The application of spectral analysis in the determination of wave loads on vertical breakwaters

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Preface

This is my Msc. report of my graduation project at Delft University of Technology, hydraulic section of the faculty of Civil Engineering.

This thesis deals with a method to determine horizontal wave forces on vertical breakwaters. It is a part of an extensive program, partly financed by the European Union, to look for new design tools for vertical breakwaters.

This report contains the theory, which is necessary to determine the forces with this design method, comparisons with test results and the final conclusions, in this order.

I would like to thank the members of my graduation committee for their support and advises to complete this work: Prof. drs. ir. J.K. Vrijling, Dr. ir. J.W. van der Meer and Ir. W.H. Tutuarima.

A.P. Verweij
Delft, July 1997
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APPENDICES
Summary

Vertical breakwaters are used in the design of harbours to create a protective area. This protective area can be an approach channel or the harbour itself. Because of large new projects, such as Maasvlakte II in the Netherlands and the increasing sizes of ships, many harbours have to be placed in deep water. Deep water requires higher breakwaters. For many years, in Europe the only breakwaters built were rubble mound breakwaters. They consist of rock material and have a more or less trapezoidal shape. With increasing depth this type of breakwaters becomes expensive and the vertical breakwaters seem to be the better alternative.

To create new and better design tools for vertical breakwaters a research program has been initiated with financial support of the European Union. European research institutes and universities are doing extensive research in this program. This report presents the results of the research of an alternative method to determine horizontal wave forces on a vertical breakwater. Due to the horizontal wave forces many failure mechanisms, such as sliding and overturning, can occur.

The alternative method, presented in this report, calculates with a given wave spectrum the wave force spectrum. A wave spectrum describes a wave field, by giving for a range of frequencies the contribution of each frequency (actually a regular wave with that frequency) to the total energy of the wave field. The waves that are subject of this study are long-crested, non-breaking waves, that approach the breakwater at a right angle.

The wave spectrum is transferred into the wave force spectrum by multiplication with the so called transfer function. For this calculation it is assumed that there exists a linear relation between the incoming waves and the wave forces on the breakwater. This, however, is not entirely true.

The transfer function is determined by calculating for a large number of regular waves with various periods the wave forces on the breakwater. The choice of the height for the regular waves poses difficulties which have been given extensive research.

The pressure diagram on the front wall of the vertical breakwater at a wave crest can be divided into two parts: one part from the wave crest to mean water level and one part from mean water level to the bottom or, often, the top of the berm. From the latter part a linear behaviour can be recognised, however, the former part introduces a non-linearity. There remains a dependency on the size of the height for the regular
waves. Therefore in the determination of the transfer function this wave height cannot be chosen arbitrarily. It can be stated that for a low wave height water depth ratio the non-linearity is small and the transfer function can be used easily, however for a large ratio the influence of the size of the chosen wave height cannot be neglected. Therefore two ways of calculating the transfer function are introduced in this study. One is a constant height for all regular waves in the determination of the transfer function. The other way takes the wave steepness constant over the frequency range. Some features of the latter way are that for low frequencies the values for the transfer function become unrealistically high and that for higher frequencies the influence of the non-linearity decreases.

To study the transfer function and its application many comparisons have been made with results of model tests and with the General Wave Spectrum Model (GWSM). The model test are tests performed in the design of the Eastern Scheldt Storm Surge Barrier and model tests with caissons, both performed at Delft Hydraulics. The General Wave Spectrum Model enables the generation of a wave spectrum by choosing parameters (e.g. for energy, peak frequency and left and right flank). The generated wave spectrum allows to make calculations with different types of wave spectra.

The model tests with caissons cannot give conclusive results to confirm the method of the transfer function. The tests give results for the transfer function that match the theoretical transfer function. The course of the measured transfer function seems to be best described by the transfer function with constant wave steepness. However the choice of which constant wave height or which wave steepness to use remains very difficult.

The comparison with the model tests performed in the design of the Eastern Scheldt Storm Surge Barrier show that, also, the transfer function calculated with the constant wave steepness follows the course of the model test results better in a large frequency range. The values of the test results and the theoretical results deviate a lot, but that is probably due to the schematisation made for the theoretical results.

Another comparison is made with the use of the GWSM. Beside the influence of the non-linearity of the transfer functions the influence of the shape of the spectrum is studied. The reason for this is the fact that other methods of determining the wave force neglect the influence of the shape of the wave spectrum, which appears to be not correct. Calculations with double-peaked wave spectra and wave spectra with varying steepness of the right flank prove the influence on the wave force.

A method, that is widely used to determine the wave forces on vertical breakwaters, is the method of Goda. It is a method that includes both breaking and non-breaking waves. The calculation of the wave force is made by using a representative wave height and period. Various comparisons have been made between the method of the transfer function and Goda's method.

The method of the transfer function takes the influence of the shape of the wave spectrum into account, opposite to the method of Goda. From comparative calculations with the method of Goda it shows that the influence of the shape of the wave spectrum is of importance. When more energy of a wave spectrum can be found at higher frequencies, because of a second, high frequency, peak or because of a less steep right flank, Goda gives relative high wave forces.
Concluding, the method of the transfer function is a method, that, with more research, could be well used in the design of vertical breakwaters. The influence of the shape of the wave spectrum is not neglected which results in significant differences. However it is not yet completely clear what constant wave height or wave steepness to use to calculate the transfer function. This can probably be verified with more model tests.
With more research it can be a fine design tool for vertical breakwater.
## Symbols

In the following list the symbols, used in this study, are given with an explanation. The parameters are taken from the list used in MAST-III PROVERBS. The list is a general one. Some parameters have been extended by adding characters, but without changing their general meaning.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>°</td>
<td>Structure front slope to horizontal</td>
</tr>
<tr>
<td>( \beta )</td>
<td>°</td>
<td>Coefficient in PM spectrum</td>
</tr>
<tr>
<td>( \gamma_{JS} )</td>
<td>-</td>
<td>Direction of wave propagation relative to normal to breakwater alignment</td>
</tr>
<tr>
<td>( \eta(t) )</td>
<td>m</td>
<td>Peak factor of JONSWAP spectrum</td>
</tr>
<tr>
<td>( \eta_c )</td>
<td>m</td>
<td>Surface elevation function referred to the mean water level (MWL)</td>
</tr>
<tr>
<td>( \eta_{Fourier} )</td>
<td>-</td>
<td>Wave crest elevation referred to MWL</td>
</tr>
<tr>
<td>( \theta )</td>
<td>°</td>
<td>Fourier series</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Hz</td>
<td>Direction of wave propagation</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-</td>
<td>Crossing frequency</td>
</tr>
<tr>
<td>( \rho )</td>
<td>kg/m(^3)</td>
<td>Mean of a stochastic variable</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>kg/m(^3)</td>
<td>Width of the probability density function</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>-</td>
<td>Mass density of sea water (1030 kg/m(^3))</td>
</tr>
<tr>
<td>( \sigma_a )</td>
<td>-</td>
<td>Variance of a stochastic variable</td>
</tr>
<tr>
<td>( \sigma_b )</td>
<td>-</td>
<td>Factor used in peak-enhancement function (0.07)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>-</td>
<td>Factor used in peak-enhancement function (0.09)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>-</td>
<td>Potential</td>
</tr>
<tr>
<td>( \omega )</td>
<td>rad/s</td>
<td>Phase</td>
</tr>
<tr>
<td>( \psi )</td>
<td>m/s</td>
<td>Angular frequency, ( =2\pi f )</td>
</tr>
<tr>
<td>( A )</td>
<td>m</td>
<td>Wave celerity in deep water</td>
</tr>
<tr>
<td>( C_r )</td>
<td>°</td>
<td>Coefficient of reflection</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>m/s</td>
<td>Directional spectrum</td>
</tr>
<tr>
<td>( d )</td>
<td>m</td>
<td>Water depth over berm in front of wall</td>
</tr>
<tr>
<td>( E )</td>
<td>m</td>
<td>Fourier coefficient</td>
</tr>
<tr>
<td>( E_{kin} )</td>
<td>J/m(^2)</td>
<td>Kinetic energy per unit area</td>
</tr>
</tbody>
</table>

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Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{tot}$</td>
<td>J/m²</td>
<td>Potential energy per unit area</td>
</tr>
<tr>
<td>$E_{tot}$</td>
<td>J/m²</td>
<td>Total energy per unit area</td>
</tr>
<tr>
<td>$f$</td>
<td>Hz</td>
<td>Wave frequency</td>
</tr>
<tr>
<td>$f_p$</td>
<td>Hz</td>
<td>Frequency of peak of wave energy spectrum</td>
</tr>
<tr>
<td>$F$</td>
<td></td>
<td>Fourier coefficient</td>
</tr>
<tr>
<td>$F_{Goda}$</td>
<td>N</td>
<td>Wave force according to Goda’s method</td>
</tr>
<tr>
<td>$F_h$</td>
<td>kN</td>
<td>Horizontal force on caisson</td>
</tr>
<tr>
<td>$F_{h,0.4%}$</td>
<td>kN</td>
<td>Horizontal force with an exceedance probability of 0.4%</td>
</tr>
<tr>
<td>$F_{h,5.0%}$</td>
<td>kN</td>
<td>Horizontal force with an exceedance probability of 5.0%</td>
</tr>
<tr>
<td>$F_s$</td>
<td>kN</td>
<td>Significant wave force on caisson</td>
</tr>
<tr>
<td>$g$</td>
<td>m/s²</td>
<td>Gravitational acceleration (9.81 m/s²)</td>
</tr>
<tr>
<td>$H$</td>
<td>m</td>
<td>Wave height from trough to crest</td>
</tr>
<tr>
<td>$H_0$</td>
<td>m</td>
<td>Deep water wave height</td>
</tr>
<tr>
<td>$H_{1/3}$</td>
<td>m</td>
<td>Mean height of highest 1/3 of waves in record (=Hₚ)</td>
</tr>
<tr>
<td>$H_{max}$</td>
<td>m</td>
<td>Maximum wave height in record</td>
</tr>
<tr>
<td>$H_{tm0}$</td>
<td>m</td>
<td>Significant wave height from spectral analysis, defined 4.0m₀⁰⁵</td>
</tr>
<tr>
<td>$H_s$</td>
<td>m</td>
<td>Significant wave height, average of highest one-third of wave heights (=H₁₀)</td>
</tr>
<tr>
<td>$h_s$</td>
<td>m</td>
<td>Water depth at of toe of structure</td>
</tr>
<tr>
<td>$h_d$</td>
<td>m</td>
<td>Water depth in deep water</td>
</tr>
<tr>
<td>$k$</td>
<td></td>
<td>Wave number=2π/L</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$L_o$</td>
<td>m</td>
<td>Deep water wave length - $gT^2/2\pi$</td>
</tr>
<tr>
<td>$L_p$</td>
<td>m</td>
<td>Deep water wave length related to peak ($T_p$) period</td>
</tr>
<tr>
<td>$L_{pi}$</td>
<td>m</td>
<td>Local inshore wave length related to peak period at structure, given approximately by $(gT_p^2/2\pi)[\tanh(4\pi^2h_s/gT_p^2)]^{1/2}$</td>
</tr>
<tr>
<td>$l$</td>
<td>m</td>
<td>Length along the breakwater</td>
</tr>
<tr>
<td>$m$</td>
<td></td>
<td>Form parameter General Wave Spectrum Model</td>
</tr>
<tr>
<td>$m_n$</td>
<td></td>
<td>nᵗʰ order moment of the wave energy density spectrum</td>
</tr>
<tr>
<td>$N$</td>
<td></td>
<td>Number of values</td>
</tr>
<tr>
<td>$n$</td>
<td></td>
<td>Form parameter General Wave Spectrum Model</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>-</td>
<td>Probability function</td>
</tr>
<tr>
<td>$p$</td>
<td>kPa</td>
<td>Pressure</td>
</tr>
<tr>
<td>$p_1$</td>
<td>kPa</td>
<td>Pressure at the front face of the structure at MWL</td>
</tr>
<tr>
<td>$p_2$</td>
<td>kPa</td>
<td>Extrapolated pressure below the core beneath the structure</td>
</tr>
<tr>
<td>$p_3$</td>
<td>kPa</td>
<td>Pressure at the bottom of the structure</td>
</tr>
<tr>
<td>$p_4$</td>
<td>kPa</td>
<td>Pressure at the crest of the structure</td>
</tr>
<tr>
<td>$p_*$</td>
<td>kPa</td>
<td>Pressure due to waves</td>
</tr>
<tr>
<td>$p_o$</td>
<td>kPa</td>
<td>Hydrostatic pressure</td>
</tr>
<tr>
<td>$p(x)$</td>
<td>[x]⁻¹</td>
<td>Probability density function</td>
</tr>
<tr>
<td>$R_c$</td>
<td>m</td>
<td>Crest freeboard, level of crest above static water level</td>
</tr>
<tr>
<td>$r$</td>
<td></td>
<td>Reduction factor for short-crested waves</td>
</tr>
<tr>
<td>$S(f)$</td>
<td></td>
<td>Spectral density</td>
</tr>
</tbody>
</table>
Symbols

$S_{\eta\eta}(f)$  
Surface elevation spectrum

$S_{ff}$  
Wave force spectrum

$s_p$  
Deep water wave steepness related to peak wave period $H/L_0 = 2\pi H/gT_p^2$

$T$  
(Regular) wave period

$T_{max}$  
Wave period, associated with $H_{max}$ (not statistically maximum period)

$T_p$  
Wave period of spectral peak, inverse of peak frequency

$T_R$  
Length of wave record, duration of sea state

$T_s$  
Wave period associated with $H_s$ (not statistically significant)

$t$  
Time

$u$  
Component of velocity along x-axis

$v$  
Component of velocity along y-axis

$x$  
x-axis, perpendicular to front face of caisson, $x=0$ at the face

$y$  
y-axis, longitudinal direction of the breakwater, $y=0$ in the middle of one section

$z$  
z-axis, vertical direction, positive upwards
1. Introduction

These days lots of research is done to improve the design tools for vertical breakwaters. It is a type of breakwater that is seldom used in Europe. It is, however, gaining interest, e.g. because of large projects such as Maasvlakte II in the Netherlands, where land will be reclaimed at greater depths, and because of the increasing sizes of ships that have to stay on deep water to load and unload. This research is done in the frame of MAST. In the frame of MAST (Marine Science and Technology), which is a research program financed by the European Union, in February 1996 the project PRObabilistic design tools for VERTical BreakwaterS (PROVERBS) has been started. 23 institutes from the region of the EU participate in this project, including Delft Hydraulics, Delft Geotechnics and Delft University of Technology in the Netherlands. The purpose of this project is to develop and to implement probabilistic design tools for vertical breakwaters and other types of monolithic structures where wave conditions dominate in the design.

Vertical breakwaters can fail due to several mechanisms, such as sliding and overturning. These failure mechanisms occur due to forces induced by waves attacking the breakwater. This particular study deals with the horizontal wave forces on vertical breakwaters. The application of spectral analysis is presented to determine the horizontal wave forces. The purpose of this study is to find out whether this method is reliable.

There also exists another method to determine the wave forces on a vertical breakwater. This is the method presented by Goda. It is a widely used method. A second objective of this study is to compare the method of the spectral analysis with the method of Goda.

In this report, first the relevant theory will be studied thoroughly. Checking the reliability of the method will be done by using the results of several model tests and a wave spectrum model. The results of these tests and model, the results of the theory presented in this report and results from Goda's method will be compared. At the end some conclusions will be drawn about the method of the spectral analysis and its applicability.
Introduction
2. Approach

2.1 Scope

2.1.1 Purpose of a breakwater
Breakwaters are used in the vicinity of harbours. Their purpose can be twofold. First when a harbour has to be created on a shore they can serve to create a protective area for ships to anchor or to load and unload. If the wave attack on ships in the harbour gets to severe, the normal handlings cannot take place. Ships will have to wait for conditions that are calmer. This results in economical losses. The purpose of the breakwater is to reduce the wave height to an extent that ships will not have any problems in the harbour.

Besides that purpose the breakwaters are used for approach channels. Problems in approach channels are the following:
- The wave height is too high for ships to sail safely through the relatively narrow channel.
- The longshore current is too strong for ships that approach a harbour with slow speed. Because of this slow speed it is difficult to respond to the current, that is perpendicular to the ship's direction. It is for a ship difficult or often impossible to keep within the narrow channel.
- Because of sedimentation the approach channel will become shallow. It is difficult for dredgers to keep the channel deep without the breakwater, because the wave height will be too high and the current too strong to do their work safely.

2.1.2 Different types of breakwaters
Roughly, two types of breakwaters exist. The most frequently used type is the rubble mound breakwater. The breakwater has a more or less trapezoidal shape. The core of the breakwater consists of fine material and more to the outside the grain size of the material becomes bigger. On top the armour layer has to break the waves to such an extent that much of the wave energy will be dissipated. Besides that, the armour layer has to resist the wave attack and protect the breakwater from failure. The armour layer consists of e.g. rock material or concrete elements like tetrapods or cubes.
Another type of breakwater, that is frequently used in Japan, is the vertical breakwater. This breakwater consists of a larger concrete element, a caisson, that is placed on a rubble mound berm. The height of the berm is relatively small compared to the height of the caisson. This type of breakwaters is especially suitable at large depths since the material used for a rubble mound breakwater increases quadratically with the depth. The construction of the vertical breakwater is completely different compared to the construction of the rubble mound breakwater. The concrete caissons are built at a construction site. When the construction is complete the caissons can float in the water and with the help of tugboats they are taken to the site of the breakwater. In the mean time the sea bottom is prepared and a berm of rock material is placed on top of which the caisson is sunk. The empty compartments of the caisson are filled with sand and concrete to give the caisson weight and stability during its life time.

There is also an intermediate type, called the composite breakwater. Again a caisson is used but now the berm is much higher.

![Figure 2-1 Main types of caisson breakwaters](image)

There are a lot of alternative designs for vertical breakwaters. It is possible to make a perforated front wall for the dissipation of energy. A crown wall on top of the breakwater can be used to prevent overtopping with less material. And for more stability of the breakwater rock material can be placed in front of the breakwater to dissipate wave energy.

### 2.1.3 Failure modes of the vertical breakwater

The vertical breakwater, subject of this study, has different failure modes. The most important failure modes are:

- **Sliding.** Due to the horizontal wave force the breakwater will move in a horizontal direction perpendicular to the breakwater.
- **Overturning.** The horizontal wave load and the uplift force are the cause of a moment on the breakwater that makes it turn.
- **Shear failure of the foundation.** In the foundation of the breakwater slip circles can occur when the maximum pressure in the foundation is exceeded. The result is a complete overturning of the caisson and the berm.

Also other failure modes exist. For instance erosion is a very important danger to the breakwater.

The first two mentioned failure modes occur mainly due to the horizontal force. The determination of the horizontal force is subject of this study.
2.2 Definition of the problem

Vertical breakwaters can fail due to the horizontal force on the front wall. This horizontal force comes from the waves. It is therefore important to find a good, reliable method to determine the wave loads on the breakwater. One of the most used methods is Goda's method. This method however does not make any distinction between breaking and non-breaking waves. Breaking and non-breaking waves might have a very dissimilar influence on the breakwater. To non-breaking waves the structure will respond in a quasi-static way, while wave impact due to breaking waves make the breakwater respond dynamically. Also the influence of the width of a wave spectrum is not taken into account. Goda does not include the spreading of the energy over a frequency range fully but actually only calculates with one maximum wave height. This maximum wave height is defined as the average of the highest 1/250 part of the wave heights.
The method of the linear transfer function is a method for non-breaking waves only. Its main idea is calculating the wave force spectrum from the surface elevation spectrum with the use of a transfer function. This method has never been used for the determination of the load on a vertical breakwater. Therefore this method has to be studied and a comparison has to be made with Goda's method. One problem occurs in this method. The relation between waves and forces is assumed to be linear. However the reality is different and the influence of this non-linearity is not clear for this case.

2.3 Problem

Is the method of the linear transfer function a good method to determine the wave load on a vertical breakwater? There are two subproblems. One is the fact of the non-linearity in the transfer function. The second subproblem is whether this method is a better one than Goda's method or not.

2.4 Objective

This study has to show how to use the method of the transfer function to determine the wave load on a vertical breakwater. Therefore the non-linearity has to be studied and a solution has to be given for dealing with this non-linearity. Finally it has to become clear what the difference is in performance of this method and Goda's method.

2.5 Assumptions and restrictions

2.5.1 Waves

- The waves are non-breaking. Impacts of breaking waves are excluded. This means that the response of the breakwater will be quasi-static.
- Wave fields are described by wave spectra. The wave spectrum gives the spreading of the wave energy over the frequency range. The waves are unidirectional and normally incident
- The determination of the wave pressure under a wave is based on the linear wave theory with all its assumptions and restrictions. These will be mentioned later.

2.5.2 Geometry

- The bottom in front of the breakwater is horizontal.
- The mean water levels on both sides of the breakwater are equal.
- The breakwater consists of a series of caissons. The load on only one caisson is subject of this study. The interaction between the caissons is neglected. Calculated values are given per unit of length along the breakwater.
- The caisson is placed on a berm of a certain constant height.
2.6 Strategy

In this study of the determination of horizontal forces on vertical breakwaters first the theory will be given necessary for the understanding of this method with transfer functions.

The theory of the wave spectrum will be given and it will be demonstrated how to determine the transfer function with the help of the linear wave theory. Determining the transfer function will immediately show the non-linearity between waves and forces. This non-linearity will be discussed extensively, followed by another way to determine the transfer function.

In the following part of this report two main issues will be subject of interest:
1. Study of the non-linearity of the transfer function.
2. Study of the influence of the width of the wave spectrum.

