Stellingen behorend bij het proefschrift
van H.A. Vrooman, 16 september 1991

1. Interferometrie is als meettechniek pas volledig wanneer het optisch afbeelding wordt gevolgd door een vorm van kwantitatieve beeldanalyse en wanneer het optisch systeem gedurende het meetproces door een computer systeem wordt gecontroleerd en bijgesteld. *(This thesis)*

2. Het antwoord op de vraag of digitale fase-stap spikkel interferometrie het gebruik van rekstrokjes overbodig zal maken is afhankelijk van de gewenste nauwkeurigheid, spatiële resolutie en van geld. *(This thesis, Part II)*

3. Onderzoek naar beeldbewerkingsalgoritmes, die de pixels in een van tevoren gedefiniëerde, willekeurige set in beschouwing nemen in plaats van de complete pixelset in een rechthoekige omgeving, moet één van de hoofdthema's in de toekomst zijn. *(This thesis, Part II)*

4. Voor de analyse van één enkel interferogram is een goede mathematische grootheid noodzakelijk, die gerelateerd is aan de 'lokale frequentie' op verschillende posities in het beeld. Technieken als locale Fourier transformatie, Hilbert transformatie en Wigner-Ville transformatie, bekend binnen de seismiek en spraakanalyse, zijn binnen de interferometrie niet toereikend. *(This thesis, Part III)*

5. Vanuit optisch oogpunt kan interferentie zowel constructief als destructief zijn. In de maatschappij komt meestal alleen de tweede variant voor. Hetzelfde geld voor vervorming, spanning, rek en scheurtjes.

(S. J. Gould in "Science: Good, Bad and Bogus" by M. Gardner)

8. In de beeldbewerking is de kloof tussen wetenschappelijk onderzoek en industriële toepassingen duidelijk: de industrie wil vaak informatie uit een beeld halen, die nauwelijks zichtbaar of zelfs niet aanwezig is, terwijl wetenschappers problemen genoeg hebben met het extraheren van informatie, die voor de mens duidelijk zichtbaar is.

9. Elke politieke en filosofische discussie eindigt met de vraag: "Is de mens van nature goed of slecht?" Het vraagt moed om het eerste standpunt in te nemen, wanneer een onderneming ‘onze jongens’ uitwuijt naar de golf en de volgende morgen de verkooporder voor Jordanië goedkeurt.


11. Hegel dacht terecht dat kunst zich verlaagt door het imiteren van de natuur en de psyche; de wereld moet echter niet geïmiteerd worden maar onthuld.
(René Magritte)

12. Wat is de mens? Een monument van zwakheid, een prooi van het moment, een spelring van het lot, en verder slijm en gal.
(Aristoteles)
Interferometry is only complete as a measuring technique, when the optical imaging is followed by a form of quantitative image analysis and when the optical system can be controlled and adjusted by a computer system during the interferometric measurement process. 

(This thesis)

The answer to the question if digital phase stepping speckle interferometry will eliminate the use of strain gauges in the future depends on the required accuracy, spatial resolution and on money.

(This thesis, Part II)

Image processing algorithms, that only take the pixels from a predefined, arbitrary set into account instead of the complete pixel set in a rectangular neighborhood, must be one of the main topics of future research.

(This thesis, Part II)

The analysis of a single interferogram needs a good mathematical variable, related to the ‘local frequency’ on different positions in the image. Known techniques in seismic research and speech analysis, such as local Fourier Transform, Hilbert Transform and Wigner-Ville Transform, are not adequate for interferometry.

(This thesis, Part III)

From the optical point of view, interference can be constructive or destructive. From the social point of view, interference mostly occurs in the second variant. The same yields for deformation, stress, strain and cracks.

A radiologist needs an image from which he can make good diagnostics. This is not always related to physical measures, such as signal-to-noise ratio, information contents and image quality. This makes the search for a quality measure ultimately complicated.

In science, ‘fact’ can only mean ‘confirmed to such a degree that it would be perverse to withhold provisional assent’. I suppose that apples might start to rise tomorrow, but the possibility does not merit equal time in physics classrooms.

(S. J. Gould in “Science: Good, Bad and Bogus” by M. Gardner)
8. In image processing the gap between academic research and industrial applications of image processing is obvious: industry very often wants to extract information from an image, that is nearly visible or even not present, while researchers have enormous problems to extract the information that is clearly visible to the human being.

9. Each political or philosophical discussion leads to the question: “Is a human being good or bad in nature?” It takes courage to choose for the first option if an enterprise waves ‘our boys’ goodbye to the Gulf and confirms the sales order for Jordania the next morning.

10. The fact that image processing has been used in art does not give computer scientists the permission to name their image processing workstations after Rembrandt, Appel, de Kooning or Escher, especially because this name list does not show much understanding of art.

11. Hegel was right in thinking that art lowers itself by imitating nature and psyche; the point is that the world should not be imitated but revealed.

   *(René Magritte)*

12. What is a human being? A monument of weakness, a prey of the moment, a whim of fate, and further slime and bile.

   *(Aristotle)*
QUAIN: Quantitative analyses of interferograms
QUAINT:
Quantitative analyses of interferograms

proefschrift

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natuurkundig ingenieur

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Prof. I.T. Young, PhD.,

Hoogleraar in de Beeldbewerking en Patroonherkenning,
Faculteit der Technische Natuurkunde

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Printed in The Netherlands.
aan Jip en Mirjam
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Part I

Introduction
Chapter 1

General introduction

1.1 Historical background

1.1.1 Interferometry

Interference phenomena were first described in the 17th century in the works of BOYLE, GRIMALDI and HOOKE (see BORN & WOLF [1980, p. xxii]), who observed colored fringes\(^1\) when light was reflected from thin plates and films. The first, more detailed experimental investigation of this phenomenon was due to NEWTON (see BORN & WOLF [1980, p. xxii]), who studied the pattern of fringes formed in the gap when he placed a convex lens onto a flat glass plate (NEWTON's rings). The fundamental principles of the interference of light as a phenomenon resulting from summation of light oscillations were first formulated by MICHELSON (see BORN & WOLF [1980, p. 260]). Fig. 1.1 shows schematically YOUNG's classical double-slit experiment which he performed in 1802. The light from an extended source S illuminates a slit in an opaque screen A. A second screen B with two slits is placed in the cone of rays diffracted at the slit in screen A. A viewing screen C is placed where the waves - diffracted at the slits in screen B - overlap, and interference fringes are observed on it. If one of the slits is closed, the fringes vanish, and the viewing screen C is illuminated uniformly. The appearance of the fringes

---

\(^1\) In this thesis the word fringe refers to a sinusoidal intensity variation.
cannot be explained by summation of the illuminations created on the viewing screen by each of the slits. According to YOUNG and FRESNEL, it is due to the summation of the light waves with account taken of their phases, the latter being determined by the distances from the two slits in screen B to the relevant points on the viewing screen. The spatial frequency of the fringes (their

**Fig. 1.2** Interference patterns: (a) Young's fringes and (b) a typical speckle pattern.
number per unit length) increases with increase of the angle $\alpha$. In Fig. 1.2a the fringe pattern generated in the double-slit experiment is shown. YOUNG's and FRESNEL's wave theory explained all the diverse interference phenomena, but the nature of light waves remained unclear until MAXWELL developed the foundation of the electromagnetic theory of light. Operation of the first helium-neon laser in 1961 (JAVAN, BENNETT and HERRIOTT [1961]) revealed an unexpected phenomenon: objects viewed in highly coherent light acquire a peculiar granular appearance. As illustrated in Fig. 1.2b, the detailed structure of this granularity bears no obvious relationship to the macroscopic properties of the illuminated object, but rather it appears chaotic\footnote{In this thesis the word chaotic does not refer to chaos theory. The word is used as it is defined in the dictionary} and unordered, with an irregular pattern that is best described by the methods of probability theory and statistics. The physical origin of the observed granularity, which we now know as 'laser speckle' was quickly recognized and is illustrated in Fig. 1.3.

![Fig. 1.3](image)

*Fig. 1.3 Physical origin of speckle for an imaging system.*

The surface of most objects is extremely rough at the scale of the optical wavelength ($\lambda \approx 5 \times 10^{-7} \text{ m}$). When nearly monochromatic light is reflected from such a surface, the optical wave resulting at any moderately distant point consists of many coherent components or wavelets, each arising from a different microscopic element of the surface. Interference and diffraction of the de-phased but coherent wavelets results in the granular pattern of intensity that we call speckle. Even for a perfectly corrected imaging system, the intensity at a given point can result from the coherent addition of contributions
from many independent surface areas due to the limited spatial resolution of
the system (see GOODMAN [1975, p. 10]).

In most early applications of interferometric techniques, researchers tried to
avoid speckle (see ENNOS [1975, p. 206]). With holographic interferometry
the disturbing influence of the laser speckle becomes less because of the high
resolution of the recording techniques used (holograms). Furthermore a
number of speckle reduction techniques can be used to improve the
interferometric measurements (see MCKECHNIE [1975, p. 123]). It was after
the invention of the laser and its applications in holography that the attention of
research workers was more and more attracted by the speckle effect described
above. The very particular properties of speckle and notably its fineness have
led to the development of new techniques as will be discussed in part II of this
thesis.

1.1.2 Digital image processing

Interest in digital image processing techniques dates back to the early 1920s
when digitized pictures of world news events were first transmitted by
submarine cable between New York and London. Applications of digital
image processing, however, did not become widespread until the middle 1960s,
when third-generation computers began to offer the speed and storage
capabilities required for practical implementation of image processing
algorithms.

Digital image processing has been used to determine the brightness of stars in a
picture from a telescope, to determine the structure of a virus in a microscope
image, and to produce highly accurate maps of the earth from satellite-
gathered pictures. It has been used to control a sausage slicing machine to get
equal weight slices from the irregularly shaped sausage, to design textile
patterns prior to weaving, and to help restore classic paintings. It has
applications in medicine, cartography, industry, manufacturing, printing and
publishing, cosmetics and personal grooming, and a host of scientific and
research fields including astronomy, mineral analysis, fluid mechanics, 
radioactive analysis, particle physics and ocean modeling.

In all cases, digital image processing is concerned with the computer
processing of pictures or, more generally, images that have been converted
into a digital form. In general, the purpose of digital image processing is to
enhance or improve the image in some way, or to extract information from it.
1.2 Combining two strong fields

"Interferometrie is als meettechniek pas volledig wanneer het optisch afbeelden wordt gevolgd door een vorm van kwantitatiebeeldanalyse."

L. Dorst, Ph.D. thesis [1986a]

"Interferometry is only complete as a measuring technique, when the optical imaging is followed by a form of quantitative image analysis."

(Translation)

The 1970s were a time of much commercial development in the field of optical testing. Vibration isolated tables, which were needed to make interferometric optical testing more than a laboratory curiosity, became commercially available. Several commercial interferometers were introduced and laser interferometric techniques became accepted by most people manufacturing and buying optics. The use of high-resolution recording materials, however, caused several drawbacks: the need for chemical processing, low sensitivity of light, long exposure times and susceptibility to environmental disturbances. The idea of speeding up the process by using a video system for direct recording and display of holographic interferograms occurred to several holographers around the world almost simultaneously. Similar systems were proposed by BUTTERS and LEENDERTZ [1971a, 1971b] in England, MACOVSKI [1971] in the USA and SCHWOMMA in Austria. The two last groups considered the system to be a pure extension of holographic interferometric techniques. The first group, being very active in the emerging laser speckle research at that time, considered it more a speckle technique and coined the name ESPI - Electronic Speckle Pattern Interferometry - for the technique.

During the 1980s the major thrust in interferometric optical testing was the marriage of electronics, microprocessors, and interferometers for rapid acquisition, storage and analysis of interferometric data. Low cost, microprocessor-based computers now make it possible for every optics house to have interferogram analysis capability. With improved electronic phase measurement techniques it is now possible to obtain the results of an interferometric test more quickly and accurately than simply looking at the result of an interferometric test and determining how the fringes depart from being straight and equally spaced.
At the moment a number of research groups all over the world are developing new interferometric measuring techniques. The acquired interferometric data is often processed by computers using image processing techniques to enable semi-automatic or fully automatic analysis of fringe patterns (see KERR [1987], TYRER [1985, 1986], ROBINSON [1983, 1986], MENDOZA [1988], KUJAWINSKI [1988], LØKBERG [1981-1988], ELLINGSrud [1989], NAKADATE [1980-1986] and KREIS [1979-1988]).

At the Faculty of Applied Physics (TUD), the combination of interferometry and digital image processing started in 1977 with a collaborative project between the Pattern Recognition group and the Optics Research group and the first application was submitted by the Heat Transport group. Real-time holographic interferometry was used in this group to study the convected heat transport in a flat solar collector. The analysis and interpretation of the interferograms\(^3\) was studied by CHAUDRY [1982] in the Pattern Recognition and Optics Research group. The image processing techniques used were essentially 1-dimensional. Further research in the field of quantitative analysis of interferograms and the introduction of 2D techniques was done by DORST [1986c].

\[\text{Optical Processing} \]

\[\text{Static Approach} \]

\[\text{Digital Image Processing} \]

\[\text{Dynamic Approach} \]

\[\text{Fig. 1.4 Digital Image Processing related to Optical Processing.}\]

The work described in this thesis is the continuation of his research. Next to static\(^4\) interferometry (analysis of single interferograms or interference patterns stored in computer memory), dynamic interferometry is studied. The

\(^3\) In this thesis the word interferogram refers to a photographic recording of an interference pattern.

