Chapter

Modeling Departure Time Choice with Stochastic Networks

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Abstract Stochastic supply and fluctuating travel demand lead to stochasticity in travel times and travel costs experienced by travelers from time to time within a day and at the same time from day to day. Many studies show that travel time unreliability has significant impacts on traveler’s choice behavior under uncertainty. Different theories are applicable to describe and model traveler’s choice behavior under uncertainty. This paper presents several behavioral models based on different behavioral hypotheses under uncertainty in the framework of utility theory. An analytical approach is employed to investigate the generality and the relationships among the different behavioral models for modeling traveler’s route/departure time choice behaviors under uncertainty. This paper shows the principal equivalence of different behavioral models and the condition at which the equivalence is maintained. The parameters of the utility components are explored with a theoretical approach and a simulation-based approach. This paper explores which utility components should be incorporated in the behavioral model, which has substantial meaning for the researchers and practitioners to gain insights into the relationships of different behavioral models for modeling traveler’s choice behavior under uncertainty. It is concluded that the mean variance approach is a special case of the scheduling approach and of the generalized utility approach. The generalized utility approach is equivalent to the scheduling approach under a certain condition.

1. Introduction

In transport networks, due to the stochastic supply and fluctuating travel demand, travel times and travel costs experienced by travelers within-day and over days are stochastic (Emam and Al-Deek 2005; Tu 2008). The uncertainty and diversity in travelers’ behaviors (for instance driving behavior) lead to variations and unpre-
dictability in travel times as well, thus in travel costs. Travel time variability has significant impacts on traveler’s choice behavior. Travelers’ choice behavior, for instance route choice, departure time choice and mode choice, under uncertainty has gained increasing attention. Especially in the context of uncertainty, departure time adaptation appears to be one the most important behavioral changes for the sake of attempting to arrive on time at work and to reduce the schedule delay, and to minimize the disutility by departing earlier (Li et al. 2009b). Many studies (Van Amelsfort et al. 2008) have been carried out on analyzing the impacts of travel time variability on travelers’ departure time/route choice behavior and on how to model their choice behaviors under uncertainty. Several empirical studies (e.g., (Abdel-Aty et al. 1996), (Kazimi et al. 2000), (Lam 2000), (Ghosh 2001)) suggest that travelers are interested not only in travel time saving but also reduction of travel time variability. Most travelers prefer reliable travel times. The reason why the travel time variability is so important can be explained at least from two aspects: the anxiety or stress caused by uncertainty, additional cognitive burden associated with planning activities, and sensitivity to the consequences of the uncertainty, for instance late arrivals, etc. (Bates et al. 2001).

Currently, many different theories are applicable to describe and model human choice behavior under uncertainty, such as expected utility theory, prospect theory, cumulative prospect theory, extended prospect theory (Van de Kaa 2008), etc. The difference of expected utility theory from other theories is that it does not account for the cognitive tasks in traveler’s decision making. However, in general the expected utility theory is sufficiently widely accepted as useful to deal with choices made under uncertainty. In addition, the expected utility theory can be straightforwardly applied to the transport situation and traffic assignment models. Therefore, this paper is interested in working in the framework of (expected) utility theory. For research on the choice behavior under uncertainty using prospect theory, we refer to for example (Avineri and Prashker 2006; Van de Kaa 2008). For a review of different theories for modeling traveler’s choice behavior under uncertainty, we refer to for example (Fujii and Kitamura 2004).

