Morphodynamic Computations under Tidal Conditions

A Survey of Simplified Solution Techniques

December 1993

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Summary

In this report a model has been derived to simulate morphological behaviour under tidal conditions. The model is based on the set of equations of motion and mass-balance for both water and sediment. The study focused on the underlying assumption that the kinematic response of the water movement on the bottom variation is much stronger than the dynamic response. Application of the transport-field method (chapter 4) as suggested by De Vriend (1985) has been examined. The performed study is a next step towards a more efficient way of morphodynamic modelling and may contribute to a more thorough understanding of morphological models and processes in tidal regions.

Acknowledgement

This study is submitted as partial fulfilment of the requirements for the degree of Civil Engineer at Delft University of Technology, Faculty of Civil Engineering. I would like to express my gratitude to all those who supported me and especially to my thesis-committee, in particular Prof.dr.ir. M. de Vries, Dr.ir. Z.B. Wang and Dr. R.J. Fokkink for their assistance and patience and to Prof.dr.ir. H.J. de Vriend, whose suggestions were indispensable.

Delft, December 1993
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1 General introduction

1.1 Analysis

Morphological change is a result of interaction between water movement and bed topography. Time-dependent variations of riverbeds have been investigated throughout the years. De Vries (1959) has shown that to describe morphological processes in rivers the flow can be considered as quasi-steady (for moderate Froude numbers), since the adjustment of the bed level is much slower than the adjustment of the water level. In tidal rivers, differently from (most) rivers without tidal influence, the water discharge cannot be considered as quasi-steady, but is dominated by different harmonic constituents of the tidal motion, generated by the attractions of the moon and the sun on open waters.

Time-scale difference between the water movement (hours) and the morphological processes (several years) complicate mathematical modelling of tidal channels. Due to this time-scale range, real-time simulation of morphological change remains very difficult. In spite of improved hardware facilities the use of existing models for solving practical problems still is very expensive.

Several investigators have devoted much attention to the determination of the cross-sectional equilibrium state of estuaries. Most theories are based on empirical relations between cross-section and tidal prism (O’Brien, 1931) or other hydraulic parameters (mean-track velocity; Gerritsen, 1985). Van Velzen (1986) has shown an analytical solution to describe a dynamic equilibrium state under certain assumptions.

1.2 Morphodynamic modelling

Morphological change is a result of the interaction between the water movement and the topography of the loose bed. To describe this interaction and the time and space dependent morphological change mathematical models are used. One type of mathematical models are dynamic models using the complete hydrodynamic differential equations; the equations of mass-balance and motion for water and sediment.

These models consist of three modules: (fig. 1.1)

a. Hydraulic module
b. Sediment transport module
c. Bed-level change module

The morphological processes are modelled as an iterative process; Flow characteristics influence the topography (geometry), leading to different flow characteristics, etc.
fig. 1.1 General lay-out morphological computer models

1.3 Concept of the model

Morphodynamic models for tidal regions have been limited applied in practice. Two main causes are (Wang, 1992):

- Morphodynamic models are expensive. For non-tidal river problems only a quasi-steady flow field has to be computed, whereas for tidal regions the hydraulic module has to be run for a complete tidal period to be able to compute the net sediment transport rate.

- Knowledge on the behaviour of morphodynamic models in tidal regions is limited. The development of these models is still in an early stage.

The mathematical concept of simulation of (long-term) morphological processes under tidal conditions is based on two important assumptions:

a) The morphological time-scale is much larger than a tidal period. Bed-level evolution is small per tidal cycle.

b) Kinematic response (mathematical translation of the law of conservation of mass) of the water movement on the bottom variation is much stronger than the dynamic response (mathematical translation of the law of conservation of momentum).

These assumptions are based on observations in nature and by others gained experiences with existing (empirical) two-dimensional models.

To decrease the number of runs of the flow module (solving the set of hydrodynamic equations) two concepts are distinguished:
a) Averaging of differential equations

To improve the efficiency of the hydraulic module, Krol (1990) proposed a method utilizing asymptotic expansion to derive a consistent scheme for one-dimensional morphological tide-averaged computations. This method, based on an averaging method of differential equations, reduces the problem to a boundary-value problem. Fokkink (1992) implemented this semi-analytical approach and tested it for more realistic cases.

b) Continuity correction

The second assumption (regarding the response of the water movement on bed level variation) reduces the number of hydraulic computations with help of the continuity correction (current velocity * depth is invariant).

In case the continuity correction is implemented in the dynamic model, two main concepts are distinguished by De Vriend (1985).

1) Transport-field method (Chapter 4)

In this method the continuity correction is implemented explicitly. The velocity profile is adapted to the new bed level by means of the continuity correction. The derivative of the sediment transport rate (power law of the velocity) defines the bed-level evolution.

2) Bed-disturbance method (Chapter 6)

In this method the continuity equation for sediment has been reduced to an advection-diffusion equation with source term, taken into consideration the continuity equation of water.

The methods proposed by De Vriend (1985) are described and implemented in a numerical model and tested for one-dimensional cases (chapter 5), expecting to find mathematical approximations (simplifications) to improve computer efficiency in case of dynamic modelling under tidal conditions.
2 Physics of estuaries

2.1 Introduction

An estuary can be defined as a body of water connecting a source of fresh water with a tidal sea or basin and extends over the length of tidal action. An estuary considered as a body of water is bounded by four boundaries (Karssen and Wang, 1991):

- Upstream: river inflow
- Downstream: tidal motion, salt intrusion
- Water surface: wind generated waves and forces
- Bed: bed forms inducing resistance

The morphological behaviour is influenced by sediment influx from the river and the sediment exchange with the sea (Wang, 1989).

The compounded influences of factors involved, i.e. the complex geometry, the tidal flows and mixing induced by density differences makes estuarine behaviour a very difficult subject for analytical description.

2.2 Tidal motion

From mathematical point of view Isaac Newton has been the first who discussed the origin of tides, the impact of the complex relations between the celestial bodies and the sea. The tidal motion at sea influences the water movement in estuaries strongly. A fluctuating water level at sea causes an inward progressive tidal wave.

A tidal wave penetrating a river is affected by several factors; the convergence of the channel, reflection of energy by the sides and head of the estuary and frictional dissipation of energy caused by the shallow water. The mutual interference of the three is complex and consequently difficult to predict.

Distortion of the tide by friction and other non-linear interactions leads to tidal asymmetry. At tidal rivers with a certain river run-off this asymmetry causes the flood period to be shorter than the ebb period. Amongst others tidal asymmetry leads to an estuary which consists of a number of channels (flood and ebb channels). These channels (gullies) may be separated by shoals, but often interconnections occur. The characterization of these channels is mainly determined by the river run-off. Channels for which the total amount of water during ebb and flood is equal are neutral channels (Dronkers, 1964).
2.3 Geometry

The geometry of tidal rivers (or estuaries) is strongly influenced by the tidal motion. It can be shown by a rather simple analysis (De Vries, 1988) that due to tidal motion at sea the cross-sectional area of the river increases in downstream direction. In case the river banks are not fixed (which is most common in nature), tidal motion induces a trumpet-like shape estuary (fig. 2.1). If on the other hand the banks are fixed, depth will decrease landwards.

\[ \text{fig 2.1 Trumpet-like shape estuary} \]

2.4 Stratification

Density currents caused by variations in salinity generate additional currents. These additional currents may affect the flow pattern and hence the sediment transport, mainly the transport of fine sediments. The degree of mixing can be related to the ratio of energy dissipation and available power represented by the Richardson-Estuary Number \( (R_{iE}) \) (Fischer, 1976):

\[
R_{iE} = \frac{\Delta \rho \ g \ a \ Q_r}{\rho \ A \ u_i^3}
\]

\( \Delta \rho \) = density difference between salt and fresh water
\( a \) = water depth
\( Q_r \) = river run-off
\( A \) = cross-sectional area at entrance
\( u_i \) = characteristic current velocity of tidal motion
Based on this number well mixed, partially mixed and salt wedge estuaries can be distinguished. For prismatic estuaries:

Well mixed: \( Ri_E < 0.08 \)
Partially mixed: \( 0.08 < Ri_E < 0.8 \)
Salt wedge: \( Ri_E > 0.8 \)

In this study only well mixed estuaries are considered. In that case the influence of the gradient of the velocity in \( x \)-direction caused by the horizontal density difference is also neglected because of its minor importance compared to the gradient induced by the tidal motion.

2.5 Sediment transport

The sediment transport rate depends on the water motion (current and waves) and sediment characteristics (availability, mass density, grain size).

Bed material transport can be divided into two parts.

a) Bed load; the amount of material transported in a thin layer above the bottom is directly related to the local flow conditions.

b) Suspended load; the amount of material transported above the layer mentioned is kept in suspension by turbulence and is often described by an advection-diffusion equation.

Most existing transport formulas are based on empirical relations between decisive variables like (critical) shear stress, relative density and grain size. Well-known formulas are given by Meyer-Peter Müller (1948), Engelund Hansen (1967), Einstein Brown (1950) and Van Rijn (1984).
3 Dynamic model

3.1 Introduction

In order to simplify the complex analysis of morphodynamic computations some restrictions have been made:

- One-dimensional model. Cross-sectional averaged parameters are considered.
- Rate and direction of the sediment transport is related to local flow conditions.
- Only tidal action is considered. Hence, the influence of wind-driven waves and additional currents on the morphological evolution is not considered.
- The transport formula is a power law of the velocity and includes the effect of gravity near the bottom by a bottom-slope component ($\beta$).
- Non-cohesive uniform sediment is supposed to be present.
- Fixed banks are postulated.

3.2 Hydraulic module

3.2.1 Numerical model

To simulate the water movement in estuaries DUFLOW is used. DUFLOW is a computer package for the simulation of one-dimensional unsteady flow in branched open water courses, jointly developed by three organizations of The Netherlands:

IHE International Institute for Hydraulic and Environmental Engineering, Delft
DGW Tidal Waters Division, Ministry of Public Works, The Hague
TUD Faculty of Civil Engineering, Delft University of Technology, Delft

The model is based on the shallow water equations (De Saint-Venant) of continuity and momentum.

Equation of continuity

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (3.1)$$

Equation of momentum

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\alpha Q^2}{A_s} \right) + gA_s \frac{\partial h}{\partial x} + \frac{gQ|Q|}{C^2 A_s R} = 0 \quad (3.2)$$
\[ B = \text{width constant} \]

\[ h(x,t) = \text{water level} \]
\[ a(x,t) = \text{water depth} \]
\[ z(x,t) = \text{bed level} \]
\[ Q(x,t) = \text{discharge} \]
\[ s(x,t) = \text{sediment transport} \]

**fig. 3.1 Schematization of the estuary**

DUFLOW computes water levels and discharges at the nodes and velocities at the middle of the sections of the network.