\(^4\) In this thesis the words static and dynamic are redefined by the author. They do not refer to static and dynamic techniques in optical processing.
most important feature of dynamic interferometry is the coupling of the computer system to the optical system (Fig. 1.4). Parts of the optical system can thus be controlled by the computer system. Digital phase stepping interferometry (see Chapter 3, 4, 5 and 6) is an example of such a dynamic interferometric technique. As we shall see, static interferometry (see Chapter 7, 8 and 9) often produces a number of problems and ambiguities whose solutions are time consuming and computationally intensive. The dynamic approach is computationally less complex due to the possibility of controlling and adjusting the optical set-up. Concluding this section I would propose a new formulation for interferometric measurement:

"Interferometry is only complete as a measuring technique, when the optical imaging is followed by a form of quantitative image analysis and when the optical system can be controlled and adjusted by a computer system during the interferometric measurement process."
Chapter 2

Theoretical aspects of interferometry

This chapter describes the theoretical aspects of interferometric measurements and analysis of interference patterns. Some basic theory of interfering waves is given; Different interferometric recording and processing techniques are described.

2.1 Interference of light

2.1.1 Fundamental relationships

A number of interference phenomena are known that can be interpreted consistently only in terms of quantum optics. In this section, however, the notions of the classical electromagnetic theory of light are used, according to which a monochromatic, linearly polarized light wave can be represented in the complex form

$$\tilde{E}(x,y,z,t) = \tilde{a}(x,y,z) e^{-i [\omega t + \phi(x,y,z)]},$$  \hspace{1cm} (2.1)

where $\tilde{E}(x,y,z,t)$ is the electric field intensity vector, $\tilde{a}(x,y,z)$ the amplitude vector at the given point of space and $\omega$ the angular frequency. The angular frequency is related to the ordinary frequency $v$, the period of oscillation $T$ and the wavelength $\lambda$ by the expression
Theoretical aspects of interferometry

$$\omega = 2\pi \nu = \frac{2\pi}{T} = \frac{2\pi c}{\lambda},$$

(2.2)

where $c$ is the speed of light in the given medium. A planar light wave is also characterized by the wave vector $\vec{k}$, whose direction coincides with the direction of propagation of the wave. The magnitude of the wave vector $\vec{k}$ is

$$k = \frac{2\pi}{\lambda} = \frac{n\omega}{c_0} = \frac{\omega}{c},$$

(2.3)

where $n$ is the refractive index of the medium, equal to the ratio of the phase speeds in a vacuum ($c_0$) and in the given medium ($c$). The quantity $\phi(x,y,z)$ is the initial phase at the given point of space. If we consider the form of the function $\phi(x,y,z)$ two special waves are to be mentioned. A wave is called planar if at any moment of time the surfaces of equal phase are planes. It can be shown that for a planar wave

$$\phi(x,y,z) = \phi(\vec{r}) = \delta - \vec{r} \cdot \vec{k},$$

(2.4)

where $\delta$ is the initial phase of the oscillations at moment $t = 0$ at the point $\vec{r} = (0,0,0)$. For a spherical wave the phase is constant for all points at a distance $r$ from the origin of the Cartesian grid. Thus

$$\phi(x,y,z) = \phi(r) = \delta - rk$$

(2.5)

The action of a light wave on a detector of radiation (e.g. when taking a photograph) is determined by its intensity (the value of the energy flux density averaged over many periods of the light fluctuations). It can be shown that this intensity is determined by the square of the length of the amplitude vector $a$:

$$I(x,y,z) = |a(x,y,z)|^2.$$  

(2.6)

All the information concerning the phase of the wave is lost.

2.1.2 Fundamental aspects of interference phenomena

If several light waves simultaneously propagate in space, then, in accordance with the principle of superposition which follows from the linearity of
Maxwell's equations (see BORN & WOLF [1980, p.1]), the resultant field can be written in the form

$$\vec{E}(x,y,z,t) = \sum_i \vec{E}_i(x,y,z,t).$$

(2.7)

To illustrate the principle of interference, let us consider the superposition of two plane waves of identical frequency characterized by the wave vectors $\vec{k}_1$ and $\vec{k}_2$ in the plane of drawing (see Fig. 2.1).

**Fig. 2.1 Addition of plane waves of the same frequency.**

If we assume that the electric field intensity vectors for both waves are perpendicular to the plane of drawing and the waves are homogeneous (Fig. 2.1), i.e. $a_1$ and $a_2$ are independent of $r$, it can be shown that the resultant intensity of the two interfering waves equals

$$I(r) = a_1^2 + a_2^2 + 2a_1a_2\cos[\vec{r} \cdot (\vec{k}_1 - \vec{k}_2) - (\delta_1 - \delta_2)].$$

(2.8)

It follows that the total intensity changes periodically, reaching maxima at points for which

$$\vec{r} \cdot (\vec{k}_1 - \vec{k}_2) - (\delta_1 - \delta_2) = 2m\pi,$$

(2.9)

where $m$ is an integer and minima at points for which

$$\vec{r} \cdot (\vec{k}_1 - \vec{k}_2) - (\delta_1 - \delta_2) = (2m + 1)\pi.$$

(2.10)
Theoretical aspects of interferometry

In the case of \((\delta_1 - \delta_2) = 0\), Eqs. (2.9) - (2.10) are equations of a family of planes perpendicular to the vector \(\mathbf{k} = k_1 - k_2\). If the difference \((\delta_1 - \delta_2)\) is constant in time, then a stable interference pattern is observed. The phase of light oscillations emitted by any real source changes chaotically with time. Consequently, if the sources of the light that forms the plane waves being considered are independent, then \((\delta_1 - \delta_2)\) changes chaotically, leading to a blurring of the interference pattern. If the time of measurement is sufficiently great, the cosine term in Eq. (2.8) will equal zero, leading to a simple addition of intensities. Thus, if fringes are to be observed, it is of great importance that the difference \((\delta_1 - \delta_2)\) remains constant. This is assured if the interfering waves are part of the same primary wave (time division or amplitude division). Whether or not \((\delta_1 - \delta_2)\) is constant, depends on the temporal coherence of the light source. The path difference at which the interference pattern vanishes is usually called the coherence length of the light source and equals in the case of a laser \(\lambda^2/\Delta \lambda\) (see BORN & WOLF [1980, p. 333]), where \(\lambda\) is the wavelength of laser and \(\Delta \lambda\) is the spectral width of the light emitted by the laser (see Section 4.2 for typical values for the coherence length).

The fringe contrast or fringe visibility of an interference pattern is defined by MICHELSON as (see BORN & WOLF [1980, p. 267])

\[
v = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},
\]

(2.11)

where \(I_{\text{max}}\) and \(I_{\text{min}}\) denote the minimum and maximum intensity in the observed part of the interference pattern. It follows from Eq. (2.8) that for two homogeneous plane waves with the same frequency the fringe visibility becomes (see BORN & WOLF [1980, p. 505])

\[
v = \frac{2\sqrt{\alpha}}{\alpha + 1},
\]

(2.12)

where \(\alpha = I_1/I_2\) is the ratio of the intensities of the interfering waves. In a more general case the fringe visibility becomes

\[
v = \frac{2\gamma \sqrt{\alpha}}{\alpha + 1},
\]

(2.13)
where $\gamma$ is the mutual coherence of the interfering waves ($\gamma = 1$ for completely coherent waves). The mutual coherence can be seen as the autocorrelation of the laser light wave (see BORN & WOLF [1980, p. 499]). In general the fringe visibility differs from point to point in the image. So the fringe modulation will be a function of $x$ and $y$. In general, if two propagating waves interfere, the resulting intensity distribution $I(x,y,t)$ at the detector plane can be written as:

$$I(x,y,t) = I_1(x,y) + I_2(x,y) + 2\sqrt{I_1(x,y)I_2(x,y)} \cos[\phi(x,y,t)], \quad (2.14)$$

where $I_1(x,y)$ and $I_2(x,y)$ are the intensities of the interfering waves and $\phi(x,y,t)$ the phase difference between the interfering waves, or as:

$$I(x,y,t) = A(x,y) + B(x,y) \cos[\phi(x,y,t)], \quad (2.15)$$

$$= A(x,y) \{ 1 + m(x,y) \cos[\phi(x,y,t)] \}, \quad (2.16)$$

where $A(x,y)$ is the background intensity, $B(x,y)$ the modulation intensity amplitude, and $m(x,y)$ the fringe modulation at location $(x,y)$. If there is no time dependence Eq. (2.15) becomes:

$$I(x,y) = A(x,y) + B(x,y) \cos[\phi(x,y)], \quad (2.17)$$

Substitution of Eq. (2.16) in Eq. (2.11) yields

$$\nu = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{A (1 + m) - A (1 - m)}{A (1 + m) + A (1 - m)} = m \quad (2.18)$$

Thus, the fringe modulation $m(x,y)$ equals the fringe visibility or fringe contrast.

2.1.3 The essence of interferometric measurements

With interferometric measurements it is possible to measure the phase difference between two interfering waves. Because the phase difference is proportional to the optical path difference (OPD) between the two interferometer branches.
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\[ \phi(x,y) = \frac{2\pi \text{OPD}}{\lambda} = \frac{2\pi \Delta(sn)}{\lambda}, \]  

(2.19)

where \( s \) is the path length and \( n \) the index of refraction of the medium the light travels through.

A number of physical properties can be accurately measured using interferometry. If \( s \) is varied, quantities such as shape, vibration, deformation, stress, strain, cracks, de-lamination and particle velocity can be measured. If \( n \) is varied temperature distributions and density distributions can be measured. A detailed analysis of these phenomena goes beyond the scope of this thesis. A number of applications will be discussed in Chapter 5.

2.2 Several interferometric measuring techniques

2.2.1 Classical optical interferometry

In the case of 'nice' wavefronts travelling through the interferometer (passing specularly reflecting and smooth transparent objects), one speaks of classical optical interferometry. There are a number of different techniques and, consequently, a number of different classical interferometers. Interferometers are usually classified according to the number of interfering beams and to the way of dividing the initial beam. Twin-wave, triple-wave and multiple-wave interferometers are distinguished according to the number of interfering beams. According to the way the initial beam is divided, interferometers are classified as instruments in which the beams are formed from different sections of the initial wavefront (wavefront division) and instruments in which the entire surface of the initial wavefront participates in the formation of each of the interfering waves (amplitude division). Several applications of classical optical interferometry are:

- **study of the shape of wavefronts**
  If a wave, distorted for example by a mirror, interferes with a plane reference wave an interference pattern is formed. By studying this pattern the deviation from the ideal shape of the mirror can be found (see BORN & WOLF [1980, p. 304]). Not only flat mirrors, but also other optical elements, such as transparent plates, prisms and lenses can be studied. Other transparent phase inhomogeneities - air streams, shock
waves, plasmas - can be studied in a similar way (see TWYMAN and GREEN [1916], TWYMAN [1918], WYANT [1985], BRUNING [1974]).

- **metrological applications**
  Accurate multiple-wave interferometric methods are used for measuring wavelengths. FABRY, PEROT and BENOIT (see BORN & WOLF [1980, p. 367]) measured the wavelength of the red line in cadmium. Later, the wavelength of the red line of cadmium was compared very accurately with that of the orange line of one of the krypton isotopes, which is now the basic unit of length instead of the old standard - the meter. At present, laser interferometers are used in high-precision machine tools for marking workpieces, in controlling the cutting of diffraction gratings, etc (see BUTTERS [1971b, 1978], ENNOS [1975, p. 203]).

- **spectroscopic applications**
  It can be shown (see OSTROVSKY, BUTUSOV and OSTROVSKAYA [1980] and BORN & WOLF [1980, p. 267]) that the visibility of a fringe pattern is related to the spectrum of the light used. Consequently, by studying the change of visibility while moving the mirror of a Michelson interferometer, we can determine the spectrum of the source. Investigations of this kind were first performed by FIZEAU [1862], who determined the structure of the sodium doublet according to the periodic change of the visibility of Newton's rings (see NEWTON [1979, p.193]).

### 2.2.2 Holographic interferometry

If two waves coming from a *diffusely scattering* object in different positions are to be compared, one of the waves has to be stored in a medium. A hologram is often used to record the interference pattern. A basic property of a hologram is that not only is the distribution of the irradiance in the light wave falling on it registered, but also the distribution of the phase of the object wave relative to that of a reference wave. Information on the amplitude of the object wave is recorded in the form of the visibility of the interference pattern and information on the phase in the form of the shape and frequency of the interference fringes (OSTROVSKY [1980, p.65]). As a result, a hologram when illuminated by the reference wave reconstructs a copy of the object wave with all its amplitude and phase details.
Holographic interferometry is defined as the interferometric comparison of two or more waves, at least one of which is holographically reconstructed. We consider here briefly three techniques of holographic interferometry.

- **double-exposure holographic interferometry**
  With double-exposure holographic interferometry, two holograms corresponding to two states of the same object are consecutively recorded on a photographic plate. At the reconstruction the two waves that are holographic replicas of waves that existed at different moments are reconstructed and interfere.

