INFLUENCE OF AMBIENT AIR PRESSURE ON IMPACT PRESSURE CAUSED BY BREAKING WAVES

Constantinos Moutzouris
Internal report no. 10-79
INFLUENCE OF AMBIENT AIR PRESSURE
ON IMPACT PRESSURES CAUSED BY BREAKING WAVES

by
CONSTANTINOS M. OUTZOURIS
Research Fellow
Delft University of Technology
Department of Civil Engineering
Delft - The Netherlands

Internal report no. 10-79
6.3.2.4. Rotation  
6.3.2.5. Radial oscillations  
6.3.3. Modeling of the air pocket  
6.3.3.1. Air pocket radius  
6.3.3.2. Flow pattern  
6.3.3.3. Pressure  
7. TOTAL PRESSURE ON THE STRUCTURE  

PART IV: PROBABILISTIC ANALYSIS OF THE PRESSURES ON THE STRUCTURE  
8.1. Introduction  
8.2. Maximum recorded pressure in one wave period  
8.3. Minimum recorded pressure in one wave period  
8.4. First peak of the pressure  

PART V: CONCLUSIONS  
9. CONCLUSIONS - FURTHER WORK  
10. ACKNOWLEDGEMENTS  

PART VI: APPENDICES  
A1. PHOTOGRAPHICAL DESCRIPTION OF THE FLOW IN THE BREAKING ZONE  
A2. PRESSURE TIME HISTORIES  
A3. LIST OF SYMBOLS  
A4. LIST OF REFERENCES
LIST OF FIGURES

Fig. 3.1: Definition sketch of the experiments
Fig. 4.1: Zones of a sloping maritime structure
Fig. 5.1: Typical pressure-time histories recorded by the transducers on the structure
Fig. 5.2: Decomposition of a pressure-time history
Fig. 6.1: Evolution of the water surface as function of time in the breaking zone of the structure
Fig. 6.2: Evolution of the water height over the transducers as function of time in one wave period.
Fig. 6.3: Water height over the transducers
Fig. 6.4: Horizontal movement of the water jet from the wave breaking
Fig. 6.5: Evolution of $h_{\text{max}}$ and $h_{\text{min}}$ in the after-breaking zone of a sloping structure (from 24).
Fig. 6.6: Definition sketch of the water jet movement
Fig. 6.7: Pressure-head due to the water jet impact
Fig. 6.8: Some shock pressures as they were recorded by the three transducers
Fig. 6.9: Correlation between the damping coefficient and the shock pressure
Fig. 6.10: Damping coefficient $\beta$ versus peak pressure $P_{sh}$
Fig. 6.11: Decomposition of the pressure $I$
Fig. 6.12: Formation of the air pocket during a wave breaking
Fig. 6.13: Decrease of the cross-section of the air pocket (exper.)
Fig. 6.14: Decrease of the diameter of the air pocket (exper.)
Fig. 6.15: Horizontal movement of the air pocket (exper.)
Fig. 6.16: Evolution of the height of the water layer below the air pocket (exper.)
Fig. 6.17: Rotation of the air pocket
Fig. 6.18: Pressure oscillations due to the air pocket vibrations (exper.)
Fig. 6.19: Amplitude of the pressure oscillations (exper.)
Fig. 6.20: Frequency of the pressure oscillations (exper.)
Fig. 6.21: Frequency of the air pocket vibrations (theor.)
Fig. 6.22: Initial frequency of the air pocket vibrations (theor.)
Fig. 6.23: Minimum speed of propagation of the sound in the down-rushing layer (theor.)
Fig. 6.24: Evolution of the height of the water layer above the air pocket (exper.)
Fig. 6.25: Amplitude of the pressure oscillations (theor.)
Fig. 7.1: Pressure-heads on the structure from the three factors (theor.)
Fig. 8.1: Characteristic statistical values of $P_{\text{max}}$ and $P_{\text{min}}$
Fig. 8.2: Significant values of $P_{\text{max}}$ and $P_{\text{min}}$
Fig. 8.3: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0 / L_0 = 0.08$)
Fig. 8.4: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0 / L_0 = 0.14$)
Fig. 8.5: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0 / L_0 = 0.19$)
Fig. 8.6: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ under a shift of the time scale
Fig. 8.7: Cumulative distributions of $P_1$

LIST OF TABLES

Table 3.1: Examined waves and ambient pressures
Table 8.1: Characteristic statistical values of $P_{\text{max}}$
Table 8.2: Characteristic statistical values of $P_{\text{min}}$
Table 8.3: Characteristic statistical values of $P_1$
Part I: INTRODUCTION

1. INTRODUCTION

Engineers are interested in the dynamics of the interface water-structure. In case of breaking of water waves on a structure high positive and sometimes negative pressures of very short duration occur.

Not only the maxima and minima of the pressures on the structure are important to a designing engineer, but also the pressure-time history: failure of the structure may occur when for the first time the pressure reaches a certain level or when the accumulation of small damages due to lower pressures reaches a certain level.

The breaking of water waves on the structure is a process in a three-phases-system, because almost always air is entrapped in the mass of the breaking wave. The large number of parameters which control the developed pressures can be classified in three groups, according to the three phases water, solid air:

- water and wave parameters such as water depth, wave period and height.
- structure parameters such as geometry of the interface, material constants
- air parameters such as ambient pressure.

Many of the parameters are not controllable and change continuously. For this reason there is no pressure history identical to a previous one. Only general trends remain the same. As most of the loads on a structure, pressures due to the wave breaking are stochastic.

Until recently it was considered that the wave parameters were the most important: the breaking mechanism depends mainly on them. Recent experimental indications show that the air parameters have also an important role in the wave breaking loads. Air pockets and bubbles entrapped in the water mass during the breaking influence the pressure. The dynamics of the interface air-water are important: some oscillating pressures on the structure are due to the vibrations of a major air pocket.

In scale model investigations of impact pressures caused by waves breaking on or against a structure, the compressibility of the air is normally not scaled down. Thus, scale effects are to be expected when the model results are converted into larger-scale prototype values according to Froude's
model law. One way of investigating the nature and magnitude of such effects is to perform pressure measurements for the same structure and incident waves but with different ambient air pressures, since the compressibility of the air varies with the pressure.

It is with this objective that the Board of Maritime Works in the Netherlands commissioned the Delft Hydraulics Laboratory to carry out an investigation on the pressures occurring during the breaking of water waves on a sloping structure with different input waves and two ambient pressures: atmospheric and near vacuum. Experiments were performed by the Delft Hydraulics Laboratory in a small wave flume of the Laboratory of Fluid Mechanics of the Civil Engineering Department, Delft University of Technology, in which the ambient air pressure could be varied. A report on the experiments, the results obtained, and interpretations thereof, is in preparation within the Delft Hydraulics Laboratory.

In the meantime, the experimental data obtained were made available to the author, who was then a Research Fellow at the Department of Civil Engineering, for purposes of analysis. Such analysis was to be carried out independently, and was in no way a partial fulfillment of the contractual obligations of the Delft Hydraulics Laboratory. Thus, although the data used in the present study were collected by the Delft Hydraulics Laboratory, the interpretations thereof and conclusions derived from them as stated in the present report are the author's own.

The present report contains the results of a theoretical analysis of the pressures on a sloping structure due to the wave breaking and an analysis of data from the above mentioned experiments. Special attention is paid to the role of the entrapped air pocket and of the ambient pressure. A strict overall modelling of the pressures is impossible. For this reason the mechanism and general trends of the pressure history are analysed and described by means of partial deterministic models, which seems to be a quite realistic approach. On the other hand, the values of the pressures are described statistically, which is a more reasonable approach because of their stochastic character.

The plan of the report is as follows:
- A brief analysis of existing approaches to the problem is given first.
- A phenomenological description of the wave propagation and breaking on a sloping structure is then made.
Typical pressure-time signals are presented, as they have been recorded. Each one of them is the result of a superposition of partial pressure-time histories due to a number of loading factors.

The main loading factors are analysed and deterministic modeling is attempted.

A statistical analysis of some characteristic values of the pressure is finally made.

2. EXISTING STUDIES ON THE PRESSURES DUE TO THE WAVE BREAKING

2.1 Introduction

A quite large number of studies has been made, both experimental and theoretical, on the pressures occurring on a structure due to the wave breaking on it. The influence of three groups of parameters has been more or less checked in these studies:

- The wave parameters:
  They are the most extensively examined, although there are not enough results relating the pressure with the wave parameters.

- The structure parameters:
  In most of the studies the structure-water interface is plane and vertical.

- The air parameters:
  Some theoretical studies exist in which it is tried to model the influence of the entrapped air. In some experimental studies the influence of the ambient pressure and of the thickness of the entrapped air layer on the shock pressure is checked.

Some of the most pioneering studies are now briefly reviewed. They are divided into two categories according to the scope of this investigation: those which ignore the entrapped air and those which take it into account.

2.2. Studies ignoring the entrapped air

Early experimental studies on the pressures during the wave breaking were carried out without sensitive transducers. For that reason many details of the pressure response, such as the influence of the entrapped air, were not detected. Examples of such early studies are the works of Gaillard [12], Hiroi [15] and de Rouvill-Besson-Pétry [31] on constructed walls and break-
waters. Larras [18] was the first to collect experimental data in a laboratory flume. All these experimental works did not present any relation between the pressure and the wave parameters.

Some other investigators collected experimental data and attempted to develop empirical relations based on these data:

Rundgren [32] reports the results of an experimental study on the breaking wave pressures on a wall. He concludes that the largest peak $P_{\text{max}}$ of the pressure-time history, due to the shock, is related to the deep-water wave height $H_o$ and length $L_o$. Based on his results and the results of other investigators the author proposes the following expression:

$$\frac{P_{\text{max}}}{\varepsilon_w H_o} = C_1 \cdot \ln \left( \frac{H_o}{L_o} \right) + C_2$$

where $\varepsilon_w$ is the unit weight of water, $C_1$ and $C_2$ two constants.

Minikin [20] proposes an equation for $P_{\text{max}}$ caused by breaking waves on vertical breakwaters placed on the top of a slope. This equation is derived from the experimental data of Bagnold and his own:

$$P_{\text{max}} = 102.4 h_t (1 + \frac{h_t}{h_o}) \frac{H_o}{L_o} \text{ in ton/m}^2$$

where $h_t$ is the water depth at the toe of the wall and $h_o$ the water depth at the toe of the sloping bottom.

According to Minikin, $P_{\text{max}}$ occurs always at the still water level. At other points on the water-structure surface the pressure $P$ is given by:

$$P = P_{\text{max}} (1 - \frac{2X}{H_o})^2$$

where $X$ is the distance of the considered point from the still water level.

Minikin's equation is very widely used in coastal engineering.

Nagai [26] performed extensive experiments on the breaking wave pressures on a vertical wall placed also on the top of a slope. He assumes the shock pressure to be due to the instantaneous momentum change during the impact and presents a theoretical expression for $P_{\text{max}}$. A constant included in this expression is evaluated from his experimental data which he divides into two categories: pressures of ordinary breaking waves and pressures of extraordinary breaking waves. Extraordinary breaking waves are defined as waves
which load the structure with extraordinary high shock pressures. Nagai does not specify the limit between the two categories. Finally he proposes:

\[ P_{\text{max}} = 20 + 500 \varepsilon w \frac{h_t}{h_o} \frac{H_o}{L_o} \] in gr/cm²

for ordinary breaking waves and

\[ \frac{h_t}{h_o} \cdot \frac{H_o}{L_o} \leq 0.22 \]

and

\[ P_{\text{max}} = 280 (0.04 + \frac{h_t^2}{h_o} \frac{H_o}{L_o})^{1/3} \] in gr/cm²

for extraordinary breaking waves, where \( h_t \) and \( h_o \) are the same as in Minikin's approach.

It is noted that the numerical coefficients in Nagai's expressions have all kind of dimensions.

Concerning the vertical distribution of the pressure over the wall, he distinguishes two types of distributions. In the first type, \( P_{\text{max}} \) occurs at the still water level and the pressure is distributed as:

\[ P = P_{\text{max}} (1 - \frac{X}{H_o})^2 \]

where \( X \) is the distance from the still water level measured in both directions on the structure-water interface. In the second type \( P_{\text{max}} \) occurs at the toe of the wall and the pressure is distributed as:

\[ P = P_{\text{max}} (1 - \frac{X}{2H_o})^2 \]

where \( X \) is measured in one direction.

Nagai writes that the vertical distribution of the pressure over the wall is affected by the shape of the breakwater and the behavior of the breaking wave. He does not specify under which conditions the two proposed distributions appear.

Garcia [13] makes an attempt to establish an empirical relation between \( P_{\text{max}} \) and the wave length and height:

\[ P_{\text{max}} = \varepsilon w \frac{H_o}{L_o} \]
where \( f(H_0) \) is a function to be defined from experimental data. Finally he proposes:

\[
P_{\text{max}} = 50 \varepsilon^{2/3} E_o^{1/3}
\]

where \( E_o \) is the deep-water wave energy per unit crest length:

\[
E_o = \frac{1}{8} \varepsilon_w H_0^2 L_0
\]

The location of the maximum shock pressure is found to depend on the slope in front of the wall, on the water depth \( h_t \) at the toe of the wall and finally on \( \frac{L_0}{L_0} \).

The vertical distribution of \( P_{\text{max}} \) is as follows, according to Garcia:

\[
P = P_{\text{max}} (1 - \frac{2X}{h_t})^2 \quad \text{above the location of } P_{\text{max}}
\]

\[
P = P_{\text{max}} (1 - \frac{1.5X}{h_t})^2 \quad \text{below the location of } P_{\text{max}}
\]

The Waterways Experiment Station Formula [16] is based on the shock fronts which appear during the breaking. They are due to the non-linear character of the impact on solids and represent surfaces of discontinuity crossed by the flow.

During the breaking two shock fronts are created: the one propagates in the water with a speed \( C_w \) and the other in the structure with a speed \( C_s \). Ahead of the first front the water is moving with a constant velocity \( V_{sh} \) and ahead of the second one the structure is at rest while behind it the structure moves with a constant velocity \( V_{st} \). The pressure behind both shock fronts represents the shock pressure \( P_{\text{max}} \).

Conservation of mass and momentum at the two fronts gives finally:

\[
P_{\text{max}} = \frac{\rho_s C_s}{\rho_w (V_{sh} + C_w) + \rho_s C_s} \rho_w (V_{sh} + C_w) V_{sh} = \frac{\rho_s C_s}{\rho_w C_w + \rho_s C_s} \rho_w C_w V_{sh}
\]

where \( \rho_w (\rho_s) \) is the mass density of water (structure), \( C_w (C_s) \) is the speed of sound in water (structure), and \( V_{sh} \) is the velocity of impact.

When the shock front is travelling from the point of impact to the nearest free surface the wave is a compression one. After its reflection on the free surface it becomes a tension one. The duration of the shock pressure is equal to the time necessary for a wave to travel with the sound speed \( C_w \)
in water from the impact to the free surface and then back. Kamel [16] compared the shock pressures as they are predicted by the above formula with experimental data obtained by dropping a plate into a water mass. The impact surface was parallel to the water surface. According to the author, W.E.S. formula predicts values much larger than the experimental ones. The discrepancy is due to the presence of entrapped air between the accelerated plate and the water surface. Von Karman [35] gives a simple expression for the maximum shock pressure $P_{\text{max}}$ which can occur during a water hammer impact:

$$P_{\text{max}} = \rho_w V_{\text{sh}} \gamma$$

where $V_{\text{sh}}$ is the velocity of impact.

2.3. Studies concerning the entrapped air

Some investigations have been carried out in the direction of the influence of the entrapped air during the breaking on the pressures loading the structure.

Bagnold [2] was the first to propose a mathematical model. It is a water-hammer model according to which a thin layer of air of thickness $d$ is entrapped and compressed between the face of the breaking wave and the structure. The compressing water mass has a unit cross-section area and a length $K$. It moves with the same velocity $V_{\text{sh}}$ as the striking wave front. The compression is adiabatic and the ambient pressure $p_o$ is equal to the atmospheric one. The shock pressure $P_{\text{max}}$ is due to the sudden reduction of the water mass momentum.

The equation of motion of the water mass over the air layer is:

$$\rho_w K \frac{d^2 y}{dt^2} - p_o \frac{dy}{y} + p_o = 0$$

where $\gamma$ is the adiabatic constant of the air and $y$ the distance in the direction of compression. This equation is integrated graphically and gives pressure-time curves from which $P_{\text{max}}$ is taken:

$$P_{\text{max}} = \frac{2.7 \rho_w V_{\text{sh}}^2}{dK}$$

$K$ is approximately equal to $H_o/5$. 
Bagnold's model is the first to take into account the air pocket entrapped in the water mass during the breaking. It considerably simplifies the real mechanism of breaking and predicts values $P_{\text{max}}$ much higher than the observed ones.

Weggel and Maxwell [36] propose a different model for the temporal and spatial distribution of the shock pressure. They assume that the air is uniformly mixed with the water and not that the two phases are separated, as it was in the model of Bagnold. They use two equations for the two-dimensional conservation of momentum, the equation of continuity and an equation of state for the compressible mixture. They assume that both the bottom and free surface are horizontal and that the free surface is independent of time. A disturbance representing the shock is introduced in the interface structure-water. The spatial and temporal variations of the disturbance are followed with a numerical solution of the equations. The numerical results compare favorably with experimental data collected in a laboratory flume.

