing</td>
</tr>
<tr>
<td></td>
<td>grade 20</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>grade 25</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>grade 30</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>grade 40</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>grade &gt; 50</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>CCBA 68</td>
<td>all grades</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>DIN 1045</td>
<td>B15</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>general</td>
<td>20</td>
<td>25-30</td>
</tr>
<tr>
<td></td>
<td>flat structures</td>
<td>15</td>
<td>20-25</td>
</tr>
<tr>
<td></td>
<td>B25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>general</td>
<td>15</td>
<td>20-25</td>
</tr>
<tr>
<td></td>
<td>flat structures</td>
<td>10</td>
<td>15-20</td>
</tr>
<tr>
<td>VB 1974</td>
<td>all grades</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>slabs</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>walls</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>beams</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>columns</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

Minimum cover shall not be less than 15 mm and maximum seize aggregate plus 5 mm.

If inspection is impossible: to increase values with 5 mm.
4.15 Minimum area of reinforcement

<table>
<thead>
<tr>
<th></th>
<th>Slabs</th>
<th>Beams</th>
<th>Walls</th>
<th>Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S220</td>
<td>S400</td>
<td>S220</td>
<td>S400</td>
</tr>
<tr>
<td>CEB</td>
<td>0,0025</td>
<td>0,0015</td>
<td>0,0025</td>
<td>0,0015</td>
</tr>
<tr>
<td>CP 110</td>
<td>0,0025</td>
<td>0,0015</td>
<td>0,0025</td>
<td>0,0015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CCBA 68</td>
<td>depends on concrete grades</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0,0014</td>
<td>0,0008-</td>
<td>0,0014-</td>
<td>0,0008-</td>
</tr>
<tr>
<td></td>
<td>0,003</td>
<td>0,0016</td>
<td>0,003</td>
<td>0,0016</td>
</tr>
<tr>
<td>DIN 1045</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VB 1974</td>
<td>0,002</td>
<td>0,0015</td>
<td>0,002</td>
<td>0,0015</td>
</tr>
</tbody>
</table>

The minimum areas of reinforcement with regard to slabs, beams and walls do not differ much.

The required minimum reinforcement in the columns however differs much, which finds expression in the example on page 47.
5 Comparison calculation

5.1 Design methods of plane, unbraced frames

It is mentioned in and permitted by all investigated codes (except the French code) to analyse this type of structure by non-linear elastic methods (normally by means of a computer). Most codes permit the use of simplified methods for solution by hand calculations.

CEB  First order moments by linear elastic design
Second order moments are replaced by moments produced by equivalent horizontal forces acting at the top of each storey (Iterative method F-∆).

CP 110  First order moments by linear elastic design
Second order moments by means of additional eccentricity dependent on effective column-height.

CCBA 68  like CP 110

DIN 1045 First order moments by linear elastic design
Second order moments by means of additional eccentricity of substitute, imperfectly restrained columns

VB 1974 First order moments by linear elastic design
Second order moments by means of additional eccentricity of substitute cross-shaped parts of the frame.
CEB

Load distribution first order
Linear elastic theory slope 1/200

\( \lambda < 25 \)

Slenderness
\( 25 < \lambda < 140 \)

Non-linear elastic design

Iterative method
\( F - \Delta \)

Determination of real stiffness of elements

Iterative method
\( F - \Delta \)

Second order eccentricity \( e_2 \)

\( 50 < \lambda < 140 \)

Additional eccentricity \( e_c \)

Determination of the sections
\( e_{\text{tot}} = e_0 + e_2 + e_c \)

CP 110

Load distribution first order
Linear elastic theory

Slender columns
\( \lambda < 40 \)

Yes

Short columns

No second order effects

\( \lambda < 70 \)

Yes

No second order effects in beams

No second order effects

\( e^o \)

\( e^o + e^2 + e^c \)
load distribution first order linear elastic theory

determination

1.0 and λ

λ < 35

no second order effects

35 < λ ≤ 50

additional eccentricity e₂ (1)

determination of reinforcement of columns

\[ e_{\text{tot}} = e_0 + e_2 \]

determination of the reinforcement of the beams

\[ e_{\text{tot}} = e_0 \]

50 < λ ≤ 150

additional eccentricity e₂ (2)

λ > 45

determination of reinforcement

λ ≤ 45

determination of the reinforcement of the beams

DIN 1045

load distribution first order linear elastic theory

determination

1.0 and λ

λ > 20

determination of e_⊂/d for decisive section

λ ≤ 20

no second order effects

λ < 70

no e_⊂/d > 35

yes e_⊂/d > 35

additional eccentricity e₂

λ > 70

determination of reinforcement

additional eccentricity e_c

determination of the reinforcement of the beams

non-linear elastic design

no second order effects
Load distribution first order
linear elastic theory

determination points of
contraflexure
(only wind load)

assume hinges at the points
of contraflexure and divide
the construction into cross-
shaped elements

not possible

non-linear
elastic
design

determination of $e_1$ (first order)
determination $e_2$ (second order)

assess reinforcement in beams
to determine the influence
of the beam on the total ecc.