According to the DUFLOW manual the equations are discretized using the implicit four-point Preissmann differencing-scheme (fig. 3.2).

Discretized equation of continuity

\[
B_j^{*} \frac{h_j^{n+1} - h_j^n}{\Delta t} + \frac{Q_j^{n+1} - Q_j^n}{\Delta x} = 0 \quad (3.3)
\]

Discretized equation of momentum

\[
\frac{Q_j^{n+1} - Q_j^n}{\Delta t} + \frac{[Q_j^n - Q_{j+1}^{n+1}]}{\Delta x} + gA_j^{*} \frac{h_j^{n+1} - h_j^n}{\Delta x} + g \frac{Q_j^{n+1} Q_{j+1}^{n+1}}{(C^2AR)_j^{*+1/2}} = 0 \quad (3.4)
\]

The set of equations is solved using a Gauss elimination.
3.2.2 Boundary conditions

The condition at the sea boundary is represented by harmonic terms, describing the tidal motion. Neglecting the $M_2$-constituent, this condition yields:

$$\eta(0,t) = \sum_{n=1}^{\infty} M_{2n} \cos(n \omega t - \theta_n)$$  \hspace{1cm} (3.5)

where

- $\eta$ = sea surface elevation above mean water level
- $M_{2n}$ = amplitude of $M_{2n}$ constituent
- $\omega$ = angular velocity: $2\pi/T$
- $T$ = tidal period of $M_2$ constituent
- $\theta_n$ = phase of $M_{2n}$ constituent

To solve the equations the discharge at the river boundary has to be known as a function of time

$$Q = Q(t)$$  \hspace{1cm} (3.6)
3.2.3 Numerical analysis

The length of time step and spatial step is determined by the conditions derived from the stability analysis. Stability analysis in terms of sinusoidal solutions is due to Von Neumann. Based on this analysis the amplification factor is derived (appendix I).

\[
\rho = \frac{e^{+y^2\theta(1-\theta)} - \frac{r\Delta t}{2} \pm \sqrt{\frac{y^2 + r^2\Delta t^2}{4}}}{e^{-y^2\theta^2}}
\]  

where

- \( \rho \) = amplification factor
- \( \epsilon \) = \( 1 + \theta r \Delta t \)
- \( y^2 \) = \( 4 \alpha^2 \beta^2 / \alpha^2 \)
- \( \sigma \) = \( \Delta t / \Delta x \) gA/B
- \( \alpha \) = \( e^\epsilon + 1 \)
- \( \beta \) = \( e^\epsilon - 1 \)
- \( \xi \) = \( k \Delta x \)
- \( k \) = \( 2\pi / L \)
- \( L \) = wavelength

Without friction \((r=0)\) and for tidal waves \((\xi \ll 1 \rightarrow \text{tg}\xi = \xi)\) the amplification factor reduces to

\[
\rho = \frac{1 - \sigma^2 \xi^2 \delta(1 - \theta) \pm \sqrt{-\sigma^2 \xi^2}}{1 + \theta^2 \sigma^2 \xi^2}
\]  

A general condition for numerical stability is that the amplification factor in absolute value should be less than unity.

\[
|\rho| \leq 1
\]

It shows that the implicit Preissmann method is unconditionally stable, provided the factor \( \theta \) is chosen in the interval \( \frac{1}{2} \leq \theta \leq 1 \).

The numerical parameters (spatial step and time step) are derived by estimating the accuracy with which long waves are reproduced numerically. In case there is no bottom friction \((r=0)\) the damping factor (the ratio of the amplitudes of the analytic solution (Vreugdenhil, 1989) and numerical solution after \( n \) time steps) can be computed by

\[
d(n) = |\rho|^n
\]

where

- \( d(n) \) = damping factor
- \( n \) = \( T/\Delta t \)
The damping factor is computed for different values of $\theta$ and $n$ (fig. 3.3). To have an accurate solution the damping factor should be near unity. For $\theta = 0.55$ numerical damping is represented accurately by $\Delta t/T \leq 0.03$. The spatial step is limited by the representation of the tidal wave; $\Delta x/L \leq 1/25$.

![Numerical Amplitude](image)

**fig 3.3 Numerical damping**

### 3.3 Sediment transport module

At this stage no specific transport formula is chosen. The transport formula depends on the local flow conditions and is represented by a power law of the current velocity and the down slope effect of the bed.

$$s = m u^n \left(1 - \beta \frac{\partial z}{\partial x}\right)$$  \hspace{1cm} (3.11)
The down-slope coefficient ($\beta$) influences the magnitude of the transport rate; it might be clear that sediment moves easier downhill than uphill. The down-slope coefficient can be regarded as a diffusion coefficient. The $\beta$-coefficient can be approximated by a (constant) theoretical value of $0.05C^2/g$ according to Struiksma (1988). For river problems this yields $\beta = 10$.

The net sediment-transport ($s_{\text{net}}$) is computed by averaging the sediment transport over one tidal period

$$s_{\text{net}} = \frac{1}{T} \int_0^T s \, dt$$  \hspace{1cm} (3.12)

The integral equation of the net sediment transport is approximated applying the trapezoidal rule

$$s_{\text{net}} = \frac{1}{2} \sum_{i=0}^{T-1} \frac{1}{2} \Delta t [s(i \Delta t) + s((i+1) \Delta t)]$$  \hspace{1cm} (3.13)

3.4 Bed-level change module

3.4.1 Numerical model

Time dependent variation of the bottom level can be obtained by the continuity equation of sediment

$$\frac{\partial z}{\partial t} + \frac{\partial s}{\partial x} = 0$$  \hspace{1cm} (3.14)

Because of the small celerity of the propagation of disturbances, equation (3.14) can be discretized using an explicit method. The equation is discretized using the modified Lax-scheme (fig. 3.4).

$$z_j^{n+1} - \frac{1}{2} \left[ \alpha (z_{j+1}^n + z_{j-1}^n) + (1 - \alpha) z_j^n \right] \Delta t + \frac{1}{2} \left( s_k^n - s_{k-1}^n \right) = 0$$  \hspace{1cm} (3.15)

where
- $\alpha$ = Lax parameter
- $j$ = nodal number
- $k$ = section number
3.4.2 Boundary conditions

To be able to predict the time-related evolution of the bed level, it is necessary to know initial conditions and boundary conditions at both ends of the model. The initial conditions are given by the bed level at each node.

The number of boundary conditions equals the number of incoming characteristics. In the described test computations the condition prescribed at the incoming characteristic boundary yields: (also section 5.2 and 5.3)

\[ z(t) = \text{constant} \]  

(3.16)

However, at the other boundary the modified Lax-scheme cannot be used. At this boundary a weak boundary condition is used, setting the second order derivative with respect to time and space equal to zero.

\[ \frac{\partial^2 z}{\partial t \partial x} = 0 \]  

(3.17)

or

\[ \frac{\partial z}{\partial x} = \text{constant}(t) \]  

(3.18)

In numerical form

\[ z_{j}^{n+1} = z_{j}^{n} + (z_{j+1}^{n+1} - z_{j-1}^{n+1}) \]  

(3.19)
3.4.3 Numerical Analysis

An advection-diffusion equation can be derived by combining eqs. 3.11 and 3.14 (appendix II).

$$\frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} - D \frac{\partial^2 z}{\partial x^2} = 0$$  \hfill (3.20)

The stability analysis is again based on the Von Neumann analysis. Applying the Von Neumann analysis to this equation, the amplification factor becomes (appendix II)

$$\rho = 1 + \sigma (\cos \xi - 1) + \lambda (\cos \xi - 1) - \sigma \sin \xi$$  \hfill (3.21)

where

- $\sigma = c\Delta t/\Delta x$
- $c = \rho s_{net}/\alpha_0$ \hspace{1em} ($s_{net}$: net sediment transport)
- $\lambda = 2D\Delta t/\Delta x^2$
- $D = \beta s_{tot}$ \hspace{1em} ($s_{tot}$: total ebb and flood transport)

Taken into account the general condition for numerical stability

$$|\rho| \leq 1$$  \hfill (3.22)

it is shown (appendix II) that this method is stable provided:

$$\sigma^2 \leq \alpha + \lambda \leq 1$$  \hfill (3.23)

This condition provides a relation between $\Delta t$ en $\Delta x$. The morphological time-step can be derived taken into account the spatial step of the hydraulic module.
4 Transport-field method

4.1 Introduction

The different alternatives based upon this concept have in common that adjustments by way of continuity correction of the water movement are discounted in the transport module. The bed level change is computed using the bed level change module.

The general outline of this concept is represented in fig. 4.1.

![Flow diagram transport field method](image)

4.2 Extrapolation in Time

The first concept which is discussed is also the most simplified. The influence of the bed-level variation on the water movement is not taken into account for a number of successive tides, neither into the continuity equation nor into the momentum equation of the hydraulic module.

The sediment transport rate is computed, assuming the bed to be fixed. The bed evolution is derived from the continuity equation for sediment (eq. 3.14).

\[
\frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} = 0
\]  

(3.14)
Using the general concept (shown in fig. 4.1) the bed level evolution can be computed by integrating the continuity equation over the morphological time-interval \((\Delta t_m)\).

\[
    z|_{t_m} - z|_0 = - \int_{t_0}^{t_m} \frac{\partial \delta}{\partial x} dt
\]  

(4.1)

To compute the net effect of the tidal motion on the bed evolution it is possible to integrate the sediment transport over the time-interval in which one is interested. The bed evolution is governed by computation of the derivative of the sediment transport rate.

\[
    z|_{t_m} - z|_0 = - \frac{\partial}{\partial x} \int_{t_0}^{t_m} s dt
\]  

(4.2)

Assumed the tidal motion is quasi-periodic, the bed level variation for \(N\) tidal periods can be derived

\[
    z|_{t_m} - z|_0 = - N \left[ \frac{\partial}{\partial x} \int_0^T s dt \right]
\]  

(4.3)

Here \(\Delta t_m = t_m - t_0 = NT\)

The integral of the sediment transport is approximated by means of the trapezoidal rule. In numerical form the bed evolution can be written as

\[
    \Delta z = - \frac{N \Delta t}{2} \sum_{i=0}^{L-1} \frac{s(i \Delta t) + s((i+1) \Delta t)}{\Delta x}
\]  

(4.4)

The adapted bed level is used to compute the velocity field by means of the continuity correction

\[
    u(\Delta t_m + t_0)[h(t_0) - z(\Delta t_m + t_0)] = u(t_0)[h(t_0) - z(t_0)]
\]  

(4.5)

The velocity field is used to derive the transport rate for the time-interval \(NT \leq t \leq (N+1)T\). The transport gradient is used to compute the bed level evolution etc. (fig 4.2).
4.2 Extrapolation in time

The number \(N\) of successive tides is determined by the desired accuracy and the time-interval in which the morphological computation is executed. This method will also be referred to as the Euler Method.

