THESIS REPORT

On Dynamic Fatigue Loads on Composite Downlines in Offshore Service

P.P. Rabe

Designing an analysis tool based on finite element analysis and drawing qualitative conclusions about fatigue life.
## Revision log

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<th>Revision date</th>
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<th>page</th>
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Abbreviations

AOG Airborne Oil & Gas
C.o.G. Centre of Gravity
D.o.F. Degree of Freedom
ICD Interface control document
ID Inner Diameter
LE Lead engineer
MEG Mono-Ethylene Glycol
OD Outer Diameter
PL Project leader
PLET Pipeline End Termination
PLR Pig Launcher/Receiver
RF Reserve Factor
SF Safety Factor
UD Uni-Directional

List of symbols

\( v_c(z) \) \hspace{2cm} \text{Current speed (function of depth coordinate } z) \\
\zeta \) \hspace{2cm} \text{Water surface elevation} \\
\sigma_{11} \) \hspace{2cm} \text{Normal stress, } 1-1 \text{ direction (2-2 and 3-3 direction similar)} \\
\sigma_{\text{all}} \) \hspace{2cm} \text{Allowable stress} \\
\sigma_vM \) \hspace{2cm} \text{Von Mises stress} \\
\tau_{13} \) \hspace{2cm} \text{Shear stress, } 1-3 \text{ direction} \\
\Delta \) \hspace{2cm} \text{‘Is defined as’} \\
a \) \hspace{2cm} \text{Added mass} \\
b \) \hspace{2cm} \text{Hydrodynamic spring coefficient} \\
c \) \hspace{2cm} \text{Hydrodynamic damping coefficient} \\
C_d \) \hspace{2cm} \text{Drag coefficient} \\
C_m \) \hspace{2cm} \text{Added mass coefficient} \\
D \) \hspace{2cm} \text{Diameter} \\
E \) \hspace{2cm} \text{Young’s modulus} \\
F \) \hspace{2cm} \text{Force} \\
g \) \hspace{2cm} \text{Gravitational acceleration: } 9.81 \text{ m/s}^2 \\
H \) \hspace{2cm} \text{Wave height } (H = 2\zeta \text{ for Airy wave theory})
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>$H_s$</td>
<td>Significant wave height</td>
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<td>Wave number</td>
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<td>Length</td>
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<td>$S$</td>
<td>Variance Density Spectrum</td>
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<td>$T$</td>
<td>Tension</td>
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<td>$T_p$</td>
<td>Peak period</td>
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<td>$\alpha, \beta, \theta$</td>
<td>Angle</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic Viscosity</td>
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<tr>
<td>$\omega$</td>
<td>Cyclic Frequency</td>
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<tr>
<td>$r$</td>
<td>Radius</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<td>$\epsilon$</td>
<td>Strain or phase shift</td>
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</tbody>
</table>
Abstract

Offshore oil & gas production is an industry where breaking records has become a necessity. So-called unconventional oil in ultra-deep water (2000 m+) requires rethinking of current production techniques, as steel tubing begins to buckle under its own weight. Other problems arise from corrosion of pipelines at all depths, requiring expensive maintenance schedules and production interruptions. Airborne Oil & Gas’ (AOG) novel Thermoplastic Composite Pipe (TCP) promises to relieve many of these problems. A light-weight GFRP pipe with excellent corrosion resistance, spooling capability and outstanding mechanical performance figures, it promises to become the go-to technology for many, if not all, of the offshore exploration and production (E&P) majors.

Offshore, downlines and risers experience heavy loads due to moving vessels/topsides. Such loads induce fatigue in these risers, and TCP must be subjected to careful analysis of the nature of these loads. This is the main topic of this thesis. Developing methods (and a simulation tool to apply them) to (conservatively) predict fatigue loads on TCP risers for various wave climate conditions.

A method based on the concepts of Response Amplitude Operators (RAO’s) is developed to simulate vessel motions for simple (unidirectional) wave climates, as well as for complex combinations of spatially distributed components making up ‘confused’ wave climates, which may consist of any combination of wind- and/or swell-induced wave components. These vessel motions are consequently used as input for a dynamic analysis in Abaqus, a well-known finite-element package available at AOG.

Stresses within the TCP as simulated by Abaqus are used in a simplified fatigue model, yielding qualitative results regarding fatigue performance of TCP in offshore service under varying circumstances. Rainflow counting is used to determine the number, and magnitude, of fatigue cycles experienced, while fatigue life is evaluated by assuming a homogeneous material for the TCP. The homogeneous assumption does not hold for actual TCP, which is highly non-linear in material behaviour, but serves as a proof of concept of the tool developed and invites for further development of the methods presented to account for such non-linearities.

Findings include good agreement between the proposed model and theoretical results for vessel motions. An important effect of currents on the dynamics and stability of the system is discussed. Furthermore, the derogatory effect of increasing wave heights and failure to correctly ‘weathervane’ the vessel (i.e., failure to meet incoming waves head-on) on the proposed fatigue life is discussed. Increasing wave height by 150% seems to more than halve fatigue life, while changing the direction of wave-impact upon the vessel by 45 degrees seems to have similarly serious impact, although a more extensive analysis programme is required to confirm such statements.
# Table of Contents

Revision log .................................................................................................................. 1

Abbreviations .................................................................................................................. 2

List of symbols .................................................................................................................. 2

1 **Global Introduction and Reference Case** ...................................................................... 8
   1.1 Problem Introduction .............................................................................................. 8
   1.2 Airborne’s Thermoplastic Composite Pipe .............................................................. 9
   1.3 Reference Case ......................................................................................................... 9

2 **Task Formulation** ....................................................................................................... 12
   2.1 Clarifications ........................................................................................................... 12

3 **Qualitative Modelling Considerations** ................................................................... 14
   3.1 Newton’s Second Law of Motion ........................................................................... 14
      3.1.1 Further Considerations .................................................................................... 14
   3.2 Linear Techniques and RAO’s ............................................................................... 15
      3.2.1 RAO’s in Frequency Domain Analyses ............................................................ 15
      3.2.2 RAO’s in Time Domain Analyses .................................................................... 16
      3.2.3 RAO-based Analysis Conclusions ................................................................... 17
   3.3 Coupled v. Uncoupled Vessel Dynamics – Case Study ........................................... 17
   3.4 Comparing Modelling Approaches ....................................................................... 19
   3.5 The Theory Behind Orcaflex .................................................................................. 19
      3.5.1 Orcaflex Statics ............................................................................................... 20
      3.5.2 Orcaflex (Line) Dynamics ............................................................................... 21
   3.6 Decisions Regarding Modelling Techniques ............................................................ 22

4 **Wave records and Wave Energy Spectra** ................................................................ 23
   4.1 Fourier Analysis Introduction ............................................................................... 23
   4.2 Fourier Transform of Surface Elevation Records .................................................... 25
   4.3 Standardized Wave Variance Density Spectra ........................................................ 26
      4.3.1 Significant Wave Height, Peak Period & Fetch ............................................... 27
      4.3.2 Bretschneider & Pierson-Moskowitz Spectra .................................................... 28
      4.3.3 The JONSWAP Spectrum ............................................................................... 28
      4.3.4 Two-peak Spectra & Torsethaugen Spectrum .................................................. 30
   4.4 Directional Spectra ................................................................................................. 31
      4.4.1 On the Spreading Parameter $s$ ....................................................................... 34
   4.5 Geographical Areas of Applicability of Standard Wave Variance Density Spectra ...... 34
   4.6 Spectra and Statistics – Mathematical Moments ...................................................... 35

5 **Environmental & Operational Influences** ............................................................... 37
   5.1 Currents & the Morison Equation .......................................................................... 37
      5.1.1 Hydrodynamic Loads on Slender Pipes ............................................................ 37
      5.1.2 Determining the $CM$ and $CD$ coefficients ..................................................... 38
      5.1.3 Types of current and current profiles ............................................................... 40
<table>
<thead>
<tr>
<th></th>
<th>Section Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Developing the Torsethaugen Spectrum</td>
<td>98</td>
</tr>
<tr>
<td>D</td>
<td>On the Mechanics of Thick-Walled Tubulars</td>
<td>101</td>
</tr>
<tr>
<td>E</td>
<td>Illustrative Abaqus Renders</td>
<td>103</td>
</tr>
<tr>
<td>10</td>
<td>BIBLIOGRAPHY</td>
<td>106</td>
</tr>
</tbody>
</table>
1 Global Introduction and Reference Case

1.1 Problem Introduction

“The age of easy oil is over.”

It is a credo that has been repeated, by businessmen and specialists from the industry alike, for several years now. In its “Energy Outlook 2035”, BP projects a global 41% increase in energy demand over 2012 by 2035, with a 1.7% increase per annum over the years 2015-2025. In this forecast, the role oil and gas (especially gas) play in the total energy mix is increasing as well, by 0.8%, c.q. 1.9% per annum, respectively [1]. It is worth noting that other upstream companies, such as Shell, come to very similar conclusions [2]. Actual supplies are not (yet) the problem. ‘Fracking’ and other new or improved technologies have unlocked vast quantities of tight shale gas in America and tar sands in Canada and Venezuela, among others. Daring entrepreneurs like BP’s former chief executive, Tony Hayward, are looking for crude in some of the most politically unstable regions in the world, unlocking even those reserves [3]. Proven global oil reserves have thus increased over the past ten to fifteen years. However, these reserves often consist of so-called ‘unconventional oil’ and they require increasingly complex technology and investments to economically produce. Following this trend of increasing demand and more expensive supply, crude reserves located offshore at extreme water depths become economically (and technically) feasible, where just a few short years ago, they might not have been.

This is where AOG’s (Airborne Oil & Gas) products’ strength shine through. A fully bonded composite downline/riser system for use offshore offers several key advantages, including light weight; absence of sensitivity to corrosion; and low pressure drops over significant lengths of pipe, due to extremely low internal surface roughness. The product is new and almost unique in the world, and due to its many promises curiosity among prominent clients is abound. Indeed, Airborne’s Thermoplastic Composite Pipe, or TCP as it is commonly abbreviated, downlines/risers appear to be well on their way to greatness. With this increasing scope, the company obviously aims to take control of as much of the front-end engineering as possible and cover all aspects of the design.

Offshore, fatigue is a major source of concern. Wind, waves, currents, cold, salt and more all contribute to an extremely harsh environment. It is essential then, to assess fatigue life of any offshore structure correctly before delivering a pipeline transporting volatile hydrocarbons. To this end, AOG intends to design a simulation tool that can conservatively simulate (predict) live offshore loads on a downline/riser. Having such a tool would greatly help accurately assess fatigue life of a downline during both the tender phases and design phases of new, and indeed existing, products, increasing confidence in the finished pipe even further. At the same time, dependency on the client to deliver representative (and accurate) data is very much reduced. It is this proposed ‘simulation tool’ that is the basis for this master’s thesis.

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1 Unconventional oil; oil that is produced from very challenging rock formations/locations. Increasingly common due to decreasing availability of ‘conventional’ oil through sheer exhaustion of supplies.
1.2 Airborne’s Thermoplastic Composite Pipe

*Thermoplastic Composite Pipe*, or *TCP*, is the name given to AOG’s flagship product. It is the main subject of this thesis and some general understanding of what it is seems very much in order. It is a new generation of pipeline, offering a host of advantages over traditional steel pipelines and the brainchild of Airborne International, with their years of experience in composite construction, and Shell, who were looking into new technologies for solving problems that were starting to get ever more severe with existing products.

From a technical point of view, TCP is in essence a glass pipe. Glass fibre filaments in a thermoplastic matrix (the Thermoplastic Composite part of TCP) make for a structure that may be redesigned depending on application and demand by client, and features some key benefits over traditional pipelines. Thermoplastic composite pipe does not rust (!), is readily compatible with most chemicals used in E&P operations, is very lightweight, spoolable with very small minimum bend radius, easy to install in the field and features very low pressure drops of flowing products thanks to a virtually smooth inner surface. It can withstand enormous pressure fluctuations and handle internal vacuum at rated depth. Because of its light weight, it theoretically enables production in waters so deep that traditional steel pipelines would buckle under their own weight. It is the answer then, too many of the offshore industries’ recent struggles, and with an increasing list of interested parties, clients, and Shell’s recently acquired stake in the company [4], the promises have never been higher.

![Render of AOG’s Thermoplastic Composite Pipe. Clearly visible are the plastic liner (black), thick glass fibre core (grey) and protective plastic coating (yellow). No metal is employed in the construction of these pipes.](image)

1.3 Reference Case

While the tool is intended to be flexible and capable of handling different situations based on different input values, AOG’s idea for such a tool stems from a specific project. This project concerned an AOG-built composite downline of just over 2100 meters length used for pre-commissioning of an 18” subsea flowline. The client in this case (further also only referred to by the anonymous title ‘client’ — due to contractual obligations) provided *Orcaflex*-analyses of several different scenarios, including deployment and active service. As mentioned in section (1.1), AOG would like to have analytical capabilities independent of any third party (e.g., the client) to gain initial insight into operational loads and in order to be capable of assessing the fatigue life of future downlines. This chapter serves to explain a typical situation we will be considering, and what problems of interest arise from it. This will hopefully strengthen both the readers understanding of the purpose and of the scope of this thesis.
For the reference case under consideration, a downline is lowered from a construction/support vessel until it hovers some 50 meters above the sea floor. It is weighed down at its submerged end with a lifting collar, end fitting, several bend restrictors and some heavy chain, to counteract the positive buoyancy TCP exhibits when filled with air or nitrogen. It is important to realize that the downline never actually reaches the ocean floor or the PLET/PLR. It only serves as a way of transporting products (several options are possible, from Nitrogen to Seawater, MEG$^2$, and many more) from the ship to about 50 meter above seafloor level. At this level, the downline terminates in an ‘adapter piece’ that connect the downline to a bundle of flexibles. This ‘adapter’ has several subsea configuration options, depending on the specific design. It might be connected via a either nylon rope or heavy mooring chain to a mudmat lying on the sea floor, which is burdened by an additional clump weight to resist excessive lateral movement of the TCP due to current. It might also simply be loaded with a heavy clump weight to achieve tension in the TCP. For the design of the software, the latter is assumed for the remainder of this document. The ‘flexibles’ mentioned briefly are small-diameter ‘hoses’, which take over from the downline as transport lines for the product. They have several weights/buoyancy modules attached to them where appropriate, to ensure an ‘s-shape’ type configuration underwater. This ensures bending strains in the flexibles to stay below critical values. These flexibles, collaterally called the ‘bundle’, are the hoses connected to the actual hardware on the seafloor. For visual reference of this, please see Figure 1.3.

The task at hand, further explained in chapter 2, is thus to design, write and test (verify) a computer program that can somehow simulate the vessel’s movement, taking into account relevant environmental influences and variables, determine the stresses and loads these movements cause and conclude what effect different circumstances have on the fatigue life of a ‘downline’. The main focus is on the analysis of the occurring forces in this thesis, with the fatigue life being calculated in an illustrative way for this thesis. This problem is clearly of a very dynamic nature, due to environmental forces influencing the vessel on the sea surface, as well as the downline and attached hardware. Waves, wind, currents, swell and others might all play a role. However, to what extent, if at all (significantly), is one of the important issues we will attempt to answer before constructing the actual analysis tool. Once we have an approximate answer to this, the quantitative question becomes important. That is; quantitatively speaking, what forces do all the different influences exert on (and in) our downline? To answer any meaningful questions on fatigue, all of this is of importance and we will attempt to answer all of these questions in the remainder of this thesis.

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$^2$ MEG: short for Mono-Ethylene Glycol. A desiccant often used in the offshore industry to prevent hydrate-plug forming in pipelines.
Figure 1.3 - Schematic Downline application as explained in section 1.3. The vessel itself is not depicted. Chute, TCP downline and possible underwater configuration are all clearly visible.
2 Task Formulation

The final goal of this thesis remains, broadly, the primary goal set forth by AOG in their initial proposal as received by the author on July 15th, 2014:

“[The goal is to] Provide AOG with a comprehensive tool to obtain conservative (but realistic) estimates of dynamic loads to which the TCP is subjected to on the overboarding chute for a downline application.”

The boundary conditions for the assignment read (quote):

“The tool shall be based on one of the following software packages:

- (MS Excel)
- SciLab
- Python/Abaqus

Input parameters for the tool shall include:

- Location of chute (side or stern of vessel)
- Deployed length of downline (i.e. water depth)
- Metocean Data (Hs [significant wave height], swell data, etcetera)
- Downline properties (weight, stiffness, etcetera)
- Vessel RAO data
- ...

In conversations taking place between the author and AOG after having received this information, it was agreed bilaterally to use this information as the basis for the final thesis (i.e., this very text).

2.1 Clarifications

The original assignment statement yields a reasonable scope of work. By this, it is meant that the assignment states a clear goal (the development of a tool for reviewing loads). However, both the ‘what exactly’ and the ‘how’ are still a bit fuzzy. What is meant by ‘dynamic loads’? And how will this tool work exactly? Identifying options and important parameters (also reviewing the parameters indicated by AOG) will hopefully ensure not missing valuable information and options, while allowing for a smooth design process later on. Let’s talk about the ‘what exactly’ first.

What exactly is it we are after? Why is it worth the trouble to occupy an employee for nine months with this particular assignment? The answer is fatigue. For a downline design that functions reliably over the course of one operation, one month or ten years, it is vitally important to understand possible failure mechanisms. Fatigue plays a major role here, and AOG has methods to analyse it. It should be clear from here on out that the actual fatigue model used by AOG is not under consideration here. For now, it is merely AOG’s wish to obtain relevant data to use with its available analytical tools. This is our scope – our ultimate goal, even more so than the original statement of the assignment at the very beginning of this chapter. Nevertheless, we will develop a simplified fatigue model for a homogeneous material to be able to compare results, at least on a qualitative basis.
From an initial analysis and interviews with AOG personnel, it became clear exactly what data is desired (needed). In no particular order, it includes:

1. Tensile stresses
2. Moments
3. Phase shifts for all these components relative to one another

Quite simply put, more available data is better. For a detailed reconstruction of fatigue damage occurring in the TCP downlines, an accurate picture needs to be painted of the occurring stresses and strains. We will now try to rewrite our original goal, using a tactic from [5]. The new goal reads:

"[To] Provide AOG with a comprehensive tool to obtain conservative (but realistic) estimates of dynamic loads to which the TCP is subjected to on the overboarding chute for a downline application by modelling all significantly contributing phenomena."

It was decided after some debate, that Python would be the main software tool to be used (instead of Microsoft Excel or SciLab). Python is a proven, high-level programming language that is completely open-source. There are many, many free toolboxes available for the language for all kinds of applications and its popularity means that all manner of support is available on internet fora. It is an ideal candidate for the task at hand, with the added benefit of being the scripting language of Simulia’s Abaqus FEA, the second important software tool we will be working with.

Abaqus FEA is a famous finite element analysis programme, especially renowned for its more-than-capable composites module. It is available within AOG and will be used to carry out the complex, non-linear dynamic analyses to follow. As mentioned before, Python is the scripting language of Abaqus, meaning we will be capable of automating a lot of the work we need to do. Python can handle the pre- and post-processing, while Abaqus will be responsible for the finite element calculations.
3 Qualitative Modelling Considerations

This chapter is intended to give some consideration to the options we have for analysing a problem such as the one we have at hand. We will also take a look at some industry-standard reference software (Orcaflex), which unfortunately is not available to the author, and how it handles certain phenomena. Finally, we will draw some conclusions from this that will enable us to model the problem from scratch.

As far as an approach to the problem of a vessel’s motion in wind, waves and weather is concerned, two options lie before us. One is a very rigorous approach that allows for extensive modelling and coupling of phenomena, as well as precise implementation of mooring and other forces; while the other is considerably more accessible, since a lot of the hard math has already been done for us. We will discuss both, and see why the former is of impractical nature to the project at hand.

3.1 Newton’s Second Law of Motion

The first approach to predicting vessel motion is based on Newton’s world-famous second law of motion: force equals mass times acceleration [6] (page 102). For hydrodynamic purposes, it must be altered a bit, to read [7] (page 6-10):

\[ F(t) = (m + a) \frac{d^2x(t)}{dt^2} + b \frac{dx(t)}{dt} + cx(t) \]  

(1)

... Where the variables, partially to be explained later, are:

\[ x(t) = \text{Vessel's centre of Gravity's instantaneous position [m]} \]
\[ m = \text{Mass of vessel [kg]} \]
\[ a = \text{Added mass [kg]} \]
\[ b = \text{Hydrodynamic damping coefficient [N s/m]} \]
\[ c = \text{Linear spring coefficient [N/m]} \]

Since this is the fundamental equation governing all Newtonian motion, it is an extremely powerful tool. By adding to the ‘\( F \)’ term in the above equation, virtually any external influence may be taken into account. Examples might include wind, waves, current, accidental forces, hurricane loads and non-linear mooring characteristics. As long as an appropriate model is available to calculate a force of interest acting on the vessel, it may readily be added to the equation. Then, after some considerable (numerical) mathematics, time-records of vessel movements may be generated.

3.1.1 Further Considerations

All this comes at a downside, unfortunately. The required coefficients \( a \) (added mass) and \( b \) (hydrodynamic damping) are rather difficult to calculate. Added mass (often misrepresented as ‘water mass moving along with the moving vessel’) is a tool allowing us to approximate the effects of a moving body in water, which thereby disturbs the local pressure field; by adding to the physical mass term present in the Newtonian equations of motion [8]. In good approximation, assuming the moving body to have a bit more mass than physically present (the added mass), accounts for part of the hydrodynamic reaction force experienced by a body accelerating in a fluid [7] (page 6-11).
Hydrodynamic damping is, just as the name suggests, a damping coefficient, representing the momentum-absorbing capabilities of water. The problem is, that both damping coefficients and added mass coefficients are functions of the frequency of the incoming wave and have to be determined for each degree of freedom (three, c.q. six, depending on the type of analysis: 2- or 3-dimensional), plus a few coupling (off-diagonal) terms. This means that for a single vessel, two large sets (matrices) of \( a \) & \( b \) are required. The generation of these matrices is left to very specialized software, based often on the so-called ‘panel method’, making their availability a real issue for a tool that is supposed to be user-friendly and generate quick results. In addition, software that can correctly implement this data is complex and involves many steps, increasing the likelihood of making significant errors.

### 3.2 Linear Techniques and RAO’s

The other option is using a linear analysis and analysing the response of a system in either the frequency- or the time-domain. The so-called Response Amplitude Operators (or RAO’s) are our helpers. Essentially, an RAO is a transfer function, calculating vessel response by multiplying the incoming wave height by an appropriate factor and then time shifting the response with a ‘lag factor’, or phase difference, which must also be specified in advance. Using RAO’s, the response of a vessel to a first-order (high frequency) wave load can be analysed directly, via spectral analysis in the frequency domain, or in the time domain.

