DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AEROSPACE ENGINEERING

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GROWTH IN AIRCRAFT MATERIAL**

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PREDICTION METHODS FOR FATIGUE CRACK GROWTH IN AIRCRAFT MATERIAL

by J. Schijve (Department of Aerospace Engineering, Delft University of Technology, Kluyverweg 1, Delft, The Netherlands)

ABSTRACT: In the first part of the paper a survey is given of relevant knowledge on fatigue crack growth and qualitative and quantitative understanding of predictions. Aspects of cycle-by-cycle predictions and characteristic K prediction methods are discussed. In the second part recent work on prediction problems is reported. This includes (1) crack growth under flight-simulation loading with crack closure measurements, (2) predictions for flight-simulation loading based on a constant crack opening stress level, and (3) crack growth under pure random loading with different S_{rms} -values, two irregularities and two crest factors.

The random load tests were also carried out to explore the usefulness of K_{rms} . The paper is concluded with some indications for future research and a number of conclusions.

KEY WORDS: Crack propagation, fatigue, predictions, flight-simulation loading, random loading, crack closure.

Nomenclature and units

a	(semi) crack length
Δa	increment of a in one cycle
C	geometry factor, or crest factor
CA	constant amplitude
COD	crack opening displacement
da/dn	crack growth rate
K	stress intensity factor
k	irregularity factor
kc	kilocycle
P	load
R	S_{min}/S_{max}
S_{mf}	mean stress in flight

S_{op} crack opening stress

t thickness

VA variable amplitude

W specimen width

γ S_m/S_{rms}

$$1 \text{ MN/m}^2 = 145.04 \text{ psi} = 0.1020 \text{ kg/mm}^2$$

$$1 \text{ MN/m}^{3/2} = 910.05 \text{ psi} \sqrt{\text{in}} = 3.2246 \text{ kg/mm}^{3/2}$$

INTRODUCTION

Fatigue cracks in aircraft structures and materials are still an urgent problem for reasons of both safety and economy. Unfortunately prediction of crack growth is a complicated problem because many aspects are involved. One significant feature is the complex nature of load-time histories occurring in service. Progress of improving prediction techniques is made slowly. However, modern experimental techniques and computer facilities suggest that further improvements are possible. In the first part of this paper a survey is given of the present state of knowledge on fatigue crack growth including qualitative and quantitative understanding of predictions. Reference is made in this part to developments in experimental techniques and indications obtained from fatigue of aircraft structures.

The second part is dealing with some recent work on prediction fatigue crack growth in aluminum alloys, carried out in the Department of Aerospace Engineering in Delft.

This includes:

1. Crack growth under flight-simulation loading with crack closure measurements.
2. Crack growth predictions for flight-simulation loading based on a constant crack opening stress level.
3. Crack growth under random loading to explore the usefulness of K_{rms} and to observe effects of irregularity and crest factor.

The paper concludes with a brief discussion on the relevance of research programs for solving the problem of crack growth prediction technique and a summary of some conclusions.

ASPECTS OF THE PROBLEM OF FATIGUE CRACK GROWTH

Any newcomer in the field of fatigue must be overwhelmed by the vast amount of literature. In Ref. 1 Mann compiled about 6000 references over the period 1951-1960 and probably more appeared in years afterwards. The subject index of Mann's book illustrates the abundant variety of aspects associated with fatigue problems.

Numerous papers on theoretical or experimental studies refer to practical problems, but usually this is done in a superficial and indirect way. The impression emerges (right or wrong) that fatigue and fracture mechanics have become disciplines in themselves with a practical significance which should be self-evident. It is a relevant question then to see which problems we try to solve and which problems we should try to solve. This point will be elaborated in this paper to some extent with respect to fatigue crack growth. For this purpose some aspects of the state of knowledge have to be recapitulated. Headlines are listed below.

Aspects of research on fatigue crack growth

- laboratory observations on fatigue crack growth
- theoretical prediction methods
- developments in experimental procedures
- service experience
- practical relevance of research programs

The first four items are discussed hereafter, while the last one is briefly touched upon in the discussion at the end of this paper.

Laboratory observations on fatigue crack growth

It is convenient to divide fatigue life into two periods:

- crack initiation period
- crack growth period

The life is completed by final failure in the last cycle. Microscopical studies have shown that crack initiation occurs early in the fatigue life if not immediately. However, microcrack growth usually is a slow process for a long time and this may be considered as part of the

initiation period. The crack growth period then starts when a macro-crack is present. Although it is difficult to define the transition from the initiation period to the crack growth period some comments may clarify the idea. In the first period microcrack growth is still a local process with hardly any effect on macroscopic stress and strain distributions. However, in the second period fatigue is no longer a localized surface phenomenon and stress and strain distributions are significantly affected by the crack. Bulk properties of the material become important. One significant conclusion is: Predictions of crack initiation life on one hand and predictions of crack growth on the other hand require different prediction techniques. For predictions on the initiation period K_t -factors and a Neuber-type analysis can be relevant. However, for crack growth predictions other means should be adopted.

A number of relevant aspects of macrocrack growth is listed below.