Sellars [33] proposes an expression for the maximum shock pressure $P_{\text{max}}$ during the impact of a liquid-air mixture on an elastic structure:

$$P_{\text{max}} = \frac{P_{\text{aa}}}{2} \left( 1 + c \frac{V_{\text{sh}} \rho_e C_e}{P_{\text{aa}}} - \frac{c}{\delta} \right) + \left( 1 + c \frac{V_{\text{sh}} \rho_e C_e}{P_{\text{aa}}} \right)^2 \frac{4c}{\delta} 0.5$$

where $P_{\text{aa}}$ is the ambient pressure, $V_{\text{sh}}$ the velocity at the impact, $\rho_e$ the mass density of pure liquid, $C_e$ the sound speed in pure liquid, $c$ the structure impedance ratio and $\delta$ the liquid-air mixture volumetric impedance ratio. Coefficient $c$ is equal to 9.1 for a rigid structure and to 0.1 for a flexible structure.

Führbötter [11] arrives at a semi-empirical formula for the shock pressure $P_{\text{max}}$ due to the wave attack on a structure.

$$P_{\text{max}} = \rho_w V_{\text{sh}} C_w \left( \frac{C_w}{V_{\text{sh}}} \Delta \right)^{1/3}$$

where $\rho_w$ is the water density, $V_{\text{sh}}$ the impact velocity, $C_w$ the sound speed in water and $\Delta$ the impact-number defined as:

$$\Delta = \left( \frac{\rho_w}{\rho_{\text{aa}}} \right)^{2/3} \frac{R_i}{d}$$
where $E_w$ and $E_a$ are the elasticity or Young's modulus of water and air, $R_i$ the hydraulic radius of the impact zone and $d$ the thickness of the air cushion.

The time of pressure rise $t_{sh}$ is given by:

$$t_{sh} = \frac{R_i}{(V_{sh} \cdot C_w^2)^{1/3} \cdot \Delta^{1/2}}$$

There are not many experimental results concerning the influence of the air cushion and of the ambient pressure on the shock pressure due to the wave breaking. Conclusions have been drawn from experiments where the shock was produced by the impact of a rigid surface on a water layer. The results of these experiments were extrapolated to the wave breaking, which is an impact between two water masses. Here are some of these experimental results:

Kamel [16] carried out experiments in a tank filled with still water. A plate was released from a certain position and the pressure was recorded. The same experiments were conducted with a disturbed water surface. He concludes that the shock pressure increases when there is no air and decreases with increasing air layer thickness.

Bagnold [2] writes that according to his experiments the pressure due to the shock is larger when the entrapped air cushion is thinner. The shock pressure due to identical waves differs from one experiment to the other because of small irregularities on the wave front but the impulse is rather constant.

Richert [29] writes that the largest pressure on the structure always occurs where the entrapped air cushion was initially situated. If the entrapped air cushion is thinner, the maximum shock pressure will be higher and the duration shorter.

According to Ross [30], the air, which is always entrapped by the irregularities of the wave front and as bubbles in the water mass, is compressed. Its cushioning effect lowers the top pressure and increases the duration of the pressure.

Negative pressures show that the air had been compressed so much that in re-expanding it threw the water back to cause the pressure of the trapped
air to drop negative. A regular vibration of the pressure indicates repeated contractions and expansions of a bubble.

Acknowledged if Chen [1] conducted experiments in a vacuum tank in order to investigate the effect of the air on impact loads of breaking waves. The impact was produced on a still water surface by a flat plate with rings. Different volumes of air were entrapped between the falling plate and the water.

The experiments show that a part of the pressure is due to the entrapped air and that the shock pressure decreases with reduction in the volume of the entrapped air and in the ambient pressure. Even when air was totally removed, water hammer conditions were never found.

It is noted that the conclusion of Ackerman and Chen according to which the shock pressure due to the impact decreases with the volume of the entrapped air is in contradiction with all the previously reviewed results.

In conclusion, if the results concerning the influence of the entrapped air and of the ambient pressure on the shock pressure could be extrapolated to the wave breaking, it should be concluded that:

- the air parameters have an important role in the pressures on the structure due to the wave breaking
- the entrapped air has a cushion effect on the shock pressure which increases with the quantity of the air and with the ambient pressure.

3. EXPERIMENTAL SET-UP

Experiments were conducted by the Delft Hydraulics Laboratory in a short wave laboratory flume with horizontal bottom of the Delft University of Technology. The transversal section of the flume is composed of an exterior metallic section and an interior section of Perspex material. The interior section has a width of 50 cm and a height of 60 cm.

Short waves of uniform period T were created by the translation movement of a plane generator situated at one end of the flume. A straight sloping structure of concrete was placed at the other end (see fig. 3.1). The slope was 1 : 6 (α = 9.46°). The distance between the mean position of the moving part of the generator and the toe of the structure was 556 cm. A filter composed of wire screens of 160 cm length was placed at a distance of 70 cm from the generator.
The maximum displacement of the generator from its mean position was kept constant during the experiments and equal to 30.1 cm. Three wave gauges measured the wave height along the horizontal part of the flume. Irregularities due to the wave generator and to the relatively small distance between the generator and the structure caused the wave height $H_0$ of the generated waves to be not very uniform along the flume. For the different values of $T$ used in this study the waves had a height between 5 and 8 cm. Because of the imprecision in the values of the wave height the steepness $\gamma_o (= H_o / L_o)$ of the waves will not be mentioned in the rest of the report.

The height of the still water level SWL over the horizontal bottom is called $h_o$ and the wave length $L_o (= \frac{gT^2}{2\pi} \text{ th} \frac{2\pi h_o}{L_o})$. The ratio $h_o / L_o$ is called the initial relative depth.

Three differential pressure transducers, model PDCK 20, Druck Ltd. recorded the pressures on the structure during the wave breaking. One side of the membrane was under the pressure due to the wave breaking and the other side under the pressure due to the ambient air.

The three transducers were placed on an aluminium plate of dimensions 150 x 119 x 18 mm, which was fixed in the breaking zone of the structure:

- The first transducer, called trans. 1, was at a height $h_1 = 32.5$ cm above the horizontal bottom
- The second transducer, called trans. 2, was placed at a height $h_2 = 33.5$ cm at a distance of 6 cm from trans. 1
- The third transducer, called trans. 3, was at a height $h_3 = 34.0$ cm above the horizontal bottom and at a distance of 3 cm from trans. 2.

The pressure range $p_r$ of the transducers was $\pm 10$ psi. Their natural frequency was 2500 $p_r \frac{1}{Hz} = 7900$ Hz.

The frequency of the system aluminum plate-transducer (without water)
approximately 2700 Hz. The natural frequency of the transducer fixed on the plate under the water was quite high and did not influence the rising times of the recorded pressure.

The signals from the three transducers were recorded on magnetic tapes and some of them on photosensitive Kodak paper.

Normal speed films (60 frames/sec) and high speed films (400 frames/sec) were made of the breaking process.

Water and air parameters were checked during the experiments: waves were generated with different values of $h_o$ and $L_o$ under atmospheric pressure, called atm. press., and under near vacuum conditions, called vacuum. The so-called vacuum corresponded in reality with an ambient pressure equal to $\sim 2\%$ of the atmospheric pressure. Of the various combinations of $h_o$ and $L_o$ used in the experimental study, only three are examined in this report, under both ambient pressures. They are listed in table 3.1:

<table>
<thead>
<tr>
<th>$h_o$</th>
<th>$T$</th>
<th>$L_o$</th>
<th>$h_o/L_o$</th>
<th>ambient pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.0 cm</td>
<td>1.30 sec</td>
<td>218 cm</td>
<td>0.19</td>
<td>atm. pres.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>vacuum</td>
</tr>
<tr>
<td>41.8</td>
<td>1.62</td>
<td>293</td>
<td>0.14</td>
<td>atm. pres.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>vacuum</td>
</tr>
<tr>
<td>42.0</td>
<td>2.75</td>
<td>537</td>
<td>0.08</td>
<td>atm. pres.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>vacuum</td>
</tr>
</tbody>
</table>

Table 3.1: Examined waves and ambient pressures
Part II: WAVE PROPAGATION AND BREAKING ON A SLOPING STRUCTURE

4. PHENOMENOLOGICAL DESCRIPTION OF THE WAVE PROPAGATION AND BREAKING

A quantitative description of the breaking of water waves and of the associated phenomena does not exist. Knowledge of the water particle motion and velocity as function of time is necessary for such a model. The old criterion according to which breaking starts when the maximum horizontal velocity of the water particles in the wave crest becomes larger than the wave celerity has been quite well verified.

A qualitative description of the wave propagation and breaking on a sloping structure is given in the following:

The characteristics of short water waves arriving from the open sea and propagating shoreward on a sloping structure change gradually. Vertical and horizontal deformations appear and increase as the water depth decreases. The waves become unstable and finally break when instability becomes irreversible.

Two radical transformations of the flow pattern occur during the propagation: the stable oscillatory flow over the lower part of the structure is transformed into an unstable oscillatory flow which finally changes into a rapidly changing up and down-rushing flow.

The total propagation zone on the structure can be divided into three main zones, corresponding to the three types of flow: stability zone, instability zone and final zone (see fig. 4.1).

![Fig. 4.1: Zones of a sloping maritime structure]
Their lengths have been found correlated with the characteristics of regular waves and the structure slope [22]:

- In the stability zone the waves remain stable, their height decreases, reflection and deformations are small.

- In the instability zone could be divided into two smaller zones: the pre-breaking zone and the breaking zone. In the pre-breaking zone [23] the waves become unstable, deformations and reflection increase. Wave height increases also and shows a maximum value, called breaking height $H_n$, at the breaking line $x_b$ after which it decreases rapidly. The location where the breaking first appears on the wave profile depends on the initial wave steepness, relative depth and structure slopes: as initial wave steepness decreases or/and initial relative depth increases it moves from the top of the profile towards its lower part and disappears (total reflection). Breakings located mainly on the top or in the upper part of the wave profile are considered in the rest of this report.

The length of the pre-breaking zone increases with decreasing structure slope. It shows a maximum value for a certain wave steepness.

The breaking line is the limit between the pre-breaking and breaking zones.

In case of a breaking starting on the top of the wave profile, a water jet is created at the breaking line. It moves shorewards in the breaking zone [24] and strikes the water layer arriving from the final zone at $x_s$. The point $x_s$ is taken to define the end of the breaking zone. Between the water jet and the water layer a major air pocket is entrapped. The water jet over the air pocket has a shoreward horizontal velocity and a vertical one due to the gravity acceleration. The water layer below the air pocket moves seawards. A circulation appears around the air pocket. At the same time, it is entrained shorewards by the up-rushing water mass. The air pocket soon takes the form of a circular vortex moving shorewards. The cross section decreases with time, because of a mass exchange with the ambient water: small air bubbles are continuously created and dispersed in the water mass. They follow a circular flow path due to the air pocket circulation.

At the end of the air pocket life the water height over the structure increases as if an explosion had taken place in the water mass. At the same time small air bubbles are thrown behind like a jet.
Many small air bubbles are also created and dispersed in the water during the impact. They rise to the free surface, forming foam.

- In the final zone, the fallen water mass is reflected upwards by the water layer and then falls again. A second air pocket is formed sometimes. Small vortices are generated in the water mass. The down-rushing movement of the water layer changes into uprushing. A kind of solitary wave is created by the fallen water mass. It propagates shorewards superimposed on the up-rushing layer. When kinetic energy has been totally transformed into potential, the system up-rushing water layer solitary wave starts moving seawards. The waterline oscillates between \( x_{p,\text{max}} \) and \( x_{p,\text{min}} \). The water line of the still water is at \( x = x_0 \).

The described flow in the breaking zone and at the beginning of the final zone is important for the rest of this study. The photographs of Appendix A1 show the successive phases of this flow. They present the evolution of the flow as function of time for the three chosen values of initial relative depth under both ambient pressure. Concerning the time scale, \( t = 0 \) corresponds with the moment at which the water jet strikes the water layer.

Some general conclusions from the photographs and the films are the following:

a. Influence of the initial relative depth \( h_o/L_o \)

- The air pocket life is shorter in case of \( h_o/L_o = 0.14 \) than in case of \( 0.18 \). In the first case the air pocket was easily distinguished up to \( t = 0.20 \) sec. and in the second case up to \( t = 0.25 \) sec. In case of \( h_o/L_o = 0.08 \), there is hardly any air pocket. The life of the air pocket is proportional to its initial size.

- The breaking zone becomes larger when \( h_o/L_o \) increases. In case of \( h_o/L_o = 0.08 \) there is hardly any water jet: the crest of the wave falls on the shoreward wave front.

- The solitary wave in the final zone becomes higher with increasing \( h_o/L_o \).

- The first part of the final zone is much more turbulent in case of \( h_o/L_o = 0.08 \).

b. Influence of the ambient pressure

- The air pocket disappears much sooner under vacuum than under atmospheric conditions. There are less bubbles created. The circulation around the air pocket is more easily distinguished.
The evolution of the water surface with time does not seem to be modified by the ambient pressure.

5. PRESSURES ON THE STRUCTURE DUE TO THE WAVE BREAKING

Pressure-time signals, as they were recorded by the three transducers in the breaking zone during one wave period under atmospheric and vacuum conditions, are shown in Appendix A2. All of them show a very short and steep rising time after the beginning at t = 0. It is followed by a decay time. During that time (in case of atmospheric conditions) oscillations appear. The pressure passes to negative values and then again to the positive ones. Finally, it decreases very regularly.

A typical pressure history is shown in Fig. 5.1. It was recorded during breaking under atmospheric conditions in order to be able to record the influence of the air pocket.

A qualitative analysis of a typical pressure history will now be made. The goal is to establish relations between some characteristic points of the diagram and the phases of a wave breaking as they have been described earlier.

The pressure diagram starts at t = 0, which is the moment of the striking between the water mass from the breaking and the down-rushing water layer. The pressure rises after t = 0 because of the increasing water height over the transducers. A shock wave due to the impact arrives to the transducers very soon after t = 0. The pressure on the structure due to the shock wave is then superimposed on the water height resulting in a steep pressure diagram.

The first peak $P_1$ of the pressure appears at $t = t_k$. At $t_k$ ends the rising time of the pressure from the shock wave. Its beginning is not always easy to distinguish. Irregularities and small air bubbles in the falling water mass sometimes give an irregular form to the diagram during the rising time. In general, $t_k$ is equal to the rising time of the shock pressure on the structure plus the time necessary for the shock to reach the structure. In some records it is possible to distinguish the moment at which the shock pressure arrives at the transducer: the pressure shows suddenly a much steeper slope than the initial one, which was due to increasing water height only.

Between $t = 0$ and $t_k$, an air pocket is formed in the water mass. The air pocket is compressed by the water. Compression starts at $t = t_b$. The pressure increases and shows a second peak. Vibrations of the air pocket follow the initial compression giving an oscillatory form to the pressure diagram. The oscillations are superimposed on a decreasing pressure due to
the decreasing water height, and to the decay of the shock pressure on the structure. Another factor contributing to the decreasing pressure is the air pocket: when it passes over the transducers the water height is reduced to the heights above and below the air mass.

Oscillations are clearer at the transducer 1 because the air pocket was formed over it. Frequency and amplitude decrease with time.

From a certain moment oscillations become less clear because the air pocket section has decreased much. The end of oscillations is not always easy to distinguish on the records.

In case of vacuum conditions, the pressure diagram does not show any oscillations due to the air pocket. It might be due either to a non-existence of vibrations of the air pocket under vacuum or to a very small frequency of vibrations which does not permit to distinguish the pressure oscillations on the record.

After the end of the decay time of the shock pressure and of the air pocket oscillations, the pressure diagram follows the evolution of the water height of the transducers. It passes to positive values and then decreases regularly to a minimum at $t = T$, which is equal to the minimum height of the water.

It is obvious from the analysis given above that the pressures on the structure are mainly due to three loading factors:
- the water layer over the structure
- the shock pressure due to the impact
- the air pocket oscillations.

Each one of them loads the structure. If it is assumed that the superposition is linear, then for the time being it is possible to decompose a typical pressure history into two partial pressure histories (see fig. 5.2):
- The first one, called pressure history I, shows a rising time to a peak and then a decay time during which it reaches negative values. After a certain time it passes into positive values and then it decreases regularly. It is due to the shock pressure and to the water layer.
- The second one, called pressure history II, has an oscillatory form and is damped. It is due to the air pocket vibrations.

In the following sections it will be tried to decompose further the pressure I and to make a mathematical model for the pressure II.
Fig. 5.1: Typical pressure-time histories recorded by the transducers on the structure

\[ \frac{h_o}{L_o} = 0.14 \]

atm. pres.

Fig. 5.2: Decomposition of a pressure-time history
Part III: DETERMINISTIC MODELING OF THE PRESSURES ON THE STRUCTURES

6. FACTORS LOADING THE STRUCTURE DURING THE WAVE BREAKING

6.1. Water layer over the breaking zone of the structure

6.1.1. Introduction

The height $h$ of the water layer over the breaking zone of the structure contributes actively to the formation of the pressures recorded by the transducers during the breaking, as it is analysed in par. 5. Because of the wave motion, $h$ changes in time and in space.