In the framework of utility theory, various behavioral models have been proposed in the literature to model traveler’s choice behavior under uncertainty. In general, there are two approaches developed to model the choice behavior under uncertainty, namely the mean-variance approach (Jackson and Jucker 1981) and the scheduling approach (Small 1982). Li, et al. (Li 2009; Li et al. 2009b) proposed a new generalized utility function based on the equivalence analysis of the two approaches for modeling travelers’ departure time/route choice behavior under uncertainty. These three behavioral models have substantial relations among each other and might be equivalent under certain conditions. However, the equivalent conditions and how the parameters of different utility components are related still remain unknown. The generality and the relationships of the mean-variance
approach, the scheduling approach and the generalized utility approach still need to be investigated and verified. This paper will show the equivalency of the mean-variance approach, the scheduling approach, and the generalized utility approach to model travelers’ departure time/route choice behavior under uncertainty. The parameters in different behavioral models will be investigated and which utility components should be incorporated in the utility function will be verified based on theoretical analysis. This paper has a substantial meaning for the researchers and practitioners working in the filed of behavioral modeling, who normally compare the effects of different behavioral models based on empirical data without fully understanding the differences and the relationships of these different approaches.

This paper will firstly present several different behavioral models based on different behavioral hypotheses on how travelers might cope with the uncertainty in their route/departure time choice decisions. Then the relationships of the different behavioral models are investigated with a theoretical approach. The equivalence among different utility function specifications, especially between the mean-variance approach, the scheduling approach, and the generalized utility function and the related parameter values are analytically explored. After that an investigation of the distributions of the parameters maintaining the equivalence of different behavioral models are explored based on a simulation-based approach, which is employed to obtain the time-dependent travel time distributions with stochastic networks. The generality of different behavioral models will be clarified. Finally some conclusions are drawn and future research is discussed.

2. Behavioral models based on different behavioral hypotheses under uncertainty

This section will present several widely applied behavioral models in the framework of utility theory, which aims for modeling travelers’ departure time/route choice behavior under uncertainty. The relationships of different models will be explored afterwards.

A hypothesis of a ‘safety margin’ being selected by travelers has once been specified by (Gaver 1968; Knight 1974), which assumed that travelers make their choice decisions by considering the expected travel time and adding an extra time budget, the so-called safety margin, to cope with the uncertainties. Travelers might also trade off their expected travel time and travel time variability based on their past experiences. However with these hypotheses, the impacts of travel time variability on scheduling costs and travelers’ reaction on the scheduling cost uncertainty are not explicitly modeled, which turns out to be very important in representing travelers’ choice behavior under uncertainty (Small 1982).
A mean-variance approach has been proposed (Jackson and Jucker 1981) (see Equation (1)), in which the individual utility function is composed of expected travel time and travel time variance (or standard deviation of travel time). It hypothesizes that travelers are interested not only in travel time savings but also a reduction of travel time variability. Variability of travel time is usually measured by the standard deviation of travel time (Small et al. 1999). The mean-variance approach does not explicitly model the effect of travel time variability on scheduling decisions.

\[
u^{\text{od}}_{p,t} = \alpha \cdot E\left[\tilde{\tau}^{\text{od}}_{p,t}\right] + \beta \cdot \text{Std}\left[\tilde{\tau}^{\text{od}}_{p,t}\right], \forall(o,d),p,t,\tag{1}\]

where \(u^{\text{od}}_{p,t} \), \(\tilde{\tau}^{\text{od}}_{p,t}\) denote the individual’s (dis)utility and stochastic travel times on route \(p\) between OD pair \((o,d)\) departing at \(t\) under uncertainty respectively. In this paper, all stochastic variables are denoted with a tilde. \(E\left[\tilde{\tau}^{\text{od}}_{p,t}\right]\) and \(\text{Std}\left[\tilde{\tau}^{\text{od}}_{p,t}\right]\) denote the expectation of travelers’ experienced travel times and the standard deviation of travel times on route \(p\) between OD pair \((o,d)\) departing at \(t\) respectively. Parameters \(\alpha\) and \(\beta\) are value of time and value of reliability respectively.

Due to the stochastic properties of travel times, a traveler departing everyday at the same time instant \(t\) may arrive early, late or on time as preferred, largely relying on the traffic situations on that specific day. Travel time variability directly leads to uncertainty in arrival times at the destination. Departure time shifts are very important reactions of travelers to travel time unreliability, since travel time unreliability causes uncertainty of their arrival time, which cause punishment for delays. Travelers attempt to depart earlier in order to arrive at the destination on time and reduce the schedule delay costs. Several empirical studies show that scheduling costs play a major role in the timing of departures (Small 1982).