calculation of the reinforcement
of the beam

calculated reinforcement $>$
assessed reinforcement?

no

don't proceed

yes

determination of reinforcement
in columns
5.2 Office building

As an example an unbraced bearing structure of an office building has been analysed.
The structure has been divided into a series of plane frames and has been simplified for solution by hand calculations.
The dimensions, materials and loads are determined in advance.
The differences between the first order moments, shear forces etc., based on linear elastic design, are small and mainly caused by the differences of the bending resistance of the beams.

After that the amounts of reinforcement of the different members have been determined.
With these theoretical quantities of reinforcement the ratio failure load-working load has been determined.
Finally the horizontal deflection of the frames has been compared.
5.3 General data

Geometrical properties of the structure are illustrated on page 51.

Materials:
- concrete: characteristic cylinder strength 18 N/mm²
- reinforcement: S 400

Actions:
- Permanent Loads:
  - dead weight of reinforced concrete: 24 kN/m³;
  - finish + pipes + separations: 1,5 kN/m²;
  - front wall load (inclusive beam): 10 kN/m';
  - side wall load (inclusive beam): 16 kN/m';

- Live loads:
  - live load: 4 kN/m² on floors, 2 kN/m² on roof;
  - wind load: 1 kN/m² equally divided over height and only perpendicular to the respective walls, suction included.

Determination of actions
- Characteristic values of actions
  Permanent loads on floors:
  - weight of floor 0,125 x 24 = 3 kN/m²
  - finish, pipes and separations = 1,5
    \[
    \frac{4,5 \text{ kN/m}^2}{2}
    \]

  - Loads on beams of central bays:
    - permanent loads 3,60 x 4,50 = 16,20 kN/m'
    - weight of beam 0,3 x 0,375 x 24 = 2,70
      \[
      \frac{18,90 \text{ kN/m'}}{2}
      \]
    - live loads floor beam 3,60 x 4,0 = 14,40 kN/m'
    - live loads roof beam 3,60 x 2,0 = 7,20 kN/m'
Weight of front wall and columns:
- weight of columns 3,60 x 0,5 x 0,5 x 24 = 21,6 kN
- weight of frontwall and beam 3,6 x 10 = 36,0 kN

\[ F_{k1} = 57,6 \text{ kN} \]
\[ F_{k2} = 21,6 \text{ kN} \]

Wind load:
- floors: 3,6 x 3,6 x 1
- roof: \( \frac{1}{2} \times 3,6 \times 3,6 \times 1 \)

\[ W_{k1} = 12,96 \text{ kN} \]
\[ W_{k2} = 6,50 \text{ kN} \]

The following values have been used.

<table>
<thead>
<tr>
<th></th>
<th>( \gamma_g )</th>
<th>( \gamma_q )</th>
<th>( \gamma_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEB</td>
<td>1,35</td>
<td>1,50</td>
<td>1,5</td>
</tr>
<tr>
<td>CP 110</td>
<td>1,20</td>
<td>1,20</td>
<td>1,20</td>
</tr>
<tr>
<td>CCBA '68</td>
<td>1,00</td>
<td>1,20/1,00</td>
<td>1,00</td>
</tr>
<tr>
<td>DIN 1045</td>
<td>1,75</td>
<td>1,75</td>
<td>1,75</td>
</tr>
<tr>
<td>VB 1974</td>
<td>1,70</td>
<td>1,70</td>
<td>1,70</td>
</tr>
</tbody>
</table>
GEOMETRICAL PROPERTIES

39600

11 x 3600

section a-a
5.4 Ratio Total Moments - First Order Moments

\[ \frac{M_{\text{total}}}{M_{\text{first}}} \]

left façade column  |  central columns  |  right façade column

- - - - CEB
- - - - CP 110
- - - - DIN 1045
- - - - CCBA 68
- - - - VB 1974
5.5 MOMENTS IN BEAMS

According to CCBA and CP no second order effects in beams (in this case)
### Theoretical Quantities of Reinforcement

Added per section for 8 floors (mm²)

