Measuring Brownian motion of MFM-tips

&

Characterizing mechanical properties of mono-crystalline diamond resonators

T. A. Baart

July 2011

Report Rotation 3 Casimir specialisation
Supervisors: prof. M.D. Lukin, S.J. Kolkowitz, dr. Q.P. Unterreithmeier
Harvard University
Department of Physics
Quantum Optics group
Abstract

In this project two main goals have been pursued: (1) measure the Brownian motion of a magnetic force microscopy-tip (MFM-tip) using either standard optical interferometry or through transmission modulation of a tapered fiber, and (2) measure the mechanical properties (quality factor and resonant frequency) of mono-crystalline diamond resonators.

The results of the first experiment can be used to verify a similar experiment that currently tries to measure the Brownian motion of the same MFM-tip, but then in a more complicated setup that couples the motion of the resonator to the state of a nitrogen vacancy center (NV-center, i.e. an example of a spin qubit). The measurements shown in this report are in very good agreement with the theoretical expected value of Brownian motion and can be made using either optical interferometry of through tapered fibers. The NV-experiment has not yet been conclusive, so that comparison has not yet been made.

The second goal is interesting as it will be the first time that someone tries to measure the mechanical properties of mono-crystalline diamond resonators. It is expected that the mono-crystallinity will lead to high quality factors. Besides, the resonators themselves can now contain spin qubits (NV’s ‘live’ inside diamond) allowing one to perform new types of experiments to couple a qubit to a resonator. This report will show two frequency spectra of such resonators measured through optical interferometry in a vacuum chamber. The resonant frequencies agree with the expected value from Euler-Bernoulli beam-theory. The precise shape of the resonances is not yet fully understood. It is non-Lorentzian and seems to consist of two slightly separated peaks. It is therefore not yet possible to give a conclusion on the possible high quality factors of such resonators. Recommendations are given on how to improve the experiment and continue this line of research.
# Contents

1 Introduction: mechanical resonators as the bridge between distant spin qubits ........................................ 1  
1.1 Goals of this project .................................................................................................................. 2  
1.1.1 Verification of measured Brownian motion with the NV-setup ........................................... 2  
1.1.2 Measuring the resonant frequencies and quality factors of mono-crystalline diamond beams .................................................................................................................. 3  
1.2 Outline of this report ................................................................................................................ 4  

2 Theoretical background and experimental setups ........................................................................ 5  
2.1 Optical interferometry: making an optical cavity using the cleaved surface of an optical fiber .................................................................................................................. 5  
2.1.1 (1) Directly cleaving an optical fiber and bringing its surface close to the resonator ............ 5  
2.1.2 (2) Focussing the light from a regular non-angular cleaved fiber connector onto the resonator by using a microscope objective ......................................................... 7  
2.2 Modulating the transmission of a tapered fiber by perturbing its evanescent field ......... 7  
2.3 Euler-Bernoulli beam theory .................................................................................................... 9  
2.4 Brownian motion ..................................................................................................................... 10  

3 Measurement results .................................................................................................................. 12  
3.1 Measuring the Brownian motion of a MFM-tip with a cleaved fiber ....................................... 12  
3.2 Measuring Brownian motion of a MFM-tip using a tapered fiber ........................................ 13  
3.3 Measuring the quality factor of a MFM-tip in vacuum .......................................................... 15  
3.4 Measuring the resonant frequencies of diamond beams ......................................................... 17  
3.4.1 Results on doubly clamped diamond beams with a triangular cross-section ............. 17  
3.4.2 Results on singly clamped diamond beams with a rectangular cross-section .......... 17  

4 Conclusions and outlook ........................................................................................................... 22  
4.1 Summary of the results .............................................................................................................. 22  
4.2 Future directions/recommendations ......................................................................................... 22  
4.2.1 The cleaved fiber setup .......................................................................................................... 23  
4.2.2 The tapered fiber setup .......................................................................................................... 23  
4.2.3 The diamond resonators ...................................................................................................... 23  

5 Acknowledgements ..................................................................................................................... 25  

6 Appendix .................................................................................................................................. 27  
6.1 Effective mass .......................................................................................................................... 27  
6.2 Details on positioning the laser beam and other specific settings for a measurement .......... 27
Chapter 1

Introduction: mechanical resonators as the bridge between distant spin qubits

The controlled and coherent manipulation of spin qubits is an important challenge in modern science. Coherent control over multiple coupled qubits is a prerequisite to be able to perform non-trivial quantum computations. However, quantum information processing requires long lived quantum states which typically implies that these states are well isolated from their environment thereby only allowing strong interactions between neighbouring systems. Mechanical resonators could fill the gap between long distant spins and act as a transducer to effectively couple multiple spins to each other in a scalable way [1].

Great advances have been made in the fabrication of and manipulation of nanoelectromechanical systems. Large quality factors are attained in for e.g. SiN-membranes [2, 3] and recently a mechanical resonator has been cooled to the ground state [4]. We are now entering a regime where it is possible to create the first building blocks described in the proposal by P. Rable et al. [1]. In this proposal nanomechanical resonators are magnetically coupled to electron spin qubits. The spin qubit is controlled by local microwave fields. The strong magnetic-field gradient $G_m$ produced by the magnetic tip of the resonator results in a spin-resonator interaction of the form $H_{sr} = \hbar / 2 (a \sigma_x + a^\dagger \sigma_z)$. Here $a$ ($a^\dagger$) are annihilation (creation) operators for the fundamental vibrational mode of frequency $\omega_r$, and $\lambda = g_s \mu_B G_m a_0 / \hbar$, where $g_s \simeq 2$ and $\mu_B$ the Bohr magneton, is the Zeeman shift associated with the zero-point motion $a_0 = \sqrt{\hbar / 2 m \omega_r}$). Intuitively this coupling can be understood by realizing that the changing magnetic field (due to the oscillation of the resonator) induces shifts in the precession rates of the spin qubit.

This process ‘only’ couples a single spin qubit to a single resonator. To induce long range interactions the mechanical resonators themselves will be charged and interact capacitively with nearby wires interconnecting them. Fig. 1.1 depicts the basic idea behind this proposal. It is expected that this proposal allows the coupling of spins over a distance of approx. 100 $\mu$m. Another advantage is that the resonators can couple to different type of spin qubits; thereby allowing you to couple different systems that each have their own advantages.

At this moment S.J. Kolkowitz and Q.P. Unterreithmeier (Lukin Group, Harvard University) are coupling a single magnetic force microscopy tip (MFM-tip) to a nitrogen-vacancy center (NV-center). They have chosen for NV-centers as they can be individually addressed, optically polarized and detected, and exhibit excellent coherence properties even at room temperature [5–7]. The current status of this work is that it is possible to detect the driven motion of the MFM-tip, the next aim is to measure the Brownian motion of this tip under ambient conditions. This allows
Figure 1.1: a) Spin-dependent displacements of the magnetic tip due to magnetic interactions between the spin qubit and the magnetic field gradient. The state of the spin qubit is converted into an electric dipole proportional to the charge on the resonator. b) Coupling two qubits: the differences in electrostatic energies between the different spin configurations lead to an effective spin-spin interaction. c) Schematic for scalably coupling multiple qubits: in this scenario the resonators are coupled through capacitive coupling with nearby wires. [1]

them to set up the first building block of the proposal by magnetically coupling a single resonator (MFM-tip) to a spin qubit (the NV-center).