#### 3.2.1 RAO’s in Frequency Domain Analyses

Building upon the short introduction above, we could for example consider heave motions as response of a vessel under influence of a regular sinusoidal wave (according to Airy wave theory). It may be calculated, in the frequency domain, as follows [7] (page 6-25):

\[
S_{k,\zeta}(\omega) = \left[ \frac{\eta_k(\omega)}{\zeta} \right]^2 S_\zeta(\omega)
\]

... Where

- \( S_{k,\zeta}(\omega) \) = Response spectrum of the \( k \)-th mode of rigid body movement under influence of \( \zeta \) \([m^2s^3]\)
- \( \zeta \) = Instantaneous water surface elevation (function of time) \([m]\)
- \( \eta_k \) = Magnitude of RAO (frequency dependent) \([m/m]\)
- \( S_\zeta(\omega) \) = Wave Variance Density Spectrum (frequency dependent) \([m^2s]\)

In the above, note that the notation for the RAO, \( \eta_k/\zeta \), is the direct definition of a transfer function. It is the amplitude of the response in the \( k \)-th mode (\( k = 1...6 \) for 3-dimensional analysis) divided by the magnitude of the incoming wave component with circular frequency \( \omega \) (meaning the RAO’s are frequency dependent). Its unit is therefore ‘meter per meter’ (m/m). The concept of (response) spectra as used here is explained in more detail in section 4.2, as it is a very important concept in this thesis and for (offshore) engineering in general.

The output of such a calculation is a response spectrum, as briefly mentioned in the previous paragraph. By using the mathematical moments (see section 4.6) of these functions, probabilistic analysis is immediately possible. Results are obtained fast and (advanced) knowledge of the mathematics behind it are not required. The latter would be especially true if the statistical analysis had been automated using a computer program.

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3 Units of \([m^2s]\) for translational degrees of freedom. For rotational degrees of freedom, they are \([rad^2s]\).
This approach, however, is only valid for narrow-banded spectra such as Pierson-Moskowitz or JONSWAP spectra (see section 4.3). For spectra containing energy over a wider range of frequencies (like most confused-sea spectra, or the Torsethaugen spectrum), equation (2) loses (significant amounts of) accuracy!

3.2.2 RAO’s in Time Domain Analyses

The second option is to use the concept of RAO’s in a time-domain analysis. Here, the point is to simulate real-time behaviour, rather than calculating the statistical characteristics of the response. The disadvantage being that for a ‘complete picture’, long periods of time will have to be calculated to get a reliable statistical picture. The very big advantage, on the other hand, is that several other forces/effects may be included in the analysis, by extending/adding to the model. Also, time-domain analyses are valid for broad-spectrum-domain processes, unlike their frequency-based counterpart (see section 3.2.1). The calculation method is as follows.

Following the basic definition of an RAO, we may calculate the linear response of a vessel to a single incoming wave of amplitude $\zeta$ [m] and frequency $\omega$ [rad/s] from radial direction $\theta$ [rad] using the following relation [7] (page 6-41; reformulated):

$$x(t) = \zeta \cdot RAO_x(\omega, \theta) \cdot \cos(\omega t + RAO_{xp}[\omega t])$$  \hspace{1cm} (3)

... where

- $x(t) = \text{Any of the six fundamental degrees of freedom [m] or [rad]}$
- $RAO_x(\omega, \theta) = \text{Magnitude of RAO (function of frequency and direction) [m/m]}$
- $RAO_{xp}(\omega, \theta) = \text{Phase lag associated with the RAO [rad/m]}$

Now, two of the most fundamental characteristics of linear systems to extend equation (3) to usable (real-life) sea-states are called upon [9] (page 33; reformulated):

1. **Scalability:** If a wave of amplitude ‘$L$’ and frequency ‘$\omega$’ ($L \sin(\omega t)$) produces a response of the vessel of amplitude ‘$R$’ and the same frequency plus a certain time lag ‘$\phi$’ ($R \sin(\omega t + \phi)$), then a wave of magnitude ‘$aL$’ and frequency $\omega$ will produce a response ‘$aR$’, again with frequency $\omega$ and the same time lag; $aR \sin(\omega t + \phi)$.

2. **Superposition:** If an incoming wave $L_1 \sin(\omega_1 t)$ produces a vessel response of $R_1 \sin(\omega_1 t + \phi_1)$ and a second incoming wave $L_2 \sin(\omega_2 t)$ produces a response $R_2 \sin(\omega_2 t + \phi_2)$, then the combination of the two incoming waves $L_1 \sin(\omega_1 t) + L_2 \sin(\omega_2 t)$ will produce the sum of the individual responses; $R_1 \sin(\omega_1 t + \phi_1) + R_2 \sin(\omega_2 t + \phi_2)$.

We will make heavy use of these two fundamental principles for the final product, since the above combined means that we can write for the response of a vessel:

$$X(t) = \sum_{n=1}^{m} \sum_{i=1}^{j} \zeta_n \cdot RAO_x(\omega_n, \theta_i) \cdot \cos(\omega_n t + RAO_{xp}[\omega_n \theta_i] + \epsilon_n)$$  \hspace{1cm} (4)

... where

$$X(t) = \text{‘Complete’ response of vessel in random sea-state for random D.o.F. [m]}$$

---

4 ‘Complete’ here refers to the response due to incoming waves, first-order response, only!
\[ m = \text{Number of wave frequencies present } [-] \]
\[ j = \text{Number of incoming wave directions considered } [-] \]
\[ \epsilon_n = \text{Phase lag associated with wave component 'n'} [rad] \]

That is, for a wave climate\(^5\) modelled as a superposition of a large number of pure sine waves of varying amplitude and frequency, \(\zeta\) and \(\omega\), respectively\(^6\), travelling in different directions, \(\theta\), the motions of a vessel can be approximated by calculating the response to each individual component (using equation (3)) and summing them.

### 3.2.3 RAO-based Analysis Conclusions

So where is the downside to this approach? The RAO-based analysis outlined in this section is 100\% linear. RAO’s are derived from added mass and damping coefficients, are therefore frequency- and direction-of-incoming-wave-dependant and only applicable to first order wave effects. Wind and current are not included. Neither are things like (non-linear) mooring (any mooring effects having significant effects in the wave-frequency domain must be accounted for in the RAO’s) nor wave-drift (second-order wave) forces. Non-linear roll damping, important for accurate movement predictions in a few modes (most notably of course; roll), is a tricky phenomenon to predict [10] (abstract) and is neglected as well. One option we have to deal with phenomena like these are to determine a displacement function and add it to the displacement we get from applying our RAO’s. For the case of current, for example, this might be a set offset from the ‘zero displacement’ position. Second-order wave forces, very important for traditionally moored vessels (no dynamic positioning) have to be accounted for in a similar fashion, but here a constant offset is not adequate. This means a completely new model has to be written to account for this dynamic low-frequency drift, which requires its own set of RAO’s (called Quadratic Transfer Functions, or QTF’s; [7] (page 9-28 onwards)). The influence of these second-order wave forces diminishes significantly when using dynamic positioning, by the way. This is because Dynamic Positioning Controllers usually only allow displacements in a pre-set field (a circle of radius in the order of 10 or 20 meters perhaps) around the desired location on the sea surface. When considering water depths of (several) thousand meters, these relatively small displacements are likely to be reasonably negligible, and in this thesis we will continue working with this assumption.

Finally, after summing all our calculated displacements, we have no way of knowing how accurate our simulation is without field-tests or more advanced calculations. This kind of simulation, where the movement due to several interacting forces is considered independently of one another, is called ‘uncoupled’ and it is only possible if there is only weak interaction between the different forces at play [11] (abstract). This does not mean that it is never any good, as we will explore in section 3.3. The disadvantages of inaccurate extreme values, discussed in section 3.1.1, also apply here of course - as soon as we include secondary effects not taken into account by the simple spectral analysis. Another disadvantage is that it is difficult to obtain an accurate set of RAO’s [12] (page 6), yet they are very important for obtaining accurate results [13] (page 154). Results are by no means necessarily ‘inaccurate’ and the principle of RAO’s is widely applied in the industry, but it should not be forgotten that we are dealing with a linearized model of reality, as so often.

### 3.3 Coupled v. Uncoupled Vessel Dynamics – Case Study

This section is largely a literature review of an original paper by Heurtier, Le Buhan et al., (2001) entitled “Coupled Dynamic Response of Moored FPSO with Risers”[11]. Within this section, reference is made to this document whenever nothing else is mentioned.

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\(^5\) ‘Wave climate’ is a term used throughout this thesis to designate the current set of wave conditions at sea; wave height, frequencies, and directions. Currents are excluded from this definition.

\(^6\) This is the essence of the ‘random-phase/amplitude model for water surface elevation. See section 4.2 for more detail.
In the aforementioned document, a system is under review that contains many of the features we identified as ‘external influences’ in section 3.1, and is therefore quite representative of our own situation. A review seems in good order, since no matter which of the two approaches to simulate vessel movements discussed so far in this chapter is chosen, the choice between a ‘coupled’ (full Newtonian) and a ‘uncoupled’ (RAO-based) approach remains. We will also encounter some inherent difficulties with both methods. Simply put, we have to make a choice whether we mathematically fully couple all our equations (and solve for all of them at the same time) to take mutual influences into account; or don’t do so, solve for several different phenomena separately and add the results (i.e., applying the principle of superposition).

Both options might yield acceptable results, and [11] discuss the differences. In their paper, a large hull (an FPSO, to be exact), connected to mooring lines and several risers (playing the part of our downline), is subjected to wave loads. Both first- and second-order influences are taken into account. Since we are dealing with a traditionally moored vessel in this reference paper, the choice to incorporate second-order effects is almost mandatory due to the crucially important second order phenomena associated with such a mooring system. For the uncoupled approach, first- and second-order wave loads are considered completely independent of one another. The ‘memory effect’, accounting for waves that have already past and arising from the transformation from the frequency to the time domain, is neglected. Motion of the vessel is decomposed into two terms; \( x(t) = x_{WF}(t) + x_{LF}(t) \), where subscripts ‘WF’ and ‘LF’ stand for Wave Frequency and Low Frequency motion, respectively. The wave-frequency part is then assessed using the RAO-approach as discussed in section 3.2 and summarized in equation (4); all incoming waves, with certain frequency and amplitude, cause a certain response of the vessel with the same frequency and an amplitude equal to the incoming wave’s multiplied by the correct RAO. For a more detailed discussion regarding this and the associated mathematics, see also [14] (page 6 onwards).

The low-frequency part of the system is modelled using a modified version of equation (1) to take into account mooring forces and wind and current loads. The problem of the ‘unknown’ added mass and damping coefficients is dealt with by taking an asymptotic value (\( \lim(\omega \rightarrow 0) \)) for the former and a custom ‘low frequencies damping matrix’ for the latter (which is not specified in detail, but mentioned to account for ‘friction and drag forces on the mooring lines’; the phenomena we can’t capture with RAO’s!). The resulting equation from the paper to be solved is a form of the classic Newtonian force equation;

\[
(m + a[0]) \frac{d^2 x_{LF}(t)}{dt^2} + b \frac{dx_{LF}(t)}{dt} + c x_{LF}(t) = F_{ext}(t)
\]

... Where all coefficients are as defined for equation (1), but in matrix form to account for all six degrees of freedom plus coupling terms. All external forces are captured in the rightmost vector \( F_{ext}(t) \). These include wind and current, mooring and of course the low-frequency wave forces we were after in the first place. The so-called \( b \)-matrix, or damping matrix, might be a problem for our own approach as no such data is likely to be available in practice. As always, \( x(t) \) denotes the instantaneous position (and orientation) of the vessel’s centre of gravity while the ‘zero’ argument given for the added mass matrix indicates that we have chosen the limit for \( \omega \rightarrow 0 \), as explained before. From the previously mentioned off-diagonal coupling terms, it is clear that the calculations are not completely uncoupled. Merely the high- and low-frequency parts are separate. In the low frequency equation, equation (5), quite a few phenomena are considered simultaneously in the right-most term and we still need additional hydrodynamic data (the \( a, b \) and \( k \) matrices), as well as relevant modelling parameters for wind, current and mooring forces (possibly including mooring line inertia and damping effects) to carry out the calculation. In this paper review, we accept the author’s data at face value and simply compare results.

Lastly, an even more simplified analysis, called quasi-static, was also carried out by the authors of our reference paper. For this method, the force the mooring system exerts on the vessel is pre-calculated from static analyses as a function of position. Per time-step, the force exerted on the vessel is then added to the external force term on the right in equation (5) by linear interpolation, based on the vessel’s position on the current time step. This means neglecting all mooring line dynamics, such as added inertia and mooring line damping, as well as the coupling effect between lines and vessel. For a system where the ship has much
greater mass than the mooring system and the water depth is not too great, this seems an acceptable simplification.

In the paper, it is concluded that the difference in results between the fully-coupled dynamic analysis and the uncoupled analysis are most pronounced in the case of surge motion. This is because the dynamic mooring line damping plays a major role in correctly predicting this component. Differences in predicted displacement from the mean position may be in the order of 100% for some situations. In case of a dynamically positioned ship, again, this kind of problem might be rendered completely mute, since the sum-frequencies (second-order phenomena) hardly get a chance to influence results; they are assumed to be largely neutralized by the dynamic-positioning controller. It is further concluded that line tensions are also badly predicted with uncoupled approaches, so here a coupled analysis is likely to pay off.

Lastly, When considering the implications of coupled versus uncoupled analyses, another document worth considering is DNV’s Recommended Practice F205 (2010); “Global Performance Analysis of Deepwater Floating Structures” [15]. Here, it is remarked that essentially the only contribution that can be taken into account properly in an uncoupled analysis is the (quote; pages 7-8) “Static restoring force from the mooring and riser system as a function of floater offset”. The “Current loading and its effects on the restoring force of the mooring and riser system”; “Damping from mooring and riser system due to dynamics, current, etc.”; and the “Additional inertia forces due to the mooring and riser system” may be approximated, while “seafloor friction (if mooring lines and/or risers have bottom contact)” and “friction forces due to hull/riser contact” can according to DNV, generally not be accounted for with an uncoupled analysis.

### 3.4 Comparing Modelling Approaches

The big upside of the linear approach from section (3.2) is the much bigger chance of finding a good set of RAO’s than finding a full set of added mass and hydrodynamic damping coefficients, as well as increased mathematic simplicity and user-friendliness. As mentioned before, added mass and damping coefficients are hard to obtain and very specific information not likely to be freely or even commercially immediately available for the vessel of interest. This, coupled with some encouraging results regarding the accuracy of (partly) uncoupled simulations as presented in section (3.3) does encourage choosing the second (linear) approach. The consequence would be, that we might lose some accuracy in our calculations, due to the absence of certain effects in the final model, such as coupling and second-order hydrodynamics. However, this loss should be very manageable since the vessel under consideration seems to dominate dynamic response of the downline with its deadweight" in excess of 6000 metric tonnes alone. In comparison, the submerged weight of the downline (2.7 \( \text{kg/m} \) submerged downline) is roughly 2.7 \( \text{kg/m} \) \( \times \) 2100 \( \text{m} \) = 5.67 \( \text{tonnes} \), so less than 1 percent of deadweight (remember, the actual weight of the operational ship will very likely be higher than its deadweight). The fact that all operations will take place from a vessel that will be dynamically positioned, instead of being held in place by a physical mooring system, means the accuracy of a RAO-type analysis is likely to increase further because there is no mooring line mass (inertia), mooring line damping (usually an important effect) or coupling effect between vessel and lines to worry about; see [11]. Adding to this the fact that downline will never deployed in heavy seas and we may reasonably assume to be operating in an environment that is sufficiently accurate described by linear (RAO) theory. We therefore make the choice to continue with the latter for the rest of the project.

### 3.5 The Theory Behind Orcaflex

Orcaflex is a very popular offshore analysis software package, incorporating finite element analysis, as well as quite advanced hydrodynamic computations. All this is integrated into a very nice and easy graphical

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7 "Deadweight”; abbreviated ‘DWT’. Defined as weight of fully loaded ship minus weight of empty ship. A measure of how much useful cargo, consumables and personnel may be transported.
user interface, allowing for quick and easy results of quite complex problems. The usual disadvantages to a high-level engineering tool apply; inexperienced or poorly educated users will be presented with seemingly correct-looking results, even if the analysis carried out was set up poorly and the results anything but reliable. Another disadvantage is the cost of the program, at some €1500 per month.

However, Orcaflex can claim many votes of confidence from large firms and experienced engineers, as well as from its own developers, who have several references to verification papers. It seems, therefore, not a bad idea to have a look at Orcaflex’ engine behind the scenes, the physics-theoretical framework. So, in the following section, we will have a look at Orcaflex’ handling of (catenary) lines and ship movement. For all these steps, we will make some annotations regarding the applicability of the present theory to our own future model.

Of course, and it must be said for the sake of academic completeness, it is explicitly noted that Orcaflex is not taken to be ‘the one truth’. The software creates models of reality and attempts to solve them mathematically. As such, it is an approximation of the truth, albeit based on scientific findings and years of experiments and attempts at validations. The following is therefore intended as a contribution to the literature studies of this thesis. It is simply a review of techniques currently employed by an industry-leader and a source of inspiration for our own work.

3.5.1 Orcaflex Statics

The first step in conducting a proper dynamic analysis in Orcaflex is getting the static part right. This is because a static equilibrium is the position which a physical configuration takes on on its own, under the influence of time-independent forces such as gravity. A sketch of the real-world situation is created on screen, the correct elements such as lines, risers, vessels and buoys are connected, gravity, friction, own (submerged) weight and any other time-independent forces are applied.

![Illustrative example of Orcaflex’ graphical user interface. Source: www.orcina.com](image)

The mathematics that follow are described (in words) below. This first step is highly necessary for Orcaflex, because it utilizes a finite-element model to calculate forces, moments and movements and we are in absolute
need of an accurate initial position from which we can calculate deviations. There are some options regarding the calculation, but the general idea is as follows [13] (page 123):

1. From starting coordinates for each vessel/buoy, calculate free hanging configuration for each line according to a specific chosen model. Assume line ends fixed.

2. Calculate out of balance loads for each vessel/buoy due to equilibrium positions from step 1, estimate new position for body.

3. Iterate until out of balance forces converge to zero, within specified tolerance.

These preliminary solutions do not take into account bending stiffness or rotational stiffness. This is fine, since it is just the preliminary assessment serving as set-up for a more detailed (finite element) analysis, which converges to the ‘true’ static solution. As it turns out, for the model under design, an assumption regarding initial geometries/configurations at the beginning of the simulation is indeed quite necessary. In order to make the model converge, it turned out to be necessary to perturbate the equilibrium position at least twice, before converging to the final static configuration.

3.5.2 *Orcaflex (Line) Dynamics*

The next step is the dynamic part, related to the (pipe)lines in the model. It is much more cumbersome than the static part, but nevertheless builds on the framework previously used for the latter. By this we mean that the model solved for the dynamic equations stays the same: masses for vessels and buoys, connected to the finite element representations of mooring lines and risers (which has already been generated if the ‘full static’ part of the static analysis was applied). The main difference then, is the fact that Newton’s equation of motion is used as presented in chapter 3.1. Reformulated a bit, it reads

\[ (m + a[0]) \frac{d^2x_{LF}(t)}{dt^2} = F_{ext}(t) - b \frac{dx_{LF}(t)}{dt} - kx_{LF}(t) \]  

...(6)

... Where \( x(t) \) is the displacement vector, dependent on time; while \( m, c \) and \( k \) represent mass, damping and stiffness matrices respectively. \( F(t) \) represents the external force vector. Note that is the exact same equation used by [11].

The tactic here is to solve for the acceleration vector and then use forward-Euler integration (an explicit integration approach) to gain the speed and position data. To put this in formula, the new position and velocity of a mass after a time step \( \Delta \), meaning at time \( t + \Delta t \), are [13] (page 129):

\[ x(t + \Delta t) = x(t) + \frac{dx(t)}{dt} \Delta t \]

\[ \frac{x(t + \Delta t)}{dt} = \frac{dx(t)}{dt} + \frac{d^2x(t)}{dt^2} \Delta t \]  

...(7)

After each step, all positions and orientations are known again, and these new positions are used as starting points for the next time step (iteration). Disadvantage of this method is the required timestep (it is very small). Alternatively, using an implicit integration approach is possible as well in Orcaflex. This means using an interactive solution for each timestep, but is usually stable over much longer timesteps, meaning in practice, it’s often faster (unless very non-linear phenomena are being simulated, such as impact simulation). The implicit algorithm used by Orcaflex is called the ‘Generalised-\( \alpha \) algorithm’, which features ‘numerical damping’
to suppress unwanted high-frequency modes in the solution [13] (page 129). For our approach to this concept, see the discussion on material damping (section 5.4).

Orcaflex, being specifically designed to model structures in moving water to a high degree of fidelity, includes a lot of additional features/influences in its model. These include, but are not limited to, such diverse phenomena as seabed-interaction with mooring chains, fluid-flow in submerged pipelines, current- and wave-induced forces and ‘clashing’ events between mooring lines/risers. We will eventually be able to include some of the more relevant features mentioned here, while others will have to be left for future development. This is either due to the complexity of some of the features, or because we don’t expect the event to have any additional value for the data we are after. Seabed friction – for example – is a highly complex, very much non-linear phenomenon. It includes things like physically deforming the soil while the line drags across it and many more phenomena, which is why even Orcaflex does not attempt a full simulation but is content with a simplified model. Furthermore, we don’t expect any contact with the seabed at all. This feature will therefore not be included. The same argument holds for clashing analysis (also not included). Drag forces on the other hand will have to be included. We expect drag to have a major impact on largely free-hanging downline of comparably low weight, and in chapter 5.1 this topic will be treated in much more detail.

Vessel motions in Orcaflex are determined using the superposition principle explained in section 3.2.2. On top of these motions, however, Orcaflex can superimpose secondary (low/high frequency) motions, which are calculated by solving a set of Newtonian equations of motion that take into account a wide range of (second-order) effects. These include (mooring) line dynamics, soil interactions, current and wind loads and more. Detailed models for all these phenomena exist and their evaluation is no easy task, although differences in difficulty certainly exist from model to model. The main conclusion here must be that superposition is a valid approach to vessel motion simulation.

3.6 Decisions Regarding Modelling Techniques

Taking all this into consideration together, we see that the industry-standard software uses quite a few ideas we had ourselves in the beginning of the chapter. We feel strengthened in our belief that the proposed approach will yield acceptable results and continue with the following strategy choices for the design of the software tool:

1. Use a linear, uncoupled, response analysis for the vessel’s motions, based on the concept of RAO’s.

2. Do so in the TIME domain, for added flexibility with the model and capability to capture broad-spectrum sea-states.
4 Wave records and Wave Energy Spectra

Looking out over the open ocean, it is an easy thing to start suspecting the movements of the water surface to be completely chaotic (or, put differently: random). This 'suspicion' appears to be wholly justified, as scientists and engineers have always had to turn to probabilistic techniques to more or less accurately describe realistic wave climates. The random character of the water surface elevation simply does not seem to lend itself to an explicit mathematical approach. However, this doesn’t mean that we can’t handle it. While engineers tend to like deterministic descriptions of their problems, the fact is that we are capable of dealing with such random phenomena as (ocean) waves mathematically, which opens many, many doors. Wave climates everywhere in the world can be accurately represented by replicating not the exact waves present on our location of interest, but conditions which are statistically indistinguishable from the ones we have in real life. Using this approach, a computer model may simulate local wave conditions with a high degree of fidelity to real life, enabling accurate engineering simulations and reliable results.