In order to determine the contribution of the water height to the total pressure, it is necessary to investigate the evolution of the water surface as function of time in one wave period in the breaking zone of the structure. This evolution will be investigated in the following paragraphs and an empirical relation representing the evolution of $h$ will be proposed.

6.1.2. Evolution in time of the water layer height

The evolution of the water surface as function of time in one wave period in the breaking zone of the structure was studied from the films. Fig. 6.1. shows such an evolution. It can be seen that:

On the down-rushing water layer (1) a mass of water is added (2). This is due to the water jet formed at the breaking and results in a modification of the flow pattern: the water mass starts up-rushing. A part of the fallen water mass is reflected upward: because of this reflection a part of the breaking zone shows minimum water height over it (around the point s in (3), (4) and (5)). The water height decreases seaward of s and increases shoreward of it. At the same time s moves shoreward. After the reflected water mass has reached a maximum height (5), it starts falling again (6). A solitary wave is created by the perturbation due to the fallen water mass. It propagates shoreward on the up-rushing water mass while its height and celerity decrease. After the end of the up-rushing movement, the flow direction changes again: the water mass starts down-rushing and the solitary wave propagating seawards, although its height is much reduced (7), (8). After the passage of the solitary wave the water height over the breaking zone decreases gradually before the
Fig. 6.1: Evolution of the water surface as function of time in the breaking zone of the structure.
water mass from the new breaking arrives (9), (10), (11).

The evolution of the water height $h$ over the transducers as function of time in one wave period was followed on the films made during the experiments (see fig. 6.2.):

$h$ shows a minimum value $h_{\text{min}}$ at $t = 0$: the down-rushing water mass takes a minimum height the moment before the striking with the water jet. After $t = 0$, $h$ increases very rapidly, because of the added water mass. It reaches a maximum $h_{\text{max}}$ at $t = t_{\text{mwl}}$ and then starts decreasing because of gravity action.

A quantity of water mass is reflected upward and the water height shows a second peak. This second peak appears on the pressure recorded by a transducer only in case this transducer is situated shoreward of $s$. It might be larger than the first peak: it depends on the reflection conditions.

After the second peak, $h$ decreases because of the up-rushing movement. A third peak of $h$ is shown when the solitary wave passes over the transducer after which it decreases gradually to the minimum value $h_{\text{min}}$.

Many small irregularities are present in the diagram of $h(t)$. They are due to all kinds of small perturbations in the breaking zone, such as air bubbles and vibrations of the major air pocket.

For a certain slope of the structure and a certain wave on it, the evolution in time of $h$ depends very much on the breaking line and breaking height. In case of modifications at the breaking line and height from one wave to the other, the water height-time history is modified, mainly between $t = 0$ and the third peak. But the general tendency between $h_{\text{max}}$ and $h_{\text{min}}$ remains the same.

The minimum value $h_{\text{min}}$ at a certain point of the breaking zone depends on the location of the SWL, the wave characteristics and the slope of the structure. It is slightly affected by the modifications in the breaking line due to the irregularities of the input waves. $h_{\text{max}}$ depends on the same parameters as $h_{\text{min}}$ and furthermore on the fallen water mass and the third peak depends on the solitary wave: both of them depend on a larger number of parameters than $h_{\text{max}}$ and $h_{\text{min}}$.

The films showed that for a certain generated wave the values of $h_{\text{max}}$ and $h_{\text{min}}$ are less scattered than the values of the second and third peaks: they are much more deterministic.

The ambient pressure does not influence the evolution in time of the water
Fig. 6.2: Evolution of the water height over the transducers as function of time in one wave period.
layer height.

In order to approach the water height-time history by means of a deter-eministic law, the most deterministic values of the history must be used.
According to the above given analysis, these values are \( h_{\text{max}} \) and \( h_{\text{min}} \). In such a case the history can be considered as a superposition of a curve which increases abruptly from \( h_{\text{min}} \) to \( h_{\text{max}} \) and then decreases regularly from \( h_{\text{max}} \) to \( h_{\text{min}} \), of two peaks due to the water mass reflection and to the solitary wave passage and of many small irregularities.

The main and most deterministic contribution to the water height-time history formation comes from the curve of a regularly decreasing slope (see fig. 6.3). It can be modeled as follows:

Between \( t = 0 \) and \( t_{\text{mwL}} \) the water height \( h \) increases nearly linearly:

\[
(6.1) \quad h = h_{\text{min}} + (h_{\text{max}} - h_{\text{min}}) \cdot \frac{t}{t_{\text{mwL}}} \quad \text{for} \quad 0 < t < t_{\text{mwL}}
\]

Between \( t = t_{\text{mwL}} \) and \( T \) the water height \( h \) decreases and the slope of the curve decreases faster when \( h \) is larger. It could be represented by:

\[
(6.2) \quad \frac{dh}{dt} = - Bh \quad \text{for} \quad t_{\text{mwL}} < t < T
\]

where \( B \) is a constant independent of \( t \). Equ. (6.2) yields:

\[
(6.2) \quad \log h = - Bt + C
\]

where \( C \) is a constant that can be evaluated either from the initial condition \((t = t_{\text{mwL}}, h = h_{\text{max}})\) or from the final one \((t = T, h = h_{\text{min}})\).

If the initial condition is used equ. (6.2) gives:

\[
\frac{h}{h_{\text{max}}} = 10^B(t_{\text{mwL}} - t)
\]

or:

\[
(6.3) \quad \frac{h}{h_{\text{max}}} = a(t - t_{\text{mwL}})/(T - t_{\text{mwL}}) \quad \text{for} \quad t_{\text{mwL}} < t < T
\]

Some waves of \( \frac{h}{L_0} = 0.14 \) under both atmospheric and vacuum conditions were studied again from the films. It turned out that equ. (6.3) with \( a = 0.04 \) is quite close to the experimental data for the three transducers and the two ambient pressures (see fig. 6.3). It is believed that \( a \) depends on the wave characteristics. The experimental data of this study did not permit to establish any relation concerning \( a \).
Fig. 6.3: Water height over the transducers
For \( t = T \) eq. (6.3) gives:

\[
\frac{h_{\min}}{h_{\max}} = a \cdot h_{\max}
\]

Substituting into eqn. (6.1):

\[
(6.1) \quad \frac{h}{h_{\max}} = a + (1 - a) \frac{t}{t_{mw}} \quad \text{for} \quad 0 \leq t \leq t_{mw}
\]

Equation (6.1) and (6.3) represent approximately the evolution of the water height over the transducers. Both of them need the values of \( t_{mw} \) and \( h_{\max} \). \( h_{\max} \) is studied in the following section 6.1.3.

It must be noted there that the pressure-head \( h_s \) on the structure due to the water layer height is not exactly equal to \( h \). When the air pocket passes over the transducers the water height is reduced by the air pocket (see par. 6.3.2.2.). The maximum value of this reduction is equal to the diameter of the air pocket at the moment it passes over the transducer.

### 6.1.3 Maximum and minimum height of the water layer

The maximum \( h_{\max} \) and minimum \( h_{\min} \) values of the water height in the after-breaking zone of a sloping structure have been studied by Moutzouris [24] and found correlated with the wave characteristics, the slope of the structure and the location in the zone. Some results from [24] are reported here:

- \( h_{\max} \) shows maximum value \( h_{\max,b} \) at \( x_b \) at the moment before the breaking starts. \( h_{\max,b}/H \) and \( h_{\min,b}/H \) increase with decreasing structure slope, initial wave steepness and relative water depth. \( h_{\max} \) and \( h_{\min} \) decrease shorewards of \( x_p \) and show minimum values at the end of the breaking zone \( x_s \). In case of plunging breaking \( x_s \) is situated at the zone of the structure struck by the water jet. \( h_{\max} \) shows its \( h_{\max,s} \) value when the reflected water mass reaches its heighest elevation. \( h_{\max,s} \) and \( h_{\min,s} \) are found to increase with the structure slope and to be linearly related to the distance \( (x_s - x_0) \) (different \( x \) are defined in fig. 4.1). \( h_{\max} \) increases shorewards of \( x_s \) because of the water mass reflection and then decreases linearly to 0 at \( x_{p,max} \). \( h_{\min} \) decreases linearly between \( x_s \) and \( x_{p,min} \).

In the breaking zone, which is the most important to this study, \( h_{\max} \)
was evaluated quite accurately as follows:
The water jet created at the breaking line moves in the breaking zone
with a horizontal velocity $U_f$ and a vertical one $V_f$.
- $U_f$ shows a constant value $U_{fo}$ in the time interval between the beginning
of the water jet movement and the moment at which it strikes the down-
washing water layer. After the end of the impact, the water mass speeds
up: $U_f$ initially increases and then starts decreasing. (In fig. 6.4 is
shown the horizontal movement of a water jet front, as it has been filmed
during the experiments of the present study).
- $V_f$ is due to the gravity. It is found very close to an expression $V_f = \frac{1}{2} gt^2$, where $g$ is the acceleration of gravity and $t$ the time starting
at the moment of the breaking.
The constant horizontal velocity $U_{fo}$ and the continuously increasing
vertical velocity $V_f$ give a parabolic trajectory to the water jet front.
The length of the breaking zone $l_s = x_s - x_b$ and the evolution of $h_{max}$
in this zone can be evaluated from the equation of the parabola:

\[
(6.4) \quad l_s = \tan \alpha \cdot \frac{U_{fo}}{g} \left( -1 + \frac{2g}{U_{fo}^2} \cdot \frac{h_{max,b}}{\tan \alpha} \right)^{0.5}
\]

\[
(6.5) \quad h_{max} = h_{max,b} - l_s \cdot \tan \alpha - \frac{1}{2} g \frac{x^2}{U_{fo}^2} \quad \text{for} \quad 0 < x < l_s
\]

The axes are shown in fig. 6.6.
Equ. (6.4) and (6.5) were quite close to the experimental data of 24.
Only the experimental values of $h_{max}$ around $x_s$ were larger than the
computed ones because of the water mass reflection. The horizontal
velocity $U_{fo}$ was introduced as follows:

\[
(6.6) \quad U_{fo} = \left( g h_{max,b} \right)^{0.5}
\]

Equ. (6.4), (6.5) and (6.6) need the value of $h_{max,b}$.
Fig. (6.5) is taken from [24]. It shows the evolution of $h_{max}$ and $h_{min}$
in the after-breaking zone of a structure with a slope $\alpha = 15^\circ$.

6.2. Shock pressure due to the impact of the water jet

6.2.1. Introduction

The water jet created at the breaking line strikes the down-rushing water
Fig. 6.4: Horizontal movement of the water jet from the wave breaking

Fig. 6.5: Evolution of $h_{\text{max}}$ and $h_{\text{min}}$ in the after-breaking zone of a sloping structure (from [24]).
layer. A shock pressure occurs due to the rapid change of the momentum of the fallen water mass. An elastic shock wave is created which propagates through the water-air bubbles mixture towards the structure. The structure responds to the shock wave which arrives after it has been considerably attenuated.

It is obvious that the down-rushing water layer has a very useful role: the shock pressure developed on the water surface is not as high as it would have been on the less deformable surface of the structure and furthermore it is attenuated while propagating through the layer.

On the other hand, the same water layer creates two nearly insurmountable difficulties in the strict mathematical modeling of the shock pressure on the structure. The first is the evaluation of the shock pressure occurring on a surface during a non-ideal impact and the second is the attenuation in a mixture of water and randomly dispersed air bubbles.

Another difficulty in evaluating the shock pressure on the water surface is created by the presence of air bubbles on the front of the water jet. They reduce the surface of impact but at the same time have a cushion effect on the shock.

Two directions can be followed to evaluate the shock pressure that finally loads the structure: idealising the reality and processing on experimental data. Both of them will be tried in the following sections.

6.2.2. Shock pressure on the surface of the down-rushing water layer

The shock pressure developed during the impact of the water jet with the down-rushing water layer is due to the very rapid change of momentum of the striking water mass and depends much on the mass of the striking water and on the velocity at the impact.

An attempt will be made to approach the shock pressure by means of an impact momentum approach. Such an approach can give an expression for the shock pressure. But this expression will be an order-of-magnitude one and not an exact one because:

- the striking mass is not exactly known
- the impact is not an ideal one (it would be the case if the breaking wave was a translatory mass and the down-rushing water mass a rigid surface).

The striking water jet has a horizontal velocity \( U_f \) and a vertical one \( V_f \) due to the acceleration of gravity as it is analysed in section 6.1.3. Let
the velocity at the impact be \( V \); before the impact \( V \) is equal to \( V_{sh} \) and after the impact equal to 0.

Let the striking mass be \( m \). By definition \( m \) has a length \( l_w = l_s - l_a \) (see fig. 6.6) and a height \( h_{sh} \) averaged over \( l_w \) and written as:

\[
(6.7) \quad \bar{h}_{sh} = \frac{\int_0^1 \bar{y} \, dx + \frac{1}{2}(l_s - l_a)^2 \tan \alpha}{l_s - l_a}
\]

The system of axes is shown in fig. 6.6.

The force \( F \) developed during the impact is:

\[
F = \frac{d(mV)}{dt}
\]

\( F \) can be considered as an average pressure \( p_s \) on a length \( l_w \)

\[
(6.8) \quad p_s l_w = \bar{h}_{sh} l_w \, \rho \, \frac{dV}{dt}
\]

Assuming that the water jet follows a parabolic trajectory in the breaking zone:

\[
(6.9) \quad y = h_{max,b} - h_{min,s} - l_s \tan \alpha - \frac{1}{2} \frac{g}{U_0^2} \frac{x^2}{l_s - l_a}
\]

Substitution of \( y \) into equ. (6.7) yields:

\[
(6.10) \quad \bar{h}_{sh} = h_{max,b} - h_{min,s} - l_s \tan \alpha - \frac{1}{6} \frac{g}{U_0^2} \left( \frac{l_s^3 - l_a^3}{l_s - l_a} + \frac{1}{2}(l_s - l_a) \tan \alpha \right)
\]

Concerning \( p_s \), it is assumed that it increases linearly with time between the beginning of the impact and the end of it at \( t = t_{sh} \), showing a maximum value \( p_{sh} \):

\[
p_s = p_{sh} \frac{t}{t_{sh}}
\]

Equ. (6.8) is now written as:

\[
\frac{p_{sh}}{t_{sh}} \int_{t_sh}^{t_{sh}} t \, dt = - \bar{h}_{sh} \rho \int_{t_sh}^{0} \frac{dV}{t_{sh}}
\]

or

\[
(6.11) \quad p_{sh} = 2 \bar{h}_{sh} \rho \frac{V_{sh}}{t_{sh}}
\]
where $V_{sh}$ is the velocity of water jet before the impact:

\[(6.12) \quad V_{sh} = U_{fo} \sin \alpha + g \frac{l_s}{U_{fo}} \cos \alpha\]

Substitution from equ. (6.12) into equ. (6.11) yields:

\[(6.13) \quad p_{sh} = 2 \frac{h_{sh} \omega}{U_{fo} \cdot \sin \alpha + g \frac{l_s}{U_{fo}} \cos \alpha/t_{sh}}\]

or, in pressure-head:

\[(6.13) \quad h_{sh} = 2 \frac{h_{sh} \omega}{g} \cdot \frac{U_{fo} \cdot \sin \alpha + g \frac{l_s}{U_{fo}} \cos \alpha}{t_{sh}}\]

$l_s$ is obtained from equ. (6.4), $U_{fo}$ from equ. (6.6) and $h_{sh}$ from equ. (6.10).

In order to compute $h_{sh}$ according to equ. (6.13), it is necessary to know $l_a$, $h_{max}$, $h_{min}$, and $t_{sh}$. The difference ($h_{max} - h_{min}$) can be taken equal to $h_b$. In fig. 6.7 is plotted $h_{sh}$ vs. $t_{sh}$ as function of $l_a$, $h_{max}$, and $h_{min}$. It depends much more on $l_a$ than on $h_{max}$, and $h_{min}$. It has not been possible to establish any relation between $l_a$ and the wave characteristics. The rising time $t_{sh}$ can be evaluated only experimentally.

It is not possible to evaluate the experimental values of $p_{sh}$ and $t_{sh}$, because the recorded shock pressures were always superimposed on the pressures due to the water layer (see par. 6.1). Besides, recorded shock pressures are the values of $p_{sh}$ as they arrived at the transducers after being attenuated by the down-rushing layer.

6.2.3. Transmission of the shock pressure through the down-rushing water layer

The shock pressure-head $h_{sh}$ occurs on the surface of the down-rushing water layer. It propagates through the mass of the layer and arrives at the structure attenuated as $h_{sh}$. It is assumed for the moment that the layer is a one-phase liquid. A phenomenological description of the propagation of the shock wave could be based on the seismic methods used in applied geophysics.