- **time-averaging holographic interferometry**
  With time-averaging holographic interferometry vibrations can be studied. During a certain illumination time the object is vibrating and the holographic plate is illuminated. If the illumination time is much greater than the vibration period dark and light areas are visible during reconstruction of the hologram, corresponding to areas of constant vibration amplitude. The brightest areas correspond to zero vibration amplitude. In the case of time-averaging interferometry the observed fringes are described by a Bessel function (see ERF [1974]):

\[
I(x,y) = A(x,y) J_0^2 \left[ \frac{4\pi}{\lambda} a_o(x,y) \right],
\]

(2.20)

where \( J_0^2 \) is the square of the Bessel function of the first kind and zero order, \( a_o(x,y) \) the average vibration amplitude and \( A(x,y) \) the intensity with no vibration present.

- **real-time holographic interferometry**
  The third technique is real-time holographic interferometry. The object wave coming from the object is recorded on a holographic plate. After development of the hologram it is placed exactly in the same position in the optical set-up. The reconstructed wave interferes with the object wave coming from the object on this moment. If the object deforms the interference pattern changes. In this way a 'live image' of the object deformation is visible.

A disadvantage of the first two techniques is that one ends up with one interference pattern. This interference pattern has to be analyzed if numerical
information is needed (quantitative interferometry). Part III of this thesis deals with the analysis of such a single interference pattern.

A few applications of holographic interferometry are (see VEST [1979]):

- measurement of strain, stress and bending moments (opaque objects).
- measurement of mechanical vibrations (opaque objects).
- medical and dental research (opaque objects).
- aerodynamics and flow visualization (transparent objects).
- plasma diagnostics (transparent objects).
- heat and mass transfer (transparent objects).
- stress analysis (transparent objects).

As indicated in Chapter 1, static interferometry (the analysis of a single interferogram) often produces a number of problems whose solutions are time consuming and computationally intensive.

2.2.3 Speckle interferometry

In the last 30 years a number of interferometric techniques based on the speckle phenomena have been developed. A few of them, described by STETSON [1970-1990], GOODMAN [1975] and VEST [1979] are:

- speckle photography
  Most speckle-interferometric techniques suppose that individual speckles in the image plane are displaced by no more than their own diameter (see Section 3.3.2), because the techniques are based on speckle correlation. The techniques are therefore limited to the measurement of very small lateral displacements unless the speckle size is made large by reducing the lens aperture. For the case in which the speckle does translate by more than a speckle diameter, however, correlation methods can still be used, but in this instance to measure the displacement of a localized area of pattern. Measurement methods based on this principle are in general called speckle photography. Speckle photography is used by ARCHBOLD [1970] to record in-plane surface displacement.

- double-exposure speckle interferometry
- shearing speckle interferometry
- real-time speckle interferometry
- speckle averaging techniques
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The intensity \( I(x,y) \) in an interference pattern generated by a speckle interferometer can be expressed as:

\[
I(x,y) = A(x,y) + B(x,y)\cos[\phi_s(x,y) - \phi_r(x,y)],
\]

(2.21)

where \( A(x,y) \) is the background intensity, \( B(x,y) \) the modulation intensity amplitude (containing random speckle intensities), \( \phi_s(x,y) \) the phase of the object beam (random speckle phase) and \( \phi_r(x,y) \) the phase of the reference beam at the detector plane.

2.2.4 Electronic Speckle Pattern Interferometry

About 20 years ago BUTTERS [1977] started using a video system to record, process and display the speckle interference patterns. Although a number of researchers considered the techniques developed as an extension of holographic techniques, BUTTERS and LEENDERTZ [1978] came up with the name ESPI, which stands for Electronic Speckle Pattern Interferometry. In the case of deformation measurement two speckle interference patterns (one before and one after deformation) are subtracted. From Eq. (2.20) it can be seen that the difference of the two intensities is:

\[
I_1 - I_2 = B \left[ \cos[\phi_s - \phi_r] - \cos[\phi_s - \phi_r + \Delta \phi] \right]
\]

\[
= -2B \sin[\phi_s - \phi_r + \frac{1}{2}\Delta \phi] \sin[\frac{1}{2}\Delta \phi],
\]

(2.22)

where \( I_1, I_2, B, \phi_s, \phi_r \) and \( \Delta \phi \) are all functions of the pixel coordinates \( x \) and \( y \). The term \( \sin[\frac{1}{2}\Delta \phi] \) contains the information about the phase change related to the object deformation. The other sine term also contains the random speckle phase values. If we square the intensity difference and then take an ensemble average over \( \phi_s \) (in which all values between 0 and \( 2\pi \) occur with equal probability) we get:

\[
<(I_1 - I_2)^2> = 4B^2 \sin^2[\frac{1}{2}\Delta \phi] \int_0^{2\pi} \sin^2[\phi_s - \phi_r + \frac{1}{2}\Delta \phi] \, d\phi_s
\]

\[
= 2B^2 \sin^2[\frac{1}{2}\Delta \phi]
\]
Section 2.2  Several interferometric measuring techniques

\[ = B^2 \{1 - \cos(\Delta \phi)\}. \] (2.23)

We can see the correspondence with holographic interferometry. An interference pattern with sinusoidal fringes occurs. Eq. (2.23) depends on the phase difference between exposures rather than the phase difference between object and reference beams. The images in ESPI are very noisy due to the random speckle phases and intensities. Another difference between holographic and speckle interferometry is the resolution of the recording medium. The resolution of a video camera is much less than the resolution of a hologram. This means that speckle size has to be relatively large compared to holographic techniques.

A few applications of ESPI are:

• non-destructive testing (see VIKHAGEN [1990]).
• vibration analysis (see LØKBERG [1976b, 1984]).
• strain analysis (see WINTHER [1988]).

In most of the techniques, described in this section, the analysis of the observed interference patterns is purely qualitative. One counts the fringes or detects the fringe discontinuities or other artefacts in the interference pattern to draw conclusions about the observed object or phenomenon. Next, some quantitative techniques are described, i.e. techniques that produce measurements.

2.2.5 The analysis of a single interferogram

As will be described in Chapter 4, a single interferogram can be analyzed using digital image processing techniques to compute the relevant information that caused the interference. This analysis is called static interferometry, because after the interference pattern has been recorded on, for example, a photograph (an interferogram), the interferometer is not present any more. The interferometric set-up is not used during the processing of the interferogram. The basic information in an interferogram is the intensity given by Eq. (2.14), from which \( \phi(x,y) \) is to be computed. Unfortunately, the intensity in a real interferogram is much more complicated than that expressed in this equation. Optical noise, diffraction and unwanted interference distort the intensity in a complex way. A real interferogram can be characterized by one or more of the following:

• broken, discontinuous and split fringes
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- varying contrast
- extraneous fringes
- broad cloudlike fringes
- closely spaced fringes
- lack of a known reference position
- unknown fringe sign
- background shading
- speckle noise
- diffraction noise
- quantization noise
- areas blocked by opaque objects

With image processing techniques that are described in Part III of this thesis, a number of different patterns can be analyzed. The techniques are very useful for analyzing interferograms that were recorded some time ago and are now stored in an archive.

2.2.6 Phase measuring techniques

Some interferometric techniques are based on measuring the phase of the light waves travelling through the interferometer. Examples are:

- heterodyne speckle interferometry

In heterodyne methods, the relative phase increases linearly in time, and the interference phase is measured electronically at the beat frequency of the reconstructed wave fields. Heterodyne holographic interferometry offers high spatial resolution and interpolation up to 1/1000 of a fringe. It requires, however, sophisticated electronic equipment and mechanical scanning of the image by photo-detectors. We can express Eq. (2.15) as:

\[
I(x,y,t) = A(x,y) + B(x,y) \cos[\Delta \omega t + \phi(x,y) + \delta],
\]

where \(\phi(x,y)\) is the phase difference between the two interfering waves, \(\Delta \omega\) the frequency difference between the two interfering waves and \(\delta\) an additional constant phase. In classical interferometry \(\Delta \omega = 0\) and the interference fringes are independent of time. In heterodyne interferometry the two optical frequencies are chosen to differ a small amount and the interference phase \(\phi(x,y)\) is transformed into a phase of the beat frequency signal. The beat frequency has to be chosen low
Section 2.2  Several interferometric measuring techniques

enough (< 100 MHz) to be resolved by an opto-electronic detector. With this method the phase $\phi(x,y)$ is measured locally by scanning the image mechanically. (see DÖNDLIKER [1985])

- **phase-stepping speckle interferometry**
  In phase-stepping speckle interferometry (also called quasi-heterodyne interferometry), the linear increase in $\omega$ is replaced by a stepwise change of $\delta$. To determine the phase $\phi(x,y)$, at least three interference patterns are generated and processed. The intensity in the image is then by:

$$I_i(x,y) = A(x,y) + B(x,y) \cos[\phi(x,y) + \delta_i], \quad (2.25)$$

which for a given $(x,y)$ corresponds for $i = 1, 2, 3$ to a system of three equations with three unknown values: the background intensity $A(x,y)$, the modulation term $B(x,y)$ and the phase $\phi(x,y)$ (see CREATH [1988]).

- **phase-shifting speckle interferometry**
  Instead of phase stepping, phase shifting is possible. In this case the constant phase $\delta$ is shifted linearly in time and during this, at least three integrated intensity images are acquired by the video system (see CREATH [1988]).

The phase step in the last two techniques can be introduced using a piezoelectric transducer or with a Wollaston prism. The phase-stepping and phase-shifting technique will be described in more detail in Chapter 5.

2.2.7  Static versus dynamic interferometry

One speaks of dynamic interferometry if the interferometer is coupled to the digital computer. Thus, during the analysis, the interferometric configuration can be changed and differing interference patterns can be formed, ‘grabbed’ and used during processing. Another nice feature of dynamic interferometry is that it is possible to measure the intensities $I_1(x,y)$ and $I_2(x,y)$ of the two interfering beams to estimate the background intensity and fringe modulation. As will be shown, the accuracy of dynamic interferometric methods can be much higher than the accuracy of the static analysis of interference patterns. In Part II of this thesis one dynamic interferometric technique will be discussed in detail, namely digital phase-stepping speckle interferometry. A number of algorithms will be described to solve the problems occurring in
Theoretical aspects of interferometry

speckle interferometry. The phase-stepping techniques and other dynamic techniques will be discussed briefly.
Part II

The Dynamic Approach
Dynamic interferometry

3.1 Introduction

Digital phase-stepping interferometry using digitization of the image and subsequent digital image-processing techniques offers advantages when high accuracy or numerical processing of the data is needed. The phase-stepping technique can be used in classical, speckle or holographic interferometry. In this thesis only classical and speckle interferometry will be discussed. Speckle interferometry is a non-contact measuring technique that can be used to determine the displacement at points on the surface of a diffusely reflecting object without the need of recording a hologram. The use of a phase-stepping technique for direct phase evaluation of speckle intensity patterns has been reported by CREATHT [1985b], NAKADATE and SAITO [1985], ROBINSON and WILLIAMS [1986].

3.2 Phase stepping and phase shifting

3.2.1 Means of determining phase

There are many ways to determine the phase of a wavefront. For all techniques a temporal phase modulation (or relative phase shift between the object and reference beams in an interferometer) is introduced to perform the measurement. By measuring the interferogram intensity as the phase is shifted, the phase of the wavefront can be determined with the aid of
electronics or a computer. Many different algorithms have been used for the
determination of the wavefront phase. A number of them will be discussed in
this section. Most algorithms can be characterized by the number of intensity
patterns used and are therefore called $n$-bucket algorithms, where $n$ stands for
the number of intensity patterns used. A common algorithm for phase
calculations is the four-bucket technique described in the next section.

3.2.2 The four-bucket algorithm

The application of a reference beam phase-stepping technique allows the
calculation of the phase modulo $2\pi$ at each point on the surface of the object.
The intensity distribution $I_i(x,y)$ of the $i$-th interference pattern generated by a
phase-stepping interferometer can be expressed as (Chapter 2, Eq. (2.25)):

$$I_i(x,y) = A(x,y) + B(x,y)\cos [\phi(x,y) + \delta_i], \quad (3.1)$$

where $A(x,y)$ is the background intensity, $B(x,y)$ the modulation intensity
amplitude, $\phi(x,y)$ the phase due to the object displacement and $\delta_i$ the $i$-th
reference beam phase step. Using Eq. (2.14), $A(x,y)$ and $B(x,y)$ can be
expressed in the intensities of the object and reference beam in the interferometer by:

$$A(x,y) = I_r(x,y) + I_o(x,y) \quad (3.2)$$

and

$$B(x,y) = 2\sqrt{I_r(x,y)I_o(x,y)}, \quad (3.3)$$

where $I_o(x,y)$ is the intensity of the object beam and $I_r(x,y)$ the intensity of the
reference beam. The value of $\phi(x,y)$ can be determined from four phase
stepped interference patterns with $\delta_i = 0, \pi/2, \pi, 3\pi/2$ radians (see CREATHTH
[1986]):

$$I_1(x,y) = A(x,y) + B(x,y)\cos [\phi(x,y)],$$

$$I_2(x,y) = A(x,y) + B(x,y)\sin [\phi(x,y)],$$

$$I_3(x,y) = A(x,y) - B(x,y)\cos [\phi(x,y)],$$

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Phase stepping and phase shifting

\[ I_4(x,y) = A(x,y) - B(x,y) \sin \left[ \phi(x,y) \right], \quad (3.4) \]

resulting in:

\[ \phi(x,y) = \arctan \left[ \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \right], \quad (3.5) \]

By taking the sign of the numerator and the denominator in Eq. (3.5) into account, \( \phi(x,y) \) can be determined modulo 2\( \pi \) radians. From Eq. (2.25) it can be seen that three intensity patterns are sufficient to compute the phase. The four-bucket technique, however, is less sensitive to phase-step errors (see CREATH [1986]).