These statistical representations are the subject of this chapter. We, like the rest of the scientific and engineering community, call them spectra (plural of spectrum), and the method to obtain them from real-world data ‘spectral analysis’. In short, we analyse a wave elevation record (obtained from a buoy or other in-situ method) mathematically, decomposing the signal into a finite number of (co)sine-waves using Fourier Transformations. This method, named after the famous French mathematician Jean-Baptiste Fourier, will be briefly explained in the following paragraphs to refresh the reader’s memory and ease understanding of the concept of spectra. We then use the information gained from the Fourier-Transform to gain a statistical description of the original wave record. Thus, we will go from a given wave-record to its spectrum, and from this spectrum back to a simulated wave-record! This shall be clarified in a more mathematical sense in the following section. The results arising from this theory will enable us to simulate accurately a 'real' sea-state, with parameters that may be chosen by the end-user, so as to enable simulations of all manner of weather.

4.1 Fourier Analysis Introduction

The (proven) idea we will be using for analysis of ocean surface elevation records, is that any repeating signal, \( x(t) \), may be represented as an infinite sum of cosine waves as follows [9] (page 102):

\[
x(t) = \zeta_0 + \sum_{k=1}^{\infty} \zeta_k \cdot \cos(k\omega_0 t + \epsilon_k)
\]

... where

\[
\begin{align*}
\zeta_0 &= \text{average of the original record (zero for our purposes) \, [m]} \\
\zeta_k &= \text{amplitude of the } k\text{-th harmonic component \, [m]} \\
\omega_0 &= \frac{2\pi}{T}; \text{ the fundamental frequency \, [rad/s]} \\
\epsilon_k &= \text{phase shift associated with the } k\text{-th component \, [rad]}
\end{align*}
\]

8 Jean-Baptiste Joseph Fourier (*1768 †1830); physicist and mathematician best known for his investigations of the Fourier transform (which bears his name to honour his achievements) and its application to heat-transfer and vibration problems.

9 'Accurately' here meaning the generation of random wave climates that adhere to the parameters entered by the end-user. These parameters are usually significant wave-height and peak period. See section 4.3.1 for a short discussion of these terms.
This is known as the (cosine-with-phase form of the) *Fourier series* of the original signal. The mathematical definition of the word ‘repeating’ in ‘repeating signal’ is that $x(t + a) = x(t)$ for any $a \neq \infty$. Basically, we represent the signal by its ‘frequency content’; a number of sine waves, each with its own amplitude and frequency. All of them added together will return the original signal. This powerful concept is explained visually in Figure 4.1. Here, a function $y(t) = 2 \sin(2\pi t) + \sin(4\pi t) + 0.5 \sin(8\pi t)$ is plotted, along with its three separate terms. The sum of these terms yields the original signal (top left), along the ideas presented here. The boundary conditions for signals for which this has proven to be possible are extremely general in a mathematical sense, covering virtually all situations arising in engineering purposes (including some classes of discontinuous functions) and surely the smooth, continuous functions we expect to find to describe wave elevations. To be more precise, equation (8) holds for a repeating signal, if the following so-called Dirichlet Conditions [9] (page 102) hold true:

1. $x(t)$ is absolutely integrable over any period; that is,
   \[ \int_{a}^{a+T} |x(t)| dt < \infty \]  
   \[ (9) \]

2. $x(t)$ has only a finite number of maxima and minima over any period;

3. $x(t)$ has only a finite number of discontinuities over any period.

As mentioned before, these conditions are pretty much certain to be met for surface elevation records, and we will consider them fulfilled for the rest of this chapter. The tool that allows us to convert a given signal to a form of equation (8), is called the *Fourier Transform*. It is readily available in most mathematical analysis packages (often called ‘FFT’, or Fast Fourier Transform), and Python is no exception. We will not go deeper into the specific mathematics of Fourier Transforms as it is a big, and very much off-topic subject, but will show some of its uses in the next section. The important part to take from this paragraph is that we will model our ocean surface elevation as being the sum of a large number of cosine terms, with amplitudes, frequencies and phase

![Figure 4.1 – Visual 'intuitive' demonstration of 'frequency content' principle. The arbitrary original signal may be decomposed into its harmonic components. Adding them back together appears to yield back the original signal again. In this simple example, only three components are needed to do so, but in general it is possible to decompose almost any signal exactly, by using an infinite number of terms.](image-url)
lags to be determined later by Fourier analysis of measured real-life situations. That is, our model of an ocean surface may be modelled as:

\[ \eta(t) = \sum_{k=1}^{n} \zeta_k \cdot \cos(\omega_k t + \epsilon_k) \]  

(10)

... where

\[ \eta(t) = \text{Instantaneous water surface elevation} \]

... and the other terms defined as for equation (8). Here, we only use \( n \) frequency components (versus the infinite number we need for an arbitrary signal by equation (8))\(^{10}\). When truncating the ‘infinite’ number of terms required for a true Fourier series, the resulting signal is an approximation of the original, but for signals that contain no extremely high frequencies or discontinuities, it is a rather good one. The \( \zeta_0 \) term from equation (8) vanishes, since we assume a zero-average for the surface elevation. This model of a dynamic ocean surface is called the ‘random-phase amplitude model’ [16] (page 33). It will be the basis for all simulations, and is therefore a very important concept in this thesis!

4.2 Fourier Transform of Surface Elevation Records

![Realized wave spectrum (JONSWAP type)](image)

\( \text{Figure 4.2 - Example of surface elevation record.} \)

Building upon the previous paragraph, we will take a look at what a Fourier analysis yields when applied to a random signal (and in this case, the random surface elevation record depicted in Figure 4.2). We will now develop the concept of Variance Density Spectra in a rather compact form, as it is a very basic (but very important!) concept in offshore engineering. Some background however, is still necessary in order to be able to understand some of the ideas behind the validation of our tool, and in particular for paragraph 8.3.2.

\(^{10}\) For a discussion on the accuracy of a truncated Fourier series, or a N-point Discrete Fourier Transform, the interested reader is referred to [7] (examples 4.12, 4.13)
A Fourier analysis always yields a set of (complex) numbers that may be interpreted as representing the amplitudes and phase lags (or the \( \zeta_k \)'s and the \( \epsilon_k \)'s) from equation (8), per discrete frequency step. This frequency step (resolution) is defined as \( \Delta f = 1/D \), which means that only increasing the length of the record under analysis may enhance it. The phases (\( \epsilon_k \)'s) turn out to be randomly and uniformly distributed between 0 and \( 2\pi \) radians for surface elevation records [7] (page 5-48), and we therefore forget the calculated values themselves. Later on, when generating a wave climate ourselves, we will remember the uniform \([0, 2\pi]\) distribution to generate new values from this. Section 7.2.2 deals with this.

The amplitudes (or \( \zeta_k \)'s) may be plotted versus their respective frequencies and the result looks a bit like Figure 4.3. Usually however, when dealing with surface elevation records, we won’t plot the amplitudes themselves, but rather \( 1/2 \cdot \zeta_k^2/\Delta f \) (again versus frequency), where \( 1/2 \cdot \zeta_k^2 \) is the variance of a harmonic wave with amplitude \( \zeta_k \). The reasons for this are very well explained by [16] (page 32) and are twofold. First of all, the variance is a “more relevant statistical quantity than the amplitude”. For example, in contrast to the amplitude, the variance of the sum is the sum of the variance [17] (page 140). This holds for uncorrelated variables, which is an assumption that can be made for individual wave-components that are not too steep and not in very shallow waters [16] (page 35). The second very good reason is that one can show that the physical quantity ‘energy’ (of the waves) is linked to variance! This means that a link can be made between statistics and physical properties of the wave; a very useful idea. The division by \( \Delta f \) ensures that the discrete amplitude-plot gets a continuous character for all frequencies, not just the discrete \( \Delta f \) steps. This is very useful for plotting purposes and when looking to ‘sample’ from the constructed plot at some random frequency \( f \) not necessarily in the set \( n \cdot \Delta f \), \( n \in \mathbb{N} \).

When plotting these transformed amplitudes (variances) as described above, we get what is called a variance density spectrum (VDS), a very important term. It is a representation of a sea-surface in the frequency domain, and the basis for generating our own seaways. Several examples (plots) will be given in the next sections.

### 4.3 Standardized Wave Variance Density Spectra

In order to be able to characterize oceans around the world from the convenience and comfort of a desktop anywhere else, several (variance density) spectra have been standardized and published in relevant
literature. These came to be by obtaining a large number of surface elevation plots of a certain part of the world’s oceans and studying the impact of several parameters on the average values of the resulting variance density spectra. The results are constructed spectra that are parameterised in two (or a few more) variables and describe local wave climates quite well. In offshore engineering, these standards are used extensively for design purposes and this practice is recognized by the major authors of engineering codices/recommended practices (e.g., DNV, API\(^{11}\)). Three examples are the Pierson-Moskowitz, JONSWAP and Torsethaugen spectra. They all have their uses and ranges of applicability. Choosing the correct one is very important for a correct analysis of offshore fatigue behaviour, and we will present each spectrum briefly and discuss its merits and problems. An excellent reference for several different spectra discussed here is [18]. Based on the definitions of these standard spectra, we will eventually generate a more-or-less random sea-state, adhering to certain probabilistic boundary conditions set by the used standard VDS, to simulate a real-life wave climate as encountered during actual in-situ operations.

4.3.1 Significant Wave Height, Peak Period & Fetch

Before delving into specific types of spectra found in the literature, we will first have to take a look at some basic concepts used to describe wave climates. The first, and probably most important one characterises wave height. It is called the *significant wave height*, usually denoted as \(H_s\) or \(H_{1/3}\) and with unit ‘length’ (meters). We will stick to \(H_s\) for the remainder of this text. The ‘\(1/3\)’ in the alternative notation references to the way the significant wave height is defined; as the average of the one-third highest waves.

The second important concept is that of *peak period*, usually denoted by \(T_p\) and with unit ‘time’ (seconds). Waves are characterized by more than just height, they have a period as well. In its simplest form, ocean waves may be thought of as sinusoidal waves of certain amplitude and period (frequency), travelling along the water surface. A real-world ocean surface then consists of many of these harmonic waves, superimposed on one another. The wave frequency which carries the most energy (i.e. the frequency of the harmonic with largest

\[ T = \frac{2\pi}{\omega} \]

\[ H_{1/3} = 8.0 [m] \]

*Figure 4.4 - Schematic representation of peak frequency (and, equivalently, of peak period) of a variance density spectrum. \(T = 2\pi/\omega\).*

\(^{11}\) DNV, API: Det Norske Veritas, American Petroleum Institute. Both publish widely used ‘Recommended Practices’ for the offshore and offshore energy sectors.
amplitude) is called the peak frequency, or, equivalently, the peak period. The latter is often supplied in design documents and is an input parameter for basically all the pre-defined wave energy spectra we will discuss in section 4.3. For visual reference, please see Figure 4.4. In this figure, the peak frequency in radians per second is indicated (it is approximately 0.523 rad/s).

Another characterisation of wave climates sometimes used is the ‘zero-up crossing period’, which is defined as the average time interval between two successive up-crossings through the mean sea-level. It is denoted by either $T_z$ or alternatively as $T_2$.

The third important concept is that of fetch, unit ‘length’ (kilometres). Fetch is the number characterizing how many kilometres of water is between the point of interest and the nearest significant landmass, downwind. It is of interest because the wind has had this many kilometres of distance/time to influence the surface waves. It is possible to predict what approximate wave climate will appear on a location, given water depth, wind speed and fetch using a diagram, though we have no direct use for it here. The interested reader however is referred to [19] (page 2-87).

4.3.2 Bretschneider & Pierson-Moskowitz Spectra

The Bretschneider spectrum, or modified two-parameter Pierson-Moskowitz wave spectrum, is defined as [7] (page 5-44):

$$S_c(\omega) = \frac{173 H_s^2}{T_1^4} \omega^{-5} \exp \left( -\frac{692}{T_1} \omega^{-4} \right)$$

... where

$H_s =$ Significant wave height [m]

$T_1 = 0.772 T_p =$ mean centroid wave period [s]

$\omega =$ wave frequency [rad/s].

It is one of the oldest spectra in use and especially suited for open sea areas, where the wind has had ample time to fully develop the ocean waves and works best for high(er) sea-states. It was accepted as the standard for seakeeping calculations and model experiments by the 12th International Towing Tank Conference (ITTC) in 1969. The original one-parameter Pierson-Moskowitz spectrum may be obtained by fixing the relationship between the significant wave height and the average wave period in equation 11 as follows [7] (page 5-44):

$$T_1 = 3.86 \sqrt{H_s}$$

4.3.3 The JONSWAP Spectrum

The JONSWAP spectrum, short for JOint North Sea Wave Project spectrum, has been developed over two years from data extracted from locations stretching a large part of the northern North Sea. It therefore works extremely well for sites around this area. Speaking more generally, it works well for describing fetch-limited\(^\text{12}\) situations and has been advised to be used by the ITTC in 1984 for such situations. It is well known to any offshore-engineer and sees frequent use. Mathematically, it is defined as [7] (page 5-44):

$$S_c(\omega) = \frac{320 H_s^2}{T_p^4} \omega^{-5} \exp \left( -\frac{1950}{T_p^4} \omega^{-4} \right) \gamma^A$$

\(^{12}\)fetch-limited; situations where the geography of the area is such, that the fetch does not exceed some 150 to 250 km.
… where

\[ \gamma = 3.3; \text{(Unless noted otherwise)} \]
\[ A = \exp \left( -\frac{\omega}{\omega_p} \right)^2 \]
\[ \omega_p = \frac{2\pi}{T_p} \]
\[ \sigma = \begin{cases} 0.07; & \omega < \omega_p \\ 0.09; & \omega > \omega_p \end{cases} \]

The JONSWAP spectrum has a validity range characterised by the factor \( T_p/\sqrt{H_s} \). The model is expected to approximate reality to a good degree for \( 3.6 < T_p/\sqrt{H_s} < 5 \) and results obtained from values outside this range should be used with increased caution [18] (page 33). This validity range is displayed in a plot in Figure 4.5. Like the Bretschneider spectrum, it works best for elevated sea states.

\[ \text{Figure 4.5 – ‘Validity Range’ for the JONSWAP spectrum displayed graphically. The surface enclosed by the two horizontal surfaces represents the ‘valid’ area.} \]

Sometimes, the ‘peakedness factor’ \( \gamma \) is varied as well, to introduce a third free parameter. Its mean value is 3.3, which is an acceptable substitute if nothing else is specified. However, Det Norske Veritas recommends the following more specific values be used in the absence of any further information [18];

\[ \gamma = 5; \text{ for } T_p/\sqrt{H_s} \leq 3.6; \]
\[ \gamma = \exp \left( 5.75 - 1.15 \frac{T_p}{\sqrt{H_s}} \right); \text{ for } 3.6 < T_p/\sqrt{H_s} < 5; \]
\[ \gamma = 1; \text{ for } 5 \leq T_p/\sqrt{H_s} \]
At this point, it is worth noting that for $\gamma = 1$, the JONSWAP spectrum reduces to the Pierson-Moskowitz spectrum. An illustrative example of the shape of a JONSWAP spectrum is given in Figure 4.4.

### 4.3.4 Two-peak Spectra & Torsethaugen Spectrum

Note: In the literature on this subject, the notations $H_S'$ (capital letter S) is used for a certain function. Due to the great potential to confuse this notation with our own $H_s$ (small letter s) for significant wave height, we will deviate from the literature and denote $H_S'$ (capital S) as $H_T$ (T for Torsethaugen). Kindly take into consideration when consulting references.

The general idea behind two-peak spectra is to account for both local, wind-generated, seas and swell. Swell are waves that have travelled outside the area where they were generated (perhaps by a distant storm). This is due to the fact that waves of different wave lengths travel at different speeds. A storm system generated hundreds or even thousands of kilometres away therefore starts to disintegrate into its various frequencies, and the components start to cross the water by themselves, rather than with their brethren of various frequencies they were generated with [16] (page 128). This is the explanation for the fact that the spectral representation of such a swell component looks very different from the spectrum that would be obtained when measuring the original storm waves that are the far-away source for exactly this same swell. The combination of wind seas and swell then, yields a typically double (or more!) -peaked spectrum, for which special approaches are necessary. The importance of double-peaked spectra lies in the fact that a) they might represent reality much more closely for some situations and b) they contain energy in very different frequencies than their single-peaked counterparts. For a dynamic analysis, this could make a big difference! Especially calmer sea states often contain a distinct swell/other second component. Here, the Torsethaugen spectrum offers a solution. Since no pipe installations would ever be carried out in storm-type conditions, we quickly see the added value of this type of spectrum.

One commonly used spectrum to account for local waves as well as swell is the Torsethaugen two-peak spectrum. It was developed for Norwegian waters and is characterized by five parameters; $H_T$, $T_p$, $\gamma$, $N$ and $M$. All of these are dependent on $H_s$ and $T_p$, and determined by means of regression analysis and curve fitting to collected in-situ data. We will use a simplified version of the original formulation of Torsethaugen's spectrum, in line with DNV recommended practices [18] and Torsethaugen & Haver [20]. The parameters $M$ and $N$ are then taken as constants (both have value 4) and do not return in the final equations. It will be clear from the following that even a simplified version of the spectrum is much more complicated than the single-peak spectra we have considered so far. Part of the reason is that the available model relies heavily on empirical techniques. This is where a computer tool excels of course, since all of the complications may be automated in a robust
The model is fully developed in Appendix C. To get an impression, a typical Torsethaugen spectrum looks like Figure 4.6.

One note of caution is necessary. The Torsethaugen spectrum assumes both wave systems to approach from the same incidence angle, and as such might not always approximate reality to a sufficient degree. Often times, swell waves come from a very different angle than the wind-generated ones. The combination of two single-peaked spectra, with different incidence angles is the best solution in such cases. See also section 4.4.

### 4.4 Directional Spectra

The wave energy spectra we have seen so far are just one dimensional in nature; meaning the value of the parametric spectrum function depends on just one variable (cyclic frequency). In real life, this is not the case. Looking out over the open ocean (especially at calm sea states) from a certain point, one would conclude that the prevailing wave pattern is, to a certain extent, also depending on the radial direction you are looking at the horizontal (water surface) plane. Another way of realizing this must be correct is looking out over the open ocean and finding at least one pair of waves that intersect each other at more or less perpendicular angles. Such a pair cannot be explained by the 1-dimensional theory explained thus far.

This dependence is captured in so-called *directional* wave spectra, of which a representative example is depicted here.
Collins, [21], showed in his paper that the effects of directional wave spreading might account for some 20% overestimation of the significant wave height during refraction\(^ {13} \) events when not taken into account. Forristall [22] showed that directionality must surely be taken into account when considering storm-wave flow velocities. Not doing so means over predicting the velocities (which is not necessarily a bad thing since it would lead to more conservative/safer designs). Furthermore, as many companies specify a certain measure of wave directionality when reporting on metocean conditions, the need to have a model for it becomes clear even without these two very good arguments. Another very good reason for directional spectra is that so-called confused seas, the prevailing sea-state during calm weather (when virtually all pipe installation operations will take place), where no single prevailing direction for waves can be found, require directional spreading to be modelled correctly. The energy added to the system by the wave components not in the main direction might induce dynamic responses very much different from the ones caused by waves from the ‘zero’-direction.

There are several equations used to describe directional spectra. They are all independent of the spectrum at hand, since the basic premise is to multiply the one-dimensional spectrum with a function ‘\( D \)’, which depends on both radial direction \( \theta \) and frequency \( f \) (and sometimes on direction only). Mathematically, this becomes [23] (page 2):

\[ E(f, \theta) = E(f) \cdot D(f, \theta) \]  

… where

\( E(f, \theta) = \) Directional Spectral Density Function  
\( E(f) = \) One-Dimensional Spectral Density Function  
\( D(f, \theta) = \) Angular Spreading Function  
\( f = \) Frequency \([Hz]\)  
\( \theta = \) Direction \([rad]\)

\(^{13}\) Refraction; the phenomenon where an initially straight wave front changes speed non-uniformly over its breadth due to shoaling water ways, thereby aligning itself with an approaching coastal profile.
The important thing here is not to forget that the integral over all frequencies of the wave-spectrum represents wave energy. Since it would obviously violate the conservation of energy principle to create energy out of thin air, we need to make sure that the directional spectrum contains no more (or less) of it than the original non-directional one. This means that [23] (page 3):

\[
\int_{0}^{\infty} E(f) \, df = \int_{0}^{\infty} \int_{-\pi}^{\pi} E(f) \cdot D(f, \theta) \, d\theta \, df
\] (15)

For the spreading function, several options are found in the literature, though all are a bit speculative in nature ([16]; page 164). Probably the simplest reads [23] (page 3):

\[
D(\theta) = \begin{cases} 
\frac{2\pi}{\cos^2(\theta - \theta_0)}; & |\theta - \theta_0| < \frac{\pi}{2} \\
0; & \text{else}
\end{cases}
\] (16)

This function is independent of frequency but is used a lot. It is defined with the mean wave direction \(\theta_0\) set at the mean wave direction. Different formulations of the above can be formulated, as long as the total wave energy stays the same! We will use a slightly adjusted version since it was recommended in the client’s original metocean assessment. It reads

\[
D(\theta) = \begin{cases} 
\kappa \cos^n(\theta - \theta_0); & |\theta - \theta_0| < \frac{\pi}{2} \\
0; & \text{else}
\end{cases}
\] (17)

We take \(n\) to be either two or four, and determine \(\kappa\) so that it meets our energy requirement. This results in us differentiating between two cases; \(n = 1, 2\). From equation (15), it is obvious that we require

\[
\int_{-\pi/2}^{\pi/2} D(\theta) \, d\theta = 1
\] (18)

… in order to meet the energy requirement. Since

\[
\kappa \int_{-\pi/2}^{\pi/2} \cos^2(\theta) \, d\theta = \frac{\pi}{2}
\] (19)

… we conclude that

\[
\Rightarrow \kappa = \frac{2}{\pi} \quad \text{(for } n = 2)\]

(20)

Just as was found in equation (16). Moving on with the case for \(n = 4\), we of course still require equation (18) to hold. We get:

\[
\kappa \int_{-\pi/2}^{\pi/2} \cos^4(\theta) \, d\theta = \frac{3\pi}{8}
\] (21)
Some more complicated spreading functions exist as well. One of these, proposed by Longuet-Higgins, et al. [23] (page 3) incorporates a spreading parameter \( s \), which allows for more or less directionally ‘focused’ models. It reads:

\[
D(\theta) = \left( \frac{2^{(2s-1)}}{\pi} \right) \left( \frac{\Gamma^2(s + 1)}{\Gamma(2s + 1)} \right) \cos^{2s} \left( \frac{\theta - \theta_0}{2} \right)
\]

... where

\[\Gamma(s) = \text{Gamma function}^{14} \]
\[s = \text{spreading parameter} [-]\]

The integral over the interval \(-\pi \leq \theta \leq \pi\) for this function is, by mathematical design, always unity. A very extensive and mathematically thorough reference on directional spectra and wave climates as stochastic functions in general is [24] (chapter 2.5).