Some observations on fatigue crack growth

- early initiation
- striations
- Δa during load increase only
- stress ratio (R) effects
- load sequence effects (interaction effects)
- crack closure
- environmental effects

Striations are a wellknown feature by now, frequently used to indicate the nature of service fatigue failures. Striation patterns have shown that some crack front advancement (Δa) occurs in every load cycle. This has prompted prediction techniques to calculate Δa for each cycle (see later). Originally it was proposed by Paris, Gomez and Anderson (Ref. 2) that the crack rate was a function of the range of the stress intensity factor (ΔK), which was approximated by a simple power relation:

$$\Delta a = \frac{da}{dn} = C \Delta K^m \quad (1a)$$

It was observed later that the effect of the stress ratio $R (= S_{\min}/S_{\max})$

could not be ignored. The so-called Forman equation is a wellknown relation including this effect:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1 - R) K_c - \Delta K} \quad (1b)$$

More empirical relations can be found in the literature, but a derivation on physical arguments appears to be very problematic.

Some decades ago it was observed that peak loads produced large delays of crack growth. An illustration from Ref. 3 is given in Figure 1. Such delays have led to the definition of so-called interaction effects. It implies that Δa in a load cycle will depend on what occurred in the preceding cycles. Similarly a load cycle will affect Δa in subsequent cycles. Actually interaction effects have to be expected, because Δa will depend on such factors like crack tip blunting, shear lip developments, crack closure, cyclic strain hardening and residual stresses around the crack tip, factors produced by the preceding load history (Ref. 4). Apparently the phenomenon can be rather complex.

Sequence effects are the result of interactions. In Figure 1 the same peak load cycle is applied in two different sequences (A: pos. $\rightarrow$ neg., B: neg. $\rightarrow$ pos.) and the growth delays are highly different. It may be argued that the example in Figure 1 is not a good one, because the maximum load range A_1A_2 (downwards) occurs in case A but not in case B. Similarly B_1B_2 (upwards) occurs in case B but not in case A. This distinction should be made because it is expected that crack extension (Δa) will occur during increasing load only. This was recently confirmed by work of Bowles (Ref. 5) who developed a new technique for fatigue crack tip observations. The distinction should also be expected from considerations on reversed plasticity and crack closure (see later).

Crack closure has now become a wellknown phenomenon, but it remains a remarkable fact that we overlooked it for so many years until Elber in 1968 discovered this phenomenon (Ref. 6). Until that time crack growth delays were explained by residual compressive stresses in the crack tip area ahead of the crack. Fatigue crack growth should be considered as the result of cyclic slip at the crack tip and the conversion of this

microplasticity into crack extension (Ref. 7), which can be activated by the environment (Ref. 8). It should not be expected that residual stress will affect slip, but it may well promote the conversion of slip into crack extension. It is important to realize that this requires the crack tip to be open. As soon as crack closure has been removed a stress singularity at the crack tip is present. It is difficult then to see how residual stresses ahead of the tip could be important. The more essential part of the question is whether the crack is open or not, and this depends on the plastic deformations left in the wake of the crack.

Finally reference should be made to environmental effects on fatigue crack growth. Obviously the effect will depend on the material/environment combination. Our knowledge of crack growth under corrosive conditions is increasing, but the subject is too extensive to be summarized here. One observation should be mentioned. For crack growth in aluminum alloys the aggressive component of normal air is water vapor. Small amounts are sufficient to produce a saturated damaging effect under load frequencies occurring in practice. From this argument and from the observation that crack extension occurs under increasing load only, it can be deduced that an accelerated test to simulate service loading should be allowed (Ref. 9). This was confirmed by flight-simulation tests at 10 Hz, 1 Hz and 0.1 Hz, which gave the same crack growth rates. Such a "time compression" is probably not allowed for mild steel in salt water environments.

Theoretical prediction techniques

The major aspects to be recognized here are shown below.

Type of loading

- constant-amplitude (CA) loading
- variable-amplitude (VA) loading

Geometry of crack front

- straight crack front (plane problem)
- curved crack front (3-dimensional problem)

Type of prediction method

- cycle-by-cycle method
- characteristic K method

The most simple problem is the prediction of a simple through crack in a structure of sheet material under constant-amplitude loading (CA-loading). For the material concerned crack growth data should be available as graphs or analytical relations, representing:

$$\frac{da}{dn} = f(\Delta K, R) \quad (2)$$

Many data have been published, but it is also easy to determine the relationship in a fairly small number of tests on simple specimens. Secondly K-values for the case of application are required. Sometimes K-values can directly be obtained from handbooks (Refs. 10-12). In other cases calculations have to be made, either by clever superpositions of known solutions, or by finite element methods. Several successful applications have been reported in the literature.

A relatively simple three dimensional problem is a semi-elliptical surface crack in the centre of a plate specimen. The value of K varies along the crack front. Reasonably accurate K-values seem to be available for semi-elliptical cracks (Ref. 13). Unfortunately, if the crack grows according to Eq. (2) the shape will not remain semi-elliptical. As a result, the problem of a semi-elliptical crack, which is so easily specified, requires already a fairly elaborate amount of calculations to predict how the curved crack front will move onwards.

Variable-amplitude loading (VA-loading) offers significant prediction problems in view of the interaction effects defined before. Several methods have been proposed (reviewed in Ref. 4) and two main lines will be recapitulated here.