1.1 Goals of this project

1.1.1 Verification of measured Brownian motion with the NV-setup

The first goal of the project described in this report is the verification of the measured Brownian motion through a different measurement setup. This should exclude the possibility that the NV-setup artificially created the measured motion. Different ways have been devised to achieve this verification and details will be described in Sections 2.1.1, 2.1.2 and 2.2. In summary two different detection methods have been used: (1) optical interference, and (2) transmission modulation of a tapered optical fiber. It’s assumed for the introduction that the reader is familiar with using optical interference as a detection method, but a short description is given in Section 2.1.1. Most readers will probably be less familiar with option (2), so Fig. 1.2 demonstrates the working principle. At the location of the taper the light waves transmitted through the fiber will for a large part propagate along, but outside the optical fiber. Any object close to the taper will block these light waves and thereby modulate the total transmission of the fiber depending on its distance to the taper. By looking at the transmission modulation one can detect the mechanical properties of the resonator.

Why did we explore two different methods?: the first method requires more optics and alignment than the second, but it’s relatively easy to build as the production of tapered optical fibers is not trivial. The second method has the advantage that it in principle could allow for on-chip detection if the tapered fiber is engineered close to the resonator during the fabrication process, see Fig. 1.3 (a) for an example by [8].
1.1. GOALS OF THIS PROJECT

Figure 1.2: Schematic overview to measure the motion of a resonator with a tapered optical fiber. The light propagating outside the taper will be blocked by the nearby resonator and thereby modulate the overall transmission of the fiber. Details can be found in Section 2.2.

![Schematic overview](image)

Figure 1.3: (a) An example of the use of a tapered fiber to measure displacements of a nearby resonator [8]. (b) SEM-picture of a singly clamped diamond resonator with a thickness of approximately 2 micron, thickness of 100 nm and length of 10 micron (fabricated by P. Maletinsky, Yacoby group, Harvard University).

1.1.2 Measuring the resonant frequencies and quality factors of mono-crystalline diamond beams

Next to the goal of measuring Brownian motion, there is the important goal of characterizing the mechanical properties of mechanical resonators made from mono-crystalline diamond. See Fig. 1.3 (b) for an example of a singly clamped diamond resonator. Diamond as a material has the highest know Young’s modulus which translates to a higher frequency for a given device size. Higher frequencies can be an advantage when performing quantum mechanical experiments at low temperature (it’s for example easier to reach the ground state). Other advantages of diamond include its high thermal conductivity (allowing efficient thermalization of nanostructures) and its high chemical resistance (no oxidation of the surface which leads to losses in the resonator).

In this experiment, *mono-crystalline* diamond will be used to create mechanical resonators and characterize their quality factor ($Q$) and resonant frequency ($f_0$), which to our knowledge will be the first time ever. Similar experiments have been performed on nano-crystalline diamond (i.e poly-crystalline diamond composed of small grains (size up to 100 nm) of mono-crystalline diamond) reporting quality factors in the range of $10^3 - 10^4$ [9–11]. The motivation to go to mono-crystalline diamond is the expectation that such a structure will (1) have less loss mechanisms (e.g. friction between the grains in nano-crystalline diamond), and (2) it should allow longer coherence times ($T2$) for NV-centers inside the resonator.

(1) is interesting as the loss mechanisms determine the quality factor of the resonator, and thereby set the time-scale for your experiment; the larger the $Q$, the easier it will be to reach the strong coupling regime between your resonator and the spin qubit. (2) is important as the presence of long coherent NV-centers can be used for several applications. One example of such an application can be found in an adjustment of the current experiment of S.J. Kolkowitz and Q.P. Unterreithmeier. As the NV is now inside the resonator, the presence of a fixed external magnet with a high
field gradient will be enough to couple the resonator to the NV: movement of the resonator will just as in the previous case induce a Zeeman shift in the energy levels of the NV-center, see Fig. 1.4. This scenario could possibly also work without the external magnet, if the changes in strain in the diamond due to movement of the beam induce large enough shifts in the energy levels of the NV-center. It is known that the energy shifts induced by strain are more prominent for the first excited state of the NV (orbital doublet) than for the ground state (orbital singlet) [12, 13]. Unfortunately the lifetime of the excited state is much shorter than the lifetime of the ground state. At the time of writing this report we have not yet calculated whether the strain will be large enough to couple the ground state of the NV to the movement of the beam. Another interesting application would be the usage of these resonators as AFM-tips whereby the inclusion of the NV allows one to perform magnetometry in a similar way as described in [14]. The small dimensions of an NV-center allow you to probe the local magnetic field in a high spatial resolution.

For the current state of the project we are specifically interested in the resonant frequencies and the quality factors of these diamond beams. The values of $f_0$ and $Q$ will determine whether it is possible in the future to strongly couple NV-centers inside these beams to the external magnet.

### 1.2 Outline of this report

This report presents several methods of measuring the Brownian motion of MFM-tips and characterizing the mechanical properties of diamond beams.

**Chapter 2** will give a more formal basis of the detection schemes used in this project and contains a combination of theory and practical issues. It will end with two short sections on the theory needed to calculate the expected resonant frequencies of the diamond beams and the amplitude of motion from Brownian motion.

**Chapter 3** contains an overview of the experimental results showing (a) the measured Brownian motion of a MFM-tip using either option (1) of the cleaved fiber or (2) of the tapered fiber, (b) the quality factor of the MFM-tips in vacuum and (c) results on mono-crystalline diamond beams.

**Chapter 4** summarizes the results and contains recommendations to extend this line of research.

This research has been performed starting March 2011 until the end of July 2011 as part of my third master rotation for the Casimir pre-PhD track.
Chapter 2

Theoretical background and experimental setups

In this project three different methods have been used to detect mechanical motion. This chapter will first briefly describe the theory needed to understand each method and discuss some of the practical issues. Next the Euler-Bernoulli beam theory will be briefly discussed to calculate the expected resonant frequencies for the diamond beams. The chapter will conclude with a section on Brownian motion.

2.1 Optical interferometry: making an optical cavity using the cleaved surface of an optical fiber

Two different ways will be used to make an optical interferometer with the use of the cleaved surface of an optical fiber: (1) directly cleaving an optical fiber and bringing its surface close to the resonator, and (2) focussing the light from a regular non-angular cleaved fiber connector onto the resonator using a microscope objective. Both methods will be described in this section.

2.1.1 (1) Directly cleaving an optical fiber and bringing its surface close to the resonator

Figure 2.1 shows the basic setup required for making an optical cavity between an optical fiber and a mechanical resonator. The optical fiber is prepared by removing its coating and cleaving it with a diamond wedge scribe to form an almost perfectly flat cleave at an angle of 90 degrees. The result is a fiber-probe that transmits most light (around 96%) and reflects the rest directly back into the fiber.

The transmitted light will reflect on the surface of the resonator and when reaching the fiber-probe either be transmitted into the fiber, or reflected again back to the resonator to repeat this cycle. Summing all possible pathways leads to the following expression for the intensity of the light, \( I \), reflected at the other end of the fiber to the detector:

\[
I = \left| r_f + (t_f t_{fp} e^{-2k_0 L})/(1 - r_{fp} e^{-2k_0 L}) \right|^2;
\] (2.1)

Where \( r_f \) (\( t_f \)) is the amplitude of reflectance (transmittance) when leaving the fiber, similarly for \( r_{fp} \) (\( t_{fp} \)) when entering the fiber, \( k_0 \) is the wavevector \( 2\pi/\lambda \) in air and \( L \) is the distance between the resonator and the fiber. For simplicity perfect reflectance of the resonator is assumed.