4.4.1 On the Spreading Parameter \( s \)

Several sources give guidance on the use of the spreading parameter ‘\( s \)’ for use in equation (23). A smaller value (maybe 2 or 4) corresponds to increasingly random seas (broader spectra); while larger values of up to 40 or so approach a one-dimensional (i.e., a much more focused) spectrum. When dealing with real analyses, the following guidelines are available for ‘good practice’. [19] (page 2-105) Recommends the following:

“For simple analysis, where a constant value of \( s \), independent of frequency is used, the value \( s = 10 \) is recommended […].”

Furthermore, the American Petroleum Institute (API) recommends [19] (page 2-105):

“In API RP2A (1993) the commentary on fatigue analysis suggests values of \( n \) of 2 for wind-driven seas or 4 for limited-fetch situations (which corresponds to values of \( s \) of 4 and 8 approximately).”

As a last reference, [18] (page 35) state:

“[…] typical values for wind sea are \( s = 4 \) to \( s = 9 \). If used for swell, \( s > 13 \) is more appropriate.

4.5 Geographical Areas of Applicability of Standard Wave Variance Density Spectra

Guidelines for what Spectra to use are available in Offshore Engineering Handbooks. For example, [25] (page 112) recommends the following:

\[\Gamma(n) \triangleq (n - 1)!\]

---

14 Gamma function, defined as [44]: \( \Gamma(n) = (n - 1)! \)
<table>
<thead>
<tr>
<th>Location</th>
<th>Operational</th>
<th>Survival</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gulf of Mexico</td>
<td>Pierson-Moskowitz</td>
<td>P-M or JONSWAP</td>
</tr>
<tr>
<td>North Sea</td>
<td>JONSWAP</td>
<td>JONSWAP</td>
</tr>
<tr>
<td>Northern North Sea</td>
<td>JONSWAP</td>
<td>JONSWAP</td>
</tr>
<tr>
<td>Offshore Brazil</td>
<td>Pierson-Moskowitz</td>
<td>P-M or JONSWAP</td>
</tr>
<tr>
<td>Western Australia</td>
<td>Pierson-Moskowitz</td>
<td>Pierson-Moskowitz</td>
</tr>
<tr>
<td>Offshore Newfoundland</td>
<td>Pierson-Moskowitz</td>
<td>P-M or JONSWAP</td>
</tr>
<tr>
<td>West Africa</td>
<td>Pierson-Moskowitz</td>
<td>Pierson-Moskowitz</td>
</tr>
</tbody>
</table>

Where it should be noted that by ‘operational’, calmer sea states are meant. This is the kind of environment typical for fatigue analyses. However, since both the Pierson-Moskowitz and the JONSWAP spectrum are simple one-peak spectra, their applicability could be called into question for confused (calm) seas. The Torsethaugen spectrum probably approximates calm seas better, in this respect. See section 4.3.4.

For survival conditions (very violent wave climates), the one-peaked spectra gain much in accuracy and become an excellent (and very much industry accepted) choice.

Furthermore, for values of $\gamma$ used by the JONSWAP model; the following recommendations are made [25] (page 112):

<table>
<thead>
<tr>
<th>Location</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>North Sea or North Atlantic</td>
<td>3.3</td>
</tr>
<tr>
<td>Northern North Sea</td>
<td>Up to 7</td>
</tr>
<tr>
<td>Offshore West Africa</td>
<td>1.5 ± .5</td>
</tr>
<tr>
<td>Gulf of Mexico</td>
<td>1 for $H_s \leq 6.5$ m</td>
</tr>
<tr>
<td></td>
<td>2 for $H_s &gt; 6.5$ m</td>
</tr>
<tr>
<td>Offshore Brazil</td>
<td>1-2</td>
</tr>
</tbody>
</table>

When considering the guidelines given in section 4.3.3 regarding the $\gamma$ parameter, it is good to remember that values mentioned in the latter paragraph are intended for use when no site-specific location is available [18] (page 34, section 3.5.5.5). Specific data for a given location is to be given preference over these general rules, then.

### 4.6 Spectra and Statistics – Mathematical Moments

Mathematical moments are a very useful tool when dealing with statistics. They define some of the most often considered probability parameters (mean, variance, etc. [26]) in terms of the Probability Density Functions of the stochastic variable under consideration in a very accessible manner. When considering Wave Variance Density Spectra, moments can do the same; making the link between them and the world of statistics. Here, the concept is used to check if our spectrum has been well-defined and whether the numerics work out. The $n$-th order moment $m_n$ is defined as [7] (page 5-41):

$$m_n \triangleq \int_0^\infty \omega^n \cdot S_\xi(\omega) \ d\omega$$

The most important relations between moments and statistics are the ones between moments and variance of the water surface elevation, the significant wave height, mean centroid wave-period ($T_1$) and mean zero-crossing wave period ($T_2$). They are, in this order [7] (page 5-42):
\[ \sigma^2 = \sqrt{m_0 \zeta} \quad (25) \]

\[ H_s = 4 \sqrt{m_0 \zeta} \quad (26) \]

\[ T_1 = 2\pi \frac{m_0 \zeta}{\sqrt{m_1 \zeta}} \quad (27) \]

\[ T_2 = 2\pi \frac{m_0 \zeta}{\sqrt{m_2 \zeta}} \quad (28) \]

The concept of moments of spectra is used in several sections of this text, mainly for verification purposes. Especially sections of chapter 8 rely on the idea.
5 Environmental & Operational Influences

5.1 Currents & the Morison Equation

Currents can play an important role in fatigue loading cases. The UK Health & Safety Executive (1995) recommends that current should be taken into account if the current magnitude is comparable with the wave orbital velocity for those waves that make the greatest contribution to the fatigue damage [19]. It is further noted that currents will “generally be more important for small diameter members”, such as production risers or TLP tethers. We will take current into account for our model, and the following sub-sections (page 2-85).

5.1.1 Hydrodynamic Loads on Slender Pipes

Currents acting on a slender pipe (or more generally speaking: fluid flows perpendicular to the axis of a cylinder) have been studied extensively, not least because it has potentially large influence on the dynamics and fatigue life of structures as important and widely used as offshore risers. The results are still inconclusive in an exact, physical, sense of the word, but some very good engineering approximations have been made. An example of such an approximation is the drag formula, which calculates the (average) force exerted on a cylinder of ‘infinite’ length per meter, due to a fluid-flow perpendicular to the axis of the cylinder. It reads [27]:

\[ F_{\text{drag}}(t) = \frac{1}{2} \rho C_D D^2 \cdot u(t)^2 \]  \hspace{1cm} (29)

... where

\[ F_{\text{drag}} = \text{Force on cylinder per meter length} \left[ \frac{N}{m} \right] \]
\[ C_D = \text{Drag coefficient, subject to further investigation} \left[ - \right] \]
\[ D = \text{Diameter of cylinder} \left[ m \right] \]
\[ u(t) = \text{Time-dependent speed of fluid moving past} \left[ \frac{m}{s} \right] \]

The classical drag equation works best for stationary flows (no accelerations of the fluid flow) and predicts the viscous component of the drag force. It is the basis for a more complete solution, which takes into account not just the viscous friction, but also the inertia term, which is predicted by potential flow theory. This second equation, named after its inventor, adds two hydrodynamic force terms to get one answer for the total drag force on slim cylinders subjected to oscillating flows. It is, of course, the Morison equation, known and used by offshore engineers across the globe. It reads [7] (page 12-8):

\[ F_{\text{Mor.}}(t) = \frac{\pi}{4} \rho C_M D^2 \cdot \frac{du(t)}{dt} + \frac{1}{2} \rho C_D D \cdot u(t) \left| u(t) \right| \] \hspace{1cm} (30)

... where

\[ C_M = \text{Inertia coefficient, subject to further investigation} \left[ - \right] \]
\[ C_D = \text{Drag coefficient, subject to further investigation} \left[ - \right] \]
\[ D = \text{Diameter of Cylinder} \left[ m \right] \]

The first term is the inertia term, accounting for the inertia forces the moving water particles exert on the structure because they are forced to divert away from it. It is proportional to the time-derivative of speed of the water particles, and therefore, of their acceleration.
The second term then, is the classical drag equation with one small alteration. Since the equation is supposed to work for alternating flows, negative forces (in opposite to the positive direction) must be accounted for. This in turn is achieved by taking the product $u(t)|u(t)|$, rather than the simple square of velocity. This works because the original sign of $u(t)$ is preserved.

It is worth noting that some areas are particularly prone to large currents, or currents with a very predominant direction, due to bathymetry\textsuperscript{15}.

### 5.1.2 Determining the $C_M$ and $C_D$ coefficients

The two parameters used in determining the two components of the Morison force are determined experimentally, and the final result of these experiments may, according to [7] (page 12-17), be summarized as follows:

<table>
<thead>
<tr>
<th>$KC$</th>
<th>$C_D$</th>
<th>$C_M$</th>
<th>$C_D$</th>
<th>$C_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt;10$</td>
<td>1.2</td>
<td>2.0</td>
<td>0.6</td>
<td>2.0</td>
</tr>
<tr>
<td>$\geq10$</td>
<td>1.2</td>
<td>1.5</td>
<td>0.6</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 5.1 - $C_D$ and $C_M$ coefficients as function of Reynolds number and KC-number according to [7] (page 12-17).

\[ KC \triangleq \frac{2\pi \zeta_a}{D} \quad \text{(Deep water}\textsuperscript{16} \text{ only)} \]  

\[ KC \triangleq \frac{\dot{r}_m T}{D} \quad \text{(Lower part of risers in deep water)} \]

\[ \zeta_a = \text{Wave amplitude [m]} \]
\[ D = \text{Cylinder diameter [m]} \]

This definition makes sense for surface waters, where there indeed is a wave (i.e., a $\zeta_a$) to influence the TCP. For much larger depths, the surface waves have virtually no direct effect on the lower parts of the riser. For these cases, it is important to realize that even if the flow does not exhibit oscillatory behaviour, the motion of the TCP itself might. In essence, the relative motion between the water particles and the TCP are what’s important. DNV then recommends the following formulation for the KC-number for an oscillating structure in still water [18] (paragraph 6.6.1.6):

\[ KC \triangleq \frac{\dot{r}_m T}{D} \quad \text{(Lower part of risers in deep water)} \]  

\[ \zeta_a = \text{Wave amplitude [m]} \]
\[ D = \text{Cylinder diameter [m]} \]

\textsuperscript{15} Bathymetry; the geographical/topological layout of the ocean floor.

\textsuperscript{16} Deep water: In offshore engineering terms, water is considered to be ‘deep’ if the water depth $h$ is more than half the wave length $\lambda$ [6] (page 5-2). This condition is fulfilled quite quickly as typically, the dominant ocean waves (the ones with some serious energy in the system) seldom exceed some 200 meters (this would be a value for storm waves) in length and we will work with the assumption that our waters are in fact ‘deep’, unless stated otherwise.
\[ \dot{r}_m = \text{Maximum velocity of the riser (DNV notation)} \quad [m/s] \]
\[ T = \text{Period of oscillation} \quad [s] \]
\[ D = \text{Cylinder diameter} \quad [m] \]

Re is the well-known Reynolds number, defined simply as \([7]\) (page 12-14):

\[ Re \triangleq \frac{\nu D}{\nu} \quad (33) \]

... where

\[ \nu = \text{Total flow velocity} \quad [m/s] \]
\[ \nu = \text{Kinematic fluid viscosity} \quad [m^2/s] \]

Note that for seawater of typical salinity of 35 PSU\(^{17}\), the kinematic viscosity is in the range of \(0.80 \times 10^{-6}\) to \(1.80 \times 10^{-6}\) \([m^2/s]\) (see appendix B2).

From this discussion it should be clear that for a proper analysis of all parameters involved, we might have to update the \(C_D\) and \(C_M\) parameters during the process. However, when looking at some typical parameters for our model, we see that our Reynolds number is practically always above \(10^5\), while our KC-number practically always is below 10, which sets our \(C_D\) and \(C_M\) at and \(\frac{\nu}{\nu}\), respectively. This may be substantiated as follows. We have the following very representative geometry/values:

\[ D = 0.15 \quad [m] \]
\[ \nu = 1.35 \times 10^{-6} \quad [Pa \cdot s] \]
\[ \nu = 0.1 \ldots 2.5 \quad [m/s] \]

... where \(\nu\) is the relative velocity between the fluid and the submerged pipe. That is, the vector sum of water particle speed and TCP movement. This value is our Ansatz, but more than 2.5 meters per second of relative velocity seems unlikely, since maximum current speeds for our reference case do not appear to exceed some 1.5 meter per second at the surface (see section 8) and movement of the TCP does not appear to exceed 1 meter per second from test runs performed. Using these values, we get Reynolds numbers between approximately \(1.1 \times 10^4\) and \(2.8 \times 10^5\). The critical flipping point, where the Reynolds-number first exceeds \(10^5\), and thereby moves from the first two column to the latter two in Table 5.1, is at approximately \(\nu = 0.9 \quad [m/s]\). This is a low enough number, that we will assume it to always be exceeded in our model, partly also because the nature of the model forces us to make a choice. The main reason why this assumption is necessary, is that within Abaqus, updating the \(C_M/C_D\) values during an analysis presents a formidable challenge. Please do note, that for \(C_M\), the difference does not matter (it is valued at the same numerical value, for all \(0 < Re < \infty\) for constant KC-number according to Table 5.1). For current around 1 meter per second, we are nearly certain to exceed the 0.9 m/s threshold that would validate the choice made here.

Next, we require a prediction of the KC-number. As mentioned in the last paragraph, we exclude significant wave heights larger than 4 meters from the current discussion. From (confidential) metocean data available to the author, it is concluded that, for parameters chosen within a fair range of probability of occurrence belonging to significant wave heights of 0 to 4 meters, the corresponding peak periods \(T_p\), should have a range of approximately 6 to 16 seconds. When considering JONSWAP spectra with these values, we can determine the amplitudes corresponding to these peak periods and call them ‘peak amplitudes’. We feel confident in taking these values as ‘representative’ or perhaps ‘dominant’, because the values for \(\zeta_i\) belonging to the peak-period values in the VDS bring with them the most energy out of all wave components. The assumption that these will contribute most to the system response is one made by, for example, DNV as well in their discussion on structural damping [28] (page 65).

\(^{17}\) PSU; Practical Salinity Units, measure of the salinity of seawater.
The values found above for the wave component amplitudes mean, by equation (31), values for KC of between approximately

\[ 7.25 < KC < 155 \]

Thanks due to both rather large amplitudes in the upper limit for \( \zeta_i \) and a rather thin pipe. We feel confident in saying that we exceed a KC-number of 10 for this specific (yet representative) reference model and conclude that:

\[
\begin{align*}
C_M &= 0.6 [-] \\
C_D &= 1.5 [-]
\end{align*}
\]  

(34)

When radically changing geometries of the model, it would be a good idea to recalculate these values, and point for improvement in future versions of the model might be a true update of these values per section of pipe per time step.

5.1.3 Types of current and current profiles

Actual current profiles are not easily predicted because sub-surface current measuring is not an easy task, and up to some few short years ago, it was generally believed that hardly any current existed below some 1000 meters. Nowadays, this is known not to be the case. Though currents on the surface are mainly generated by wind affecting the surface waters, tides and variations of atmospheric pressure; different phenomena do exist which affect currents up to very large depths and also the region near the ocean floor [25] (page 118). Phenomena which generate currents may include such diverse phenomena as:

- \textit{Wind-generated} currents (Currents caused by wind shear at the water surface.)
- \textit{Tidal} currents (Caused by regular astronomical tides. Weak in deep water, but not near shorelines.)
- **Circulation currents**, loop currents (Caused by natural oceanic circulation and eddies\(^{18}\), respectively.)
- **Soliton currents** (caused by sub-surface waves due to variations in water-density.)

In our model, we assume a current profile consisting of a tidal/circulation component. We adhere to DNV’s Recommended Practice C205 (Environmental Conditions and Loads) [18]. In this text, paragraph 4.1.4, we read: “When detailed field measurements are not available, the variation in […] tidal current velocity with depth may be modelled as a simple power law, assuming uni-directional current” and write for the tidal/circulation current speed:

\[
\vec{v}_{c,tide}(z) = \vec{v}_{c,tide}(0) \left( \frac{d + z}{d} \right)^\alpha \quad \text{for } z \leq 0
\]  

(35)

… where

\[
\vec{v}_{c,tide}(0) = \text{Tidal current velocity at the still water level [m/s]}
\]
\[
d = \text{Water depth to still water level [m]}
\]
\[
z = \text{distance from still water level, positive upward [m]}
\]
\[
\alpha = \text{Exponent (typically 1/7) [-]}
\]

In these and the following equation, note the use of the vector notation, \(\vec{v}\). This we do to drive home the fact that currents are not just a function of depth, but also of radial incoming direction. In our final model, this has to (and will) be taken into account.

Another component often found in currents, but not currently included in the final tool, is the surface wind-generated current component, for which we may write [18]:

\[
\vec{v}_{c,wind}(z) = \vec{v}_{c,wind}(0) \left( \frac{d_0 + z}{d_0} \right) \quad \text{for } -d_0 \leq z \leq 0
\]  

(36)

… where

\[
\vec{v}_{c,wind}(0) = k \cdot \vec{U}_{1h,10m}
\]
\[
k = 0.15 - 0.30
\]
\[
\vec{U}_{1h,10m} = \text{1 hour sustained wind speed at height 10m above sea level}
\]
\[
d_0 = \text{Reference depth for wind generated current; } d_0 = 50m
\]

… And sustained wind speed is defined as the one-hour average wind-speed, in this case at 10 meter above sea level (wind speed is a function of height above ground level).

Now, we could say that the combined current speed is the vector sum of both components:

\[
\vec{v}_c(z) = \vec{v}_{c,tide}(z) + \vec{v}_{c,wind}(z)
\]  

(37)

This would be the total (vector) current experienced by the TCP.

Strictly speaking, combining currents and waves would have to lead to corrections being made for orbital water particle movement. See [18], page 54 onwards. This in turn might influence the loads near the surface,

---

\(^{18}\) Eddies; “The swirling of a fluid and the reverse current created when the fluid flows past an obstacle” [Wikipedia]. Large ‘chunks’ of flow break of from the main oceanic circulation and form large (or small!) rotating fluid-flow systems on their own, which may persist for months.
where wave induced water-particle movement is relevant. This might be an interesting question for further research but is not included in the final version of our tool.

5.2 Waves

Waves are responsible for all motion of the ‘vessel’ being simulated. The description of wave climates has been completed in earlier chapters, but one aspect has not. It is the fact that waves move around water particles in orbital paths, which influences the dynamic behaviour of the pipe in the surf zone, down to a few meters of water depth. According to Airy wave theory, the horizontal part of orbital velocity of a water particle in a single harmonic wave adheres to [7] (page 5-14):

\[
 u(t, z, \omega) = \zeta \omega \cdot \exp(k \cdot z) \cdot \cos(-\omega t)
\]

(38)

… where

\[
 k = \frac{\omega^2}{g} \text{ (deep water)}
\]

\[
 z = \text{depth coordinate, negative downwards}
\]

From this equation, using a formulation of the Morison equation, the influence of orbital velocities of water particles due to waves on the TCP might be calculated. However, these equations are based on free-streaming water. For the current considerations, waves might be coming from any and all directions and \textit{meet the vessel tensioning the TCP}. The effect the vessel has on the already very complex kinematics of the water particles is very hard to predict. Perhaps standing waves are generated. What happens if the waves hit starboard but the TCP is strung from the portside? For this thesis, it was decided to not take the effect of these orbital velocities into account due to the very complex nature of the problem.

5.3 Internal Pressure & Flow

Internal flow is a difficult subject that changes the fundamental equations of motion governing the downline’s vibrations in several ways. When one has to assume unstable flow regimes (such as would be the case when transporting both oil and gas/any multi-phase product), the influence becomes even more intricate (and therefore, much more difficult to simulate). In this thesis, fluid flow will be taken into account, and its basic effects on the dynamics of a hanging pipe will be presented in this section. We will restrict our discussion to steady internal single-phase pressure/flows. Strictly speaking, the following three terms appear in the equation of transversal motion for a cantilevered pipe conveying fluid of mass per meter \( M \) with velocity \( U \) [29] (page 60):

\[
 MU^2 \frac{\partial^2 w}{\partial x^2} + 2MU \frac{\partial^2 w}{\partial x \partial t} + M \frac{\partial^2 w}{\partial t^2}
\]

(39)

… where \( x \) is the axial coordinate along the main axis of the pipe and \( w(x, t) \) is the transversal displacement function perpendicular to the main axis. The first term represents the flexural restoring force of the traveling fluid on bent section of pipe. The second term is associated with centrifugal forces arising from the fluid flowing in curved parts, and the last term is associated with Coriolis forces; or with rotations of the earth while the contents of the pipe attempt to travel in a straight line. However, these terms are not considered further in this thesis.

What is considered is that internal pressure has the effect of changing the ‘observed’ tension in a pipeline. The resulting ‘new’ tension, called ‘effective tension’, is defined as:
\[ T_{\text{eff}} = T_w - P_i A_i + P_e A_e \]

... where

\[
\begin{align*}
T_{\text{eff}} & = \text{Effective Tension [N]} \\
T_w & = \text{Wall Tension ('normal' tensile forces; } \sigma A) [N] \\
P_i A_i & = \text{Internal pressure times internal surface area per meter length [N]} \\
P_e A_e & = \text{External pressure times external surface area per meter length [N]}
\end{align*}
\]

For phenomena like Euler buckling of the pipeline (like a beam in compression), this effective tension is the governing parameter; if it becomes negative, the pipe will buckle. Some design codes (e.g., API RP1111 [1998]) in fact specify allowable design stresses in terms of this effective pressure. The physical effect of internal pressure is that the pipe will expand radially. Due to the Poisson’s ratio however, this means that the pressurized segment will also decrease in length. This is the intuitive logic behind the fact that the effective tension in fact is lower than the ‘regular’ wall tension (for larger internal than external pressure). The big side note must be made in this specific case, however, that this kind of behaviour is not guaranteed to be directly occurring for composite products. This will depend on the exact lay-up of the fibres within the TCP and would ideally have to be analysed on a case-by-case basis.

Paidoussis [29] (page 325) even goes so far as to comments (on the stability of deep-water risers):

“Because of their great lengths, measured in kilometres, flexible risers may generally be considered to be hoses or strings, pipe-strings, thus neglecting flexural restoring forces. [...] they are like any other strings, a limp strand of spaghetti, the configuration of which is solely determined by the imposed tension [...] internal and external pressure, gravity and internal flow effects.”

The clear message is that (effective) tension is major contributor to the (dynamic) behaviour of a downline and a good model should include provisions for this phenomenon. Abaqus provides a good tool by means of specially formulated ‘pipe’ elements that allow for internal pressures and the calculations of effective tensions.

5.4 Material Damping

In order to more accurately describe the behaviour of our TCP, one could consider a material-damping model. After all, it seems very plausible that the thick-walled composite structure dissipates some energy when forced to oscillate, as all materials do to some extent. The main question then obviously becomes: how much exactly? In the flexible riser market, Airborne’s TCP is a special product, in that it is a bonded flexible. Most competitors supply unbonded variants, which consist of several layers of material to carry out specific tasks such as pressure containment or tensile strengthening. Complicated internal damping behaviour is a direct consequence of this construction, based on the idea that for certain modes of movement (such as bending), the inner layers start moving relatively to each other. This causes a lot of dry (Coulomb) friction between the layers themselves [30]. Significant research has been carried out to determine the damping parameters for such pipes, and Larsen [31] concludes that the difference in damping models/parameters used is one of the big factors contributing to difference in results for dynamic analyses of (idealized, constant engineering-parameter) flexible risers.