Cycle-by-cycle calculations start from simple crack length additions:

$$a = a_0 + \sum \Delta a_i \quad (3)$$

where a_0 is the initial crack length and Δa_i is the crack length increment in cycle (i), and these increments have to be predicted. Two crack growth models (Willenborg, Ref. 14, Wheeler, Ref. 15) are accounting for interaction effects by:

$$\Delta a_i = \beta_i \left(\frac{da}{dn} \right)_{CA} \quad (4)$$

with:

$$\left(\frac{da}{dn} \right)_{CA} = f(\Delta K_i, R_i) \quad (5)$$

similar to Eq. (2), while β_i accounts for the interactions. In both methods β_i is assumed to depend on plastic zone sizes associated with load cycle (i) and the preceding cycles. Simple assumptions are made for this purpose. As a result crack growth delays after a peak load are obtained, but negative interactions (crack growth accelerations) can not be predicted, although they do occur (Ref. 3).

From a physical point of view crack closure appears to offer better arguments for interaction effects (Ref. 16). Then equations (4) and (5) have to be replaced by:

$$\Delta a_i = \left(\frac{da}{dn} \right)_{CA} = f(\Delta K_{eff,i}) \quad (6)$$

which is the relationship between crack rate and ΔK_{eff} as proposed by Elber (Ref. 6). The problem still remains to calculate the crack-opening stress level (S_{op}) in cycle (i) from which $\Delta K_{eff,i}$ has to be deduced:

$$\Delta K_{eff} = C \cdot (S_{max} - S_{op}) \cdot \sqrt{\pi a} = C \Delta S_{eff} \sqrt{\pi a} \quad (7)$$

Different delays in Figure 1 for case A and case B can be understood if crack closure is considered. In case B the load range $B_1 B_2$ will cause a large plastic zone. If the crack tip is penetrating into this zone it will meet with a high crack opening stress level and significant growth retardation will occur. In case A an equally large plastic zone will occur at peak load A_1 and the crack tip will be plastically opened. The load range $A_1 A_2$ will then cause a considerable reversion of plasticity ahead of the crack tip. As a result lower crack opening stress levels will follow later on and the delay is much smaller. Both analytical studies on cyclic plasticity at the crack tip (Refs. 17-19) and some experimental evidence (Ref. 20, 21) confirm the argumentation. Some work is now going on to introduce crack closure into cycle-by-cycle calculations

for complex load time histories (Refs. 21-23). It is easily understood that elasto-plastic calculations for each load cycle is a rather elaborate procedure. It then seems reasonable to look also for acceptable simplifications.

Elber (Ref. 24) measured S_{op} during random load tests with a short return period and he found it to be approximately constant during a test. He then defined effective stress ranges of the random loading by:

$$\Delta S_{eff} = S_{max} - S_{op} \quad \text{if } S_{min} < S_{op} \quad (8a)$$

and

$$\Delta S_{eff} = S_{max} - S_{min} \quad \text{if } S_{min} > S_{op} \quad (8b)$$

It means that those parts of stress ranges which are above S_{op} are supposed to be effective. Subsequent derivations of equations in Ref. 24 are not rather explicit, but it is equivalent to substituting ΔS_{eff} according to Eqs. (8a) and (8b) into Eq. (7) and ΔK_{eff} -values thus obtained into Eq. (6). No further interactions are supposed to occur. Later on in this paper the idea will be carried on somewhat further in view of application to flight-simulation test results. It should be pointed out here that an approximately constant S_{op} can be applicable only if the VA-loading may be considered to be stationary as discussed in Ref. 25 (lecture IV). It implies that load sequence properties should be constant, while crack growth on a macro scale should not show apparent discontinuities (i.e. delays observable from the crack growth curve).

Characteristic K methods were adopted in the literature for random loading and flight-simulation loading. It requires that the VA-loading is stationary and fully characterized by a single stress level, S_{char} . A characteristic K-value is easily defined:

$$K_{char} = C S_{char} \sqrt{\pi a} \quad (9)$$

The crack rate should then be a function of K_{char} only. Paris (Ref. 26) adopted this idea for random loading, for which Eq. (9) becomes:

$$K_{rms} = C S_{rms} \sqrt{\pi a} \quad (10)$$

Crack growth results can then be represented by:

$$\frac{da}{dn} = f(K_{rms}) \quad (11)$$

and this has found some confirmation in the literature (Refs, 27-29). It should be clear that Eq. (11) represents empirical results, which can be used for prediction purposes if the same type of random loading applies, including the same S_{rms}/S_m ratio. Writing Eq. (11) in a more general form:

$$\frac{da}{dn} = f(K_{char}) \quad (12)$$

an inherent advantage and disadvantage of the characteristic K method will be recognized:

1. Any change of a nominal or local stress level in a structure does usually not affect the character of the VA-loading in service. Effects of such changes on crack growth rates can thus be derived from an empirical relation like Eq. (12) obtained from simple laboratory specimens.
2. The disadvantage is that the empirical relation in Eq. (12) has to be established for each relevant VA-loading and unfortunately many types of VA-loadings occur in various structures. For instance, random loadings are not of the same type if the spectral density functions are different, or if the crest factor or the S_{rms}/S_m ratio are different (discussed later).

The applicability of Eq. (12) to flight-simulation loading was checked fairly extensively at the National Aerospace Laboratory. The mean stress in flight, S_{mf} was adopted as the characteristic stress level to define K_{mf} analogous to Eq. (10). It was hoped to find a unique correlation between da/dn (growth rate per flight) and K_{mf} but the first results were not very positive (Ref. 30). It was argued that similar crack rates to be obtained require similar K values, and in addition similar dK/da values to account for the preceding history. Usually the requirements of both similar K and similar dK/da are incompatible. Later on more data of flight simulation tests were compiled by Wanhill (Ref. 31). He concluded that K_{mf} could be used for crack growth prediction as a first approximation, provided the flight-simulation loading is stationary. He emphasized that K_{mf} cannot account for load spectrum variations, such as

different truncation levels, different numbers of low-amplitude cycles, etc.