For typical values of \( r_f = r_{fp} = 0.2 \) and \( \lambda = 670 \text{ nm} \) the intensity as a function of \( L \) is depicted in Fig. 2.2 (a). As the finesse of this cavity is low, the intensity modulation can be approximated by...
CHAPTER 2. THEORETICAL BACKGROUND AND EXPERIMENTAL SETUPS

Figure 2.1: Schematic overview of the setup for directly cleaving an optical fiber and bringing its surface close to the resonator. In this picture the dimensions for the MFM-tip used are shown. The cleaved fiber consists of a.o. a core where the mode field diameter of the used light is 4.6 µm; the overall thickness of the cleaved fiber after removing its coating is 125 µm. Of the incoming light, a fraction \( t_f \) is transmitted in the direction of the resonator and a fraction \( r_f \) is directly reflected back into the fiber. The resonator will reflect a fraction \( r_{\text{resonator}} \). When the reflected light from the resonator reaches the cleaved surface it will be partly reflected back to the resonator \( (r_{fp}) \) and partly transmitted back into the fiber \( (t_{fp}) \). The sum of all these processes leads to an interference pattern as described by Eq. 2.1 and displayed in Fig. 2.2.

A sine. This measurement system can be calibrated by driving the resonator (with a piezo) in such a way that its amplitude of motion is at least a quarter of the used wavelength. In that scenario each cycle will span more than half a fringe in the intensity modulation and the peak-to-peak value in voltage, \( V_{pp} \), can be deduced. \( V_{pp} \) gives an estimate of the maximal sensitivity [V/m] of the system by calculating the maximum slope of the intensity modulation from \( \frac{\Delta I}{\lambda} \frac{V_{pp}}{\text{amplification}} \) where a sine model is assumed and the ‘amplification’ denotes the amplification-value of the used amplifiers. An example is shown in Fig. 2.2 (b).

![Figure 2.2: (a) Measured intensity \( I \) as a function of \( L \) for \( r_f = r_{fp} = 0.2 \) and \( \lambda = 670 \) nm. (b) Method of calibration: if the amplitude of motion of the driven resonator is at least a quarter of the used wavelength, each cycle will span more than half a fringe in the intensity modulation and the peak-to-peak voltage can be converted to the maximum sensitivity of the system. In this example the maximum sensitivity was 6.71 V/m. The red line is a fit of 4 sines with frequency \( f_0 \), \( 2f_0 \), \( 3f_0 \) and \( 4f_0 \). By mounting the cleaved fiber on a xyz-stage with piezos for the fine tuning, one can tune the distance \( L \) such that you are in the region of maximum sensitivity. Overall, this is a quick method to characterize resonators under ambient conditions.](image-url)
2.1.2 (2) Focussing the light from a regular non-angular cleaved fiber connector onto the resonator by using a microscope objective

This method is based on the same principle as the one just described, the only difference is that one uses a microscope objective to focus the light from the cleaved end of the fiber on the resonator. The working distance (WD) of the objective determines the required distance between the objective and the MFM-tip and can easily be 1 cm (much larger than option (1)); this is typically enough to look at the resonator in a vacuum chamber through a plastic window. Depending on the size of the resonators, a focussed spot might certainly give you more reflected light from the resonator compared to option (1) where the mode field diameter of the light is already 4.6 $\mu$m. An interference pattern will arise from the light internally reflected at the non-angular cleaved fiber connector and the light reflected from the MFM-tip. See Fig. 2.3 for a schematic overview.

Alternative way of generating an interference pattern

Next to the method of creating interference with the reflected light from the cleaved end of an optical fiber, it should also be possible to create an interference pattern between the resonator and a nearby reflective substrate, see Fig. 2.4 (a). This method is insensitive to distance variations between the microscope objective and the sample. The disadvantage is that the signal strength is likely to be less if the substrate and the resonator are rather rough or have non-flat structures. To change the measurement setup to this ‘modus’, one needs to replace the non-angular cleaved fiber connector with an angular cleaved fiber connector (which should have almost no internal reflections at the location of the cleave). This method has been tried on the doubly clamped diamond beams as shown in Fig. 2.4 (b) (see Section 2.3 for more information on these beams). These had rather rough underlying substrates and had a triangular cross-section.

2.2 Modulating the transmission of a tapered fiber by perturbing its evanescent field

The third method uses a totally different mechanism of detection: modulation of the evanescent field of a tapered fiber. A tapered fiber can be created by pulling on a single mode optical fiber whilst heating it with a flame. This allows you to shrink the diameter $D$ of the fiber from 125 micron to approximately 800 nm, see Fig. 2.5. By sending light with a wavelength $> D$, a large portion of the light will travel as an evanescent wave outside the fiber itself. If the fiber’s
CHAPTER 2. THEORETICAL BACKGROUND AND EXPERIMENTAL SETUPS

Figure 2.4: (a) Alternative way of creating an interference pattern: the incoming light will partly be reflected and partly transmitted at the location of the resonator (for diamond 17% should be reflected). The transmitted light will partly reflect at the substrate and the returning wavefront will either reflect or transmit again at the interface of the resonator. The transmitted light will interfere with the directly reflected light from the incoming beam. Incorporating multiple reflections leads to a similar formula as Eq. 2.1. Motion of the resonator will translate in a changing distance $L$ and therefore a modulation in the measured intensity. (b) SEM picture of a doubly clamped diamond beam with a length of approx. 20 $\mu$m, width of approx. 400 nm and angle of etching of 45 degrees. This diamond beam has been made by B.J. Shields and N.P. de Leon (Lukin group, Harvard University).

Figure 2.5: Schematic overview of a tapered fiber. A resonator brought closely to the center of the taper will perturb the evanescent wave (depicted by the red cylinder) and thereby modulate the transmission of the fiber.

shape is altered adiabatically the light will completely enter the single mode fiber of 125 micron again after it has passed the taper. However, if a high-index dielectric such as silicon is present in the vicinity of the taper; the evanescent tail couples to the dielectric and thereby reduces the transmission $T$ through the fiber. By measuring the transmission of the fiber as a function of time, one can determine the frequency spectrum of the resonator. This method preferably uses a large wavelength laser source (we used $\lambda = 1550$ nm), and should allow you to bring the fiber relatively close to the resonator. The evanescent field decays on a length scale of $(\lambda/2\pi)/\sqrt{n^2-1} \approx 238$ nm where $n = 1.44$ is the refractive index of the optical fiber. The previous two methods don’t suffer from such a small length scale restriction. The advantage of this method is however that a substantially less amount of optics is needed and it’s less sensitive to the overall alignment with respect to the resonator. In principe this method should allow on chip devices, where the tapered fiber is incorporated next to the resonator of interest during the fabrication process.
2.3 Euler-Bernoulli beam theory

In this project two types of resonators have been measured: I doubly clamped beams with a triangular cross-section (e.g. in Fig. 2.4 (b)) and II singly clamped beams with a rectangular cross-section (e.g. in Fig. 1.3 (b)). This section will briefly describe how the Euler-Bernoulli beam theory has been implemented to find an estimated value of their resonant frequency. For I the shape of the beam is assumed to be triangular with \( b \) the width of the beam, \( h \) the height and \( \phi \) the angle as defined in Fig. 2.6. After this calculation it is quickly possible to derive the formula for the rectangular beams.