<table>
<thead>
<tr>
<th>PLACE</th>
<th>CEB</th>
<th>CP 110</th>
<th>CCBA 68</th>
<th>DIN 1045</th>
<th>VB 1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.500</td>
<td>8.000</td>
<td>8.400</td>
<td>16.300</td>
<td>13.400</td>
</tr>
<tr>
<td>2</td>
<td>4.000</td>
<td>4.100</td>
<td>4.300</td>
<td>5.500</td>
<td>4.600</td>
</tr>
<tr>
<td>3</td>
<td>10.600</td>
<td>8.300</td>
<td>8.700</td>
<td>9.200</td>
<td>12.000</td>
</tr>
<tr>
<td>4</td>
<td>4.800</td>
<td>4.000</td>
<td>4.000</td>
<td>4.500</td>
<td>5.600</td>
</tr>
<tr>
<td>5</td>
<td>2.200</td>
<td>1.400</td>
<td>1.400</td>
<td>2.100</td>
<td>3.000</td>
</tr>
<tr>
<td><strong>Total beams</strong></td>
<td>32.400</td>
<td>26.200</td>
<td>26.800</td>
<td>37.600</td>
<td>38.000</td>
</tr>
<tr>
<td>facade column</td>
<td>12.550</td>
<td>7.320</td>
<td>22.800</td>
<td>8.400</td>
<td>6.000</td>
</tr>
<tr>
<td>central column</td>
<td>13.325</td>
<td>7.320</td>
<td>7.300</td>
<td>7.200</td>
<td>6.000</td>
</tr>
<tr>
<td><strong>Total columns</strong></td>
<td>25.875</td>
<td>14.460</td>
<td>30.100</td>
<td>15.600</td>
<td>12.000</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>57.975</td>
<td>40.660</td>
<td>56.900</td>
<td>53.200</td>
<td>50.000</td>
</tr>
</tbody>
</table>
5.7 HORIZONTAL DISPLACEMENT

- CEBulk
- CP 110
- CCBA 68
- DIN 1045
- VB 1974
5.8 Ratio failure load-working load

Each structure, with the amounts of reinforcement as shown in page 54, has been submitted to increasing loads (live load, dead load and wind load) and calculated with the help of a computer.

Depending on the amount and the situation of the reinforcement, plastic hinges develop.

The situation of these hinges with the accessory load-factors are shown on page 58.

If at one section the ultimate strain of the concrete or of the steel has been attained, the failure load is supposed to be reached.

Material properties:

\[
\begin{align*}
\overline{\sigma}_c (N/mm^2) & \quad \overline{\sigma}_s (N/mm^2) \\
2.5 & \quad 10 \quad 19 \quad 19 \quad 100 \\
35 & \quad \xi'_{c,\%} \quad \xi_{s,\%} \\
18 & \quad 400 \\
\text{CONCRETE} & \quad \text{STEEL}
\end{align*}
\]
RATIO FAILURE LOAD – WORKING LOAD

CEB

VB 1974

DIN 1045

CCBA 68

CP 110

plastic hinge and accessory load-factor

failure and accessory load-factor
Conclusions

The differences between the corresponding articles are rather divergent and can not be discussed in generality.

With regard to the comparison calculations the following can be noticed:

a. Load distribution
- The differences between the first-order moments, shear forces etc., based on linear elastic design, are small and mainly caused by the differences of the bending resistance of the beams (effective width T beams).
- The second-order moments, calculated according to the schemes on the pages 44, 45 and 46, differ much. These moments are shown on the pages 52 and 53. According to CCBA 68 and CP 110 no second-order moments have to be taken into account at the beams.

b. Reinforcement
The theoretical quantities of reinforcement are shown on page 54.

The differences are mainly caused by:
- the calculated second-order moments
- the load factors
- the prescribed minimum reinforcement at the columns (except for the façade columns of the French structure all columns have been provided of minimum reinforcement).

c. Relation failure load-working load
The bearing structures have been checked by means of a computer program STANIL. This program is especially suitable to non-linear elastic design of plane frames.

By increasing the loads each time with 10%, the ratio failure load-working load has been determined.

If at one section the ultimate strain of the concrete or of the steel has been attained, the failure load is supposed to be reached.

By cracking of the beams at the span, the moments near the supports increase. Because of the higher percentage of reinforcement near the supports calculated according to CEB, DIN 1045 and VB 1974, the plastic hinges develop especially in the span, as supposed by the English and French structures.

All structures have attained the load factor required.

A reason for the low value (1,3) according to CP 110 will be:
- the load factors are low (1,2)
- the wind loads in England are considerable higher than 1 kN/m².

d. Deflection
The horizontal displacements at the top of the buildings (γ = 1,0) amount to a maximum of \( \frac{1}{600} \) of the total height of the building.
The effect of the amounts of reinforcement in the beams is evident.

d. Calculations
According to all codes simplified methods can be used for analysis by hand, except according to CEB.
The P-Δ method is labourious and by using this method a computer is indispensable.

e. Main differences
- The methods and formulae for determination of second-order effects.
- The fact that according to some codes it is not necessary to introduce these effects in the beams.
- The required minimum areas of reinforcement.

Note. The difficulty of the comparison calculations is that it is not possible to determine the relation between the adopted live loads and the loadfactors according to the several codes.