The created shock wave, considered to be plane, propagates in a liquid and arrives at a plane boundary separating the liquid from a solid. It is partly reflected and partly transmitted. Compressive stresses are developed in the solid. In case of oblique incidence (in our case the angle of incidence is $\alpha$), shear stresses also appear [14]. Transmitted stress field in the solid
Fig. 6.6: Definition sketch of the water jet movement

Fig. 6.7: Pressure-head from the water jet impact
contains the so-called S and P waves:

S is a cubical dilatation wave moving with a velocity:

\[ C_S = \left\{ (\lambda + 2\mu)/\rho \right\}^{0.5} \]

where \( \lambda \) and \( \mu \) are the Lamé's constants of the medium defined as:

\[ \lambda = \frac{\sigma E}{(1 + \sigma)(1 - 2\sigma)} , \quad \mu = \frac{E}{2(1 + \sigma)} \]

where \( E \) is the Young's modulus and \( \sigma \) the Poisson's ratio of the medium. P is a rotational wave propagating with a velocity:

\[ C_P = \left( \frac{\mu}{\rho} \right)^{0.5} \]

Both S and P propagate through the whole mass of the medium and are attenuated roughly as \( x^{-1} \) where \( x \) is the distance from the epicentre.

Ergin [10] computed the reflection and transmission coefficients for a plane compressional wave incident on a fluid-solid boundary with \( \mu = \lambda \) for both of them. He used the Knott equations [14] which are based on a ray path progress of the front. According to these computations and for an angle of incidence equal to 9.46° the energy of the reflected P wave is about 65% of the incident energy. The energy of the transmitted P(S) wave in the structure is about 30% (6%) of the incident energy.

According to the above description, a transducer placed on the water-structure interface should record a pressure-time history with two peaks due to the transmitted P wave and to the incident shock wave itself.

It has been assumed that the shock wave from the impact is plane. This should be true if the two water surfaces were plane during the impact. It is more realistic to consider that small irregularities on both surfaces give a curvature to the front of the created shock wave. If that is true, besides the P and S waves a third type of wave will be created when the shock wave arrives to the structure: A Stoneley wave, which is a type of Rayleigh wave and propagates along the surface of an elastic solid. It arises from the diffraction of the curved front and attenuates roughly as \( x^{-\frac{1}{2}} \). Its velocity is much smaller than \( C_S \) or \( C_P \), which means that it will be recorded by the transducer after the transmitted P wave.

It is possible that a peak recorded by the transducers during the decay time (see par. 5) is due to a Stonely wave. This peak has many similarities
with the peak recorded by Roever and Vining [14] and attributed to a Stoneley wave.

Up to now it has been assumed that the water layer is a one-phase liquid. In reality, when the shock wave starts propagating the water layer has been already transformed into a mixture of water and air bubbles created during the impact. The air bubbles have different sizes and are randomly dispersed in the mixture.

A first effect of the air bubbles is to decrease the speed $C_m$ of the wave propagation in the mixture. For an isothermal process in an homogeneous medium $C_m$ is defined as:

$$C_m = \left( \frac{dp}{d\rho_m} \right)^{0.5}$$

where $p$ is the pressure and $\rho_m$ the mass density of the mixture. $C_m$ depends very much on the air volume fraction $\delta$ defined as the ratio of the air volume in a unit volume of mixture. For $\delta = 0$, $C_m$ is approximately equal to 1500 m/s and for $\delta = 1\%$ $C_m$ decreases to about 1 m/s.

A second effect of the air bubbles is an increase of the shock wave attenuation: A shock pressure propagating through a one-phase liquid is highly damped by diffusion. Damping is higher and more irregular when air bubbles are dispersed in the liquid due to reflections on the interfaces. According to Noordzij [23], an initial step function changes into an error function and finally disappears.

In order to evaluate the shock pressure which arrives at the transducers on the water-structure interface, it is necessary to know the mechanism of attenuation through the two-phase mixture. For this reason it will be tried to arrive at some results concerning the law of attenuation of a shock pressure through the down-rushing layer using the records of the three transducers. These results can be obtained by examining how some major shock pressures arrived at the three transducers. The problem is to isolate such shock pressures on the records from other superimposed minor shock pressures.

Unfortunately it is not possible to use the shock pressure from the impact because in most of the recorded pressure histories small parasitic pressures due to oscillations of air bubbles and the pressure due to the water height were superimposed on the main shock pressure. But many times the shocks from the initial compression of the air pocket were recorded by the three transducers without any other superposition. Some of them are shown in
in fig. 6.8 as they were recorded by the transducers, but their epicentres are not known.

The pressures in fig. 6.8 change from one transducer to the other. It is due to the attenuation. The law of attenuation could be obtained by fitting different physically meaningful curves to the sets of corresponding values and finding the one which should fit to all of them.

Coefficients in the mathematical expression of a curve, which fits to a number of sets of experimental points, are parameters showing different values according to the different sets of points. Best fitting is obtained when the curve represents well all the sets of points and the parameters are well defined and correlated with a small number of fundamental parameters.

In our case, the number of experimental points in each set is three. One of the parameters is related with the distance X from the epicentre of the shock pressure. Another parameter might be related with the maximum value shown by the shock pressure at the epicentre.

The pressures shown in fig. 6.8 were processed according to several mathematical expressions. Best fitting was obtained by

\[(6.14) \quad p = p_0 \exp (-\beta |X|)\]

Three parameters are involved in equ. (6.14): \(p_0\), which is the (theoretical) shock pressure at the epicentre of the shock on the structure and \(\beta\), which is the damping coefficient. Distance \(|X|\) is measured on the structure surface starting from the epicentre (see definition sketch in fig. 6.8).

In most of the experiments the epicentre of the shock was located shoreward of trans. 3. Sometimes it was located between trans. 3 and trans. 2.

Parameter \(\beta\) was found related to \(p_0\) (see fig. 6.9):

\[(6.15) \quad \beta = 0.0018 p_0 + 0.13\]

Coefficient 0.13 has dimensions \([L]^{-1}\). Coefficient 0.0018 has dimensions \([ML^{-1}T^{-2}]^{-1}\). The range of \(\beta\) was found to be quite narrow (from 0.16 to 0.32) although \(p_0\) varied between 15 and 107 cm w. Ambient pressure had no influence on \(\beta\).

Equ. (6.14) shows that a shock pressure decays exponentially with the distance in both directions on the structure surface. It is the type of decay shown in many physical processes.

Some values of the first peak \(P_1\) (see par. 5) of the recorded pressure-
time histories were processed according to the same law (6.14). $\beta$ showed the same dependence on $P_1$ (see fig. 6.10) but in a more dispersed way. The dispersion is attributed to the fact that $P_1$ is due partly to a shock pressure and partly to the height of the water layer. It is interesting to underline the influence of the ambient pressure on $\beta$:

- under vacuum conditions, $\beta$ was much less dispersed, +0.16 to +0.28 although the range of $p_0$ was also large: 35 to 110 cm w.

More generally speaking, equ. (6.14) represents the distribution over the surface of a structure of a shock or peak pressure applied against the structure at a certain point called epicentre. The values of pressure corresponding to the different points do not occur at the same moment: there are retardations due to the time necessary for the shock wave to reach the different points.

The damping coefficient $\beta$ was found quite well correlated with the pressure $p_0$ at the epicentre (equ. 6.15), although the water-air bubbles mixture was changing from one experiment to the other. It might be due to the fact that the correlation was based on shock pressures which occurred when the quantity of air bubbles had significantly decreased. A part of the dispersion of the values of $\beta$ vs $P_{sh}$ might be due to the considerable quantity of air bubbles during the impact.

Dickson [9] writes that an impulse wave behaves like a high frequency wave and that the energy attenuation constant $M$ per unit length of mixture is given by:

$$M = \frac{2A\delta(1 - \delta)}{C_m}$$

where $A$ is the area of air-water interface, $\delta$ is the volume ratio air-mixture and $C_m$ the speed of impulse waves in the mixture given by:

$$C_m = (kp)^{0.5}$$

where $k$ a constant depending on the mixture and $P$ the pressure.

The laws of decay of a shock pressure on a vertical wall according to Nagai and Minikin are reported in par. 2.

Some observations of Leendertse, Mitsuyasu and Weggel-Maxwell [36] indicate that the decay of a pressure against a wall is exponential.
trans. 3

*Fig. 6.8: Some shock pressures as they were recorded by the three transducers*

*Fig. 6.9: Correlation between the damping coefficient and the shock pressure*

*Fig. 6.10: Damping coefficient $\beta$ versus peak pressure $P_1$*
6.2.4. On the pressure on the structure due to shock pressure on the downrushing water layer

The shock pressure-head from the impact arrives at the structure as $h'_{sh}$. The records of the pressure of this study do not permit to evaluate the pressure histories on the structure from $h'_{sh}$. It may be deduced from pressure $I$.

As it is analysed in section 5, pressure $I$ is due to the superposition of the pressures on the structure from the shock developed during the impact and from the water layer over the structure. The pressure from the water layer is due to the height of the layer and to the accelerations of the water particles before the movement of the layer becomes parallel to the structure.

The experiments, as they have been carried out, do not permit a further decomposition of the pressure $I$: it would be possible only if the evolution of the water height over the transducers and the pressure due to the accelerations of the particles of every recorded wave were known. In such a case, the difference between pressure $I$ and the corresponding pressure from the water layer would represent the pressure on the structure from the shock pressure $h'_{sh}$.

Nevertheless, a rough idea of the pressure due to the shock can be obtained after assuming that the water height history of fig. 6.2 is a quite typical one and subtracting it from pressure $I$. Resulting pressures on the structure due to the shock pressure $h'_{sh}$ and to the accelerations for two recorded waves are shown in fig. 6.11 (small irregularities and oscillations in the diagrams have been omitted). They show a rising time to a peak corresponding to a compression of the structure when the shock wave arrives. After the peak, which is distributed according to equ. (6.14), pressure decreases to negative values during the decay time. In some cases they oscillate once or twice after the decay time (residual oscillation). In general a high damping is present due to the internal dissipation of energy.

The negative values shown by the pressure histories resulting from the subtractions are sometimes larger in absolute value than the positive peaks. This could never happen in reality if the histories were due to only the shock pressure. It is mostly due to high accelerations of water particles directed upwards and partly to the fact that the decrease of the loading water height due to the presence of the air pocket over the transducers has been neglected. In order to have a more realistic diagram the water
Fig. 6.11: Decomposition of pressure I
height must be reduced during the passage of the air pocket over a transducer to only the height of the water layer above and below the air pocket. This reduction would result in much lower negative values and maybe in higher positive ones.

6.3. Air pocket

6.3.1. Introduction

A major air pocket is entrapped in the water as follows: After the beginning of the wave breaking a water jet appears. It moves shorewards with an initial horizontal velocity $U_{fo}$ (see §6.1). Under the influence of the gravity and of $U_{fo}$ the water jet follows a nearly parabolic trajectory and strikes with the down-rushing water mass (see (1) and (2) of fig. 6.12). Between the plane surface of the down-rushing water mass and the parabolic surface of the water jet, air is entrapped. The seaward front of the air pocket is closed and nearly circular. The shoreward front shows an angular point (3), (4) from where small air bubbles escape during the first moments after the air pocket formation. An explanation of the air escape could be the following: the radius of curvature is smaller on the shoreward front of the air pocket than on the seaward one. The pressure $p_w$ due to the water layer above the air pocket is roughly the same over the two fronts. Continuity of normal stresses in the interface air/water makes that the pressure $p_a$ in the pocket is $p_a - p_w = \sigma_w/R$, where $\sigma_w$ is the surface tension of water and $R$ the radius of curvature. From this equation it is easily deduced that just after the air pocket formation, the air pressure near the shoreward front is higher than near the seaward front. This results in flow from the angular point, from where air bubbles escape. Due to the air escape and to the surface tension the air pocket changes rapidly section. It takes the form of a pear (5), then an elliptical section and finally a circular one (6), (7). Meanwhile the section decreases because of permanent escape of air by means of small air bubbles. The section remains circular up to the moment the whole air pocket disappears. The last moments of the air pressure are very difficult to follow.
Fig. 6.12: Formation of the air pocket during a wave breaking
The duration of existence of the air pocket is much higher under atmospheric pressure than under vacuum conditions.

The movements of the air pocket and the pressures due to them are studied in the following sections from the experimental data. A mathematical modeling of the pressures is also made.

6.3.2. Movements of the air pocket

Five main movements of the air pocket were detected in the films:

1. A continuous decrease of the section
2. A horizontal movement
3. A vertical movement
4. A rotation
5. A radial vibration.

The five movements are now analysed.

6.3.2.1. Decrease of the section

The air pocket section decreases from the first moment. To follow the evolution of the section on the films is a quite difficult task, especially during the last moments of existence: the interface air/water is very dim.

The decrease of the cross-sectional area $S$ of an air pocket under both atmospheric and vacuum conditions is shown in fig. 6.13, as it has been followed from the films: $S$ decreases slowly at the beginning and quickly at the end of the air pocket life. The scatter in the measured values of $S$ is due to the not-so-clear air/water interface. It might be also due to the radial oscillations of the air pocket, as will be described later. The air pocket life $t_{oa}$ is shorter under vacuum conditions.

The evolution of the air pocket diameter $D$ ($= 2R$) has been studied starting from the moment the air pocket showed circular section. In fig. 6.14 are shown the diameter-time histories corresponding to the air pockets of fig. 6.13. It can be distinguished that $D$ decreases nearly linearly in case of vacuum conditions.

Assuming the air pocket to show circular section right after it is formed, the experimental results shown in fig. 6.13 and 6.14 indicate that $S$ and $D$ decreases according to the following expressions:
where $S_o$ is the initial cross-sectional area, $D_o$ the initial value of $D$, corresponding with $S_o$, $v$ a constant initially equal to 1 (linear decrease I) and after that changing to 2 (second order decrease II). The duration of the linear decrease is equal to $t_I$ and the duration of the second order decrease is equal to $t_{II}$ where $t_{II} = t_{os} - t_I$. It is possible that $v$ increases furthermore at the end of the air pocket life. Coefficient $\lambda$ is defined as follows:

$$\lambda = \lambda_I = \{1 - (S_I/S_0)^{0.5}\}/t_I \quad \text{for} \quad 0 \leq t \leq t_I$$

$$\lambda = \lambda_{II} = 1/t_{II}^2 \quad \text{for} \quad t_I \leq t \leq t_{os}$$

where $S_I$ is the value of $S$ at $t = t_I$.

Substituting equ. (6.18) into equ. (6.16) and (6.17):

$$S = S_o(1 - \lambda_I t)^2 \quad \text{for} \quad 0 \leq t \leq t_I$$

$$S = S_o(1 - \lambda_I t)^2(1 - \lambda_{II}(t - t_I)^2)^2 \quad \text{for} \quad t_I \leq t \leq t_{os}$$

$$D = D_o(1 - \lambda_I t) \quad \text{for} \quad 0 \leq t \leq t_I$$

$$D = D_o(1 - \lambda_I t)(1 - \lambda_{II}(t - t_I)^2) \quad \text{for} \quad t_I \leq t \leq t_{os}$$

The experimental results of fig. 6.13 and 6.14 are represented accurately by equ. (6.19) and (6.20), with the following values of $t_I$ and $S_I$:

$$t_I = 0.12 \text{ sec} , \quad S_I = 23.6 \text{ cm}^2 \quad \text{under atmospheric conditions}$$

$$t_I = 0.07 \text{ sec} , \quad S_I = 13.4 \text{ cm}^2 \quad \text{under vacuum}.$$ 

Substitution of equ. (6.21) into equ. (6.18) yields:

$$\lambda_I = 1.17 \text{ sec}^{-1} , \quad \lambda_{II} = 75.61 \text{ sec}^{-2} \quad \text{under atmospheric conditions}$$

$$\lambda_I = 6.39 \text{ sec}^{-1} , \quad \lambda_{II} = 156.25 \text{ sec}^{-2} \quad \text{under vacuum}.$$
Equ. (6.19) and (6.20) with the values of $\lambda$ from equ. (6.22) are also plotted in fig. 6.13 and 6.14.

Equ. (6.19) and (6.20) combined with equ. (6.18) can be used to describe the evolution of $S$ and $D$. Both of them need the values of $S_o$, $S_i$, $t_i$ and $t_{os}$. It does not seem possible for the moment to evaluate $S_o$, $S_i$, $t_i$ and $t_{os}$ as functions of the wave characteristics, because the real mechanism of water/air exchange through the interface is not known.

Keeping the coefficient $v$ constant and equal to 1 or 2 during the entire existence of the air pocket would reduce the number of inputs from 4 to 2 ($t_{os}$ and $S_o$):

For $v = 1$, equ. (6.18), (6.19) and (6.20) are written as:

\[(6.18') \quad \lambda = 1/t_{os}\]
\[(6.19') \quad S = S_o(1 - \lambda t)^2 \quad \text{for } 0 \leq t \leq t_{os}\]
\[(6.20') \quad D = D_o(1 - \lambda t)\]

For $v = 2$:

\[(6.18''') \quad \lambda = 1/t_{os}^2\]
\[(6.19''') \quad S = S_o(1 - \lambda t^2)^2\]
\[(6.20''') \quad D = D_o(1 - \lambda t^2)^2\]

The two last sets of equations are also plotted in fig. 6.13 and fig. 6.14. Equ. (6.19') and (6.20') are much closer to the experimental results than equ. (6.19''') and (6.20''').

6.3.2.2. Horizontal movement

The bulk horizontal movement of the air pocket is due to the up-rushing water movement. The air pocket drifts shorewards. A rough approximation of the evolution of the up-rushing velocity could be obtained by following the movement of the air pocket.