One starts with the Euler-Bernoulli beam equation for a beam with no load:

\[
\frac{\partial^4 V(x,t)}{\partial x^4} + \rho A \frac{\partial^2 V(x,t)}{\partial t^2} = 0 \tag{2.2}
\]

Where \( V(x,t) \) describes the deflection of the beam in the \( y \)-direction (beam is along \( x \)), \( \rho \) is the density of the material, \( A \) the cross-section of the beam, \( E \) the young modulus and \( I \) the second moment of area (which is \( bh^3/36 \) for a triangular beam). Assuming that the deflection will be of the form \( V(x,t) = V(x) \cos(\omega t - \phi) \), gives the following differential equation:

\[
\frac{\partial^4 V(x)}{\partial x^4} - \frac{\rho A \omega^2}{EI} \frac{\partial^2 V(x)}{\partial t^2} = 0 \tag{2.3}
\]

Typically one renames the term \( \frac{\rho A \omega^2}{EI} \) to \( \lambda^4 \). Then \( V(x) \) should be of the form:

\[
V(x) = c_1 \sinh(\lambda x) + c_2 \cosh(\lambda x) + c_3 \sin(\lambda x) + c_4 \cos(\lambda x) \tag{2.4}
\]

Applying the boundary conditions for a doubly clamped beam: \( V(x = 0) = V(x = L) = 0 \) and \( \frac{\partial V(x)}{\partial x}|_{x=0} = \frac{\partial V(x)}{\partial x}|_{x=L} = 0 \) leads to the characteristic equation \( \cosh(\lambda L) \cos(\lambda L) - 1 = 0 \). Solving this equation gives values for \( \lambda \) from which the resonant frequencies \( f = \sqrt{\frac{EI\lambda^2}{2\pi\rho A}/2\pi = \sqrt{\frac{E(\lambda L)^2}{2\pi\rho A}} \cdot \frac{b \tan(\phi)}{L^2}} \) follow. The smallest value of \( \lambda L = 4.73 \); Table 2.1 give estimates for the ground frequencies of beams varying in size from 5 to 15 \( \mu \)m, widths of 200 to 400 nm and etched under an angle of 45 degrees. The properties of bulk diamond are assumed: \( \rho = 3.5 \cdot 10^3 \) kg m\(^{-3}\) and \( E = 1220 \cdot 10^9 \) Pa.

### Table 2.1: Estimated ground frequencies [MHz] for varying widths \( b \) and lengths \( L \) of doubly clamped diamond resonators for an angle \( \phi \) of 45 degrees.

<table>
<thead>
<tr>
<th>( b/L )</th>
<th>5 ( \mu )m</th>
<th>10 ( \mu )m</th>
<th>15 ( \mu )m</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 nm</td>
<td>63</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>300 nm</td>
<td>94</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>400 nm</td>
<td>125</td>
<td>31</td>
<td>14</td>
</tr>
</tbody>
</table>

Singly clamped beams with rectangular cross-section

For the singly clamped beams with a rectangular cross-section the resonant frequencies can quickly
be found by replacing the second moment of area with \( I = bh^3/12 \) and changing the boundary conditions to \( V(x=0) = 0, \frac{\partial V(x)}{\partial x} \big|_{x=0} = 0, \frac{\partial^2 V(x)}{\partial x^2} \big|_{x=L} = 0 \) and \( \frac{\partial^2 V(x)}{\partial x^2} \big|_{x=L} = 0 \) (leading to a smallest value for \( XL \) of 1.875). For the resonator shown in Fig. 1.3 (b) one would get an estimated frequency of around 3.6 MHz.

The final important parameter for these resonators will be their quality factor. The quality factor is defined as \( Q = \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} \). For large values of \( Q \) it can easily be deduced from the frequency spectrum by \( Q = \frac{f_0}{\Delta f} \), with \( f_0 \) the center frequency and \( \Delta f \) the range of frequencies for which the energy is at least half its peak value. The \( Q \)-factor determines the time scale on which a resonator will decay/how much it is coupled to the environment, thereby setting the amount of time it can interact with other systems (such as NV-centers).

It has been decided to not estimate the \( Q \)-factor beforehand. In principle it should be possible to calculate these but from private communications with Q.P. Unterreithmeier and J.D. Thompson I deduced that these theoretical values tend to not agree with experiment that well. The time scale of this internship would not allow for a thorough estimate.

### 2.4 Brownian motion

As described in the introduction, one of the motivations for this research is to measure the Brownian motion of a MFM-tip in a different way than with NV’s. This allows us to compare the two results for a consistency check. This section will describe how to derive the amplitude of motion versus frequency spectrum of a resonator performing Brownian motion. It is a summary of the findings reported in [15].

The differential equation describing the amplitude of motion, \( x(t) \), of a resonator under influence of a randomly fluctuating force \( a(t) \) reads:

\[
\frac{d^2 x(t)}{dt^2} = -\gamma \frac{dx(t)}{dt} - k \frac{m}{m} x(t) + \frac{a(t)}{m} \tag{2.5}
\]

Where \( m \gamma \frac{dx(t)}{dt} \) describes the friction force, \( kx(t) \) the restoring spring force and \( m \) the effective mass of the resonator (\( m \) is calculated at the end of this section). Next a fourier transform is applied:

\[
-\omega^2 X(\omega) = -i\omega X(\omega) - k \frac{m}{m} X(\omega) + A \tag{2.6}
\]

Where \( X(\omega) \) is the fourier transform of \( x(t) \) and the fourier transform of \( a(t) \) is a constant: \( A \), which is to be determined later on to fit the equipartition theorem. Solving for \( X(\omega) \) and using \( k = m\omega_0^2 \) (with \( \omega_0 \) the resonant frequency of the resonator) gives:

\[
X(\omega) = \frac{A}{m(\omega_0^2 - \omega^2 + i\gamma\omega)} \tag{2.7}
\]

To fix the value of \( A \) we apply the equipartition theorem which states that \( 1/2m\omega_0^2 \langle x^2 \rangle = k_b T/2 \) with \( k_b \) the Boltzmann constant and \( T \) the temperature:

\[
1/2m\omega_0^2 \int_0^\infty X^2 \, d\omega = k_b T/2 \tag{2.8}
\]

Resulting in \( A = \sqrt{3m k_b T} \). Typically \( \gamma \) is replaced with \( \rightarrow \omega_0/Q \); this is consistent with the definition for \( Q \) given in section 2.3 for \( Q \gg 1 \) and more easy to implement as one generally measures the quality factor and the resonant frequency of a resonator (and not \( \gamma \) directly). This leads to the following frequency spectrum of a resonator performing Brownian motion:

\[
X(\omega)^2 = \frac{f_0 k_b Q T}{2m \pi^3 (f_0^4 Q^2 + f_0^4 Q^2 + f_0^2 (1 - 2Q^2))} \tag{2.9}
\]
Here $X(\omega)$ has the units of [m Hz^{-0.5}]. This equation allows us to calibrate the system; or if the system can be calibrated in a different way, tell us whether we are truly seeing Brownian motion.

**The effective mass of the resonator**

Not every part of the resonator will oscillate with the same amplitude: this depends on the boundary conditions. To correct for this the $m$ used in Eq. 2.9 is the *effective* mass of the resonator. Calculations for the effective mass are described in the Appendix 6.1.
Chapter 3

Measurement results

This chapter describes the measurement results attained during this experiment. The first and second section describe the measurement of Brownian motion of a MFM-tip using optical interferometry and transmission modulation of a tapered fiber respectively. The third section demonstrates the working principle of the vacuum setup on the same MFM-tip. Section 4 concludes with the frequency spectra measured on diamond resonators.