4.3 Predictor-corrector method

To achieve higher accuracy a predictor-corrector method can be proposed instead of extrapolation in time. As shown extrapolation in time can be regarded as a method explicit in time. Only the bed-level evolution of the preceding time-interval is taken into account.

The method described here computes the bed evolution based on a multi-step method like Heun or Runge-Kutta. Figure 4.3 shows a graphical representation of this method.
From the continuity equation for sediment (eq. 3.14) it follows

$$ z|_{t_n} = z|_{t_0} - \frac{\partial}{\partial x} \int_{t_0}^{t_n} s(t, z(t)) \, dt $$

(4.6)

In case the Heun-method is used the new bed level can be derived based on the *trapezoidal rule*

$$ z|_{t_n} = z|_{t_0} - \frac{\partial}{\partial x} \left[ \int_{t_0}^{t_n} s(z(t)) \, dt \right] = z|_{t_0} - \frac{1}{2} N \left[ s_{\text{net}}(0) + s_{\text{net}}(NT) \right] $$

(4.7)

if $t_0 = 0$ and again $\Delta t_n = NT$.

The net transport rate $s(NT)$ is a function of $u(NT)$ and $z(NT)$. The unknown $z(NT)$ is predicted by $z^E(NT)$ by means of the *Euler Method*:

$$ z^E(NT) = z(0) - NT \frac{\partial}{\partial x} s_{\text{net}}(0) $$

(4.8)

The velocity field ($u(NT)$) is computed by means of the continuity equation

$$ u(NT) [h(0) - z^E(NT)] = u(0) [h(0) - z(0)] $$

(4.9)

The derived sediment transport rate $s^E(NT)$ is substituted in eq. 4.7 yielding

$$ z|_{t_n} = z|_{t_0} - \frac{1}{2} N \frac{\partial}{\partial x} \left[ s_{\text{net}}(0) + s_{\text{net}}^E(NT) \right] $$

(4.10)

In other words the bed evolution for one morphological time-interval is computed by centring in time the predicted solutions for $t = NT$ and $t = 2NT$

$$ z'(NT) = \frac{z(0) + z(2NT)}{2} $$

(4.11)

The bed level obtained is now used to derive the bed-level evolution for the proceeding time-interval ($NT \leq t \leq 2NT$), using the same concept.
5 Test computations

5.1 Introduction

The different solution techniques mentioned in chapter 4 remain to be tested. In this chapter two schematic cases are considered: a tidal basin (section 5.2) and a tidal river (section 5.3). The purpose of these computations is to examine the difference in morphological behaviour by means of the hydrodynamic equation and the continuity correction.

5.2 Tidal Basin

5.2.1 Basin lay-out

The first series of computations concern a simple one-dimensional case. A tidal basin is schematised to be a rectangular channel of length \( L \) and a constant width. Initially with a constant bed level with respect to the reference level. The basin is closed at one end. The other end is connected to the open sea.

As explained before (chapter 3) only tidal action is considered.

5.2.2 Boundary conditions

Hydraulic module

At the closed boundary no flux is present. The boundary condition is given by

\[
Q(L, t) = 0
\]
The condition at the sea boundary is described by a semi-diurnal ($M_2$) vertical tide. The amplitude of the $M_2$ tide is by far the largest of the astronomical tidal components (Dronkers, 1986). The net transport per unit of wave energy produced by the semi-diurnal tide is the largest (Seelig and Sorensen, 1978). Overtides (tidal constituents of which the angular speed is an exact multiple of the astronomical constituent) caused by propagation of the tidal wave into shallower waters are generated by this constituent. The influence of the overtides as well as that of the tidal cycle (neap-spring tide) is not considered yet.

Boundary condition sea level:

$$ h(x,t) = h_0 + M_2 \cos(\omega t - \theta) $$

(5.2)

Numerical parameters of the hydraulic module:

- **Spatial step:** $\Delta x = 500$ [m]
- **Initial bed level:** $z = 1$ [m]
- **Time step:** $\Delta t = 600$ [s]
- **Amplitude sea boundary:** $M_2 = 2$ [m]
- **Length basin:** $L = 10,000$ [m]
- **Tidal phase:** $\theta = 90^\circ$
- **Width basin:** $B = 1,000$ [m]
- **Angular velocity:** $\omega = 12.5$ [hours]
- **Mean water level:** $h_0 = 6$ [m]
- **Chézy coefficient:** $C = 50$ [m$^{1/2}$/s]

The Courant number $((ga)^{1/2} \Delta t/\Delta x)$ is smaller than 10, which is small enough to have an accurate tidal flow computation.

The computational results of DUFLOW (fig. 5.2) are used for input data of the transport and the bed level module.
Discharge and water level curve (beginning, middle, and end) section.

23
Transport and Bed-level module

The transport rate is proportional to the velocity raised to some power \( p (p \geq 3, \text{Engelund-Hanssen}; p = 5) \).

Numerical parameters:

Grain size: \( D = 200 \mu [m] \)

Bed-slope coefficient: \( \beta = 100 \)

Spatial step: \( \Delta x = 500 [m] \)

The net transport (fig. 5.3) and the ratio \( R \) (net transport \( s_{\text{flood}} - s_{\text{ebb}} \) divided by total transport \( s_{\text{flood}} + s_{\text{ebb}} \), fig. 5.4) indicate flood dominance (positive value). However, since the values of \( R \) are much smaller than unity, it obviously is not strongly flood-dominated. Flood dominance results in a net-inward transport, causing the basin to shoal on a larger time-scale.

**fig. 5.3** Net transport per section per tidal cycle

**fig. 5.4** \( R \)-ratio per section per tidal cycle
Although the boundary condition is symmetric and the length of the basin is too short to cause significance distortion of the tidal wave the resulting transport is inward. The minor flood dominance is a result of the internally generated overtides due to non-linear terms of the momentum equation and the phase lag between discharge and water depth (Wang, 1992).

The conditions at both ends for the bed-level module depends on the direction of the net transport. Net transport inwards means that there is one incoming characteristic, at the sea boundary. At this end the bed level is fixed

\[ z(0, t) = z(0, 0) = \text{constant} \]  
(5.3)

The Lax-scheme cannot be applied at the closed boundary. Instead an adapted scheme is used, setting the second order derivative with respect to time and space equal to zero.

\[ \frac{\partial^2 z}{\partial t \partial x} = 0 \quad \text{or} \quad z_{j}^{n+1} = z_{j}^{n} + (z_{j-1}^{n} - z_{j+1}^{n}) \]  
(5.4)

The morphological time-step \((\Delta t_m)\) is stipulated by the stability condition of the explicit numerical scheme

\[ \sigma^2 \leq \alpha + \lambda \leq 1 \]  
(5.5)

Diffusion plays a significant role in both the stability and accuracy analysis. Accuracy is investigated by determining the truncation error. The truncation error can be derived by substituting the solution of the differential equation into the difference equation. Developing the Lax-scheme into Taylor series, numerical diffusion \((D_{num} = (\alpha - \sigma^2) \Delta x^2 / 2\Delta t_m)\) will be found. The left-hand condition of eq. 5.5 can be derived taken into consideration that the apparent diffusion \((D + D_{num})\) has to have a positive value. Numerical diffusion disappears in case the Lax-parameter equals the square of the Courant-number (Lax-Wendrof).

To be able to compare the solution techniques, different values for the morphological time-step will be considered. Using different morphological time-steps, the contribution of the Lax-parameter (introducing pseudo viscosity) will not remain the same. The Lax-parameter has to be adapted to the morphological time-step in order to compute with equal viscosity. To avoid this problem, the Lax-parameter is set equal to zero. The influence of the Lax-parameter will be taken into consideration in sub-section 5.2.7.

When the Lax-parameter is not taken into consideration the stability condition becomes

\[ \left( \frac{p S_{net} \Delta t_m}{a} \right)^2 \Delta x^2 \Delta t_m \leq 1 \]  
(5.6)

The apparent diffusion has to be near to the physical diffusion from accuracy point of view

\[ D_{num} \ll D \quad \text{or} \quad \Delta t_m \ll \frac{2 \beta S_{net}}{\frac{p S_{net}}{a}} \quad \forall \quad \alpha = 0 \]  
(5.7)
5.2.3 Middle-long term evolution

To be able to compare the morphological computations it is recommended to define dimensionless parameters. The model has three dimensions, viz. length, depth and time.

In case nodes and sections are considered only depth and time have to be scaled. Depth (or bed level) is scaled by: 

\[ z = Z \frac{z' - z_{0,0}'}{a_{0,0}} \]

where:
- \( z' \) = computed bed level with respect to datum
- \( z_{0,0}' \) = initial bed level at sea boundary \((x = 0) = 1 \) [m]
- \( a_{0,0} \) = initial water depth at sea boundary = \( 5 \) [m]

Scaling time is more difficult. An apparently logical quantity to take is the celerity of the bed. Time can be scaled by the travelling time of a sand wave. Hence, time is scaled by: 

\[ t = T \frac{T'}{c} = \frac{L}{p_{s_{\text{surf}}}} \cdot \frac{1}{a_0} = 350,000 \text{ [tidal periods]} \]

where:
- \( L \) = length of the basin
- \( c \) = initial celerity at sea boundary \((c = p_{s_{\text{surf}}}/a_0)\)

The first morphological computations that are considered concern middle-long term \((t' = 0.02)\) shoaling. In fig 5.5 the reference computation is given. In this case the bed-level change has been computed for one tidal period, after which the total system of hydrodynamic equations has been solved etc..

![Bed-level evolution reference computation](image)

Although the bed-level change is rather small (due to the rather small gradient of the net transport), the development of a shock wave can be clearly observed. To describe the numerical
behaviour of shock waves central differencing is not unstable in general. The sediment transport is computed in the middle of the sections, as the bed-level change is computed in the nodes. The scheme may cause oscillations, which is a form of inaccuracy (sub-section 5.3.3)

The computation has been compared to the results for different values for the morphological time-step \( t_m \) (fig. 5.6).