In fig. 6.15 is shown the horizontal movement of the center of the air pocket studied in §6.3.2.1., as well as the horizontal movements of their
Fig. 6.13: Decrease of the cross-section of the air pocket (exper.)

Fig. 6.14: Decrease of the diameter of the air pocket (exper.)
seaward and shoreward fronts: which finally decrease to 0, at the end of the air pocket life. Under atmospheric conditions the initial constant value $U_c^{co}$ of $U_c$ is equal to about 95 cm/s and under vacuum it is about 88 cm/s.

It is noted that the center and the two fronts of the air pocket move with constant velocities during the period which corresponds more or less with the period of linear decrease of the air pocket section.

The seaward front of the air pocket moves faster than the center which moves faster than the shoreward front. It is due to the continuous decrease of the air pocket volume which results in an additional negative (positive) velocity $-|\dot{R}| (|\dot{R}|)$ to the shoreward (seaward) front of the air pocket.

The horizontal velocity $U_d (U_u)$ of the shoreward (seaward) front of the air pocket could be written as:

$U_d = U_c - |\dot{R}|$  \hspace{1cm} (6.23)

$U_u = U_c + |\dot{R}|$  \hspace{1cm} (6.24)

The difference between the constant values of $U_u$ and $U_d$ in fig. 6.15 was compared to $2|\dot{R}| (= D_o l)^2$: $(U_u - U_d)$ was found larger than $2|\dot{R}|$. This could be attributed to the transformation of the elliptical section of the air pocket to a circular one: the major axis decreases faster than the minor one. The decrease of the major axis results in an additional horizontal velocity to the seaward front.

6.3.2.3. Vertical movement

The air pocket moves upwards quite slowly. Apart from buoyancy, the vertical movement is possibly due to a lifting force known as Magnus effect. A way to study this vertical movement is to follow the evolution of the height $h_d$ of the water layer between the lower point of the air pocket and the structure. If there was no vertical movement, $h_d$ should decrease during the horizontal movement of the air pocket with the constant velocity $U_c^{co}$ according to the following expression:

$h_d = h_d^{o} - U_c^{co} t \tan \alpha + R_o \lambda_I t$ for $0 \leq t \leq t_I$ \hspace{1cm} (6.25)

where $h_d^{o}$ is the initial value of $h_d$. The second term in the second member is due to the horizontal movement of the air pocket and the third term due to the volume decrease.

In fig. 6.16 is shown the $h_d$-time history, as it appeared in the films. It refers to the air studied previously. Equ. (6.25) is also plotted. The
Fig. 6.15: Horizontal movement of the air pocket (exper.)
Fig. 6.16: Evolution of the height of the water layer below the air pocket (exper.)

Fig. 6.17: Rotation of the air pocket
shift between the experimental values of $h_d$ and equ. (6.25) is due to the upward movement of the air pocket.

6.3.2.4. Rotation

Before the breaking, the water particles of the down-rushing layer move seawards (see (1) in fig. 6.17).

During the breaking, the water particles of the down-rushing layer continue moving seawards and the water jet moves shorewards (see (2)).

The combination of movements of the water particles above and below the air pocket results in a rotation of the air pocket around its longitudinal axis (see (3) and (4)).

6.3.2.5. Radial vibrations

The pressure from the water layer and the shock perturbs the air pocket by compressing it. The initial compression results in a radial deformation. A radial expansion follows, etc. The successive radial compressions and expansions of the air pocket are called radial vibrations of the air pocket and result in oscillating pressures in the water mass. They propagate and load the structure with an oscillating pressure-time history (pressure II) which starts at $t = t_b$ (see fig. 6.18), and ends at $t = t_e$. The difference $t_e - t_b$ is called $t_{os}$.

Pressure oscillations do not appear under vacuum conditions either because they do not exist or because the frequency is very low.

The difference in pressure between a maximum (minimum) and the next minimum (maximum) value of the pressure is assumed to be the amplitude of the oscillations at the moment of the zero down (up) - crossing time between the two considered values. The difference in time between these two values is assumed to be equal to half of the period of oscillations and to correspond with the same zero.

Some conclusions concerning the amplitude and frequency of the air pocket vibrations can be based on the recorded oscillating pressures II, after assuming the process of transformation of the vibrations into pressure to be linear:

The amplitude of radial vibrations oscillates between maximum and minimum values (see fig. 6.19), while its mean value decreases with time. When the oscillating amplitude reaches the zero level for the first time, the air pocket disappears.
Fig. 6.18: Pressure oscillations due to the air pocket

Fig. 6.19: Amplitude of the pressure oscillations (exper.)

Fig. 6.20: Frequency of the pressure oscillations (exper.)
The frequency $f$ increases with time (see fig. 6.20) especially during the last moments, which means that $f$ increases with decreasing air pocket radius. Initial frequency $f_0$ depends on initial $R_0$; $f_0$ increases with decreasing $R_0$.

Oscillations last always longer at trans. 3 than at trans. 1 because the air pocket moves shorewards: it disappears closer to trans. 3 than to trans. 1.

6.3.3. Modelling of the air pocket

A modelling of the pressures due to the air pocket vibrations (pressures II) is now made assuming the air pocket to be cylindrical with circular section, starting at $t = 0$.

The radial decrease and oscillations are first studied.

A simulation of the flow pattern around the air pocket is then made.

Pressures are finally evaluated.

6.3.3.1. Air pocket radius

The radius of the air pocket oscillates. Let $R_0$ be the initial, $R_m$ the mean value and $R$ the instantaneous value of the radius.

$R_m$ decreases from $R_0$ at $t = 0$ to $0$ at $t = t_{os}$. The function $1 - R_m/R_0$ is continuously increasing from 0 to 1 and may be expanded into a power series of a small parameter, like $t/T$ for $0 < t < t_{os}$:

$$1 - \frac{R_m}{R_0} = \sum_{\mu=1}^{n} \lambda'_\mu \left(\frac{t}{T}\right)^\mu$$

or:

$$R_m = R_0 \left(1 - \sum_{\mu=1}^{n} \lambda'_\mu \left(\frac{t}{T}\right)^\mu\right)$$

The coefficients $\lambda'_\mu$ must be defined using some conditions. For this reason two conditions are now introduced:

The first condition is an initial one according to the observation that the section decreases very slowly at the beginning (see fig. 6.13).

It is assumed that the slope of $R_m(t)$ is equal to 0 at $t = 0$: $R_m = 0$ for $t = t_{os}$.

The second condition is a final one:
\[ R_m = 0 \text{ for } t = t_{os} \]

Ratio \( t/T \) remains small for \( 0 \leq t \leq t_{os} \): according to the filmed waves \( t_{os}/T \) was smaller than 0.15, which means that \((t/T)^n\) for \( n \geq 3 \) and \( 0 < t < t_{os} \) is too small compared to \( t/T \). The two conditions combined with equ. (6.26) permit to write:

\[
\begin{align*}
(6.26') \quad & \lambda_1 = 0 \\
& \lambda_2 = T^2/t_{os}^2
\end{align*}
\]

Using equ. (6.26) and (6.26'),

\[ R_m = R_0 (1 - t^2/t_{os}^2) \]

or:

\[ (6.27) \quad R_m = R_0 (1 - \lambda t^2) \]

where:

\[ \lambda = 1/t_{os}^2 \]

It is interesting to note that equ. (6.27) is identical to the empirically defined equ. (6.20'').

Radial vibrations start at \( t = t_b \). A compression due to the breaking wave gives the initial perturbation to the air pocket. After the initial perturbation the air pocket starts vibrating. It is assumed that the oscillating part of \( R \) is given by:

\[ (6.28) \quad R - R_m = \varepsilon(t') = A \exp (\phi t') = A \exp \{ \text{Re}(\phi t') \} \{ \cos \text{Im}(\phi t') + \\
+ i \sin \text{Im}(\phi t') \} \]

where \( t' = t - t_l \) the time, \( t_l \) the end of the initial compression, \( \phi \) a function of \( t' \) and \( A \) a constant depending on the breaking conditions and representing the decrease of the air pocket radius. The frequency \( f \) of oscillations is given by:

\[ (6.29) \quad f = \frac{\text{Im}(\phi)}{2\pi} \]
Amplitude and period of vibrations decrease continuously (see 6.3.2.5). Equ. (6.27) and (6.28) give:

\[ R = R_0 \left(1 - \lambda t^2\right) + \varepsilon(t') \]

\( \varepsilon(t') \) is assumed to be very small compared to \( R_m \):

\[ \varepsilon(t') \ll R_m \]

To first order in \( \varepsilon(t')/R_m \):

\[ \frac{1}{R} = \frac{1}{R_0 \left(1 - \lambda t^2\right)} \left\{1 - \frac{\varepsilon(t')}{R_0 \left(1 - \lambda t^2\right)}\right\} \]

\[ \frac{R_0}{R_0 + \varepsilon(t')} = 1 - \frac{\varepsilon(t')}{R_0} \quad \text{and} \quad \left(\frac{R_0}{R_0 + \varepsilon(t')}\right)^2 = 1 - 2 \frac{\varepsilon(t')}{R_0} \]

Above written relations will be used in the following sections.

6.3.3.2. Flow pattern

The flow pattern around the decreasing section of the air pocket is examined now. The influence of the vibrations will be introduced later.

The flow is composed of a shoreward uniform flow with a horizontal velocity due to the breaking wave and of a clockwise circulation around the air cylinder due to the combined effect of the seaward movement of the breaking wave. Described flow is similar to the two-dimension flow around a circular cylinder turning around its longitudinal axis. This flow can be simulated by the combined flow of a two-dimensional doublet with a free vortex in a uniform stream.

Let the axis \( x \) be in the direction of the uniform stream (positive shorewards), the axis \( y \) vertical (positive upwards), the origin of axis at the center of the circular section, \( \Gamma \) the circulation around the cylinder \( \mu \) the strength of the doublet inserted at the origin 0 with its axis in the \( x \)-direction and \( U_0 \) the horizontal velocity of the uniform flow in the direction of positive \( x \) (see following sketch).
If $U_c$ is the velocity of entrainment of the air pocket by the up-rushing water mass and the flow is assumed to be irrotational, the complex potential function of the superimposed flows is written as:

$$W = \phi + i\psi = (U_o - U_c)Z + \frac{\mu}{2\pi Z} + i \frac{\Gamma}{2\pi} \ln \frac{Z}{R_m}$$

where $Z(= x + iy)$ is a complex number, $\phi$ the velocity potential and $\psi$ the stream function:

$$\phi = (U_o - U_c) \left( x + i\left( \frac{\mu x}{2\pi(x^2 + y^2)} - \frac{\Gamma}{2\pi} \arctg \frac{y}{x} \right) \right)$$
$$\psi = (U_o - U_c) y - \frac{\mu y}{2\pi(x^2 + y^2)} + \frac{\Gamma}{2\pi} \ln \frac{(x^2 + y^2)^{0.5}}{R_m}$$

In writing the complex potential function it is assumed that the zero streamline is situated at a radius $R_m$, which means that:

$$\psi = 0 \text{ for } x^2 + y^2 = R_m^2$$

which is possible only when:

$$\mu = 2\pi(U_o - U_c)R_m$$

The stream function is now written as:

$$\psi = (U_o - U_c) y - (U_o - U_c) \frac{R_m^2}{(x^2 + y^2)^{0.5}} + \frac{\Gamma}{2\pi} \ln \frac{(x^2 + y^2)^{0.5}}{R_m}$$

If $U_w$ and $V_w$ are the horizontal and vertical components of velocity of the water:

$$U_w = -\psi'_x = -U_o + U_c + (U_o - U_c) \frac{R_m^2}{x^2 + y^2} \frac{x^2 - y^2}{(x^2 + y^2)^{0.5}} - \frac{\Gamma}{2\pi} \frac{y}{x^2 + y^2}$$
$$V_w = \psi'_x = (U_o - U_c) \frac{R_m^2}{(x^2 + y^2)^{0.5}} \frac{2xy}{x^2 + y^2} + \frac{\Gamma}{2\pi} \frac{x}{x^2 + y^2}$$

At a point $(x, y)$ in the water, situated on the interface water/air:

$$U_w = -U_o + U_c + (U_o - U_c) \frac{R_m^2}{x^2 + y^2} \frac{x^2 - y^2}{R_m^2} - \frac{\Gamma}{2\pi} \frac{y}{R_m^2}$$
$$V_w = (U_o - U_c) \frac{R_m^2}{x^2 + y^2} + \frac{\Gamma}{2\pi R_m^2} x$$
Total velocity at the same point \((x,y)\):

\[
(6.33) \quad V_w = \frac{R}{2\pi R_m} + \frac{1}{R_m} \left( \frac{2(U_o - U_c)}{y} \right)
\]

Equ. (6.33) will be used in the following sections.

### 6.3.3.3. Pressure

The pressures on both sides of the water/air interface are now examined:

The pressure on the interface from the side of the water is due partly to the flow examined in 6.3.3.2. and partly to the vibrations of the air pocket.

The pressure \(P_{wf}\) due to the flow at a point \(f\) on the interface is evaluated utilizing the Bernoulli-Euler equation:

\[
(6.34) \quad P_{wf} = \varepsilon_w h_{br} + p_{aa} - \varepsilon_w \zeta_f + \frac{\rho_w U_o^2}{2} - \frac{\rho_w V_w^2}{2} - \varepsilon_w (x_f - x_b) \tan \alpha
\]

where \(\varepsilon_w\) is the unit weight of water, \(\rho_w\) the mass density of water, \(g\) the gravitational acceleration, \(h_{br}\) the elevation of the water surface above the bottom at the breaking line, \(\zeta_f\) the elevation of the point \(f\) above the bottom and \(p_{aa}\) the pressure of the ambient air. The terms \(\varepsilon_w h_{br} + p_{aa} \frac{\rho_w U_o^2}{2}\) respectively denote the static and the dynamic pressure in the undisturbed stream at the breaking line. The elevation \(\zeta_f\) is written as:

\[
(6.35) \quad \zeta_f = h_f - h_u - Y - \varepsilon(t') \quad \text{with} \quad 0 \leq Y \leq 2R_m
\]

where \(h_f\) is the elevation of the water surface above the bottom at the point \(f\), \(h_u\) the height of the water layer above the highest point of the air pocket and \(Y(= - y + R_m)\) the vertical distance of the point \(f\) from the highest point of the air pocket (see following sketch).
Due to the radial vibrations a pressure $p_{wo}$ occurs on the interface. According to acoustic theory such a pressure on the surface of a pulsating sphere in infinite medium is \[6\\]:

\[
(6.36) \quad p_{wo} = \frac{\phi \rho_w R e(t')}{1 + \phi R_m/C_m}
\]

where $C_m$ is the speed of sound in the medium (air/water mixture). A similar relation is adopted in the present study on the interface. A corrective term must be introduced into the expression of the pressure on the interface. This correction $p_{wc}$ is necessary because in the examined pattern of flow the uniform stream has been considered with straight streamlines. In reality the streamlines are substantially curved, due to the gravity effect on the water jet. According to Chow \[7\\] the pressure correction on an interface with convex flow over it is negative and given by:

\[
(6.37) \quad p_{wc} = -\frac{\rho_w h V^2}{R_m}
\]

where $h_c$ is the height of the flowing water layer with curved streamlines above the interface. In the present case $h_c$ is the height of the water layer above the point $f$ and depends on $Y$: it shows a value equal to $h_u$ at the highest point of the air pocket and increases with $Y$:

\[
(6.38) \quad h_c = h_u + h_{cy}
\]

where $h_{cy}$ is the part of $h_c$ which depends on $Y$ and shows value equal to 0 for $y = 0$.

Concerning the distance $(x_f - x_b)$, it is written as:

\[
(6.38') \quad x_f - x_b = l_o + U_c t + (2Y R_m - Y^2)^{0.5}
\]

where $l_o$ is the horizontal distance between the center of the air pocket and the breaking line $x = x_b$ at $t = 0$.