3.1 Measuring the Brownian motion of a MFM-tip with a cleaved fiber

These measurements have been performed as described in Section 2.1.1: directly cleaving an optical fiber and bringing its surface close to the resonator. Before each measurement a calibration was made of the system and by looking at the demodulated signal from the lock-in detector (Zurich Instrument H2FLI) the distance between the optical fiber and the resonator was tuned such that the system was in its most sensitive point. The drift of the system was characterized and it turned out that it was possible to keep the signal within approx. 20% of its most sensitive point for more than 5 minutes. The cleave of the used fiber had a reflectance of 2.1% (theoretically this value can reach a maximum of 4%).

The results for two different measurements are shown in Fig. 3.1. (a) is the result of 40 averages at a bandwidth of 20 Hz and took approx. 21 minutes. (b) is the result of 10 averages at the same bandwidth and took 5 minutes. Both measurements were done with the driving signal of the lock-in detector mechanically decoupled from the system to exclude any possible drive. The red lines are a fit to Eq. 2.7 where the free parameters are $Q, f_0, A/m$ and an additional offset. The blue lines are the theoretical expected spectra using the fitted values for $Q$ and $f_0$ and an estimate for the mass of the resonators from their specifications (length = 240 $\mu$m, width = 30 $\mu$m and thickness = 2.7 $\mu$m, material: silicon, leading to 45.3 nanogram).

Fig. 3.1 (a) shows a measured rms-amplitude $A_{rms}$ of 41.5 pm, and a theoretically expected $A_{rms}$ of 39.8 pm. The red fit returns a quality factor of 200 and a center frequency of 76.5 kHz. The noise floor is 0.81 pm Hz$^{-0.5}$. Fig. 3.1 (b) shows a measured rms-amplitude $A_{rms}$ of 38.3 pm, and a theoretically expected $A_{rms}$ of 39.8 pm. The red fit returns a quality factor of 193 and a center frequency of 76.5 kHz. The noise floor is 0.79 pm Hz$^{-0.5}$.

It must be noted that the center frequency is somewhat lower for driven motion, for a very low drive the center frequency lies at 76.3 kHz. It is unknown what's causing this.

These results clearly demonstrate that this relatively simple experimental setup can be used to verify the measured Brownian motion measured with a NV-center.
3.2 Measuring Brownian Motion of a MFM-tip using a Tapered Fiber

These measurements have been performed as described in Section 2.2. The tapered fiber itself was made from a single mode fiber (SM980-5.8-125) on a setup designed by T.G. Tiecke (Lukin group, Harvard University). This setup consisted of two motorized stages to which you could attach either side of the fiber. Another motorized stage allowed you to position the flame. For the tapered fibers used in this experiment the position of the flame was kept fixed at the center of the two other motorized stages. These two stages were programmed to symmetrically pull an exponential decaying profile for the thickness of the taper. The best results (i.e. a thin taper and being able to make one taper within 5 hours) were attained for a total length of 22 mm at a speed of 1 mm/s.

The total flame consisted out of a large dark blue flame with a smaller light blue flame inside (butane gas was used). The best results were attained by positioning the fiber at the top of the inner light blue flame; at that location the airflow was low (preventing the breaking of the taper). At the end of the pulling-procedure one can use the turbulent part of the flame to make the taper even thinner. This procedure will make the shape of the taper non-exponential (and thereby less adiabatic) and has a high chance of breaking the taper; however, if one succeeds one can get a very thin taper and thereby maximize the field strength outside the fiber. See Fig. 3.2 (a) for an SEM-image of the thinnest tapered fiber made during this experiment (D = 733 nm). A taper with a diameter of 840 nm was used to produce the results shown in Fig. 3.3.

After the complete procedure the transmission of the fiber has typically decreased by at least 50%. Using the cleaved fiber setup described in Section 3.1 the voltages applied to the piezos on the xyz-stage could be converted to a distance. Using this coarse calibration (piezos are known to have a lot of hysteresis) it could be determined that it takes approx. 2.3 µm for the transmission of the fiber to go from 100% (the value for the resonator far away) to 20.4% (resonator touching the tapered fiber). See Fig. 3.2 (b) for a photograph of the tapered fiber close to the MFM-tip. Figure 3.3 shows the measured Brownian motion on the same MFM-tip as used in Section 3.1. The measurement took 16 minutes for 40 averages at a bandwidth of 20 Hz. The results is Q of 212 and a center frequency of 76.5 kHz. Please note that in this case the y-axis is given in V^2.

Figure 3.1: The measured undriven amplitude of motion of a MFM-tip using a cleaved fiber. (a) The measured (red fit) $A_{rms}$ is 41.5 pm, the theoretically expected (blue line) $A_{rms}$ is 39.8 pm. The red fit returns a quality factor of 200 and a center frequency of 76.5 kHz. The noise floor is 0.81 pm Hz^{-0.5}. 40 averages, bandwidth 20 Hz. (b) The measured (red fit) $A_{rms}$ is 38.3 pm, the theoretically expected (blue line) $A_{rms}$ is 39.8 pm. The red fit returns a quality factor of 193 and a center frequency of 76.5 kHz. The noise floor is 0.79 pm Hz^{-0.5}. 10 averages, bandwidth 20 Hz.
which is proportional to $\langle x^2 \rangle$. It is not possible to calibrate this system, and therefore the noise floor is calculated on the assumption that this spectrum corresponds to the theoretically expected curve for Brownian motion returning a value of $1.1 \text{ pm Hz}^{-0.5}$.

From the ‘piezo voltage to distance-calibration’ mentioned earlier, it can be deduced that during this measurement the resonator was on the order of 100 nm away from the sticking point to the tapered fiber.

These results can directly be compared to the results from Section 3.1 as the same MFM-tip has been used. The center frequencies are in perfect agreement. The quality factor measured with the tapered fiber is however 5% larger. Another measurement of 50 averages even reveals a $Q$ of 285 (same $f_0$, noise floor 1.3 pm Hz$^{-0.5}$) and it is not known where this increase in $Q$ comes from. It was planned to repeat this experiment later on on a better isolated optical table (which was not yet available) to find the cause for this difference, but by the time the table had arrived the overall transmission of this fiber had gone to zero (see the next section on ‘practical problems’).

Another difference with the measurements from Section 3.1 is the noise floor which is approx. 1.5 times larger. It was also planned to investigate this with the new optical table by looking at the noise of the laser itself (the two experiments required different lasers), comparing noise levels between different optical tables and measuring the noise floor of the photodetectors. For the same reason as just described this part of the experiment has not been performed.

**Practical problems encountered with the tapered fibers**

Although the use of tapered fibers does not require any complicated optics and alignment procedures, there are some other disadvantages that arise. The most important ones will be summarized here for future reference:

- The transmission modulation works best when the resonator is close to the taper; this is however also exactly the regime where mechanical levels of noise (e.g. vibrations from the environment) combined with Vanderwaals (and possible dielectric forces) make it very likely that the resonator will stick to the taper. Fortunately the tapered fibers are relatively robust (they are strong enough to break the MFM-tip when you push them). It does however require that your setup has the option to retract the taper far enough, and that might not be the case if one designs on-chip devices with the fiber incorporated. It also means that one has to monitor during a measurement whether a fiber has ‘collapsed’. This problem is completely absent in the case of cleaved fibers.
3.3 Measuring the quality factor of a MFM-tip in vacuum

In the future the experiment that S.J. Kolkowitz and Q.P. Unterreithmeier are currently performing might be done under vacuum conditions. Therefore the method described in Section 2.1.2 (focussing the light from a regular non-angular cleaved fiber connector onto the resonator by using a microscope objective) has been performed on the MFM-tip. This experiment also provides a good test of the setup before going to the diamond resonators.