**fig. 5.6**  
Middle-long term evolution by means of hydrodynamic equation

Since the bed-level changes are small (max. bed-level change \( O(0.012) \)), the apparent differences between the computations are neglected. The influence of the momentum equation is regarded by applying the continuity correction for two morphological time-intervals: \( T \) and \( 700 \, T \) (fig. 5.7)

**fig. 5.7**  
Middle-long term evolution by means of continuity correction

Both figures illustrate that the influence of the morphological time-step can be neglected for this middle-long period. In case the morphological evolution is not dependent on the morphological time-step it is obvious that the predictor-corrector method described in section 4.3 is not more accurate. Figure 5.8 illustrates small deviations between both solution techniques.
Based on the results of these test computations (until $t' = 0.02$) it can be concluded that in case of small flood-dominance the bed-level evolution does not depend on the morphological time-step and the influence of the momentum equation can be neglected.

5.2.4 Long-term evolution

Since the middle-long term computations did not bring clear differences, the application of the solution techniques for long-term morphological evolution will be considered.

Based on well-known empirical relations (eg. O'Brien, 1931) and 1-D equilibrium test computations (Wang, 1992) the equilibrium state tends to approach to a linear bed-level. In case of the shoaling process in the tidal basin the equilibrium state is not reached. Figure 5.9 shows a shock-wave, propagating and growing in time. Second-order waves tend to develop (peak bed-level) and drying occurs. The DUFLOW computer package is not able to execute its computation in case the bed-level is higher than the lowest water level.
Again the influence of the morphological time-step can be neglected. The morphological process computed for $\Delta t_m = 700\ T$ is used as reference computation. In fig. 5.10 and 5.11 the process is compared to the application of the continuity correction.

**BED-LEVEL EVOLUTION**

*continuity correction; $t_m = 700\ T$*

![Graph 1](image1.png)

*fig. 5.10  Long-term evolution by means of continuity correction*

**BED-LEVEL EVOLUTION**

*hydro.comp. versus cont. corr.*

![Graph 2](image2.png)

*fig. 5.11  Comparison hydrodynamic computation and continuity correction*

Figure 5.11 illustrates that the shoaling process is similar for both solution techniques. However a clear time-lag appears in case of the continuity correction. This is illustrated in fig. 5.12.
This time lag can be explained by the influence of the non-linear terms in the momentum equation. It seems obvious to compute the velocity field by means of a 'storage basin contemplation' since the length of the model is less than 1/20 of the tidal wave-length.

The continuity and momentum equation can be rewritten by integrating both equations over a small distance (Δx). For each time they hold

**continuity equation:**

\[ Q(x_2) - Q(x_1) = -\Delta F \frac{dh}{dt} \] (5.8)

where: Storage area \(\Delta F = B\Delta x\)

This gives a linear decreasing velocity field in case of an initially horizontal bed.

**momentum equation:**

\[ h(x_2) - h(x_1) = \frac{1}{gA_x} \left[ \frac{\partial Q}{\partial t} \frac{\Delta x}{Q} + \frac{Q^2}{A_x} \left| \frac{Q}{A_x} \right| \frac{\partial Q}{\partial x} \right] - \frac{Q^2}{C^2 A_x^2 R} \] (5.9)

The contribution to the fall depends on the sign and magnitude of discharge and gradient of discharge. Since the sign of discharge changes with changing flow direction the contribution of the advective term to the fall has the same sign for both flow directions (ebb and flood).

**Ebb, negative Q:** \(Q_2^2 < Q_1^2\)

**Flood, positive Q:** \(Q_2^2 < Q_1^2\)

Apparently the contribution of the advective term to the fall is positive for both periods.
The contribution of the friction term, which is the most important of the two (fig. 5.14), is negative during flood and positive during ebb. In case of a linearly increasing bed-level, the velocity adapted proportional to the changing depth (continuity correction) does not equal the velocity computed by the hydrodynamic equations.

The cross section decreases in x-direction. Compared to the horizontal bed-level it results in an increasing contribution of the non-linear terms and in particular the friction term. Tidal distortion increases (fig. 5.15) since both terms are quadratic with regard to the discharge. The net transport and its gradient increase.
fig. 5.14 Contribution to fall by terms of dynamic equation in case of a horizontal bed-level
fig. 5.15 Contribution to fall by terms of dynamic equation in case of a linear bed-level
If only the final bathymetry is considered it seems that application of the continuity correction approaches a similar profile as in case of the reference computation. In case the morphological process is described, the tests show that it is not necessary to compute the hydraulic module for each morphological time step. In fig 5.16 the relative error of the bed-level change (in case the difference between the computations is more than 0(0.005)) by means of the hydrodynamic computation and the continuity correction is represented.

\[ \text{relative error} = \frac{\Delta z_{ce} - \Delta z_{hc}}{\Delta z_{hc}} \]  

(5.10)

In fig. 5.16 it can be observed that the shoaling process is more or less similar for both solution techniques until \( t' = 0.1 \). Based on these results and the small relative error (for \( t' = 0.1 \)) illustrated in fig. 5.16 it is assumed that the continuity correction can be applied until \( t' = 0.1 \) until the hydrodynamic module will be run (comb.comp.) (fig 5.17).

In that case the relative error is halved. Since the relative error for \( t' = 0.3 \) still is rather
significant, the same computation has been executed for $\Delta t' = 0.02$. Figure 5.18 shows the results of this computation, the maximum relative error is in order of ten percent. In case of morphological computations this is sufficient accurate.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{relative_error.png}
\caption{Relative error by means of hydrodynamic computation $\Delta t' = 0.02$}
\end{figure}

It is found that the non-linear terms of the momentum equation mainly influence the time-scale of the shoaling process and not the process itself. It is obvious that tidal asymmetry is generated internally by non-linear terms, in particular the friction term. In the next sub-section the influence of the boundary condition of the hydraulic module at the sea-end is examined.

### 5.2.5 Sensitivity on the boundary condition of the hydraulic module

In the previous sub-section it has been demonstrated that the time-scale of the process is fixed by higher harmonics, causing tidal asymmetry. In case the boundary condition at the sea end is determined by the $M_2$ constituent, higher harmonics are generated by the non-linear terms internally. In this sub-section the boundary condition is given by the $M_2$ and $M_4$ constituent.

Shallow water constituents (higher harmonics) are generated by effects which are described by non-linear terms of the dynamic equations which govern the propagation of the tides in the oceans, coastal waters and estuaries (Dronkers, 1964). Of the overtides the $M_4$ component with a quarter-diurnal frequency and an amplitude of 10% of that of the $M_2$ component is the most important along the Dutch coast (Wang, 1992).

The boundary condition becomes:

$$h(x,t) = h_0 + M_2 \cos(\omega t - \theta_2) + M_4 \cos(2\omega t - \theta_2)$$

The relative phase of the $M_4$ constituent is given by $2\theta_2 - \theta_2$. A positive value indicates a lead of $M_4$ over $M_2$, a negative value indicates a lag. DiLorenzo illustrated the impact of the relative phase on the type of distortion, which on its turn has a large impact on the morphological evolution. Figures 5.21 - 5.24 indicate the type of distortion (after Boon and Byrne, 1981). The distortion varies between symmetric (equal rise and fall duration, unequal maxima and minima)
and anti-symmetric (unequal rise and fall duration, equal maxima and minima). Positive symmetric curves (fig. 5.21) have larger maxima than minima whereas positive anti-symmetric curves (fig. 5.24) give longer rise than fall durations. The influence on the morphological evolution for different values of the relative phase is illustrated by the net transport (fig. 5.19) and the R-ratio (fig. 5.20).

The time scale \( T \) has been adapted to the changing net transport-rate. Hence,

\[
T = \frac{L}{c} \approx 20,000 \text{ tidal periods}
\]

Apparently the contribution of the \( M_4 \)-constituent strongly influences the time-scale. Both scales differ a factor 17.5.

Maximum flood-dominance occurs for a relative phase of 90 degrees. In this case the flood period is shorter than the ebb period. To meet the continuity equation the flood velocity is more intense than the ebb velocity, causing a higher flood-transport than ebb-transport.
Fig. 5.21  Tidal distortion; positive symmetric

Fig. 5.22  Tidal distortion; negative anti-symmetric

Fig. 5.23  Tidal distortion; negative symmetric

Fig. 5.24  Tidal distortion; positive anti-symmetric
The computation by means of the hydrodynamic equation ($\Delta t_m = 100$) is used as reference computation (fig. 5.25) and compared to the results by means of the continuity correction (fig. 5.26).

**BED-LEVEL EVOLUTION**

**fig. 5.25** Long-term evolution by means of hydrodynamic equation

**BED-LEVEL EVOLUTION**

**Comparison hydrodynamic computation and continuity correction**

The influence of the $M_4$ constituent, causing a negative anti-symmetric curve, is significant. The time scale on which the basin is shoaling decreases by approximately by a factor 8 to 10 (fig.5.27). The influence of the internally generated distortion is less important or in other words the non-linear terms of the momentum equation play a less significant role. The distortion due to the hydraulic boundary condition is dominant. Hence, application of the hydrodynamic computation and the continuity correction show small differences. The morphological process, in particular the time-scale is fixed by the boundary conditions of the hydraulic module.

The largest difference occurs at the wave front. Again this can be explained by the role of the non-linear terms (although of minor importance). The celerity of the shock wave is a function of the net transport ($c = ps/\alpha$). The net transport on its turn depends on tidal asymmetry, generated
internally and by the boundary conditions. In case of computation by means of the hydrodynamic equations the non-linear terms influence tidal asymmetry. In this case the net transport increases, hence the celerity is larger.

To be able to compare both computations by means of the $M_2$- and $(M_2+M_4)$-constituent both computations are related to the first time-scale ($T = 350,000$ tidal periods).

**fig. 5.27** Comparison hydrodynamic computation $M_2$- and $(M_2+M_4)$-constituent

Summarized it can be concluded that the number of hydrodynamic computations that have to be executed to reach a (more or less) similar profile as for the reference computation, strongly depends on the boundary conditions of the hydraulic module and the relative importance of the non-linear terms of the momentum equation. The net transport fixes the time scale of the process, but not the equilibrium state. Wang (1992) illustrated that the equilibrium state depends on the boundary conditions of the bed-level module.
5.2.6 Disturbed equilibrium profile

The conclusion of the last sub-section will be examined in this sub-section. The initial profile is a disturbed equilibrium profile. Again the tests have been executed by means of the hydrodynamic equation and continuity correction for different values for the morphological time-step.