A scheduling approach based on the expected utility theory (Polak 1987) has originally been specified (Noland and Small 1995), which hypothesizes that scheduling delay cost plays a very important role in the timing of departures under uncertainty. A related (dis)utility function may be formulated as:

\[
u^{\text{od}}_{p,t} = \alpha \cdot E\left[\tilde{\tau}^{\text{od}}_{p,t}\right] + \gamma_1 \cdot E\left[\text{PAT} - t + \tilde{\tau}^{\text{od}}_{p,t}\right] + \gamma_2 \cdot E\left[t + \tilde{\tau}^{\text{od}}_{p,t} - \text{PAT}\right], \forall(o,d),p,t,\tag{2}\]

\(\text{PAT}\) denotes the preferred arrival time. The second and the third component are the expected schedule delay costs of being early and late respectively. Function \((\cdot)^+\) is equivalent to \(\max 0, x\), since there is either early schedule delay or
late schedule delay on a specific day. It can never occur that travelers experience both delay costs at the same time. Parameters $\alpha$, $\gamma_1$, and $\gamma_2$ represent value of time, value of early schedule delay and late schedule delay respectively. It has been shown that scheduling costs explain the aversion to uncertain travel times (Small et al. 1995). They conclude that in models with a fully specified set of scheduling costs, it is unnecessary to add an additional disutility for unreliability of travel time. Expected scheduling costs account for all the aversion to travel time uncertainty.

However it is found that scheduling delay costs cannot capture the travel time unreliability completely (Noland and Polak 2002; Van Amelsfort et al. 2008). Besides a scheduling effect, travel time unreliability appears to be a separate source of travel disutility. Travelers may not only consider the expected travel time (i.e. duration of their trip), the expected schedule delay costs, but also the variability in travel times as an indicator representing travelers’ perceived uncertainty. Therefore an alternative utility function might be expressed as:

$$u_p^o t = \alpha \cdot E \left[ \hat{t}_p^o t \right] + \gamma_1 \cdot E \left[ \text{PAT} - t + \hat{r}_p^o t \right] + \gamma_2 \cdot E \left[ t + \hat{r}_p^o t - \text{PAT} \right] + \beta \cdot \text{Std} \left[ \hat{t}_p^o t \right], \ \forall (o,d), p, t,\ (3)$$

This utility function has the variability term on the basis of the scheduling approach.

Li, et al. (Li 2009; Li et al. 2009b) has proposed a new generalized utility function based on the equivalence of the mean-variance approach and the scheduling approach. It assumes travelers are aware of the distributive properties of their perceived travel time distributions at a departure time instant $t$ based on their past cumulative experiences under uncertainty. They hypothesize that travelers attempt to minimize a weighted function of the expected travel time, the schedule delay costs, which is derived based on their perceived expected travel time, and the variability in travel time, and schedule delays, which can be formulated as:

$$u_p^o t = \alpha \cdot E \left[ \hat{t}_p^o t \right] + \gamma_1 \cdot \text{PAT} - t + E \left[ \hat{r}_p^o t \right] + \gamma_2 \cdot t + E \left[ \hat{r}_p^o t \right] - \text{PAT} + \beta \cdot \text{Std} \left[ \hat{t}_p^o t \right], \ \forall (o,d), p, t,\ (4)$$

where $\beta$ can be a constant or a distribution. The generalized utility function is different from the mean variance approach since it explicitly accounts for the scheduling effects. It differs from the scheduling approach in the way that the
scheduling delays are based on the expected travel time and travel time reliability effects are captured explicitly. Travelers make a trade-off between all the disutility components.

Due to the complexity of human decision making, travelers make their choice decisions based on different reasoning about the traffic situation and perceive different utilities. With regard to travelers’ mental representations, different measures, different indicators and different disutility components can be chosen, thus different models based on the hypotheses can be adopted to represent travelers’ choice behavior under uncertainty.