Optimizing the spot size

The objective used (10X Olympus Plan Achromat Objective) has a NA of 0.25 and an effective focal length of $f = 18$ mm. Combining the following two equations:

$$\text{NA} = \frac{D}{2f} \quad (3.1)$$

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_r}\right)^2} \quad (3.2)$$

where $D$ is the diameter of the inner aperture of the objective, $w(z)$ is the width of a focussed Gaussian beam profile as a function of the distance $z$ from its focal point and $z_r = \frac{\pi w_0^2}{\lambda}$. The smallest spot size, $w_0$, is found by calculating $D$ from Eq. 3.1 and replacing $w(f)$ with $D/2$. For the objective used this leads to a $1/e^2$-radius (the radius for which the intensity is $1/e^2$ of the intensity at the center of the beam) of 853 nm. Fig. 3.4 (a) shows the measured spot size with a $1/e^2$-radius of 1.6 μm. It’s not perfect, but close enough to the theoretical limit given the dimensions of the MFM-tip.
Figure 3.4: (a) Measured spot size used for this experiment. A gaussian fit returns a $1/e^2$-radius of 1.6 $\mu$m. The x-axis denotes pixel number, where each pixel corresponds to a known size of 0.1344 $\mu$m. (b) Amplitude of motion spectrum of the same MFM-tip used for the experiments in Sections 3.1 and 3.2. This measurement was performed at a pressure of approx. 66 mTorr and at a driving amplitude of 0.2 mV. The bandwidth was 4.9 Hz and the result consists of 20 averages. The red fit is a fit to Eq. 2.9 returning a $Q = 4100$ and a $f_0 = 76.7$ kHz.

<table>
<thead>
<tr>
<th>Measurement nr.</th>
<th>Driving amplitude</th>
<th>Pressure</th>
<th>$f_0$ [Hz]</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 mV</td>
<td>68 mTorr</td>
<td>76680</td>
<td>3666</td>
</tr>
<tr>
<td>2</td>
<td>0.5 mV</td>
<td>66 mTorr</td>
<td>76681</td>
<td>3874</td>
</tr>
<tr>
<td>3</td>
<td>0.2 mV</td>
<td>66 mTorr</td>
<td>76679</td>
<td>4106</td>
</tr>
<tr>
<td>4</td>
<td>1 mV</td>
<td>5.99-7.34 Torr</td>
<td>76680</td>
<td>1480</td>
</tr>
<tr>
<td>5</td>
<td>0.5 mV</td>
<td>7.34-6.48 Torr</td>
<td>76680</td>
<td>986</td>
</tr>
</tbody>
</table>

Table 3.1: Overview of the results attained from measuring the MFM-tip in vacuum. The pressure for measurement nr. 4 was reached by closing the valve to the vacuum chamber and turning of the roughing pump. In the case of measurement nr. 5 the valve was closed with the roughing pump on. In both scenario’s the first pressure mentioned is the starting pressure at the beginning of the measurement.

Measuring the MFM-tips in vacuum
The $Q$ and $f_0$ of the MFM-tips have been measured for different pressures and different driving amplitudes on the piezo. A typical spectrum is shown in Fig. 3.4 (b) and an overview of all the results is shown in Table 3.1. The same MFM-tip as described in Sections 3.1 and 3.2 was used. If we compare these results with those from Sections 3.1 and 3.2 we see that the $Q$ has certainly increased by an order of magnitude for a pressure of 66 mTorr. From these measurements it can not be deduced whether we have reached the limiting value of $Q$ as a function of air pressure. It could be that decreasing the pressure even more will increase the $Q$ further. At a certain point the $Q$ will no longer be limited by the friction with air molecules, but by clamping losses and internal losses due to bending and stretching of the material.

Another difference with the earlier measurements can be found in the value of the resonant frequency. It seems to have shifted from 76.3 kHz\footnote{This was the value for driven motion mentioned in Section 3.1} to 76.7 kHz. In principle you would expect the resonant frequency in vacuum to be slightly larger as the resonator effectively carries less mass from the surrounding air molecules that could stitch to it. This hypothesis seems to be confirmed by the fact that a vented chamber gives a center frequency of 76.3 kHz again.

Table 3.1 also reveals that $Q$ depends on the driving amplitude. Measurements 1 till 3 indicate that a smaller driving amplitude increases the $Q$, while measurement 4 and 5 show the opposite
3.4. MEASURING THE RESONANT FREQUENCIES OF DIAMOND BEAMS

result. It is known that for large driving amplitudes one will reach the non-linear regime of the resonator, but that does not seem to be the case from the measured spectra (they should become asymmetric). If one is interested in the relation between driving amplitude and the value of $Q$, it would be an easy experiment to take more data points and extract the behavior. This was however not one of the goals of this project.

3.4 Measuring the resonant frequencies of diamond beams

3.4.1 Results on doubly clamped diamond beams with a triangular cross-section

Measurements on various doubly clamped diamond beams have been performed. Lengths of 5 and 20 micron and angles of 45 and 60 degrees have been tried, all with a width between 200-400 nm. The first measurements were performed using a setup designed to measure interference between the resonator and the underlying substrate (see final paragraph of Section 2.1.2); this should be less sensitive to possible distance variations between the microscope objective and the sample. These measurements showed no distinctive peaks in the frequency spectrum. The same was true for looking directly at the interference between the resonator and the non-angular cleaved fiber. Varies settings for the bandwidth and the driving power of the actuation piezo have been tried.

Possible reasons why no signal was found include:

1. The spot size of the laser had a $1/e^2$ diameter of 3.2 $\mu$m which is rather large compared to the width of the resonators (max. 400 nm). So most of the reflected signal will be from the non-resonating substrate. A higher NA objective could improve this situation.

2. The resonators have a triangular cross-section: light will not follow a straight path through the resonator to the substrate and back to the objective. This makes it more difficult to create an interference pattern between the resonator and the substrate. It must be noted though that as these resonators are smaller than the wavelength, the geometric optics picture is not directly applicable.

3. The underlying substrate is far from nice and flat. See Fig. 3.5 for an example. This adds up to the previous argument that quite some light will never reflect to the objective. During this internship the fabrication process of these resonators made rapid progress and at the end of the project much cleaner resonators had been made. It should now be possible to get rid of the triangular shape directly underneath the resonator by asymmetrically etching from both sides.

Another interesting feature measured was the fact that when focussing the laser-spot one on of the anchor-points of the resonators such as in Fig. 2.4 (b) (which are more similar in size to the laser spot), that the amount of reflected light is 2-3 times higher as the amount of light reflected from the surrounding diamond substrate (both were focussed at their respective distance). SEM-images show no significant difference in roughness between the surface of the anchor-points or the surrounding substrate. A possible explanation could be that there is still some aluminium left from the fabrication process on the anchor-points, but a lot of cleaning processes have taken place after the application the aluminium layer.

3.4.2 Results on singly clamped diamond beams with a rectangular cross-section

During the period that the doubly clamped resonators were fabricated by B.J. Shields and N.P. de Leon, P. Maletinsky designed a whole array of singly (and also doubly) clamped resonators with a rectangular cross-section and a width on the order of 2 $\mu$m. See Fig. 3.6 (a) for an example of such
CHAPTER 3. MEASUREMENT RESULTS

Figure 3.5: SEM picture showing the roughness and shape of the substrate directly underneath the resonators.