Experimental developments

Some developments have had a most significant impact on fatigue research programs. Especially the combination of closed loop fatigue machines with computer controlled programs should be mentioned here. A closed loop electro-hydraulic loading system can apply any load time history that can be generated as an electrical command signal. Computer controlled signal generation has led to most versatile possibilities for programming of load-time histories. It includes aspects as load sequences, wave shapes and frequency. Moreover any random sequence of loads can exactly be reproduced in subsequent tests. Flight-simulation tests and random load tests can now be adopted for a variety of testing purposes. It has already initiated a standardizing of two different types of flight-simulation loading (TWIST, Ref. 32, and FALSTAFF, Ref. 33) and probably more will follow. It is also easily understood that the characteristic K method requires basic material test data under relevant loading histories. Without computer controlled fatigue machines this would be practically impossible.

Another development to be mentioned here is the automatic measurement of crack length by the electrical potential drop method. Each type of specimen has to be calibrated for this purpose. Computer control is possible again. We are now in a position to obtain crack growth data for any type of load sequence by employing routine procedures.

The above comments are made here because they are relevant to solving prediction problems. Nowadays computer controlled fatigue machines are frequently used to check existing prediction techniques for realistic specimens. This might narrow the gap between experimental data and the real structure.

Service experience

In view of the extensive literature on fatigue prediction techniques one might hope that service experience on fatigue problems is well documented in the literature. This is only partly true. The table below will help to

classify the position.

Crack growth in real structures

Origin of cracks: - incidental cases (corrosion pits, damage, etc.)
- systematic occurrence (bolt and rivet holes, fillets, etc.)

Source of information: - aircraft in service
- full-scale test
- component tests

Most information in the literature is on the origin of cracks. If cracks are found in service it is a highly practical question to know whether it is an isolated case or a symptomatic one. Solutions to cure the problem will depend on the answer to this question. Remedial actions are taken as soon as a crack is noted because it should not be there. As a result crack growth data from service experience are highly exceptional, and moreover the load history will not be accurately known. The best data are coming from full-scale tests and component testing, usually carried out to prove the fail-safety of a new structure. Such data are scattered through the literature and usually the description is not sufficiently complete to be used by other investigators for checking prediction methods. Moreover, real structures do not have the geometrical simplicity of laboratory specimens. Nevertheless, a co-ordinated program to compile available data of crack growth in structures with relevant information on the structure and its fatigue loading, would be most instructive to set the scene of the prediction problem.

RESULTS OF SOME RECENT CRACK GROWTH STUDIES IN DELFT

Crack growth under flight-simulation loading

Tests were carried out on sheet specimens ($t = 2$ mm, $W = 100$ mm) of 2024-T3 Alclad and 7075-T6 Clad material. A standardized flight-simulation loading (TWIST, Ref. 32) was used. Different truncation levels were adopted and the wellknown effect of faster crack growth for lower truncation levels was found (see Fig. 2). During these tests numerous crack closure measurements were made with a COD meter.

It was hoped that the tests would indicate higher crack closure levels if the truncation level was higher. This would offer an explanation for the truncation effect and perhaps a basis for improved prediction techniques based on crack closure. Unfortunately the crack closure measurements showed a rather chaotic picture which did not allow a simple evaluation (Ref. 34). However, two lessons could be learned from the results:

- (1) The most severe flights with the highest maximum loads significantly changed the crack closure level.
- (2) The COD meter was located in the center of the specimen during all measurements, since locating the meter near the crack tip for all measurements is very elaborate. Unfortunately a determination of S_{op} becomes less accurate.

Examples of crack closure measurements are shown in Figure 3. During a severe flight (No. 2936) a significantly enlarged plastic zone is formed ($r_p \sim [K_{max}/S_{0.2}]^2/\pi = 1.45$ mm) and plastic crack tip blunting will occur. As a result S_{op} is relatively low immediately after this flight (A_1 in Fig. 3), and the non-linear behavior below A_1 shows that the crack is not fully closed under the compressive load applied. After the severe flight the crack tip has to grow into the newly formed plastic zone and this implies that S_{op} now is very high, see point C_2 after flight No. 3180. The crack has grown then from 12.1 mm to 12.5 mm ($\Delta a = 0.4$ mm). The P-COD record contains two linear parts (A_2B_2 and C_2D_2) from which the first one (A_2B_2) is parallel to A_1B_1 . Equal compliances indicate equal crack lengths, which means that the crack was open until $a = 12.1$ mm during the load increase A_2B_2 . However, the very last part of the crack (from $a = 12.1$ mm to $a = 12.5$ mm) was still closed during

A_2B_2 , while it was opened during the load increase B_2C_2 . During further crack growth S_{op} is decreasing (points $C_3 \rightarrow C_4 \rightarrow C_5 \rightarrow C_6$) but then another severe flight occurs (No. 3841). The same process is repeated and after flight No. 4105 even three linear parts can be observed: A_7B_7 parallel to A_1B_1 corresponding to $a = 12.1$ mm, C_7D_7 parallel to C_6D_6 corresponding to $a = 17.5$ mm, E_7F_7 corresponding to $a = 20.1$ mm. The successive openings are schematically shown in Figure 4.