The general consensus is to use modal damping (also known as Rayleigh damping) to account for structural dissipation [31] [28]. Rayleigh damping has several advantages, most of which are of computational nature. The model calls for two damping parameters, \( \alpha \) and \( \beta \), which represent the so-called ‘mass proportional’ and ‘stiffness proportional’ damping coefficients respectively. The damping coefficient \( \xi \) is then calculated as follows:

\[
\xi(\omega) = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2}
\]
... Where usually, we neglect the mass proportional damping term (the first term in equation (41)) because this term would give us damping for the rigid body modes of movement as well. In these modes, where there is no deformation of the object under consideration, material damping should equate to zero [28]. A value for the damping ‘\(\beta\)’ of ‘5% at the dominant frequency’ is often called for [32] [31], but without substantial justification.

For the case of bonded flexible risers, no experimental data for material damping behaviour was found during a literature review. It is a novel field of science and engineering, and as such warrants extensive research on its own. Paidoussis [29] (pages 119 onwards) makes some interesting comments regarding the dynamic stability of pipes conveying fluid, with and without material damping by the way, but the scope of that text is beyond this thesis.

A lot of research has been carried out on damping of simple composite structures (beams), with simple composite lay-ups (unidirectional, for example), but the authors of these papers note the complex relationships the damping behaviour exhibits. For example, temperature and humidity both affect the damping behaviour of the material. The damping parameters for such simple parts are found to be very small for modes in direction of the fibres, and slightly less small in transversal modes [33]. However, all of this unfortunately still tells us very little about the kind of damping behaviour we can expect for the complex product at hand.

The option to add Rayleigh damping has been included in the final tool, but the two corresponding parameters are set to zero by default, and the option disabled as a whole if no parameters are specified to preserve computing capacity. A proper (experimental) analysis of the material damping properties would be a nice follow-up research question. Alternatively, one could (for the sake of having a model of any kind at all, although this is hardly a scientific notion) abide by the ‘5% at dominant frequency/first eigenmode rule’ by having Abaqus calculate the first flexural/most relevant eigenfrequency and tune the damping parameter \(\beta\) to evaluate to 0.05 at this frequency. Note that the eigenfrequency should be calculated at the effective tension in this case, rather than the static one. This tension varies along the length of a long pipe, further complicating matters.

As a last remark, the user might be tempted to include some artificial material damping in order to suppress high-frequency ‘noise’ in the results. Such non-physical oscillations could be the result of discretization and do not represent anything ‘real’. However, it has been demonstrated that this strategy (adding Rayleigh damping

![Figure 5.2 - Example of an unbonded flexible riser. The different inner layers that cause so much friction are clearly visible and labeled. Source: http://fps.nov.com/subsea/flexibles/dynamic-flexible-risers (National Oilwell Varco).](image)
to suppress the noise) mainly damps out the ‘middle’ modes, leaving response in the low frequency modes and high frequency modes intact. It is suggested to use ‘numerical damping’ (i.e., damping inherent to the integration algorithm) instead [34] (page 420). Abaqus/Implicit (as used for our analyses) provides a certain degree of numerical damping by default [35] (section 6.3.2).
6 A Simplified Fatigue Model

6.1.1 Motivation

While the results from our analyses are primarily displacements, stresses and strains that will be used by AOG in a complex finite element model that incorporates composite plies and advanced material non-linearities and failure mechanisms, for this thesis the interpretation of results will be of a more basic nature. The idea is to develop a general fatigue model for a fictional material with fatigue characteristics resembling those of simple construction steels. We will construct a so-called S-N curve for this material (also called a Wöhler curve), taking into account the significant constant stresses that are present, and determine a ‘fatigue life’ for the TCP under specific conditions. We use quotes around the words fatigue life to indicate that it is not an actual tangible result that could be copied one-to-one into a design report. The reason for this, is that we will be using the stresses acquired from our Abaqus analyses (which are based on engineering parameters and dimensions of the functional composite TCP) but couple these to fatigue parameters associated with our fictional material. This is done for several reasons. First of all, parts of the theory behind these steps could be adopted/extended for use with composites. Secondly, setups like the ones we have for this project (Figure 1.3) are not unheard of in the world of steel tubing. A good example of a metal ‘equivalent’ for TCP is so-called (Steel) Coiled Tubing, used for, among others, interventions in the offshore oil and gas industry [36]. Coiled tubing is a long length of continuous steel pipe spooled onto a drum on which it may readily be transported (offshore). Once on the worksite, it is reeled off and straightened to be used. The idea, from a concept/application point-of-view then, is not so very different from the TCP concept and we see the relevance of the proposed fatigue model. Fatigue analyses could be carried out on this coiled tubing in the very same manner as done for TCP using the tools designed alongside this thesis. Merely the geometries and engineering parameters would have to be adjusted.

As mentioned in the previous paragraph, a non-existent material resembling steel will be considered for fatigue analyses. While the original idea was to use actual steel properties instead of ‘designing’ a new material to compare results against one another here, steel is simply too resilient to be damaged in fatigue with the results acquired from the Abaqus analyses carried out using TCP engineering parameters and geometries. Because of the comparatively low Young’s modulus of TCP, resulting stresses are too low to damage actual steel in fatigue. This means, that while the following theories concerning Wöhler curves, the Palmgren-Miner rule and Rainflow counting are all very valid engineering theories with real-world applications and successes, the numerical results arising from their application using the proposed fictional material are slightly artificial and are intended as a proof of concept and a means of quantitative comparison rather than a set of directly applicable data.

We will be using the Miner’s rule for additive fatigue damage, while adjusting the curves in the S-N diagrams to take into account the mean (tensile) stresses. These are deemed important because mean tensile stresses are thought to open up (micro)cracks that may then easily propagate, develop into macro-cracks, and eventually lead to failure of the component. This means that increased positive (tensile) mean stresses shorten fatigue life. Mean negative (compressive) stresses are thought to have the reverse effect, effectively increasing fatigue life [37] (page 151). We will furthermore use an equivalent-stress fatigue theory and apply the found equivalent stresses to a uni-axial S-N curve. The additional comment must be made here that this is deemed appropriate, because the principal stresses from results were checked against one another and found to be orders of magnitude apart. The first principal stress thereby dominates the (fatigue) response. The equivalent-stress approach mentioned, will be the Maximum Principal Stress Theory [37] (page 171). See section 6.1.4 for more details. This approach is in contrast to more advanced methods, such as the Path Dependant Maximum Range algorithm [38] that takes into account the contributions from several stress components separately, as well as their mutual phase differences. Such a contribution to this work, while likely valuable, is left to others.
6.1.2 Constructing the Full-Reversal S-N Curve

The basis for the proposed fatigue model is an S-N curve, also known as a Wöhler curve. It plots for each constant stress amplitude the number of cycles until theoretical fatigue-induced failure. The confidence flowing from such plots is based on experimental research, and the curves as such do not convey absolute truths but rather the expected amount of cycles to failure, in a probabilistic sense. Such a curve may be constructed for comparatively simple (homogeneous) materials in the following manner.

The basis for the design of this fatigue model will be a non-existent material with ultimate strength equal to that of steel grade API X42 and endurance strength close to half that of its real-life counterpart. X42 is an American designation for a standard grade of steel, set forth by the American Petroleum Institute. The comparable European standard, according to [39], would be grade S275. From [40] (Table 3-1), we find the following values for S275’s ultimate tensile strength $\sigma_{u, S275}$ and endurance strength $\sigma_{e, S275}$:

$$\begin{align*}
\sigma_{u, S275} &= 430 \text{ [MPa]} \\
\sigma_{e, S275} &= 170 \text{ [MPa]}
\end{align*}$$

(42)

Our fictional material will maintain the value for ultimate stress and have an adjusted endurance strength: This adjustment makes the material more susceptible to onset of fatigue, thereby ensuring actual fatigue damage.

$$\begin{align*}
\sigma_{u, mat} &= 430 \text{ [MPa]} \\
\sigma_{e, mat} &= 50 \text{ [MPa]}
\end{align*}$$

(43)

... where subscript $mat$ stands simply for ‘material’ and will be the notation used for our fictional design-material. Now, from [41] (page 144), we adopt the Basquin relation:

$$\sigma_{f, N}^k N = \text{constant}$$

(44)

... where

$$\begin{align*}
\sigma_{f, N} &= \text{Fatigue stress amplitude at N cycles [MPa]} \\
N &= \text{Number of cycles to failure [-]} \\
k &= \text{coefficient determining slope of S-N Curve [-]}
\end{align*}$$

(45)

The Basquin relation states that the S-N curve evaluates to a straight line when plotted on a log-log scale for a large range of cycles $N$. The slope of this curve then equals $-1/k$. In this simplified fatigue model, we assume the Basquin relation to hold from $N = 10^1$ to $N = 10^6$. $N = 10^6 - 10^7$ is often agreed to be the number of cycles where the infinite life regime starts [42] (page 61); [37] (page 119). This is the number of cycles corresponding to the above mentioned endurance strength $\sigma_e$ mentioned in equation (42). The consequence of this is, that we do not expect any cases of fatigue failure to occur once $N = 10^6$ cycles have been completed without fatigue failures. Since it is established that we are looking for a straight line in the S-N curve, only one more point is required to be evaluated besides the fatigue limit $\sigma_e$ at $N = 10^6$. A concept from [42] is utilized to achieve this. In this reference document (page 66), another point called $\sigma_m$ (subscript $m$ here not denoting ‘mean’ stress!) is defined at $N = 10^3$ as follows:

$$\sigma_m = 0.75 \cdot \sigma_y$$

(46)

This evaluates to $\sigma_m = 322.5$ Mpa. Now, the $k$-value can be determined by application of equation (44):

$$\sigma_{f, N}^k N = \text{constant}$$

---

19 Endurance strength $\sigma_e$: stress level below which stress cycles are hypothesized not to cause any fatigue damage to a material (meaning that an infinite fatigue life is prophesized under such loading). Material property to be determined experimentally, with some empirical relations available if no experimental data is available for consultation [44] (page 61).
\[(170 \cdot 10^6)^k \cdot 10^6 = (322.5 \cdot 10^6)^k \cdot 10^3\]  

Resulting in:

\[k \approx 10.8\]  

As a last feature to the S-N plot, we consider an argument made in [37] (page 150) on variable amplitude loading. The thought is, that when a sample is subjected to variable amplitude loading, small cycles that do not reach the endurance strength do cause fatigue damage when following larger cycles. They are thought to further open up cracks that resulted from larger stress cycles, even when they are not capable of initially causing cracks. The Miner-Halbach model takes this into account by extending the fatigue curve beyond the endurance stress with a decreased slope. This new slope is defined as [37] (page 151):

\[k_{\text{new}} = 2k - 1\]

The constructed S-N curve is printed here as Figure 6.1.

6.1.3 Accounting for Mean Stress Effects

Since the model under consideration is, in essence, a long beam with a large mass attached to it, a rather large constant pre-tension is expected. As explained in section 6.1.1, this influence is expected to shorten fatigue life by adding to forces that attempt to open up cracks in the material. Out of several models to take this into account, [37] (page 154) suggest to use the Smith-Watson-Topper (SWT) for general applications. It reads:

\[\sigma_{e,m} = \sqrt{\sigma_{\text{max}} \cdot \sigma_m}\]  

... where
\[ \sigma_{e,m} = \text{Endurance strength for zero-average mean stress [MPa]} \]
\[ \sigma_{\text{max}} = \text{Magnitude of endurance strength (avg. plus amplitude) [MPa]} \]
\[ \sigma_a = \text{Amplitude of endurance strength given certain mean stress \( \neq 0 \) [MPa]} \]

Using this corrected endurance stress value, the whole S-N graph is shifted vertically to account for this new value. So, for an example mean stress of 150 MPa, Figure 6.2 would result.

\[ \sum \frac{n_i}{N_{f,i}} \geq D_{PM} \quad (50) \]

... where

\[ n_i = \text{Amount of cycles of amplitude } i [-] \]
\[ N_{f,i} = \text{Amount of cycles to failure for amplitude } i [-] \]
\[ D_{PM} = \text{cut-off value where fatigue failure is predicted [-]} \quad (51) \]
The value for $D_{PM}$, though often quoted as being equal to unity, has been shown to be a stochastic variable varying from 0.15 to 1.06; but for mechanical designs, a constant value of 0.3 is recommended [37] (page 150). This is the value we will use for our own considerations. The values for $N_{f,i}$ follow from the Wöhler curves defined in sections 6.1.2 and 6.1.3 for a given material.

The last variable, the number of cycles of a specific amplitude range endured, requires the analysis of the loading signal (stress history) by an algorithm called ‘Rainflow counting’. Several variations on the exact method of implementation exist, but the underlying thought is always to identify ‘closed hysteresis loops’ [43]. The idea is that hysteresis, irreversibilities that arise during stress cycles, contributes to fatigue by dissipating energy inside the material sample and therefore are a good indicator for quantifying fatigue damage [37] (page 90). Using Rainflow counting, we may analyse a given stress signal and subdivide it into several ‘bins’ (ranges) of stress amplitudes. These bins (groups of stress cycles with comparable magnitudes) may then be assigned a contribution to the total fatigue damage using equation 50. From this, a fatigue ‘life’ may be predicted by assuming the analysed stress signal to be representative of the load experienced by the component over its entire lifespan and linearly extrapolating it. For example, assume a total evaluated fatigue damage $D = 0.02$ for a signal of duration $T$ of 1 hour. The expected fatigue life $f$ of the loaded component (with a cut-off value $D_{PM}$ equal to 0.3) is then:

$$f = \frac{D_{PM} \cdot T}{0.02} \cdot 1 \text{ [h]} = 15 \text{ [h]}$$

For more details regarding the exact implementation and algorithm of the Rainflow code, as well as a representative example of the output generated from it, see section 8.4 and [43].

![Diagram](image_url)

*Figure 6.3 - Schematic representation of the fatigue evaluation procedure. From the original stress signal a) via Rainflow counting output b) to quantitative damage evaluation through Wöhler curves c). Image own work, inspired by [41] (page 84).*
One final question remains. That is, what stress signal to analyse for fatigue analysis? Several options lay before us, such as principal stresses, shear stresses, or some kind of equivalent stress. Based on [37] (page 171), we choose to compare fatigue performance (fatigue initiation) based on von Mises stress. We therefore say to adopt an *equivalent stress approach*; [37] (page 171).
7 Design of the Software Tool

Using all of the information above, this chapter is dedicated to explain what the resulting software tool does in more detail. As explained in the introduction, we are interested in fatigue-inducing load on the pipe while in offshore service on a steel chute. The fatigue life is a function of the load patterns (magnitudes, phases) the pipe is subjected to. Multi-axial fatigue of a composite, thick-walled structure is no easy subject and the literature does not agree on any single method of analysis (in fact, it does not agree on any method whatsoever). Airborne makes good use of the classic Miner's rule for cumulative damage and as such is very much interested in the load spectra induced onto the chute at the contact point, as well as the exact location of the contact point itself. This chapter describes how we analyse a typical situation, what general algorithms we use, what limitations and assumptions are in place and how we end up with some useful results.

For general understanding of the tool, we will make use of flowchart. In a single sentence, we calculate a response of the vessel under consideration, use this as our input for an Abaqus model, let Abaqus do all the hard calculations, retrieve relevant information for post processing and carry out said post-processing. In diagram form:

![Flowchart](image)

Figure 7.1 - Global flowchart of the main steps carried out and methods used for an analysis.

7.1 Model Assumptions & Limitations

As any model, the one we use here has some limitations. These are necessary, because a model that approaches real-world situations with more fidelity nearly always means more complexity in modelling, testing, verifying and use, which in turn means a much larger investment in time from the developer's part. Furthermore, some assumptions are perfectly reasonable within the scope of physical reality and would have negligible influence on results even if the added time was taken to implement the relevant phenomena. The most important assumptions are, in a nutshell:
1. The vessel is assumed to behave in a linear fashion, and in response to wave-frequency forces only. Second order forces (motions) are assumed to be compensated by a Dynamic Positioning system. This implicitly means that we are assuming quite calm weather and sea-states, which is very reasonable given that we are considering an actual stimulation/workover/pre-commissioning operation. These would never be carried out in heavy seas.

2. The mass (i.e., the reaction forces) of the TCP and clump weight and all appendages does not influence the motion of the vessel (uncoupled analysis).

3. Influences of wind and current on the vessel are entirely compensated for by the dynamic positioning system. However, a steady current with a power-law profile (see section 5.1.3) may be specified to influence the subsea section of the TCP. Topside excitation (i.e., vessel motion) is not affected by this current.

4. The TCP is assumed to be of a uniform ‘reference’ material (not actual GFRP), with a single Young’s Modulus and Shear Modulus. These values are taken to approximate the actual product’s engineering parameters. Actual values supplied to customers are not listed due to reasons of confidentiality. If operating temperature is taken to be constant, this is a reasonable assumption [AOG internal source].

5. The ship has zero (forward) speed, which is very much reasonable given that we are analysing an active operation.

6. Calm weather assumed (see also item one). This means reasonably low waves, wind and current speeds. Airy linear wave theory is used, meaning wave climates may be modelled as superpositions of sine waves [16] (chapter 5, page 107). ‘Deep water’ is also assumed (see footnote 16, page 38, for a definition of ‘deep waters’).

Within these assumptions, we will show the tool to yield very satisfactory results and reliable analyses. The design philosophy for the tool was always to keep it as modular as possible. This means writing smaller snippets of code and wrapping them up as functions, rather than having one huge pile of code execute all at once. This ensures flexibility not only in the execution of the tool (functions may be rearranged, omitted or run all sequentially, in any order) but also in the inner design. Crashing functions are generally easier to identify and fix, and adding to the script by writing another function or procedure is relatively straightforward.

### 7.2 Pre-Processing

Pre-processing in our case means generating the response of the vessel to be fed into Abaqus. This in itself is quite an extensive process, taking up some 1500 lines of code. This is mainly because all of the data has to be read, stored in an orderly fashion and so many eventualities/cases have to be taken into account. When this thesis is completed, all employees at AOG with sufficient basic knowledge should be able to initiate runs without having to debug the code because one or two parameters changed. The basic steps in the pre-processing phase are described schematically in Figure 7.2 below.
7.2.1 Generating a Random Sea

The analysis starts with the basic parameters required to generate a wave climate. The user can choose from three options for this. The most basic input is a standard Variance Density Spectrum (VDS), characterised by the significant wave height and peak period. The three kinds of standard VDS available are the JONSWAP, Bretschneider (or two-parameter Pierson-Moskowitz), and Torsethaugen (double peaked) spectra as discussed in section 4.3. A choice for any of the three simple models implies increasingly agitated sea-states, as single-wave-direction sea-states only occur to some satisfactory degree of accuracy during large storms, with significant wave height of several meters. For the purpose of pipe-laying, one would never operate in such conditions, and such an approach to modelling a wave-climate is much more suitable when considering survival conditions for an entire offshore platform or a large vessel.

For this reason, option two is available. By calling a different function, the user may generate a VDS with the additional dimension ‘radial direction’. A standard VDS (one of the three listed in the previous paragraph) is multiplied by a specific spreading function. Here, too, three options are available. They are: ‘cos-second’, ‘cos-fourth’ and a more flexible (and more complex) function by Longuet and Higgins (LH). For the mathematical details of these functions, please refer to paragraph 4.4. The result is a two-dimensional VDS, varying not only in frequency but also in radial direction. This already is a more rigorous approach to the kind of situation one might encounter in a real-life intervention operation. The wave energy is spread out over several incoming directions, simulating an increasingly confused (or random) sea.

Lastly, the third option combines several instances of ‘option 2’ climates into one, making for a truly random character. By allowing any combination of default VDS, with any kind of spreading function and with any value for an ‘average incoming direction’, truly confused seas can be represented. For example, it is possible to have one ‘swell’ direction (best modelled by a Bretschneider spectrum), defined as coming in from around 80 degrees off bow. This swell might be approximated by a ‘cos-fourth’ spreading function. The prevailing wind direction, however, is port to starboard (270 degrees of bow) and the resulting waves (modelled by a JONSWAP spectrum) are quite narrow banded, thus best represented using LH-type spreading with a high parameter \( s \) (say, \( s = 10 \)). This combination is possible and easy to enter. In the following graphic figure, note that even when ‘no’ spreading function is chosen, the figure still holds. The ‘spreading function’ in that case is simply \( D(\theta) = 1 \).
7.2.2 Generating Actual Waves from VDS

Since we are looking for a simulation in the time-domain, the generated VDS has to be converted into an actual time-trace of the water surface elevation. Here, we come back to the random-amplitude/phase model discussed in section 4.2. For convenience, it is reprinted here. The ocean surface may be described by the sum

\[ \eta(t) = \sum_{i=1}^{n} \zeta_i \cdot \cos(\omega_i t + \epsilon_i) \]  

The first step then, is to ‘sample’ the constructed Variance Density Spectrum at regular frequency intervals \( \Delta \omega \). This frequency resolution, as well as the exact range of frequencies to be sampled, may be set by the user. From these sampled points \( \zeta_i \), we get the amplitudes \( \zeta_i \) we require by transforming the sampled values. Since in a VDS we plot \( 1/2 \cdot \zeta_i^2 / \Delta \omega \) versus frequency \( \omega \), we take

\[ \zeta_i = \sqrt{2 \cdot \zeta_i \Delta \omega} \]  

The corresponding frequencies are obviously the frequencies that we sampled the VDS at. The corresponding phases we discarded when constructing our VDS, but we did note that they are uniformly distributed with range \( [-\pi, \pi] \). So, we sample a uniform distribution with this range at regular intervals and get an appropriate set of phases. Note that this is acceptable practice because the statistics of the signal constructed in this way adhere to the VDS used, regardless of the values for the phases we obtain.

In case of a 2-dimensional VDS, we sample not just frequency, but radial direction as well. Since for a 2D VDS, we plot \( 1/2 \cdot \zeta_i^2 / (\Delta \omega \Delta \theta) \) versus frequency \( \omega \) and versus radial direction \( \theta \), the amplitudes then become

\[ \zeta_i = \sqrt{2 \cdot \zeta_i \Delta \omega \Delta \theta} \]
7.2.3 Extracting RAO's from Excel file

The next step in the process is to extract all available RAO's from an Excel file provided by the user. Since there is no rigid convention on how to supply RAO's, the data files provided by third parties may have several lay-outs and use different kinds of definitions/units for the data itself. The tool can deal with most of them, provided the lay-out adheres to some basic rules. The names of thirteen categories necessary for a complete set are hardcoded into the script and the function that extracts the RAO's, called ‘Get_RAO’, relies on these names to generate an accurate and consistent data set to pass on to the rest of the script. They are the six degrees of freedom (six entries), their respective phase lags (again six, totalling twelve) and the corresponding frequencies (totalling thirteen). The function can deal with any combination of the six degrees of freedom provided (whether it's a complete set or just, for example, 'heave', 'roll' and 'pitch' RAOs are provided) and with single directions or several directions (needed for confused seas) RAO's.