The first issue is a study of how to use the transfer functions. The question arises very quickly: how important is the non-linearity between the waves and the forces induced by the waves? In relation to this non-linearity a second type of transfer function will be presented.

The second mentioned issue is actually a comparison between the method of the transfer function and Goda's method. In order to make a clear comparison first Goda's method will be discussed. After this discussion it will be clear that Goda does not take into account the shape of a wave spectrum but only one maximum wave height with its period. The distribution of the wave energy over the frequency range can be found in the method of the transfer function.

For the study of the two mentioned issues three different cases will be used:
1. General Wave Spectrum Model: This model makes it possible to 'create' a wave spectrum. Different wave spectra will be determined and with the transfer functions wave force spectra will be calculated. The wave spectra will vary in width and place on the frequency-axis.
2. Model tests with caissons performed at Delft Hydraulics: These are quite recent tests, performed to determine wave forces on vertical breakwaters. Also the transfer functions were derived. This enables a comparison between the theory and test results.
3. Model tests for the design of the Eastern Scheldt Storm Surge Barrier. For the design of the Storm Surge Barrier the method with the transfer functions was used to determine the wave loads. The theory was tested with model tests performed by Delft Hydraulics. In the results of the model tests transfer functions are given, that will be used to compare with the theory presented in this report.

Afterwards conclusions will be drawn concerning the application of the spectral analysis and transfer functions in the determination of wave loads on vertical breakwaters, the choice between the two types of transfer functions and how to determine this choice in a proper way. The comparison with Goda's method must result in a conclusion concerning the differences between the two methods.
3. Theory

3.1 Introduction

The method of the transfer function can be used to calculate the wave force on a breakwater. In the following two figures the idea of this method is shown. Multiplying the given surface elevation spectrum with the transfer function will give a wave force spectrum.

In fact two kinds of information are necessary for the calculation of the wave force on the breakwater. In the first place the wave spectrum is necessary. The wave spectrum is the input of the linear system. So first wave spectra will be explained in this chapter with their important parameters. The basis of the wave spectrum is the theory about stochastic variables. A wave spectrum is basically a variance spectrum. Spectral analysis enables the derivation of a wave spectrum from a set of data from wave measurements and will therefore be explained also.

The second kind of information necessary is the geometry of breakwater and foreshore. Some characteristic values of the geometry are used to determine the transfer function. In the assumed linear system the transfer function transfers the surface elevation spectrum (input) into the wave force spectrum (output). The transfer function is based on the linear wave theory and therefore this theory will be explained first. When it is known how to calculate the wave pressures the forces on the breakwater can be calculated. From these forces the transfer function can be derived.
The three pictures in Figure 3-1 show the idea of the method of the transfer function. The first picture is a wave spectrum. It describes a wave field. The horizontal axis gives the frequency, the vertical axis the contribution per unit of frequency to the total variance in the wave field.

The second picture is the actual transfer function, with again the frequency on the horizontal axis. The vertical axis gives for a certain frequency the wave force amplitude per unit surface elevation amplitude.

The last picture is the result of the multiplication of the two function values of each frequency. It is the wave force spectrum. It is basically the same picture as the one for the wave spectrum. Now the vertical axis gives the contribution per unit of frequency to the total wave force variance.

*Figure 3-1 Calculating the wave force spectrum with a transfer function*
The three pictures in Figure 3-2 are similar to the ones on the foregoing page. The only difference is that now the wave spectrum is double-peaked. One of the important features of the method of the transfer function becomes apparent immediately. The second peak with the highest peak period is very much damped by the transfer function, while the first one is getting sharper.

Figure 3-2 Calculating a wave force spectrum with a transfer function from a double-peaked wave spectrum
3.2 Wave spectrum

3.2.1 Statistical properties of stationary Gaussian processes

3.2.1.1 Momentary values
Battjes [2] gives a description of stationary Gaussian processes. By first introducing the stochastic variables and then the stochastic processes he is able to give the statistical properties of stationary Gaussian processes. Given the fact that a stochastic process \( \{x(t)\} \) is Gaussian he states that \( x(t) \), a momentary stochastic value, is Gaussian distributed. The mean of the process is per definition zero and using the fact that the variance is equal to \( m_0 \), of which the proof will be given later (section 3.2.4.1), the probability density function of \( x \) at an arbitrary instant is:

\[
p(x) = \frac{1}{\sqrt{2\pi m_0}} e^{-\frac{x^2}{2m_0}}
\]

Eq. 3-1

3.2.1.2 Maxima
Besides the distribution of the momentary values it is also useful to know the distribution of the maxima. To that end the width of the spectral density function is looked at. The definition of the width is:

\[
\rho = \frac{m_2}{\sqrt{m_n m_1}}
\]

Eq. 3-2

The parameters \( m_n \) are called moments. Many important properties of the wave spectrum can be expressed in the moments \( m_n [2] \). The \( n^{th} \) moment of the spectrum is given by:

\[
m_n = \int_{-\infty}^{\infty} f^n S_\eta (f) df
\]

Eq. 3-3

The value for \( \rho \) is always between 0 and 1. When the wave spectrum is wide (\( \rho \to 0 \)), there will be both positive and negative maxima and the probability density function of the heights of the maxima will be almost equal to the one of the momentary value, i.e. the Gaussian one.

It is found empirically that in case of wind waves the spectrum becomes narrow and the value for \( \rho \) will go to 1.

The fact that a spectrum is narrow signifies that there will only be positive maxima. The
number of maxima will be the same as the number of the upward or downward zero-crossings.

To derive an expression for the wave height first the result of Rice [3] is taken, in which he states that the mean frequency ($\lambda_0$), with which the value of a function of a stationary Gaussian process $\{x_t\}$ with an arbitrary spectrum, crosses a level $x$ in upward or downward direction, is given by:

$$\lambda_x = \left(\frac{m_x}{m_0}\right)^{\frac{1}{2}} \exp\left(-\frac{x^2}{2m_0}\right)$$  \hspace{1cm} \text{Eq. 3-4}$$

Secondly the assumption is made of a narrow spectrum, so that there are no negative maxima or positive minima and consequently there are as many maxima as upward or downward zero-crossings ($\lambda_0$ per unit of time). Even so there are as many maxima above a certain positive level $x$ as there are upgoing crossings through that level ($\lambda_x$ per unit of time). The result for a narrow spectrum is:

$$\frac{\text{number of maxima with } x_m > x}{\text{number of maxima}} = \frac{\lambda_x}{\lambda_0}$$  \hspace{1cm} \text{Eq. 3-5}$$

Using the equation for the mean frequency ($\lambda_0$) and interpreting the first part of the last equation as the probability of exceeding level $x$ by the maximum $x_m$, leads to:

$$Q_{x_m} = \Pr\{x_m > x\} = \exp\left(-\frac{x^2}{2m_0}\right)$$  \hspace{1cm} \text{Eq. 3-6}$$

$$P_{x_m} = \Pr\{x_m \leq x\} = 1 - \exp\left(-\frac{x^2}{2m_0}\right)$$  \hspace{1cm} \text{Eq. 3-7}$$

$$p_{x_m} = \frac{x}{m_0} \exp\left(-\frac{x^2}{2m_0}\right)$$  \hspace{1cm} \text{Eq. 3-8}$$

The probability density function of the heights of the maxima is called the Rayleigh probability density function. In most general form the Rayleigh probability density function and distribution function are the following:

$$p(\eta) = 0 \hspace{1cm} \text{for } \eta \leq 0$$

$$= \eta e^{-\frac{1}{2}\eta^2} \hspace{1cm} \text{for } \eta \geq 0$$  \hspace{1cm} \text{Eq. 3-9}$$

$$P(\eta) = 1 \hspace{1cm} \text{for } \eta \leq 0$$

$$= e^{-\frac{1}{2}\eta^2} \hspace{1cm} \text{for } \eta \geq 0$$  \hspace{1cm} \text{Eq. 3-10}$$

In a Gaussian process with a narrow spectrum the heights of the maxima are Rayleigh distributed.
Usually only one maximum is taken in the interval between two zero-crossings. Secondary maxima are not taken into account. For a narrow spectrum there are as upward or downward crossings through a certain positive level as there are upward or downward crossings through the zero-level. Between a upward crossing through the zero-level and the following downward crossing there will be only one maximum.

The next step is the description of a sine wave in case of a narrow spectrum. The wave height is given as twice the height of the accompanying maximum above the mean level:

\[
Pr\{H > H\} = Pr\{\eta_m > H\} = Pr\{\eta_m > \frac{1}{2} H\} \tag{3-11}
\]

\[
P_H(H) = Pr\{H > H\} = \exp\left(-\frac{H^2}{8m_0}\right) \tag{3-12}
\]

\[
p_H(H) = \frac{H}{4m_0} \exp\left(-\frac{H^2}{8m_0}\right) \tag{3-13}
\]

### 3.2.2 Spectral analysis

A sea with waves generated by wind does not give the impression of a simple sine wave. However it is possible to model this wave field as the sum of many sine waves [21].

When data are given of the time-varying water-level of the sea during a certain time \(T_R\) (say 30 minutes), the data can be separated with Fourier analysis in a number of sine waves with each its own period, amplitude and phase. The total elevation can be written as:

\[
\eta_{\text{Fourier}}(t) = \sum_{n=1}^N E_n \cos(2\pi f_n t) + F_n \sin(2\pi f_n t) \tag{3-14}
\]

in which: \(f_n = f_T = n/T_R\).

The Fourier analysis of a data set gives the values for \(E_n\) and \(F_n\) for \(n=1,2,\ldots\):

\[
E_n = \frac{1}{T_R \tau_k} \int x(t) \cos(2\pi f_n t) \, dt \tag{3-15}
\]

\[
F_n = \frac{1}{T_R \tau_k} \int x(t) \sin(2\pi f_n t) \, dt \tag{3-16}
\]

The index \(n\) denotes one single wave with a period that fits \(n\) times in the time domain of the data set, so that \(T_n = T_R/n\) (\(n=1, 2, 3\ldots\)). The function \(\eta_{\text{Fourier}}(t)\) is an approximation of the measured wave field \(x(t)\). Using the following two expressions the data of the wave field can be written in a new form:
\[ A_n = \sqrt{E_n^2 + F_n^2} \quad \text{Eq. 3-17} \]
\[ \tan \phi_n = \frac{F_n}{E_n} \quad \text{Eq. 3-18} \]
\[ \eta_{\text{Fourier}}(t) = \sum_{n=1}^{N} A_n \cos(2\pi f_n t + \phi_n) = \sum_{n=1}^{N} A_n \cos(\omega_n t + \phi_n) \quad \text{Eq. 3-19} \]

For each frequency a value for the amplitude \( A_n \) and the phase \( \phi_n \) is found.
The next step in determining the wave spectrum is made by using the Random Phase Model [3,21]. For one sine wave like

\[ \eta_i(t) = A_i \sin(\omega_i t + \phi_i) \quad \text{Eq. 3-20} \]

the mean and the variance can be found, assuming that \( \phi \) is random with an uniform probability density and amplitude that is stochastic:

\[ \mu\{\eta_i(t)\} = \int_0^{2\pi} \eta_i(t) p(\phi) d\phi = \int_0^{2\pi} A_i \sin(\omega_i t + \phi) \frac{1}{2\pi} d\phi = 0 \quad \text{Eq. 3-21} \]
\[ \sigma^2\{\eta_i(t)\} = \int_0^{2\pi} \eta_i^2(t) p(\phi) d\phi = \int_0^{2\pi} A_i^2 \sin^2(\omega_i t + \phi) \frac{1}{2\pi} d\phi = \frac{A_i^2}{2} \quad \text{Eq. 3-22} \]

When a wave field is described as the sum of a great number of sine waves like:

\[ \eta(t) = \sum_{i=1}^{I} A_i \sin(\omega_i t + \phi_i) = \sum \eta_i(t) \quad \text{Eq. 3-23} \]

the mean and the variance of the wave field become:

\[ \mu\{\eta(t)\} = \sum \mu\{\eta_i(t)\} = 0 \quad \text{Eq. 3-24} \]
\[ \sigma^2\{\eta(t)\} = \sum \sigma^2\{\eta_i(t)\} = \sum \frac{1}{2} A_i^2 \quad \text{Eq. 3-25} \]

With Fourier analysis the process has been described as the sum of a number of sine waves with different frequencies, each with its own amplitude. The value of the amplitude of one sine wave squared and divided by 2 is the contribution of that sine wave with given frequency to the total variance of the wave field.

### 3.2.3 Energy spectrum

The spectrum found above can get a second meaning. In physical processes, like the one that is subject of this study, an additional dimension is given to the variance spectrum. The determined spectrum has on the x-axis the frequency [Hz] and on the y-axis \( A_i^2 f \) [m^2·Hz⁻¹]. There is an easy link between variance and energy. The total energy in a wave field is equal to the sum of the kinetic energy and the potential energy:
\[ E_{\text{tot}} = E_{\text{kin}} + E_{\text{pot}} = \frac{1}{2} \rho g \bar{\eta}^2 + \frac{1}{2} \rho g \bar{\eta}^2 = \rho g \bar{\eta}^2 = \frac{1}{2} \rho g \bar{A}_i^2 = \rho g \text{var}(\eta) \] Eq. 3-26

The link between the variance and the energy is made by the constants \( \rho_w \) (density of the water) and \( g \) (gravitational acceleration). The expression for the total energy per unit of surface has the dimension \([\text{J/m}^2]\).

The area under the spectrum \( \rho g S_{\eta \eta} \) gives the total energy per unit horizontal area in the wave field. The area under the spectrum in the frequency range \( \Delta f \) gives the contribution of that range of frequencies to the total energy in the wave field. This is also the reason that the variance spectrum is often called the energy spectrum \( (\rho g S_{\eta \eta}(\eta)) \).

### 3.2.4 Parameters of the wave spectrum

#### 3.2.4.1 Area under the spectrum

For a lot of calculations it is convenient to know the 0th moment of the wave spectrum. This is because the area under the spectrum, which is the 0th order moment of the wave spectrum, is equal to the total variance of the surface elevation:

\[ \sigma^2 = \text{var}(\eta) = \int_0^\infty S_{\eta \eta}(f)df = \int_0^\infty f^0 S_{\eta \eta}(f)df = m_0 \] Eq. 3-27

#### 3.2.4.2 Significant wave height

The significant wave height is the mean value of the one third highest wave heights. This value is often used to describe the typical wave in a wave field.

It is formally defined as:

\[ H_s = H_{1/3} = \frac{\int_{H_s}^\infty p_H(H)dH}{\int_{H_s}^\infty p_H(H)dH} \] Eq. 3-28

\[ H_s = 4.004 \sqrt{m_0} \approx 4 \sqrt{m_0} \] Eq. 3-29

This equation gives the direct relation between the significant wave height and the probability density function of the individual wave heights. Therefore the significant wave height can also be found from the area under the surface elevation spectrum.
The quotient of the 1" order moment and the 0" order moment of the probability density function for the wave heights over the interval of the one third part of the highest wave heights gives the main point of that area, which is per definition the significant wave height. This quotient has to be found with a numerical calculation. The value $H_s$ is the value for the wave height that is exceeded by one third part of the waves. This value can be calculated with the known expressions for the Rayleigh distribution.

With these results a new expression for the Rayleigh distribution is the following:

$$P_H(H) = Pr(H > H_s) \equiv \exp \left\{ -2 \left( \frac{H}{H_s} \right)^2 \right\}$$

Eq. 3-30

3.2.4.3 Peak frequency

The peak frequency is the frequency with the largest spectral density. Usually the peak frequency or the significant wave height is directly present in the expression for the wave spectrum. The other one is used implicitly via for instance the area under the spectrum $m_p$.

3.2.5 Directional spectrum

So far the wave spectra were one-dimensional spectra. With a directional spectrum, it is possible to describe short-crested waves [4]. The study of the effect of short-crestedness on wave forces on long structures starts with the following assumptions:

- The structure is long relative to the wave lengths.
- The water is of constant mean depth.
- The structure is exposed to a random, statistically stationary and uniform, incoming wave field.
- This wave field is given by a wave spectrum.
- The wave motions and the induced forces are assumed to be described by linear equations.

With these assumptions, which are mainly the same as already given before, the force due to short-crested incoming waves should be compared to the force due to a long-crested wave field of the same frequency spectrum.
The one-dimensional frequency spectrum of the previous chapters can be rewritten in the form:

\[ S_{m}(f) = \frac{1}{\pi} \int_{-\pi}^{\pi} S_{m}(f, \theta) d\theta \]  \hspace{1cm} \text{Eq. 3-31} 

This can also be given in a different way. The distribution of the energy over the directions for one frequency are given by the normalised directional distribution function:

\[ D(\theta | f) = \frac{S_{m}(f, \theta)}{S_{m}(f)} \]  \hspace{1cm} \text{Eq. 3-32} 

The integral over all angles is equal to 1:

\[ \int_{-\pi}^{\pi} D(\theta | f) d\theta = 1 \quad 0 < f < \infty \]  \hspace{1cm} \text{Eq. 3-33} 

This directional spectrum gives the relative energy density for the directions in case of a fixed frequency. The distribution function can be written in different ways. For wind-driven waves often the following function is used [4]:

\[ D(\theta | f) = \frac{2}{\pi} \cos^{2}(\theta - \bar{\theta}) \quad \text{for} \quad \left| \theta - \bar{\theta} \right| < \frac{\pi}{2} \quad \text{and} \quad 0 < f < \infty \]  \hspace{1cm} \text{Eq. 3-34} 

### 3.3 Transfer function

In section 3.3.1 the linear wave theory will be used to describe waves and the pressures under a wave. With the linear theory it is possible to determine the transfer function. The transfer function enables the calculation of the wave force spectrum from the given wave spectrum. The first step is to determine the wave force against the wall for a regular wave. The total force against the wall is found by integration of the pressure over the depth. This has to be done for a number of frequencies in such way that with a given wave spectrum and geometry a wave force spectrum can be determined as the product of the wave spectrum and the transfer function.

### 3.3.1 Linear wave theory

#### 3.3.1.1 Travelling waves

Linear wave theory [1] assumes relatively small disturbances of the water-level, which means that the wave height \( H \) is small compared to the wave length \( L \) and the water depth \( h \). The consequence is the omitting of non-linear terms.

When a travelling sine wave with height \( H \) en length \( L \) is considered, the wave has a certain period \( T \) and celerity \( c \) compared to a co-ordinate system O-xyz. The horizontal x-axis is chosen to be in the wave direction, the also horizontal y-axis at a right angle to the x-axis and the vertical z-axis perpendicular to the plane through the
x and y-axis, the water surface. The water surface is the reference plane \( z = 0 \). The wave with a certain period \( T \), celerity \( c \) and length \( L \) behaves according to the expression:

\[
L = c \cdot T \tag{3-35}
\]

The following relation describes the travelling wave in one (x-)direction:

\[
\eta(x,t) = \frac{1}{2} H \sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{L}\right) = A \sin(\omega t - kx) \tag{3-36}
\]

The pressure can be calculated with Bernoulli's equation:

\[
p = -\rho_0 g z - \rho_0 \frac{\partial \phi}{\partial t} - \frac{1}{2} \rho_0 \mathbf{q}^2 = p_0 - \rho_0 \frac{\partial \phi}{\partial t} - \frac{1}{2} \rho_0 \mathbf{q}^2 = p_0 - p \tag{3-37}
\]

The first term in the right hand side of the equation, \( p_0 \), is the hydrostatic pressure. The other term \( p \), describes the influence of the waves to the total pressure. In the linear approach, which is discussed here, the term \( \frac{1}{2} \mathbf{q}^2 \) (where \( \mathbf{q} \) is the resultant of the particles speed) will be neglected compared to the other wave term.

![Figure 3-5 Pressures under a wave](image)

Substitution of the potential flow \( \phi \), the pressure \( p \) under a wave can be calculated [1,8]. Therefore also the dispersion relation has to be used. The dispersion relation, which is a relation between the frequency and the wave number, is a characteristic of a free water movement. The expression for the potential flow and the dispersion relation are given, followed by the expression for the wave pressure induced by waves.

Potential flow:

\[
\phi(x,z,t) = \frac{\omega A}{k} \frac{\cosh k(h+z)}{\sinh kh} \cos(\omega t - kx) \tag{3-38}
\]
Dispersion relation:
\[ \omega^2 = gk \tanh kh \]  
\[ p = -\rho_w g z + \rho_g A_s \frac{\cosh kh}{\cosh kh} \sin(\omega t - kx) \]  
Eq. 3-39  
Eq. 3-40

3.3.1.2 Standing waves against a vertical wall
When a wave in water with constant depth is approaching a vertical wall, this wave will be reflected. The incoming wave has an amplitude \( A_i \) and a horizontal particle speed \( u_i \) (the \( i \) stands for 'incoming'):
\[ \eta_i = A_i \sin(\omega t - kx) \]  
\[ u_i = \hat{u}_i \sin(\omega t - kx) \]  
Eq. 3-41  
Eq. 3-42

When the co-ordinate system is chosen in such way that the wall has the co-ordinate \( x=0 \) the condition \( u=0 \) in \( x=0 \) holds. For the reflected waves the following relations hold (\( r \) means 'reflected'):
\[ \eta_r = A_r \sin(\omega t + kx + \phi) \]  
\[ u_r = -\hat{u}_r \sin(\omega t + kx + \phi) \]  
Eq. 3-43  
Eq. 3-44

The condition \( u=0 \) in \( x=0 \) means that the horizontal particle speed of the reflected wave is opposite to the horizontal particle speed of the incoming wave \( \hat{u}_r = \hat{u}_i \) and that the phase difference \( \phi = 0 \). The result is full reflection. The reflected wave can be superposed on the incoming wave and this gives an amplitude \( A_s = 2 \cdot A_i \). The water-level development and the pressure due to the wave can then be described with:
\[ \eta = A_s \cos kx \sin \omega t \]  
\[ p_s = \rho_w g A_s \frac{\cosh kh}{\cosh kh} \cos kx \sin \omega t \]  
Eq. 3-45  
Eq. 3-46

The result of this linear approach is a standing wave. This result holds whenever there is a vertical and impermeable wall and therefore full reflection. In other cases through permeability or roughness of the wall the reflection will not be 100%. The reflection coefficient \( C_r \) expresses the extent of reflection. This coefficient is defined as the ratio of the amplitude of the reflected wave to the incoming wave \( A_r/A_i \), and has a value between 0 and 1. The resulting amplitude of \( \eta \) varies between \( A_s = A_i + A_r = (1 + C_r) A_i \), and \( A_s = A_r - A_i = (1 - C_r) A_i \).