3.2.3  Phase shifting

In addition to the phase-stepping technique, phase shifting can be used to measure the phase. In this case the phase change that is introduced shifts linearly from a start to an end value. During this shift the required interference patterns are recorded. The detector array will integrate the intensity in time. Eq. (3.1) becomes in that case:

\[ I_i(x,y) = \frac{1}{2\tau} \int_{t_i - \tau}^{t_i + \tau} A(x,y) + B(x,y) \cos \left[ \phi(x,y) + g(t) \right] dt \quad (3.6) \]

with

\[ g(t) = \frac{2\pi \nu t}{\lambda}, \]

where \( \nu \) is the phase shift velocity (see Chapter 4) and \( \lambda \) is the wavelength of the laser light. After integration of this expression, the recorded intensities are:

\[ I_i(x,y) = A(x,y) + B(x,y) \ \text{sinc} \left[ \Delta \right] \ \cos \left[ \phi(x,y) + \delta_i \right], \quad (3.7) \]

where

\[ \delta_i = g(t_i), \ \Delta = \frac{2\pi \nu \tau}{\lambda} \ \text{and} \ \text{sinc} \left[ \Delta \right] = \frac{\sin \left[ \Delta \right]}{\Delta} \]
So, the only difference between phase stepping and phase shifting is a reduction of modulation intensity. If \( \tau = 0 \), we have phase stepping and the sinc function becomes one (\( \Delta = 0 \)). At the other extreme, if \( 2\nu \tau = \lambda \), the sinc term is zero (\( \Delta = \pi \)) and there is no modulation of the intensity. If for example \( 2\nu \tau = \lambda/4 \), the sinc term becomes about 0.9 (\( \Delta = \pi/4 \)). Note that the effect of integrating is very small. An advantage of the phase shifting technique is the higher speed with which the four interference patterns can be recorded. In the case of phase stepping, one has to wait one or two video frame times for the vibrations of the moving mirror to disappear. In the case of an electro-optical phase stepper this artefact would not be present. In the case of speckle interferometry it is important to ‘grab’ the data fast to diminish or eliminate the influence of environmental disturbances on the phase such as mechanical vibrations, temperature changes, air flow and acoustical disturbances.

### 3.2.4 Other phase measuring algorithms

In addition to the four-bucket technique, a few other phase measuring techniques should be mentioned here. They differ principally in their sensitivity to small random errors in \( \delta_i \), which we have until now considered to be constant.

- **The three-bucket algorithm**
  Since a minimum of three interference patterns is needed to reconstruct a wavefront (there are three unknowns in Eq. (3.1)), the phase can be calculated from a phase step of \( \pi/2 \) per exposure with \( \delta_i = \pi/4, 3\pi/4 \) and \( 5\pi/4 \) radians. The phase at each detector point is then simply:

\[
\phi(x,y) = \text{arctan}\left[\frac{I_3(x,y) - I_2(x,y)}{I_1(x,y) - I_2(x,y)}\right].
\]

This three-bucket algorithm is the simplest to use, but it is also the most sensitive to an error in the reference phase step (see CREATH [1986]).

- **The Carré algorithm**
  The three- and four-bucket algorithms assume that the phase step is known either by calibrating the phase stepper or by measuring the phase step. CARRÉ [1966] developed a technique of phase measurement which is independent of the size of the phase step. It assumes that the phase is
stepped by an amount $\alpha$ between consecutive intensity measurements to yield four intensity equations. It can be shown that the phase at each point can be calculated from these intensities by:

$$\phi = \arctan \left[ \sqrt{\frac{3(I_2 - I_3) - (I_1 - I_4)}{(I_2 + I_3) - (I_1 + I_4)}} \right] \cdot \frac{\sqrt{(I_2 - I_3) + (I_1 - I_4)}}{(I_2 + I_3) - (I_1 + I_4)}.$$  \hfill (3.9)

For convenience, the $x,y$-dependency is omitted here. A big advantage is that the phase step does not need to be calibrated. However, the Carré algorithm is computationally more complex.

- **The averaging-three-and-three algorithm**
  This technique was developed by SCHWIDER [1983] and is based on the averaging of two three-bucket phase measurements. One realization of this technique involves taking four measurements as in the four-bucket technique, calculating the phase using Eq. (3.8) for the first three buckets, and averaging this with the phase calculated using Eq. (3.8) with the last three of the four buckets. This can be written as:

$$\phi(x,y) = \frac{\arctan \left[ \frac{I_3(x,y) - I_2(x,y)}{I_1(x,y) - I_2(x,y)} \right] + \arctan \left[ \frac{I_4(x,y) - I_3(x,y)}{I_2(x,y) - I_3(x,y)} \right]}{2}. \hfill (3.10)$$

This technique has the advantage of being simple to calculate, and yet has the ability to average out errors (see CREATHT [1986]).

- **The five-bucket algorithm**
  An early technique used for phase measurement utilizes methods of communication theory to perform synchronous detection. To synchronously detect a noisy signal, the signal is multiplied with sinusoidal and cosinusoidal signals of the same frequency and averaged over many periods of oscillation. The method of synchronous detection applied by BRUNING [1974] to phase measurement can be extracted from the least square approximation results of GREIVENKAMP [1984], when the phase steps are chosen such that $N$ measurements are equally spaced over
Digital phase stepping speckle interferometry

one modulation period. With phase steps \( \delta_i = i2\pi/N \), with \( i = 1, \ldots, N \), the phase is given by:

\[
\phi(x,y) = \arctan \left[ \frac{\sum I_i(x,y) \sin[\delta_i]}{\sum I_i(x,y) \cos[\delta_i]} \right].
\]  

(3.11)

CREATH [1986] has compared these phase stepping techniques. She discusses the influence of errors such as phase-shifter miscalibration and non-linearity. Also, detector non-linearity has been investigated.

3.3 Laser speckle characteristics

3.3.1 First-order statistics

It can be shown (see GOODMAN [1975]) that the intensity and the phase in a polarized speckle pattern obey the following statistics. The marginal probability density function of the intensity \( I \) is:

\[
p_I(I) = \frac{1}{<I>} e^{-\frac{I}{<I>}} \quad (I \geq 0),
\]

(3.12)

where \( <I> \) is the average intensity. Of special importance are the second moment and the variance of the intensity. It can be shown that

\[
<I>^2 = 2<I>^2
\]

(3.13)

and

\[
\sigma_I = <I^2> - <I>^2 = <I>-^2.
\]

(3.14)

Thus the standard deviation \( \sigma_I \) of a polarized speckle pattern is equal to the mean intensity. A reasonable measure of the contrast of a speckle pattern is the ratio:
Section 3.3 Laser speckle characteristics

\[ C_s = \frac{\sigma_I}{\langle I \rangle} \]  \hspace{1cm} (3.15)

Using this definition, we see that the contrast of a polarized speckle pattern is always unity. The marginal density function for the speckle phase \( \theta \) is given by:

\[ p_{\theta}(\theta) = \frac{1}{2\pi} \quad (-\pi \leq \theta < \pi). \]  \hspace{1cm} (3.16)

So the intensity at a point in a polarized speckle pattern obeys negative exponential statistics, while the phase obeys uniform statistics. It can be shown that the intensity and phase are statistically independent at any given point.

3.3.2 Speckle size and second-order statistics

The size of laser speckles, a statistical average of the distance between adjacent regions of maximum and minimum brightness, is always related to the aperture angle that the radiation giving rise to it subtends at the plane defining

![Diagram of laser illumination](image)

**Fig. 3.1** Formation of (objective) speckle by scattering of coherent light from a circular region of diameter \( D \).

the speckle field (see ENNOS [1975]). A reasonable measure of the 'average width' of speckle is the width of the autocorrelation function of intensity

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$R_l(\Delta x, \Delta y)$. Two different situations can be considered. In the case of free space geometry and a uniform and square scattering spot with dimensions $L \times L$, the autocorrelation function of the intensity in the observation plane is (see GOODMAN [1975]):

$$R_l(\Delta x, \Delta y) = \langle I \rangle^2 \left[ 1 + \frac{\text{sinc}^2 L \Delta x}{\lambda z} \cdot \frac{\text{sinc}^2 L \Delta y}{\lambda z} \right].$$  \hspace{1cm} (3.17)

where $\lambda$ is the wavelength of the laser light, $\langle I \rangle$ is the mean intensity

\textbf{Fig. 3.2}  \hspace{0.5cm} \textit{Formation of (subjective) speckle by collecting the scattered radiation field with a lens and focussing it on a screen}\n
and $z$ is the distance from the object surface to the observation plane. The size of the speckles can reasonably be taken to be the value of $\Delta x$ where $\text{sinc}^2 (L \Delta x, \lambda z)$ first falls to zero. Denoting this distance $\delta_{sL}$, we have

$$\delta_{sL} = \frac{\lambda z}{L}.$$ \hspace{1cm} (3.18)

The size (diameter) $\delta_{sL}$ of speckles formed on a screen at a distance $z$, by scattering of coherent light from a circular region of diameter $D$ (Fig. 3.1), is given approximately by:
Section 3.3 Laser speckle characteristics

\[ \delta_{s1} = \frac{1.2 \lambda z}{D}. \]  

(3.19)

For the usual case of an imaging system with a circular lens pupil of diameter \( D \), it can be shown (see GOODMAN [1975]) that the autocorrelation function of the speckle pattern is:

\[
R_f(r) = \langle I \rangle^2 \left[ 1 + \left| J_1 \left( \frac{\pi D r}{\lambda z} \right) \right|^2 \right],
\]

(3.20)

where \( r = \sqrt{(\Delta x)^2 + (\Delta y)^2} \) is the distance between two points in the speckle field and \( z \) is the distance from the object surface to the observation plane. The first minimum of the Bessel function yields the speckle size \( \delta_{s2} \). Thus, if the speckle field is formed by collecting the scattered radiation field with a lens and focussing it on a screen (Fig. 3.2), the speckle size is related to the effective numerical aperture N.A. of the lens by:

\[ \delta_{s2} = \frac{1.22 \lambda z}{D} = \frac{0.6 \lambda}{\text{N.A.}}, \]

(3.21)

where the numerical aperture N.A. = \( n \sin [\alpha] \). \( n \) is the index of refraction (1.0 in vacuum). The speckle size is very important in digital speckle interferometry.

3.3.3 Integrated and blurred speckle patterns

In Section 3.3.1 the intensity distribution is given for a single, fully polarized, speckle field. This distribution is purely theoretical. In practice a number of factors influences the intensity distribution of the recorded image. In the experimental measurement of the intensity in a speckle pattern, the detector aperture (see Chapter 4) always has a finite size. Hence the measured intensity is always a somewhat smoothed or integrated version of the ideal point-intensity, and the statistics of the blurred speckle will be somewhat different from the ideal statistics described in the previous sections. in this thesis. ENNOS [1975] gives an approximation for the statistical distribution function of the intensity \( I_b \) in a blurred speckle field:
Digital phase stepping speckle interferometry

\[ p_{I_b}(I_b) = \frac{\left( \frac{M}{<I_s>} \right) I_b M^{-1} e^{-\frac{I_b}{<I_s>}}}{\Gamma(M)} \]

(3.22)

where \(<I_s>\) is the true mean intensity of the speckle pattern and \(\Gamma(M)\) the gamma function with argument \(M\). The parameter \(M\) is related to the ratio of the speckle size and the detector aperture size and to the weighting function of a detector element. If the detector aperture is very large (compared to the speckle size), one can say that \(M\) is approximately the number of speckles within one detector element. If the detector aperture is very small, \(M\) is approximately unity. In that case the measured intensity at the detector element is influenced by one ‘speckle cell’. In Fig. 3.3a the distribution function \(p_{I_b}(I_b)\) is shown for different values of \(M\). In Fig. 3.3b some experimental curves for different speckle sizes are shown. The (approximate) speckle sizes are given with the curves. It is very complicated to compute the value of the parameter \(M\) for a given experimental set-up (see GOODMAN

![Fig. 3.3](image-url)
Section 3.3  Laser speckle characteristics

(1975)). For the case of $\sigma = 3 \, \mu m$, a very rough estimation yields $M = 100/9 = 11$. For the case of $\sigma = 12 \, \mu m$, the speckle size becomes larger than the physical size of the detector aperture and $M$ will approach unity.

3.3.4 Interference with a uniform reference beam

If a speckle field interferes with a coherent, uniform reference beam, the resulting statistical distribution function for the intensity $I_c$ becomes:

$$p_{I_c}(I_c) = \frac{1}{<I_s>} J_0 \left( 2 \sqrt{\frac{2 \sqrt{I_{dr}}}{<I_s>}} \right) e^{-\frac{(I_c + I_r)}{<I_s>}} ,$$  

(3.23)

where $I_r$ is the intensity of the uniform coherent reference beam, $<I_s>$ the mean intensity of the speckle pattern alone and $J_0$ the Bessel function of zero order.

![Graphs showing probability density function and contrast of speckle + background](image)

**Fig. 3.4** (a) Probability density function ($<I_s>$ $p_{I_c}$) of a speckle pattern added to a uniform coherent background for different beam ratios ($I_r/<I_s>$) and (b) the contrast of such a pattern as a function of the beam ratio.

of the first kind. In Fig. 3.4a distributions of such an interference pattern for various beam ratio’s $r = I_r/<I_s>$ are shown. The important feature of the curve with $r = 1$ is that it does not differ greatly from the distribution curve of

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Digital phase stepping speckle interferometry

brightness is still zero, so that the speckle texture can be readily recognized. It can be shown (see GOODMAN [1975]) that the mean size of the speckles approximately doubles, when a reference field is introduced. The contrast of the pattern can be expressed as:

$$C_s = \frac{\sqrt{1 + 2r}}{1 + r},$$  \hspace{1cm} (3.24)

which is plotted in Fig. 3.4b.