Furthermore, the data columns in the provided Excel file may be sorted by degree of freedom first, then by incoming direction, or the other way around. The rotational RAO's may be provided in units \([\text{rad}/\text{m}]\) or in dimensionless form; \([\text{rad/rad}]\). The phase lags may be defined in either \([\text{deg}]\) or \([\text{rad}]\). In each case, Get_RAO returns a set of RAOs that is neatly sorted in a so-called dictionary (a common data structure within Python). This data set is then interpolated using a function from Python’s scientific package ‘Scipy’ and the results from this interpolation are a set of \(n\) functions, for \(n\) degrees of freedom provided in the Excel file, ready to be called upon by the rest of the code. If the provided RAOs are provided over a single direction only, the interpolation functions take only frequency \(\omega\) as input variable; otherwise they take frequency and radial direction \(\theta\). The return value is always an interpolated value of the requested RAO at given frequency (and direction if applicable).

Figure 7.4 - The Get_RAO function is, perhaps surprisingly, one of the more elaborate ones in the program.

7.2.4 Calculate Vessel Response

The next step is to calculate the vessel response in the time domain. We have all the data we need from the previous two steps and the theory from paragraph 3.2.2 (specifically, equation (3)). When a one-dimensional VDS was specified, the program will accept an incoming angle-argument. The RAOs belonging
to that angle are interpolated from the given set of RAOs and the response is calculated as a sum of cosine terms; one for each cosine term that is present in the formulation for the water surface elevation as stated in equation (3). In this manner, all frequencies specified by the VDS are taken into account. The total number of terms \( N \) calculated per degree of freedom is hence equal to

\[
N = \frac{\Omega}{\Delta\omega}
\]  

(56)

... where

\[ \Omega = \text{Frequency range under consideration (e.g., 0..2.5 [rad/s])} \]
\[ \Delta\omega = \text{Frequency resolution specified [rad/s]} \]

If a two-dimensional VDS was specified, the function checks whether a set of RAOs was specified that is defined over several angles. If this is the case, the response calculation, per degree of freedom, proceeds as for the one-dimensional case, but it is now the sum of not only all frequency terms, but also of all incoming directions. The total number of terms \( N \) for one degree of freedom is thus

\[
N = \frac{\Omega}{\Delta\omega} \cdot \frac{\theta}{\Delta\theta}
\]  

(57)

... where

\[ \theta = \text{Radial range under consideration (e.g., \(-\pi/4..\pi/4\) [rad])} \]
\[ \Delta\theta = \text{Radial Resolution Specified [rad]} \]

It should be noted that for fine resolutions, the number of loops needed to generate sea states quickly increases. A 100 second time signal with time resolution 0.2 seconds, and frequencies from 0 to 2.5 radians per second in 0.05 radian per second step requires \( 100/0.2 \times 2.5/0.05 = 25,000 \) loops. A 1000 second signal with time resolution 0.1 seconds, frequency resolution 0.005 radians per second, and radial resolution of 0.1 radian over the entire circle (\(2\pi\) radians) requires \( 310,000,000 \) loops! This increases is very much notable when doing calculations.

### 7.3 Abaqus Implementation of Selected Features

While some features are being incorporated in a rather straightforward manner in Abaqus, others are a bit more complicated and require some justification. In this context, we will discuss the hydrodynamic damping and current drag forces used in the final model.

#### 7.3.1 Hydrodynamic (Morison) Damping

The famous Morison equation was introduced in section 5.1.1. It is important to recognize that this equation is obviously not just usable for currents on stationary cylinders; a moving cylinder in stationary fluid has the same net effect (a damping effect, in the latter case). The equation consists of two terms, and both need a place in our model. Abaqus appears to offer a convenient solution with a simple ‘submerged beam’ option without further explanation for beam elements. Fluid density and added mass coefficients for translational and axial movements may be specified. However, upon checking the theory manual, it is discovered that, quite deceptively, only the inertial term is included from the Morison equation. Quoting from the Abaqus Analysis User’s Guide (section 29.3.5):
"When a beam is fully or partially submerged, the effect of the surrounding fluid can be modelled as an additional distributed inertia on the beam."

The text goes on to specify the added inertia $I$ per unit length of beam as

$$I = \pi r^2 \rho C_M$$

... where

$r = \text{Radius of pipe element [m]}$

$\rho = \text{Density of surrounding fluid [kg/m}^3]$

$C_M = \text{Added mass coefficient [-]}$

Comparing to the inertia term from our definition of the Morison equation (30):

$$F_{Mor, In} = \frac{\pi}{4} \rho C_M D^2 \cdot \frac{du(t)}{dt}$$

We obviously would like to equate the two. Realizing that the ‘added inertia’ Abaqus takes into account must be multiplied by acceleration to give the added force experienced by the member (Newton; force equals mass times acceleration), we see that the following must hold true (which it does immediately):

$$\pi r^2 \rho C_M = \frac{\pi}{4} \rho C_M D^2$$

We conclude, that the $C_M$ factor Abaqus requests is the same as we determined in section 5.1.2 and we can immediately supply it to the analysis.

The second term, the viscous drag term, is a bit more complicated, since Abaqus does not provide an easy implementation. It was finally decided to model the viscous drag by applying a large number of (linear) dampers over the length of the beam in the horizontal plane. In this manner, the quadratic term appearing in the drag term is not captured. Properly implementing the non-linear damping would require writing a so-called subroutine in FORTRAN, compiling it outside of the main program and adding it to the analysis. This was deemed too time-consuming. The task now is to find a suitable Damping coefficient to model the drag as accurately as possible. Remembering the original formulation of the viscous term:

$$F_{Mor, \text{drag}} = \frac{1}{2} \rho C_D D \cdot |u(t)|$$
We can plot the Force for some reference values of $D, \rho$, and $C_D$ as determined in section 5.1.2. We realize that the governing equation for a linear damper is equal to $F = C \cdot u(t)$ and that we have to linearize the Morison term with the only available parameter, $C$. In the following Figure 7.5, both the non-linear force and two linearizations are plotted. We see that when choosing a smaller value for $C$, we get much better results at lower (relative) velocities. No matter what we choose, the values between original and linearization converge quickly for larger velocities. This is of course due to the quadratic nature of the original.

In making a choice for this reference case, we opt for the lower value of 95 $[N \cdot s/m]$, since we don’t expect any extreme values in the calm weathers we are operating in. We therefore follow the same logic as employed for the determination of our added mass/drag coefficients in section 5.1.2. As in this section, for radically different geometries/circumstances, we would have to rethink our strategy!

An error estimation between the original and our linearization yields

<table>
<thead>
<tr>
<th></th>
<th>0.0 [m/s]</th>
<th>0.5 [m/s]</th>
<th>1.0 [m/s]</th>
<th>1.5 [m/s]</th>
<th>3.0 [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-linear $[N/m]$</td>
<td>0.0</td>
<td>25.0</td>
<td>99.94</td>
<td>225.0</td>
<td>900.0</td>
</tr>
<tr>
<td>Linear $[N/m]$</td>
<td>0.0</td>
<td>47.5</td>
<td>95.0</td>
<td>142.5</td>
<td>285.0</td>
</tr>
<tr>
<td>Quotient</td>
<td>1.00</td>
<td>0.53</td>
<td>1.05</td>
<td>1.58</td>
<td>3.00</td>
</tr>
</tbody>
</table>

We see that the error we make is unfortunately quite large, but that we can tune the range where the results are quite acceptable. For the time being, we will have to accept this approach. A proper FORTRAN routine might improve the tool in this regard. As a last notion, it must be added that the damping factor actually used by the model is divided by the number of dampers used per meter of pipe, since we are talking about drag forces per meter submerged pipe.

Figure 7.5 - Original Morison non-linear force (viscous drag component only; blue) plotted against two linearizations. Red (300 $[N \cdot s/m]$) and green (95 $[N \cdot s/m]$).
7.3.2 Viscous Axial Damping

Obviously, the pipe experiences viscous drag not only in transversal, but also in axial direction. For a formulation for an approximate magnitude for this, we take a look at [13] (page 183) and find for the drag force in axial direction:

$$F_z = \frac{1}{2} \cdot \rho (\pi DL) C_{D,A} u_z |u_z|$$  \hspace{1cm} (62)

... where

- \( P \) = Proportion of L that is submerged
- \( L \) = Length of Pipe
- \( C_{D,A} \) = Axial drag coefficient
- \( u_z(t) \) = Vertical velocity

...and where the parameters are adapted to the notations used consistently in this document. Immediately, the distinct resemblance between equations (61) and (62) is clear. This does not surprise, since the drag term of the Morison equation deals with viscous drag of a flow on a cylinder as well. The only terms that have changed are the new axial drag coefficient \( C_{D,A} \) and the addition of the number \( \pi \) in the formulation to account for the surface area of the pipe, rather than its diameter \( D \) only. We take the same approach to implementing this phenomenon as in section 7.3.1 and attempt to approximate the axial viscous force with linear dampers distributed over the length of the pipe.

One problem that will have to be solved for this approach to work, is the determination of the coefficient \( C_{D,A} \). We refer to [44] (table B-2, page 148) where different Drag coefficients are listed for various geometries. For thin circular cylinders (large length over diameter quotient), we appear to see a trend for the coefficient to tend to unity. For our model, we accept the extrapolation and set \( C_{D,A} \) to 1. While this is not a perfectly scientific approach to the problem, we seem to be faced with a lack of data regarding this problem and a limitation in the amount of time and available tools to implement a different model. Plotting the full non-linear term versus two linearisations, exactly as done in the previous section, we get the following Figure 7.6.

Based on observations from previous simulations, we don’t expect to see excessive vertical speeds (or even much bigger than 1 meter per second). We choose the first approximation and end up with a linearized damping coefficient of 128 [N \cdot s/m]. As was the case for transversal damping, care must be taken to normalize this number to the amount of nodes (damping elements!) used in the model of the TCP per meter.

7.3.3 Current Forces

Current forces are assumed to be constant in our model, and current velocity distributed according to the power-law profile explained in section 5.1.3. The drag forces are calculated using the full non-linear drag term from the Morison equation and are distributed over the depth of the pipe according to the 1/7-power-law profile. The inertia term is neglected, since we are dealing with a steady current. Another reason why we feel that this is acceptable practice is because we have a KC-number (defined according to Journée & Massie [7], page 12-21)

$$KC = 2\pi \cdot \frac{U}{D}$$  \hspace{1cm} (63)

... which is quite big (>45) even at low(er) current speeds \( U \) (approx. 1 [m/s]). For such large values, the inertia term may be neglected (again, [7]). The incoming direction of the current may be specified for the model,
independent of the incoming wave direction, although in physical reality, at least the top few meters of the water column will most likely show a significant correlation between the prevailing wind direction and the direction of the current.

![Non-Linear Axial Drag vs. Linearisations](image)

**Figure 7.6** - Original axial non-linear viscous drag force (blue), plotted against two linearisation. Red (210 [N \cdot s/m]) and green (128 [N \cdot s/m]).

### 7.4 Dealing with Very Large Lengths of TCP

An important fact to take into consideration for a finite element model of a downline is the length of such a pipeline. Potentially (and in reality), the length of the TCP under consideration can exceed 2000 meters. This is especially true because TCP is designed, and marketed expressly, to be able to reach very deep waters. The light weight of the product coupled with high axial strength makes it the ideal candidate for very deep water applications. For any finite element model, this is a source of possible convergence issues, as well as very much increased costs in computing power (and therefore, in time needed to run the model). While attempting direct dynamic simulations of large lengths of TCP (1000 m or more) during the design phase of the software, the model would either fail to converge to the required ‘pre-analysis equilibrium’ (the equivalent of Orcaflex’ static step, see section 3.5.1) or take a very long time to converge during the actual dynamic analysis. In short, the process was simply not practical to work with.

A solution is found by breaking down the system under consideration to its essential parts. A long string of TCP may be approximated as a long spring. The logic behind this is that any pipe or beam has certain global engineering parameters such as section area $A$, Young’s modulus $E$, density $\rho$ and axial length $L$. Since any beam in linear first approximation experiences an elongation $\delta$ as a function of applied axial force $P$ according to [45] (page 124):

$$\delta = \frac{PL}{AE}$$

We can find a spring constant $k$ of
Given this useful feature, it is possible to approximate a large length of free-hanging pipe/beam as a much shorter spring with equivalent spring coefficient and with a weight at its end to account for the mass of the replaced length of TCP. For visual reference of this idea, see Figure 7.7.

\[
k^{-1} = \frac{\delta}{P} = \frac{L}{AE}
\]  

(65)

This model is the basis for dealing with the long length of submerged TCP. The decision to approximate the system in this manner is justified in part by remembering the initial goal of the software set forth by AOG. We are primarily interested in the behaviour and loads on the TCP at the chute. The subsurface behaviour is thus expected to be modelled in such a way that it contributes to the movement of the pipe at the very top in a correct manner, but does not have to perfectly represent the entire subsurface behaviour.

The weakness in the model proposed here is the fact that we replace a ‘spring’ with distributed mass by a massless spring with a single point-mass at its end. The dynamic behaviour is not, generally speaking, the very same. The latter model only has one degree of freedom (when considering axial motion) since there is only one mass. The former has ‘infinite’ degrees of freedom. A closer approximation then, would be a model with several masses and several spring, as depicted in Figure 7.8 on the left. This model is however not perfectly equivalent to the one proposed (depicted on the right). The difference is that we have introduced another degree of freedom by adding the second mass. When performing a dynamical eigenmode analysis on this system, it can be shown that both systems can move equivalently (first eigenmode of the left system, only eigenmode for system on the right), but the system with multiple masses has additional options of movement by being excited in eigenmodes other than the first. For a more elaborate discussion of forced vibrations of multi-degree of freedom systems, see for example [46] (chapter 5.2).

Based on the requirement of having to model the correct behaviour at the chute rather than over the entire subsea length, we accept the ‘spring model’ proposed here. The idea is that the spring model is a good approximation of the influences of the length of pipe we do not wish to model due to resource constraints, on the very top section of the same pipe (that is, the pipe section at the chute).
Of course, the first few tens of meters must be modelled as an actual pipe, with associated bending stiffness, internal pressures etcetera to be able to assess internal stresses and contact events between TCP and chute. The final Abaqus model then, is a combination of both. A chute is modelled first, and attached to it a length of actual ‘pipe’ (which is a Timoshenko beam, with specially formulated pipe-elements that allow for all kinds of typical influences to be included\(^\text{20}\)). Attached to this length of pipe-beam is the ‘spring’ model discussed in this section. Attached to this spring again is the clump weight that has to account for both the mass of the length of neglected pipe, as well as for the deadweight that is attached to the TCP in a real-world situation to keep it tensioned under water.

### 7.5 (Non-Linear) Roll Damping

Of all the six fundamental degrees of freedom for a ship, rotation along the length of the ship, called ‘roll’ plays a special role. Since vessels by approximation tend to have cylindrical undersides, there is very little ‘potential damping’ associated with roll motions. That is, the underwater shape of the ship does not disturb the pressure field significantly when performing roll motions. As a consequence, roll motions are often largely damped by a viscous force, which results in quite non-linear roll-damping [7] (page 6-14). Furthermore, roll damping in general is rather usually rather low. This results in very sharp (only lightly damped) peaks in plots of vessel RAO’s at the vessel’s fundamental roll eigenfrequency, especially for beam waves. Large roll motions may be the consequence when these frequencies are excited and this is one of the best reasons to opt for weathervaning, or positioning a vessel’s bow into the waves, when performing any kind of operation or ‘riding out’ a storm. Models for equivalent linear damping coefficients can call for a damping coefficient dependent on frequency as well as amplitude of oscillation, complicating simulations [7] (page 6-15). While this problem could be solved by introducing a second or third set of RAO’s for roll motion (making response dependant on frequency, incoming wave direction and roll amplitude), such data is not available at the present. For the tool currently under development, the choice has been made to continue with a single available set of roll damping.

\(^{20}\) See [36] (section 29.3.3) for more details on pipe elements and Timoshenko beams in Abaqus.
and accept the inaccuracy resulting from this. The error made is accepted, since the linear approach very likely results in higher responses than the non-linear approach would. We can see this from an equation given in [7] (page 6-15; reprinted at the end of this paragraph) for an equivalent linear damping coefficient that the coefficient increases with increasing \( \phi_a \) (roll amplitude). A higher roll damping coefficient would result in more damped motions and lower loads for many cases. Given the fact that conservative loads were requested as output from this programme, the choice seems justifiable. For reference, the original equation for a linear equivalent roll damping coefficient \( b^{eq} \), dependent on roll amplitude:

\[
b^{eq} = b_1 + \frac{8}{3\pi} \cdot \omega \cdot \phi_a \cdot b_2^2 + \frac{3}{4} \cdot \omega^2 \cdot \phi_a^2 \cdot b_3^3
\]  

(66)
8 Verification

The purpose of this chapter is to present some findings that verify the implementation of the techniques presented in the previous chapters into the programme under design. The main Python file consists of a grand total of 5000 lines of code and since AOG intends to actually use the tool for design purposes, a thorough error check seems to be more than appropriate.

8.1 Wave Climate Generation – 1-Dimensional

The most basic feature of the script is to be able to generate a ‘random’ sea-state based on a standard Variance Density Spectrum (JONSWAP, Bretschneider or Torsethaugen). For any real-life sea-state, the Probability Density Function (PDF) of a surface-elevation statistical analysis should approach a Gaussian distribution (for sufficiently long samples), by the Central Limit Theorem. We can see that this must hold true by remembering how we construct our ‘random’ sea-state. We take \( N \) amplitudes, sampled from our standard wave VDS; \( N \) phase lags, sampled from a uniform distribution; and construct \( N \) different sine waves, which we superimpose on one another. Quoting [17] (page 195):

“For a large number of independent identically distributed random variables, \( X_1, \ldots, X_n \), with finite variance, the average \( \bar{X}_n \) approximately has a normal distribution, no matter what the distribution of the \( X_i \) is.”

Look at our construction process, we can see that this indeed is the case for our process. The PDF of our generated wave-elevation record should approach a Gaussian PDF. Furthermore, we may even predict the standard deviation and average, the two governing parameters for a Gaussian PDF. From [7] (page 5-41):

\[
\sigma_\zeta = \sqrt{m_{0\zeta}} \quad (67)
\]

... where

\[
\sigma_\zeta = \text{Standard deviation of water surface elevation}
\]

\[
m_{0\zeta} = \text{Zeroth mathematical moment of area under spectral curve}
\]

... and in general the distributions \( n \)-th moment is defined as [7] (page 5-41):

\[
m_{n\zeta} \triangleq \int_0^\infty \omega^n \cdot S_\zeta(\omega) \cdot d\omega \quad (68)
\]

The average, \( \mu \), must be zero, since ‘the expectation of the sum is the sum of the expectations’ (see section 4.2, or [17] page 140), and the average of each individual sine is zero by definition; that is,

\[
E\left[\sum_{i=1}^n X_i\right] = \sum_{i=1}^n E[X_i] \quad (69)
\]

---

\(^{21}\) For a general sine wave, \( x(t) = A \cdot \sin(2\pi f t + \theta) \), its PDF is defined as

\[
P(x) = \begin{cases} 
\pi \sqrt{2\sigma^2 - x^2}^{-1} & |x| < A \\
0 & |x| \geq A 
\end{cases}
\]

phase angle \( \theta \) is considered a random variable with uniform distribution over \([-\pi, +\pi]\), which is exactly what we have in our case. The point here is, that all sine waves have the same type of distribution and the Central Limit Theorem applies [41].
\( X_i = \text{Random stochastic variable with finite variance} \)

For the actual verification, all types of available spectra have been scrutinised, with a sample time of 1500 seconds. [7] recommend a duration of the sample of "at least 100 times the longest observed period of the waves of the record". Since the lowest frequency we expect is around 0.5 radians/second, the lowest period is around \( \frac{2\pi}{0.5} \approx 12.5 \text{[sec]} \). A duration of 1500 seconds is within the desired range, then. The significant wave height and peak period parameters were chosen as 2 meters and 7 seconds, respectively. The original JONSWAP spectrum used, and generated surface-elevation based on it are displayed below.

*Figure 8.1 – The JONSWAP Variance Density Spectrum used for the verification processes in this chapter. \( H_{1/3} = 2 \text{[m]}, T_{\text{peak}} = 7 \text{[s]} \).*

*Figure 8.2 – The 1500-second water surface elevation generated from the JONSWAP spectrum depicted in Figure 8.1.*
This is the signal that was analysed statistically. ‘Statistically’ in this case means, that a histogram of the signal values was constructed, normalized so that the total area under the histogram equals one. This is in essence a plot of the PDF of the signal. When a Gaussian PDF is superimposed on this plot with mean and standard deviation equal to the theoretical values described above, the two should – visually – match. This resulted in the following three plots, one for Bretschneider, JONSWAP, and Torsethaugen each.

**Figure 8.4** - Histogram for instantaneous water surface elevation generated from Bretschneider VDS.

**Figure 8.3** - Histogram for instantaneous water surface elevation generated from JONSWAP VDS.
Theoretical values for averages and standard deviations from equation (67):

<table>
<thead>
<tr>
<th></th>
<th>( \xi )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bretschneider</td>
<td>0</td>
<td>0.49478</td>
</tr>
<tr>
<td>JONSWAP</td>
<td>0</td>
<td>0.40415</td>
</tr>
<tr>
<td>Torsethaugen</td>
<td>0</td>
<td>1.23272</td>
</tr>
</tbody>
</table>

Table 8.1 - Theoretical values for average and standard deviation for Gaussian Distributions generated from instantaneous water surface elevations. Applies to Figure 8.4, Figure 8.3 and Figure 8.5, respectively.

These are the same values that were used to construct the Gaussian PDF’s, plotted in green, in Figure 8.3, Figure 8.4 and Figure 8.5. Just by visual inspection, the results match up very adequately. The link between the different spectra available to the end user and the wave climates generated from them, is therefore said to be demonstrated. Furthermore, the generated wave climates appear to have the statistical properties we expect from them.

8.2 Wave Climate Generation – 2-Dimensional

The verification of the model for radially varying wave climates relies on the principle that the total energy content of a wave climate generated in this way (by realizing from a 2-dimensional VDS) must be equal to the total energy content of a wave climate generated by realization from a 1-dimensional VDS. This relation was used in defining our spatial distribution function in paragraph 4.4 and we build upon this useful idea in this section.