The models we have presented are mainly for purpose of our interest to investigate the relationship between the typical mean-variance approach, the scheduling approach, and the generalized utility approach. There are many other models based on different hypotheses on travelers’ departure time/route choice behavior under uncertainty, which are not enumerated in this paper.

Based on above models representing travelers’ choice behavior under uncertainty, some questions come up, for instance which utility function performs best, which disutility components should be included, what are the relationships among the different utility functions, is there any better utility function and forth alike. Already in 1994, the relationship between the expected utility approach and the mean-standard deviation approach has been analyzed (Senna 1994), but in his work, the utility does not account for any scheduling costs. Some analytical investigations on the scheduling utility function (2) has been performed by (Fosgerau and Karlstrom 2008), which showed that in case the travel time distribution is independent of departure times, the expected scheduling cost is linear in mean and standard deviation of travel time by assuming normally distributed travel time. The equivalence between the travel time budget approach and the scheduling approach has been demonstrated by (Siu and Lo 2007) for a given travel time distribution. The travel time budget approach is proposed by (Lo et al. 2006), which is defined based on the probability requirement of arrivals within the travel time budget. This approach shares a similar form as the mean-variance approach, but carries an entirely different interpretation. In case the travel time distribution is known, the parameter of the standard deviation in the travel time budget function can be derived given a probability that a trip completes within the travel time budget. The next section will analytically explore the relationships of the different behavioral models to answer the above questions on utility component and its parameter values.
3. Analytical investigation on the relationships of behavioral modeling approaches

As discussed in the previous section, various behavioral models are applicable to model travelers’ choice behavior under uncertainty. This section will investigate the relationships among different behavioral models using a theoretical approach.

We will present the equivalence of the mean-variance approach and the scheduling approach conditional on departure times, which is the same conclusion as from (Fosgerau and Karlstrom 2008), but for general travel time distributions.

Let’s start with the scheduling approach based on Formula (2). Its utility function assumes linearity of schedule delays, which means that travelers have constant values for early and late schedule delays. With this utility function, it is assumed that travelers have perfect knowledge on the distributive properties of their future travel times and the schedule delay early and late due to their past experiences, for all departure time instants and all available routes.

It has been proven analytically by our previous research (Li 2009; Li et al. 2009b) that if schedule delay is a linear function, then for any departure time $t$ expected schedule delay on route $p$ between OD pair $(o, d)$ is a linear function of schedule delay with expected travel time on route $p$ between OD pair $(o, d)$ when departing at $t$ and standard deviation of travel time on route $p$ between OD pair $(o, d)$ when departing at $t$. This can be mathematically expressed as:

\[
E\left[\text{PAT} - t + \tilde{\tau}^{\text{sd}}_p \ t \right] = \text{PAT} - t + E\left[\tilde{\tau}^{\text{sd}}_p \ t \right] + \delta_1 \cdot \text{Std}\left[\tilde{\tau}^{\text{sd}}_p \ t \right] \\
E\left[t + \tilde{\tau}^{\text{sd}}_p \ t - \text{PAT} \right] = t + E\left[\tilde{\tau}^{\text{sd}}_p \ t \right] - \text{PAT} + \delta_2 \cdot \text{Std}\left[\tilde{\tau}^{\text{sd}}_p \ t \right]
\]

with $0 \leq \delta_1, \delta_2 \leq \frac{1}{2}$, $\forall (o,d), p,t$.

where $\delta_1$ and $\delta_2$ are parameters of standard deviation maintaining the equivalence to the expected schedule delay early and late respectively. These two parameters are constant for any departure time instant $t$. From the analytical analyses, $\delta_1$ and $\delta_2$ are bounded and smaller than 0.5. In case the travel time distribution $\tilde{\tau}_p^{\text{sd}}$ at any departure time instant $t$ follows a normal distribution, $\delta_1$ and $\delta_2$ can be derived directly and expressed mathematically. $E\left[\tilde{\tau}^{\text{sd}}_p \ t \right]$ and $\text{Std}\left[\tilde{\tau}^{\text{sd}}_p \ t \right]$ denote the expectation and standard deviation of the stochastic travel times respectively on route $p$ between OD pair $(o, d)$ when departing at $t$.