In a practical digital speckle interferometric set-up, one will always have a combination of the effects due to blurred speckle and the addition of a uniform reference beam.

3.3.5 Using laser speckle in interferometric measurements

The intensity distribution $I_i(x,y)$ of the $i$-th interference pattern generated by a phase-stepping speckle interferometer can be expressed as:

$$I_i(x,y) = A(x,y) + B(x,y)\cos [\phi_o(x,y) + \phi_s(x,y) + \delta_i],$$ \hspace{1cm} (3.25)

where $A(x,y)$ is the background intensity, $B(x,y)$ the modulation intensity amplitude, $\phi_o(x,y)$ the phase change due to the object displacement, $\phi_s(x,y)$ the random phase difference between object and reference beam at zero displacement and $\delta_i$ the reference beam phase step. The value of $\phi_o(x,y) + \phi_s(x,y)$ can be calculated using the four-bucket technique described in Section 3.2.1:

$$\phi_o(x,y) + \phi_s(x,y) = \arctan \left[ \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)} \right].$$ \hspace{1cm} (3.26)

Because of the random phase distribution in a speckle field, the calculated phase values have to be subtracted from the corresponding values of a reference phase measurement to give a deterministic result. Assuming that the speckle phase $\phi_s(x,y)$ does not change, subtracting yields the phase change of the object beam:

$$\Delta \phi_o(x,y) = \phi_o,after(x,y) - \phi_o,before(x,y).$$ \hspace{1cm} (3.27)
The calculated phase change $\Delta \phi_o(x,y)$ is related to the displacement of each object surface point.

### 3.3.6 Speckle decorrelation

In Section 3.3.5 we assumed that the speckle phase does not change during the recording of the interference patterns. Due to the object deformation, however, a phenomena called speckle decorrelation occurs. This causes a change in the speckle phase $\phi_s(x,y)$ and the speckle intensity $I_s(x,y)$ at a detector point. Speckle decorrelation can be explained by considering the light cone scattered from an object surface point and collected by the optical system (Fig. 3.5). All light rays inside the cone will contribute to the speckle at the corresponding detector point. When the object is slightly rotated, the light cone will be displaced, shifting out a number of light rays and shifting a number of different rays. Out-of-plane as well as in-plane object displacement causes speckle decorrelation. The change in fringe visibility due to speckle decorrelation has been studied by JONES and WYKES [1977] and by ENNOS [1975]. The influence of speckle decorrelation on the phase measurements, described in this thesis, is described in more detail by MAAS [1991].
Chapter 4

Optical system, hardware and software

In this chapter a description will be given of the computer systems, software, optics and other equipment used to perform the phase-stepping speckle interferometric technique described in Chapter 3 and the subsequent image-processing techniques described in Chapter 5.

4.1 Introduction

To perform interferometric measurements such as displacement, strain and shape measurements using digital phase-stepping (speckle) interferometry, an experimental system was developed. The experimental system (Fig. 4.1) consists of a speckle interferometer coupled to a VME/MC68000 microcomputer (Ironics 12.5 MHz) used for digitizing and processing the interference patterns and for controlling a mirror position in the optical arrangement. The system in Fig. 4.1 was initially developed for interferometric deformation measurements and strain analysis. Any kind of diffusely scattering object up to dimensions of 50 x 50 x 50 cm$^3$ can now be positioned in the optical set-up. At the moment the optical system has been made smaller, more flexible and less sensitive to environmental influences (see Section 4.3). With a few changes classical interferometric measurements can also be performed with the system described in this chapter.
4.2 The laser

4.2.1 Argon-ion laser

For the work described in this thesis an Argon-ion laser is used at an output power of approximately 100 mW and a wavelength of 514.5 nm. This laser is the most cumbersome and most expensive part of the experimental system. A big cooling device is needed and an input power of 10 kW provides approximately 1 W of output power.

The coherence length $l_c$ of a laser is approximately equal to $\lambda^2/\Delta \lambda = c/\Delta \nu$, where $\lambda$ is the wavelength, $c$ the speed and $\nu$ the frequency of the laser light. Typically $\Delta \nu \approx 3$ GHz, $c = 3 \times 10^8$ m/s, yielding $l_c \approx 10$ cm. To increase the coherence length of the laser, an etalon is used, which narrows the spectral width to approximately 1 line. In that case $\Delta \nu$ becomes about 3MHz resulting in a coherence length of $l_c \approx 100$ m. In the latter case the need for interferometer branches of approximately equal length becomes less important. The laser should be stabilized, however, to avoid phase shifts due to a wavelength shift (see Section 2.2.3). It takes about 30 minutes for the Argon ion laser to become stable.
4.2.2 Using a semi-conductor laser

One of the goals of this project was the development of a transportable, cheap measuring system for industrial purposes. To realize this a small semi-conductor laser is crucial. At the moment there is a substantial growth in the industrial development of semi-conductor lasers. Diode lasers have several advantages over conventional gas lasers. First, they are very small, and require only relatively low voltages (about 15 V) as compared with voltages of about 1 kV required to power gas lasers. Second, they can have coherence lengths of several meters, without the use of an etalon. Finally, the costs of diode lasers should be less and the lifetimes longer than those of gas lasers (see WYKES and FLANAGAN [1987]). To perform speckle interferometric measurements, however, an output power less than 1 W will be insufficient, especially when big object surfaces (0.5 m x 0.5 m) must be analyzed. With speckle averaging techniques (see Section 5.2.3) a lower output power will be sufficient.

4.3 The interferometer

4.3.1 General description

In a speckle interferometer the used laser beam is splitted into two beams; one beam is redirected to the object and the other beam (reference beam) is redirected to the detector. The object illumination direction can be changed by blocking one of the two illuminating beams (see Section 4.3). Optical fibers were used to redirect the different beams in the optical arrangement. In front of the detector the reference and object beam are recombined to produce the speckle interference pattern. The optical arrangement for a speckle interferometer and the optimization of such an interferometer is described in more detail by MAAS [1991].

4.3.1 The sensitivity vector

An important feature of an interferometric set-up is the sensitivity vector. In Fig. 4.2 a schematic drawing of a displacement measurement is shown to explain the relation between the measured phase change $\Delta \phi_o$ in the interferometer (see Chapter 3) and the amount of displacement $L$ of an object
point. Vectors \( \mathbf{R} \) and \( \mathbf{r}_1 \) lie in the plane defined by \( \mathbf{O}, \mathbf{P}, \) and \( \mathbf{Q} \), and \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \) are the propagation vectors of the light illuminating \( \mathbf{P} \) and the light scattered towards the observer, respectively. It can be shown (see VEST [1979, p. 70]) that the phase difference measured by the observer is

\[
\Delta \phi_o = (\mathbf{k}_2 - \mathbf{k}_1) \cdot (\mathbf{r}_1 - \mathbf{r}_2) + \Delta \mathbf{k}_1 \cdot \mathbf{r}_2 + \Delta \mathbf{k}_2 \cdot (\mathbf{R} - \mathbf{r}_2). \tag{4.1}
\]

In practical systems the magnitudes of \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) are much larger than \( \mathbf{L} \), so for practical purposes \( \Delta \mathbf{k}_1 \perp \mathbf{r}_2 \) and \( \Delta \mathbf{k}_2 \perp (\mathbf{R} - \mathbf{r}_2) \). Because of these relations the last two scalar products in Eq. (4.1) vanish, and we arrive at the relation:

\[
\Delta \phi_o = (\mathbf{k}_2 - \mathbf{k}_1) \cdot \mathbf{L} = K \cdot \mathbf{L}, \tag{4.2}
\]

where \( K = \mathbf{k}_2 - \mathbf{k}_1 \) is called the sensitivity vector. Thus, with interferometric displacement measurements, one measures the component of the displacement vector \( \mathbf{L} \) in the direction of the sensitivity vector \( \mathbf{K} \). In the interferometric set-up, shown in Fig. 4.1, the object illumination direction can be altered by alternatively blocking one of the illuminating beams. Changing the

---

**Fig. 4.2** The sensitivity vector of an interferometric displacement measurement.
illumination direction changes the sensitivity vector of the interferometric system.

4.4 The piezo-electric translator

Phase stepping and phase shifting in the reference beam is achieved using a mirror in the reference beam mounted on a Burleigh piezo-electric translator (PZT). In the case of phase-stepping the mirror in the reference beam is located at four distinct positions to introduce the additional phase shifts $\delta_j$. With the phase-shifting technique a sawtooth voltage is set on the piezo-electric transducer. The mirror is translated normally to its surface with constant velocity from a start to an end position, again and again. During this translation the required interference patterns are recorded. A study of the influence of non-linearities of the PZT on the phase measurements is done by AI [1987] and V. WINGERDEN [1991]

4.5 The CCD-camera

The camera that was used in the experimental system was a MX high resolution CCD-camera (HTH$^2$) with the NXA 1011 frame transfer chip (Philips$^3$). Specifications of the camera are given in table 4.1. The pixels of the camera have an aspect-ratio vertical:horizontal of about 3:2 (the camera

<table>
<thead>
<tr>
<th>Number of pixels</th>
<th>604 (H) x 575 (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pixel size</td>
<td>10 $\mu$m x 15.6 $\mu$m</td>
</tr>
<tr>
<td>Light sensitive area</td>
<td>7.5 mm diagonal</td>
</tr>
<tr>
<td>Resolution*</td>
<td>460 (H) X 450 (V)</td>
</tr>
<tr>
<td>Light sensitivity</td>
<td>3 lux with F=1.4</td>
</tr>
<tr>
<td>Signal-to-noise ratio</td>
<td>&gt; 48 dB</td>
</tr>
<tr>
<td>External sync</td>
<td>possible</td>
</tr>
<tr>
<td>Gamma correction</td>
<td>gamma = 1 or 0.45</td>
</tr>
<tr>
<td>Power consumption</td>
<td>1.9 W</td>
</tr>
</tbody>
</table>

*Table 4.1 Specifications of the used HTH CCD-camera. *This resolution is specified by the manufacturer. It is lower than the actual number of pixels due to inherent camera characteristics.
was used in the interlaced mode, so the effective vertical size of a pixel was 7.8 μm. The computed phase change \( \Delta \phi(x,y) \) is corrected by interpolation in the horizontal direction to get the correct phase values.

The camera was connected to the framegrabber in a pixel synchronous mode. The internal clock of the camera was switched off by connecting the camera to a clock signal with a frequency greater than 22.5 MHz. This clock signal was provided by a Datacube MaxScan Board (see Section 4.6.2). The elimination of video jitter, thus achieved, causes a significant reduction of the noise in the digitized interference patterns.

### 4.6 The Ironics computer system

#### 4.6.1 AD/DA converter board

A DataTranslation digital-to-analog convertor has been used to control the piezo-electric transducer. Two modes have been used to control the Burleigh transducer which in turn controls the mirror in the reference beam of the speckle interferometer. In the first mode the DA-convertor has been used as a switch to start and stop the ramp voltage fed to the PZT. This mode has been used during phase shifting. Putting a voltage on the input of the PZT switches the voltage ramp on and off. In the second mode the input voltage was used to place the mirror in a predefined position. This mode has been used during phase stepping.

During the acquisition of the interference patterns different values were fed to the DA-convertor to produce the desired phase steps of \( k\pi/2 \). The resolution of the DA-convertor is 12 bits and the maximum displacement of the piezo is 2 μm, leading to a displacement resolution of 0.5 nm. A mirror displacement of about 65 nm leads to a phase step of \( \pi/2 \).

#### 4.6.2 Datacube image processing modules

The microcomputer is equipped with a number of Datacube image processing modules. In Fig. 4.3a the pipeline configured to measure the phase change \( \Delta \phi \) modulo \( 2\pi \) radians is shown. The pipeline consists of the following modules:

- one framegrabber to digitize the interference patterns (MaxScan SC).
Section 4.7 General- and special-purpose software

- a 2.0 Mbyte and a 0.5 Mbyte ROI-stores (ROI = Region Of Interest) to store arrays of pixels (RS0.5 and RS2.0).
- two signal processors to perform simple arithmetic on pixel streams (MaxSp's MS1 and MS2).
- one multiplexer containing a 256 x 256 8 bit hardware look-up table (MaxMux MU).
- one display module for displaying the results and graphical needs (MaxGraph GH).

In Fig. 4.3b another scheme is shown, illustrating the different connections between the modules. The initialization of each module is done by the Ironics host computer.

![Diagram](image)

**Fig. 4.3a** A configuration of Datacube image-processing modules to compute the phase change on a 512 x 512 grid in 240 ms.

### 4.6.3 Pixel-synchronous grabbing

To improve the accuracy of the measurements, a pixel-synchronous mode has been used during the acquisition of the interference patterns. Each pixel of the CCD-array has been transferred to one array location on the MaxScan acquisition board. This acquisition board has the ability to control the HTH-camera. Instead of an internal camera pixel clock, the clock of the MaxScan board was used to digitize the video-signal. The pixel-synchronous acquisition, however, leads to non-square pixels, because the pixel size is determined by the
physical pixel size of the CCD-array. With the equipment used it was not possible to use square pixels in combination with pixel-synchronous acquisition. Because the speckle size almost equals the pixel size, it was important to choose for the pixel-synchronous acquisition mode. A significant reduction of video jitter has been achieved by this technique. The standard deviation of the detected intensities decreased from about 6 (standard acquisition) to 3 (pixel-synchronous acquisition).