We have previously seen that the total energy of a wave elevation record is the total area under its VDS. This means that we can calculate the area under a JONSWAP VDS with given parameters \( H_s \) and \( T_{\text{peak}} \) and call this value the total energy of a signal realized from this spectrum. Then, when calculating the 1-dimensional VDS for all surface elevation plots realized from a single 2-dimensional VDS, one can calculate the area (energy!) under all these and sum them up. The single value calculated in this manner should approximately
be equal to the energy calculated for the 1-dimensional JONSWAP spectrum mentioned before. If this turns out to be true, we will have shown that our simulated ocean surface does not contain more or less energy than a simple 1-dimensional model. Since these signals are the input we will use for our dynamic analysis, this is a crucial check. Putting the above to the test, we shall take an example JONSWAP spectrum with parameters

\[
\begin{align*}
H_e &= 1 \,[\text{m}] \\
T_{\text{peak}} &= 5 \,[\text{s}]
\end{align*}
\]

The total energy content (i.e., area under the JONSWAP curve) is, by numerical integration, equal to approximately

\[
E_{\text{JONSWAP}} \approx 0.04015
\]  
(70)

Regarding the 2-dimensional counterpart of this spectrum then, one has to consider the very same ‘parent-spectrum’ (JONSWAP; 1 [m] and 5 [s], respectively) with an arbitrary spreading function. For this example, the cos-squared function was chosen. See section 4.4 for a definition and details for this function.

32 different surface elevations were realized from this new spectrum, in 32 different radial directions (approximately representing the range -90 degrees to +90 degrees off the dominant wave direction in steps of 5.6 degrees, or 0.1 radians). When constructing the VDS of all these 32 components, calculating the area under each and summing all these areas, we get the following value for the total area under all curves (and therefore, the total energy of all components together) of

\[
E_{2D-\text{JONSWAP}} \approx 0.03997
\]  
(71)

Which represents a quickly defined ‘accuracy’ of

\[
\frac{E_{\text{JONSWAP}}}{E_{2D-\text{JONSWAP}}} \approx 1.0045
\]  
(72)

… or a nearly perfect ‘score’ of 1. The deviation is less than a half percent, which we can attribute to numerical inaccuracy. It is worth noting that different simulated situations have yielded similarly accurate results. From this, one can conclude that; i) the 2-dimensional wave climate contains the expected amount of energy summed over all frequencies and directions; and ii) since the surface elevation traces are generated by the
same algorithm as used for the 1-dimensional case, which was proven to be mathematically robust in section 8.1, the entire 2-dimensional wave climate generation module behaves as expected.

8.3 Vessel Response

8.3.1 Unity-RAO Check

The second important feature to check, is the simulated vessel response. This is the second major step the tool takes and we will analyse the fidelity to theoretical results from two perspectives. First, we remember that, as described in chapter 7, the response is calculated by multiplying the incoming simulated waves by the appropriate RAO and then translating the resulting wave in time by the phase lag belonging to this RAO; see equation (3). The easiest check to see if the algorithm holds up, then, would be to simply set all RAO magnitudes to unity and all corresponding phase lags to zero. Then, all responses should be an exact copy of the original input.

We shall consider both modes, for translational and rotational degrees of freedom. The RAO’s are considered to be supplied in a default \([\text{rad}/\text{m}]\) format, negating the need to convert them first. We use a pure sine-wave (one frequency) as the input signal for our model and expect the exact same sine to be the output of our analysis, as described above. The results were identical for all translational and rotational DOF, respectively, so we will only show graphs for one of each category. First, the reference signal, with period \(\omega = 1\ [\text{rad}/\text{s}]\) and amplitude \(\zeta = 1\ [\text{m}]\) (Figure 8.8). The results, with unity RAO and zero phase lag, are displayed here.

![Selection of Radial Components](image-url)

*Figure 8.6 - Selection of three water surface elevation plots for different radial directions. The red signal clearly has larger amplitudes than especially the blue signal, which indicates that the red surface elevation belongs to a radial direction closer to the ‘primary’ one. These three where among the 32 used for the verification of the 2-dimensional wave climate generation as discussed in section 8.2.*
Figure 8.8 - Illustration of unity RAO's used in section 8.3.1 to prove correct implementation of vessel response algorithm.

Figure 8.7 - Test wave climate consisting of a single wave with amplitude 1 meter and frequency 1 rad/s. Used as input for 'unity-RAO check'.
The surge response looks to be an exact copy of the input – and it is. We are happy with this result. For the roll response, we see an amplitude of roughly 57. This may be explained by realizing that we are looking for translational degree of freedom here, and the RAO was defined as ‘radians per meter input’. Then we would expect a return value for the amplitude of \(1 \cdot \frac{180}{\pi} \approx 57 \, \text{deg}\), exactly what we are observing! Note that the return values must be in radians since Abaqus only accepts rotations denoted in radians (i.e., *not* degrees). For this plot, our software converts the results to degrees since more people have a proper sense of the magnitude of ‘one degree’ than they do of ‘one radian’. A warning regarding this conversion is printed onto the plot of the rotational degrees of freedom by default. In Figure 8.9, it is located on the bottom left (but too small to read properly in this printed version).
8.3.2 Frequency Domain Verification

The second verification we may carry out tests a few more aspects of the code, like the proper summing of all components as well as some theory verification. Using equation (2) and the built-in capability of our tool to construct Variance Density Spectra of a given time-trace (such as a generated vessel response), we should be able to make a sound argument. The process here is as follows. First, a response VDS is calculated from a random input VDS (one-dimensional). Let’s say of a JONSWAP type, with random significant wave height and peak period. Then, using a given set of RAOs, we may calculate a theoretical response VDS and plot it, for each of the six degrees of freedom (given availability of RAOs for all six D.o.F.).

Next, we shall simulate a decent amount of time of vessel response under this input VDS and reconstruct a VDS from the response traces. If our code did its work, the two plots should match up quite nicely. For the purpose of the test depicted here, the following parameters were used:

- $H_s = 2 \text{ [m]}$
- $T_{peak} = 7 \text{ [s]}$
- Duration = 1500 [s]
- Spectrum type: JONSWAP

The simulated response is not depicted here, but the far more relevant result of the Theoretical response Variance Density Spectra versus the ones constructed from the respective response traces is. From a visual inspection, we see that the response matches up very nicely. With this, we are confident that our code produces reliable results.

![Image](image.png)

*Figure 8.11 - In this figure, the theoretical response spectra (blue, dotted) for all six degrees of freedom are plotted along with the Variance Density Spectra constructed from the responses that were calculated for simulated wave climate with given RAOs. The source VDS for the wave climate used is displayed for the surge, sway and heave degrees of freedom only, to get a sense of the scale. For the rotational degrees of freedom, the units on the vertical axis differ by definition and the source spectrum is omitted.*
For the translational degrees of freedom, our tool automatically plots the original input VDS in red. We do this as a reality check, to see if the theoretical response VDS is of the same order of magnitude as the input. As long as the RAOs are in the order of magnitude one, this is obviously a valid check. We can observe both the red and the blue spectra nicely in one plot. In this example case, the RAO for surge has a magnitude so low that it might as well be zero. The result is that we can barely make out the response VDS (being the product of the nearly-zero RAO and the input spectrum) at the very bottom, on the x-axis. The author has checked, however, that when zoomed in far enough, we get plots similar to those found for the other RAOs. For the rotational degrees of freedom, the input VDS is not plotted, since the units are not the same for translational and rotational degree of freedom variance density spectra ([m²/s] versus [rad²/s]).

We notice very decent visual agreement between the theoretical and the reconstructed spectra, but also quite a lot of scatter around what appears to be the average of the curve; which we claim is the theoretical response spectrum. Is this observation in line with our claim that we abide by the theory? The short answer is 'yes', and the reasoning is as follows. Remember that we are only considering a single wave-elevation record, of duration 'D' for our analysis above. This means, that when we are constructing a VDS from this signal, we are constructing the function \( S(\omega) \) as follows:

\[
S(\omega) = \frac{1}{\Delta \omega} \left( \frac{1}{2} \zeta_i^2 \right) \tag{73}
\]

... where \( \zeta_i = \text{The amplitude belonging to frequency } \omega_i \)

This looks like a valid VDS, but we must not forget how the standard spectra are actually constructed; by analysing a lot of these traces and calculating the average of all of the results. This means that a standard VDS is a function assigning an expected value to the square of the amplitude \( \zeta_i \) belonging to frequency \( \omega_i \). Equation (73), constructed from a single wave record is thus a raw estimate of the real (original) VDS. Since any one component of our generated wave climate can be written as

\[
\zeta_i \cos(\omega_i t + \epsilon_i) = A_i \cos(\omega_i t) + B_i \sin(\omega_i t) \tag{74}
\]

... with \( A_i, B_i \) Gaussian distributed with the same mean and standard deviation (see [16], page 34), \( \zeta_i = \sqrt{A_i^2 + B_i^2} \), and is therefore Rayleigh distributed\(^{23}\). It can then be shown that the distribution of \( \zeta_i^2 \) is an exponential distribution and that the error in estimating the values in the constructed VDS is in the order of 100%! Holthuijsen [16] is an excellent reference on this subject.

Indeed, if we were to reconstruct the VDS for several time traces realized from the same source spectrum, we would get a much closer fit to the source. This is done in Figure 8.13, where 50 time traces of 400 seconds each were analysed by Fast-Fourier Transform, the values averaged and constructed into a single VDS. Note that this was done on time traces generated directly from a JONSWAP spectrum (essentially, these are surface elevations as we use them in our simulations as well) and not on response traces. Obviously, for the argument made in this section, this matters little. The fit is much closer to the expectation line, the original JONSWAP spectrum.

\(^{22}\) One proof of this is acquired simply enough by application of the cosine addition formula: \( \cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b) \). Note that \( \cos(b) \) and \( \sin(b) \) are constants for constant argument \( b \). May also be proven by Euler's relation \([42]\).

\(^{23}\) Any two-dimensional vector with components that are Gaussian distributed, independent, have the same variance and are centred at zero is Rayleigh distributed \([54]\).
Figure 8.12 - SAE Grapple Skidder Torsion History reference case (see section 8.4). It is the benchmark signal used to check the Rainflow implementation against.

Figure 8.13 - This is the average VDS, constructed from 50 different time traces that were generated from the same JONSWAP spectrum. Note the much closer fit to the blue line (original spectrum) than achieved in Figure 8.11.
8.4 Rainflow Post-Processing

The Rainflow capabilities of our tool are the most important post-processing method we have available, so a verification of this module seems very well in order. The algorithm used stems from a widely-known and often referenced paper on the subject by Downing & Socie [43]. In words, the algorithm is quite simple. In fact, it may be summarised in just four steps, as explained in the original document by Downing. The challenge, of course, is to implement it into Python code; free of bugs. The best way to check for correct implementation seems to be to check against existing tools/analyses, so that is what we will do. In a very important paper by Dong et al., [38], a novel cycle counting method called PDMR is presented. The authors compare their work against a standard reference case they call the “SAE grapple skidder load history”, and remark that the results

![Cycle Histogram and Cumulative Distribution Function](image1)

*Figure 8.15 - Rainflow analysis results of reference signal presented above. The plot is presented on a log-scale. The blue bars represent a histogram of occurring stress cycles of given magnitude; the green line represents the cumulative occurrence of all values, also on a log-scale.*

![Bin of Stress Range](image2)

*Figure 8.14 - Reference Rainflow analysis, carried out on SAE grapple signal depicted above. Two different algorithms with the same outcomes. Note the striking resemblance to Figure 8.15. Reprinted with permission from Dr. Pingsha Dong, one of the original authors of [35].*
generated by their novel approach should equal the ones generated from a regular Rainflow algorithm in this particular case. This load-history, apparently, is a well-known reference case, for which several Rainflow benchmark results are available. The original data was retrieved from [47] and, for reference purposes, is plotted here. The quantities plotted appear to be lbs (pounds) versus time, but it hardly matters since Rainflow counting is a path-independent method and the results from an analysis are 'number of cycles', which is a dimensionless quantity. Now comes the important part of comparing our results with the accepted reference. Dong et al. give such a reference, along with their own results of the PDMR algorithm. The relevant figure is reprinted here with permission from the original author. We see that the two figures match exactly, except for a few single cycles with less than 10^0 (one) occurrence in the higher stress ranges. This result, together with a rigorous check and careful implementation of the code makes a compelling case for correct implementation of the Rainflow code.
9 Sensitivity Analysis & Results

One of the most useful consequences of this work for AOG, is that it is now possible to carry out sensitivity studies on various parameters that might affect fatigue performance. The results from such a study promise interesting results for AOG, as well as from an academic point of view. The setup for this sensitivity study will be choosing a location of interest around the world, look at some typical data for current, wind waves and swell and deviate from the averages for some of these parameters. Doing simulations runs for multiple parameter combinations and consequently analysing the resulting fatigue performance with the simplified model developed in chapter 6 promises insights into the kind of real-life performance that can be expected from an actual TCP.

For reasons of data-availability, we choose a site north-west offshore Norway, which is characterized by significant sea-states year-round and with a distinct swell component nearly always present. The source for the numerical values used for these comparison unfortunately cannot be revealed due to reasons of confidentiality. Furthermore, we agree to not consider significant wave heights over 3-4 meters, because no operation using the TCP is expected to be carried out in more severe weather states. The main parameters to be subjected to a sensitivity analysis are the wave heights and peak periods, as well as various combinations of wind-driven seas and swell systems approaching from different directions.

The Wind-driven components will be modelled with a JONSWAP spectrum and a cos-4th spreading function (section 4.4). Swell components are, by definition, long-crested24. They have travelled far from the original weather system that originally caused them and the waves arrive more or less parallel to one another and are spectrally narrow-banded. This means very little spatial distribution of the swell energy and consequently, a sharp spreading function is required. We choose the flexible Longuet-Higgins function (again, section 4.4) with a very high spreading parameter $s$ of 30. This is an attempt to model a nearly parallel wave front. Support from the literature for such sharp swell-crests can, for example be found in [18] (page 35), and in [16] (page 47).

9.1 Sensitivity Study Setup

The setup of the proposed sensitivity study is to define a default model and deviate from this default in terms of a few parameters (one at a time). The default model defines the geometric properties of the model. We define as the basis for all simulations (unless noted otherwise):

$$
\begin{align*}
OD &= 0.13 \ [m] \\
ID &= 0.076 \ [m] \\
v_c(0) &= 1 \ [m/s] \\
\theta_{curr.} &= 0 \ [deg] \\
L_0 &= 70 \ [m] \\
M &= 6500 \ [kg] \\
\theta_{chute} &= 180 \ [deg] \\
x, y, z &= -45, 5, 6 \ [m] \\
T &= 900 \ [s] \\
L_s &= 1500 \ [m]
\end{align*}
$$

(75)

... where

---

24 Long-crested waves; waves not intersected by wave components travelling in a direction not parallel to the original wave, resulting in theoretically very long, unbroken, wave fronts. Implies very little to no (significant) spatial spreading of the wave energy and therefore a very narrow-banded (speaking in terms of radial direction) spreading function.
\[ v_c(0) = \text{Current velocity at water surface} \]
\[ \theta_{\text{curr}} = \text{Incidence angle of current (0 = stern to bow)} \]
\[ L_0 = \text{Length of pipe (beam) modelled by pipe elements} \]
\[ M = \text{Mass of clump weight} \]
\[ \theta_{\text{chute}} = \text{Physical orientation of chute on vessel} \]
\[ x, y, z = \text{x,y,z coordinates of chute on vessel relative to C.o.G} \]
\[ T = \text{Duration of simulation} \]
\[ L_s = \text{Length of additional pipe emulated by 'spring' model} \]

The primary factor to alter per simulation is the exact nature of the wave climate that is present. The testing programme is summarized in the following Table 9.1.

<table>
<thead>
<tr>
<th>Case number</th>
<th>Wind Component ( \text{H}<em>{\text{sig}} ) [m], ( T</em>{\text{peak}} ) [s]</th>
<th>Wind Component spread [-], ( \theta_{\text{w,0}} ) [deg]</th>
<th>Swell Component ( \text{H}<em>{\text{sig}} ) [m], ( T</em>{\text{peak}} ) [s]</th>
<th>Swell Component spread [-], ( \theta_{\text{s,0}} ) [deg]</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 0</td>
<td>1.0, 10.0</td>
<td>LH(^25) (s = 30), 90</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>2</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 45</td>
<td>1.0, 10.0</td>
<td>LH (s = 30), 90</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>3</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 90</td>
<td>1.0, 10.0</td>
<td>LH (s = 30), 0</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>4</td>
<td>3.0, 9.0</td>
<td>Cos-4th, 0</td>
<td>1.5, 12.5</td>
<td>LH (s = 30), 90</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>5</td>
<td>3.0, 9.0</td>
<td>Cos-4th, 45</td>
<td>1.5, 12.5</td>
<td>LH (s = 30), 90</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>6</td>
<td>3.0, 9.0</td>
<td>Cos-4th, 90</td>
<td>1.5, 12.5</td>
<td>LH (s = 30), 0</td>
<td>Chute 180°</td>
</tr>
<tr>
<td>7</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 0</td>
<td>1.0, 10.0</td>
<td>LH (s = 30), 90</td>
<td>Chute 90°</td>
</tr>
<tr>
<td>8</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 45</td>
<td>1.0, 10.0</td>
<td>LH (s = 30), 90</td>
<td>Chute 90°</td>
</tr>
<tr>
<td>9</td>
<td>2.0, 8.0</td>
<td>Cos-4th, 90</td>
<td>1.0, 10.0</td>
<td>LH (s = 30), 0</td>
<td>Chute 90°</td>
</tr>
</tbody>
</table>

Table 9.1 - Sensitivity study testing programme. Parameters of interest are mainly the different incidence angles for wind- and swell components of waves, but also the influence of changing wave heights. Cases 7 through 9 have a different orientation of the chute on the vessel (beam versus aft mounted). Cases are subdivided into three groups of three, for easy analysis of results in small batches.

Here, all the theory discussed comes together. We analyse the first principal stresses for some highly stressed areas according to methods developed in section 6.1.4 and will draw both qualitative as well as quantitative conclusions from these considerations. Points of interests are especially the elements making contact with the chute. The small bend radius of the TCP on the chute results in large bending stresses that are expected to dominate fatigue behaviour.

Since we are dealing with thick-walled pipe, the stresses will vary over the cross-section (see Appendix D for details on the mechanics). Abaqus defines so-called section points, for which stress output is given. They are visualised in Figure 9.1. By default, only for points 2, 8, 14 and 20 output is generated. However, the results presented here are available for section points 2, 7, 8, 9, 13, 14, 15 and 20. These locations were chosen intentionally to get a good picture of stress variations over the height and breadth of a section of TCP. Because of the large bending stresses induced on the chute, the difference in stress levels for (especially) points 13, 14 and 15 is very relevant. For case 1, the difference in average maximum principal stress is roughly a factor 1.6(!). The differences are illustrated in Figure 9.2. Notable is the lack of phase differences between the radially increasing components and the significant difference in stress magnitude, the latter clearly indicating the need for a thick-walled model that allows for such variations.

\(^{25}\) LH; Longuet-Higgins type spreading function, spreading parameter \( s \) denoted in brackets. See section 4.4.
For a good fatigue analysis, one is interested in the elements most heavily loaded in terms of fatigue. In Figure 9.3, time-stress plots of the six most heavily loaded elements are given. For all these traces, the corresponding fatigue life was calculated to determine the most critical stress. Element seven might be a good intuitive candidate because for this element, the largest absolute von Mises stress is calculated. However, in terms of fatigue life, this element lasts a calculated 315 days, versus just 1.4 days for element 10, which turned out to be the most critical element. The reason for this is that for fatigue, the magnitude of the stress cycles (corrected to account for large average stresses), rather than the largest average itself is the determining factor. Picking the most critical element then is not generally possible by ‘intuition’. For all results presented in the next section, the most critical element has been determined.

**Figure 9.1** - Default 'section points' in Abaqus for thick-walled pipe elements. Quantities such as stress and strain may be requested for these locations. For the current comparisons, output was usually requested for the following points: 2, 7, 8, 9, 13, 14, 15, 20.

For a good fatigue analysis, one is interested in the elements most heavily loaded in terms of fatigue. In Figure 9.3, time-stress plots of the six most heavily loaded elements are given. For all these traces, the corresponding fatigue life was calculated to determine the most critical stress. Element seven might be a good intuitive candidate because for this element, the largest absolute von Mises stress is calculated. However, in terms of fatigue life, this element lasts a calculated 315 days, versus just 1.4 days for element 10, which turned out to be the most critical element. The reason for this is that for fatigue, the magnitude of the stress cycles (corrected to account for large average stresses), rather than the largest average itself is the determining factor. Picking the most critical element then is not generally possible by ‘intuition’. For all results presented in the next section, the most critical element has been determined.

**Figure 9.2** - Von Mises stress-time history for one element on the chute, case 3. The relation between the different colors (output for different section points at the same axial coordinate along the beam element) and Figure 9.1 is as follows, going down. Blue: point 15; green: point 14; gold: point 2; purple: point 13; yellow: point 7; red: point 8; marine blue: point 9; brown: point 20. It is clear that the topmost section points see the strongest loads due to heavy bending, while points 7,8,9 are more or less on the neutral axis and are much less heavily loaded.
9.2 Analysis Results

We begin our considerations for the first three cases. Wave heights are moderate, there is a steady current of one meter per second from the stern and the main parameter to change is the incoming direction of the wind-waves and swell-waves. All this is summarized in Table 9.1.

Analysing the von-Mises stress at the top of the pipe, for the most critically loaded element, the following table summarizes the results for the first group of three cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{V,\text{mean}}$ [MPa]</th>
<th>$\sigma_{V,\text{std. dev.}}$ [MPa]</th>
<th>Calculated Fatigue Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>157.10</td>
<td>7.81</td>
<td>4.06 days</td>
</tr>
<tr>
<td>2</td>
<td>154.05</td>
<td>9.89</td>
<td>1.89 days</td>
</tr>
<tr>
<td>3</td>
<td>153.62</td>
<td>9.72</td>
<td>2.11 days</td>
</tr>
</tbody>
</table>

Table 9.2 - Fatigue Analysis results for cases 1 through 3.

Note that the standard deviation of the stress is included for comparison purposes. A standard deviation along an average gives a much more complete picture than an average by itself. The general trends observed here are not unexpected. When considering case 1 as the base-case, changing the wind-wave component to be inbound from a 45 degree angle (case 2), the average loads may stay roughly equal, but the response is nevertheless much more violent as indicated by the increased standard deviation. As a consequence, the
fatigue life is more than halved. The reason for this is very most likely that bow-quartering waves obviously excite some degrees of freedom much stronger (roll, sway) than bow-waves do. These additional movements cause an increase in stresses experienced and decreased fatigue life. Case 2 even appears slightly worse than case 3 (beam-waves), though the mean and standard deviation of the stress are nearly identical and the difference may not be very relevant, statistically speaking.

For the second group, we repeat the experiments from the first group but increase the wave heights of both the wind component and the swell component (see Table 9.1). The results are summarized here.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{vM,mean}$ [MPa]</th>
<th>$\sigma_{vM}$ std. dev. [MPa]</th>
<th>Calculated Fatigue Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>169.31</td>
<td>10.28</td>
<td>1.40 days</td>
</tr>
<tr>
<td>5</td>
<td>168.28</td>
<td>12.69</td>
<td>0.45 days</td>
</tr>
<tr>
<td>6</td>
<td>156.24</td>
<td>12.60</td>
<td>0.80 days</td>
</tr>
</tbody>
</table>

Table 9.3 - Fatigue Analysis results for cases 4 through 6.