Figure 3.5: SEM picture showing the roughness and shape of the substrate directly underneath the resonators. These resonators have two advantages: (1) the width of the resonators is more comparable to the width of the laser-spot, (2) if the beam spot is not positioned on a resonator, you don’t get a reflection (there is no underlying substrate). This allows you to more easily position the laser spot (details on positioning the laser spot and other specific settings can be found in the Appendix 6.2). A typical measurement on this array is shown in Fig. 3.7 (a). This measurement has been taken at the end of resonator A. The expected resonant frequency is between 2.8-5.3 MHz (main uncertainty comes from the thickness variation). There are many distinct peaks visible, but there should in principle only be one (and possible higher modes). By moving the beam spot along the resonator, the relative signal (compared to the other peaks) from the two peaks around 4 MHz (3.96 and 4.00 MHz) decreased making one of those the likely candidate for being the resonance from resonator A.

Figure 3.6: (a) A zoom in of part of the array designed by P. Maletinsky. Within the array one can distinguish ‘squares’ that each contain a mix of singly and doubly clamped resonators of varying size. The total array consisted of approximately 50 of these squares. (b) ‘A’ and ‘B’ denote the resonator used for the measurement results shown in Fig. 3.7 and 3.8. Resonator A has a length of approx. 16 µm and a thickness between 240 and 450 nm (corresponding to 2.8-5.3 MHz), B has a length of about 14 µm and a thickness between 360 and 530 nm (5.5-8.1 MHz).
3.4. MEASURING THE RESONANT FREQUENCIES OF DIAMOND BEAMS

The most likely explanation for this rather large collection of resonances comes from the structure of the diamond slab itself. The length scale of the ‘squares’ inside the slab are rather similar to the length scale of the resonators, and the driving piezo is attached to the bottom of this slab. So the piezo is able to excite the whole slab which can have many modes to vibrate in. The solution to this problem was to cut out the two resonators shown in Fig. 3.6 (b) and put them on a separate sample as shown in Fig. 3.7 (b).

Figure 3.7: (a) Measurement result from measuring at the end of resonator A from Fig. 3.6. (b) The first three resonators (including A and B) have been cut out of the large slab from Fig. 3.6 and put on a separate sample. The whitish color on some parts of the resonators is from the metal deposition used to ‘glue’ the sample to the chip.

Repeating the measurements on A and B gives the results shown in Fig. 3.8 (a) and (b). In this case distinct single peaks are seen on the resonators; the frequency of A is 3.83 MHz and of B 5.99 MHz. These values lie in the theoretically expected regime of 2.8-5.3 and 5.5-8.1 MHz respectively. One of the questions that can now be answered if the frequency of 3.83 MHz was indeed visible in Fig. 3.7 (a) when the resonator was still part of the full slab. The answer is very likely yes. Those two frequencies lay around 3.96 and 4.00 MHz and during the cutting of the diamond slab and ‘gluing’ it to a new substrate, some metal got deposited on the resonators thereby increasing the mass of the resonator and decreasing its resonant frequency. So one of them was likely caused directly by the resonator.

Zooming in on the resonances of A and B gives the results as shown in Fig. 3.9 (a) and (b) respectively. The frequency spectra clearly don’t look like a single lorentzian, but seem to be split in two separate frequency parts. Similar results are attained for decreasing the driving actuation up to the noise limit. The phase diagrams also indicate that we are not looking at a ‘nice’ single resonance, which should show a 180 degrees phase shift. We checked for hysteresis but none has been found. This indicates that the system behaves in a linear way.

At the moment of writing of this report we do not yet know what the explanation for the shape of these resonances is. We came up with several possibilities but none have yet been confirmed. These possibilities include:

- It could be that the second resonance comes from another vibrating mode, either a torsion mode or the mode perpendicular to the bending mode. A torsion mode is unlikely as these in general have much higher frequencies than the first bending mode. A perpendicular mode...
CHAPTER 3. MEASUREMENT RESULTS

Figure 3.8: (a) Frequency spectrum from resonator A from Fig. 3.7 (b). (b) Frequency spectrum from resonator B from Fig. 3.7 (b). The increase in background-signal for frequencies larger than 5.5 MHz is an intrinsic problem caused by the used lock-in amplifier and is also present when the laser is turned off.

will also have a higher frequency due to the width-thickness ratio of about 5:1 (leading to a 5x higher frequency).

- The crystal axis is not perfectly aligned with the resonator axis. This should in principle lead to a different effective Young’s modulus and thereby a different frequency, but it won’t lead to two different modes.

- It could be that the piezo’s response is non-uniform in this frequency range.

- If the laser power is high enough, it could drive the resonator and thereby influence its response as a function of frequency. The laser power used for this experiment was 700 µW; much lower than the typical power expected to drive these systems (several mW’s).

There was no time to take more measurements on other resonators so the amount of data is rather limited. There’s unfortunately also some metal vapor on top of the resonators making them far from ideal and certainly reducing the chance of seeing high quality factors resonances. The results do show that it’s possible to measure resonances from these diamond resonators.
Figure 3.9: (a) Zoom-in on the frequency spectrum from resonator A from Fig. 3.7 (b). (b) Zoom-in on the frequency spectrum from resonator B from Fig. 3.7 (b).
Chapter 4

Conclusions and outlook

4.1 Summary of the results

During this project different methods have been developed to measure the Brownian motion of a MFM-tip and characterize the mechanical properties of mono-crystalline diamond resonators in vacuum.

The Brownian motion of the MFM-tip has been measured using optical interferometry and transmission modulation of a tapered fiber. Both methods gave results in very good agreement with theory and can be used to verify an experiment where the Brownian motion of a similar MFM-tip is measured using a spin qubit (in this case a NV-center). The optical interferometry setup requires more optics and alignment than the tapered fiber setup, but once built it’s easier to take a measurement as it can be tuned to its most sensitive point without having the risk of sticking to the resonator. The noise floor is typically around 0.8 pm Hz^{-0.5}. The tapered fiber setup requires much less optics and is relatively easy to build. Most time is spent on creating the tapered fibers themselves. In this report thicknesses as small as 733 nm in diameter have been reported. The results attained agree very well with the optical interferometer. The noise floor was somewhat higher at 1.2 pm Hz^{-0.5}.

A vacuum setup for optical interferometry has been built and was first used to measure the behavior of the MFM-tip under vacuum conditions. The MFM-tip’s resonant frequency increased by 0.5% under vacuum conditions, and the quality factor increased by approx. a factor of 20 to a value of $Q \approx 4000$. Next the resonant frequencies of mono-crystalline diamond resonators have been measured. To our knowledge it’s the first time that the mechanical properties of mono-crystalline resonators are characterized. Doubly clamped resonators with a length of 5-20 µm, width 200-400 nm and a triangular cross-section revealed no results. Measurements on two singly clamped resonators with a length of 14 and 16 µm and a width of 2 µm each showed a distinct resonance within their theoretically expected range of frequencies. A zoom-in of these specific resonances reveals a complex non-lorentzian structure with a non-trivial phase diagram. At this moment the complete interpretation of these diagrams is unknown and also does not allow us to give a definitive answer to the question whether mono-crystalline resonators give rise to high quality factor devices. All in all the setup is there and with a little more effort and some time it should certainly be possible to fully characterize the mechanical properties of these diamond resonators.

4.2 Future directions/recommendations

Most of the time of this project has been spent on building the necessary setups and only at the final stage it was possible to get the first signals from the diamond resonators. There are therefore still many things that can be tried to improve the situation, and this section will give
recommendations towards that end.

### 4.2.1 The cleaved fiber setup

There is little that needs to be improved on the cleaved fiber setup. It did not require a superb alignment and quickly gave results. The best improvement would be to implement a feedback loop that always keeps the distance between the sample and the fiber fixed at the most sensitive point of the interferometer making you less sensitive to drift and the local environment.