3.3.2 Formal definition of the transfer function
In the determination of a transfer function [21] the existence of a linear system is a very important assumption. We assume therefore a system with a given input and a resulting output, which are linearly related.
For the input a sine wave is taken and due to linearity the output will then also be a sine wave, however with a different amplitude and a different phase:
Input: \[ x = \hat{x} \sin(\omega t + \phi) \]  
Eq. 3-47

Output: \[ y = \hat{x}|H| \sin(\omega t + \phi + \psi) \]  
Eq. 3-48

If the input has a random phase \( \phi \) in the interval \([0,2\pi]\) the resulting phase \( (\phi+\psi) \) will also be random in the interval \([0,2\pi]\).
The same assumption of a linear system yields for the sum of a number of waves:

Input: \[ x = \sum \hat{x}(\omega) \sin(\omega t + \phi(\omega)) \]  
Eq. 3-49

Output: \[ y = \sum \hat{x}(\omega)|H(\omega)| \sin(\omega t + \phi(\omega) + \psi(\omega)) \]  
Eq. 3-50

Using the same method for spectra as has been used in 3.2.2 for the input and the output it can be found:

Input: \[ S_{xx}(\omega) = \lim_{\Delta \omega} \frac{1}{\Delta \omega} \hat{x}^2(\omega) \]  
Eq. 3-51

Output: \[ S_{yy}(\omega) = \lim_{\Delta \omega} \frac{1}{\Delta \omega} |H(\omega)|^2 \hat{x}^2(\omega) \]  
Eq. 3-52

or:

\[ S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega) \]  
Eq. 3-53

For a linear system a transfer function \( |H(\omega)|^2 \) establishes the relation between the input and output spectra. Only the absolute value of the amplitude factor is of importance (the phase is obviously not).

### 3.3.3 Derivation of the transfer function for a vertical wall

In the case of a hydraulic structure, there often is a force coming from waves working on this structure. With only one wave taken into account, the force on the structure is easily determined by using the linear wave theory as described in section 3.3.1. However the force on the structure is coming from a wave field and not from one single wave. A wave spectrum describes the wave field. Now the wave force can also be described with a spectrum.

The transfer function is based on the linear relation between the sine wave (input) and the wave pressure against the vertical wall of the structure (output).

For the sine wave the following expression is derived in section 3.3.1.1:

\[ \eta(x, t) = A \sin(\omega t - kx) \]  
Eq. 3-54

To start the derivation of the transfer function, the wave length has to be determined for a sine wave with a certain period. The influence of the frequency can be located
in the presence of the wave length \( L \) and the wave number \( k (=2\pi/L) \) through the dispersion relation \([1]\).

\[
L = \frac{gT^2}{2\pi} \tanh \frac{2\pi h}{L}
\]  
Eq. 3-55

Given \( g, h \) and \( T \), the wave length \( L \) can be determined through iteration or available diagrams and tables. For this determination the deep water wave length is necessary:

\[
L_o = \frac{g}{2\pi} T^2
\]  
Eq. 3-56

The dispersion relation yields into:

\[
L = L_o \tanh \frac{2\pi h}{L}
\]  
Eq. 3-57

For sine waves with varying periods the wave length can be determined. In a situation of deep water \((h/L_o>0.5)\) the wave length \( L_o \) can be used. When the situation means shallow water \((h/L_o<0.05)\) the expression used reads:

\[
L = T\sqrt{gh} = f^{-1}\sqrt{gh}
\]  
Eq. 3-58

The wave length gives immediately the wave number, that is necessary for the equation of the pressure:

\[
p_+(x,z,t) = \rho_w g A_s \frac{\cosh k(h+z)}{\cosh kh} \cos(kx) \sin(\omega t)
\]  
Eq. 3-59

If the front wall of the breakwater gets the co-ordinate \( x=0 \) the value for the cosine wave will be 1 and the new equation for the wave pressure will be:

\[
p_+(z,t) = \rho_w g A_s \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t + \phi)
\]  
Eq. 3-60
Under the wave crest two areas of different pressure distribution can be recognised. The first area is the one between the wave crest and mean water-level. The pressure distribution in this area is linear. The second area is between the mean water-level and the bottom or in case of a berm the top of the berm. In this area the pressure distribution is according to the last mentioned equation.

Integration of the pressure distributions over the two areas gives the total force as the sum of two forces working on the vertical wall. The parameter $h$ gives the depth, the parameter $d$ the depth of the top of the berm below mean water-level.

$$F_1 = \frac{1}{2} \rho g A_s^2 \sin(\omega t + \phi)$$  \hspace{1cm} \text{Eq. 3-61}

$$F_2 = \rho g A_s \frac{\sinh kh - \sinh k(h-d)}{k \cosh kh} \sin(\omega t + \phi)$$  \hspace{1cm} \text{Eq. 3-62}

$$F_{tot} = \rho g \left[ \frac{1}{2} A_s^2 + \frac{\sinh kh - \sinh k(h-d)}{k \cosh kh} A_s \right] \sin(\omega t + \phi)$$  \hspace{1cm} \text{Eq. 3-63}

A very important simplification is made in the expression for the force $F_t$. Also the sine wave should be squared. However this has not been done for a reason that will be explained later.

In these expressions overtopping is supposed to be absent. The crest height is higher than the amplitude of the superposed wave amplitude.

Now the transfer function can be derived via the variances of surface elevation and wave force:
\[ \sigma^2 \{ \eta(t) \} = \int_0^{2\pi} \eta^2(\phi) p(\phi) d\phi = \int_0^{2\pi} A^2 \sin^2(\omega t + \phi) \frac{1}{2\pi} d\phi = \frac{A^2}{2} \quad \text{Eq. 3-64} \]

\[ \sigma^2 \{ F_{\text{tot}}(t) \} = \int_0^{2\pi} (F_1(t) + F_2(t))^2 p(\phi) d(\phi) \]

\[ = \int_0^{2\pi} \left[ \left( \frac{\rho_w g (1 + C_r) A}{2} \right)^2 + \left( \rho_w g (1 + C_r) AD \right)^2 \right] \frac{\sin^2(\omega t + \phi)}{2\pi} d\phi \]

\[ = \frac{1}{2} \left[ \left( \frac{\rho_w g (1 + C_r) A}{2} \right)^2 + \left( \rho_w g (1 + C_r) AD \right)^2 \right] \quad \text{Eq. 3-65} \]

\[ = \frac{A^2}{2} \left[ \left( \frac{1}{2} \rho_w g (1 + C_r)^2 A \right) + \left( \rho_w g (1 + C_r) D \right) \right]^2 \]

with:

\[ D = \frac{\sinh kh - \sinh k(h - d)}{kcosh kh} \quad \text{Eq. 3-66} \]

The integration over the phase angle \( \phi \) makes the sine disappear from the equations with \( \pi \) as a result. This also means that the influence of the time disappears. The term \( A^2/2 \) in the equation for the variance of the wave force is written separately, because this factor is equal to the variance of the surface elevation. Dividing the variance of the surface elevation to the variance of the wave force (both for the same frequency) gives the value for the transfer function for one frequency.

In the remaining term the square of the sum of the maxima of both forces \( F_1 \) and \( F_2 \) can be recognised (i.e. when the sine has a value 1).

### 3.3.4 Non-linearity of the transfer function

In the preceding sections the formal derivation has been given for a transfer function. The proof has been given for the use of the transfer function as a way to determine the wave force spectrum out of the wave spectrum. The transfer function is formally defined as:

'\text{The transfer function is the wave force (0-top) per unit of incoming wave amplitude, as a function of the frequency } f.'\]

Implicitly the transfer function is taken to be independent of the incoming wave amplitude. However the equations for the variances of both wave height and wave force show that the transfer function is dependent on the incoming wave amplitude \( A \). In the third line of the expression for the variance of the total horizontal force the amplitude remains present in the part for the force \( F_1 \) (part above the mean water-level). The force \( F_1 \) shows a quadratic dependency on the incoming wave amplitude. For the force \( F_2 \) a linear relation exists.
To illustrate the non-linearity the pressure diagram of Figure 3-7 is taken. The area between the wave crest and mean water-level is triangular as before. The area below mean water-level is now rectangular. The following equation gives the maximum wave force on the front wall of the vertical breakwater.

\[ F_i = \frac{1}{2} \rho_w g (1 + C_r) A_i^2 + \rho_w g (1 + C_r) A_i d \]  

Eq. 3-67

A division by the amplitude \( A_i \) gives the transfer function. It is obvious that the value for \( A_i \) in the first term in the right-hand side causes the non-linearity. Because of the size of the pressure diagram below mean water-level, which is relatively big, the influence of the non-linearity will be smaller than in the next case. The next case is illustrated in Figure 3-8. Now the pressure diagram below mean water-level is curved.
The maximum wave force now becomes:

\[
F_i = \frac{1}{2} \rho_w g (1 + C_r) A_i \left[ 1 + \frac{\sinh kh - \sinh k(h - d)}{k \cosh kh} \right] \]

Eq. 3-68

The division by the amplitude \( A_i \) gives the value for the transfer function. Again \( A_i \) will be present in the first part of the equation after division. The curved wave pressure diagram shows a smaller area under mean water-level compared to the situation in Figure 3-7 and therefore a relatively larger influence of the non-linearity, which comes from the area above mean water-level.

### 3.3.5 Transfer function based on a constant wave height over the frequency range

In the section 3.3.3 the total force is determined. Calculating the total force \( F_{\text{tot}} \) on the vertical wall for one wave with a certain frequency and then dividing this force by the amplitude of the incoming wave results in the first step in determining the transfer function.

This step has to be repeated for a number of waves with different frequencies and the transfer function is found.

In this section also the non-linearity also appeared. The wave amplitude \( A \) remains in the part of the force \( F_i \). In the calculation of the transfer function this wave amplitude can be taken constant for all sine waves with different frequencies. In the following part transfer function will be calculated with constant wave heights. Figure 3-9 shows transfer functions for the geometry of the model tests with caissons at Delft Hydraulics (chapter 5). The depth in these model tests is 0.61m and the depth of the top of the berm below mean water-level is kept at 0.477m. In the calculation of the transfer functions overtopping is supposed to be absent. This means that the crest height \( R_c \) is higher then the height of the wave amplitude against the front wall.

Seven transfer functions have been calculated with different constant wave heights \( H \), ranging from 0.08m to 0.20m.
3.3.6 Transfer function based on a constant wave steepness over the frequency range

Following from the transfer function given in section 3.3.3 it is an obvious step to use a constant wave amplitude over the frequency range to determine the transfer function. However, there is a second way to determine the transfer function. The definition states that the wave amplitude has to be used. But it is of course possible to calculate the transfer function with a wave amplitude that is not constant over the frequency range. The method presented in this section uses a constant wave steepness over the frequency range.

The wave steepness relates the wave frequency to the wave height via the following equation:

\[ s = \frac{2\pi H_i}{gT_i^2} = \frac{2\pi}{g} H_i f_i^2 = \frac{4\pi}{g} A_i f_i^2 \]  \hspace{1cm} \text{Eq. 3-69}

There is one obvious restriction to this method. For low frequencies, the incoming wave height becomes very big, which means high values for the transfer function. The transfer function becomes steeper for low frequencies and the values for the transfer function are larger than the other type of transfer functions up to a certain value for the frequency.

Figure 3-10 shows five transfer functions with different constant wave steepnesses over the frequency range for the geometry of the model tests with caissons at Delft.
Hydraulics. In the previous section the relevant values for the geometry of the tests have been given. The steepnesses are 1%, 2%, 3%, 4% and 5%.

Figure 3-10 Transfer functions calculated with constant wave steepnesses ranging from $s_i=0.01$ to 0.05 over the frequency range.
3.3.7 Course of the transfer functions

The transfer function based on a constant wave steepness is compared to the transfer function based on a constant incoming wave height in Figure 3-11.

![Diagram](image)

Figure 3-11 Transfer functions based on constant wave steepnesses and constant wave heights over the frequency range

It is clear to see that for low frequencies the values for the transfer function based on constant wave steepness are larger than the values for the transfer function calculated with constant wave heights. For high frequencies the values are smaller.

The transfer function based on constant wave steepness has for low frequencies values that are unrealistically high. However the transfer function based on constant wave heights has for high frequencies also unrealistically values. For a frequency of
2.0Hz the wave length \( L_0 \) is almost 0.4m. For a wave height of 0.2m, that is taken constant over the frequency range, this means a wave steepness of 50%. So both transfer functions can only be used in a certain range.

Figure 3-12 and Figure 3-13 compare the transfer functions with different constant wave heights and the transfer functions with different constant wave steepnesses over the frequency range individually. It is obvious that in case of the transfer function based on a constant wave amplitude over the frequency range the influence of the non-linearity increases with higher frequencies. However, in contrast with this, for the transfer function based on a constant wave steepness over the frequency range, the non-linearity will have a great influence for low frequencies. For higher frequencies the influence of the non-linearity will remain constant and relatively small.

For the choice of the type of transfer function the place of the wave spectrum along the frequency axis is relevant. Wave spectrum with the energy at low frequencies will be best transferred into a wave force spectrum with the transfer function based on a constant wave height over the frequency range. For wave spectra with the energy at higher frequencies the transfer function based on a constant wave steepness will be more suitable.

![Transfer functions relative to O2 (0.08m)](image)

*Figure 3-12 Relative transfer functions with constant wave heights over the frequency range. The transfer functions have been divided by the transfer function calculated with a constant wave height of 0.08m*
In following chapters this non-linearity will be studied more. With different wave spectra at different places along the frequency axis the influence of the non-linearity will be given a closer look.

### 3.3.8 Discussion on the derivation of the transfer function

In this report a derivation of the transfer function has been used that needs more explanation. A simplification has been applied in the expression for the wave force $F_i$. Assuming a hydrostatic pressure distribution, for the wave force against a vertical wall over a depth $\eta$ the expression for the force is found:

$$F_i = \frac{1}{2} \rho g \eta^2$$

Eq. 3-70

The height $\eta$ is not a constant but is varying with the time. This variation is described by a harmonic wave. This wave is given by:

$$\eta = A \sin(\omega t + \varphi)$$

Eq. 3-71

Combining both expression gives for the varying force $F_i$:

$$F_i = \frac{1}{2} \rho g \left[ A \sin(\omega t + \varphi) \right]^2$$

$$= \frac{1}{2} \rho g A^2 \sin^2(\omega t + \varphi)$$

Eq. 3-72
In the derivation of the transfer function the sine wave has not been squared. This results in a different expression for the transfer function. This expression has the advantage that the transfer function can easily be calculated. The procedure is calculating for a number of regular waves with varying periods the wave force, dividing this force by the amplitude of the regular wave and squaring the result. In this approach it is important to realise where the spectral density can be found along the frequency axis. For very low frequencies as well as for high frequencies the values of the transfer function become quite unrealistic.

Other methods to calculate the wave force with the given wave conditions are consistent 2nd-order transfer functions. These are used in ship hydrodynamics.

3.4 Wave force spectrum

For the calculation of the wave force spectrum the wave spectrum and the transfer function are necessary. In the preceding sections it has been explained how to derive these from measurements and geometry of the breakwater and foreshore.

Just as the wave spectrum the wave force spectrum has two typical parameters. The peak frequency $f_p$ gives the frequency with the largest spectral density of the total wave force energy on the structure. The significant wave force $F_s$ can be determined in the same way as $H_s$ with the exceedance curve and the probability density function combined. Because the assumption rules that there is a linear relation between the wave spectrum and the wave force spectrum, the wave force distribution will also be Rayleigh type. But in case of the wave heights the wave height is divided by 2 for the amplitude. This procedure does not have to take place for the wave forces. This results in the following expressions for the exceedance distribution and probability density function of the wave force, with $F$ is the wave force at an arbitrary time:

$$Q_F (F) = Pr\{ F > F \} = \exp \left\{ - \frac{F^2}{2m_o} \right\} \quad \text{Eq. 3-73}$$

and

$$p_F (F) = \frac{F}{m_o} \exp \left\{ - \frac{F^2}{2m_o} \right\} \quad \text{Eq. 3-74}$$

Following the same procedure as in section 3.2.4.2 gives the significant wave force in relation to the total area under the wave force spectrum $m_o$.

$$F_s = 2\sqrt{m_o} \quad \text{Eq. 3-75}$$
4. Analysis with the General Wave Spectrum Model

4.1 Introduction

Goda [7] calculates the wave force on the vertical breakwater with an input of the maximum wave height and one characteristic wave period. One of the advantages of the method of the transfer function is that it also takes the influence of the shape of the wave spectrum into account. The course of the transfer function shows that for higher frequencies the value of the transfer function decreases, which means also lower values for the wave force spectrum. Therefore in this chapter first a double peaked spectrum will be taken as input (4.3) for the method of the transfer function and afterwards the right flank of the wave spectrum will be given different slopes (4.4).

For this purpose the General Wave Spectrum Model is quite suitable.

Figure 4-1 Transfer functions for the geometry of the model tests with caissons at Delft Hydraulics
4.2 General wave spectrum model

The General Wave Spectrum Model was originally used in the design of the Eastern Scheldt Barrier [19]. The main purpose was to model swell but it proved to be useful to describe wind wave spectra and shallow water spectra as well.

The general wave spectrum model is written as:

\[ S(f) = \alpha g^2 (2\pi)^{-i} f_p^{-m-n} f^m \quad 0 < f < f_p \]  \hspace{1cm} \text{Eq. 4-1}

\[ S(f) = \alpha g^2 (2\pi)^{-i} f^{-n} \quad f_p \leq f \leq f_h \]  \hspace{1cm} \text{Eq. 4-2}

\[ S(f) = 0 \quad f > f_h \]  \hspace{1cm} \text{Eq. 4-3}

with: \( \alpha = f(H_s, f_p, f_h, m, n) \) \hspace{1cm} \text{Eq. 4-4}

\[ \alpha = \frac{H_s^2(n-1)(f_h f_p)^{(n-1)}}{16 g^2 (2\pi)^{-i} \left( \frac{n+m}{m+l} \right) f_h^{(n-l)} - f_p^{(n-l)}} \]

\( f_h \) = cut off frequency = \( q f_p = 4 \) to \( 5 f_p \)

\( m, n \) = form parameters

\begin{figure}
\centering
\includegraphics[width=\textwidth]{general_wave_spectrum.png}
\caption{Definition sketch of the general wave spectrum}
\end{figure}

With this spectrum it is possible to describe the spectral forms of many wave spectra. By choosing the right form parameters, the P&M spectrum (Figure 4-3) and the JONSWAP spectrum can be approximated too. For that it is important to note that the positions of the P&M spectrum and the JONSWAP spectrum differs along the frequency axis for \( H_s \). The steepness \( s_p (H_s/L_p) \) for the P&M spectrum is equal to 2.55% and for the JONSWAP spectrum the steepness lies between 4% and 5%. 

A.P. Verweij
The best fit of the general wave spectrum model for the P&M spectrum is found with the values for the form parameters $m=7$ and $n=4$. For the JONSWAP spectrum these form parameters are $m=8$ and $n=5$.

When of a certain wave field the data are available for the water-level development, a spectrum can be fitted along these data with the General Wave Spectrum Model. The form parameters, which are exponents in the equations, are estimated by a regression analysis.

It is also possible to describe a double peaked spectrum. Such spectrum can for instance describe a sea of wind waves that grows on top of a swell. These double peaked spectra are modelled by adding two general wave spectra with different peak periods and other parameters. The following expression shows this situation:

$$S_{\eta\eta}(f) = S_{\text{swell}}(f, f_{p1}) + S_{\text{wind}}(f, f_{p2})$$

**Eq. 4-5**

*Figure 4-4 Double peaked spectrum with $f_{p1}=0.1\text{Hz}$ and $f_{p2}=0.2\text{Hz}***
The geometry of the Delft Hydraulics model (chapter 8) is used. This means a water depth in front of the caisson of 0.61m and the top of the berm lies 0.477m below still water level.

Besides the influence of the width of the spectrum also the influence of the transfer function, and more in particular the chosen constant wave height or constant wave steepness, will be shown. Therefore transfer functions will be calculated with constant wave heights of 0.08m, 0.12m, 0.16m and 0.20m and with constant wave steepnesses of 0.010, 0.030 and 0.050 (Figure 4-1). These values are based on the geometry of the model and possible wave spectra, shown in the next chapter.

4.3 Double-peaked spectrum

Two double-peaked spectra are generated with the General Wave Spectrum Model. For each case the parameter values will be given followed by the actual calculated significant wave forces.