Therefore, the expected schedule delay early and late can be expressed as:
\[ \gamma_1 \cdot E \left[ PAT - t + \tilde{e}_p^\text{od} - t \right] + \gamma_2 \cdot E \left[ t + \tilde{e}_p^\text{od} - t - PAT \right] \]

\[ = \gamma_1 \cdot PAT - t + E \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_2 \cdot t + E \left[ \tilde{e}_p^\text{od} - t \right] - PAT + \xi \cdot \text{Std} \left[ \tilde{e}_p^\text{od} - t \right]. \]

With \( \xi = \gamma_1 \delta_1 + \gamma_2 \delta_2 \), \( 0 \leq \delta_1, \delta_2 \leq \frac{1}{2}, \forall (o,d), p,t, \) \( (6) \)

where \( \xi \) is the parameter of the standard deviation on the basis of equivalence to the scheduling approach. It is a constant for any given departure time \( t \) and changing over departure time depending on the travel time distribution at \( t \). \( \xi \) has a range of:

\[ 0 \leq \xi \leq 1/2 \cdot \gamma_1 + \gamma_2 \] \( (7) \)

As illustrated with Formula (2), the scheduling approach can be re-formulated as following based on Equation (6):

\[ u^\text{od} \cdot t = \alpha \cdot E \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_1 \cdot E \left[ PAT - t + \tilde{e}_p^\text{od} - t \right] + \gamma_2 \cdot E \left[ t + \tilde{e}_p^\text{od} - t - PAT \right] \]

\[ = \alpha \cdot E \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_1 \cdot PAT - t + E \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_2 \cdot t + E \left[ \tilde{e}_p^\text{od} - t \right] - PAT + \xi \cdot \text{Std} \left[ \tilde{e}_p^\text{od} - t \right]. \]

\[ = \left\{ \xi - \gamma_1 \right\} E \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_1 \cdot PAT - t + \xi \cdot \text{Std} \left[ \tilde{e}_p^\text{od} - t \right] + \gamma_2 \cdot \left( -PAT \right) + \xi \cdot \text{Std} \left[ \tilde{e}_p^\text{od} - t \right] \]

With \( \xi = \left( \delta_1 + \gamma_2 \delta_2 \right) \), \( 0 \leq \delta_1, \delta_2 \leq \frac{1}{2}, \forall (o,d), p,t, \) \( (8) \)

The reformulation of the scheduling approach yields our proposed generalized utility function, as shown in Formula (4). For any given departure time instant \( t \), \( PAT \) and \( t \) are constants. As seen in Formula (7), expectation of the schedule delays can be decomposed as a linear function of expected travel time and standard deviation of travel time. Thus the mean-variance approach is a special case of the scheduling approach. The generalized utility function is also different from the mean-variance approach, since it explicitly includes the schedule delay early and late disutility.

Based on the theoretical decompositions and mathematical reformulations of the expected schedule delay early and late, it appears that the generalized
(dis)utility function (4) is different from the scheduling approach from the mathematical formulations. In case the $\beta_t$ in the generalized utility function in Formula (4) is equivalent to $\xi_t$ over departure time, the scheduling approach and the generalized utility function are equivalent. Although the generalized utility function (4) is consistent with the scheduling approach when $\beta_t$ is well chosen to be the same as $\xi_t$, their behavioral explanations are quite different. With the scheduling approach, travelers consider the expected schedule delay costs, where the average values of both schedule delay early and late are positive. It means that travelers will experience both schedule delay early and late on the same trip and will never expect to be on time, which is not logical. However, with the generalized utility function, travelers consider the schedule delay costs based on the expectation of travel time. This is more plausible and behaviorally sound since travelers based on their expected travel time anticipate that they will be either early or late or on time. Travelers are assumed to be able to know the expected travel time based on their past accumulative experiences. Then they infer when they will arrive at the destination. From analytical analysis, we only obtain that $\xi_t$ is changing over departure time and bounded as seen in Formula (7), it is not clear how $\xi_t$ distributes over time. This will be investigated in the next section.