Finally, the pressure $p_w$ from the water on the interface at the point $f$ is written as:

\[
(6.39) \quad p_w = p_{wf} + p_{wo} + p_{wc}
\]
Substitution into equ. (6.39) from equ. (6.33), (6.34), (6.35), (6.36),
(6.37), (6.38) and (6.38') yields:

\[
(6.40) \quad p_w = p_{aa} + \varepsilon_w \{h_{br} - h_f + h_u + Y + \varepsilon(t')\} + \\
+ \frac{\rho_w}{2} U_o^2 - \frac{\rho_w}{2} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 + \frac{\phi^2 \rho_w R \varepsilon(t')}{1 + \frac{\phi R_m}{C_m}} + \\
\frac{\rho_w (h_u + h_{cy})}{R_m} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 - \left( 1_o + U_c t + (2YR_m - Y^2)^{0.5} \right) \tan \alpha
\]

Assuming the air pocket to be entrained with a horizontal velocity \( U_o \),
equ. (6.41) is simplified to:

\[
(6.41) \quad p_w = p_{aa} + \varepsilon_w \{h_{br} - h_f + h_u + Y + \varepsilon(t')\} + \\
+ \frac{\rho_w}{2} U_o^2 - \frac{\rho_w}{2} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 + \frac{\phi^2 \rho_w R \varepsilon(t')}{1 + \frac{\phi R_m}{C_m}} + \\
\frac{\rho_w (h_u + h_{cy})}{R_m} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 - \left( 1_o + U_c t + (2YR_m - Y^2)^{0.5} \right) \tan \alpha
\]

The experiments showed that the air pocket takes and conserves a circular
section. Furthermore, the pressure of air in the interior of the air pocket
is uniform everywhere. These two statements and the expression of continuity
of normal stresses on an interface (see \( 6.3.1 \)) permit to conclude that
pressure \( p_w \) must be uniform everywhere around the air pocket. In other terms,
\( p_w \) must not depend on \( y \), which results in two equations:

\[
(6.42) \quad p_w = p_{aa} + \varepsilon_w \{h_{br} - h_f + h_u + \varepsilon(t')\} + \frac{\rho_w}{2} U_o^2 - \\
- \frac{\rho_w}{2} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 + \frac{\phi^2 \rho_w R \varepsilon(t')}^{1 + \frac{\phi R_m}{C_m}} - \frac{\rho_w h_u}{R_m} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 - \left( 1_o + U_c t \right) \tan \alpha
\]

\[
(6.43) \quad \varepsilon_w Y - \frac{\rho_w h_{cy}}{R_m} \left( \frac{\Gamma}{2 \pi R_m} \right)^2 - (2YR_m - Y^2)^{0.5} \tan \alpha = 0
\]

The first member of equ. (6.43) cannot be equal to a constant different
than 0 because \( h_{cy} \) must be equal to 0 for \( Y = 0 \). Equ. (6.43) is now written
as:
Pressure $p_a$ on the interface from the side of the air is evaluated as follows. It is assumed that the air is a perfect gas and the air pocket vibrations an isothermal process. The air pocket decrease is due to the formation of small air bubbles which is equivalent to a reduction of volume under constant pressure. It does not influence the pressure in the air pocket. Equation of state is written as:

$$P_a = P_{ao} \left( \frac{R_o}{R_o + \varepsilon(t')} \right)^2$$

where $P_{ao}$ is the air pressure at the moment of $R_m = R_o$.

Using eqn. (6.32):

$$p_a = P_{ao} \left( 1 - \frac{2\varepsilon(t')}{R_o} \right)$$

Pressures $p_w$ and $p_a$ are related between them by the relation of continuity of stresses on a curved interface (neglecting viscosity):

$$p_a = p_w + \frac{\sigma_w}{R}$$

where $\sigma_w$ is the surface tension of water.

Utilizing eqn. (6.45) $P_{ao}$ is written as:

$$p_{ao} = p_{aa} + \frac{\sigma_w}{R_o}$$

and eqn. (6.44) yields:

$$p_a = (p_{aa} + \frac{\sigma_w}{R_o}) \left( 1 - \frac{2\varepsilon(t')}{R_o} \right)$$

Substitution from eqn. (6.31), (6.42) and (6.47) into eqn. (6.45) yields:

$$p_{aa} = \frac{\rho_m}{2} \left( \frac{\Gamma}{2\pi R_m} \right)^2 + \frac{\rho_w}{2} \left( \frac{\Gamma}{2\pi R_m} \right)^2 + \frac{\phi^2 \rho_w R_m \varepsilon(t')}{1 + \frac{\phi R_m}{C_m}} - \frac{\rho_w u}{R_m} \left( \frac{\Gamma}{2\pi R_m} \right)^2 + \frac{\sigma_w}{R_m} \left( \frac{1 - \varepsilon(t')}{R_m} \right) - (l_o + U_c) \varepsilon(t') \tan \alpha$$
Circulation $\Gamma$ is independent of the oscillations. Equ. (6.48) must be valid even before the vibrations start. This statement results in a set of two equations:

\begin{equation}
(6.49) \quad - \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2 \varepsilon(t')}{R_0} = \varepsilon_w \varepsilon(t') + \frac{\phi^2}{\omega_m R} \varepsilon(t') - \frac{\sigma_w}{R} \varepsilon(t') \\
\frac{1 + \phi \frac{R}{C}}{R_m}
\end{equation}

\begin{equation}
(6.50) \quad \frac{\sigma_w}{R_0} = \varepsilon_w (h_{br} - h_f + h_u) + \frac{\omega_w}{2} \frac{U}{2} - \frac{c_w}{2} \left( \frac{\Gamma}{2\pi R_m} \right)^2 - \frac{\rho_w h_u}{R_m} \left( \frac{\Gamma}{2\pi R_m} \right)^2 + \\
+ \frac{\sigma_w}{R_m} - (1_o + U_c) \tan \alpha
\end{equation}

Equ. (6.49) gives information on the air pocket vibrations. It is written as:

\begin{equation}
(6.51) \quad \phi^2 \frac{R}{R_m} + \phi \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2 R_m}{R_0 C_m} + \frac{\varepsilon_w R_m}{C_m} - \frac{\sigma_w}{C R_m} + \\
+ \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2}{R_0} + \varepsilon_w - \frac{\sigma_w}{R_0} = 0
\end{equation}

and

\begin{equation}
(6.52) \quad \text{Im}[\phi] = \frac{- (p_{aa} + \frac{\sigma_w}{R_0}) \frac{2 R_m}{C_{mR}} + \frac{\varepsilon_w R_m}{C_{mR}} - \frac{\sigma_w}{C R_{mR}} + 4 \rho_w R \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2}{R_0} + \\
+ \frac{\sigma_w}{R_m} - \frac{\sigma_w}{R_0} \varepsilon_w \sqrt{0.5}}{2 p \frac{R_m}{R}}
\end{equation}

\begin{equation}
(6.53) \quad \text{Re}[\phi] = \frac{- (p_{aa} + \frac{\sigma_w}{R_0}) \frac{2 R_m}{C_{mR}} + \frac{\varepsilon_w R_m}{C_{mR}} - \frac{\sigma_w}{C R_{mR}} + 4 \rho_w R \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2}{R_0} + \\
+ \frac{\sigma_w}{R_m} - \frac{\sigma_w}{R_0} \varepsilon_w \sqrt{0.5}}{2 p \frac{R_m}{R}}
\end{equation}

According to equ. (6.29) the frequency is given by:

\begin{equation}
(6.54) \quad f = \frac{1}{4 \pi \rho_w R_m} \left\{ - (p_{aa} + \frac{\sigma_w}{R_0}) \frac{2 R_m}{C_{mR}} + \frac{\varepsilon_w R_m}{C_{mR}} - \frac{\sigma_w}{C R_{mR}} \right\} \frac{1}{2} + \\
+ 4 \rho_w R \left( p_{aa} + \frac{\sigma_w}{R_0} \right) \frac{2}{R_0} + \varepsilon_w - \frac{\sigma_w}{R_0} \sqrt{0.5}
\end{equation}

Initial frequency $f_o$ is written as:

\begin{equation}
(6.55) \quad f_o = \frac{1}{4 \pi \rho_w R_0} \left\{ - \frac{2 p_{aa}}{C_m} + \frac{\sigma_w}{C_{mR}} + \frac{\varepsilon_w R_0}{C_{mR}} \right\} \frac{1}{2} + 4 \rho_w \left( 2 p_{aa} + \frac{\sigma_w}{R_0} + \varepsilon_w R_0 \right) \sqrt{0.5}
\end{equation}
Fig. 6.21: Frequency of the air pocket vibrations (theor.)
Fig. 6.22: Initial frequency of the air pocket vibrations (theor.)

Fig. 6.23: Minimum speed of propagation of the sound in the down-rushing layer (theor.)
f depends mainly on $R_o$, $p_{aa}$, and $t$: it increases with decreasing $R_o$ and/or $p_{aa}$ and increases with increasing $t$, especially during the last moments of the air pocket life. The influence of $C_m$ on $f$ is smaller: $f$ decreases with decreasing $C_m$. The evolution of $f$ as function of time according to equ. (6.54) is shown in fig. 6.21. $R_o$ is taken equal to 3.2 (3.8 cm) under atm. pres. (vacuum): these values correspond with the air pockets examined in §6.3.2. Two values of $C_m$ are utilized: $10^4$ cm/s and $10^3$ cm/s. They do not influence the evolution of $f$ under vacuum. In fig. 6.21 is also plotted the experimental values of $f$ from fig. 6.20: the experimental values of $f$ become higher than the theoretical ones. Computed frequencies are very low under vacuum.

In fig. 6.22 are shown the values of $f_o$ according to equ. (6.55) as functions of $R_o$, $C_m$ and $p_{aa}$. Equ. (6.55) has a similarity with the expression proposed by Chapman and Pliss (6) for the natural frequency of a spherical air bubble of radius $R_o$ oscillating in the mass of a liquid, neglecting viscosity:

$$f_o = \frac{1}{2\pi R_o} \left( \frac{1}{\rho_1} \left( 3K\rho_o - \frac{2\sigma_0}{R_o} \right) \right)^{0.5}$$

where $\rho_1$ and $\sigma_1$ are the liquid density and surface tension, $K$ the polytropic exponent equal to 1 for an isothermal process and $\rho_o$ the mean ambient pressure in the liquid.

Equ. (6.56) with $p_o = p_{aa}$, $K = 1$ and $\rho_1 = \rho_w$ predicts higher initial frequencies $f_o$ than equ. (6.55) under both ambient pressures. In order to have vibrations of the air pocket, $C_m$ must be larger than a certain minimum value $C_{mo}$ defined from equ. (6.54) as follows:

$$C_{mo} = 0.5 \left( \frac{g(2h_{aa} + R_o)}{\rho_w R_o} + \frac{2\sigma_{w}}{\rho_w R_o} \right) \frac{R_m}{R_o} - \frac{\sigma_{w}}{\rho_w R_o} \cdot \frac{R_o}{R_m}^{0.5}$$

For a given value of $R_o$, $C_{mo}$ increases with $R_m$ (see fig. 6.23). This means that even if an air pocket does not vibrate right after it is formed, it will do so after a certain time, because of the continuous decrease of its radius. In other words, an air pocket in a water/air bubbles mixture containing a quantity of air bubbles larger than a certain one (corresponding with $C_{mo}$) will not vibrate unless the quantity of air bubbles and/or the radius of the air pocket is reduced to satisfy equ. (6.57). This statement is based on the dependence of $C_m$ on the quantity of air bubbles (see §6.2).
The damping of the oscillations expressed in logarithmic decrement $\Lambda$ is written as:

\begin{equation}
\Lambda = 2\pi \frac{\text{Re}\{\phi\}}{\text{Im}\{\phi\}}
\end{equation}

$\Lambda$ depends on $C_m$: it increases much with decreasing $C_m$.

Equ. (6.50) concerns the flow around the air pocket. The circulation is written as:

$$\Gamma = 2\pi R_m \left[ \frac{2\omega_w \left( \frac{1}{R_m} - \frac{1}{R_o} \right) + U_o^2 + g(h_{br} - h_f + h_u) - \frac{2(1 + U_c t)}{\rho_w} \tan \alpha}{1 + \frac{2h_u}{R_m}} \right]^{0.5}$$

The term in-between the brackets is equivalent to a circulation velocity $U_r$:

\begin{equation}
U_r = U_o^2 + g(h_{br} - h_f + h_u) - \frac{2(1 + U_c t)}{\rho_w} \tan \alpha + \frac{2\sigma_w}{\rho_w} \left( \frac{1}{R_m} - \frac{1}{R_o} \right)^{0.5} \left(1 + \frac{2h_u}{R_m}\right)^{-0.5}
\end{equation}

$U_r$ decreases initially with time and shows very large values during the last moments of the air pocket life. At $t = 0$:

$$U_{ro} = (U_o^2 + g(h_{br} - h_{fo} + h_{uo}) - \frac{21}{\rho_w} \tan \alpha)^{0.5} \left(1 + \frac{2h_{uo}}{R_o}\right)^{-0.5}$$

where $o$ indicates values at $t = 0$

$h_u$ depends on $t$. In Fig. 6.24 is shown the evolution of $h_u$ for the two air pockets examined in §6.3.2.1. It is not possible to check the obtained expression concerning $U_r$ because there are no available experimental data.

Each radial vibration of the air pocket creates a radial pressure pulse which propagates through the downrushing layer, arrives at the structure and loads it (pressure II). Pressure $p_{os}$ from the vibrations is written as:

$$p_{os} = p_a - \frac{\sigma_w}{R}$$
Substitution from equ. (6.31) and (6.47) yields:

\[(6.60) \quad p_{os} = \left(p_{aa} + \frac{\sigma_{w}}{R_o}\right)\left(1 - \frac{2\varepsilon(t')}{R_o}\right) - \frac{\sigma_{w}}{R_m} \frac{\varepsilon(t')}{R_m^2} = \]

\[= p_{aa} + \frac{\sigma_{w}}{R_o} - \frac{\sigma_{w}}{R_m} + \varepsilon(t') \cdot \left(\frac{1}{R_m^2} - \frac{2}{R_o^2}\right) - \frac{2p_{aa}}{R_o}\]

\[(t')\] is given by the real part of the second member of equ. (6.28):

\[(6.61) \quad \varepsilon(t') = A \cdot \exp\{\text{Re}\{\phi(t')\}\} \cdot \cos \text{Im}\{\phi(t')\}\]

\(a\) is the initial radial compression of the air pocket. It is due to the pressure on the air pocket during the rising time, between \(t = 0\) and \(t = t_{sh}\).
The initial compression provides the perturbation necessary to the air pocket to start vibrating. The shift of time between $t_{sh}$ and $t_b$ is equal to the time of travel of the compressing pressure to the air pocket and then to the transducer.

$a$ is evaluated from equ. (6.45) and (6.47) for $t' = 0(t = t_1)$. $p_w$ is substituted by $P'_1 + p_{aa}$, where $P'_1$ is the first peak of the pressure history, as it arrives at the air pocket:

$P'_1 + p_{aa} + \frac{\sigma_w}{R_m} = (p_{aa} + \frac{\sigma_w}{R_o})(1 + \frac{2a}{R_o})$ for $t = t_1$

The radial compression is considered to be negative. Equ. (6.62) is written as:

$A = \frac{P'_1}{p_{aa} + \frac{\sigma_w}{R_m}} \cdot \frac{\frac{R_o}{2}}{h_{aa}}$

or

$H'_1 = \frac{\frac{R_o}{2}}{h_{aa}}$

where $H'_1$ and $h_{aa}$ are the pressure-heads corresponding to $P'_1$ and $p_{aa}$.

Substituting equ. (6.61) and (6.63) into equ. (6.60) yields:

$p_{os} = p_{aa} + \frac{\sigma_w}{R_o} - \frac{\sigma_w}{R_m} + \frac{H'_1}{h_{aa}} \cdot \frac{\frac{R_o}{2}}{h_{aa}} \exp(\text{Re}(\Phi t')) \cdot \{(\frac{\frac{1}{2} - \frac{2}{2}}{R_m} - \frac{2p_{aa}}{R_o}) \cos \text{Im}(\Phi t')\}$

Equ. (6.64) needs $H'_1$, $C_m$ and $R_o$. In fig. 6.25 is shown the evolution of the second term of equ. (6.64) called $h_{os2}$. It concerns the air pocket examined in §6.3.2. $H'_1$ can be taken equal to $H_1$ (the first peak of the pressure-time history).

$h_{os}$ propagates through the water/air mixtures and arrives at the structure attenuated as $h'_{os}$.

7. TOTAL PRESSURE ON THE STRUCTURE

The pressure-head recorded by the transducers is a superposition of the pressure-heads due to the water-layer, to the shock from the impact as it arrives attenuated at the structure and to the air pocket vibrations.
Fig. 6.25: Amplitude of the pressure oscillations (theor.)
as it arrives attenuated at the structure.

According to the modelling made in §6 the following pressure heads can be obtained:

- Proposed empirical relations (6.1) and (6.3) for the pressure-head history from the water layer take into account the pressure-head from the water height without the second and third peaks. The pressure-head from the different accelerations of the water particles is not modelled, because there are no available data.

- Proposed mathematical modelling (equ. (6.64)) of the shock pressure-heads from the air pocket vibrations predicts the whole pressure-head history.

For the attenuation of the shock pressures during their propagation through the water/air bubbles mixture, expression (6.14) can be used.

Assuming the superposition of the three terms to be linear the total pressure-head \( h_{tr} \) on the structure is written as

\[
(7.1) \quad h_{tr} = h_s + h_{sh} \exp(-\beta d_{sh}) + h_{ap} \exp(-\beta d_{ap})
\]

where \( d_{sh} \) is the distance from the epicenter of the shock pressure from the impact and \( d_{ap} \) the distance from the lower part of the air pocket.

- The first term on the second member of equ. (7.1) exists during the whole wave period.
- The second term appears after \( d_{sh}/C_m \) from \( t = 0 \), time necessary to the shock wave to reach the structure.
- The third term appears after the air pocket has been compressed and the shock pressure from the first compression has arrived at the structure.

Equ. (7.1) needs the following inputs:

\( l, h_{max}, t_{mw}, l_s, h_{max,b}, h_{min,s}, t_{sh}, C_m, R_o, \beta, d_{sh}, d_{ap} \)

Their number can be reduced by the following considerations:

\( h_{max} \) is quite well evaluated by the equation of parabola between \( x_b \) and \( x_s \) (see §6.1.3)

\( t_{mw} \) can be taken equal to \( t_{sh} \), which may result in higher \( H_{sh} \).