In case 1 the model gives a swell (starting peak wave steepness \( s_p \) of 0.004) and on top of it a wind sea. Both swell and wind sea have their own \( f_p \) and \( H_s \). The \( H_s \) of the wind sea will reduce in a few steps and the energy, that comes free, will be put in the swell. This means that the total energy \( m_o \) of the wave field is kept constant and equal to 0.0016m² (\( H_s=0.159m \)). The table shows the parameter values for the five steps of diminishing \( H_s \) of the wind sea.

The parameter values for the swell are:
- \( f_{p,\text{swell}}=0.25\text{Hz} \)
- \( H_{s,\text{swell}}=0.100m \) increasing to 0.157m
- \( s_{p,\text{swell}}=0.004 \) increasing to 0.0063
- \( m=7 \)
- \( n=3.5 \)

The parameter values for the wind sea are:
- \( f_{p,\text{wind}}=0.50251\text{Hz} \)
- \( m=8 \)
- \( n=5 \)

and:

<table>
<thead>
<tr>
<th>Step</th>
<th>( H_{s,\text{swell}} ) [m]</th>
<th>( s_{p,\text{swell}} ) [-]</th>
<th>( H_{s,\text{wind}} ) [m]</th>
<th>( s_{p,\text{wind}} ) [-]</th>
<th>( H_s ) [m]</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.0040</td>
<td>0.124</td>
<td>0.020</td>
<td>0.159</td>
</tr>
<tr>
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<td>0.0050</td>
<td>0.100</td>
<td>0.016</td>
<td>0.159</td>
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<td>0.008</td>
<td>0.159</td>
</tr>
<tr>
<td>5</td>
<td>0.157</td>
<td>0.0063</td>
<td>0.025</td>
<td>0.004</td>
<td>0.159</td>
</tr>
</tbody>
</table>

*Table 4-1 Numbers for the double-peaked spectrum in the General Wave Spectrum Model (Case 1)*

In case 2 the peak frequency of the swell is chosen higher. On top of this swell again a wind sea is generated. The parameter values for the swell are:
- \( f_{p,\text{swell}}=0.80\text{Hz} \)
- $H_{s,\text{swell}} = 0.045\text{m}$ increasing to $0.059\text{m}$
- $s_{p,\text{swell}} = 0.01845$ increasing to $0.02434$
- $m = 7$
- $n = 3.5$

The parameters of the wind sea are:
- $f_{p,\text{wind}} = 1.18\text{Hz}$
- $m = 8$
- $n = 5$

and:

<table>
<thead>
<tr>
<th>Step</th>
<th>$H_{s,\text{swell}}$ [m]</th>
<th>$s_{p,\text{swell}}$ [-]</th>
<th>$H_{s,\text{wind}}$ [m]</th>
<th>$s_{p,\text{wind}}$ [-]</th>
<th>$H_h$ [m]</th>
</tr>
</thead>
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<td>0.00892</td>
<td>0.0584</td>
</tr>
</tbody>
</table>

*Table 4-2 Numbers for the double-peaked spectrum in the General Wave Spectrum Model (Case 2)*

Appendix B show for all cases the wave spectra with the calculated wave force spectra.

For each step the wave force spectrum will be calculated and from that $F_s$ will be derived. Although the spectra are double-peaked, it is assumed that the wave height distribution is still Rayleigh-type. This means that $F_s$ can easily be found ($\approx 2\sqrt{m_0}$).

The results of the calculations are presented in Table 4-3 and Table 4-4. The rows give the calculated significant wave forces for the transfer function with the constant wave steepness, respectively constant wave height, given in the first cell of the row. From left to right the significant wave height of the wind sea decreases.

The grey rows give an indication of the non-linearity of the transfer function. These percentages have been derived by dividing the values for the significant wave force calculated with the transfer function with the smallest constant wave steepness (respectively smallest constant wave height) to the significant wave force calculated with the transfer function with the largest constant wave steepness (respectively largest constant wave height).
4.3.1 Non-linearity of the transfer function

There are some clear results in the tables of the previous section relating to the influence of the chosen transfer function. One result is the fact that for decreasing $H_s$ for the wind sea the differences between the $F_s$'s calculated with transfer functions with different constant wave heights diminish. The percentages in the grey rows show this. However for the transfer functions with constant wave steepnesses the contrary can be seen. The second result is that the differences between the forces calculated with transfer functions with increasing constant wave height over the frequency range are almost constant (compare the rows in the table). For the method with the constant wave steepness over the frequency range this conclusion does not hold.
The course of the transfer functions as can be seen in Figure 4-1 can also be seen in the results for the case with the double-peaked spectrum. For low frequencies the transfer function based on constant wave steepness gives unrealistically large forces with big differences for the used wave steepness. So when the wave spectrum has energy at low frequencies the transfer function with constant wave heights over the frequency range is more suitable than the transfer function with constant wave steepness over the frequency range.

For higher frequencies the forces calculated with the transfer functions based on constant wave steepness will be lower than based on constant wave height.

**4.3.2 Influence of the shape of the wave spectrum**

Unlike Goda’s method, the method presented in this report to determine the wave load on a vertical breakwater, takes the influence of the width and the shape of the wave spectrum into account. In this section the influence of the width is studied. A constant $H_s$ is taken and the width is varied by reducing the second peak for the wind sea. All energy is shifted to lower frequencies. Because of the fact that for lower frequencies the transfer function has higher values the wave forces will increase too.

The results of Table 4-3 and Table 4-4 show this. When the values are compared by looking at the different columns the values in the right columns appear to increase. The differences between the columns however decrease.

However it is also important to notice that the interpretation of the calculated forces is quite difficult. These forces have been calculated with fictitious wave spectra. There are not data of measured wave forces available to verify the results. These calculations can only be used to get an idea of the non-linearity and of the influence of e.g. the width of the wave spectrum, as is the subject of this section.

For case 1 the transfer function with constant wave height over the frequency range gives results that show a small deviation with the chosen constant wave height compared to the other type of transfer function (grey rows in Table 4-2 and Table 4-4). The results coming from the transfer functions with constant wave steepness deviate a lot with the chosen wave steepness. It can easily be observed that when the energy is shifted to lower frequencies the calculated wave force increases.

For case 2 the same occurs. Now however, the transfer function with the constant wave steepness gives the results with smaller deviations, compared to the transfer function with constant wave height. It also produces smaller values for the significant wave forces.

With in mind the influence of the non-linearity, the results can be looked at in relation to the width of the spectrum. Therefore in case 1 the transfer function based on a constant wave height over the frequency range is looked at. For case 2 the other transfer function is subject of study.

In the following tables, Table 4-5 and Table 4-6, the results are combined for the start situation and the final situation. These results have been taken from the Table 4-3 and Table 4-4. In the last column the increase of the significant wave force is given in percentages. The differences of both tables can hardly be compared because completely different wave spectra have been taken.

When looking at the results of the following tables it is important to compare the results with Figure 4-1. The differences between the start situation and the final situation can be explained by looking at the place where the spectrum is positioned at the frequency axis. In case 1 the transfer function is not too steep at the place of
the spectrum and therefore changes will not be very big. In the second case the contrary occurs and this can be seen in the differences between start and final situation.

<table>
<thead>
<tr>
<th>( H_s ) [m]</th>
<th>( H_{s, \text{start}}=0.124 \text{m} )</th>
<th>( H_{s, \text{final}}=0.0025 \text{m} )</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>677.89kN</td>
<td>734.96kN</td>
<td>8.4</td>
</tr>
<tr>
<td>0.12</td>
<td>708.10kN</td>
<td>765.29kN</td>
<td>8.1</td>
</tr>
<tr>
<td>0.16</td>
<td>738.34kN</td>
<td>795.65kN</td>
<td>7.8</td>
</tr>
<tr>
<td>0.20</td>
<td>768.61kN</td>
<td>826.02kN</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Table 4-5 Significant wave forces for the start situation and the final situation calculated with transfer functions with constant wave heights of 0.08m, 0.12m, 0.16m and 0.20m, Case 1

<table>
<thead>
<tr>
<th>( s ), [-]</th>
<th>( H_{s, \text{start}}=0.04 \text{m} )</th>
<th>( H_{s, \text{final}}=0.01 \text{m} )</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>125.51kN</td>
<td>144.62kN</td>
<td>15.2</td>
</tr>
<tr>
<td>0.030</td>
<td>135.33kN</td>
<td>156.50kN</td>
<td>15.6</td>
</tr>
<tr>
<td>0.050</td>
<td>145.19kN</td>
<td>168.43kN</td>
<td>16.0</td>
</tr>
</tbody>
</table>

Table 4-6 Significant wave forces for the start situation and the final situation calculated with transfer functions with constant wave steepnesses of 1%, 3% and 5%, Case 2

4.4 Changing right flank of the wave spectrum

This section again deals with a wave spectrum that is generated with the General Wave Spectrum Model. The width of the wave spectrum will now be varied by changing the slope of the right flank of the wave spectrum. With the following two expressions given it is clear that the parameter \( n \) is related to this slope.

\[
S(f) = \alpha g^2 (2\pi)^{-4} f_p^{-(n+1)} f^m \quad 0 < f < f_p \quad \text{Eq. 4-6}
\]

\[
S(f) = \alpha g^2 (2\pi)^{-4} f^{-n} f_p \leq f < f_h \quad \text{Eq. 4-7}
\]

The second expression describes the right flank of the spectrum. The factor \( f^n \) is actually responsible for the slope. An increase of \( n \) means a steeper flank. A steeper flank means less energy at higher frequencies, there where the values of the transfer function are smaller.

Now the comparison between the wave spectra with changing right flank (other values for the parameter \( n \)) is only interesting when the energy for those wave spectra is constant (\( m_0 \) must be constant) and when the peak frequency for all spectra is identical.

In the following part wave forces for wave spectra \((\ell=1, 2, 3, 4)\) with four different peak frequencies \( (f_p=0.25, 0.50, 0.75 \text{ and } 1.00 \text{Hz}) \) will be calculated (Table 4-7). For each peak frequency four wave spectra will be calculated with changing right flanks. The slope of the right flank will be changed by changing the parameters in the General Wave Spectrum Model according to Table 4-8. For \( H_s \) a constant value of 0.1m will be taken.
<table>
<thead>
<tr>
<th>i</th>
<th>$f_{p,i}$ [Hz]</th>
<th>$H_{p,i}$ [m]</th>
<th>$s_{p,i}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.1</td>
<td>0.004</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>0.1</td>
<td>0.016</td>
</tr>
<tr>
<td>3</td>
<td>0.75</td>
<td>0.1</td>
<td>0.036</td>
</tr>
<tr>
<td>4</td>
<td>1.00</td>
<td>0.1</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Table 4-7 Values of the peak frequency, significant wave height and peak wave steepness for the wave spectra

<table>
<thead>
<tr>
<th>Case</th>
<th>m</th>
<th>n</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>3</td>
<td>0.016227</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>4</td>
<td>0.022086</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>5</td>
<td>0.026992</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>6</td>
<td>0.031144</td>
</tr>
</tbody>
</table>

Table 4-8 Values for m and n

The following part will give the results in graphs and values for the significant wave force $F_s$ for all spectra. Section 4.4.1 will give the results of the study of the non-linearity of the transfer functions, while section 4.4.2 will deal with the influence of the width of the spectrum.

The following tables, Table 4-9 through Table 4-12, present the calculated significant wave forces. The tables are than same as in section 4.3. The grey rows give an indication of the non-linearity and each row shows the calculated significant wave force with one transfer function with a constant wave steepness, respectively constant wave height. The columns represent from left to right spectra with increasing slope of the right flank.

One extra column has been added to the tables. This column gives the same values as in the Table 4-5 and Table 4-6, i.e. the increase of the significant wave force due to an increasing slope of the right flank.
Wave spectra with right flanks \( f^{-n} \)
for \( f_p = 0.25 \text{Hz} \)

![Wave spectra graph](image)

Figure 4-5 Wave spectra with changing right flanks, generated with the General Wave Spectrum Model \((f_p = 0.25 \text{Hz, } H_1 = 0.1 \text{m})\)

<table>
<thead>
<tr>
<th>( F_s [\text{N}] )</th>
<th>( s_i [-] )</th>
<th>( m=7, n=3 )</th>
<th>( m=7, n=4 )</th>
<th>( m=7, n=5 )</th>
<th>( m=7, n=6 )</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>487.11</td>
<td>517.36</td>
<td>534.18</td>
<td>544.88</td>
<td>11.9</td>
<td></td>
</tr>
<tr>
<td>0.030</td>
<td>665.47</td>
<td>722.23</td>
<td>757.37</td>
<td>781.47</td>
<td>17.4</td>
<td></td>
</tr>
<tr>
<td>0.050</td>
<td>849.08</td>
<td>931.99</td>
<td>985.10</td>
<td>1022.30</td>
<td>20.4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( F_s [\text{N}] )</th>
<th>( H_1 [\text{m}] )</th>
<th>( m=7, n=3 )</th>
<th>( m=7, n=4 )</th>
<th>( m=7, n=5 )</th>
<th>( m=7, n=6 )</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>438.74</td>
<td>456.13</td>
<td>463.75</td>
<td>467.65</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>0.12</td>
<td>457.09</td>
<td>474.69</td>
<td>482.37</td>
<td>486.29</td>
<td>6.4</td>
<td></td>
</tr>
<tr>
<td>0.16</td>
<td>475.46</td>
<td>493.27</td>
<td>501.00</td>
<td>504.93</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>493.85</td>
<td>511.84</td>
<td>519.62</td>
<td>523.57</td>
<td>6.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4-9 \( F_s \) values for General Wave Spectrum Model and different transfer functions \((f_p = 0.25 \text{Hz, } H_1 = 0.1 \text{m})\) and in the grey rows the differences between the significant wave forces calculated with the transfer functions with the largest and smallest constant wave steepness (respectively wave height)
Figure 4-6 Wave spectra with changing right flanks, generated with the General Wave Spectrum Model \((f_p=0.5\text{Hz}, H_r=0.1\text{m})\)

<table>
<thead>
<tr>
<th>(s_i) [-]</th>
<th>(m=7, n=3)</th>
<th>(m=7, n=4)</th>
<th>(m=7, n=5)</th>
<th>(m=7, n=6)</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>335.85</td>
<td>364.72</td>
<td>381.30</td>
<td>391.76</td>
<td>16.6</td>
</tr>
<tr>
<td>0.030</td>
<td>380.71</td>
<td>415.80</td>
<td>436.68</td>
<td>450.31</td>
<td>18.3</td>
</tr>
<tr>
<td>0.050</td>
<td>426.17</td>
<td>467.52</td>
<td>492.72</td>
<td>509.53</td>
<td>20.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(F_s) [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_i) [-]</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>0.010</td>
</tr>
<tr>
<td>0.030</td>
</tr>
<tr>
<td>0.050</td>
</tr>
</tbody>
</table>

Table 4-10 \(F_s\) values for General Wave Spectrum Model and different transfer functions \((f_p=0.5\text{Hz}, H_r=0.1\text{m})\) and in the grey rows the differences between the significant wave forces calculated with the transfer functions with the largest and smallest constant wave steepness (respectively wave height)
Figure 4-7: Wave spectra with changing right flanks, generated with the General Wave Spectrum Model ($f_{p,3}=0.75\text{Hz}$, $H_3=0.1\text{m}$)

![Wave spectra graph]

### Table 4-11: $F_s$ values for General Wave Spectrum Model and different transfer functions ($f_{p,3}=0.75\text{Hz}$, $H_3=0.1\text{m}$)

<table>
<thead>
<tr>
<th>$s_1$ [-]</th>
<th>$m=7, n=3$</th>
<th>$m=7, n=4$</th>
<th>$m=7, n=5$</th>
<th>$m=7, n=6$</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>237.73</td>
<td>263.59</td>
<td>279.79</td>
<td>290.74</td>
<td>22.3</td>
</tr>
<tr>
<td>0.030</td>
<td>258.17</td>
<td>286.77</td>
<td>304.83</td>
<td>317.15</td>
<td>22.8</td>
</tr>
<tr>
<td>0.050</td>
<td>278.72</td>
<td>310.06</td>
<td>330.00</td>
<td>343.69</td>
<td>23.3</td>
</tr>
<tr>
<td><strong>17.2%</strong></td>
<td><strong>17.6%</strong></td>
<td><strong>17.9%</strong></td>
<td><strong>18.2%</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H_3$ [m]</th>
<th>$m=7, n=3$</th>
<th>$m=7, n=4$</th>
<th>$m=7, n=5$</th>
<th>$m=7, n=6$</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>261.17</td>
<td>287.39</td>
<td>303.42</td>
<td>314.07</td>
<td>20.3</td>
</tr>
<tr>
<td>0.12</td>
<td>278.16</td>
<td>305.18</td>
<td>321.55</td>
<td>332.38</td>
<td>19.5</td>
</tr>
<tr>
<td>0.16</td>
<td>295.24</td>
<td>323.03</td>
<td>339.73</td>
<td>350.71</td>
<td>18.8</td>
</tr>
<tr>
<td>0.20</td>
<td>312.40</td>
<td>340.94</td>
<td>357.95</td>
<td>369.08</td>
<td>18.1</td>
</tr>
<tr>
<td><strong>19.6%</strong></td>
<td><strong>18.6%</strong></td>
<td><strong>18.0%</strong></td>
<td><strong>17.5%</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the grey rows, the differences between the significant wave forces calculated with the transfer functions with the largest and smallest constant wave steepness (respectively wave height).
4.4.1 Non-linearity of the transfer function

The same conclusions can be drawn as in the section about the double-peaked spectrum. Again it appears that for low frequencies the transfer functions with constant wave steepness over the frequency range gives large differences. This becomes clear in the Table 4-9 through Table 4-12 in the grey rows. However when the peak frequency of the spectrum increases, the transfer function based on a constant wave steepness will give results which are less dependent on the chosen wave steepness than the transfer function based on the constant wave height is dependent on the chosen wave height.

Table 4-12 $F_s$ values for General Wave Spectrum Model and different transfer functions ($f_o=1.0\,\text{Hz}$, $H_s=0.1\,\text{m}$) and in the grey rows the differences between the significant wave forces calculated with the transfer functions with the largest and smallest constant wave steepness (respectively wave height)
4.4.2 Influence of the shape of the wave spectrum

In order to show the influence of the shape of the wave spectrum, in the right columns of Table 4-9 through Table 4-12 the differences are given between the cases 1 and 4. It appears that for higher values of $f_p$, the influence of the width of the wave spectrum becomes more important. However the figures show also clearly that for higher peak frequency and equal wave energy the General Wave Spectrum Model generates wider wave spectra. It is therefore not possible to compare the results of the wave spectra with different peak frequencies. It is nevertheless quite well possible to compare wave spectra with the same peak frequency. The conclusion follows that for steeper right flanks the significant wave force increases. A steeper right flank means more energy at low frequencies, where the values for the transfer functions are greater. This also follows from Figure 4-1.

All spectra in the four cases have the same amount of energy, because the values for $H_s$ was kept constant. However for higher $f_p$ the calculated forces become smaller, which can be explained from the course of the transfer function. A four times higher peak frequency gives a significant wave force that is twice as small.
5. Verification of the transfer functions with model tests

5.1 Introduction

This chapter makes a link between the theoretical model and results from model tests. The tests considered are model tests with caissons [6] and model tests performed in the design of the Eastern Scheldt Storm Surge Barrier [10,16]. Both tests have been carried out at Delft Hydraulics.

The aim of this chapter, the comparison between the results of the theoretical model and test results, is to come to a choice for the transfer function. With the chosen transfer function, i.e. which type and which constant wave height or wave steepness, the comparison can be made with Goda’s method.

5.2 Model tests with caissons at Delft Hydraulics

5.2.1 General

Model tests with caissons were carried out in the basins of Delft Hydraulics (from here on denoted as DH). One of the fields of study was the determination of the wave forces on a vertical breakwater [6].

With the determination of the wave spectrum and wave force spectrum the transfer function could be derived as the quotient of both spectral densities. These results can be used to test the method of the transfer function. The purpose of this chapter is to study the non-linearity and to make a choice for the transfer function (i.e. which constant wave height or constant wave steepness) that will be used to determine the wave forces. The wave forces calculated with the chosen transfer function will be compared to wave forces calculated with Goda’s method in a following chapter.

The description of the model and the measuring devices are given in the appendix D. In the following sections first the test conditions will be given. Two test have been performed with different wave conditions. These will be discussed.

The test conditions will be followed by the calculation with transfer functions. Both types of transfer function will be treated separately. For both types first the
comparison will be made with the transfer functions derived from the model tests. Then the measured forces with given exceedance probability will be compared with the Rayleigh distribution. For the Rayleigh distribution the calculated wave force spectrum and significant wave force serves as input.

With the comparisons the choice will be made for the transfer function, followed by the comparison with Goda's method.

5.2.2 Test conditions
This part of the experiment had the aim to determine the transfer function by dividing the measured wave spectral density to the measured wave force spectral density. Only waves which approach the wall perpendicularly, were generated. The target $H_s$ had a fixed value of 0.14m. The actual measured values for $H_p$ were slightly lower. Also the water depth in front of the caisson was kept constant at 0.61m. With these numbers the target relative water depth was $h/H_p = 4.36$.

The values for $T_p$ for the wave spectra were 1.5s and 2.12s. This corresponds with target peak wave lengths of 3.5m and 7.0m respectively. With the expression for the peak wave steepness:

$$s_p = \frac{2\pi H_s}{g T_p^2}$$

Eq. 5-1

this gives values of 0.04 and 0.02 respectively.

To obtain the statistical validity, rather long test durations were used with no less than 1000 waves.

Although the target value for $H_s$ was 0.14m, the measured $H_p$’s for both spectra turned out to be slightly lower. This means also that for fixed values for $s_p$ the values for $T_p$ are slightly different (Table 5-1).