![Bed-level evolution](image)

**fig. 5.28** Middle-long term evolution by means of hydrodynamic equation

Figure 5.28 illustrates that the bed level converges to the equilibrium profile. However, it seems that a tidal flat will develop at the end of the basin. Although the boundary condition of the hydraulic module is given by the $M_2$ constituent, the morphological behaviour as well as the time-scale of the process of both methods are similar (fig. 5.29).

![Bed-level evolution](image)

**fig. 5.29** Comparison hydrodynamic computation and continuity correction

The influence of the non-linear terms can obviously be neglected. This had been expected since the disturbance is relatively small and there will be no significance change of tidal distortion, hence the gradient of the net transport will be more or less similar for both methods. The bed level does not seem to reach the theoretical equilibrium state. Apparently a tidal flat develops at the closed boundary.
5.2.7 The influence of the Lax-parameter

The main parameters causing diffusion are the bed-slope parameter (\(\beta\)) and the Lax-parameter (\(\alpha\)). The down-slope coefficient induces a diffusive term in the mass-balance equation. The Lax-parameter induces pseudo viscosity.

The computation clearly illustrated the development of a shock wave. The front of the wave gets sharper in time, hence diffusion is of minor importance compared to celerity. Even when the value of the bed-slope coefficient is much higher (to maintain numerical stability, in case of the \(M_2\) and \(M_4\) component) than the theoretical value, diffusion remains rather insignificant. Apparently the morphological behaviour does not depend on the \(\beta\)-coefficient. Wang (1992) underlines this assumption. He found although the net transport decreases with increasing \(\beta\)-coefficient, the value of the \(\beta\)-coefficient does not have significant influence on the variation of the net transport with \(x\).

Smoothening can be gained in case the Lax-parameter is used. However, the \(\alpha\)-coefficient influences the numerical diffusion. In case the numerical diffusion will exceed the physical diffusion the computation is not accurate. Again it should be noticed that numerical diffusion disappears in case the \(\alpha\)-coefficient equals the square of the Courant number. Hence, in that case the numerical computation will be more accurate or in other words the truncation error will be of higher order. Figures 5.30 and 5.31 illustrate the morphological behaviour in case the \(\alpha\)-coefficient is chosen 0.1.

![BED-LEVEL EVOLUTION](image)

**fig. 5.30** Long term evolution by means of hydrodynamic equation \(dt' = 0.02\)
The wave is clearly smoothened by the apparent influence of the diffusion and again the influence of the non-linear terms is present. At the end of the basin drying occurs however, on a much larger time-scale (dependent on the $\alpha$-coefficient (fig. 5.32)) than in case $\alpha$ is chosen to be zero.

It can be concluded that in case the Lax-parameter is applied the value should be chosen with utmost care, from stability and accuracy point of view.
5.3 Tidal River

5.3.1 Description of test case

The lay-out of the estuary is schematized to be a rectangular prismatic channel of length $L$ and a constant width.

\begin{equation}
Q_r(L, t) = \text{constant}
\end{equation} 

At the open sea the tidal wave is given by:

\begin{equation}
h(x, t) = h_0 + M_2 \cos(\omega t - \theta)
\end{equation}

Tidal motion at sea causes an inward progressive wave. Hence, the river boundary-condition should be chosen outside the area of tidal influence, otherwise the boundary condition will cause reflection of the tidal wave. The model length can be estimated, based on a simple analytical analysis (Vreugdenhil, 1989).

To illustrate the distortion of a tidal wave in case of a linear bed-level, a test computation gives the following results (fig 5.34 and fig. 5.35). Both figures clearly show distortion of the penetrating wave. Compared to the theoretical analysis of Vreugdenhil the model length had to be increased. This can be understood by taken into consideration that in the theoretical analysis it is assumed that the hydraulic radius is constant and the friction term is linearized. In the DUFLOW-package the hydraulic radius is a function of the wet-surface area and is computed for each section during each time-step.
Hydraulic module

Numerical parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time step: $\Delta t$</td>
<td>900 [s]</td>
</tr>
<tr>
<td>Spatial step: $\Delta x$</td>
<td>10,000 [m]</td>
</tr>
<tr>
<td>Model length: $L$</td>
<td>200,000 [m]</td>
</tr>
<tr>
<td>Model width: $B$</td>
<td>500 [m]</td>
</tr>
<tr>
<td>Chézy value: $C$</td>
<td>50 [m$^1$/s]</td>
</tr>
<tr>
<td>Mean water-level: $h_0$</td>
<td>10 [m]</td>
</tr>
<tr>
<td>Water-depth sea boundary:</td>
<td>$a_0 = 8$ [m]</td>
</tr>
<tr>
<td>Amplitude sea boundary:</td>
<td>$M_2 = 2$ [m]</td>
</tr>
<tr>
<td>River boundary:</td>
<td>$Q_r = 1000$ [m$^3$/s]</td>
</tr>
<tr>
<td>Bed slope: $i$</td>
<td>$2.5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

fig. 5.34 Distortion discharge (1: sea boundary; 11: middle section; 20: end section)

fig. 5.35 Distortion water level (1: sea boundary; 11: middle section; 20: end section)
Transport and bed-level module

Numerical parameters:

Grain size: $D = 200 \mu [m]$  
Bed-slope coefficient: $\beta = 100$  
Spatial step: $\Delta x = 10,000 [m]$  

The net transport (fig. 5.36) and the $R$-ratio (fig. 5.37) indicate ebb-dominance. The $R$-ratio approaches unity, hence the transport is strongly ebb-dominated.

The transport rate and gradient are highest at the entrance of the channel. This could be expected because the contribution of the tidal wave is most significant at the entrance. Here, maximum ebb-velocity occurs, hence the transport rate (velocity raised to power 5) is at maximum. The $R$-ratio illustrates that the influence of the tidal wave on the transport rate fades away. Obviously the contribution of the tidal wave on the net (ebb) sediment-transport decreases in $x$-direction. To underline this the transport rate as a function of time is given for two sections.
In contrary to the basin model, the transport is ebb dominated. The condition at the river boundary is given by

\[ z(L,t) = z(L,0) = \text{constant} \quad (5.12) \]

The numerical scheme applied to the sea boundary is the same as applied at the closed boundary in case of the tidal basin

\[ \frac{\partial^2 z}{\partial t \partial z} = 0 \quad \text{or} \quad z_{j=1}^{n+1} = z_j^n + (z_{j=1}^{n+1} - z_{j=1}^n) \quad (5.13) \]

Due to the large spatial-step and high absolute value of the R-ratio, the left-hand stability condition becomes decisive

\[ \Delta t_n \leq \frac{2\beta a^2}{p^2 S_{na} R} \quad (5.14) \]

For \( \alpha \neq 0 \) numerical diffusion will be significant, because of the large spatial-step. Hence, the Lax-parameter is chosen to be zero, taken into consideration the remark in sub-section 5.2.2. with respect to the relation to the morphological time-step.
5.3.2 Middle-long term evolution

Before any computations are executed, depth and time have to be scaled. Depth (or bed level) is scaled by:

\[ z = \frac{z' - z_{0,0}}{a_{0,0}} \]  

(5.15)

where:
- \( z' \) = computed bed level with respect to datum
- \( z_{0,0} \) = initial bed level at sea boundary (at \( x = 0 \)) = 2 [m]
- \( a_{0,0} \) = initial water depth at sea boundary = 8 [m]

Now, the celerity is considered based on the initial transport-rate at the river boundary. Hence, time is scaled by:

\[ t = \frac{T}{t'} = \frac{L}{c} \frac{L}{p \cdot s_{vel,L} / a_{0,L}} = 875,000 [\text{tidal periods}] \]  

(5.16)

where:
- \( L \) = length of the basin
- \( c \) = initial celerity at river boundary (\( c = p \cdot s_{vel,L} / a_{0,L} \))

The first computations concern middle-long term morphological behaviour. To get an impression of the bed-level development, fig. 5.39 illustrates the results based on the hydrodynamic equations.

![BED-LEVEL EVOLUTION](image)

**fig. 5.39 Bed-level evolution reference computation**

Obviously, the largest degradation takes place at the sea entrance, where the gradient of the net sediment-transport is highest. Within this time interval, no significant bed-level change occurs besides the first sections.
As in case of the basin model, the influence of the momentum equation is tested by applying the continuity correction for several morphological time-steps during the same time-interval.

![Bed-level evolution](image)

**fig. 5.40**  
*Middle-long term evolution by means of the continuity correction*

Apparently the evolution depends (although not significant) on the chosen time-step. To illustrate this the bed-level change (fig. 5.41) and the relative error for $\Delta t_m = 700 T$ (fig. 5.42) are shown for node 2 as a function of time.

![Bed-level evolution node 2](image)

**fig. 5.41**  
*Bed-level evolution node 2*
Besides the initial difference the lines run parallel. Most striking is the decrease of the relative error in time. Obviously the process tends to become similar for different values of the morphological time-step after some time. It is assumed that the deviation for the first years can be imputed to a kind of settle time. Initially the bed-level change per tidal cycle is significant. The velocity field is changed accordingly, hence the net transport decreases faster in case a smaller time-step is assumed. Obviously, morphological behaviour becomes independent of the time step in case the bed-level change per tidal cycle decreases to a certain value.

The deviation between the computations for different values of $\Delta t_m$ justifies the application of the predictor-corrector method. The solution clearly shows increasing accuracy.
The computations by means of the continuity correction are compared to the reference computation at \( t' = 0.008 \).

**fig. 5.44** Continuity correction (c.c.) versus hydrodynamic computation (h.c.)

**fig. 5.45** Continuity correction (c.c.) versus hydrodynamic computation (h.c.)
As could be expected the relative error of the computation by means of the continuity correction increases in time. The relative error for different time-steps for each technique decreases however. Due to the initial error in the computation by means of the continuity correction for $\Delta t_m = 700 \, T$ this computation seems to be more accurate than for lower values of the morphological time-step. Obviously the results depend on the morphological time-step for both solution techniques. Before any conclusions are drawn long-term behaviour is considered.

5.3.3 Long-term evolution

The middle-long term model did show clear differences between both methods. To get a first impression of the morphological behaviour and time scale, the process based on the continuity correction is considered (fig. 5.47).
Apparently oscillations (wiggles) occur. These oscillations are not related to stability but to accuracy. Wiggles or short waves with wave length $2\Delta x$ occur in case the cell Péclet number (or also indicated as the cell Reynolds number) exceeds the value two (Vreugdenhil 1989). Hence, to avoid wiggles

$$|P| = \left| \frac{c\Delta x}{D} \right| < 2 \quad (5.17)$$

where:

Celerity: \quad c = ps_{ref}/a

Diffusion: \quad D = ps_{ref}

The value tells something on the coarseness of the grid. Although these oscillations are not related to stability, it is preferable to avoid them (take into consideration the influence of the bed slope on the transport rate). Most obvious is to decrease the spatial step. However, the maximum number of sections is limited for the DUFLOW package. Another option is to try to smoothen the solution artificially. Smoothening can be achieved by means of the Lax-parameter, introducing a kind of pseudo viscosity. Figure 5.48 illustrates the bed-level evolution for different values of the Lax-parameter.