**Fig. 4.3b** A schematic picture of the Datacube image-processing modules. Each board has a number of input-, output- and timing-ports. The lines indicate the different data flows and control flows from one board to another.
4.6.4 ROI Mathematics

After digitization a sub-array of 512 x 512 8 bit pixels is sent from one module to another through the MAXbus with a pixel rate of 10 MHz. Pixel

Fig. 4.4 Time-space scheme of the computation of the phase change modulo $2\pi$ in 6 video frame times (T6 is used to stabilize the mirror). The code in the bottom left corners indicates the position of the data on the processing module. Four 512 x 512 images of 16 bits can be stored on the ROI-store. So, for example the code $2M$ means: Image is stored in the MOST significant part (8 bits) of the second array on the framestore.
stream delays due to each processing module are compensated with several delay lines on the modules. Datacube software has been used to control the image processing hardware. ROI mathematics have been used to map the correct pixels on each other after the processing by one of the hardware modules. With the ROI mathematics it is possible to define the the start and end position of the array to be processed. During our measurements 256 x 256 or 512 x 512 arrays have been used. The Datacube hardware and software, however, provide the possibilities to process and transfer every user-defined part of the image.

4.6.5 ‘Real time’ processing

Every 240 ms (6 video frame times) a new phase change modulo 2π radians is calculated with the configured pipeline. One extra frame time is needed to reset the mirror to the stable zero position. A time-space scheme of the computation, performed by the configured pipeline, is shown in Fig. 4.4.

4.7 The Apple Macintosh IIx computer system

To study the possibility of configuring an inexpensive, portable system for interferometric measurements the optical set-up is coupled to an Apple Macintosh IIx computer system, equipped with a DataTranslation framegrabber, and a DataTranslation AD/DA-convertor board. With this system the same measurements as with the Ironics computer system were performed. Major differences between the systems are:

- **less accurate digitizing with the DataTranslation framegrabber.**
- **no possibility for real time measurements.**
- **more computational power for floating point calculations** *MC68881 coprocessor*.

In combination with a semi-conductor laser, the Macintosh system will be a powerful and user-friendly system to perform quantitative analysis of interference patterns.

4.8 General- and special-purpose software

After the phase measurement on a 512 x 512 grid, further computations are done using general-purpose computers. The image processing software
environment is TCL-image\textsuperscript{8}, a software package containing basic image processing routines. For time-consuming calculations, e.g. floating point operations, a SUN3/MC68020 workstation with a MC68881 co-processor was used. All the software, described in this thesis is part of the QUAIN\textsuperscript{T} software package. The package is now running on different SUN\textsuperscript{9} systems, other UNIX systems and on the Apple Macintosh II systems integrated in the TCL-image image processing software package. A part of the package has been implemented in the TIM\textsuperscript{10} package that runs on MS-DOS\textsuperscript{11} systems.

\begin{flushleft}
1. Ironics, Santa Barbara, USA
2. HTH, Eindhoven, The Netherlands
3. Philips, Eindhoven, The Netherlands
4. Datacube, Massachusetts, USA
5. DataTranslation, Marlboro, USA
6. Burleigh Instruments, East Rochester, USA
7. Apple Computer, Cupertino, USA
8. TCL-image, TPD/TNO Delft, The Netherlands
9. Sun Microsystems Europe, Zeist, The Netherlands
10. TIM, DIFA Measuring Systems, Breda, The Netherlands
11. Microsoft Corporation, Redmond, USA
\end{flushleft}
Chapter 5

Computation of the continuous phase map

In this chapter a sequence of image processing algorithms for the analysis of interference patterns generated by a phase stepping speckle interferometer is discussed. The goal is the accurate determination of the phase change related to the displacement of an object.

5.1 Introduction

In our phase-stepping measurements the phase change is accurately calculated from eight digitized interference patterns using a four-bucket phase-stepping algorithm. Four patterns are recorded before and four patterns are recorded after deformation of the object. Digital image processing algorithms have been developed for phase unwrapping, phase restoration and phase fitting. The algorithm for phase unwrapping propagates through the image and takes several pixels into account to detect $2\pi$-steps in the phase data. Phase restoration is be used to correct the values of pixels with low accuracy. Phase fitting is applied to smooth the phase data and to compute the spatial first derivatives of the phase. During the processing steps a binary mask is used to solve the problem of invalid areas.
5.2 Computation of the phase and detection of invalid pixels

5.2.1 A 2D Look-up table to compute the phase

The determination of \( \phi_o(x,y) + \phi_s(x,y) \) in Eq. (3.10) is performed by calculating the numerator and denominator from the measured intensities and applying a 2D look-up table that outputs the phase value modulo 2\( \pi \). A characteristic feature of speckle interferometry is the loss of accuracy in points with low modulation caused by low-intensity speckles or areas with low reflectance. Consequently, the resulting image contains invalid pixels, i.e. pixels with an inaccurate or completely erroneous phase value. These pixels would disturb subsequent processing of the data. They are identified by considering the expression for the modulation intensity amplitude \( B(x,y) \):

\[
B(x,y) = \frac{1}{2} \sqrt{(I_4(x,y) - I_2(x,y))^2 + (I_1(x,y) - I_3(x,y))^2}.
\]  

(5.1)

Pixels with the same modulation intensity amplitude \( B_0 \) lie on a circle in the 2D look-up table with its center at the origin of the \((I_4 - I_2),(I_1 - I_3)\)-plane and a radius \(2B_0\). Hence, pixels with a modulation intensity amplitude below a
Section 5.2  Computation of the phase and detection of invalid pixels

![Image](image.png)

**Fig. 5.2**  (a) The 2D arctangent look-up table decoded as grey values (0-255) and (b) a speckle intensity pattern formed by a flat test object.

A certain threshold $B_T$ can be detected (see VROOMAN and MAAS [1989c]) using a circular area in the 2D look-up table with a radius $2B_T$ determining the modulation intensity amplitude threshold (Fig. 5.1). This yields a binary

![Image](image2.png)

**Fig. 5.3**  (a) The computed phase after using the four-bucket technique and (b) the mask constructed during the phase measurement. Valid pixels are represented in black and invalid pixels are represented in white.

65
image $M(x,y)$ containing a mask that will be used during further processing of the data. The size of the circular area depends on the amount of noise in the intensity data. Points with no modulation at all ($B(x,y) = 0$) should always be detected. To achieve this, a practical choice for the threshold $B_T$ is $4s_n$, where $s_n$ is the standard deviation of the noise. Based upon the Chebyshev inequality (see DAVID AND ROBERT [1958]) no more than 6% (1/16) of pixels with no modulation will be assigned a phase value independent of the noise distribution. If the noise is normally distributed the percentage will be much lower than 6%. In Fig. 5.2a the look-up table is shown decoded as grey values (0-255 grey value range; the radius of the marked area equals 12). Pixels having the maximum grey value 255 in at least one of the four interference patterns are assumed to be saturated and thus invalid. These invalid pixels are also registered in the binary image $M(x,y)$. This mask is processed during phase unwrapping and phase restoration to identify object and background areas. In Fig. 5.2b a typical recorded speckle intensity pattern is shown formed by a simple flat object in the interferometer. In Fig. 5.3a the computed phase $\phi_o(x,y) + \phi_s(x,y)$ is shown after the four-bucket algorithm with a phase step of $\pi/2$. In Fig 5.3b the mask $M(x,y)$ is shown constructed during the phase measurement. Valid pixels are represented in black and invalid pixels are represented in white.
Section 5.2 Computation of the phase and detection of invalid pixels

As was shown in Chapter 3 (Eq. (3.11)), the phase change $\Delta \phi(x,y)$ due to e.g. a displacement of the object can be computed with:

$$\Delta \phi(x,y) = \phi_{o,after}(x,y) - \phi_{o,before}(x,y),$$  
(5.2)

In Fig. 5.4a the computed $\Delta \phi(x,y)$ is shown after a tilt of the object. One line of the image is shown in Fig. 5.4b. The wrapped phase data is already visible, but the image is very noisy.

5.2.2 Computing the average modulation and phase step

Look-up tables are also used to calculate the average modulation and the average phase step. The modulation at position $(x,y)$ can be calculated using Eq. (5.1). As in Section 5.2.1, the intensity differences are used as indices for a 2D look-up table yielding the value for modulation intensity amplitude $B(x,y)$. The computed values in the image were averaged over the complete image to get an estimate for the average modulation during the measurements. With the measurements described in this thesis an average modulation of about 20 was reached (in a total range of 0-255).

To check if the phase step introduced (see Chapter 4) was correct, the average phase step can be calculated from the four speckle patterns. It is shown by JÜPTNER, KREIS and KREITLOW [1983] that the phase step $\delta_i - \delta_{i-1}$ (see Section 3.1) can be expressed as:

$$\delta_i - \delta_{i-1} = \arccos \left[ \frac{(I_1(x,y) - I_2(x,y)) + (I_3(x,y) - I_4(x,y))}{2(I_1(x,y) - I_2(x,y))} \right].$$  
(5.3)

A 2D look-up table was used to compute $\delta_i - \delta_{i-1}$. The average value of the phase step was computed to check the introduced phase step. The value supplied to the DA-converter was adjusted if the phase step deviated more than 2% from $\pi/2$ (when using the four-bucket algorithm). A number of researchers (v. WINGERDEN [1991], CREATH [1988]) has investigated the influence of an error in the phase step on the measured phase change $\Delta \phi(x,y)$.

5.2.3 Averaging different speckle patterns

To improve the phase-stepping measurements, a speckle averaging technique can be used to increase the signal-to-noise ratio. When, for example, the phase change $\Delta \phi(x,y)$ is very small or the amount of light reflected by the object is
too low, this technique is very useful. Instead of 'grabbing' four speckle interference patterns, a large number of patterns (8, 12, etc) are recorded. These intensity patterns are averaged yielding the four intensity patterns used in the four-bucket technique. The averaging of the patterns is based on a variance-criterion. Suppose there are four speckle patterns, called 'sp1', 'sp2', 'sp3' and 'sp4'. Another set of four patterns is recorded, which are called 'sp5', 'sp6', 'sp7' and 'sp8'. Then the variance is computed of 'sp1-sp5', 'sp1-sp6', 'sp1-sp7' and 'sp1-sp8'. The lowest variance determines the perfect couple. The other images are coupled in sequential (cyclic) order. So, for example, when 'sp1-sp6' has the lowest variance, the couples of images to be averaged are ('sp1','sp6'), ('sp2','sp7'), ('sp3','sp8') and ('sp4','sp5'). The effect of the averaging is a decrease of the modulation proportional to $N_a$ and a decrease of the noise proportional to $(N_a)^2$, where $N_a$ is the number of averaged speckle patterns. For a more detailed description of this averaging technique see MAAS [1991].

5.3 Phase unwrapping

5.3.1 The straightforward algorithm

Because $\phi_o(x,y) + \phi_s(x,y)$ is determined modulo $2\pi$, $2\pi$-steps and $4\pi$-steps may occur in the phase change $\Delta \phi_o(x,y)$. The removal of these steps, called phase unwrapping, is necessary to make the phase data continuous. A simple algorithm to unwrap the phase data is based on adding an offset (a multiple of $2\pi$) to each pixel value. Starting at the top left pixel of the image, the offset is set to zero. Then the first column is scanned to determine the offsets of the

![Fig. 5.5 Scanning the data line-by-line during phase unwrapping.](image)
first pixel of each row. Finally for each row the offsets are calculated starting
with the offset in the first column. The offset changes each time a $2\pi$-step is
detected. A $2\pi$-step is detected if the absolute difference between a pixel and
the previous pixel exceeds $\pi$. One can also start the unwrapping procedure at
the center of the image.

The scanning method mentioned above has been successfully used in classical
interferometry (see BRUNING [1974]). Due to speckle decorrelation and

![Image](image_url)

*Fig. 5.6  Artefacts during phase unwrapping with the 'classical' algorithm.*

noise this method is not suitable for speckle interferometry. The $2\pi$-steps can
be hidden in speckle noise causing steps to be neglected or offsets to be added
at the wrong points. In Fig. 5.6, the artefacts are shown when this algorithm is
used to unwrap the data, shown in Fig. 5.4a. Unwrapping algorithms consist,

![Diagram](image_url)

*Fig. 5.7  Scanning the data 'spirally' to take more pixels into account.*
in fact, of two steps: Scanning through the image and detecting discontinuities. These two steps are not, however, independent of each other. Different algorithms have been developed to increase the performance of each step. They will be discussed in detail in the next sections.

5.3.2 Spiral scanning of the data

We have developed an algorithm, that spirally scans the image starting at the center (Fig. 5.7). In this way more pixels can be taken into account to detect a discontinuity. The pixel value is not compared to the previous pixel, but to the mean of a set of previously unwrapped pixels in a $3 \times 3$ neighborhood. The data is spirally scanned in order to maximize the number of unwrapped pixels in that neighborhood. In most of the neighborhoods 4 unwrapped pixels can be taken into account to detect a $2\pi$-step. In our experimental system taking 3 unwrapped neighbors into account proved to be sufficient in most cases. An

![Fig. 5.8 Scanning the data using a pixel queue neglecting invalid pixels.](image)

important restriction of the straightforward algorithm and the 'spiral-scan' algorithm is that all pixels in the input image have to be valid data pixels. If
there are invalid pixels, these pixels have to be corrected before (or neglected during) phase unwrapping. The correction of invalid pixels is discussed in Section 5.6.