In Figure 3 A_2B_2 and C_2D_2 correspond to $a = 12.1$ mm and $a = 12.5$ mm respectively, i.e. the slope difference is no more than 3 percent. Although this can be observed, it will be clear that the transition points B_2 and C_2 can not be accurately indicated. In spite of this it is evident that the line $C_6C_5C_4C_3C_2$ (and also the line E_8E_7) are going upwards. It means that S_{op} during the first part of crack growth after a severe flight will be extremely high. This was confirmed in more accurate measurements during constant-amplitude tests after a peak load (results of Arkema in Ref. 4). In later flights in Figure 3 S_{op} comes down, but it is possible that the linear parts C_3D_3 , C_4D_4 and C_5D_5 (corresponding to apparently open cracks) consist of two linear parts with almost equal slope as a result of somewhat less severe flights in between. It should be realized that the very end of a crack (tenths of millimeters and even smaller) can be closed, while this cannot be observed empirically. At the same time we have to face the problem that the physical meaning of:

$$\Delta K_{eff} = K_{max} - K_{op} \quad (13)$$

as it was originally suggested by Elber (Ref. 6) might break down if a very small part of the crack tip is closed only.

The P-COD records with more than one linear part are obtained as a result of local contacts behind the crack tip (Fig. 4). This phenomenon was observed before (Ref. 20), and apparently it also follows from analytical studies (Refs. 17 and 19). It may be expected that it can be included in a cycle-by-cycle calculation in the future. At the same time it has to be recognized that the above evidence of flight-simulation tests indicates that crack closure will be rather complex then.

Secondly a formal application of ΔK_{eff} may not be fully justified any longer from a physical point of view. There is a limit to the level of

sophistication which is still feasible. Under such conditions simplified approaches, giving sufficient credit to physical observations, should be explored. The introduction of an average crack closure level for a stationary variable-amplitude loading should be considered as such an approach.

Crack growth predictions for flight-simulation tests based on a constant crack-opening stress level

In a previous NLR investigation (Ref. 35) crack growth was studied under flight-simulation loading with the following main variables:

- different truncation levels
- omission of low-amplitude cycles
- omission of ground-to-air cycles

Some other aspects studied were omission of taxiing loads in the ground-to-air cycle (GTAC), application of gust cycles in a programmed low-high-low sequence instead of a random sequence and application of all gust cycles in reversed order. Especially the first three issues implying fairly drastic load spectrum variations seem to be critical for proving the validity of a prediction model. This is one reason to adopt the results of Ref. 35 for a first exploration. The second one is that constant-amplitude test results were also obtained for the materials tested in Ref. 35.

The model to be discussed here starts from Elber's observation, which was an approximately constant crack opening stress level during pure-random-load tests (Ref. 24). Basic assumptions for the model are:

- (a) During a stationary variable-amplitude (VA) loading the stress opening stress level (S_{op}) may be regarded to be constant.
- (b) The value of the constant S_{op} under stationary VA-loading is a function of the maximum stress (S_{max}^{VA}) and the minimum stress (S_{min}^{VA}) occurring in the VA-loading. Moreover this function is the same one applicable to CA-loading.
- (c) Stress ranges are effective as far as they are above S_{op} (see Fig. 5).

The third assumption was adopted by Elber (Ref. 16), see Eqs. (8a) and (8b), but instead of the second one he used S_{op} -values measured in the random load tests for which predictions were made. However, it is thought that the second assumption on S_{op} involves some obvious elements. The

maximum load in a stationary VA-test will determine the maximum plastic zone. This zone should have a large effect on the plastic deformation left in the wake of the crack, which causes crack closure.

Some substantiation comes from simple measurements (light reflection) on plastic deformation around cracks grown under flight-simulation loading (Ref. 25, lecture II). For a flight-simulation test it implies that S_{op} will depend on the truncation level of the load spectrum. It should also be expected that the minimum stress level will be significant in view of reversed plasticity, occurring in the crack tip plastic zone.

Confirmation is offered by analytical work (Refs. 17-19). Consequently the stress ratio for stationary VA-loading defined as:

$$R_{VA} = \frac{(S_{min})_{VA}}{(S_{max})_{VA}} \quad (14)$$

should be a significant parameter to estimate $(S_{op})_{VA}$, which has been assumed to be constant. The validity of the model has been checked as yet for 2024-T3 sheet material only. For this material Elber found (Ref. 16):

$$\frac{\Delta K_{eff}}{\Delta K} = \frac{S_{max} - S_{op}}{S_{max} - S_{min}} = 0.5 + 0.4 R \quad (15)$$

which can also be written as:

$$S_{op} = S_{max} (0.5 + 0.1 R + 0.4 R^2) \quad (16)$$

The applicability of this equation on our own CA data was checked first. These data include S_a -values corresponding to R-values 0.73, 0.52, 0.23, 0.03 and -0.11. An R-effect was clearly observed, but plotting da/dn as a function of $\Delta K_{eff} = (0.5 + 0.4 R) \Delta K$ brought all data points on a single curve with a very narrow scatter band (to be published). This curve has been used for the predictions on the flight simulation tests.

According to assumption (b) equation (16) now implies:

$$(S_{op})_{VA} = (S_{max})_{VA} [0.5 + 0.1 (R)_{VA} + 0.4 (R)_{VA}^2] \quad (17)$$

where $(R)_{VA}$ follows from Eq. (14). It is noteworthy that the applicability of Eq. (17) can be checked for Elber's own random load test results, because

he reports $(S_{\max})_{VA}$ and $(S_{\min})_{VA}$. For six different random load histories in Ref. 24 the results are shown below, and although there are differences the comparison is promising.