![Bed-level evolution](image)

**fig. 5.48** Influence of Lax-parameter

It is obvious that the value of the Lax-parameter should depend on the morphological time-step. The parameter has to decrease with decreasing time-step, to assure equal influence of the viscosity factor on the outcome.

The computation by means of the continuity correction has been compared by the computational results by means of the hydrodynamic computation (fig. 5.49). The hydraulic module is computed for every 700 tidal periods, because it seemed that the solution converged to the reference computation in case of middle-long term (fig. 5.46).
Fig. 5.49 Continuity correction (c.c.) versus hydrodynamic computation (h.c.)

Obviously the influence of the non-linear terms is significant. Most striking is the difference in shape. Convex in case of the continuity correction and concave in case of the hydrodynamic computation. Apparently, the hydrodynamic computation does not seem to converge to an equilibrium profile. This process is due to the combination of boundary conditions of the hydraulic module.

The computation by means of the continuity correction will converge to some kind of equilibrium profile. Degradation of the bed level will cause both flood and ebb velocity to decrease (constant water-level assumed) until the net transport is uniform. In case of the hydrodynamic computation in contrary to the continuity correction, the maximum flood-velocity increases and the ebb velocity decreases for the first sections, causing the net transport to decrease. However, the tidal wave will be able to penetrate further, hence both flood velocity and ebb velocity will increase, causing the net transport to increase in upstream direction. Dependent on the relative importance of the boundary conditions of the hydraulic module this process will proceed, and may or may not lead to an equilibrium profile. It should be noticed that the boundary condition at the river boundary only holds for a certain time interval in case of this combination of boundary conditions.

The influence of the boundary conditions of the hydraulic module on the morphological behaviour is tested in the next sub-section.

5.3.4 Sensitivity on the boundary conditions of the hydraulic module

The boundary conditions of the hydraulic module can be separated into the (constant) discharge at the river boundary and the amplitude, number of constituents and their relative phase of the tidal wave at the sea boundary.
The maximum flood- ($Q_f$) and ebb-discharge ($Q_e$) at the sea boundary are influenced by the river run-off. In case a ratio $X$ is defined as

$$X = \frac{Q_r}{Q_f}$$

(5.18)

three phenomena are distinguished viz.

a) no dominance
b) tidal dominance
c) river dominance

These phenomena can be related to the $X$-ratio. In case:

1) $X \geq 1$ river dominance
2) $0 \leq X < 1$ physically irrelevant
3) $-1.5 < X < 0$ tidal dominance
4) $-x < X < -1.5$ no dominance
5) $X < -x$ river dominance

The $x$-value for the transition area from no-dominance to tidal dominance is not determined.

To decrease the tidal influence the amplitude at the sea boundary will be decreased to one meter. Several computations have been executed for three upstream boundary conditions.

a) $Q_r = 2000$ [m$^3$/s]; $X = -10.6$: no initial dominance
b) $Q_r = 100$ [m$^3$/s]; $X = -0.96$: tidal dominance
c) $Q_r = 5000$ [m$^3$/s]; $X = 1.54$: river dominance
ad a) \( Q_r = 2000 [\text{m}^3/\text{s}] \)

Time is scaled by means of eq. 5.16 \( T = 175,000 [\text{tidal periods}] \).

The net sediment-transport is illustrated in fig. 5.50. Obviously the penetration of the tidal wave becomes less than in the previous case because of the increasing river discharge and decreasing amplitude of the tidal wave.

![NET SEDIMENT-TRANSPORT](image)

**fig. 5.50** Net transport per section

The shape of the graph is remarkable. The net transport decreases strongly for the first sections after which it increases again. Hence, erosion can be expected for the first nine sections after which sedimentation is expected to occur. Distortion of the tidal wave causes both maximum flood and ebb velocity to decrease and converge to the stationary flow velocity at the river boundary.

![RATIO NET TRANSPORT - TOTAL TRANSPORT](image)

**fig. 5.51** R- ratio per section
Figure 5.46 illustrated that the dependence of the hydrodynamic results on the morphological time-step gradually disappeared. Hence the morphological time-step is chosen to equal 700 tidal periods. The ratio of the Péclet numbers for different boundary conditions is given by

$$\frac{|c_1 \Delta x|}{|P_1|} = \frac{|D_1|}{|c_2 \Delta x|} = \frac{|s_{nt_1} s_{nt_2}|}{|s_{nt_2} s_{nt_1}|} = \frac{|R_1|}{|R_2|}$$

(5.19)

The R-ratio of the previous paragraph ($Q_r = 1000 \text{ [m}^3\text{/s]}, M_2 = 2 \text{ [m]}$) is less than the R-ratio of this paragraph ($Q_r = 2000 \text{ [m}^3\text{/s]}, M_2 = 1 \text{ [m]}$) in other words the Péclet number will increase. Hence in case the Lax-parameter is chosen to equal zero, wiggles will again occur.

**fig. 5.52**  Bed-level evolution reference computation

It can be observed that indeed sedimentation occurs upstream and erosion downstream of node 11 until $t' = 0.4$. Afterwards the transport gradient becomes positive for the whole region, hence the bed-level will drop.

**fig. 5.53**  Continuity correction (c.c.) versus hydrodynamic computation (h.c.)
Most remarkable is the difference in process between both techniques. In case of application of the hydrodynamic equation sedimentation upstream disappears in time, where in case of continuity correction the sedimentation process increases and likely penetrates downstream. This can be underlined by presenting the bed evolution for both methods for node 2 in time (fig. 5.54).

![Bed-level evolution node 2](image)

**fig. 5.54** Bed-level evolution node 2

It can be clearly observed that in case of the continuity correction in contrary to the hydrodynamic equation ‘equilibrium’ will be reached relatively fast. The morphological behaviour in case of the continuity correction is quite remarkable. A period of erosion is proceeded by sedimentation. This process is due to the combination of boundary conditions and will always occur in case none of the boundary conditions is dominant or in other words in case the shape of the initial transport rate is similar to figure 5.50. The arguments for the different behaviour are the same as in subsection 5.3.3.

Based on the comparison of the continuity correction and the reference computation, the hydrodynamic equation has been computed at $t' = (n+1) * 0.04$ (fig. 5.55).

![Comparison hydrodynamic computation and combination computation](image)

**fig. 5.55** Comparison hydrodynamic computation and combination computation
Both computations converge to the same equilibrium profile. Figure 5.56 illustrates the variation of the transport rate for different 'equilibrium' solutions of the computational techniques.

The comparison shows that in case the Lax-parameter equals zero the net transport for the equilibrium profile indeed becomes uniform (equilibrium condition of continuity equation sediment). In case the Lax-parameter equals 0.07, in most of the sections sedimentation is expected. Sedimentation is proceeded with erosion and visa versa. In this case a dynamic equilibrium profile is defined. The latter conclusion is made under reservation and influenced by the Lax-parameter and the coarseness of the grid (see also sub-section 5.3.4). In case of the equilibrium profile of the continuity correction it is obvious that the erosion process will proceed.
b) \( Q_r = 100 \text{ [m}^3\text{/s]} \)

Time is scaled by means of eq. 5.16 \( T = 43,750 \times 10^3 \) [tidal periods].

In this case the tidal wave is dominant. The net transport rate is much less (less distortion) than in the previous case and indicates erosion for all sections.

![Net transport per section](image)

**fig. 5.57** Net transport per section

![R- ratio per section](image)

**fig. 5.58** R- ratio per section

Also the \( R \)-ratio is much less than for both previous cases and has a remarkable shape. Comparing the results of middle-long term computations by means of the hydrodynamic equation and continuity correction illustrated significant difference after \( t^* = 0.0008 \) (fig. 5.59).
Fig. 5.59 Comparison hydrodynamic computation (h.c.) and continuity correction (c.c.)

Based on these results long-term evolution by means of the different approaches have been compared (fig. 5.60), including the combination (combi) computation, solving the set of hydrodynamic equations at $t' = (n+1) \times 0.0008$.

Fig. 5.60 Comparison hydrodynamic computation (h.c.), continuity correction (c.c.) and combination computation (combi.)

The combination computation approaches the reference computation by means of the hydrodynamic equation quite well. Although the computational results of the continuity correction and reference computation differ significantly, the bed-level shape is similar. In contrary to both previous cases the continuity correction does not (yet) converge to an equilibrium profile. Obviously, like in case of the basin the process for both computations seems similar although much slower. The shape of the 'final' profile is remarkable.

The remarkable profile is tried to be explained qualitatively considering the shape of the R-ratio graph. The R-ratio changes significantly. Obviously the total transport ($s_{tot}$) changes accordingly, since the net transport ($s_{net}$) is rather uniform for the upstream sections. The proportion between
diffusion and celerity will change accordingly, most likely causing the profile.

Long-term evolution is now again considered to do research on the equilibrium profile (fig. 5.61).

![Bed-level evolution](image)

**fig. 5.61** Long term evolution by means of combination computation

Although it seems that the profile at the sea boundary converges to a kind of equilibrium profile, the other sections do not seem to converge. It can be expected that the shape of the profile will converge to a similar one as described in sub-section 5.3.3. Hence, no equilibrium situation is expected (for these boundary conditions).

\[
Q = 5000 \, \text{[m}^3/\text{s]}\]

c) \quad Q = 5000 \, \text{[m}^3/\text{s]}\]

Time is scaled by means of eq. 5.16 \( T = 28,000 \) [tidal periods].

In this case the river discharge is dominant. The sediment transport rate (fig. 5.62) indicates high transport-rates.

![Net transport](image)

**fig. 5.62** Net transport per section
The bed-level evolution is computed by means of the hydrodynamic equations (fig. 5.64).

The profile seems to converge to some kind of equilibrium profile. The same computation has been executed by means of the continuity correction and both computations have been compared (fig. 5.65).
As could have been expected both computations converge to an equilibrium profile. This is clearly illustrated at fig. 5.66.