As seen in Formula (6), the expectation of schedule delays already include travel time variability, which proves the finding from (Small et al. 1999; Hollander 2006; Van Amelsfort et al. 2008) that the expected schedule delay accounts for the aversion to travel time unreliability. Adding a separate component of variability in travel time as shown in Equation (3) together with expected schedule delay cost seems unnecessary and may overestimate the impacts of uncertainty on traveler’s departure time/route choice behavior. There will be a large correlation between the expected scheduling delays and the standard deviation, see the behavioral model formulated in Formula (3).

However, it is concluded from (Noland and Polak 2002) based on empirical studies that in some limited circumstances there appear to be some residual effects of reliability that are not completely subsumed by scheduling considerations. Similar results are also found by (Van Amelsfort et al. 2008) showing that travel time unreliability cannot be captured completely by scheduling delay costs (see the scheduling approach), and besides a scheduling effect, travel time reliability is a separate source of travel disutility. From this aspect, the generalized utility function (4) is able to overcome above shortcoming of the scheduling approach since the disutility parameter of travel time variability $\beta_t$ can be very flexible.
In conclusion, the mean variance approach (see Formula (1)) is a special case of the scheduling approach (see Formula (20)) and of the generalized utility function (see Formula (4)). The generalized utility function (4) will be equivalent to the scheduling approach in case $\beta$ is equivalent to $\xi$. The utility function proposed in Formula (3) has a large correlation between the expected scheduling delays and the standard deviation, which is not plausible. The generalized utility function is more behaviorally sound for modeling travelers’ departure time/route choice behavior under uncertainty.

4. Investigation of the relationship of utility component parameters

From the theoretical analysis of the relationship of the generalized utility approach and the scheduling approach in previous section, it is concluded that the mean of the scheduling delay can be decomposed into a sum of the scheduling delay based on the expected travel time and the standard deviation of travel time when departing at $t$. The parameters of the standard deviation, $\delta_{1}$ and $\delta_{2}$, to maintain the equivalence to the scheduling approach are bounded, which are positive and smaller than 0.5. For a given departure time $t$, the parameters are constant, but may change over time depending on the travel time distribution. Therefore, this section will mainly focus on the investigation of these two parameters to investigate under which condition the scheduling approach and the generalized utility approach are equivalent. It will not be accurate to investigate the parameters on the basis of the arbitrary distribution combinations of the travel times over departure times. Therefore, we investigate the parameters based on the equilibrium traffic condition derived from a simulation-based approach. The equilibrium will be a long term user equilibrium at which travelers make their strategic departure time/route choice decisions with stochastic networks and consideration of this by travelers. The travel time at any departure time $t$ will follow a distribution due to the stochastic capacities. Herein, we just directly take the simulation results on a small network to investigate the parameters we are interested. For detailed definition and formulation of the reliability-based long term user equilibrium with stochastic capacities, see (Li et al. 2008a; Li et al. 2008b; Li 2009; Li et al. 2009a; Li et al. 2009b).

Since different long term user equilibrium will be derived if different behavioral model is applied, then different travel time distributions will be derived as well, which might influence the parameters we obtain based on simulations. Therefore, we have applied the scheduling approach and the generalized utility function with
a fixed parameter of standard deviation $\beta$ (see Formula (4)) to estimate the long term user equilibrium with a simulation-based approach.