$(S_{op})_{VA}$ in MPa	{	Measured	104	53	87	102	56	89
		Eq. (17)	102.8	51.4	86.6	102.8	51.4	86.6
Difference (%)			-1.2	-3.0	-0.5	0.8	-8.2	-2.7

Predictions on crack growth rates for the flight-simulation tests in Ref. 35 will now be made. Before, we have required the VA-loading to be stationary in order to justify a constant S_{op} . Another advantage of the stationarity should be exploited. The crack extension in cycle (i) according to the crack closure concept adopted will be equal to:

$$\left(\frac{da}{dn}\right)_i = f(\Delta S_{eff,i}) \quad (18)$$

If there are n_i stress ranges $\Delta S_{eff,i}$ in a certain period the average crack growth rate in that period follows from:

$$\left(\frac{da}{dn}\right)_{VA} = \frac{\sum n_i \left(\frac{da}{dn}\right)_i}{\sum n_i} \quad (19)$$

In this equation $(da/dn)_{VA}$ and $(da/dn)_i$ should apply to the same a-value and da/dn data have to be derived from CA tests plotted as a function of ΔK_{eff} .

Predictions of da/dn for the test series with various types of flight-simulation loading can now be made. Steps to be followed are:

1. For each test series calculate S_{op} from $S_{\max}$ and $S_{\min}$ (eqs. 14 and 17).
2. For each possible effective stress range (Fig. 5) determine how many times (n_i) it will occur.
3. For each effective stress range determine the related crack rate from the CA data in the form $da/dn = f(\Delta K_{eff})$.
4. Combine information from steps 2 and 3 by substitution in Eq. (19) to give $(da/dn)_{VA}$.

The last steps 3 and 4 have to be repeated for a sufficient number of a-values to see how the predicted crack rate depends on crack length.

It still should be pointed out that step 2 can be done in two different

ways. One way is a simple counting analysis with S_{op} as a kind of a lower boundary condition. Another method followed here is to calculate the statistical expectation from the statistical data on flight types and gust cycles (Ref. 36).

Although this work is still being continued illustrative results can already be presented. Predicted crack growth rates are presented in Figures 6-8 together with test results. Predicted crack growth lives are obtained by integrating the inverse of the growth rate over a crack growth interval from $a = 14$ mm to $a = 50$ mm. Results are compared with test data in Figs. 9a-d. In these figures predictions based on non-interaction (ignoring crack closure) are shown also.

Effect of truncation. Figure 6 shows that there is a moderate agreement between predicted and actual crack rates. Figure 9a confirms the trend of increasing crack growth life for higher truncation levels. Apparently the trend is also indicated by the S_{op} -model. If crack closure is ignored (non-interaction) the latter figure shows that there is no predicted effect of the truncation level at all. This is not surprising because the rarely occurring high loads hardly contribute to a non-interactive damage summation.

Omission of low-amplitude cycles. Omitting low-amplitude cycles of the flight-simulation tests implied a lower number of cycles per flight. An extreme case was also investigated, i.e. to have only one positive gust per flight, that means the largest one occurring in each flight. Omitting cycles gave lower crack rates as should be expected, and this empirical trend is also predicted (Figs. 7 and 9b and c).

It should be noted that a 1:1 relation between predicted life and that result is sometimes observed in Fig. 9, does not imply that the crack growth rate is also accurately predicted (Figs. 6-8). For a reliable prediction method it should be required that the growth rate is predicted reasonably well. A good prediction of crack growth life is then obtained automatically.

Omitting ground-to-air cycles (GTAC). The results in Figures 8 and 9d are somewhat disappointing. The tests indicate a significant growth rate reduction if the GTAC are omitted. The prediction gives a small reduction only. The latter results is obtained because omitting the GTAC changes $(S_{min})_{VA}$ from -3.4 kg/mm² to $+0.4$ kg/mm², R_{VA} from -0.250 to $+0.029$ but

S_{op} from 6.80 to 6.845 kg/mm² only. The small change of S_{op} explains the results obtained. At this point it should be asked how a fairly drastic change of S_{min} can give such a small change of S_{op} . To answer this question Elber's results leading to equation (17) have been replotted here in the way as it was done by him (Fig. 10a) and in another way preferred here (Fig. 10b). Clearly enough his results do not extend any further down as $R = -0.1$ and an applicability below this value was never claimed. Figure 10b shows that Elber's function for S_{op} goes through a minimum at $R = -0.125$ and it would be rather strange if such a minimum would exist. Elasto-plastic analysis of Newman (Ref. 17) indicated the trend of the dotted line in Figure 10b which appears to be more plausible. A similar trend was also predicted by Fühling (Ref. 19). As a matter of fact other lines can be drawn through Elber's data points giving credit to the expected trend following from analytical work. From such a line a larger shift of S_{op} after removing the GTAC would have followed and a more significant effect would have been predicted as indicated by the test results.

It is thought that a systematic picture is emerging from the above results. A model with a constant crack opening stress level depending on maximum and minimum stress in a stationary VA-loading is capable of predicting the trends of significant load spectrum variations. The quantitative accuracy is still insufficient, but this may well be a consequence of insufficient knowledge about crack opening stress levels.