For the wave spectrum the standard JONSWAP spectrum was used with a peak-enhancement factor $\gamma_{ps}$ of 3.3. With the $\alpha$ coefficient in the P&M spectrum the value for $m_s$ can be changed, so that the value of $H_s$ and $T_p$ match for the given fixed steepness $s_p$. The resulting values for the coefficient are:

<table>
<thead>
<tr>
<th>Test</th>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>$s_p$ [-]</th>
<th>$\alpha$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.124</td>
<td>1.99</td>
<td>0.02</td>
<td>0.003243</td>
</tr>
<tr>
<td>2</td>
<td>0.137</td>
<td>1.48</td>
<td>0.04</td>
<td>0.01304</td>
</tr>
</tbody>
</table>

Table 5-1 Values for the wave spectra for the model tests at DH
The figure gives the wave spectra for the target $H_s$ of 0.14m ($T_o$ of 1.5s and 2.12s) as well as for the measured parameters of the wave spectrum (Table 5-1). The measured wave spectrum is of course not as smooth as the one generated with the measured parameters.

Figure 5-1 JONSWAP spectra (the legend gives $T_o$ and $H_s$)

5.2.3 Influence of the non-linearity and type of transfer function
The wave forces on the vertical wall will be calculated with the theory presented in this report. A water depth of 0.61m is taken and the top of the berm lies 0.477m below mean water-level. In the determination of the transfer function wave overtopping is supposed to be absent; this means that the crest height will always be higher than the maximum of the wave amplitude against the wall. The measured wave spectra are used as input. Other relevant variables are the gravity acceleration $g$ (9.81m/s$^2$) and the density of the water $\rho$ (1000kg/m$^3$). For the reflection coefficient $C_r$ a value of 0.95 is taken.

5.2.3.1 Transfer functions with constant wave height over the frequency range
In previous sections it was shown that the relation between the incoming waves and the wave forces is non-linear. Consequently the value of the transfer function and that of the computed wave forces depend on the wave height that is used to determine the transfer function.
To investigate the influence of this non-linearity for this special case, transfer functions have been determined with different constant wave heights of 0.08m, 0.10m, 0.12m, 0.14m, 0.16m, 0.18m and 0.20m for the geometry of the model and long-crested waves.
Transfer functions (wave height constant over freq. range) and JONSWAP spectra

![Graph showing transfer functions and JONSWAP spectra](image)

**Figure 5-2** Transfer functions calculated with constant wave heights \(H = 0.08\text{m}, 0.12\text{m} \text{ and } 0.16\text{m}\) over the frequency range and the two wave spectra of test 1 and 2.

For the two different wave spectra the wave force spectra are calculated using the transfer functions with different constant wave heights over the frequency range. The following figures show the wave force spectra for test 1 and test 2.

![Wave force spectra graph](image)

**Figure 5-3** Wave force spectra calculated with wave spectrum with \(T_p = 1.99\text{s}\) and \(H_p = 0.124\text{m}\) (test 1) and transfer functions with varying constant wave height.
The results of the determination of the wave force spectra can also be given in a table. The values for $F_s$ are given for the transfer functions calculated with constant wave heights ranging from 0.08 m to 0.20 m. In Table 5-2 also values have been given for $F_s/H_s$. These values can be compared with the results calculations with regular waves later in this report. In those calculations the wave spectra have been transformed to regular waves. The results of those calculations given as $F/H$, are actually values for the transfer function, or better transfer numbers. Comparing the values for $F_s/H_s$ and $F/H$ gives an indication of the influence of the shape of the transfer function.

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>$T_p=1.99s$, $H_s=0.124m$ (test 1)</th>
<th>$T_p=1.48s$, $H_s=0.137m$ (test 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_s$ [m]</td>
<td>$F_s$ [N]</td>
<td>$F_s$ [N]</td>
</tr>
<tr>
<td>0.08</td>
<td>482.92</td>
<td>451.24</td>
</tr>
<tr>
<td>0.12</td>
<td>505.79</td>
<td>476.28</td>
</tr>
<tr>
<td>0.16</td>
<td>528.67</td>
<td>501.38</td>
</tr>
<tr>
<td>0.20</td>
<td>551.57</td>
<td>526.52</td>
</tr>
</tbody>
</table>

Table 5-2 $F_s$ and $F_s/H_s$ for DH-tests and transfer functions with constant wave heights over the frequency range. The rows represent the different transfer functions.

The values for $F_s$ calculated with the transfer function with the highest and the lowest constant wave height differ for the tests 1 and 2 14.2% and 16.7% respectively.

5.2.3.2 Comparison DH-tests and transfer functions with constant wave height over the frequency range

5.2.3.2.1 Transfer functions
The comparison is made between transfer functions derived from the test results and transfer functions from the theory. The former transfer functions are derived by dividing the wave spectrum to the wave force spectrum. Since significant energy is
present in a small frequency range, only there the values for the transfer function are interesting and valuable. For test 1 this range goes from approximately 0.25Hz to 1.00Hz. For test 2 this range lies between approximately 0.4Hz and 1.3Hz. In the remaining parts of the frequency axis there is only little energy present and small fluctuations will give huge values for the transfer function. The lines in Figure 5-5 and Figure 5-6 with the description 'frame' and 'pressure cells' indicate the results from the model tests. The two types of metering the wave forces are explained in the appendix D.

The appendix D gives also the figures of the measured wave force spectrum and wave spectrum, used to calculate the transfer functions.

Figure 5-5 Transfer functions according to theory for constant wave heights (0.08m, 0.10m, 0.16m and 0.20m) over the frequency range and according to DH test 1 (T_p=1.99s and H_s=0.124m)
Although the measured transfer function fluctuates a lot, it can be seen that the theoretical transfer function has the same course as the measured transfer functions. The calculated transfer function gives some mean course of the measured ones. The fluctuations come from the fact that the wave spectrum and wave force spectrum are not really smooth. But in general the results are, also in comparison with the theoretical transfer function, quite satisfying.

The transfer functions for both test are presented separately. However theoretically the transfer functions should have the same values and courses, since the geometry is the input in the determination of the transfer function and not the wave spectrum. But the transfer functions from the DH test are derived from the spectra by division and will therefore be valid only at a certain range (that have already been given).

5.2.3.2.2 Distribution of wave forces

With the theory here presented, based on the linear wave theory, the values for $F_s$ can be determined as given in Table 5-2. In the Vinjé basin the wave forces were measured with pressure cells and a caisson in a measuring frame. The results are presented in a slightly different way. The forces are given as values which are exceeded a certain number of times. The number of waves that were measured was 1000. The force $F_{0.1\%}$ means the highest recorded force, $F_{0.4\%}$ means the fourth highest wave. $F_{1250}$ is the mean of the four highest recorded forces in a 1000 waves long test. For the two tests the following results have been found:
Table 5-3 Forces measured at the DH-tests

The results of Table 5-3 can be compared with the results of the calculations with the transfer functions. The results of the calculations with the spectral analysis are forces that fit the Rayleigh distribution. So the results for $m_0$ or $F_s$ (previous section) can be used to describe an exceedance curve of the individual wave forces:

$$Q_E(F) = Pr\{F > F\} = \exp\left(-\frac{F^2}{2m_0}\right) = \exp\left(-2\left(\frac{F}{F_s}\right)^2\right)$$

Eq. 5-2

With this distribution the forces with exceedance probabilities of 0.1%, 0.2%, 0.4%, 1.0%, 2.0% and 5.0% have been calculated. These can be compared with the forces above.

Only the last column of Table 5-3 gives relevant information. Because of the fact that the number of waves is small, the 5.0% exceedance probability value has the smaller statistical error. In the appendix D the complete tables are given. The results of the two model tests are compared to values calculated with the Rayleigh distribution. The first (grey) rows in Table 5-4 give the forces with an exceedance probability of 5% resulting from the model tests. The other rows present the same forces coming from the Rayleigh distribution.

Table 5-4 Forces with 5.0% exceedance probability in the grey rows (DH) and Rayleigh distribution (calculated). The last four rows give each the results for the transfer function with one constant wave height

The loads acting on the tested structures have been analysed statistically. In [6] a two-parameter Weibull distribution has been fitted to the measured horizontal forces with exceedance probability of 5%, 2%, 1%, 0.4%, 0.2% and 0.1% (the latter value is the maximum measured force). In the following expression for the exceedance probability of the horizontal wave forces, $a$ is the scale parameter, $b$ the shape parameter. Fitting of a line through the forces with a certain exceedance probability resulted in the values for the shape parameters $b$ and scale parameter $a$. The values for these parameters are reported in Table 5-5.
\[ P(F) = e^{-(\frac{F}{a})^b} \]  

Eq. 5-3

<table>
<thead>
<tr>
<th>Test</th>
<th>Pressure cells (offshore)</th>
<th>Caisson frame (onshore)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>b 3.11</td>
<td>a 595.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>b 2.03</td>
</tr>
<tr>
<td>2</td>
<td>1.07</td>
<td>165.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>319.0</td>
</tr>
</tbody>
</table>

Table 5-5 Scale and shape parameter of the Weibull distribution function for the horizontal forces of the tests 1 and 2

The Weibull function fitted to the results of test 2 shows low correlation with the data. Figure 5-7 and Figure 5-8 show exceedance curves of the horizontal forces for test 1 respectively test 2. With the terms 'onshore' and 'offshore' the lines are denoted that have been fitted through the forces measured with the measuring frame, respectively the pressure cells. The other lines are exceedance curves of the results of the calculations with the transfer functions. The significant wave force is used in the expression for the Rayleigh distribution function. This function is drawn in the figures for the results of seven different transfer functions.

![Exceedance curves of the horizontal force test 1](image)

Figure 5-7 Exceedance curves for the horizontal force of the measured empirical distribution versus the theoretical distribution (test 1)
5.2.3.3 Conclusions

The comparison in Figure 5-5 through Figure 5-8 and the additional comments result in a few conclusions and remarks for the transfer functions calculated with constant wave heights over the frequency range:

- In the comparative figures with the transfer functions, the theoretical transfer functions approach the measured transfer functions to some extend. The only interesting range is the one with the spectral density. Especially for test 1 the theoretical transfer function gives a good average course. However the measured transfer functions fluctuate too much to give a definite answer to the question which constant wave height to use. Therefore the constant wave height is chosen to be the same as the significant wave height of the wave spectrum for both tests.

- The distribution functions resulting from the model test 1 ($T_p=1.99s, H_s=0.124m$) at DH show some similarity with the results from the calculations with the transfer functions. Especially the model test results from the measuring frame are well described by the results from the transfer function.

- The distribution functions resulting from the model test 2 ($T_p=1.48s, H_s=0.137m$) at DH deviate quite a lot from the results calculated with the transfer functions. The measuring frame gives much lower values. The pressure cells give results that do not match with the Rayleigh distribution.

- The values in the right part of Figure 5-7 and Figure 5-8, that is with the smallest probability of exceedance, are rather inaccurate for the model test results. It is therefore difficult to draw a conclusion from those results.

- The values for the wave forces calculated with transfer functions in both graphs do not differ very much. The difference of energy of the wave spectra nullify the difference caused by the varying peak frequencies.
The final conclusion, therefore, is the same as the first mentioned above: use the significant wave height of the incoming wave field as constant wave height for the transfer function.

5.2.3.4 Transfer functions with constant wave steepness over the frequency range

Now the transfer function will be determined in a different way. Instead of a constant wave height, here the wave steepness is taken as a constant over the frequency range. This means that with the following relation and a fixed wave steepness a wave height can be determined for each wave period:

\[
H_i = \frac{sg}{2\pi f_i^2}
\]  
Eq. 5-4

The wave steepnesses considered are 1%, 2%, 3%, 4% and 5%. There is one obvious restriction to this method. For low frequencies, the incoming wave height becomes unrealistically high, which means very large values for the transfer function.

![Transfer functions (wave steepness constant over freq. range) and JONSWAP spectra](image)

Figure 5-9 Transfer functions calculated with constant wave steepnesses ranging from \(s_i=2\%\) to \(4\%\) over the frequency range and the two wave spectra of test 1 and 2

The following two figures show the calculated wave force spectra for test 1 and test 2.
For the two used spectra in the model tests Table 5-6 gives the following values for $F_s$ calculated with the transfer functions with constant wave steepness. In this table also values have been given for $F_s/H_s$. 
Table 5-6 $F_s$ and $F_s/H_s$ for DH-tests and transfer functions with constant wave steepness over the frequency range. The rows represent the different transfer functions.

The values for $F_s$ for the transfer functions with $s_i$ 0.010 and 0.050 differ 27.8% and 19.1% for the two tests respectively.

5.2.3.5 Comparison DH-tests and transfer functions with constant wave steepness over the frequency range

5.2.3.5.1 Transfer functions

The same holds for these transfer functions as has been said in the section about the transfer functions based on constant wave heights. The ranges with relevant spectral density have again been indicated in the figures.

Figure 5-12 Transfer functions according to theory for constant wave steepnesses (2%, 3% and 4%) over the frequency range and according to DH test 1 ($T_p=1.99s$ and $H_s=0.124m$)
It is obvious that with higher frequencies the influence of the non-linearity becomes smaller. However the ranges of spectral density of the wave spectra are not at that part of the frequency range, but at lower frequencies. So the advantage of this type of transfer function does not work in the range of interest.

The transfer functions based on constant wave steepnesses however seem to follow the measured transfer functions to a certain extent. There are some fluctuations in the measured transfer functions. These fluctuations have maxima that increase with lower frequencies. This tendency of increasing maxima is followed by the calculated transfer function.

5.2.3.5.2 Distribution of wave forces

The same procedure with the Rayleigh distribution will be followed as in section 5.2.3.2.2. The meaning of the different forces has already been explained as well as the relevance of only the 5% exceedance probability. Table 5-7 gives some values for the calculation with this method. The appendix D contains the complete results. Again the data coming from the model test at DH [6] are added. Figure 5-14 and Figure 5-15, which are of the same kind as in the section 5.2.3.2.2, give some indication of the results. The results coming from test 2 again deviate considerably from the results of the calculation, however less spectacularly.
Table 5-7 Forces with 5.0% exceedance probability in the grey rows (DH) and Rayleigh distribution (calculated). The last three rows give each the results for the transfer function with one constant wave steepness.

<table>
<thead>
<tr>
<th>Measuring method/transfer function</th>
<th>$F_{h,5.0%}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame (T_p=1.99s, H_s=0.124m)</td>
<td>test 1: 699</td>
</tr>
<tr>
<td></td>
<td>test 2: 513</td>
</tr>
<tr>
<td>Pressure cells</td>
<td>831</td>
</tr>
<tr>
<td>s_t=0.020</td>
<td>615</td>
</tr>
<tr>
<td>s_t=0.030</td>
<td>654</td>
</tr>
<tr>
<td>s_t=0.040</td>
<td>695</td>
</tr>
<tr>
<td>s_t=0.050</td>
<td>520</td>
</tr>
<tr>
<td>s_t=0.060</td>
<td>540</td>
</tr>
<tr>
<td>s_t=0.070</td>
<td>565</td>
</tr>
<tr>
<td>s_t=0.080</td>
<td>590</td>
</tr>
</tbody>
</table>

Figure 5-14 Exceedance curves for the horizontal force of the measured empirical distribution versus the theoretical distribution (test 1)
5.2.3.6 Conclusions

- Figure 5-12 and Figure 5-13 show that in the areas of spectral density the theoretical transfer functions describe the measured transfer function quite satisfying. For test 1 as well as test 2 the theoretical transfer functions give good mean courses. Test 2 gives the better results. The measured transfer functions show that, although they fluctuate a lot, the top values of the ‘peaks’ of the transfer functions increase with decreasing frequency. The transfer function based on constant wave steepness describes this phenomenon rather well.

- Again the distribution functions resulting from the model test 1 \((T_p=1.99s, H_s=0.124m)\) at DH show some similarity with the results from the calculations with the transfer functions. Especially the model test results from the measuring frame are well described by the results from the transfer function. The distribution function of the results of the transfer function with a value for the constant wave steepness of 4% seems to follow the distribution function coming from the results of the measuring frame.

- The distribution functions resulting from the model test 2 \((T_p=1.48s, H_s=0.137m)\) at DH deviate quite a lot from the results calculated with the transfer functions. The measuring frame gives much lower values. The pressure cells give results that do not match with the Rayleigh distribution.

- The values in the right part of the graph, that is with the smallest probability of exceedance, are rather inaccurate for the model test results. It is therefore difficult to draw a conclusion from those results.

- The values for the wave forces calculated with transfer functions in both graphs do not differ very much. Although the wave spectra have different peak frequencies, the difference of energy of the wave spectra nullify the difference caused by the varying peak frequencies.
At this point a general conclusion has to be drawn about what transfer function to use for calculations of the wave forces. A choice has to be made for the constant wave steepness to use in the determination of the transfer function. Although the results of this chapter are quite satisfying regarding course of the transfer functions and wave forces, a preference for a certain wave steepness did not emerge. Therefore wave steepnesses are chosen that are related to the incoming wave field. The peak wave steepness of the incoming wave field is used. This means 2% for test 1 and 4% for test 2.

5.2.4 Influence of the shape of the wave spectrum

After the study of the previous chapter concerning the shape of the wave spectrum, in this section another analysis will be made of the influence of the shape of the wave spectrum. Therefore the measured wave spectra described in section 5.2.2 have been slightly changed. Instead of a spreading of the energy over a frequency range the energy has been put at one frequency, the peak frequency of the spectra. In fact now regular waves are examined with a period equal to the peak period and a wave height equal to the significant wave height of the wave spectrum.

5.2.4.1 Transfer functions with constant wave height over the frequency range

The results are given in Table 5-8. It is obvious that the results coming from the wave with the smallest period $T$, do not differ very much from the ones coming from the other wave. However the steepness of the lines shows the influence of the size of the used wave height for the transfer function.

![Graph showing forces induced by regular waves]

*Figure 5-16 The influence of the used $H_i$ for the transfer function on the $F$ for regular waves (test 1: $H=0.124m$ and $T=1.99s$, test 2: $H=0.137m$ and $T=1.48s$)*
Verification of the transfer functions with model tests

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>T=1.99s, H=0.124m (test 1)</th>
<th>T=1.48s, H=0.137m (test 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_i [m]</td>
<td>F [N]</td>
<td>F/H [N/m]</td>
</tr>
<tr>
<td>0.08</td>
<td>514.86</td>
<td>4152</td>
</tr>
<tr>
<td>0.12</td>
<td>537.99</td>
<td>4339</td>
</tr>
<tr>
<td>0.16</td>
<td>561.11</td>
<td>4525</td>
</tr>
<tr>
<td>0.20</td>
<td>584.24</td>
<td>4712</td>
</tr>
</tbody>
</table>

Table 5-8 F and F/H for regular waves with transfer functions with constant wave heights over the frequency range

The values for F/H calculated with the transfer function with the highest and the lowest constant wave height differ for the tests 1 and 2 13.5% and 15.6% respectively.

5.2.4.2 Transfer functions with constant wave steepness over the frequency range

In this section the wave forces are calculated for the situation with transfer functions based on constant wave steepnesses over the frequency range.

![Diagram](forces_induced_by_regular_waves.png)

*Figure 5-17 The influence of the used s, for the transfer function on the F for regular waves (Test 1: H=0.124m and T=1.99s, Test 2: H=0.137m and T=1.48s)*

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>T=1.99s, H=0.124m (test 1)</th>
<th>T=1.48s, H=0.137m (test 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_i [-]</td>
<td>F [N]</td>
<td>F/H [N/m]</td>
</tr>
<tr>
<td>0.010</td>
<td>504.35</td>
<td>4067</td>
</tr>
<tr>
<td>0.020</td>
<td>540.10</td>
<td>4356</td>
</tr>
<tr>
<td>0.030</td>
<td>575.85</td>
<td>4644</td>
</tr>
<tr>
<td>0.040</td>
<td>611.60</td>
<td>4932</td>
</tr>
<tr>
<td>0.050</td>
<td>647.35</td>
<td>5221</td>
</tr>
</tbody>
</table>

Table 5-9 F and F/H for regular waves with transfer functions with constant wave steepnesses over the frequency range

The values for F/H calculated with the transfer function with the biggest and the smallest wave steepness differ for the tests 1 and 2 28.4% and 18.9% respectively.
5.2.4.3 Comparison between the results

In section 5.2.4.1 and 5.2.4.2 the transfer functions have been used to calculate the horizontal forces for the situation when all energy is put at one frequency. The sections 5.2.3.1 and 5.2.3.4 give the results of the calculations of the wave force spectrum from the wave spectrum. In the following two tables the results have been combined. The results with the energy concentrated at one frequency are called 'Dirac' type. The grey rows show the differences between the forces calculated with the transfer functions with the smallest and the largest wave steepness respectively wave height. It gives an idea of the influence of the non-linearity.

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>( F ) and ( F_s ) [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_s = 1.99s ), ( H_s = 0.124m ) (test 1)</td>
<td>( T_s = 1.48s ), ( H_s = 0.137m ) (test 2)</td>
</tr>
<tr>
<td>( s, [-] )</td>
<td>'Dirac' type</td>
</tr>
<tr>
<td>0.010</td>
<td>504.35</td>
</tr>
<tr>
<td>0.020</td>
<td>540.10</td>
</tr>
<tr>
<td>0.030</td>
<td>575.85</td>
</tr>
<tr>
<td>0.040</td>
<td>611.60</td>
</tr>
<tr>
<td>0.050</td>
<td>647.35</td>
</tr>
<tr>
<td><strong>28.4%</strong></td>
<td><strong>27.8%</strong></td>
</tr>
</tbody>
</table>

Table 5-10 Values for \( F \) ('Dirac' type) and \( F_s \) (wave spectrum) calculated with transfer functions with constant wave steepness over the frequency range.