5.3.3 Using a pixel queue or pixel buckets

The computation of a continuous phase map can be improved by excluding the invalid pixels from the unwrap procedure. We have developed an algorithm based on a pixel queue (see v. VLIET and VERWER [1988]) to scan the data like a fluid spreading over the object but around invalid pixels (Fig. 5.8). A pixel queue is a one-dimensional data structure. Pixel addresses are stored on one side of the queue and fetched from the other side. In Fig. 5.9 a flowchart of the algorithm is shown. In Fig. 5.10 the result of the algorithm performed on the data shown in Fig. 5.4a is shown (after a contrast stretch to get a grey

Fig. 5.9 A flowchart of the phase unwrapping algorithm using a pixel queue.
value range of 0-255). A few remarks must be made. If no valid start pixel can be found in the 3 x 3 neighborhood of the start location, the procedure stops and another start location has to be selected. If separate areas occur in the image, i.e. areas that are not connected by a path of valid pixels, each area has to be processed by choosing an appropriate start location.

Only pixels that are 4-connected to (share an edge with) the current pixel are put on the pixel queue; the processing therefore propagates in a diamond shape. If the 8-connected neighbors (pixels that share a corner or edge with the current pixel) are taken into account a square propagation occurs. Because errors are more likely at the corners of a propagation front (less pixels are present in the 3 x 3 neighborhood to be considered during step detection), new algorithms have been developed that provide a circular propagation front (see VERWER, VERBEEK and DEKKER [1989]). These algorithms are based on 'pixel buckets' instead of a 'pixel queue'. With the use of 'pixel buckets' it is possible to use metrics for which the distance from one pixel to another differs from the 'chessboard' distance or 'city block' distance. The advantage of this technique - the use of an alternative metric - is that better approximation to the Euclidian metric may be achieved on the digital grid. For example the 'Chamfer distance' (see Fig. 5.11) can be used with coefficients 5 (horizontal and vertical), 7 (diagonal) and 11 (knight's move) and achieves a distance

*Fig. 5.10 The computed phase after using the unwrapping technique described in Section 5.3.3.*
measure that deviates from ordinary Euclidian distance by less than 1.79 % (see BORGEFORS [1983], VERWER [1991]).

\[
\begin{array}{ccc}
2 & 1 & 2 \\
1 & x & 1 \\
2 & 1 & 2 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
1 & x & 1 \\
1 & 1 & 1 \\
\end{array}
\]

City block metric \hspace{2cm} Chessboard metric

\[
\begin{array}{ccc}
7 & 5 & 7 \\
5 & x & 5 \\
7 & 5 & 7 \\
\end{array}
\quad
\begin{array}{ccc}
11 & 11 \\
11 & 7 & 5 & 7 & 11 \\
11 & 7 & 5 & 7 & 11 \\
11 & 11 \\
\end{array}
\]

3 x 3 Chamfer metric \hspace{2cm} 5 x 5 Chamfer metric

*Fig. 5.11* A schematic drawing of local distances on a digital grid.

The unwrapping of a pixel is done by adding or subtracting $2\pi$ until the difference with the average of the values of the unwrapped pixels in a 3 x 3 neighborhood is less than $\pi$. During the phase unwrapping the mask $M(x,y)$ is used to identify valid pixels. After the phase unwrapping, $M(x,y)$ stays ‘zero’ (valid pixel) if the phase is unwrapped at $(x,y)$ and becomes ‘one’ (invalid pixel) at the remaining locations.

### 5.3.4 The step detection criterion / detection tolerance

There are several ways to detect a discontinuity in the phase data. The step detection criterion depends of course on the way the data is scanned. In the straightforward algorithm the pixel preceding the current pixel is used as a reference. If the difference between the grey value of the current pixel and the grey value of the preceding exceeds $\pi$, a step is detected. In the case of the
‘spiral scan’ a number of unwrapped pixels (mostly 3 or 4) are averaged and the average value is used in the detection step. And, as we saw in Section 5.3.3, the unwrapped pixels in a 3 x 3 neighborhood are used in the case of the more advanced algorithms.

Because there are still situations that can go wrong (when the derivative of the phase data is high or much noise is present), a tolerance factor was introduced in the detection criterion. Instead of:

\[
\text{no step present if } \quad |g - g_r| < \pi \\
\text{step present if } \quad |g - g_r| \geq \pi, \quad (5.4)
\]

where \( g \) is the grey value of the current pixel and \( g_r \) is the reference grey value (the mean value of the valid pixels in the 3 x 3 neighborhood), the following criteria are used:

\[
\begin{align*}
\text{no step present if } & \quad |g - g_r| < F_t \pi \\
\text{unreliable pixel if } & \quad F_t \pi \leq |g - g_r| < (2 - F_t) \pi \\
\text{step present if } & \quad |g - g_r| \geq (2 - F_t) \pi, \quad (5.5)
\end{align*}
\]

Fig. 5.12  (a) Using a tolerance factor: Unreliable areas during the detection of 2\( \pi \)-steps depending on a tolerance factor. (b) The slope of the ‘fringes’ has an upper limit in order to have a successful unwrapping procedure.
where $F_t$ is a tolerance factor ($0 \leq F_t \leq 1$). If $F_t = 0$, all pixels are assumed to have an inaccurate value. If $F_t = 1$ all pixels are assumed to have an accurate value. But, there are always inaccurate phase values due to speckle decorrelation (see Section 3.3.6), which are not detected by the circular area in the look-up table. In practice $F_t = 0.5$ seems to be a reasonable choice (see Section 5.3.5). In Fig. 5.12 the idea of using a tolerance factor is illustrated. When a tolerance factor is used, unreliable pixels are considered to be invalid and the corresponding locations in the $M(x,y)$ are set to ‘one’. So the number of invalid pixels increases, when a tolerance factor less than 1 is used. Another effect of the tolerance factor is that the number of fringes ($2\pi$-steps) along a straight line in the image has an upper limit. Consider Fig. 5.12b. In the worst case only one neighboring pixel of the pixel to be unwrapped is a valid pixel. In that case the increase in grey value ($s$) has to be less then $F_t \pi$, otherwise it will be considered an unreliable pixel. This condition leads to:

**Fig. 5.13** Tolerance factor versus the number of fringes that can be unwrapped successfully measured for a $256 \times 256$ image. The straight line is the theoretical function.
where $N_p$ is the number of pixels along the line and $N_f$ is the number of fringes to be unwrapped. The number of fringes that can be successfully unwrapped as a function of the tolerance factor for a 256 x 256 computer generated interference pattern is shown in Fig. 5.13. At a value of about $N_f = 64$, a limit is reached. This seems to be the upper limit for this phase unwrapping technique. This upper limit can be explained by the fact that at least 3 pixels per fringe are needed for this phase unwrapping technique.

5.3.5 Performance of the unwrapping algorithms

Which artefacts are introduced by the different unwrapping algorithms?

In Fig. 5.6 the artefacts caused by the classical unwrapping algorithm were shown. It is possible to reduce these kind of artefacts by applying a median filter technique (see PRATT [1978, p. 330]). In that case, however, valid data points are also filtered. Further, when the artefacts extend over more than one line (when, for example, an error occurs during the unwrapping of a column),

**Fig. 5.14** (a) Artefacts occurring during unwrapping: Artefacts generated by the spiral-scan algorithm and (b) artefacts due to the pixel queue unwrapping technique.
Section 5.3  Phase unwrapping

a larger window size for the median filter is needed, which will in turn influence the data even more. The classical algorithm performs very poorly on speckle interference patterns. A completely unwrapped pattern was never successfully computed during test measurements.

As indicated in Sections 5.3.2 and 5.3.1, an important restriction of the straightforward algorithm and the ‘spiral-scan’ algorithm is that all pixels in the input image have to be valid data pixels, while with the algorithm using the pixel queue, invalid pixels are neglected during phase unwrapping. The ‘spiral-scan’ algorithm, however, performs much better than the

![Figure 5.15](image)

**Fig. 5.15**  (a) The number of invalid pixels introduced as a function of $B_T$ and (b) the total number of invalid pixels due to the modulation threshold and the tolerance factor versus the number of fringes in the image.

straightforward algorithm. The kind of artefacts generated by the the ‘spiral-scan’ and ‘pixel queue’ techniques are shown in Fig. 5.14. In these cases a complete area will be erroneous instead of just a pattern of lines. All artefacts represent an accumulating effect. If the unwrapping goes wrong at a cluster of pixels, the error propagates in the scan direction.

**How many invalid pixels are introduced during the computation of the phase?**

The look-up table with the marked area at the center and the use of a tolerance factor have improved the unwrapping technique. To get an idea of how many
invalid pixels are introduced in the image by using the modulation amplitude threshold (see Section 5.2.1), the number of invalid pixels was measured for different speckle interference patterns as a function of $B_T$. The result is shown in Fig 5.15a. Curves 1 and 2 are measured for two 256 x 256 images with a different speckle size. The speckle size of the interference pattern in image1 was approximately 3 µm and the speckle size for image2 was about 12 µm. Because of the difference in speckle size, the average modulation (see Section 5.2.2) also differs. The average modulation for image1 was 17.6 and for image2 it was 28.0. The image for which curve 3 was computed was an image in which a large part was 'background' (modulation ≈ 0). The average modulation in the complete image was 7.6. The curve quickly reaches a point where the entire 'background' has become invalid (as it should).

**How many invalid pixels can be tolerated during this unwrapping technique?**

If one starts with an empty mask (all pixels 'zero') and the pixels in the mask are randomly set to 'one' (uniformly distributed), the right border of the image is connected to the left border of the image with an 8-connected line of pixels at the level of about 40 % invalid pixels (experimentally determined). At that point the object in the image splits into two and will not be unwrapped in one pass. If one increases the number of pixels, the objects in the image splits into a large number of small, connected areas. Unwrapping then becomes far too complicated. It will also become very difficult to select a start position in the image that contains a valid data pixel. In a speckle pattern the invalid pixels do not have to be distributed uniformly over the image; they may cluster in patterns with a large speckle size. Further, unwrapping errors are more likely to occur when there are only one or two valid neighbors. The maximum number of invalid pixels that can be tolerated is therefore about 30 %. Experiments have shown that this number is a proper guide line.

**How many invalid pixels are introduced due to the tolerance factor?**

Both the use of a modulation amplitude threshold and the use of a tolerance factor will increase the number of invalid pixels. The number of invalid pixels due to the modulation amplitude threshold will be known before phase unwrapping (after applying the 2D look-up table). The number of invalid pixels due to the tolerance factor increases during the phase unwrapping. In Fig 5.15b two plots are shown of the measured number of invalid pixels versus the number of fringes in the image with a tolerance factor $F_T = 0.5$ (for $B_T = 0$ and $B_T = 4$). The maximum number of fringes that can still be unwrapped is
experimentally found to be about 35. If \( B_T = 4 \) and \( F_I = 0.5 \) the number of invalid pixels after unwrapping 35 fringes is about 30\% (see Fig 5.15b). An increase of the number of fringes causes too many errors and only a small part around the start point will be unwrapped. The other pixels are all considered invalid.

5.4 Phase masking and restoration

5.4.1 Nearest neighbor substitution

Two methods are used to restore the phase value of invalid pixels. The first one substitutes the value of an invalid pixel with the value of a valid neighboring pixel (3 x 3 neighborhood). This restoration technique can be performed before phase unwrapping. Averaging neighboring pixels is not possible because \( 2\pi \)-steps may be present. If no valid pixel is available in the neighborhood, the pixel remains invalid. A repeated application of this technique would eventually correct all invalid pixels, but the systematic error in the output data would increase.

5.4.2 Averaging valid pixels in the neighborhood

The phase unwrapping algorithm described in Section 5.5.3 permits invalid pixels to exist in its input image. After phase unwrapping, these pixels can be replaced by the average of the valid pixels in their 3 x 3 neighborhood. As with the previously discussed restoration technique, a number of iterations can

![Fig. 5.16 Processing of the original mask (left) to separate objects from the background. First, an 8-connected erosion is performed to remove small objects (middle) and then an 8-connected dilation is performed to restore the big objects (right).](image)

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be performed to restore the data. This phase restoration is not necessary if further processing steps make use of the mask \(M(x,y)\) to identify invalid pixels. For each pixel that is restored, the related location in \(M(x,y)\) is set to zero. This corresponds to an 8-connected binary erosion (see ROSENFELD and KAK [1982]) of the mask in \(M(x,y)\). After a number of iterations small clusters of invalid pixels (dark speckles) disappear and large clusters (dark areas) are eroded. After phase unwrapping and phase restoration, the same number of 8-connected binary dilations (see ROSENFELD and KAK [1982]) can be applied to the mask in \(M(x,y)\) to produce a mask that separates the objects from the background (Fig. 5.16).

5.5 Phase smoothing and computation of derivatives; Phase fitting to valid data points

Because of the occurrence of invalid pixels, a convolution filter is inappropriate to determine the first derivatives of the phase with respect to \(x\)- and \(y\)-coordinates. We use a linear least squares (LLS) fit of a plane to the set of valid pixels in a rectangular neighborhood of each valid pixel, yielding the derivatives with respect to both coordinates at the same time. This enables us to measure the in-plane strain near invalid areas. As will be shown, this method can also be applied successfully as a smoothing filter.