The transport rate of both 'equilibrium' solutions shows that only in case of the hydrodynamic computations the profile converged to a real equilibrium profile.
Summarized it can be concluded that in these cases of no dominance and river dominance the bed-level is likely to converge to a kind of equilibrium profile. In case of a tidal-dominated river obviously no equilibrium profile exists. The time the continuity correction can be applied is not only a function of the relatively changing depth, but is also influenced by the generation of tidal distortion due to the non-linear terms in the momentum equation.

5.3.5 Down-slope versus pseudo viscosity

The Lax-parameter ($\alpha$) and bed-slope coefficient ($\beta$) should be applied with care. As observed and mentioned before both might influence the outcome of the computations significant, because of their defensive character. It should be heard in mind that the origin of both parameters is different. The $\beta$-coefficient has a physical background where as the $\alpha$-parameter is purely numerical.

$\beta$-coefficient

The down-slope coefficient seems a logical physical parameter. According to Struiksma (1988) the value of this parameter is only a function of the bed-roughness expressed in the Chézy value ($C$). However, the contribution of the bed-slope (roughness) is also accounted for during the flow-velocity computation. This should be taken into consideration in case an acceptable value is to be chosen. Especially in case of steep bed-profiles the contribution of this parameter has to be examined. Obviously the $\beta$-parameter cannot be freely chosen as large as needed to maintain numerical stability or to avoid wiggles.

$\alpha$-parameter

In both cases the $\alpha$-parameter clearly influenced the bed-level evolution. Based on the continuity equation of sediment (3.14) the equilibrium profile is reached in case the transport derivative equals zero.
However, in case the Lax-scheme is applied this (physical) condition is not sufficient. The Lax-scheme can be rewritten as

\[ z_j^{n+1} - z_j^n = \alpha \left( \frac{z_{j+1}^n + z_{j-1}^n}{2} - z_j^n \right) - \frac{\Delta s_n}{\Delta x} \Delta t \]  

(5.20)

Obviously the condition \( \Delta s/\Delta x = 0 \) is not sufficient to guarantee equilibrium under all circumstances. Only in case of a linear or horizontal equilibrium bed-level the transport gradient will satisfy the physical condition. If \( \Delta s/\Delta x = 0 \) and the bed is neither horizontal nor linear, the bed level will change. Hence, the net transport will not longer be uniform, causing the bed level to develop. In this case a kind of dynamic equilibrium profile may be defined. In general the equilibrium profile will satisfy:

\[ \alpha \left( \frac{z_{j+1}^n + z_{j-1}^n}{2} - z_j^n \right) = \frac{\Delta s_n}{\Delta x} \Delta t \]  

(5.21)

It has to be noticed that the coarseness of the spatial grid may play an important role. This can be explained in case different bed-shapes are considered.

In case of a convex profile (fig. 5.68)

\[ z_j^n > \frac{z_{j+1}^n + z_{j-1}^n}{2} \]  

(5.22)

Hence the left-hand term in equation 5.21 becomes negative. Three different possibilities, besides the equilibrium solution may arise.

a) In case the transport gradient is positive, the expected erosion will be intensified due to the numerical error

b) In case the transport gradient is negative, two possibilities should be distinguished
1) The modulus of the right-hand term is less than that of the left-hand term of equation 5.21. In this case the numerical error will slow down the erosion process.

2) In case the modulus of the right-hand term is larger than that of the left-hand term of equation 5.21, the numerical error even prevents erosion and sedimentation will occur.

Of these possibilities the most worrying phenomenon is that of b2, because this holds against physics. The occurrence of this problem will increase according as the equilibrium profile will be approached. Besides the value of the $\alpha$-parameter, the coarseness of the spatial grid is important. In case of a concave profile the opposite holds.
6 Bed disturbance method

6.1 Introduction

In the description of the transport-field method the continuity correction has been applied explicitly. The adapted velocity profile is used to derive the sediment transport rate, which in turn is used to compute the bed evolution.

In contrary to this method, the continuity correction has been applied implicitly in the bed-disturbance method. In this method the continuity equation of water is substituted into the continuity equation of sediment, again assuming that the kinematic response is stronger than the dynamic response (fig. 6.1)

![fig. 6.1 Bed disturbance method](image)

6.2 Advection-diffusion equation

6.2.1 Derivation of the equation

The continuity equation can be rewritten as

\[- \frac{\partial z}{\partial t} = (1 - \beta \frac{\partial z}{\partial x}) \frac{\partial s'}{\partial x} - \beta s' \frac{\partial^2 z}{\partial x^2} \] (6.1)

Neglecting products of derivatives yields

\[- \frac{\partial z}{\partial t} = \frac{\partial s'}{\partial x} - \beta s' \frac{\partial^2 z}{\partial x^2} \] (6.2)
in which

\[ s' = m u^p \]  \hspace{1cm} (6.3)

The derivative of this equation in \( x \)-direction can be rewritten as

\[ \frac{\partial s'}{\partial x} = \frac{ds'}{du} \frac{\partial u}{\partial x} \]  \hspace{1cm} (6.4)

**Taken into consideration** the assumptions that

1) the water level variation is invariant for bed level changes
2) the response of the velocity on the bed level change can be computed by means of the *continuity correction*
3) the bed level changes are small per time-step

the derivative of the velocity can be governed from the continuity equation of water

\[ \frac{\partial h}{\partial t} - \frac{\partial z}{\partial x} + a \frac{\partial u}{\partial x} + u \frac{\partial a}{\partial x} = 0 \]  \hspace{1cm} (6.5)

or

\[ \frac{\partial u}{\partial x} = - \frac{1}{a} \left[ \frac{\partial h}{\partial t} - \frac{\partial z}{\partial t} + u \left( \frac{\partial h}{\partial x} - \frac{\partial z}{\partial x} \right) \right] \]  \hspace{1cm} (6.6)

The derivative of the velocity is substituted into the modified continuity equation for sediment. In this way an advection-diffusion equation with source term is derived.

\[ (1 - \frac{c}{u}) \frac{\partial z}{\partial t} + c_b \frac{\partial z}{\partial x} - D \frac{\partial^2 z}{\partial x^2} = \frac{c}{u} \left( \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right) \]  \hspace{1cm} (6.7)

in which

\[ c = p s_{mc}/a \]
\[ D = \beta s_{se} \]

**Taken into consideration** that in general \( p s/q << 1 \) the eq. yields

\[ \frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} - D \frac{\partial^2 z}{\partial x^2} = \frac{c_b}{u} \left( \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \right) \]  \hspace{1cm} (5.8)
6.2.2 Application of the advection-diffusion equation

Application of the advection-diffusion equation for (middle) long-term morphological modelling seems to be very promising (De Vriend, 1993; Wang, 1992).

In case the source term is neglected, three concepts are distinguished.

a) Linear advection-equation

In case the influence of the bed-level slope on the transport rate is not taken into consideration the diffusion term will vanish. This gives

$$\frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} = 0$$  \hspace{1cm} (5.9)

b) Linear advection-diffusion equation

In case the bed-level slope is taken into consideration a diffusion term will be introduced. The diffusion-coefficient and celerity are assumed to be constants.

$$\frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} - D \frac{\partial^2 z}{\partial x^2} = 0$$  \hspace{1cm} (5.10)

c) Non-linear advection-diffusion equation

In this case the dependence of both diffusion and celerity on the bed-level is taken into consideration.

$$\frac{\partial z}{\partial t} + c(z) \frac{\partial z}{\partial x} - \frac{\partial}{\partial x} \left( D(z) \frac{\partial z}{\partial x} \right) = 0$$  \hspace{1cm} (5.11)

Different techniques to solve these equations numerically will have to be examined and compared. Goddijn (1986) showed that to solve the linear advection-diffusion equation the 1-step Stone-Brian implicit seems most promising. In that case a set of linear differential equations has to be solved. In case of the non-linear advection-diffusion equation a two-step FTCS + Crank-Nicholson seems most appropriate.

These models will have to be tested and (if possible) compared to the results of chapter 5.
7. Summary, Main Conclusions and Recommendations

7.1. Summary and main conclusions

In this report a model has been derived to simulate morphological behaviour under tidal conditions. The model is based on the set of equations of motion and mass-balance for both water and sediment. The aim of the research was to find mathematical simplifications to reduce computing time. The study focused on the underlying assumption that the kinematic response of the water movement on the bottom variation is much stronger than the dynamic response. The application of the transport-field method (chapter 4) as suggested by De Vriend (1985) has been examined. Two strongly schematized cases have been considered viz. a basin model (section 5.2) and a tidal river model (section 5.3).

Basin model

In case of the basin model the morphological system is purely tide-driven. The net transport is inwards, causing the basin to shoal. In case of the main tidal-constituent ($M_2$), flood-dominance is caused by internal generated overtides and the phase lag between discharge and water depth. The small gradient of the net sediment-transport causes small bed-level changes per morphological time-step. Obviously in case of small transport gradients the computational results do not depend on the morphological time-step.

Because of the small bed-level changes per morphological time-step the continuity correction (velocity * depth is invariant) does not show significant differences compared to the reference computation for middle-long term periods. On long-term behaviour the non-linear terms of the momentum equation mainly influence the time-scale of the process rather than the development of the bathymetry.

The net transport averaged over a tidal period is very sensitive to the amplitudes and relative phases of the tidal constituents. The number of tidal constituents and their amplitudes strongly influence the time scale of the morphological development, rather than the bathymetric development. The internal generated overtides become less important. The difference in time scale between computations by means of the hydrodynamic equations and the continuity correction decreases.

None of the computations have shown an absolute equilibrium state (net transport is zero everywhere). However, the bathymetry develops in the same way in all computations. This might indicate that a kind of equilibrium state exists. If the numerical parameters (bed-slope coefficient, Lax-parameter, boundary conditions bed-level module, etc.) are properly chosen, the computation might converge to a linear equilibrium state.

Numerical oscillations tend to occur when a steep front is present. The numerical oscillations tell something on the coarseness of the grid. These oscillations occur in shallow regions, where they might cause drying, which disturb the morphological development. These oscillations can be avoided by adjustment of the spatial grid or addition of diffusion, by means of the bed-slope coefficient or the Lax-parameter.

Application of the predictor-corrector method did not show higher accuracy than the Euler-method in case of small transport-gradients.
Tidal river model

In case of the tidal-river model the net transport is ebb dominated. The net sediment-transport depends on tidal distortion. Tidal distortion is strongly influenced by the ratio of the amplitudes of the tidal constituents and the river discharge. Dependent on the ratio of the maximum ebb- and flood-discharge the bed-level will or will not converge to an equilibrium state.