\( h_{max,b} \) can be evaluated from one of the existing correlations between \( h_{max,b} \) and the wave characteristics

\( h_{min,s} \) is approximately equal to \( h_{min,b} \)

\( C_m \) can be taken equal to 1000 cm/s

\( \beta \) can be evaluated from equ. (6.15)
Furthermore, coefficient \( a \) was evaluated to be equal to 0.04 for the present experiments. Radius \( R_0 \) is related with the distance \( l_a \): the air pocket derives from the air entrapped between the down-rushing layer of length \( l_a \) and the parabolic trajectory of the water jet (see fig. 6.6).

In fig. 7.1 are shown indicatively the three terms of equ. (7.1) and the resulting \( h_{tr} \).

Fig. 7.1: Pressure-heads on the structure from the three factors
Part IV: PROBABILISTIC ANALYSIS OF THE PRESSURES ON THE STRUCTURE

8.1. Introduction

In this experimental study of the breaking wave pressures on a structure, several input waves were adopted, as has been stated in par. 3. Identical experiments were performed many times under atmospheric and vacuum conditions. Pressures were recorded by transducers placed in three different places on the structure.

Pressures recorded by the three transducers between \( t = 0 \) and the third peak of the water layer height continually differed from one experiment to the other although the input wave and the ambient pressure were the same: the pressure-time history is a stochastic process, as most of the loads on a structure. Nevertheless, pressures showed the same general trends in every experiment, because major parameters such as wave characteristics, structure parameters and ambient pressure were under control. But the values of the pressure and minor trends were random, because of fluctuations in variables not under control. Such variables are the interaction between the down-rushing water and the arriving wave, the impact due to the irregular configuration of the striking water surface, the air bubbles in the water mass and their fluctuations.

Finally, the process of the pressure formation at the breaking is a deterministic one in its general trends and a random one in its values. An approach to the deterministic features of the process is made. It seems to be the most realistic treatment for some characteristics of the pressure-time history.

In the statistical description that follows it is tried to determine the frequency with which a certain pressure situation occurs. Pressure responses from the three transducers were recorded in a time interval equivalent to approximately 60 wave periods for the three examined relative depths under atmospheric and vacuum conditions.

The chosen time interval is assumed to be sufficiently long, which means that a statistical description according to that interval is quite accurate. On the other hand, the time interval is considered to satisfy the following condition [4]: it is so short that the process within it may be considered to be a stationary one.

A process is called stationary when its statistical parameters (ensemble average, variance, probability distribution etc.) are independent of time.
The stationary character of the recorded process will be checked in a following section by calculating the statistical parameters and their evolution under a shift of the time scale.

The wideness of the pressure-time histories depends on the frequency of the used filter in the recording equipment:

- With a high-pass filter the histories are typical wide-band processes: between a zero-upcrossing and the next zero down-crossing sometimes many maxima and minima appear. In case of vacuum conditions the process becomes less wide-banded because the air-pocket oscillations disappear. The frequency of zero-level crossings is also random under atmospheric conditions.

- With a low-pass filter (~50 Hz) the processes change to narrow-band ones: one or two zero up-crossings, maxima and minima of the pressure appear in one wave period, depending on the location of the transducer and on the input wave. In such a case the pressure-time history corresponds more or less with pressure without the small irregularities.

The following characteristic values of the pressure-time histories are analysed statistically:

\[ P_1 \] which is the first peak of the pressure due to the shock and to the water layer.

\[ P_{\text{max}} \] which is the largest positive peak in one wave period

\[ P_{\text{min}} \] negative

\[ P_{\text{max}}, P_{\text{min}} \] and \[ P_1 \] are measured as pressure-heads.

The modelling method for \[ P_1 \] is the population of pressure within a record measured at the end of regularly recurring time intervals because the difference in time between two successive \[ P_1 \] is approximately one wave period: \[ P_1 \] appears after every impact of the jet created by the wave breaking. The wave period is the regularly occurring time interval.

Concerning \[ P_{\text{max}} \] (\[ P_{\text{min}} \]), the modelling method could be of the type maximum (minimum) value per wave cycle in statistically stationary processes.

8.2. Maximum recorded pressure in one wave period

In many of the recorded waves \[ P_{\text{max}} \] appears soon after \( t = 0 \). In these cases \[ P_{\text{max}} \] is due to the superposition of the pressures from the impact,
the water height, accelerations of the water particles and the initial compression of the air pocket (in case of atmospheric pressure only).

In the rest of the recorded waves, \( P_{\text{max}} \) appears some time after \( t = 0 \) and is random in value as well as in generating factors.

Some characteristic values of \( P_{\text{max}} \), with interesting significance are shown in table 8.1 and in fig. 8.1 and 8.2. They concern the three transducers, the two ambient pressures and the three relative depths examined. \( \bar{P}_{\text{max}} \) is the ensemble average of the population and \( P_{\text{max},s} \) the significant maximum pressure, defined as the ensemble average of the highest one-third fraction of \( P_{\text{max}} \) values. \( \text{max} P_{\text{max}} \) (\( \text{min} P_{\text{min}} \)) is the maximum (minimum) recorded \( P_{\text{max}} \) value.

The values of \( P_{\text{max}} \) for each combination of transducer and \( h_o/L_o \) become more scattered about the ensemble average as the ambient pressure decreases. In any case, \( \bar{P}_{\text{max}} \) and \( P_{\text{max},s} \) show larger values as the ambient pressure decreases. This could be due to the decreasing cushion effect of the entrapped air.

Concerning the influence of \( h_o/L_o \), it seems that \( P_{\text{max}} \) shows larger values as \( h_o/L_o \) decreases, especially in case of \( h_o/L_o = 0.08 \).

Trans. 1 recorded smaller values of \( P_{\text{max}} \) than the other two transducers. It is due to the fact that in many cases \( P_{\text{max}} \) is mainly due to the impact, which impact in most of the experiments was situated in the zone of the structure between trans. 3 and trans. 2.

Cumulative distributions of \( P_{\text{max}} \), for the three transducers, the two ambient pressures and the three relative depths examined are shown in fig. 8.3, 8.4 and 8.5 where \( F(P_{\text{max}}) \) is the probability of non-exceedence of the value \( P_{\text{max}} \). The experimental data indicate that \( F(P_{\text{max}}) \) is of the following form, mainly at trans. 1 and trans. 2

\[
F(P_{\text{max}}) = 1 - \exp \left[ -\frac{\pi}{4} \left( \frac{P_{\text{max}}}{\bar{P}_{\text{max}}} \right)^n \right]
\]

Coefficient \( n \) depends on \( h_o/L_o \) and the ambient pressure but does not show any dependence on the location of the transducer. It is found that \( n \) increases with decreasing ambient pressure and shows the following values under atmospheric (vacuum) conditions:

\[
\begin{align*}
\frac{h_o}{L_o} & = 0.19 & n = 3.2 (3.8) \\
0.14 & & 2.4 (3.5) \\
0.08 & & 2.7
\end{align*}
\]
In any case, \( n \) shows values superior than 2, which is the case of the Rayleigh distribution \([28]\). It is noted here that according to Rice and Longuet-Higgins the distribution of the maximum values of a narrow-band process is almost of the Rayleigh type.

Two other statistical parameters of importance are ratios \( P_{\text{max}} / \bar{P} \) and \( \max P_{\text{max}} / \bar{P}_{\text{max}} \):

- Ratio \( P_{\text{max}} / \bar{P} \) is found to increase with decreasing \( h_0 / L_0 \): for \( h_0 / L_0 = 0.19 \) it is equal to 1.23-1.33, for \( h_0 / L_0 = 0.14 \) to 1.30-1.43 and for \( h_0 / L_0 = 0.08 \) to 1.35-1.64. There is no net influence of the pressure of the ambient air.

- Ratio \( \max P_{\text{max}} / \bar{P}_{\text{max}} \), as recorded by the three transducers under atmospheric pressure, is in quite good agreement (maximum difference 11%) with the values, as computed by the following expression:

\[
\left( \frac{\max P_{\text{max}}}{\bar{P}_{\text{max}}} \right) = 1.20 \left( \ln k \right)^{0.5} + 0.25 \left( \ln k \right)^{-0.5}
\]

where \( k \) is the number of independent values of \( P_{\text{max}} \). In case of vacuum conditions, there are larger differences between recorded and computed values.

Some comments on the Rayleigh distribution need to be done at this point. It is well known that the Rayleigh distribution was applied successfully in some processes with sea waves: The distribution of the maximum positive sea-surface elevations \( n_{\text{max}} \) between a zero up-crossing and the next zero down-crossing is found to be quite well represented by a Rayleigh distribution, in case of narrow spectrum. According to Battjes \([3]\), the results of Koelé-de Bruyn and Titov show that the Rayleigh distribution for \( n_{\text{max}} \) is satisfying even for spectra with \( \nu = 0.63 \) to 0.77, where \( \nu \) is the narrowness of the spectrum (equal to 1 in case of narrow spectrum). Furthermore, the Rayleigh distribution is found also in good agreement with data concerning the wave height \( H \) (difference between maximum and minimum water surface elevation between two successive zero up-crossings) in case of narrow and less narrow spectra.

Two objections could be raised in comparing relations derived from the Rayleigh distribution to data from a process such as the pressure-time histories from the breaking of waves. They concern the two fundamental assumptions made in deriving the Rayleigh distribution: Gaussian process and narrow spectrum. By definition, a Gaussian process is the result of the superposition of a large number of independent random processes with no significant contribution by one of them. But pressure histories from the
breaking are results of the superposition of the pressures from the three loading factors mentioned earlier in this paper. Although these processes are random their number is not large. Furthermore, there is a certain dependence among them: the pressure from the striking water mass depends on the wave height at the breaking. On the same wave height depends also the radius of the air pocket on which depends the pressure from the air pocket oscillations.

Nevertheless, some of the results reported earlier in this section are quite close to theoretical results, as derived from the Rayleigh distribution: Ratio \( n_{\text{max}} / \bar{n}_{\text{max}} \) is found approximately equal to 1.59. Longuet-Higgins [19] derived a relation for the expected maximum value of \( H \) in a record containing \( k \) independent wave heights. He used the Rayleigh distribution supposing that \( k \) is quite large and arrived at the following probability of non-exceedence of \( H \):

\[
F(H) = \exp \left[ -k \exp \left( -\frac{\pi}{4} \left( \frac{H}{\bar{H}} \right)^2 \right) \right]
\]

From this distribution he evaluated the expected maximum value \( \max H \) in the sample of \( k \) wave heights:

\[
E(\max H) = 1.13 \left( \ln k \right)^{0.5} + 0.29 \left( \ln k \right)^{-0.5}
\]

where \( \bar{H} \) is the ensemble average of the values of \( H \).

Concerning the narrowness of the process, the pressure-time histories, as recorded by the transducers, are not narrowbanded, as analysed earlier in this paper. If only the value \( P_{\text{max}} \) in the interval between a crossing of the zero pressure level with positive slope (called zero up-crossing) and the next crossing of the same level with negative slope (called zero down-crossing). In such a case the number of zero up-crossings is equal to the number of \( P_{\text{max}} \), which means that the spectrum is narrow. The Rayleigh distribution has been derived after making the assumption that the spectrum is narrow. It is possible that the distribution is not critically depending on the narrowness of the spectrum. The same statement was made by Longuet-Higgins concerning the sea wave heights.

An important point to investigate now is the stationary character of \( P_{\text{max}} \). A random process is said to be stationary if its probability distribution is invariant under a shift of time[8]. This implies that the statistical averages based on the probability distribution (ensemble average, variance) are independent of time.
The probability distributions of $P_{\text{max}}$ for ensembles of records of 15 to 20 successive waves with a shift of the time scale equal to 5 wave periods are shown in fig. 8.6. It is at trans. 1 that they show the maximum invariance under the shift of the time. Concerning the influence of the ambient pressure, it seems that there is a larger shift under vacuum conditions.

Although the examined ensembles are not quite large it is concluded that $P_{\text{max}}$ is a quasi-stationary stochastic process.

8.3. MINIMUM RECORDED PRESSURE DURING ONE WAVE PERIOD ($P_{\text{min}}$)

In many of the recorded waves $P_{\text{min}}$ appears at $t = 0$ and is due to only the height of the down-rushing water layer, which is a rather deterministic process (see par. 6.1). In the rest of the recorded waves $P_{\text{min}}$ appears after $t = 0$ and is due to an expansion of the air pocket during the decay time of the pressure-time history I.

These observations prepare the reader to expect smaller scattering in the values of $P_{\text{min}}$ than in the values of $P_{\text{max}}$.

Certain characteristic statistical values of $P_{\text{min}}$ are shown in table 8.2, fig. 8.2 and 8.3. The values of $P_{\text{min}}$ are more scattered about the ensemble average in case of atmospheric conditions. It is due to the existence of an oscillating air pocket: the size and the place of formation are quasi-random which result in more scattered negative values in all the transducers.

The results concerning $P_{\text{min}}$ do not show any correlation with $h_o/L_o$. Ensemble averages and significant values increase from trans. 3 to trans. 1 because the height over the bottom of the downrushing water mass increases. Ensemble averages are close to the lowest height of water over the transducers, as it was measured on the films.

The cumulative distributions of $P_{\text{min}}$ are shown in fig. 8.3, 8.4 and 8.5. They are steeper in case of vacuum conditions and become almost vertical straight lines when $h_o'/L_o = 0.08$, because $P_{\text{min}}$ appears always at $t = 0$.

Neither Rayleigh distribution nor equ. (8.1) represent well $P_{\text{min}}$. It might be explained by the decreased stochastic character of $P_{\text{min}}$. For this reason the Rayleigh distribution was compared to the distribution of a more stochastic ensemble of $P_{\text{min}}$ values: This ensemble included the minimum recorded negative values which appeared after $t = 0$. It was found that the distribution of this ensemble was much closer to a Rayleigh distribution than the initial one but not as close as $P_{\text{max}}$. 
Ratio $P_{\text{min},s}/P_{\text{min}}$ shows values between 1.00 and 1.40.

Concerning the stationary character of $P_{\text{min}}$, it is in transducer 1 that the population of $P_{\text{min}}$ shows the minimum variance under a shift of time (see fig. 8.6).

8.4. FIRST PEAK OF THE PRESSURE HISTORY ($P_1$)

Pressure $P_1$ is due to the water height and the shock pressure of the water jet. It occurs more or less at the same time in the wave period which means that the part of $P_1$ due to the water height is almost a constant. The most stochastic process contributing to $P_1$ is the shock.

Characteristic values of $P_1$ are shown in table 8.3. A first conclusion is that the values of $P_1$ are more scattered than the values of $P_{\text{max}}$. The contrary should be more normal because of the larger number of processes superimposed in $P_{\text{max}}$. A second conclusion is that $P_1$ at transducer 1 is more scattered than at the other transducers and that scattering increases with decreasing $h_o/L_o$.

Concerning the influence of $h_o/L_o$ on $P_1$ at the three transducers: $P_1$ shows larger values at the transducer which is situated closer to the impact zone. In this respect, it can be said that in case of $h_o/L_o = 0.14$ the impact is situated between trans. 3 and trans. 2.

$P_1$ decreases with increasing ambient pressure. It is due to the higher cushion effect of the small air bubbles entrapped in the water jet front.