<table>
<thead>
<tr>
<th>Transfer function</th>
<th>( F ) and ( F_s ) [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_s = 1.99s ), ( H_s = 0.124m ) (test 1)</td>
<td>( T_s = 1.48s ), ( H_s = 0.137m ) (test 2)</td>
</tr>
<tr>
<td>( H_s, [m] )</td>
<td>'Dirac' type</td>
</tr>
<tr>
<td>0.08</td>
<td>514.86</td>
</tr>
<tr>
<td>0.12</td>
<td>537.99</td>
</tr>
<tr>
<td>0.16</td>
<td>561.11</td>
</tr>
<tr>
<td>0.20</td>
<td>584.24</td>
</tr>
<tr>
<td><strong>13.5%</strong></td>
<td><strong>14.2%</strong></td>
</tr>
</tbody>
</table>

Table 5-11 Values for \( F \) ('Dirac' type) and \( F_s \) (wave spectrum) calculated with transfer functions with constant wave height over the frequency range.

In the mentioned sections also the values \( F/H \) and \( F_s/H_s \) have been given. In case of regular waves these values are actually transfer numbers, a value for the transfer function for one frequency.

Both transfer functions give different results, but the differences between spectra and regular waves are for both methods almost the same: less than 10% compared to the accompanying significant wave force for both methods.
5.3 Model tests for the design of the Eastern Scheldt Storm Surge Barrier

5.3.1 General
Many model tests were performed in the design of the Eastern Scheldt Storm Surge Barrier. Some of those tests are useful in this study, especially the tests regarding the loads from a perpendicular wave attack for a closed barrier in the deep section of ‘Roompot’ described in report M1422-I [18] and the loads from a perpendicular wave attack (regular and irregular waves) for a closed barrier in a deep and shallow section described in report M1469 [10]. In order to calculate the forces on the barrier also the spectral analysis with transfer functions has been used. To validate this method the results of the model tests were also presented as transfer functions. Besides that the computer program QS-GOLF was made to determine the wave forces through spectral analysis and transfer functions. QS-GOLF is a program of the Directorate General of Public Works and Water Management, Rijkswaterstaat, designed to calculate horizontal forces and moments on a closed barrier as a result of perpendicular wave attack. The results of the model tests had to confirm the results of QS-GOLF too.

Considering the long names of the model tests, from here on the report codes M1422-la and M1469 are used to denote the different model tests.

M1422 concerns model tests to determine the wave forces on two parts of the storm surge barrier in the Eastern Scheldt. These parts of the barrier are located in the deepest part of the line in the Roompot and the northern wells in the Hammen. These locations are chosen as probably normative for the maximum dimensions of the well. The tests that are used for the comparison with the theory of the transfer functions are those of the Roompot (M1422-la).

The main purpose of the study M1469 is testing the transfer functions of the forces on a closed barrier, which are calculated with the program QS-GOLF on the basis of the linear wave theory. The forces concern only the part of the barrier above the berm. Besides the determination of the forces on the breakwater, also the reflection coefficient, necessary for QS-GOLF, has been determination and the linearity of the relation between the force and the wave height has been studied.

All values in the next section are given in prototype scale, although the test were of course performed at model scale.

5.3.2 Theoretical model
For the geometries of the model tests 1422-la and 1469, described in the following sections, the transfer functions are determined as described in preceding chapters. These transfer functions are compared to the results of the model tests in order to validate the theory.

In the determination of the transfer functions two types can be discerned: 1. Transfer function with constant wave height over the frequency range. For this determination constant wave heights are used with $H$ is 1.0m, 1.5m, 2.0m, 2.5m, 3.0m, 3.5m and 4.0m.
2. Transfer function with constant wave steepness over the frequency range. The used wave steepnesses are 1.5%, 2%, 2.5%, 3%, 3.5%, 4% and 4.5%.

For the reflection coefficient a value of 1 is taken and for the $\rho 1000 \text{kg/m}^3$.

In the following sections the comparisons are made.

### 5.3.3 Model test 1422-la

#### 5.3.3.1 Water-levels and wave spectra

For the design of the structure of the storm surge barrier the wave forces have been determined for five different water-levels. These water-levels were $+3.50 \text{m}$, $+4.50 \text{m}$, $+5.50 \text{m}$, $+6.50 \text{m}$ and $+7.50 \text{m}$ NAP.

To determine the head difference over the barrier a water-level at the Eastern Scheldt side of $-1.70 \text{m}$ NAP was taken (although a value of $-1.50 \text{m}$ NAP was planned).

Then for every water-level two wave spectra were generated: spectrum A and spectrum B. These spectra had to have only one peak.

<table>
<thead>
<tr>
<th>Test</th>
<th>Spectrum</th>
<th>Water-level [m +NAP]</th>
<th>$\alpha$ [-]</th>
<th>$H_s$ [m]</th>
<th>$f_p$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sea side</td>
<td>Eastern Scheldt side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>A</td>
<td>3.5</td>
<td>-1.7</td>
<td>0.85</td>
<td>3.63</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>3.5</td>
<td>-1.7</td>
<td>0.85</td>
<td>3.87</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>4.5</td>
<td>-1.7</td>
<td>0.69</td>
<td>4.58</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>4.5</td>
<td>-1.7</td>
<td>0.65</td>
<td>4.87</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>5.5</td>
<td>-1.7</td>
<td>0.52</td>
<td>4.82</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>5.5</td>
<td>-1.7</td>
<td>0.53</td>
<td>5.16</td>
</tr>
<tr>
<td>7</td>
<td>A</td>
<td>6.5</td>
<td>-1.7</td>
<td>0.51</td>
<td>5.68</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>6.5</td>
<td>-1.7</td>
<td>0.45</td>
<td>5.37</td>
</tr>
<tr>
<td>9</td>
<td>A</td>
<td>7.5</td>
<td>-1.7</td>
<td>0.44</td>
<td>5.80</td>
</tr>
<tr>
<td>10</td>
<td>B</td>
<td>7.5</td>
<td>-1.7</td>
<td>0.43</td>
<td>6.07</td>
</tr>
</tbody>
</table>

*Table 5-12 Wave spectra and water-levels*

#### 5.3.3.2 The results

Before the start of the tests the instruments were adjusted without the head difference. Then when the head difference was prepared the instruments were put to the zero-level again. After this procedure the wave paddle was started. In this way only the pressure by waves was determined, not the pressure because of the head difference.

The results are given as the quotient of the wave force and the incoming wave height. This value is not squared as is done in the other parts of this study.

For the tests with a water-level higher than the wall, the relation between the waves and forces becomes linear and all transfer functions will have the same values. This is a rough approach, because the columns are higher than the wall.
The following figures give the transfer functions for the different water-levels. These transfer functions are the ones with constant wave height over the frequency range, constant wave steepness over the frequency range and determined from the model test results.

For the water-levels +3.50m and +4.50m NAP the non-linear behaviour can be seen in the first two figures. For the other three water-levels the upper part of the pressure diagram is omitted and therefore all transfer functions are equal.

For increasing water-level the calculated transfer functions describe the measured transfer functions better. The differences can be explained by the neglectings of the columns of the barrier.

The course of the measured transfer function is best described by the transfer function with constant wave steepness, because of the rather steep slope.

Figure 5-18 Transfer functions for water-level +3.5m NAP

Figure 5-19 Transfer functions for water-level +4.5m NAP
Verification of the transfer functions with model tests

![Graphs showing transfer functions for water-level +5.5m NAP, +6.5m NAP, and +7.5m NAP with various frequency (f [Hz]) and response (O [radians]) values.](image)

Figure 5-20: Transfer functions for water-level +5.5m NAP
Figure 5-21: Transfer functions for water-level +6.5m NAP
Figure 5-22: Transfer functions for water-level +7.5m NAP
5.3.4 Model test 1469

5.3.4.1 Water-levels and wave spectra

For the deep section three values for $H_s$ have been adjusted for each water-level. The tests with the highest waves did not produce much new information and have therefore been cancelled for the tests in the shallow section.

The wave spectra used had the following properties:
- an extreme steep left flank,
- a $T_s/T_r$ ratio between 1.4 and 1.5,
- the spectra were supposed to have one single top.

Table 5-13 and Table 5-14 give the values for the adjusted wave spectra. The actually incoming wave spectra are slightly different according to the influence of the reflection coefficient.

<table>
<thead>
<tr>
<th>Test</th>
<th>Water-level sea side [m + NAP]</th>
<th>$\alpha$ [-]</th>
<th>$H_s$ [m]</th>
<th>$f_s$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>201</td>
<td>7.5</td>
<td>0.51</td>
<td>5.40</td>
<td>0.079</td>
</tr>
<tr>
<td>202</td>
<td>7.5</td>
<td>0.50</td>
<td>4.60</td>
<td>0.083</td>
</tr>
<tr>
<td>203</td>
<td>7.5</td>
<td>0.44</td>
<td>3.50</td>
<td>0.094</td>
</tr>
<tr>
<td>204</td>
<td>5.5</td>
<td>0.62</td>
<td>4.75</td>
<td>0.084</td>
</tr>
<tr>
<td>205</td>
<td>5.5</td>
<td>0.58</td>
<td>3.35</td>
<td>0.099</td>
</tr>
<tr>
<td>206</td>
<td>5.5</td>
<td>0.55</td>
<td>2.60</td>
<td>0.109</td>
</tr>
<tr>
<td>207</td>
<td>3.5</td>
<td>0.68</td>
<td>3.65</td>
<td>0.095</td>
</tr>
<tr>
<td>208</td>
<td>3.5</td>
<td>0.70</td>
<td>2.50</td>
<td>0.109</td>
</tr>
<tr>
<td>209</td>
<td>3.5</td>
<td>0.62</td>
<td>1.35</td>
<td>0.139</td>
</tr>
</tbody>
</table>

*Table 5-13 Wave spectra for the deep section*

<table>
<thead>
<tr>
<th>Test</th>
<th>Water-level sea side [m + NAP]</th>
<th>$\alpha$ [-]</th>
<th>$H_s$ [m]</th>
<th>$f_s$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>401</td>
<td>7.5</td>
<td>0.57</td>
<td>4.05</td>
<td>0.082</td>
</tr>
<tr>
<td>402</td>
<td>7.5</td>
<td>0.54</td>
<td>3.10</td>
<td>0.097</td>
</tr>
<tr>
<td>403</td>
<td>5.5</td>
<td>0.77</td>
<td>3.15</td>
<td>0.100</td>
</tr>
<tr>
<td>404</td>
<td>5.5</td>
<td>0.72</td>
<td>2.40</td>
<td>0.116</td>
</tr>
<tr>
<td>405</td>
<td>3.5</td>
<td>0.89</td>
<td>2.35</td>
<td>0.109</td>
</tr>
<tr>
<td>406</td>
<td>3.5</td>
<td>0.71</td>
<td>1.40</td>
<td>0.137</td>
</tr>
</tbody>
</table>

*Table 5-14 Wave spectra for the shallow section*

5.3.4.2 Description QS-GOLF

For the deep section of model test M1469 also values are given of QS-GOLF. Two-dimensional quasi-static wave forces and moments on a structure or parts of a structure are calculated by this program. The basis of this program is the linear wave theory. This means that QS-GOLF contains all limitations the linear wave theory contains.

The basic equation of QS-GOLF is the one that describes the pressure development as a result of the wave motion according to the 1st-order-theory as a function of place and time:
\[ p = \rho g \frac{H \cosh kd}{2 \cosh kh} \left[ (1 + \alpha) \cos kx \sin \omega t + (1 - \alpha) \sin kx \cos \omega t \right] \]  

Eq. 5-5

The equation describes the pressure distribution below mean water-level and above the top of the berm. Above mean water-level the triangular pressure diagram is taken. Integration of the pressure over the total surface gives the total wave force.

The determination of the transfer function for a certain structure, geometry and water-level by means of QS-GOLF comes down to calculating for a number of wave periods the wave force (0-top) per unit of wave height. For each frequency the parameters have to be known. Connecting the calculated points results into the transfer function.

5.3.4.3 The results

Before the start of the tests the instruments were adjusted without the head difference. Then when the head difference was prepared the instruments were put to the zero-level again. After this procedure the wave paddle was started.

A period of 15 minutes is necessary for the wave pattern to become stable. The value for \( H_s \) becomes constant due to successive reflection against structure and wave paddle. Now the measurements started and during the time of 1000 water waves the forces were measured.

Several wave spectra have been generated with different \( H_s \) and \( T_p \) for the three water-levels at sea side (+3.5m, +5.5m, +7.5m NAP). Again the tests have been performed in both sections.

The transfer function has been calculated for several points on the frequency-axis from the spectral densities of the considered wave forces and the incoming wave heights.

\[ \frac{F}{H} = A \frac{S_{ff}}{\sqrt{S_{yy}}} \]  

Eq. 5-6

With the factor \( A \) the value for the transfer function, calculated from the incoming wave height, determined at 15m from the structure, is standardised to the value, that goes with the mean incoming wave height in the flume.

Figure 5-23 through Figure 5-25 give the transfer functions for the shallow section. Figure 5-26 through Figure 5-28 show them for the deep section. Only the transfer functions for water-level +3.50m NAP have a distinction in the type of transfer function. The other two still water-levels are above the crest level resulting in the omitting of the part of the pressure diagram between still water-level and the wave top. This part is responsible for the non-linear behaviour and for the different types of transfer function. So with these water-levels there cannot be discerned any distinction between the transfer function.

For the shallow section the measured transfer functions are much bigger than the calculated ones. With increasing water-levels the differences diminish. In the deep section the calculated transfer function describe the measured transfer functions rather well. The differences can be explained from the drastic schematising in which the columns of the barrier are neglected.
The transfer functions with constant wave steepness describe the measured transfer functions best, because of the corresponding slope.
Verification of the transfer functions with model tests

Figure 5-23 Transfer functions for the shallow section and water-level +3.50m NAP

Figure 5-24 Transfer functions for the shallow section and water-level +5.50m NAP

Figure 5-25 Transfer functions for the shallow section and water-level +7.50m NAP
Figure 5-26 Transfer functions for the deep section and water-level +3.50m NAP

Figure 5-27 Transfer functions for the deep section and water-level +5.50m NAP

Figure 5-28 Transfer functions for the deep section and water-level +7.50m NAP
5.3.5 General conclusions

- The model tests with caissons are described rather well by the theoretical transfer function. The transfer function with the constant wave steepness gives the impression to produce the best results.
- The comparisons of the theoretical transfer functions with the measured transfer functions of the design of the Eastern Scheldt Storm Surge Barrier do not produce a satisfying answer to the question which transfer function to use. The course of the measured transfer functions is best described by the transfer functions with a constant wave steepness. However, the difference between both measured and theoretical transfer function amounts to a factor 1.5-2.

These remarks lead to the idea that a comparison with Goda’s method is only interesting for the model tests with caissons. Therefore, both transfer functions will be used to calculate wave forces on the vertical breakwater. For the constant wave height the significant wave height of the incoming wave field will be used, for the constant wave steepness the peak wave steepness of the wave field will be the input of the calculation of the transfer function.

The following results will be compared with Goda’s method:

<table>
<thead>
<tr>
<th>Fs [N]</th>
<th>O² (Hₕ=Hₛ)</th>
<th>O² (sₛ=Hₛ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1 (Tₛ=1.99s, Hₛ=0.124m, sₛ=0.02)</td>
<td>508.07</td>
<td>502.15</td>
</tr>
<tr>
<td>Test 2 (Tₛ=1.48s, Hₛ=0.137m, sₛ=0.04)</td>
<td>486.94</td>
<td>481.82</td>
</tr>
</tbody>
</table>

Table 5-15 Significant wave forces to use in the comparison with Goda’s method. The second column gives the results of the calculation with transfer functions with constant wave height (Hₕ=Hₛ), the third column with transfer functions with constant wave steepness (sₛ=Hₛ)
6. Comparison with Goda's formula

6.1 General
In the previous chapter the results of calculations with the theoretical transfer functions have been verified with the results of the model tests with caissons and tests used in the design of the Eastern Scheldt Storm Surge Barrier, carried out at Delft Hydraulics.
It had to be concluded that only the former test results were useful for the comparison of the results of the calculations with the transfer functions with the results of Goda's method. In this chapter the comparison will be made.

Therefore, first Goda's formula will be explained. The following section gives results of the research done by Bruining [5] and Van der Meer et al. [11]. With this theory it is possible to make the comparison between both methods followed by some conclusions.

6.2 Goda's formula for wave pressures under a wave crest

6.2.1 General
In this section, first the wave pressure formulas proposed by Goda [7] for the design of vertical breakwaters will be discussed. Goda assumes the existence of a trapezoidal pressure distribution along a vertical wall, regardless of whether the waves are breaking or non-breaking. Secondly, results of extra research by Bruining [5] and Van der Meer et al. [11] are added. The research comprised the reliability of Goda's formula in relation to the distribution of the wave forces.

In Figure 6-1 the relevant parameters as well as the shape of the pressure distribution are given. In this figure, \( h \) denotes the water depth in front of the breakwater, \( d \) the depth above the armour layer of the rubble foundation, \( h' \) the distance from the design water level to the bottom of the upright section and \( h_c \) the crest elevation of the breakwater above the design water level. In the following part the relevant factors in the calculation of the wave pressure are given.
1. Design wave:
In this analysis it is important to take the highest wave in the design sea state. Since in this analysis only the non-breaking waves have to be employed the significant wave height seaward of the surf zone is taken and not a highest wave within the surf zone. So for the maximum wave height and the maximum wave period the following two equations have to be used:

\[ H_{\text{max}} = 1.8 H_s \]  \hspace{2cm} \text{Eq. 6-1}
\[ T_{\text{max}} = T_s = 0.9 T_p \]  \hspace{2cm} \text{Eq. 6-2}

Goda states that "the height \( H_{\text{max}} \) is a probabilistic quantity. But to avoid possible confusion in design, a definite value of \( H_{\text{max}} = 1.8 H_s \) is recommended in consideration of the performance of many prototype breakwaters as well as with regard to the accuracy of the wave pressure estimation. Certainly there remains the possibility that one or two waves exceeding \( 1.8 H_s \) will hit the site of the breakwater when storm waves equivalent to the design condition attack. But the distance of sliding of an upright section, if it were to slide, would be very small. It should be remarked, however, that the prescription \( H_{\text{max}} = 1.8 H_s \) is a recommendation and not a rule. The design engineer can use his judgement in choosing another value, such as \( H_{\text{max}} = 1.6 H_s \), \( H_{\text{max}} = 2.0 H_s \), or some other value." [7]

2. Elevation to which the pressure is exerted:

\[ \eta^* = 0.75 (1 + \cos \beta) H_{\text{max}} \]  \hspace{2cm} \text{Eq. 6-3}

In this equation \( \beta \) denotes the angle between the direction of wave approach and a line normal to the breakwater. The wave rotation should be rotated by an amount of up to \( 15^\circ \) toward the line normal to the breakwater from the principal wave direction in view of the uncertainty in the estimation of the design wave direction. However in this study only waves attacking the breakwater at a right angle are used.

3. Wave pressure on the front of a vertical wall:
\[ p_1 = \frac{1}{2} \left( 1 + \cos \beta \right) \left( \alpha_1 + \alpha_2 \cos^2 \beta \right) w_0 H_{\text{max}} \]  
\[ p_2 = \frac{p_1}{\cosh(2\pi h/L)} \]  
\[ p_3 = \alpha_3 p_1 \]  
\[ p_4 = \begin{cases} 
  p_1 \left( 1 - \frac{h}{\eta} \right) & : \eta > h_c \\
  0 & : \eta \leq h_c 
\end{cases} \]  
\[ \alpha_1 = 0.6 + \frac{1}{2} \left[ \frac{4\pi h/L}{\sinh(4\pi h/L)} \right]^2 \]  
\[ \alpha_2 = \min \left[ \frac{h_b - d}{3\eta_h} \left( \frac{H_{\text{max}}}{d} \right)^2 \frac{2d}{H_{\text{max}}} \right] \]  
\[ \alpha_3 = 1 - \frac{h'}{h} \left[ 1 - \frac{1}{\cosh(2\pi h/L)} \right] \]  

in which:

\[ \eta = \frac{H_{\text{max}}}{d} \]

The value of the coefficient \( \alpha_1 \) can be read of from the diagram of Figure 6-2. The value of \( 1/cosh(2\pi h/L) \) for the coefficient \( \alpha_2 \) can be determined from the diagram of Figure 6-3. In both diagrams the \( L_0 \) stands for the wave length corresponding to the significant wave period in deep water.

Having calculated the pressures at different levels the total horizontal force can quite easily be calculated since the pressure distribution diagram is trapezoidal. This results in the following equation for the total horizontal force:

\[ F = \frac{1}{2} \left( p_1 + p_3 \right) h' + \frac{1}{2} \left( p_1 + p_4 \right) h_c^2 \]  

\[ \text{Figure 6-2 Calculation diagrams for the parameter } \alpha 1 \]
with $h_c^*$ is the minimum value of $\eta^*$ and $h_c$.

6.2.2 The bias and scatter in the measurements
Van der Meer et al. [11] re-analysed nine selected cases of projects at the Danish Hydraulic Institute and Delft Hydraulics in order to come to an expression for the reliability of Goda’s method for calculation of forces and moments on vertical breakwaters. Using Goda’s method probabilistic level II design calculations were made with the found reliability.