In case of tidal dominance the computations by means of the continuity correction show the same phenomenon as in case of the basin model, because of the rather small net transport. The application of the continuity correction in case of higher net-transport (no- and river dominance) indicate a different bathymetry development. The final equilibrium state by means of the continuity correction differs significantly from that by means of the hydrodynamic equation.

General

Application time of the continuity correction is not only a function of the ratio between bed-level change and water depth and but also depends on the generation of tidal distortion.

The value of the Lax-parameter and the bed-slope coefficient should be chosen with care, because both are considered to be of more importance than in case of non-tidal rivers.

Computation time of the morphological evolution strongly depends on the number of runs of the hydraulic module. For the test cases (20 sections), computation time of the hydraulic module for one tidal period is of order $O(40-50 \text{ [s]})$, whereas computation by means of the continuity correction takes less than one second. Obviously, computation time for both the hydraulic module and the transport and bed-level module increases with increasing number of sections. (These numbers are based on a 80486 processor, 50 MHz).

7.2. Recommendations for further research

To obtain a thorough understanding of the problem more research is needed. Therefore the following suggestions are recommended for the near future:

- The method described in chapter 6 should be implemented in a computer program and tested for the same cases as described in chapter 5.

- Because of the limited size of the DUFlow-package another hydraulic program should be used.

- The influence of the boundary conditions of the bed-level module on the morphological development should be examined, taken into consideration the possibility of two incoming characteristics

- Other numerical schemes than the Lax-scheme should be examined and compared to the computational results of chapter 5.

- Computational results based on the methods proposed by De Vriend (1985) should be compared to those of the method proposed by Krol and (1990) implemented by Fokkink (1992).
### Main Symbols

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<th>Symbol</th>
<th>Description</th>
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Bibliography


Appendix I Hydraulic Module

Stability analysis

The partial differential equations can be written as a system of equations, replacing the derivatives by finite differencing equations. Reference \((x_j, t_n, \theta^x)\)

The continuity equation

\[
B^* \left( \frac{h_{j+1/2}^{n+1} - h_{j-1/2}^n}{\Delta t} + \frac{Q_{j+1}^{n+1} - Q_j^n}{\Delta x} \right) = 0 \quad (A.1)
\]

The momentum equation:

\[
\frac{Q_{j+1/2}^{n+1} - Q_{j-1/2}^n}{\Delta t} + \alpha \left[ \frac{Q_j^n}{A_{j+1}} - \frac{Q_j^n}{A_j} \right] + gA_{j+1/2} \left( \frac{h_{j+1}^{n+1} - h_j^n}{\Delta x} + g \frac{|Q_{j+1/2}^{n+1}|}{(C^2A R)_{j+1/2}} \right) = 0 \quad (A.2)
\]

Since waterlevel and discharge are not defined in the point of reference the equations have to be rewritten. To be able to estimate the (relative) damping of the amplitude of the solution, the momentum equation is linearized.

\[
B^* \left[ \frac{h_{j+1}^{n+1} - h_j^n}{\Delta t} + \frac{h_{j-1}^{n+1} - h_j^n}{\Delta t} \right] + \theta \frac{Q_{j+1}^{n+1} - Q_j^n}{\Delta x} + (1 - \theta) \frac{Q_j^n - Q_{j-1}^n}{\Delta x} = 0 \quad (A.3)
\]

\[
\frac{1}{2} \left[ \frac{Q_{j+1}^{n+1} - Q_j^n}{\Delta t} + \frac{Q_j^n - Q_{j-1}^n}{\Delta t} \right] + e \frac{h_{j+1}^{n+1} - h_j^n}{\Delta x} + e \frac{Q_{j+1}^{n+1} - Q_j^n}{\Delta x} + 0 \frac{Q_j^n - Q_{j-1}^n}{\Delta x} = 0 \quad (A.4)
\]

For stability analysis the Von Neumann analysis is used, assuming sinusoidal solutions

\[
h_j^n = h^e e^{ikx_j} = h^n e^{i\theta^x}
\]

\[
Q_j^n = Q^n e^{i\theta^x}
\]

Defining the amplification factor \(\rho\)

\[
h^{n+1} = \rho h^n \Rightarrow Q^{n+1} = \rho Q^n
\]
the following set of equations is derived

\[ B[e^{\theta(p-1)+p-1}]h^* + \frac{2\Delta \ell}{\Delta x} [e^{\theta(p_\theta-\theta+1)-p_\theta+\theta-1}]Q^* = 0 \]

\[ gA^2 \Delta \ell [e^{\theta(p_\theta-\theta+1)-p_\theta+\theta-1}]h^* + [e^{\theta(p-1)+p-1+\Delta \ell \ell [e^{\theta(p_\theta-\theta+1)+p_\theta+\theta-1}]}Q^* = 0 \]

(A.7)

Non-trivial solutions are derived if the determinant of the coefficient matrix equals zero

\[ \det \begin{bmatrix} B[e^{\theta(p-1)+p-1}] & \frac{2\Delta \ell}{\Delta x} [e^{\theta(p_\theta-\theta+1)-p_\theta+\theta-1}] \\
& gA^2 \Delta \ell [e^{\theta(p_\theta-\theta+1)-p_\theta+\theta-1}] & e^{\theta(p-1)+p-1+\Delta \ell \ell [e^{\theta(p_\theta-\theta+1)+p_\theta+\theta-1}]} \end{bmatrix} = 0 \]

(A.8)

Solving above system leads to the following expression of the amplification factor

\[ \rho = \frac{e + \gamma^2 \theta(1-\theta) - \frac{r \Delta \ell}{2} \sqrt{\gamma^2 + \frac{r^2 \Delta \ell^2}{4}}}{e - \theta^2 \gamma^2} \]

(A.9)

or

\[ \rho = \frac{e - 4 \sigma^2 \tan^2 \frac{1}{2} \xi \theta(1-\theta) - \frac{r \Delta \ell}{2} \sqrt{\frac{r^2 \Delta \ell^2}{4} - 4 \sigma^2 \tan^2 \frac{1}{2} \xi}}{e + \theta^2 4 \sigma^2 \tan^2 \frac{1}{2} \xi} \]

(A.10)

The relative damping is calculated without friction \(r=0\) for reasons of simplicity. For a tidal wave \(\xi \ll 1 \Rightarrow \tan \xi = \xi\) can be applied. The following complex function is derived

\[ \rho = \frac{1 - \sigma^2 \xi^2 \theta(1-\theta) \pm \sqrt{-\sigma^2 \xi^2}}{1 + \theta^2 \sigma^2 \xi^2} \]

(A.11)

Applying the condition

\[ |\rho| \leq 1 \]

(A.12)

it can be easily shown that the method is unconditionally stable for \(0.5 \leq \theta \leq 1\)
The relative damping can be governed by using

\[ \sigma^2 \xi^2 = \left( c \left( \frac{\Delta t}{\Delta x} \right) \right)^2 = \left( \frac{2\pi}{n} \right)^2 \]

with \( n = \frac{T}{\Delta t} \)

leading to

\[ |\rho|^n = \frac{1 - \left( \frac{2\pi}{n} \right)^2 \Theta(1 - \Theta) \pm i \left( \frac{2\pi}{n} \right)}{1 + \theta^2 \left( \frac{2\pi}{n} \right)^2} \]

or

\[ |\rho|^n = \left| \frac{1 - \left( \frac{2\pi}{n} \right)^2 \Theta(1 - \Theta)^2 + \left( \frac{2\pi}{n} \right)^2}{1 + \theta^2 \left( \frac{2\pi}{n} \right)^2} \right|^\frac{n}{2} \]
Appendix II Bed level module

Stability analysis

The basic equation of continuity of sediment per unit of width is used

$$\frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} = 0$$  (A.16)

The transport formula used depends on the local hydraulic conditions of which \(z\) and \(u\) vary in place and time. The formula expresses bed material load, implying that the adaption length is much smaller than the spatial grid.

$$s = mu^p(1 - \beta \frac{\partial z}{\partial x})$$  (A.17)

Applying the rigid lid approximation the following equation is derived

$$\frac{\partial s}{\partial x} = \frac{ps_{net}}{a_0} (1 - \beta \frac{\partial z}{\partial x}) \frac{\partial z}{\partial x} - \beta ps_{net} \frac{\partial^2 z}{\partial x^2}$$  (A.18)

Combining A.16 and A.18 and neglecting products of derivatives the following advection diffusion equation can be derived

$$\frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} - D \frac{\partial^2 z}{\partial x^2} = 0$$  (A.19)

The derivatives are replaced by finite differences using the Lax-scheme

$$z_j^{n+1} = \frac{1}{2} \alpha (z_{j+1}^n + z_{j-1}^n) + (1 - \alpha)z_j^n + c \frac{z_{j+1}^n - z_{j-1}^n}{2\Delta x} - D \frac{z_{j+1}^n - 2z_j^n + z_{j-1}^n}{\Delta x^2} = 0$$  (A.20)

The Von Neumann method is used again, introducing sinusoidal solutions. It can be easily shown that

$$\rho \frac{1}{\Delta \tau} \left[ \frac{1}{2} \alpha (e^{i\xi} + e^{-i\xi} + (1 - \alpha)) \right] + c \frac{e^{i\xi} - e^{-i\xi}}{2\Delta x} - D \frac{e^{i\xi} - 2 + e^{-i\xi}}{\Delta x^2} = 0$$  (A.21)
Rewriting the exponential equation in terms of Fourier the equation becomes

\[ \rho - \alpha \cos \xi - (1 - \alpha) + \sigma i \sin \xi - \lambda (\cos \xi - 1) = 0 \]  

(A.22)

Again using the condition of stability

\[ |\rho| \leq 1 \quad 0 \leq \xi \leq \pi \]  

(A.23)

\[ [1 + \alpha (\cos \xi - 1) + \lambda (\cos \xi - 1)]^2 + (\sigma i \sin \xi)^2 \leq 1 \]  

(A.24)

dividing by \((\cos \xi - 1)\)

\[ 2 \alpha + 2 \lambda + (\cos \xi - 1)(\alpha + \lambda)^2 + (-1 - \cos \xi) \sigma^2 \geq 0 \]  

(A.25)

\[ \xi = 0 \Rightarrow \cos \xi = 1 \implies 2 \alpha + 2 \lambda - 2 \sigma^2 \geq 0 \]  

(A.26)

\[ \xi = \pi \Rightarrow \cos \xi = -1 \implies 2 \alpha + 2 \lambda - 2(\alpha + \lambda)^2 \geq 0 \]  

(A.27)

Combining both equations the condition of stability yields

\[ \sigma^2 \leq \alpha + \lambda \leq 1 \]  

(A.28)