### 4.2.2 The tapered fiber setup

The tapered fibers for this experiment have been made using butane gas and under non-vacuum conditions. This is not the best way to make these fibers, although it suffices to measure the Brownian motion of a MFM-tip. It would be better to use a non-carbon containing gas such as hydrogen to prevent carbon deposits on the surface of the optical fiber\(^1\). Keeping the fiber in vacuum as much as possible prevents the accumulation of dirt on the fiber and thereby conserves the transmission properties of the fiber. The current fiber lost a factor of 7 in overall transmission in a week, and did not transmit any light after a month. It would also still be nice to figure out what caused the somewhat higher noise floor for the tapered fibers.

### 4.2.3 The diamond resonators

The measurements for the diamond resonators can still use quite some improvements:

- Also apply a feedback loop that fixes the distance of the probe with respect to the sample to keep the interferometer in its most sensitive point during a measurement.

- Make sure that the coherence length of the laser is long enough. The distance between the end of the cleaved fiber and the sample was relatively large (total path length to and fro was on the order of 1m).

- For the current measurements the increase in background signal from the lock-in detector was not dominant in the frequency region of interest. There are however also resonators that will have a frequency of interest in the region where the lock-in detector already gives large signal itself. It would be useful if this problem could be resolved (a very likely partial cause is cross-talk between the external in- and output coax-cables going to the setup).

- Decrease the spot size of the laser for the smaller resonators. A high NA-objective (0.70) is available in the lab with a large enough working distance (10.0 mm); it’s very sensitive to alignment with the sample though and absorbs almost 50% of the light (each way) due to its coatings.

- Increase the laser power. The signal reaching the photodiode from the singly clamped resonators was currently around 10 \( \mu W \). It was enough to make the measurements, but in the case of the smaller doubly clamped resonators it may certainly help. This experiment was limited by the noise from the amplifiers, which is in principle not the best regime to be given the fact that more laser power isn’t too expensive (the laser used for this experiment was 4.5 mW).

- Use dielectric actuation to drive the resonators. In the current setup the whole diamond slab is actuated thereby exciting modes from the slab itself and nearby resonators. By writing local gate structures close to the resonator of interest, one can create an electric field gradient and polarize the resonator. This allows you to create an attractive force that can be modulated at high frequencies and specifically address a single resonator \(^{16}\).

---

\(^1\) We did not have a permit to use hydrogen gas in the lab.
• It would be really helpful to repeat the experiments on clean resonators (without a metal deposition). This is also the plan for the near future.

• Last but not least a theoretical problem: calculate whether the changes in strain by a vibrating resonator containing an NV, are large enough to couple the ground state of the NV to the state of the mechanical resonator.
Chapter 5

Acknowledgements

Time has gone fast and I am absolutely positive that this will be the internship that I will remember as the most special one of my life as a student. Harvard University is a very special place to be and I am still impressed by the amount of smart people and the amount of funding available to perform research. This internship was my first time to go abroad and I would especially like to thank Shimon and Quirin for me accepting me in their lab and making me be a part of the group. In the beginning it took me some time to get adjusted to the lack of technicians, but together with Shimon and Quirin I am now capable of aligning a setup in a day instead of a week. I really was a complete newbie in the field of optics when I arrived (ah, an optical fiber has a cladding?...) but I’m glad they were willing to invest the necessary time in me so that I could get my setup up and running. And when the setup wouldn’t work at once, we would just go outside and play emergency frisbee in the yard. And special thanks of course to introducing me to the song ‘It’s Friday’ from Rebecca Black; I’ll keep playing it every Friday now :P. I really hope you guys will be able to measure Brownian motion soon with your NV’s, you deserve it!

Misha, I was honored to part of your team. I am impressed by the type of people you manage to attract and the amount of creative ideas that are produced by the Lukin group. It reinforced my feeling that Physics is the place to be for me. Thanks again for sponsoring my trip to the Gordon Conference on Atomic Physics; it really helps me to start seeing all the interconnections between the different fields of physics.

Ronald, thanks for giving me the opportunity to go to such a nice location! A few years ago I would not have even dared to dream that it would be possible for me to end up at a place like this. You even gave me the option to choose between four places and quickly arranged an internship at the location of my first choice.

Tobias and Jeff, wow, you know a lot about optics. I’ll be honest with you that you have certainly set a landmark for what I would like to achieve during my own PhD. You have saved me a lot of time during my project by giving tips on how to for example check that your beam is collimated, and by almost always having the parts I just need. Good luck with trapping your atom!

Steve, I really appreciated the way you were always willing to help with questions. Even though you were not my supervisor at all and have a lot of projects running you were still willing to answer my questions. Many thanks also for introducing me to fly-fishing and next time I will buy a permit ;P.

This internship would not have been possible by the funding I have received from:

- Justus and Louise van Effen-fund.
- Hendrik Casimir prize given by the widow of H.B.G. Casimir: Mrs. Casimir-Jonker.
• Imtech Bachelor Grant given by the company Imtech.
• Nanoscale Science and Engineering Center International travel fund.
• TNW-fund given by the TU Delft.

I am very grateful to the people who have made the above funding possible.

Last but not least I would like to thank my parents for the support they have given me during my studies and have always trusted me in making my own decisions.
6.1 Effective mass

As mentioned at the end of Section 2.4, the mass used to calculate the amplitude of motion should be the effective mass. The effective mass corrects for the fact that not every part of the resonator oscillates with the same amplitude. For a singly clamped beam (corresponding to the MFM-tips) the effective mass can be calculated as follows:

Using the same procedure as in section 2.3 for a fixed-free beam, the normalized amplitude (i.e. \( V(0) = 0 \) and \( V(L) = 1 \)) of the oscillating beam is given by \( V(x) = c_1(\sinh(\lambda L \cdot x/L) - \sin(\lambda L \cdot x/L)) + c_2(\cosh(\lambda L \cdot x/L) - \cos(\lambda L \cdot x/L)) \) where \( \lambda L = 1.875 \), \( c_1 = -0.3671 \) and \( c_2 = 0.500 \).

The energy stored in an oscillation is \( \frac{1}{2} \int dA \rho \langle v^2(s) \rangle \), where \( s \) is the distance along the resonator, \( A \) the cross-section, \( \rho = m/(L A) \) the density of the material and \( v(s) \) the maximum velocity at position \( s \):

\[
\frac{1}{2} \int_{s=0}^{s=L} \frac{Am}{LA} v_0^2 V(x)^2 ds = 0.125mv_0^2
\]

The Brownian motion thus equals that of a particle with effective mass \( m_{eff} = 0.25 m_{total} \). For a doubly clamped beam (corresponding to one of the diamond ones) a similar calculation leads to an effective mass of \( 0.40 m_{total} \).

6.2 Details on positioning the laser beam and other specific settings for a measurement

This short section is meant for people that would like to use the same setup and repeat the experiment. It enumerates things to keep in mind to attain the best results.

- After placing the sample and connecting the vacuum hose: adjust the angle of the vacuum chamber with respect to the laser probe to correct for any tilt of the sample holder.
- The fine-tuning of the laser spot position with respect to the resonators is done by using the piezo’s one the xyz-stage. The course tuning can be done with the camera and by blocking the light with a piece of paper (which still allows you to see the location of the laser spot). The fine tuning is done by looking at the power meter and changing the focus + position to attain maximum reflected power.
- Choose the gain settings of the lock-in amplifier depending on the frequency of interest (see page 631 of the lock-in detector manual). The bandwidth is typically set around 10-20 Hz.
Bibliography