“In all cases, wave trains with lengths of 1000-3000 waves were considered. Based on the recordings the horizontal and vertical forces and horizontal and vertical overturning moments at the heel of the caisson were tabulated, with exceedance frequencies of 0.1%, 0.4%, 1%, 2% and 5% respectively.” [11]

The measured horizontal forces, $F_{o,h}$, were compared to the forces calculated with Goda’s method. It appeared that for the case with a vertical superstructure both forces agreed quite well. However for the cases with an inclined or curved superstructure Goda’s forces were much higher than the measured forces. The ratio between the calculated and measured force was in the order of 1.4-1.6. It was therefore concluded that in those cases the crest height should be determined at the transition from the vertical front to the inclined or curved superstructure. Now the ratios found between the calculated and measured forces were in the same order as was found in the cases with a vertical superstructure.

“The average ratio between measured ($F_{o,h}$) and calculated ($F_{Goda}$) forces and moments was based on 134 data sets for horizontal forces and moments.” [11]

The average value of the ratio between the measured horizontal force and the calculated horizontal force ($F_{o,h}$) and the standard deviation of this ratio, assuming a normal distribution, were: $\mu=0.83$ and $\sigma=0.25$.

The ratio is smaller than one, which means an overprediction by the Goda formula. The standard deviation is large and the variation coefficient, $\sigma/\mu$, amounts to 30%.
"Through the five tabulated measured horizontal, but also vertical forces, with exceedance values of 5%, 2%, 1%, 0.4% and 0.1%, a two-parameter Weibull distribution was fitted for each run (95 in total). This analysis resulted in an average shape parameter of 2.1 which corresponds closely to a Rayleigh distribution for the higher wave forces. The reliability of this factor 2.1 could be described by a standard deviation of 0.77. The conclusion was that, considering the large scatter, the distribution of the highest wave forces can be described by a Rayleigh distribution with a shape parameter of 2:

\[ R(F) = e^{-\frac{(F)}{\lambda}} \]  

Eq. 6-12

where \( R(F) \) is the exceedance probability and \( \lambda \) the scale parameter. Results from using the Goda method were compared with measured 0.4% exceedance values." [11]

"The Goda force is not equal to 1/250=0.4%, but equal to the average of the highest 1/250-th part of the forces. Now the shape of the force distributions has been found to be a Rayleigh distribution one can calculate the ratio between the average of the highest 1/250-th part and the 0.4%. This ratio amounts to 1.084." [11]

With this factor the actual comparison between Goda and measured forces can be given as the average ratio, given above, multiplied by 1.084. This results in: \( \mu=0.90 \) and \( \sigma=0.25 \).

The scale parameter in the last equation can be based on the Goda formula and on the found bias and reliability, given as the ratios. With \( R(F_{0.4\%})=0.004 \) substituted in the last equation the scale parameter \( \lambda \) can be replaced using \( F_{0.4\%} \). Bruining [5] gives in his Master’s thesis the derivation, that also holds for the uplift force.

\[ R(F) = e^{-\frac{2.35F}{F_{0.4\%}}} \]  

Eq. 6-13

With the modified factor \( r_{FH} \), which includes \( F_{0.4\%} \), the formula becomes:

\[ R(F) = e^{-\frac{2.35F}{r_{FH}F_{0.4\%}}} \]  

Eq. 6-14

This last equation can be seen as a design formula for the exceedance curve of the highest forces, based on Goda’s method.

"In reality the maximum wave force is related to the maximum number of waves during the sea state considered and not to the 0.4% or 1/250 wave only. Taking into account the actual maximum wave force based on the actual storm duration, a second factor, \( r_N \), can be introduced:

\[ r_N = \frac{F_{max}}{F_{0.4\%}} = \frac{\ln(1/N)}{\ln(0.004)} \]  

Eq. 6-15
where \( N \) is the number of waves in the sea state. The design formula for the maximum wave force becomes then: 

\[
F_{\text{max}} = r_{rh} \cdot r_N \cdot F_{\text{Goda}}
\]

Eq. 6-16

Figure 6-4 gives a graphical overall view of the above equations. It shows an example of a wave force exceedance curve. The horizontal axis has been plotted on a Rayleigh scale which means that a Rayleigh distribution becomes a straight line in the graph. The last equation for the exceedance probability gives the exceedance curve of the highest 5% of the wave forces. The calculated value for \( F_{\text{Goda}} \) has been drawn at 0.152%, which is equal to \( F_{1/250} \), assuming a Rayleigh distribution. The difference between \( F_{\text{Goda}} \) and \( F_{0.4\%} \) is given by the ratio \( r_{rh} \), mentioned first. The actual difference between \( F_{\text{Goda}} \) and the exceedance curve is given by the modified ratio \( r_{rh} \). The 90% confidence levels can be calculated by taking into account the standard deviations given for the ratios \( r_{rh} \).

The most right point on the curve in Figure 6-4 gives the maximum wave force \( F_{\text{max}} \) for an exceedance probability of \( 1/N \).

![Figure 6-4 Distribution of horizontal forces (Weibull), including \( F_{\text{Goda}} \) and \( F_{\text{max}} \) [11]](image)

6.3 Comparison

6.3.1 Analysis with the General Wave Spectrum Model

In the chapter "Analysis with the General Wave Spectrum Model" the most important issue is the influence of the shape of the wave spectrum on the horizontal force on the breakwater. Therefore the cases of a double-peaked spectrum and a spectrum with a changing right flank have been studied.
One of the most important differences between the method of the transfer function and Goda's method is the influence of the shape of the wave spectrum. Goda takes a maximum wave height and an accompanying wave period. In the mentioned chapter the wave forces have been derived by using the method of the transfer function. These can be compared with results of Goda's formula.

### 6.3.1.1 Double-peaked spectrum

Two different cases were taken to study the influence of a double-peaked spectrum. In both cases there was a swell with a wind sea on top of it. For the calculation with Goda’s method, $H_s$ (necessary to calculate the $H_{max}$) for the total sea was taken (so the total energy). The peak period of the swell was taken for the peak period necessary for Goda’s formula.

Table 6-1 gives the input values for Goda’s method and the resulting horizontal wave forces according to Goda.

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_{p, \text{swell}}$ [s]</th>
<th>$T_{\text{max}}$ [s]</th>
<th>$H_s$ [m]</th>
<th>$H_{\text{max}}$ [m]</th>
<th>$F_{\text{Goda}}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3.6</td>
<td>0.159</td>
<td>0.2862</td>
<td>1607.42</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>1.125</td>
<td>0.0584</td>
<td>0.10512</td>
<td>262.76</td>
</tr>
</tbody>
</table>

Table 6-1 Values of both cases of double-peaked wave spectra for Goda’s method

It should be noticed that for the double-peaked spectrum one wave force was calculated with the total significant wave height and the peak frequency of the swell as input. Calculating the horizontal wave forces for swell and wind sea separately results in a bigger force on the breakwater. But this is not the practice for Goda’s method. Goda’s method does not take double-peakedness into account and the influence of that restriction has to be demonstrated here.

For the double-peaked spectrum the energy of the wind sea was shifted to the swell. The start situation and the final situation will be used to compare with Goda’s method. The significant wave forces are repeated once more for the two cases:

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>$H_{s, \text{start}}=0.124$m</th>
<th>$H_{s, \text{final}}=0.0025$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>677.89kN</td>
<td>734.96kN</td>
</tr>
<tr>
<td>0.12</td>
<td>708.10kN</td>
<td>765.29kN</td>
</tr>
<tr>
<td>0.16</td>
<td>738.34kN</td>
<td>795.65kN</td>
</tr>
<tr>
<td>0.20</td>
<td>768.61kN</td>
<td>826.02kN</td>
</tr>
</tbody>
</table>

Table 6-2 Significant wave forces for the start situation and the final situation calculated with transfer functions with constant wave heights of 0.08m, 0.12m, 0.16m and 0.20m, Case 1

<table>
<thead>
<tr>
<th>$s_1$ [-]</th>
<th>$H_{s, \text{start}}=0.04$m</th>
<th>$H_{s, \text{final}}=0.01$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>125.51kN</td>
<td>144.62kN</td>
</tr>
<tr>
<td>0.030</td>
<td>135.33kN</td>
<td>156.50kN</td>
</tr>
<tr>
<td>0.050</td>
<td>145.19kN</td>
<td>168.43kN</td>
</tr>
</tbody>
</table>

Table 6-3 Significant wave forces for the start situation and the final situation calculated with transfer functions with constant wave steepnesses of 1%, 3% and 5%, Case 2
With the expression of the Rayleigh distribution function the wave forces have to be calculated with an exceedance probability of 0.152%:

<table>
<thead>
<tr>
<th>$H_i$ [m]</th>
<th>$H_s, \text{start} = 0.124 \text{m}$</th>
<th>$H_s, \text{final} = 0.0025 \text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>1221.05kN</td>
<td>1323.85kN</td>
</tr>
<tr>
<td>0.12</td>
<td>1275.47kN</td>
<td>1378.48kN</td>
</tr>
<tr>
<td>0.16</td>
<td>1329.94kN</td>
<td>1433.17kN</td>
</tr>
<tr>
<td>0.20</td>
<td>1384.46kN</td>
<td>1487.87kN</td>
</tr>
</tbody>
</table>

Table 6-4 Wave forces with an exceedance probability of 0.152% for the start situation and the final situation calculated with transfer functions with constant wave heights of 0.08m, 0.12m, 0.16m and 0.20m, Case 1

<table>
<thead>
<tr>
<th>$s_i$ [-]</th>
<th>$H_s, \text{start} = 0.04 \text{m}$</th>
<th>$H_s, \text{final} = 0.01 \text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>226.08kN</td>
<td>260.50kN</td>
</tr>
<tr>
<td>0.030</td>
<td>243.76kN</td>
<td>281.90kN</td>
</tr>
<tr>
<td>0.050</td>
<td>261.52kN</td>
<td>303.39kN</td>
</tr>
</tbody>
</table>

Table 6-5 Wave forces with an exceedance probability of 0.152% for the start situation and the final situation calculated with transfer functions with constant wave steepnesses of 1%, 3% and 5%, Case 2

Now the ratio can be determined of the wave force according to Goda to the wave force determined with the calculations with the transfer functions:

<table>
<thead>
<tr>
<th>$H_i$ [m]</th>
<th>$H_s, \text{start} = 0.124 \text{m}$</th>
<th>$H_s, \text{final} = 0.0025 \text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>1.32</td>
<td>1.21</td>
</tr>
<tr>
<td>0.12</td>
<td>1.26</td>
<td>1.17</td>
</tr>
<tr>
<td>0.16</td>
<td>1.21</td>
<td>1.12</td>
</tr>
<tr>
<td>0.20</td>
<td>1.16</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Table 6-6 Ratio $F_{\text{Goda}}$ to the force from the calculations with the transfer functions (constant wave heights of 0.08m, 0.12m, 0.16m and 0.20m) for the start situation and the final situation, Case 1

<table>
<thead>
<tr>
<th>$s_i$ [-]</th>
<th>$H_s, \text{start} = 0.04 \text{m}$</th>
<th>$H_s, \text{final} = 0.01 \text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>1.16</td>
<td>1.00</td>
</tr>
<tr>
<td>0.030</td>
<td>1.08</td>
<td>0.93</td>
</tr>
<tr>
<td>0.050</td>
<td>1.00</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Table 6-7 Ratio $F_{\text{Goda}}$ to the force from the calculations with the transfer functions (constant wave steepnesses of 1%, 3% and 5%) for the start situation and the final situation, Case 2

Comparing the horizontal forces of the table with the forces, it is quite obvious that Goda's method gives much higher forces for case 1. When almost all the energy is shifted to lower frequencies the wave forces according to Goda and the calculations with the transfer functions agree better.
Case 2 however presents different results. Now the ratio becomes smaller and the wave forces according to Goda even become smaller with transfer functions with constant wave steepnesses of 3% and 5%. The ratios again get smaller when the energy is shifted to lower frequencies.

6.3.1.2 Changing right flank of the wave spectrum
Although several calculations have been made with the method of the transfer function to get the horizontal wave force on the breakwater for wave spectra with
changing right flanks, Goda's method requires only one calculation. The only input is again the peak period and the significant wave height resulting in the following values:

<table>
<thead>
<tr>
<th>Case</th>
<th>$T_p$ [s]</th>
<th>$T_{max}$ [s]</th>
<th>$H_s$ [m]</th>
<th>$H_{max}$ [m]</th>
<th>$F_{\text{Goda}}$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3.6</td>
<td>0.1</td>
<td>0.18</td>
<td>976.28</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.8</td>
<td>0.1</td>
<td>0.18</td>
<td>684.51</td>
</tr>
<tr>
<td>3</td>
<td>1.33</td>
<td>1.2</td>
<td>0.1</td>
<td>0.18</td>
<td>496.57</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.9</td>
<td>0.1</td>
<td>0.18</td>
<td>436.03</td>
</tr>
</tbody>
</table>

*Table 6.8 Values of the four cases of wave spectra with changing right flank calculated with Goda's method*

Considering the large number of significant wave forces that have been calculated for four cases with each four different slopes of the right flank another approach is used here. The ratio wave force according to Goda to the wave force calculated with the transfer functions is given a value 1. For this situation the significant wave force is calculated with the Rayleigh distribution function:

<table>
<thead>
<tr>
<th>Case</th>
<th>$F_{\text{Goda}}$ [N]</th>
<th>$F_s$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>976.28</td>
<td>542.00</td>
</tr>
<tr>
<td>2</td>
<td>684.51</td>
<td>380.02</td>
</tr>
<tr>
<td>3</td>
<td>496.57</td>
<td>275.58</td>
</tr>
<tr>
<td>4</td>
<td>436.03</td>
<td>242.07</td>
</tr>
</tbody>
</table>

For case 1 only the results of the transfer functions with constant wave height seem to have small influence of the non-linearity and will be studied, for the cases 2 and 3 both can be used, for case 4 the transfer function with constant wave steepness is used.

Case 1 shows larger values for the wave force coming from Goda's method. With increasing flank the differences become less. This well declarable considering the fact that the spectrum becomes more a 'Dirac' type spectrum, with one period and one wave height. This is in agreement with Goda's method.

The cases 2 and 3 give actually only larger values for the wave force for the method of the transfer function. The same phenomenon with the increasing slope of the right flank can be discerned.

Case 4 shows that, comparable to the double-peaked spectrum, Goda seems to give higher forces with increasing difference compared to the method of the transfer function, when the energy of the wave spectrum is shifted to higher frequencies.

### 6.3.2 Model tests with caissons

In the chapter "Verification of the transfer functions with model tests" the results of the calculations with the transfer functions have been compared with test results. Based on this comparison, wave forces have been determined that will be compared to the wave forces calculated with Goda's method.

The values for these wave forces are repeated in next table.
Comparison with Goda’s formula

<table>
<thead>
<tr>
<th>$F_s ,[N]$</th>
<th>$\sigma^2 ,(H=H_s)$</th>
<th>$\sigma^2 ,(s=s_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1 ($T_p=1.99s$, $H_s=0.124m$, $s_p=0.02$)</td>
<td>508.07</td>
<td>502.15</td>
</tr>
<tr>
<td>Test 2 ($T_p=1.48s$, $H_s=0.137m$, $s_p=0.04$)</td>
<td>486.94</td>
<td>481.82</td>
</tr>
</tbody>
</table>

Table 6-9 Significant wave forces to use in the comparison with Goda’s method. The second column gives the results of the calculation with transfer functions with constant wave height ($H=H_j$), the third column with transfer functions with constant wave steepness ($s=s_j$).

The values of Table 6-9 are significant wave forces. The wave forces are assumed to be Rayleigh distributed. The significant wave force is the necessary parameter for the distribution function.

Section 6.2 describes how to calculate the horizontal wave force according to Goda. The wave forces have been calculated for both test 1 and test 2. Table 6-10 gives these forces.

<table>
<thead>
<tr>
<th>Test</th>
<th>$T_p ,[s]$</th>
<th>$T_{max} ,[s]$</th>
<th>$H_s ,[m]$</th>
<th>$H_{max} ,[m]$</th>
<th>$F_{Goda} ,[N]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.99</td>
<td>1.791</td>
<td>0.124</td>
<td>0.432</td>
<td>861.04</td>
</tr>
<tr>
<td>2</td>
<td>1.48</td>
<td>1.332</td>
<td>0.137</td>
<td>0.2466</td>
<td>755.67</td>
</tr>
</tbody>
</table>

Table 6-10 Values for the input and the results of Goda’s method.

The wave force calculated with Goda’s formula is equal to $F_{1.25q}$. Assuming a Rayleigh distribution for the wave forces, the calculated value for $F_{Goda}$ is equal to the wave force with an exceedance probability of 0.152%.

With the Rayleigh distribution function and the significant wave force calculated with the method of the transfer function, the wave force is calculated at an exceedance probability of 0.152%. The check that will be made is whether or not the latter wave force is within the 90% confidence boundaries.

The wave forces according to Goda have been given in Table 6-10. With the Rayleigh distribution and the results in Table 6-9 the wave forces $F_{0.152%}$ become:

<table>
<thead>
<tr>
<th>$F_{0.152%} ,[N]$</th>
<th>$\sigma^2 ,(H=H_s)$</th>
<th>$\sigma^2 ,(s=s_p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1 ($T_p=1.99s$, $H_s=0.124m$, $s_p=0.02$)</td>
<td>915.16</td>
<td>904.50</td>
</tr>
<tr>
<td>Test 2 ($T_p=1.48s$, $H_s=0.137m$, $s_p=0.04$)</td>
<td>877.10</td>
<td>867.88</td>
</tr>
</tbody>
</table>

Table 6-11 Wave forces with an exceedance probability of 0.152%. The second column gives the results of the calculation with transfer functions with constant wave height ($H=H_j$), the third column with transfer functions with constant wave steepness ($s=s_j$).

The ratio of measured and calculated wave forces was given to be 0.90 with a standard deviation of 0.25. Using the normal distribution the 90% confidence levels can be produced. The upper confidence level is equal to $f_{Pr.90%}=1.22$, the lower confidence level $f_{Pr.90%}=0.58$.

The ratio of the wave force calculated with Goda’s formula to the wave force calculated with the method of the transfer function must stay between these confidence levels. So in contrast with Van der Meer et al. [11] the measured wave
forces have been replaced by the wave forces calculated with the method of the transfer function.

<table>
<thead>
<tr>
<th></th>
<th>( r_{FH} )</th>
<th>( O^2 ) (( H_i = H_l ))</th>
<th>( O^2 ) (( s_i = s_p ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1 (( T_p = 1.99s ), ( H_p = 0.124m ), ( s_p = 0.02 ))</td>
<td>1.06</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Test 2 (( T_p = 1.48s ), ( H_p = 0.137m ), ( s_p = 0.04 ))</td>
<td>1.16</td>
<td>1.15</td>
<td></td>
</tr>
</tbody>
</table>

Table 6-12: Ratios calculated with Goda's formula and calculated with the method of the transfer function. The second column gives the results of the calculation with transfer functions with constant wave height \( (H_i = H_l) \), the third column with transfer functions with constant wave steepness \( (s_i = s_p) \).

These ratios are well within the 90% confidence levels. Goda produces smaller values for the wave forces compared to the method of the transfer functions. Van der Meer et al. [11], however, state in their conclusions that "in general, the horizontal forces calculated by Goda's formula are about 10% higher than the corresponding measured forces." This would mean that the method of the transfer functions gives values for the wave forces that are higher than the measured forces, even higher than Goda calculates.
7. Conclusions and recommendations

With the results from the comparisons with the General Wave Spectrum Model, the model tests with caissons and the model tests for the design of the Eastern Scheldt Storm Surge Barrier at Delft Hydraulics, conclusions can be drawn concerning the application of the spectral analysis in the determination of horizontal wave forces on vertical breakwaters. The conclusions will be divided in main issues. After the conclusions the recommendations can be made for the application of this method and further analysis.

7.1 Conclusions

7.1.1 Type of transfer function
Two types have been analysed in this report: the transfer function with constant wave height over the frequency range and the transfer function with constant wave steepness over the frequency range. The first type has been used before and it proved its applicability in the design of the Eastern Scheldt Storm Surge Barrier. However the reflection coefficient had to be used to fit the theoretical results to the model test results. In this way the transfer function became steeper. The model tests with caissons and the model tests for the design of the eastern Scheldt Storm Surge Barrier also produced transfer functions with steep courses. The transfer functions with constant wave steepness have a steep course. This results in the conclusion that the transfer function with constant wave steepness is to prefer above the other type of transfer function, considering the course of the transfer function.

The analysis with the General Wave Spectrum proved the idea of the application of both types of transfer function in only a certain interval. The transfer function with the constant wave height shows little influence of the non-linearity at low frequencies and for the transfer function with constant wave steepness the opposite can be said. Besides this, for low frequencies the transfer function with constant wave steepness gives unrealistic high values. On the opposite, for high frequencies the other transfer function gives with constant wave height a unrealistic high wave steepness.
7.1.2 Choice of the constant wave height and the constant wave steepness
The comparisons gave not a conclusive answer to the size of wave height or wave steepness that can be used to determine the transfer function. The wave height now can be chosen, as a rough estimate, equal to the significant wave height of the wave field that causes the wave force on the breakwater. For the wave steepness the choice should be made for the peak wave steepness of the wave field. The model tests with caissons proved this to be a good estimation.

7.1.3 Shape of the wave spectrum
The analysis with the General Wave Spectrum Model showed that the influence of the shape of the wave spectrum is quite important for the wave force on the breakwater. A double peaked spectrum or a spectrum that is a large width gives quite evidently another wave force than a regular wave with the same energy and a period equal to the peak period (of the first peak).
This conclusion comes also from the calculations made for the model tests with caissons at DH.