Elber's formula Eq. (15) assumes S_{op} to be independent of crack length and K_{max} . However, both analytical studies (Refs. 17, 19, 37) and experimental work (Ref. 20, 38) have shown that some effects do exist. In Figure 10b Newman's curve applies to $S_{max}/S_{0.2} = 0.4$ but he found different results for other $S_{max}/S_{0.2}$ ratios.

The literature cited indicates lower S_{op}/S_{max} for high K_{max} -values. It is noteworthy then that experimental curves in Figures 6-8 merge together for large a -values corresponding to high K -values.

Crack growth under random loading

An investigation on the applicability of K_{rms} for correlating crack growth under random loading was started some time ago and is still being continued. Variables of the first test series (Ref. 39) were:

- S_{rms}
- irregularity factor k
- truncation of high amplitudes (crest factor C)

Tests were carried out on 2024-T3 Alclad sheet specimens ($t = 2$ mm, $W = 100$ mm) with a central crack. The random load was applied by computer control of a closed loop fatigue machine. For this purpose we have adopted a load signal generating procedure developed in Germany (Ref, 40). It starts from a two-dimensional density function proposed by Kowalewski, which can be written as:

$$h(M, A) dM dA = \frac{1}{I^2 \sqrt{2\pi(1 - I^2)}} \cdot \exp \left[\frac{-(M/\sigma)^2}{2(1 - I^2)} \right] \cdot \frac{A}{\sigma} \exp \left[\frac{-(A/\sigma)^2}{2I^2} \right] d \left(\frac{M}{\sigma} \right) d \left(\frac{A}{\sigma} \right) \quad (20)$$

Two ranges between two successive peak values (a maximum p and a minimum q or the reverse) is equal to $2A$ with a mean value M . The irregularity factor k ($= 1/I$) is defined as the ratio of the number of peak values and the number of mean crossings, and σ is the standard deviation of the random signal. Defining S_{rms} as the root-mean-square of $S - S_m$ implies that $S_{rms} = \sigma$. Both σ and k can be derived from the power spectral density function, $\Phi(\omega)$, assuming the random signal is Gaussian.

Equation (20) is used to fill a matrix with numbers corresponding to the frequency of occurrences of ranges between 32 p -levels (maxima) and q -levels (minima). The computer makes a random walk through the matrix, which produces a random signal satisfying eq. (20). Two samples are shown in Figure 11 for a narrow band random signal and a broader band signal.

In the tests the ratio $\gamma = S_m / S_{rms}$ was kept constant ($\gamma = 3.28$) which should be required for the applicability of $K_{rms} (= C S_{rms} \sqrt{\pi a})$. The return period of the random signal was chosen to correspond to 10^6 positive zero crossings (actually mean crossings). As a result the crest factor C , defined by:

$$C = \frac{S_{max} - S_m}{S_{rms}} \quad (21)$$

becomes $C = 5.25$. Crack growth tests were carried out at five different S_m -values. Averages of two tests are presented in Figures 12 and 13 for

broad band and narrow band loading respectively. It was hoped that both figures would confirm the applicability of:

$$\frac{da}{dn} = f(K_{rms}) \quad (22)$$

However, a small but systematic effect of S_m is observed. Higher S_m values give slightly higher crack rates. Consequently an average curve in Figures 12 and 13 representing Eq. (22) can be applicable only in an approximate way.

On the average crack rates in Figure 13 for narrow band random loading are about 1.5 times faster than for broadband random loading in Figure 12. In both figures the cycle definition for da/dn is one minimum plus one maximum. However, if the definition of one cycle is based on two mean crossings the factor 1.5 reduces to $(1.01/1.43) \times 1.5 = 1.06$ and the difference between crack rates should be considered as negligible. A similar trend was found in Ref. 28.

A somewhat more significant effect is observed if the random signal is artificially truncated until $S_{max}/S_{rms} = 3.31$, see Figures 14 and 15. In practice mechanical systems with their own characteristic response will usually damp high amplitude excitations. Since such a truncation was known to have a significantly harmful effect on crack growth under flight-simulation loading it appeared desirable to see whether this also applied to pure random loading. The results in Figure 14 again indicate some effect of S_m while Figure 15 illustrates the effect of truncation. A fairly drastic truncation increased the crack rate 1.3-2 times. This is a fairly modest effect as compared to the effect in flight-simulation tests.

Tests are now underway to apply $da/dn-K_{rms}$ data to a specimen with a terminating stringer, where the crack originates from the hole of the end rivet. Some tests with a much higher irregularity factor ($k = 3.3$) are also carried out.

DISCUSSION

In the first part of this paper physical aspects of fatigue crack growth relevant to prediction methods were surveyed. This was followed by a discussion on two different prediction methods: (1) cycle-by-cycle calculations and (2) characteristic K methods. The significance of crack closure for explaining interaction effects was emphasized. In the second part aspects of both methods were studied as part of recent test series. It is obvious that a cycle-by-cycle calculation is more universal than the characteristic K method. The problem is that a cycle-by-cycle prediction method to be accepted should include the possible occurrence of both positive and negative interactions. So far the introduction of crack closure seems to be the only available way to satisfy this requirement. However, a cycle-by-cycle calculation then becomes fairly elaborate. A sufficient simplification to avoid this was to adopt a constant crack-opening stress level, which was considered to be justified for a stationary variable-amplitude loading. Results thus obtained indicate that such a calculation model might open a useful perspective. However, it was also clear that our quantitative knowledge of crack opening stress levels is still insufficient. It is stimulating to see that this can be improved by elasto-plastic analysis, but measurements seem to be desirable as well.