We derive the parameters of the standard deviation based on the travel time distributions at the simulated equilibrium traffic condition. In order to maintain the equivalence to the scheduling approach, we derive the parameters as:

$$
\hat{\delta}_1 = \frac{E \left[ \text{PAT} - t + \tilde{t}_p^{\text{old}} t \right]^+ - \text{PAT} - t + E \left[ \tilde{t}_p^{\text{old}} t \right]^+}{\text{Std} \left[ \tilde{t}_p^{\text{old}} t \right]} \\
\hat{\delta}_2 = \frac{E \left[ t + \tilde{t}_p^{\text{old}} t - \text{PAT} \right]^+ - \text{PAT} - t + E \left[ \tilde{t}_p^{\text{old}} t \right] - \text{PAT}^+}{\text{Std} \left[ \tilde{t}_p^{\text{old}} t \right]}
$$

(9)

With the simulation-based approach, we are able to derive the travel time distribution $\tilde{t}_p^{\text{old}} t$ for any given departure time $t$ for all routes at the long term equilibrium with stochastic capacities. $\hat{\delta}_1$ and $\hat{\delta}_2$, compared to $\delta_1$ and $\delta_2$, from the theoretical analyses, denote the parameters of standard deviation derived from the simulation-based approach guaranteeing the equivalence to the scheduling approach.

Fig. 1 presents the parameters of standard deviation $\hat{\delta}_1$, $\hat{\delta}_2$, (see formula (9)) at different departure time $t$ with PAT = 80 [min] from the two user equilibrium with different behavioral models. It can be seen that indeed the parameters are bounded, which are always smaller than 0.5, which proves the analytical results given in Formula (5). It shows that indeed the parameters $\hat{\delta}_1$, $\hat{\delta}_2$ are changing over departure time depending on the travel time distribution at a specific departure time $t$. Therefore, the scheduling approach is a linear function of the mean and standard deviation only for a specific departure time $t$, while a non-linear function of the mean and standard deviation over departure times. The parameters of standard deviation follow a distribution as shown in Fig.1.
Fig. 1. Parameters of standard deviations from the decomposition of the mean schedule delay.

Fig. 2 presents the cumulative departures along departure times.

Fig. 2. Cumulative departures on route one based on different behavioral modeling.
It is noticed that all the travel demand departs during the time period from about 32 [min] to 60 [min]. However, as seen in Fig. 1 the parameters of standard deviation are positive only between the time period from 43 [min] to 52 [min] with the scheduling approach and from 44 [min] to 55 [min] with the generalized utility approach. It implies that travelers departing before 43 [min] (44 [min] with the generalized utility approach) will always arrive earlier than preferred with the scheduling approach, irrespective to the stochastic travel times they might experience. Travelers departing after 52 [min] (55 [min] for the generalized utility approach) will always arrive later than PAT with the scheduling approach, regardless of the stochastic travel times. Therefore, the mean schedule delay equals the schedule delay based on the mean travel time when departing at time $t$, expressed as following for the scheduling approach case:

$$
E\left[ PAT - t + \tilde{\tau}_d^t \right]^+ = PAT - t + E\left[ \tilde{\tau}_d^t \right]^+, \quad t < 43[\text{min}] \text{ or } t > 52[\text{min}]
$$

$$
E\left[ t + \tilde{\tau}_d^t - PAT \right]^+ = t + E\left[ \tilde{\tau}_d^t \right]^+ - PAT
$$

(10)

A surprising result is found from Fig. 1 that:

$$
\hat{\delta}_1 = \hat{\delta}_2, \quad \forall t,
$$

(11)

which means that the following holds for any departure time $t$:

$$
E\left[ PAT - t + \tilde{\tau}_d^t \right]^+ - PAT - t + E\left[ \tilde{\tau}_d^t \right]^+
$$

$$
= E\left[ t + \tilde{\tau}_d^t - PAT \right]^+ - t + E\left[ \tilde{\tau}_d^t \right]^+, \forall (o, d), p, t,
$$

(12)

It is important to prove $\delta_1 = \delta_2$ theoretically. Therefore we provide the proof in the following.

$\tilde{\tau}_d^t$ is a positive stochastic variable, following a certain distribution. Let $\tilde{x} t = PAT - t + \tilde{\tau}_d^t$ for a given departure time $t$. Then $\tilde{x} t$ is a stochastic variable as well, of which the mean value $E\tilde{x} t$ can be positive or negative since $\tilde{x} t$ can be partially positive and partially negative. Now the problem is equivalent to prove that:

$$
E\left[ \tilde{x} t \right]^+ - E\left[ \tilde{x} t \right]^+ = E\left[ -\tilde{x} t \right]^+ - E\left[ -\tilde{x} t \right]^+, \forall t
$$

(13)
Assume that \( x(t) \) follows a distribution with a probability density function \( f(t) \).