7.1.4 Comparison with the method of Goda
The method of the transfer function has definitely advantages compared to the method of Goda. It has become clear that the influence of the shape of the wave spectrum cannot be neglected. In that case the method of the transfer function gives a better result. If the wave spectrum becomes a 'Dirac' type spectrum, the situations becomes comparable for both calculation methods. However a wave spectrum usually has a certain width.
When the wave spectrum has a higher peak frequency the method of the linear transfer function gives higher forces than Goda's method. For lower frequencies the opposite occurs.
If a good choice can be made of the type of transfer function and the size of the wave height or steepness to be applied, it can be a good design tool.

7.2 Recommendations
From the mentioned conclusions follow some recommendations that give a lot of material for further research.

It is important to do more model tests or find data of already performed model tests. These should be model tests with long-crested waves that approach a vertical perpendicularly. Overtopping should be prevented. In this way a proper transfer function could be determined, especially if regular waves are applied. It would result in a better insight of the following issues:
• Choice of the type of transfer function. It needs especially research to come to a frequency range in which the transfer function can be used.
• Choice of the constant wave height and wave steepness. This did not result from this report but it is necessary for a proper application of this method. Perhaps a region of reliability of a certain wave height or wave steepness can be determined.
When there is a good insight in the long-crested waves and waves that approach the breakwater perpendicularly, the waves can be generated short-crested and at an angle to come more to a real situation.

At last it should also be studied whether upward pressures and moments can be determined in the same way. It would give more weight to this method and make it an alternative design tool for vertical breakwaters.
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Appendices

APPENDIX A STANDARD WAVE SPECTRA
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Appendix A Standard wave spectra
Pierson Moskowitz spectrum

Pierson and Moskowitz published in 1964 [17] the results of their study of the frequency spectrum for a fully-grown wave field on deep water (from now on called P&M spectrum).

\[ E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \exp \left[ -\frac{5}{4} \left( \frac{f}{f_p} \right)^{-4} \right] \]  
Eq. A-1

with: \( \alpha = \) scaling parameter (0.0081)

![Pierson Moskowitz spectra](image)

*Figure A-1 P&M spectra for different \( f_p \) (0.10Hz, 0.12Hz, 0.14Hz) and \( m_0 \)*

The spectrum can be written in a different form. Each spectrum has a peak frequency which gives the place of the peak of the spectrum on the frequency-axis. The significant wave height is related to the area of the spectrum. For the P&M spectrum the area under the spectrum is known. It can be obtained by integration of the function above:

\[ m_0 = \frac{\alpha g^2}{(2\pi)^4 5 f_p^4} \]  
Eq. A-2

This equation combined with the relation between the significant wave height and the area \( m_0 \) and then used in the expression for the P&M spectrum gives:

\[ S_{\eta\eta}(f) = \frac{\alpha g^2}{(2\pi)^4 f^4} \exp \left[ -\frac{\alpha g^2}{4\pi^4 f^4 H_s^4} \right] \]  
Eq. A-3

The following derivation shows that the P&M spectrum has a constant peak wave steepness of 2.56%.
\[ m_0 = \frac{ag^2}{(2\pi)^4 5f_p^4} \]
\[ m_0 = \frac{ag^2 T_p^4}{(2\pi)^4 5} \]
\[ \sqrt{m_0} = \sqrt{\frac{\alpha g T_p^2}{5 (2\pi)^2}} \]
\[ \sqrt{m_0} = \sqrt{\frac{\alpha L_p}{5 \cdot 2\pi}} \]
\[ 4\sqrt{m_0} = 4 \sqrt{\frac{\alpha L_p}{5 \cdot 2\pi}} = H_s \]
\[ H_s = \frac{4 \sqrt{\alpha}}{2\pi \sqrt{\frac{0.0081}{5}}} \]
\[ = \frac{2}{\pi} \sqrt{\frac{0.0081}{5}} \]
\[ = 2.56\% \]

The spectrum shows several phenomena when the significant wave height $H_s$ is taken larger:

- The area under the spectrum becomes larger due to the relation

\[ H_s = 4\sqrt{m_0} \] \hspace{1cm} \text{Eq. A-4}

- The peak frequency $f_p$ diminishes due to the constant wave steepness, independent of the value of the significant wave height. The high frequency tail is independent.

**JONSWAP spectrum**

Another standard wave spectrum is the JONSWAP spectrum [9]. In the frame of the Joint North Sea Wave Project wave spectra were measured for a period of ten weeks in 1968 and 1969 westward from the Island of Sylt, Germany, in the North Sea. Measurements were taken along a profile of 160 km in situations of offshore wind to determine the process of wave growth.

The JONSWAP spectrum is determined by multiplying the P&M spectrum with a peak-enhancement function so as to give it a sharper top. The peak-enhancement function is given to be:

\[ \gamma(f) = \gamma_0 \exp \left[ -\frac{(f-f_p)^2}{\sigma f_p^2} \right] \] \hspace{1cm} \text{Eq. A-5}

with:
- $\gamma_0$ = peak-enhancement factor, for which often the value 3.3 is given
- $\sigma = \sigma_a$ for $f<f_p$, with $\sigma_a=0.07$
- $\sigma = \sigma_b$ for $f\geq f_p$, with $\sigma_b=0.09$
Figure A-2 P&M spectrum, peak-enhancement function and JONSWAP spectrum
Appendix B Analysis with the General Wave Spectrum Model
Figure B-1 Case 1, Wave spectrum in the 1st picture, the 2nd and 3rd pictures give wave force spectra calculated with wave force spectra with constant wave heights (0.08m, 0.10m, 0.12m, 0.14m, 0.16m, 0.19m and 0.20m), respectively constant wave steepnesses (1%, 2%, 3%, 4% and 5%)
Figure B-2 Case 2, Wave spectrum in the 1st picture, the 2nd and 3rd pictures give wave force spectra calculated with wave force spectra with constant wave heights (0.08m, 0.10m, 0.12m, 0.14m, 0.16m, 0.18m and 0.20m), respectively constant wave steepnesses (1%, 2%, 3%, 4% and 5%).
Figure B-3 Case 3, Wave spectrum in the 1st picture, the 2nd and 3rd pictures give wave force spectra calculated with wave force spectra with constant wave heights (0.08m, 0.10m, 0.12m, 0.14m, 0.16m, 0.18m and 0.20m), respectively constant wave steepnesses (1%, 2%, 3%, 4% and 5%).
Figure B-4 Case 4. Wave spectrum in the 1st picture, the 2nd and 3rd pictures give wave force spectra calculated with wave force spectra with constant wave heights (0.08m, 0.10m, 0.12m, 0.14m, 0.16m, 0.18m and 0.20m), respectively constant wave steepnesses (1%, 2%, 3%, 4% and 5%).
Figure B-5 Case 5. Wave spectrum in the 1st picture, the 2nd and 3rd pictures give wave force spectra calculated with wave force spectra with constant wave heights (0.08m, 0.10m, 0.12m, 0.14m, 0.18m and 0.20m), respectively constant wave steepnesses (1%, 2%, 3%, 4% and 5%).
Appendix C Model tests with caissons at Delft Hydraulics
The model

Vinjé basin
The experiments were carried out in the Vinjé basin of DH. This basin is 26.4m wide and 23m long. The basin is equipped with an advanced multipaddle generator. In this basin a structure was placed consisting of 13 caissons, each 0.9m wide. The structure had steel curved roundheads at the two ends. The structure was fixed on a concrete bottom and linked to the floor through a permeable two layer rock basement, which was protected at the front with larger rock material. In the basin several precautions were taken in order to prevent any disturbances in the wave pattern by reflection through the basin, such as brick walls, more rock material and columns of bricks. Also dampers had to avoid those reflections. So the pressure and force measurements could be made coming from the incoming wave field that only reflected against the vertical wall. Different heights for the crests were applied, using removable crest elements. The heights were in relation to $H_s$: $R_c/H_s$ of 1.18, 1.5 and 1.63.

![Diagram](image)

*Figure C-1 Plan view of the basin layout [6]*

Measuring devices
The horizontal wave force was measured in two ways. The total horizontal force on a caisson and the horizontal pressure distribution were measured separately. In order to measure the incoming wave height, the wave conditions in front of the caisson were analysed through a set of 20 two-wire gauges placed in two rows of ten, at 1.0m offshore of the structure.
Total horizontal force

A caisson that was suspended above the floor of the basin by rigid metal plates attached to the inner section of the laboratory force metering frame was used to measure the total horizontal force.

A rigidly mounted external frame section supported the inner section of the metering frame through six suspensions with strain gauges, three in the horizontal and three in the vertical planes. Complete freedom of movement was ensured during each test. The time histories of the total horizontal force ($F_h$), the total vertical force ($F_v$) and the total overturning moment ($M$) were calculated from the signal of the strain gauges.

![Diagram](image)

*Figure C-2 The force measuring frame [6]*

Horizontal pressure distribution

Wave forces were measured by a set of 21 pressure cells placed against the wall. The position of the pressure cells enabled the study of the wave pressure distribution on the outer vertical face, on the bottom slab and along the longitudinal direction of the caisson structure. All pressure signals were combined into a total horizontal force and a total uplift force by multiplying each cell output with different factors, representing the respective working area.
Figure C-3 Vertical and longitudinal placement of the pressure cells. Front view and cross section (measures in cm) [6]
### Comparison DH-tests distribution and Rayleigh distribution

<table>
<thead>
<tr>
<th>Measuring method/transfer function</th>
<th>$F_n$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/250</td>
</tr>
<tr>
<td>Frame</td>
<td>978</td>
</tr>
<tr>
<td>Press.cells</td>
<td>1062</td>
</tr>
<tr>
<td>$H_r=0.08\text{m}$</td>
<td>866</td>
</tr>
<tr>
<td>$H_r=0.12\text{m}$</td>
<td>907</td>
</tr>
<tr>
<td>$H_r=0.16\text{m}$</td>
<td>948</td>
</tr>
<tr>
<td>$H_r=0.20\text{m}$</td>
<td>989</td>
</tr>
</tbody>
</table>

*Table C-1 Forces measured at DH and calculated with Rayleigh distribution function for test 1 ($T_s=1.99\text{s}$, $H_r=0.124\text{m}$)*

<table>
<thead>
<tr>
<th>Measuring method/transfer function</th>
<th>$F_n$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/250</td>
</tr>
<tr>
<td>Frame</td>
<td>866</td>
</tr>
<tr>
<td>Press.cells</td>
<td>980</td>
</tr>
<tr>
<td>$H_r=0.08\text{m}$</td>
<td>809</td>
</tr>
<tr>
<td>$H_r=0.12\text{m}$</td>
<td>854</td>
</tr>
<tr>
<td>$H_r=0.16\text{m}$</td>
<td>899</td>
</tr>
<tr>
<td>$H_r=0.20\text{m}$</td>
<td>944</td>
</tr>
</tbody>
</table>

*Table C-2 Forces measured at DH and calculated with Rayleigh distribution function for test 2 ($T_s=1.48\text{s}$, $H_r=0.137\text{m}$)*

<table>
<thead>
<tr>
<th>Measuring method/transfer function</th>
<th>$F_n$ [N]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/250</td>
</tr>
<tr>
<td>Frame</td>
<td>978</td>
</tr>
<tr>
<td>Press.cells</td>
<td>1062</td>
</tr>
<tr>
<td>$s=0.020$</td>
<td>900</td>
</tr>
<tr>
<td>$s=0.030$</td>
<td>958</td>
</tr>
<tr>
<td>$s=0.040$</td>
<td>1017</td>
</tr>
</tbody>
</table>

*Table C-3 Forces measured at DH and calculated with Rayleigh distribution function for test 1 ($T_s=1.99\text{s}$, $H_r=0.124\text{m}$)*
Analysis of the influence of the transfer functions

The results of the analysis of the transfer functions give good reason to look at the differences between the two transfer functions more closely. To achieve this more regular waves are taken with the same value for $H$ (0.14m) but different $T$ (1s, 1.5s, 2s, 2.5s). Again the differences are calculated between the results using the transfer functions with the highest constant wave height and the lowest constant wave height, respectively the largest constant wave steepness and the smallest constant wave steepness.

Table C-5 Values for $F$ calculated with different transfer functions and regular waves. The grey rows indicate the difference in wave force through division of the results calculated with the transfer function with the highest constant wave steepness (respectively wave height) by the results calculated with the transfer function with the smallest constant wave steepness (respectively wave height)
Measured wave spectra, wave force spectra and transfer functions
Appendix D Model tests for the design of the Eastern Scheldt Storm Surge Barrier
The basin
For both model tests the wind wave flume of Delft Hydraulics was used. The flume has a width of 2m and a length of 100m. This flume has a hydraulic wave paddle, which enables the generation of both regular and irregular waves.

![Figure D-1 Plan and cross-sections of the wind wave flume [10,18]](image)

The water level at the Eastern Scheldt side was regulated with an adjustable spillway. In this way it could be kept at a constant level of -1.50m NAP. In front of the spillway a reflection damping bank was installed to prevent disturbing reflections.

The model test 1422-la

Lay-out of the model
In this study the tests were carried for the deepest part of the Eastern Scheldt, which was located in the deepest part of the Roompot (well no R15, chosen bottom depth NAP -35m). The bottom of the flume was chosen to be horizontal. A berm was made that reached from -35m NAP to -21m NAP. On top of this berm a column was placed with a width of 4.5m and a length of 35m. The top of the column reaches to +12m NAP.
In the model one plane was installed in between two columns, that have a heart-to-heart distance of 40m. This plane reaches from -21m NAP to +5.50m NAP. The front side is 13.65m from the front of the columns.

The measures in the model tests are given for the prototype.
Measuring devices

Wave height

The wave pattern in front of the structure was registered at three places in the flume with wave gauges at 42.5m, 15m and 6m (model values). The wave pattern behind the structure was measured with only one wave gauge at 2m behind the structure (model value).

With these results the wave height distribution was determined as well as the wave spectrum. For the registration always a number of 1000 waves was taken.

To determine the reflection coefficient regular waves have to be used, since the reflection coefficient depends on water level, wave period and wave height. For the determination of the reflection coefficient of the total wave pattern, one representative regular wave has to be used. Therefore a wave has been chosen with the for that certain wave spectrum and water level accompanying $H_s$ and $T_p$.

The waves that approach the structure will be reflected by that structure and a wave pattern will occur consisting of incoming waves and reflected waves. Looking at this wave pattern it is possible to see a maximum and minimum wave height. By measuring the wave height at different places, with short distances between these places, a line can be drawn through the wave tops and the wave troughs. In this way it is possible to determine the maximum and minimum wave height and this results in the wave reflection coefficient:

$$\alpha = \frac{H_{\text{max}} - H_{\text{min}}}{H_{\text{max}} + H_{\text{min}}}$$

Eq. D-1

With this reflection coefficient it is possible to determine the incoming wave pattern from the total wave pattern through:
\[ H_{\text{incomin}g} = \gamma H_{\text{total}} \]
\[ = \frac{1}{\sqrt{1 + \alpha^2}} H_{\text{total}} \]

Eq. D-2

**Metering frame**

The centre column was not attached to the berm and together with two halves of the planes on both sides of the column it was hung in a metering frame. With this frame horizontal and vertical forces could be measured.

*Figure D-3 Metering frame [18]*

**The model test 1469**

**Lay-out of the model**

In the flume a model was placed consisting of two columns with in between the upperbeam, threshold and wall. The width of the columns was 5m and the heart-to-heart distance was 40m. This means that the wall and beams had a width of about 35m.

The tests in model 1469 consists of two parts. Because it is important to make a sensible check of QS-GOLF, one part of the test has been performed in a deep section and one in a shallow section. These two parts are first described.
Deep section

The bottom of the deep section was chosen at -34.00m NAP; the highest possible water level was +7.50m NAP. The barrier was put on a berm, reaching from the bottom at -34.00m NAP to -21.50m NAP. The banks had a slope of 1:4. The berm was almost completely impermeable.

Figure D-4 Configuration deep section [10]

Figure D-5 Placing in the flume [10]
Shallow section

In case of the shallow section there was not a berm. Only a steel plate was used. Therefore the bottom of the model was chosen at -11.45m NAP and the top of the berm at -11.00m NAP.

Figure D-6 Configuration shallow section [10]

Figure D-7 Placing in the flume [10]

Measuring devices

Wave height

At the sea side of the barrier the wave pattern was registered at three different places: 70m, 46.5m and 15m in front of the model (model measures). At the Eastern Scheldt side also a wave gauge was placed to determine the head difference over the barrier.

For the measurements to determine the reflection coefficients a number of wave gauges were put in the middle of the flume at a short distance of several meters to the structure.

The reflection coefficients were determined from wave height measurements in a standing wave pattern consisting of an incoming regular wave and the reflection of
that wave. The measurement was done before the first wave reached the structure, due to reflection against the structure and the wave paddle, the place of measurement for the second time.

From a description of the standing wave pattern by means of the linear wave theory the reflection coefficient can be determined from:

$$\alpha = \frac{H_{\text{max}} - H_{\text{min}}}{H_{\text{max}} + H_{\text{min}}}$$  \hspace{1cm} \text{Eq. D-3}

At several places (7 to 9) the wave height was determined in the axis of the flume. From the course of the wave height as a function of the distance to the reflection point $H_{\text{max}}$ and $H_{\text{min}}$ can be determined.

Because of the fact that there will be waves with short period on top of waves with longer periods two methods have been used to improve the reflection coefficients [10].

From the measured wave spectrum an incoming wave spectrum could be determined according to:

$$S_{\text{girp}} = \frac{1}{\sqrt{1 + \alpha^2}} S_{\text{mgirp}}$$  \hspace{1cm} \text{Eq. D-4}

For the reflection coefficient a value was taken that was measured for a regular wave with $H=H_0$ and $T=T_p$. This means that the $m_o$ of the spectrum changes a bit as well as the shape of the spectrum because of the fact that the reflection coefficient depends on the frequency $f$.

**Metering frame**

In order to calculate the wave force on the section of the barrier (i.e. two column halves, the wall, threshold, upperbeam and structure) the section was put in a metering frame (Figure D-8).

*Figure D-8 Metering frame [10]*
Measured transfer functions
ZEEWATERSTAND = N.A.P. + 7,5 m
ONREGELMATIGE GOLVEN

- \bar{H}_z, 1/3 \times 5,40 m (P 201)
- \bar{R}_z, 1/3 \times 4,60 m (P 202)
- \bar{H}_z, 1/3 \times 3,50 m (P 203)
- QS GOLF (\alpha = 0,45)

OVERDRACHTSFUNKTIES VAN HORIZONTALE KRACHTEN,
DIEPE SEKTIE, MODELPROEVEN EN QS GOLF

WATERLOOPKUNDIG LABORATORIUM

M.1469-1023 FIG.23
ZEEWATERSTAND N.A.P. + 5,5 m
ONREGBELMATIGE GOLVEN

\[ H_z, 1/3 \times 475 \text{ m} (P 204) \]
\[ H_z, 1/3 \times 335 \text{ m} (P 205) \]
\[ H_z, 1/3 \times 250 \text{ m} (P 206) \]
\[ QS 80LF (\alpha \times 0,55) \]

OVERDRACHTSFUNKTIES VAN HORIZONTALE KRACHTEN,
DIEPE SEKTIE, MODELPROEVEN EN QS - GOLF

WATERLOOPKUNDIG LABORATORIUM
ZEewaterstand - N.A.P. + 3,5 m
ONREBELMATIEGE BOLVEN

$H_2, 1/3 \times 3.65 \text{ m} (P\ 207)$
$H_2, 1/3 \times 2.50 \text{ m} (P\ 208)$
$H_2, 1/3 \times 1.35 \text{ m} (P\ 209)$
$QS\ GOLF\ (\alpha = 70)$

BETROUWBARE GEBIEDEN

GOLFFRÉKWENTIE $f \text{ (Hz)}$

OVERDRACHTSFUNKTIES VAN HORIZONTALE KRACHTEN,
DIEPE SEKTIE, MODELPROEVEN EN QS GOLF

WATERLOOPKUNDIG LABORATORIUM
ZEEWATERSTAND ± N.A.P. ± 7,5 m
ONREDELMATIGE GOLVEN

$\bar{H}_z, \frac{1}{3} \cdot 4,05 \text{ m (P 401 )}$

$\bar{H}_z, \frac{1}{3} \cdot 3,10 \text{ m (P 402 )}$

OVERDRACHTSFUNKTIES VAN HORIZONTALE KRACHTEN
OP EEN ONDIEPE SEKTIE ($F_H / H$)

WATERLOOPKUNDIG LABORATORIUM

M.1469- 1027 FIG.27
ZEEWATERSTAND = N.A.P. + 5.5 m
ONREGELEMATIGE GOLVEN

\( \bar{h}_2, 1/3 \times 3.15 \, \text{m (P 403)} \)
\( \bar{h}_2, 1/3 \times 2.40 \, \text{m (P 404)} \)

OVERDRACHTSFUNKTIEN VAN HORIZONTALE KRACHTEN
OP EEN ONDIEPE SEKTIE \((F_H/H)\)

WATERLOOPKUNDIG LABORATORIUM

M.1469-1028 FIG.28
OVERDRACHTSFUNKTIES VAN HORIZONTALE KRACHTEN
OP EEN ONDIEPE SEKTIE ($F_H / H$)

ZEEWATERSTAND = N.A.P. + 3,5 m
ONREGELMATIGE GOLVEN

$R_z, 1/3 \times 2,35 $ m ($P 405$)
$R_z, 1/3 \times 1,40 $ m ($P 405$)

BETROUWBARE GEBIEDEN

GOLFFREKWENTIE $f (Hz)$