An important practical point is that the major part of present research is restricted to through cracks in sheet materials. It was pointed out that cracks with curved crack fronts offer additional problems, while brief reference was made to limited information of fatigue crack growth in service. Anyhow, we know that many cracks in service have curved crack fronts. It is sufficient to refer to semi or quarter elliptical cracks at bolt holes in joints. In view of the practical relevance of such cracks it is definitely desirable to compile more data on crack growth of various types of cracks.

If we now try to summarize some indications of previous parts of the paper the following points may be observed:

- (1) Until now much research was carried out on the effects of peak loads in constant-amplitude tests or other very simple loading programs. Beyond any doubt this has been very useful to recognize and understand interaction effects. A second benefit of this type of tests

is that they stimulated analytical studies employing elasto-plastic mechanics, which proved to give most useful indications. We thus have learned to appreciate available tools to develop prediction techniques.

- (2) At the same time there is a risk to overlook the question of which problem we want to solve. That problem is to predict crack growth of cracks with curved crack fronts as well, in components with more complex geometries than a sheet specimen, under highly variable fatigue loads. Since information of crack growth under such conditions will not come from service experience, it has to be generated in the laboratory.

Some recommendations appear to be a logical outcome now:

I. Crack growth data should be produced under well specified conditions concerning the following aspects:

- (a) Materials and specimen geometry should be representative for details of aircraft structures, which are supposed to have a critical nature.
- (b) Fatigue loads to be applied should cover a variety of load-time histories which are relevant for aircraft utilization.

II. For the evaluation of prediction techniques to be applied on the above test results sufficient basic information must also be made available with respect to:

- (c) Basic crack growth data for the material concerned.
- (d) Crack closure behavior of the material.
- (e) Relevant K-values.

It will be understood that a plea is made here for approaching the problem from the practical side. If we would stick to a step by step evolution from simple cases to more complex problems there is a certain risk of never reaching a practical solution.

SOME CONCLUSIONS

1. Pertinent physical information on the process of fatigue crack growth is now available. It provide guidelines to accounting for load cycle interactions in the prediction of fatigue crack growth under variable-amplitude loading. Crack closure is an important observation in this respect.
2. A cycle-by-cycle prediction method is more universal than a method based on characteristic K values. However, the former one is more elaborate, while satisfactory methods are not really established as yet.
3. Crack closure measurements were made during flight-simulation tests on two aluminum alloy sheetmaterials. The results illustrate that crack closure can be a fairly complex phenomenon during service simulation loading. The effect of severe flights on the crack opening stress level (S_{op}) was clearly noted.
4. For the prediction of crack growth under stationary variable-amplitude loading it was proposed that: (1) S_{op} may be considered to be constant, (2) the level of S_{op} depends on the maximum and the minimum stress occurring in the variable-amplitude loading and (3) this dependence is the same one applicable to constant-amplitude loading. This method was applied to flight-simulation test results with essentially different load spectra and the results appear to be promising. However, more quantitative information on S_{op} levels is desirable.
5. Crack growth tests under pure random loading (with $S_m/S_{rms} = 3.28$) were carried out on 2024-T3 Alclad specimens. Five different S_m -values and two irregularity factors were used. As a rough approximation the growth rate for different S_m -values is a function of K_{rms} only. However, a small but systematic effect of S_m was recognized. The growth rate (mm per mean crossing) was practically similar for narrow band ($k = 1.01$) and broad band ($k = 1.43$) random loading. If the maximum stress amplitudes of the random loading were truncated a significant increase of the growth rate was observed.
6. It appears that crack growth data for more practically relevant cracks are highly desirable. Guidelines for research on this topic have been outlined briefly.

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2024-T3 Alclad, $t = 4$ mm, $W = 80$ mm

$$CA \begin{cases} S_m = 8.0 \text{ kg/mm}^2 \\ S_a = 2.5 \text{ kg/mm}^2 \end{cases}$$

Peak load cycle $S_a = 12.0 \text{ kg/mm}^2$

Figure 1 : Different delays depending on sequence in peak load cycle (Ref.3).

Figure 2 : Effect of truncation level on crack growth curves.

Figure 3 : Crack closure measurements during a flight-simulation test on a 7075-T6 sheet specimen

Figure 4 : Schematic picture of crack opening in different stages during a flight simulation test.

Figure 5 Example of flight profile (low truncation level) with S_{op} -level and effective stress ranges.

Figure 7 : Effect of omitting low-amplitude cycles on crack growth in flight - simulation tests.

Figure 6 : Effect of truncation level on crack growth rate in flight - simulation tests.

Figure 8 : Effect of omitting ground-to-air cycles on crack growth in flight-simulation tests.

Figure 9 : Crack propagation lives ($a = 14 \text{ mm}$ to $a = 50 \text{ mm}$), comparison between prediction and test results of flight-simulation tests.

Figure 10 : Crack-opening stress as a function of R . Experimental results of Elber.

Broad band ($k=1.43$)

Narrow band ($k=1.01$)

Figure 11: Samples of load histories in random load test

Figure 12 : Crack growth rates for broad band random loading

Figure 13 : Crack growth rates for narrow band random loading

Figure 14 : Crack growth rates for truncated broad band random loading

Figure 15: Effect of truncation on crack growth under random loading (curves derived from Figs 12 and 14)