Then we have:

\[
E\left[\tilde{x}(t)\right] = \int_{\mathbb{R}} x(t) \cdot f(t) \cdot dt
\]

(14)

\[
E\left[-\tilde{x}(t)\right] = -\int_{\mathbb{R}} x(t) \cdot f(t) \cdot dt
\]

(15)

\[
E\left[\tilde{x}(t)\right] = \int_{\mathbb{R}} x(t) \cdot f(t) \cdot dt + \int_{\mathbb{R}} \tilde{x}(t) \cdot f(t) \cdot dt + \int_{\mathbb{R}} \tilde{x}(t) \cdot f(t) \cdot dt
\]

(16)

If \( E\left[\tilde{x}(t)\right] \geq 0 \), the right side of Equation (13) is:

\[
E\left[\tilde{x}(t)^+\right] - E\left[\tilde{x}(t)^-\right] = E\left[\tilde{x}(t)^+\right] - E\left[\tilde{x}(t)^+\right]
\]

(17)

The left side of the Equation (13) is:

\[
E\left[-\tilde{x}(t)^+\right] - E\left[-\tilde{x}(t)^-\right] = E\left[-\tilde{x}(t)^+\right]
\]

(18)

The right side (Formula (17)) equals the left side (Formula (18)). Equation (13) is proven.

If \( E\left[\tilde{x}(t)\right] < 0 \), the right side of Equation (13) is:

\[
E\left[\tilde{x}(t)^+\right] - E\left[\tilde{x}(t)^-\right] = E\left[\tilde{x}(t)^+\right]
\]

(19)

The left side of the Equation (13) is:
The right side (Formula (19)) equals the left side (Formula (20)). Again Equation (13) is proved. Therefore, $\delta_1 = 2\delta_2$. The scheduling approach can be expressed as the generalized utility function with $\beta_1 = 2\delta_2$, where $\delta_2$ is bounded and follows a distribution as shown in Fig. 1.

\[
\begin{align*}
E[-\tilde{x} \cdot t] &= E[-\tilde{x} \cdot t] + E[\tilde{x} \cdot t] \\
&= -\int_{x_1}^{x_2} f \cdot t \cdot dt \cdot \int_{x_1}^{x_2} \tilde{x} \cdot t \cdot f \cdot t \cdot dt + \int_{x_1}^{x_2} \tilde{x} \cdot t \cdot f \cdot t \cdot dt \\
&= \int_{x_1}^{x_2} f \cdot t \cdot dt \cdot \int_{x_1}^{x_2} \tilde{x} \cdot t \cdot f \cdot t \cdot dt
\end{align*}
\]

(20)

5. Conclusions and future research

Several behavioral models based on different behavioral hypotheses for modeling traveler’s route/departure time choices under uncertainty have been presented and discussed. The generality and relationships among different behavioral models are explored analytically. It is found that for any given departure time $t$, the expected schedule delay can be decomposed into a linear function of the expected travel time and standard deviation at departure time $t$. The mean-variance approach is a special case of the scheduling approach.

The scheduling approach and the generalized utility approach are equivalent in case the parameter of standard deviation in the generalized utility function is equivalent to the parameter derived from the decomposition of the expected schedule delays, which is bounded and follows a certain distribution as shown from the simulation-based approach.

The generalized utility function is more behaviorally sound. It can capture more complete risk aversion attitude of travelers towards travel time unreliability than the scheduling approach.

There is a large correlation between the expected schedule delays and the standard deviation of travel time, adding an extra reliability component to the schedul-
ing approach may overestimate the variability effects on travelers’ choice behavior under uncertainty. The scheduling approach and the generalized utility function are more general than the mean variance approach.

For the future work, it is worthwhile to validate the generalized utility function by conducting empirical studies.

References