The distribution of equ. (8.1) is not close to the distribution of $P_1$, which is shown in fig. 8.7. Ratio $P_{1,s}/P_1$ shows values between 1.20 and 1.65. Equ. (8.1) predicts values not always close to the maximum recorded $P_1$. 
<table>
<thead>
<tr>
<th>exp</th>
<th>wave period s</th>
<th>water depth cm</th>
<th>water relat. depth</th>
<th>ambient pressure</th>
<th>ensemb. average</th>
<th>variance</th>
<th>minimum recorded press.</th>
<th>maximum recorded press.</th>
<th>significance press.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm. 20.9</td>
<td>7.4</td>
<td>11</td>
<td>51</td>
<td>27.2</td>
<td></td>
</tr>
<tr>
<td>VT7</td>
<td></td>
<td></td>
<td></td>
<td>vac. 19.0</td>
<td>6.0</td>
<td>5</td>
<td>34</td>
<td>24.6</td>
<td></td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm. 20.8</td>
<td>7.5</td>
<td>7</td>
<td>50</td>
<td>26.5</td>
<td></td>
</tr>
<tr>
<td>VT4</td>
<td></td>
<td></td>
<td></td>
<td>vac. 37.3</td>
<td>10.4</td>
<td>14</td>
<td>72</td>
<td>49.9</td>
<td></td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm. 52.0</td>
<td>20.2</td>
<td>18</td>
<td>110</td>
<td>78.0</td>
<td></td>
</tr>
<tr>
<td>VT12</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.1: Characteristic statistical values of $p_{max}$
<table>
<thead>
<tr>
<th>exp</th>
<th>wave period s</th>
<th>water depth cm</th>
<th>water relat. depth</th>
<th>ambient pressure</th>
<th>ensemb average</th>
<th>variance</th>
<th>minimum recorded press.</th>
<th>maximum recorded pressure</th>
<th>signif press.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm.</td>
<td>- 5.9</td>
<td>2.3</td>
<td>- 16</td>
<td>- 3</td>
<td>- 7.3</td>
</tr>
<tr>
<td>VT7</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 6.3</td>
<td>1.2</td>
<td>- 10</td>
<td>- 4</td>
<td>- 7.3</td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm.</td>
<td>- 7.5</td>
<td>1.7</td>
<td>- 14</td>
<td>- 4</td>
<td>- 8.1</td>
</tr>
<tr>
<td>VT4</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 7.4</td>
<td>1.7</td>
<td>- 17</td>
<td>- 4</td>
<td>- 8.1</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm.</td>
<td>- 7.5</td>
<td>0.5</td>
<td>- 10</td>
<td>- 6</td>
<td>- 8.2</td>
</tr>
<tr>
<td>VT12</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 7.5</td>
<td>0</td>
<td>- 7.5</td>
<td>- 7.5</td>
<td>- 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>exp</th>
<th>wave period s</th>
<th>water depth cm</th>
<th>water relat. depth</th>
<th>ambient pressure</th>
<th>ensemb average</th>
<th>variance</th>
<th>minimum recorded press.</th>
<th>maximum recorded pressure</th>
<th>signif press.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm.</td>
<td>- 6.8</td>
<td>1.9</td>
<td>- 13</td>
<td>- 4</td>
<td>- 8.2</td>
</tr>
<tr>
<td>VT7</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 6.0</td>
<td>1.6</td>
<td>- 12</td>
<td>- 5</td>
<td>- 9.6</td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm.</td>
<td>- 9.8</td>
<td>1.8</td>
<td>- 15</td>
<td>- 6</td>
<td>- 11.1</td>
</tr>
<tr>
<td>VT4</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 9.1</td>
<td>1.3</td>
<td>- 13</td>
<td>- 7</td>
<td>- 10.0</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm.</td>
<td>- 8.8</td>
<td>2.7</td>
<td>- 6</td>
<td>- 22</td>
<td>- 10.5</td>
</tr>
<tr>
<td>VT12</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 8</td>
<td>0</td>
<td>- 8</td>
<td>- 8</td>
<td>- 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>exp</th>
<th>wave period s</th>
<th>water depth cm</th>
<th>water relat. depth</th>
<th>ambient pressure</th>
<th>ensemb average</th>
<th>variance</th>
<th>minimum recorded press.</th>
<th>maximum recorded pressure</th>
<th>signif press.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm.</td>
<td>- 8.8</td>
<td>2.3</td>
<td>- 14</td>
<td>- 4</td>
<td>- 11.3</td>
</tr>
<tr>
<td>VT7</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 8.2</td>
<td>2.3</td>
<td>- 14</td>
<td>- 4</td>
<td>- 11.5</td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm.</td>
<td>- 9.1</td>
<td>2.7</td>
<td>- 20</td>
<td>- 4</td>
<td>- 10.3</td>
</tr>
<tr>
<td>VT4</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 9.2</td>
<td>1.6</td>
<td>- 13</td>
<td>- 5</td>
<td>- 10.2</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm.</td>
<td>- 12.6</td>
<td>4.6</td>
<td>- 6</td>
<td>- 26</td>
<td>- 17.0</td>
</tr>
<tr>
<td>VT12</td>
<td></td>
<td></td>
<td></td>
<td>vac.</td>
<td>- 8.5</td>
<td>0</td>
<td>- 8.5</td>
<td>- 8.5</td>
<td>- 8.5</td>
</tr>
</tbody>
</table>

Table 8.2: Characteristic statistical values of $P_{\text{min}}$
### Table 8.3: Characteristic statistical values of $P_1$

<table>
<thead>
<tr>
<th>Transducer 1</th>
<th>Wave Period (s)</th>
<th>Water Depth (cm)</th>
<th>Water Relat. Depth</th>
<th>Ambient Pressure</th>
<th>Ensemble Average</th>
<th>Variance</th>
<th>Minimum Recorded Pressure (atm)</th>
<th>Maximum Recorded Pressure (atm)</th>
<th>Significant Pressure (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm</td>
<td>14.4</td>
<td>7.8</td>
<td>0</td>
<td>38</td>
<td>21.8</td>
</tr>
<tr>
<td>VT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>vac</td>
<td>9.7</td>
<td>8.1</td>
<td>2</td>
<td>54</td>
<td>12.0</td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm</td>
<td>5.5</td>
<td>3.4</td>
<td>0</td>
<td>13</td>
<td>9.2</td>
</tr>
<tr>
<td>VT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>vac</td>
<td>5.6</td>
<td>5.5</td>
<td>0</td>
<td>27</td>
<td>8.6</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm</td>
<td>5.3</td>
<td>4.0</td>
<td>0</td>
<td>15</td>
<td>6.8</td>
</tr>
<tr>
<td>VT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>vac</td>
<td>11.1</td>
<td>6.3</td>
<td>1</td>
<td>27</td>
<td>14.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transducer 2</th>
<th>Wave Period (s)</th>
<th>Water Depth (cm)</th>
<th>Water Relat. Depth</th>
<th>Ambient Pressure</th>
<th>Ensemble Average</th>
<th>Variance</th>
<th>Minimum Recorded Pressure (atm)</th>
<th>Maximum Recorded Pressure (atm)</th>
<th>Significant Pressure (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm</td>
<td>16.4</td>
<td>7.2</td>
<td>6</td>
<td>49</td>
<td>22.9</td>
</tr>
<tr>
<td>VT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>vac</td>
<td>24.7</td>
<td>6.9</td>
<td>12</td>
<td>41</td>
<td>31.3</td>
</tr>
<tr>
<td>AT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm</td>
<td>14.4</td>
<td>6.1</td>
<td>0</td>
<td>27</td>
<td>20.5</td>
</tr>
<tr>
<td>VT4</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>vac</td>
<td>17.5</td>
<td>7.2</td>
<td>2</td>
<td>33</td>
<td>24.5</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm</td>
<td>21.2</td>
<td>13.0</td>
<td>0</td>
<td>58</td>
<td>27.5</td>
</tr>
<tr>
<td>VT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>vac</td>
<td>31.7</td>
<td>22.0</td>
<td>0</td>
<td>104</td>
<td>44.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transducer 3</th>
<th>Wave Period (s)</th>
<th>Water Depth (cm)</th>
<th>Water Relat. Depth</th>
<th>Ambient Pressure</th>
<th>Ensemble Average</th>
<th>Variance</th>
<th>Minimum Recorded Pressure (atm)</th>
<th>Maximum Recorded Pressure (atm)</th>
<th>Significant Pressure (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT7</td>
<td>1.30</td>
<td>41.00</td>
<td>0.19</td>
<td>atm</td>
<td>12.0</td>
<td>5.6</td>
<td>1</td>
<td>33</td>
<td>15.9</td>
</tr>
<tr>
<td>VT7</td>
<td>1.62</td>
<td>41.85</td>
<td>0.14</td>
<td>atm</td>
<td>16.7</td>
<td>4.2</td>
<td>5</td>
<td>32</td>
<td>20.1</td>
</tr>
<tr>
<td>AT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>atm</td>
<td>32.7</td>
<td>13.4</td>
<td>6</td>
<td>66</td>
<td>42.5</td>
</tr>
<tr>
<td>VT12</td>
<td>2.75</td>
<td>42.00</td>
<td>0.08</td>
<td>vac</td>
<td>42.5</td>
<td>26.9</td>
<td>7</td>
<td>102</td>
<td>62.1</td>
</tr>
</tbody>
</table>
Fig. 8.1: Characteristic statistical values of $P_{\text{max}}$ and $P_{\text{min}}$
Fig. 8.2: Significant values of $P_{\text{max}}$ and $P_{\text{min}}$
Fig. 8.3: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0/L_0 = 0.08$)
Fig. 8.4: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0/L_0 = 0.14$)
Fig. 8.5: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ ($h_0/L_0 = 0.19$)
Fig. 8.6: Cumulative distributions of $P_{\text{max}}$ and $P_{\text{min}}$ under a shift of the time scale.
Fig. 8.7: Cumulative distributions of $P_1$
9. CONCLUSIONS - FURTHER WORK

Wave breaking is a stochastic process because of the large number of parameters which influence the breaking and which are not under control. Pressure-time histories recorded in the breaking zone of a sloping structure are random in values and deterministic in form and general trends.

The analysis of a typical wave breaking and the corresponding pressure-time history shows that there are three factors which load the structure during the breaking:

- The water layer over the structure
- The shock due to the impact of the water jet from the breaking
- The air pocket entrapped in the water mass

The water layer loads the structure with the water height and acceleration of the water particles during the breaking.

The water layer height shows maximum value at the moment of the impact between the water jet from the breaking and the down-rushing layer on the structure. After this maximum value the height decreases to a minimum value, shown at the moment before the next breaking. That part of the pressure-time history is deterministic and modeled. Two minor peaks are also shown by the height: they are due to the reflection of the fallen water mass and to a solitary wave created by the fallen water mass.

Accelerations of the water particles influence also the pressure history. In order to model the pressure from these accelerations more research and data are needed.

The pressure of ambient air does not influence the pressure from the water layer.

The impact between the water jet from the breaking and the down-rushing layer or the structure results in a shock pressure. The shock pressure developed on the down-rushing layer surface propagates through the layer, which is a water/air bubbles mixture. The attenuation of the shock pressure through the mixture is found to be exponential and to depend on the distance from the epicentre of the shock.

Further research is needed on the shock pressure developed during the impact of the water jet with the down-rushing layer.
Ambient pressure influences considerably the shock pressure: small air bubbles on the water jet front have a cushion effect on the shock pressure which is higher under vacuum.

The air pocket which is entrapped in the water mass, has circular section. It moves shorewards and turns around its longitudinal axis.

The air pocket section decreases because of a mass exchange between air and water.

The pressure history on the structure from the air pocket has an oscillatory form due to radial vibrations of the air pocket. Period and amplitude of the oscillations decrease with decreasing air pocket radius.

A mathematical model is proposed for the air pocket vibrations and pressures.

Ambient pressure influences much the pressures from the air pocket: period and amplitude of oscillations of the pressure history decrease much with decreasing ambient pressure. There are practically no air pocket vibrations under vacuum.

Concerning the random characteristics of the pressure-time histories, statistical analysis of the highest positive and negative pressure peaks, as well as the first peak of the pressure is made.

The highest positive peak $P_{\text{max}}$ takes larger values with decreasing initial relative depth and is a quasi-stationary stochastic process. $P_{\text{max}}$ becomes more dispersed and less stationary when ambient pressure decreases.

The highest negative peak $P_{\text{min}}$ is much less random than $P_{\text{max}}$. There is no correlation between $P_{\text{min}}$ and initial relative depth. Cumulative distribution of $P_{\text{min}}$ becomes very steep under vacuum conditions.

The first peak of the pressure history increases with increasing ambient pressure.

Further research is needed on some other characteristic values of the pressure-time histories and on the total impulse from the breaking. There are indications that impulses can be scaled.

10. ACKNOWLEDGEMENTS

The author gratefully acknowledges the receipt of a Research Fellowship from the Delft University of Technology. Thanks are due to the Delft Hydraulics Laboratory and to the Board of Maritime Works for giving their
permission to use the data on impact pressures. Particular appreciation is expressed to Dr. J.A. Battjes of the Delft University of Technology for his comments on a draft of the present report. The writer gratefully acknowledges the long discussions on the subject of the present report with Mr. Th. van Doorn of the Delft Hydraulics Laboratory.
APPENDIX A1

PHOTOGRAPHICAL DESCRIPTION OF THE FLOW
IN THE BREAKING ZONE
$h_o/L_o = 0.14$ atm. pres.
APPENDIX A2

PRESSURE-TIME HISTORIES
\( \frac{h_0}{L_0} = 0.19 \) atm. pres.
$h_o/L_o = 0.19$ vacuum

trans. 3

trans. 3

trans. 3
$h_o/L_o = 0.08$ atm. pres.
APPENDIX A3

LIST OF SYMBOLS

- \( C_m \): speed of propagation of the shock wave in the water/air mixture
- \( C_w \): speed of propagation of the shock wave in the water
- \( C_{mo} \): minimum value of \( C_m \) for air pocket vibrations
- \( D \): diameter of the air pocket
- \( f \): frequency of vibrations of the air pocket
- \( f_o \): initial frequency of vibrations of the air pocket
- \( F \): force
- \( F(a) \): probability of non-exceedance of \( a \)
- \( g \): acceleration of gravity
- \( h \): height of the water layer over the sloping structure
- \( h_c \): height of the water layer with curved streamlines over the interface
- \( h_d \): height of the water layer between the structure and the lowest point of the air pocket
- \( h_{do} \): initial value of \( h_d \)
- \( h_i \): \((i = 1, 2, 3)\) height of the transducer \( i \) over the horizontal bottom of the flume
- \( h_{aa} \): pressure head due to the ambient air
- \( h_{br} \): height of the water layer over the structure at the breaking line
- \( h_{cy} \): part of \( h_c \) depending on the depth from the water surface
- \( h_{os}, h'_{os} \): pressure-head on the water mass (structure) due to the vibrating air pocket
- \( h_{sh}, (h'_{sh}) \): pressure-head on the water surface (structure) due to the shock from the water jet impact
- \( h_i \): \((i = 1, 2, 3)\) height of the still water level over the transducer \( i \)
- \( h_{tr} \): total pressure-head on the structure
- \( h_{sh} \): average height of the striking water mass
- \( h_{max}(h_{min}) \): maximum (minimum) value shown by \( h \) in one wave period
- \( h_{max,b}(h_{min,b}) \): maximum (minimum) value shown by \( h_{br} \) in one wave period at \( x_b \)
- \( h_{max,s}(h_{min,s}) \): maximum (minimum) value shown by \( h \) in one wave period at \( x_s \)
- \( H \): wave height
- \( H_b \): " " at \( x_b \)
- \( H_o \): " " over the horizontal bottom of the flume
- \( H_1(H'_1) \): first peak of the total pressure-head on the structure (as it arrives at the air pocket)
length occupied initially by the entrapped air

length of the breaking zone

length of the impact zone

\[ L_0 = \frac{\alpha T^2}{2} \] wave length over the horizontal bottom of the flume.

mass

maximum sea surface elevation from the still water level

significant value of a population of \( n_{\text{max}} \)

" " " " " " " " water "

averaged pressure due to the shock on the water surface at a moment during the impact

pressure due to the ambient air

value of \( p_a \) at the moment of the air pocket formation

pressure on the water/air interface due to the air pocket vibrations

maximum value of \( p_s \) reached at the end of the impact \( t_{\text{sh}} \)

pressure correction due to the convex flow around the air pocket vibrations

atmospheric pressure

first peak of the pressure-time history

largest positive peak of the pressure-time history

largest negative peak of the pressure-time history

instantaneous radius of the air pocket

initial radius of the air pocket

mean value of the oscillating radius of the air pocket

cross-sectional area of the air pocket

initial value of \( S \)

time scale

time at the beginning of the air pocket vibrations

time at the end of the air pocket vibrations

time at the beginning of the breaking

duration of the air pocket vibrations

duration of the shock from the water jet

duration at the first peak shown by the water layer height time history

time at the first peak shown by pressure \( \Pi \)

duration of the linear decrease of the air pocket section.

duration of the second order decrease of the air pocket section

wave period

horizontal, vertical velocity of the air pocket center

horizontal velocity of the shoreward, seaward front of the air pocket
$U_f, V_f$ horizontal, vertical velocity of the water jet from the breaking
$U_{co}$ initial constant value of $U_c$
$U_{fo}$ initial constant value of $U_f$
$U_o$ horizontal velocity of the uniform flow
$V_w(u_w, v_w)$ total (horizontal, vertical) velocity of the water particles on the water surface
$V_{sh}$ velocity at the impact
$W$ complex potential function
$x, y$ horizontal, vertical axis
$x_b$ horizontal distance between the breaking line and the toe of the structure
$x_{p,max}$ horizontal distance between the maximum water line on the structure and the toe of the structure
$x_{p,min}$ horizontal distance between the minimum water line on the structure and the toe of the structure
$X$ distance measured on the structure surface from the shock epicentre
$Y$ ($= R_m - y$) vertical distance measured from the highest point of the air pocket
$z$ complex number
$\alpha$ slope of the structure
$A$ initial decrease of the air pocket radius due to the initial compression
$\beta$ damping coefficient of the shock wave propagation in the mixture
$\Gamma$ circulation around the air pocket
$\varepsilon_w$ unit weight of the water
$\varepsilon(t')$ oscillating part of $R$
$E$ Young's modulus
$\zeta_f$ elevation of the point $f$ from the bottom
$\lambda, \lambda'$ coefficient of decrease of the air pocket section
$\lambda_1, \lambda_2, \lambda_3$ during linear (second) order decrease
$\lambda, \mu$ Lamé's constants
$\Lambda$ log. decrement of the air pocket radius
$\mu$ vortex strength
$\mu_w, \mu_v$ viscosities of water
$\nu$ narrowness of spectrum
$\rho$ mass density
$\rho_w$ mass density of water
APPENDIX A4
LIST OF REFERENCES


110

[16] KAMEL, A.M. << Water wave pressures on seawalls and breakwaters >> Research Report No 2-10, Feb. 1968, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mi.