Again, we clearly see a trend in decreasing fatigue life with cases. As for the first group, bow-quartering waves (case 5) have the biggest impact on fatigue life, with beam-waves being slightly less detrimental to performance. For this group, the difference in performance between head-waves and bow-quartering waves (so between cases 4 and 5) is even worse than for the difference between case 1 and 2. Just over a third of fatigue life is left. For this elevated sea state, some clashing between the TCP and the chute’s side-walls could be observed, whereas this was not the case for cases 1-3. See Figure 9.4 for a visual explanation of this ‘clashing’.

The last group of results that will be presented is a copy of cases 1 through 3, with the difference that the chute on which the TCP rests is oriented towards the right-hand side of the vessel, rather than towards the stern as in the previous cases. In terms of the default parameters set in section 9.1, we set $\theta_{\text{theta}} = 90 \textdegree$.

\[ (77) \]

A key comment that must be made here is that current directions have not changed and are still set to flow stern to bow. Raw results are printed here.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma_{vM,mean}$ [MPa]</th>
<th>$\sigma_{vM}$ std. dev. [MPa]</th>
<th>Calculated Fatigue Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>176.98</td>
<td>7.50</td>
<td>1.12 days</td>
</tr>
<tr>
<td>8</td>
<td>176.01</td>
<td>9.23</td>
<td>0.93 days</td>
</tr>
<tr>
<td>9</td>
<td>178.10</td>
<td>8.43</td>
<td>0.70 days</td>
</tr>
</tbody>
</table>

Table 9.4 - Fatigue Analysis results for cases 7 through 9.

Immediately, it is clear that fatigue life is affected a great deal when compared to the nearly identical cases 1-3. For the ‘why’ however, we need to take a closer look at the simulation results. When considering that the chute is now oriented towards port and the current flows from stern to bow, it becomes obvious that the current now acts perpendicular to the main axis of the chute. As a consequence, the current, having substantial effect on the free-hanging geometry of the TCP, pushes the pipe into the sidewalls of the chute and causes additional dynamic effects over the length of the TCP. In Figure 9.4 a ‘contact event’ is plotted together with the von-Mises stress in several points on the cross section of the TCP. The large scatter, or noise-like, stress signal resulting from this contact contributes significantly to the cycles experienced by the material. Since the Rainflow counting algorithm counts all these very high frequency ‘noise’ cycles separately and attributes to them a certain amount of fatigue damage, theoretical fatigue life goes down quickly in the regular presence of such contact events. In appendix A, a distinct peak can be observed at the very left of the Rainflow histogram plots for all three cases. More than 100 very small cycles have been counted by the algorithm for all cases as a consequence of the scatter behaviour witnessed in Figure 9.4. Whether or not the very high frequencies observed in the stress plots around these contact events are due to physical reality or because of numerical
artefacts is an unknown, but it might be an indication of ‘stick-slip’ behaviour on the chute. In this particular case, the simulation was even stopped short of completion because the TCP eventually ‘left’ the open section of the chute which resulted in a configuration Abaqus was unable to compute an equilibrium for. Because of these instabilities, the predicted fatigue life for cases 7-9, in itself already a rather artificial quantity here as explained in section 6.1.1, must be viewed with scepticism towards its accuracy.

Despite this however, a very important conclusion may be drawn from these results. Currents matter. They contribute to the stability of the whole system and have the potential to cause very unwanted behaviour, including the TCP leaving its chute. Aligning the vessel to strong currents or not operating under conditions of strong currents is likely to be prudent.

Figure 9.4 - On the left side, von Mises stress is plotted for the ‘element under consideration’ denoted on the right side. At roughly 180 seconds (at the yellow line in left plot), a collision event occurs further down the TCP because of current. The resulting stress fluctuations from this event travel along the length of the TCP and influence the results at the element currently under consideration. Several of these collision events occur during the timespan of the simulation, such as at (roughly) 110, 130, 275 and 290 seconds. The continuous clashing eventually even crashed the simulation when the TCP ‘left the chute’ and Abaqus could not resolve the resulting geometry.
9.3 General Conclusions

Some general conclusions can be drawn from the above, although due to a lack of large datasets care must be taken in making any definitive statements or predicting any real trends. First of all, increasing sea-states indeed does lead to reduced fatigue lives. This is hardly surprising, but seeing the results first hand gives confidence in the model. When comparing cases 1 and 4, 2 and 5 & 3 and 6 (between these pairs of cases, the significant wave height, and only the significant wave height, changes by a factor 1.5), the following ratios of fatigue life are found:

<table>
<thead>
<tr>
<th>Case a</th>
<th>Case b</th>
<th>Ratio of Fatigue Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>4.2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2.6</td>
</tr>
</tbody>
</table>

*Table 9.5 - Fatigue life ratio for pairs of identical cases under increasing $H_s$.*

From this very limited data-set, it is clear that an increase in wave height of a factor 1.5 has a very large impact on fatigue life. Between 62% and 76% of lifetime is lost. Large losses should not come as a surprise. Remembering the Basquin relation (equation (44)), we see a clear exponential relation between increases in stress cycle amplitude and endurance strength. Determining the exponent for TCP under various conditions seems a promising research question for the future!

The fact that waves coming in at 45 degrees off-bow give worse fatigue performance than beam waves was not expected beforehand. This is a new conclusion for AOG that might lead to changes in the way the ‘worst-case scenario’ is determined. Performing another comparison, this time for equal sea-states but changing angle of wave-impact, the following table is obtained:

<table>
<thead>
<tr>
<th>Case a</th>
<th>Case b</th>
<th>Ratio of Fatigue Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2.2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3.11</td>
</tr>
</tbody>
</table>

*Table 9.6 - Fatigue life ratio for pairs of identical cases under changing angle of incoming wave.*

Regrettably, for this table even fewer data points are available. Cases 2 and 3 & 5 and 6 may not be directly compared because between them, the angle of incoming swell (secondary wave system) changes as well. Direct comparison is therefore not possible because circumstances have changed in more than one degree of freedom. The numbers however, again are quite large and the effect of incoming wave direction on fatigue life seems to be a fact.

The above encourages a thorough sensitivity analysis. Unfortunately, the time allotted for this master thesis passed before a decent testing programme could be set up or carried out. However, with the tools developed in the process, carrying out such a testing programme is relatively straightforward. The author would very much encourage such efforts!

Furthermore, currents are found to be a big influence on the behaviour of the model. Their potential to undermine the stability of the model should not be underestimated. Here, AOG might draw conclusions regarding operating manuals for clients, as well as considerations for the design process. More quantitative research (i.e., more simulation runs) seems appropriate, and a recommendation in AOG’s TCP operating manuals warning of strong currents seems perhaps appropriate.
9.4 Future Research Opportunities

In this thesis, a concept for analysing fatigue loads endured by TCP has been proposed and implemented. The model has a few limitations however, most of them mentioned in section 7.1. These limitations and others could be overcome with additional time and resources, and the will to improve upon this work. In no particular order, the author makes the following suggestions for such improvement.

1. Implementing non-linear roll. Seeing the influence of changing roll motion on fatigue life would be interesting indeed, especially since it was proven during evaluation of the results in section 0 that beam-waves have a detrimental effect on fatigue performance. Of course, it might be argued that the linear roll damping is likely to be smaller than the non-linear one and that the current model is simply conservative in estimating fatigue life which might be desirable; but this discussion seems worthwhile to have.

2. Replacing the linear dampers used for modelling Morison drag by a proper FORTRAN routine to catch the non-linear part of this behaviour. For lower sea-states and low to moderate current speeds, the linear approximation works quite well and this is the regime we are interested in anyway. For increased accuracy however, the non-linear terms would be a welcome addition.

3. Implementing a non-linear material model. The material used for the current analyses is a linear-elastic model, with equivalent engineering parameters obtained from AOG. While likely accurate over smaller strain-ranges, the large deformations experienced by the model on the chute are very likely to provoke a non-linear material response. While for this thesis the time allotted was not sufficient to address this concern, the qualitative approach to generating results described in the documentation regarding the fatigue model (section 6) might be converted into an approach yielding quantitative results directly usable in a real-world application scenario if the material model (and corresponding fatigue model) were to be extended.

4. Addressing the non-linear stresses seen at the contact point of chute and TCP. As seen in section, some peculiar non-linear stress cycles are observed for nodes making direct contact with the chute. The consequences of these stress cycles and their severity on fatigue life should be addressed in more detail in future work.

5. Improving the model of the sub-surface layout. The boundary condition prescribed for the submerged end of the TCP at the moment consist of a point mass representing a clump weight. Within AOG, other subsea layout have been proposed, to keep the bottom section of TCP from ‘wandering off’ due to currents. The influence of such layouts should be considered, especially when physical anchoring to the seafloor is proposed. Such layouts might cause large additional tension forces in the TCP and heavily influence loads and fatigue; especially since currents have indeed been observed to cause large offsets of the pipe in simulations.

6. Experimenting with more wave systems influencing the vessel. The code developed for this thesis can handle any kind of combination of wave systems, with random spreading function, coming in from any possible angle. Simulating seas with two or more swell components might lead to some interesting results.
A Fatigue Analysis Results from Sensitivity Study Cases

The following are the results generated by the Python Tool when requested the fatigue life of the cases discussed in chapter 9. The set order is the same for all cases. The first plot is the time trace of the von Mises stress of the element on the chute that proved to have the lowest fatigue life. The second plot is a Histogram of the Rainflow analysis results as used for the fatigue life calculations. For the latter, all cycles ranges are plotted against their absolute occurrence, together with a cumulative distribution function, on log-normal scale. An observant reader might notice that not all time traces have the same length of time. The aim was 1000 seconds for each, but due to various reasons, this could not always be achieved. Especially for cases 7-9, the analysis would crash sooner or later, cutting simulations times short.

Plots are given starting on the next page.
Key 1

Key 1 Critical Mises Stress

Time [s]

Stress [\( P_h \)]

Cycle Histogram and Cumulative Distribution Function

Number of cycles: 124

Absolute Occurrence [\( \cdot \) ]

Units Magnitude [\( \cdot \)]
Key 2 Critical Mises Stress

Cycle Histogram and Cumulative Distribution Function

Number of cycles: 194
Key 3

**Key 3 Critical Mises Stress**

![Graph showing the critical Mises stress over time](image)

**Cycle Histogram and Cumulative Distribution Function**

![Histogram showing cycle distribution](image)

Number of cycles: 138
Key 4

Key 4 Critical Mises Stress

Cycle Histogram and Cumulative Distribution Function

Number of cycles: 164
Key 6 Critical Mises Stress

Number of cycles: 169
Key 7

**Key 7 Critical Mises Stress**

- Stress [σ]
- Time [s]

**Cycle Histogram and Cumulative Distribution Function**

- Number of cycles: 338
- Units Magnitude
- Absolute Occurrence
Key 8

Key 8 Critical Mises Stress

Cycle Histogram and Cumulative Distribution Function

Number of cycles: 633
Key 9

Key 9 Critical Mises Stress

Cycle Histogram and Cumulative Distribution Function

Number of cycles: 333
B Various Details

B.1 Nomenclature and Coordinate Conventions for Ships and Other Vessels

Figure 9.5 - Accepted default directions and names for all degrees of freedom.
B.2 Physical constants of sea water and fresh water

Excerpt from [18] (Appendix F):

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>Density $\rho'$, [kg/m³]</th>
<th>Fresh water</th>
<th>Sea water (35 PSU)</th>
<th>Kinematic viscosity $\nu'$, [m²/s]</th>
<th>Fresh water (x 10⁶)</th>
<th>Sea water (35 PSU, x 10⁶)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>999.8</td>
<td>1028.0</td>
<td>1.79</td>
<td>1.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1000.0</td>
<td>1027.6</td>
<td>1.52</td>
<td>1.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>999.7</td>
<td>1026.9</td>
<td>1.31</td>
<td>1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>999.1</td>
<td>1025.9</td>
<td>1.14</td>
<td>1.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>998.2</td>
<td>1024.7</td>
<td>1.00</td>
<td>1.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>997.0</td>
<td>1023.2</td>
<td>0.89</td>
<td>0.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>995.6</td>
<td>1021.7</td>
<td>0.80</td>
<td>0.85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If more data is needed, however unlikely, the IAPWS (International Association for the properties of Water and Steam) standard for ‘Thermodynamic Properties of Seawater’ (2008) might be consulted. See www.teos-10.org.
C Developing the Torsethaugen Spectrum

All of the formulae, information and concepts in this section are sourced from [18] (Appendix A, pages 109-110)

The deceptively simple looking final formulation of the Torsethaugen spectrum may be denoted as:

\[ S_\xi(f_{jn}) = \sum_{j=1}^{2} E_j S_{jn}(f_{jn}) \]  

(78)

Basically, two spectra are added, one for wind waves and one for swell. The final function depends not on cyclic frequency (\(\omega\)) directly, but rather on a transformed frequency \(f_{jn}\), which will be explained at the end of this section. Now, in order to characterise two-peak sea states, we must first account for two possible situations:

1. Wind dominated sea; \(T_p < T_{pf}\)
2. Swell dominated sea; \(T_p > T_{pf}\).

Here, \(T_{pf}\) is the spectral peak period for fully developed seas at the location where \(T_p\) is given, and is defined as:

\[ T_{pf} = a_f H_s \]  

(79)

Where \(a_f\) depends slightly (and vaguely, even in the available literature) on fetch. We find in the literature that ‘for fetch of 370 km, \(a_f = 6.6\)’ while for ‘fetch of 100 km, \(a_f = 5.5\)’. For a review of this information and explanation how it came to be (for possible future derivations of different values), see [20].

We define the ‘primary sea system’ by choosing either option 1 or 2 above; for wind dominated seas, the wind waves spectrum is the primary system. Conversely, for swell dominated seas, the swell spectrum is the dominant one. Important is that the primary system is always denoted by \(j = 1\) in equation (78). The secondary sea system then, is always denoted by \(j = 2\).

First, we introduce a lower and upper limit for the wave period \(T_p\). The two limits, respectively named \(T_l\) and \(T_u\) are defined as:

\[ T_l = 2.0\sqrt{H_s} \]  

(80)

\[ T_u = 25 \]  

(81)

Now, dimensionless scales for spectral peak period are defined as:

\[ \epsilon_l = \frac{T_{pf} - T_p}{T_{pf} - T_l} \]  

(82)

\[ \epsilon_u = \frac{T_{pf} - T_p}{T_u - T_{pf}} \]  

(83)
Please note that for values of $T_p$ below $T_l$ or above $T_u$, both $\epsilon$ should be set to 1!

The rest of the required parameters is summarised here below, dependent on the type of dominance for the current situation (wind or swell dominated). For wind-dominated seas:

**Primary system ($j = 1$):**

\[ H_1 = R_w H_s \]  \hspace{1cm} (84)

\[ R_w = 0.3 e^{-(2\epsilon_l)^2} + 0.7 \]  \hspace{1cm} (85)

\[ T_{p,1} = T_p \]  \hspace{1cm} (86)

\[ \gamma_1 = 35.0 \left[ \frac{2\pi H_1}{g \cdot T_{p,1}^2} \right]^6 \]  \hspace{1cm} (87)

**Secondary System ($j = 2$):**

\[ H_2 = \sqrt{1 - R_w^2 H_s} \]  \hspace{1cm} (88)

\[ T_{p,2} = T_{pf} + 2.0 \]  \hspace{1cm} (89)

\[ \gamma_2 = 1 \]  \hspace{1cm} (90)

For swell dominated seas:

**Primary system ($j = 1$):**

\[ H_1 = R_s H_s \]  \hspace{1cm} (91)

\[ R_s = 0.4 e^{(\frac{\epsilon_u}{3})^2} + 0.6 \]  \hspace{1cm} (92)

\[ T_{p,1} = T_p \]  \hspace{1cm} (93)

\[ \gamma_1 = 35.0 \left[ \frac{2\pi H_s}{g \cdot T_{pf}^2} \right]^6 \left( 1 + \frac{T_p - T_{pf}}{25 - T_{pf}} \right) \]  \hspace{1cm} (94)

**Secondary system ($j = 2$):**

\[ H_2 = \sqrt{1 - R_s^2 H_s} \]  \hspace{1cm} (95)

\[ T_{p,2} = 6.6 \sqrt{H_2} \]  \hspace{1cm} (96)

\[ \gamma_2 = 1 \]  \hspace{1cm} (97)

Now, we can define the factors $E_j$ and $S_{jn}$ found in equation (78):
\[ E_j = \frac{1}{16} H_j^2 T_{p,j}; \quad j = 1, 2 \quad (98) \]

\[ S_{1n}(f_{1n}) = 3.26 \cdot A_\gamma f_{1n}^{-4} \exp(-f_{1n}^{-4}) \frac{\exp(-\frac{1}{2\sigma^2}(f_{1n}-1)^2)}{\gamma_1} \quad (99) \]

\[ S_{2n}(f_{2n}) = 3.26 \cdot f_{2n}^{-4} \exp(-f_{2n}^{-4}) \quad (100) \]

In the available literature, the unit of frequency used is Hertz. For our software tool, we are using radians per second. Therefore we have to convert using the following formulae. Please note that the resulting quantities, \( f_{jn} \), are dimensionless frequencies.

\[ f_{1n} = \frac{\omega}{2\pi} T_{p,1} \quad (101) \]

\[ f_{2n} = \frac{\omega}{2\pi} T_{p,2} \quad (102) \]

The last two variables needed are \( A_\gamma \) and \( \sigma \), where the latter is a stepwise function of frequency. The respective definitions are:

\[ A_\gamma = \frac{1 + 1.1[\ln(\gamma)]^{1.19}}{\gamma} \quad (103) \]

\[ \sigma = \begin{cases} 0.07; & f_{jn} < 1 \\ 0.09; & f_{jn} > 1 \end{cases} \quad (104) \]

By combining equations (98), (99) and (100) into (78), we get our final spectrum. An illustrative example is given in Figure 4.6.
D On the Mechanics of Thick-Walled Tubulars

The main difference occurring between ‘thick’- and ‘thin’-walled pipes (which are open-ended pressure vessels), is that in ‘thin’ pipes, the radial stress component is assumed to be constant over the vessel thickness \( t \). This consideration arises from the fact that one dimension of the object under consideration (in this case, its thickness) is much smaller than its other two. For such cases, we consider the pipe material to be in a state of plane-stress. Only three numbers then determine the complete stress situation in such a pipe in static equilibrium. They are: two normal stresses; \( \sigma_i \) & \( \sigma_j \), and one shear stress: \( \tau_{ij} \) [48]. ‘Thin’ is obviously a relative term, and most definitions found in relevant literature put the limit somewhere between a ratio of 1/10 to 1/20 \((r_i/r_o)\). In all practical cases however, TCP exceeds even the most generous limits given for thin-wall theory limits. Taking the example of the reference case used throughout this text for representative numerical values, we would have a ratio \( r_i/r_o \) of 76.8/130, which is approximately equal to 3/5. Clearly this is a ‘thick’ wall.

For a better understanding of thin walls in pipes, one can imagine a small rectangular piece of a material theoretically being cut from a thin pipe section, say 100 [mm] OD, 3 [mm] thickness. This little piece of material is a good example of a section of material that obeys the plane-stress model described here. Using this approach, it is almost trivial to come to the following relations for hoop stress and axial stress for a pipe subjected to internal pressure using static equilibrium equations. They are [45] (pages 370-371):

\[
\sigma_{axial} = \frac{P_i r}{t} \tag{105}
\]

\[
\sigma_{hoop} = \frac{P_i r}{2t} \tag{106}
\]

... where

\( P_i = \) Internal Pressure [Pa]
\( r = \) Internal Diameter [m]
\( t = \) Wall Thickness [m]
... and where the axial stress formula is based on closed end-caps. For an open-ended pipe, this equation would not do, since the static equilibrium it is based on is not valid. Since there is only one diameter present in the equations, the internal diameter $r$, there is no variation of the hoop (or radial) stresses over the thickness of the pipe. We will show that this is not the case for thick-walled tubulars.

With this very basic understanding of stresses in pipes, let's continue with the more realistic case for the scenario of TCP; thick-walled pipes. The following figure is intended to ease the understanding of the concepts of hoop, radial and axial stresses. As mentioned before, for thin-walled pipes, the radial stress component $\sigma_{radial}$ is assumed zero.

Harvey [49] (page 57) makes the argument to subdivide a thick pipe (cylinder) into concentric sections and consider a small element from such a ring. Using this concept, he developed stress relations for both the radial, as well as the tangential (hoop) stress for thick-walled pipes. The final results are [49] (page 60):

$$\sigma_{rad} = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o)a^2 b^2}{r^2(b^2 - a^2)}$$

$$\sigma_{hoop} = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o)a^2 b^2}{r^2(b^2 - a^2)}$$

... where all coefficients are clarified in the figure below. These results are known as the Lame solution.

Illustrative example of hoop-, axial-, and radial stresses. TCP is a product where thick-wall theory certainly is necessary. The exact distribution of all stress components within the thick glass fibre sandwich that makes up the TCP's functional core are, in reality, very complex due to the anisotropic nature of the fibres.

Coordinate conventions for derivation of equation of equilibrium for thick-walled pipes. Own work, reproduced from [46] (page 58; figure 2.21)
E  Illustrative Abaqus Renders

Close-up render of the metal chute (modelled as rigid shell elements) and the first few meters of the TCP. The latter is modelled as a Timoshenko beam, using specially formulated ‘pipe’-elements. This is the geometry as generated by the Python script. For this particular render, the option ‘render beam elements’ was enabled. For this reason, the pipe is properly displayed as having a hollow cross section.

Same close-up view as above, but with generated elements rendered in for the chute. The elements for the pipe are not discernible, since the tubular cross-section is superimposed on top of them. The number of elements generated is flexibly controlled from within the script by adjusting a single parameter for the chute and pipe each.
Overview of the entire model as generated by the Python script. The total length of pipe in this case was 50 meters, but is adjustable to much greater (or shorter) lengths if circumstances (water depth) dictate it.

The Abaqus analysis has started and the pipe is in the process of converging to the static equilibrium position. To achieve numerical convergence, the pipe is first protruded with a small force (100 [N]) from its equilibrium position as shown above. Two of these pre-equilibrium steps are needed. Finally, gravity is enabled and the model assumes its static resting position. From here, the dynamic analysis may begin. The color coding represents tensile stresses in the pipe. The chute is considered rigid and gets no coding.
Render from the dynamic part of the analysis. The pipe section in contact with the chute experiences severe bending and contact stresses. The contact stress is color-coded blue to red, low to high.

Same contact stress as plotted above, but with the actual pipe not rendered. This kind of plot helps to determine the exact position of the pipe on the chute. When animating this figure for the simulated duration of time, one can see the point of contact vary over the breath and length of the chute.
10 BIBLIOGRAPHY

A Word of Thanks

With this document, my academic education is coming to an end – at the very least for the near future. This being the case, I feel that I owe many people a sincere ‘thank you’. Of the ‘many’ however, especially my loving parents deserve praise of the highest order. Without your generous support over the years (perhaps even one too many) I have spent educating myself in various ways, my life would have been so much harder in so many ways. The next sentence applies to the both of you double.

So here it is. To all those that have wished me well, wanted to see me succeed and have helped me along the way to become who I am now; not just the past nine months, but over my entire time as a student:

I thank you deeply.

Philip P. Rabe,

Delft, June 30th, 2015.