![Diagram](image)

**Fig. 5.17** Fitting a plane in a rectangular neighborhood of \((x,y)\) to estimate the first spatial derivatives of the data.

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Section 5.5  Phase smoothing and computation of derivatives

For each pixel in the image a rectangular neighborhood is considered (Fig. 5.17). This neighborhood contains a set of \(N(x,y)\) valid data points \((m,n,g(m,n))\), where \((m,n)\) is the position of a data point relative to \((x,y)\) and \(g(m,n)\) is the grey value at that position. A LLS fit of a plane at location \((x,y)\) is performed by minimizing the following expression:

\[
S(x,y) = \sum (am + bn + c - g(m,n))^2. \tag{5.5}
\]

where the summation is performed over all valid data points in the rectangular neighborhood.

Minimizing Eq. (5.5) is done by solving the following set of equations:

\[
\frac{\delta S(x,y)}{\delta a} = 0, \quad \frac{\delta S(x,y)}{\delta b} = 0, \quad \frac{\delta S(x,y)}{\delta c} = 0 \tag{5.6}
\]

or, substituting Eq. (5.5):

\[
a \sum (m^2) + b \sum (nm) + c \sum (m) = \sum (mg) \]

\[
a \sum (mn) + b \sum (n^2) + c \sum (n) = \sum (ng) \tag{5.7}
\]

\[
a \sum (m) + b \sum (n) + cN = \sum (g)
\]

This matrix equation is solved for each data point, assuming its coordinates to be \((0,0)\) and its location to be at the center of a square neighborhood. If no marked pixels are present in the neighborhood, the set of equations reduces to:

\[
a = \frac{\sum (mg)}{\sum (m^2)}
\]

\[
b = \frac{\sum (ng)}{\sum (n^2)} \tag{5.8}
\]

\[
c = \frac{\sum (g)}{N}
\]
If marked pixels are present, a matrix inversion is used to solve Eq. (5.7) and the marked pixels are excluded from the set of data points. This allows the LLS fit to be performed near the edges of the object as well.

Fig. 5.18  Shifting the window over the mask with invalid pixels.

Because of the occurrence of invalid data points, all summations involved in the solution of Eq. (5.7) have to be calculated each time the window (neighborhood) shifts one pixel to the right. To reduce computation time, the summations in the window are computed using a special updating mechanism based on the use of lower order summations.

Starting at the top left of the image, the window is shifted to the right until a valid data point is located at the center of the window. For that window all summations and the number of valid data points $N(x,y)$ are calculated. For the computation of $\sum (m^2)$, $\sum (n^2)$ and $\sum (mn)$ a look-up table is used. Thus, before the processing of the image 3 tables are computed containing $\sum (m^2)$, $\sum (n^2)$ and $\sum (mn)$ for $m = 0, 1, \ldots$, ‘filter size in the $x$-direction’ and $n = 0, 1, \ldots$, ‘filter size in the $y$-direction’. Next, the window is shifted one pixel to the right and the number of invalid pixels $N(x,y)$ and the summations are updated using the values in the column added at the right-hand side and the column removed at the left-hand side. As the relative coordinates in the window change after a shift to the right, corrections are necessary after this procedure. Consider the $s \times s$ window $W(p,q)$ positioned at location $(p,q)$. This window covers $M(x,y)$ with $x = p-s$ to $p+s$ and $y = q-s$ to $q+s$. After a window shift of 1 pixel to the right the window is positioned at location $(p+1,q)$ and $\sum (m^2)$ becomes:
\[ \sum (m^2) = \sum_{n=-s}^{s} \sum_{m=-s}^{s} (m^2 M(p+1+m,q+n)) \]
\[ = \sum_{n=-s}^{s} \sum_{m=-s+1}^{s+1} (m-1)^2 M(p+m,q+n)) \quad \text{(substituting } m-1 \text{ for } m) \]
\[ = \sum_{n=-s}^{s} \sum_{m=-s}^{s} ((m-1)^2 M(p+m,q+n)) \]
\[ - \sum_{n=-s}^{s} ((-s)^2 M(p-s,q+n)) \]
\[ + \sum_{n=-s}^{s} ((+s)^2 M(p+s+1,q+n)) \]
\[ = \sum_{n=-s}^{s} \sum_{m=-s}^{s} (m^2 M(p+m,q+n)) \quad \text{(previous calculated } \sum (m^2)) \]
\[ - \sum_{n=-s}^{s} ((-s)^2 M(p-s,q+n)) \quad \text{(updating column at the left-hand side)} \]
\[ + \sum_{n=-s}^{s} ((+s)^2 M(p+s+1,q+n)) \quad \text{(updating the right-hand side)} \]
\[ - 2 \sum_{n=-s}^{s} \sum_{m=-s}^{s} (m M(p+m,q+n)) \quad \text{(previous calculated } \sum(M)) \]
\[ + \sum_{n=-s}^{s} \sum_{m=-s}^{s} (M(p+m,q+n)) \quad \text{(previous calculated } N) \]
Computation of the continuous phase map

Similar calculations can be made for the other summations. After adding and removing a column, the following corrections are therefore necessary:

\[ \Delta \{ \sum (m^2) \} = - 2 \sum (m) + N \]
\[ \Delta \{ \sum (mn) \} = - \sum (n) \]  \hspace{1cm} (5.9)
\[ \Delta \{ \sum (mg) \} = - \sum (g) \]
\[ \Delta \{ \sum (m) \} = - N \]

This updating algorithm is executed for each line in the image.

The values of \( a \) and \( b \) are estimates of the first derivatives with respect to \( x \) and \( y \). The computation of \( c \) provides a smoothing filter based upon the plane \((am + bn + c)\). If no invalid pixels are present, the computation of \( a \), \( b \) and \( c \) is equivalent to the application of convolution filters (see Eq. (5.7)) with the following kernels:

\[ a: \quad \text{kernel}_x = \frac{1}{f_a} (\ldots,-3,-2,-1,0,1,2,3,\ldots) \]
\[ \text{kernel}_y = \frac{1}{f_a} (\ldots,1,1,1,1,1,1,1,\ldots) \]

\[ b: \quad \text{kernel}_x = \frac{1}{f_b} (\ldots,1,1,1,1,1,1,1,\ldots) \]
\[ \text{kernel}_y = \frac{1}{f_b} (\ldots,-3,-2,-1,0,1,2,3,\ldots) \]

\[ c: \quad \text{kernel}_x = \frac{1}{f_c} (\ldots,1,1,1,1,1,1,\ldots) \]
\[ \text{kernel}_y = \frac{1}{f_c} (\ldots,1,1,1,1,1,1,\ldots) \]  \hspace{1cm} (5.10)

where \( f_a, f_b \) and \( f_c \) are normalization factors depending on the filter sizes in \( x \)- and \( y \)-direction. The convolution with \( \text{kernel}_x \) is performed in the \( x \)-direction and the convolution with \( \text{kernel}_y \) is performed in the \( y \)-direction. The first two convolution filters can be considered as an extended Prewitt filter (see PREWITT [1970]).
Determination of deformation, strain and shape

In this chapter applications of the image processing techniques, described in Chapters 3 and 5, are discussed. Qualitative as well as quantitative results of displacement measurement, strain analysis and shape measurement are presented.

6.1 A bottle under pressure

To investigate the possibility of performing deformation measurements with the experimental system described in Chapter 4, a bottle filled with water was placed in the interferometer. The bottle was deformed by increasing the air pressure above the water surface. The bottle was painted white to increase the reflectance. In Fig. 6.1a the recorded speckle intensity pattern of the bottle is shown (the contour of the bottle is just visible). In Fig. 6.1b the phase of each point on the surface of the bottle is shown. The phase is computed with the four-bucket phase-stepping algorithm (see Chapter 3 and 5). Pixels that belong to the background are masked because there was not enough modulation at those pixel positions. In Fig. 6.2a the measured phase change $\Delta \phi_0(x,y)$ due to the deformation of the bottle is shown (the ‘fringes’ are buried in speckle noise). In Fig. 6.2b the unwrapped phase change is shown. The invalid pixels
Determination of deformation, strain and shape

Fig. 6.1 (a) The speckle intensity pattern recorded after positioning a lemonade bottle in the interferometer and (b) the computed phase using the four-bucket phase-stepping technique.

are restored and the mask \( M(x,y) \) contains the background (see Section 5.4). Because of the grey value range 0-255, there seems to be discontinuities in the data. In Fig. 6.3a the data is shown after application of the 5 x 5 averaging

Fig. 6.2 (a) The measured phase change due to a deformation of the object and (b) the unwrapped phase change in which the invalid pixels in the object are restored and the background is completely masked.
technique described in Section 5.5. In Fig. 6.3b a single line of the data is plotted to show that the discontinuities are really removed. To get a better view of the computed data a contrast stretch is performed and iso-phase-change contours are drawn (Fig. 6.4a). Another technique to visualize the phase change is a quasi three-dimensional plot (see Section 6.2.4). A quasi three-dimensional plot of the phase change is shown in Fig. 6.4b.

For each point on the surface of the object the measured phase change $\Delta\phi_0(x,y)$ is directly related to the displacement vector $\vec{L}(x,y)$ by:

$$\Delta\phi_0(x,y) = \vec{K}(x,y) \cdot \vec{L}(x,y) = \frac{2\pi}{\lambda} \left( \vec{i}_v(x,y) + \vec{i}_l(x,y) \right) \cdot \vec{L}(x,y),$$  \hspace{1cm} (6.1)

where $\vec{K}(x,y)$ is the sensitivity vector (see Section 4.3), $\vec{i}_v(x,y)$ and $\vec{i}_l(x,y)$ are unit vectors in the illumination and viewing direction and $\lambda$ is the wavelength (see VEST [1979, p.71]).

![Image](image_url)

**Fig. 6.3** (a) The data of Fig. 6.2b after applying the smoothing technique described in Section 5.5 and (b) a single line of the data to show that the phase discontinuities are removed by phase unwrapping.

This result is generally sufficient if only a quantitative inspection of the object deformation is required. If, however, calculation of the complete three-dimensional displacement vector field is required at least three phase change measurements have to be performed. By performing three or more
measurements with independent sensitivity vectors \( \vec{K} \), the components \( L_x, L_y \) and \( L_z \) of the displacement vector \( \vec{L} \) can be calculated. If the displacement vector at one object point is known, the absolute determination of the three components at each object point is possible (see STETSON [1990]). In case of the bottle under pressure a single measurement was done, because only a qualitative investigation of the interferometric techniques was desired.

![Graphical representations of the data: (a) iso-phase-change contours (contour spacing is 24 grey levels) and (b) a quasi-3D plot of the deformation.](image)

**Fig. 6.4** Graphical representations of the data: (a) iso-phase-change contours (contour spacing is 24 grey levels) and (b) a quasi-3D plot of the deformation.

### 6.2 Strain analysis

#### 6.2.1 Basic theory

Under the influence of external forces solid bodies will be deformed, i.e. change their size and shape. Such forces can for example be load or pressure, and they are expressed locally as stresses measured as force per unit area. The effect on elastic solid state bodies will be strain concentration. The relations between strain and displacement can be developed for the simplified case of a 2D deformation (see Fig. 6.5). When the object is deformed, point \( P \) moves to \( P' \) by translations \( L_x \) and \( L_y \) in the \( x \) and \( y \) directions, respectively. If the deformation is very small, the translations of other points are given by the first two terms of Taylor series expansions, as indicated in Fig. 6.5. Extension of
this development to 3D deformation yields the complete three-dimensional description of the strain tensor.

Fig. 6.5 Small deformations of a solid. Point \( P \) translates to \( P' \) during deformation. Small displacements of other points are given by the first two terms of Taylor series expansion.

Expressed in mathematical terms, strain is a tensor (requiring nine elements) which can be represented by the matrix:

\[
S = \begin{bmatrix}
\frac{\partial L_x}{\partial x} & \frac{1}{2} \left( \frac{\partial L_x}{\partial y} + \frac{\partial L_y}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial L_x}{\partial z} + \frac{\partial L_z}{\partial x} \right) \\
\frac{1}{2} \left( \frac{\partial L_y}{\partial x} + \frac{\partial L_x}{\partial y} \right) & \frac{\partial L_y}{\partial y} & \frac{1}{2} \left( \frac{\partial L_y}{\partial z} + \frac{\partial L_z}{\partial y} \right) \\
\frac{1}{2} \left( \frac{\partial L_z}{\partial x} + \frac{\partial L_x}{\partial z} \right) & \frac{1}{2} \left( \frac{\partial L_z}{\partial y} + \frac{\partial L_y}{\partial z} \right) & \frac{\partial L_z}{\partial z}
\end{bmatrix}, \quad (6.2)
\]

where \( L_x, L_y \) and \( L_z \) are the components of the displacement vector \( \vec{L}(x,y,z) \) at the actual point \((x, y, z)\) of the body. Of these nine components, six are
independent. At any point in a solid object there are three components of normal strain:

\[ \varepsilon_x = \frac{\partial L_x}{\partial x}, \]  
(6.3a)

\[ \varepsilon_y = \frac{\partial L_y}{\partial y}, \]  
(6.3b)

\[ \varepsilon_z = \frac{\partial L_z}{\partial z}. \]  
(6.3c)

and three independent shear strains:

\[ \gamma_{xy} = \frac{\partial L_x}{\partial y} + \frac{\partial L_y}{\partial x}, \]  
(6.4a)

\[ \gamma_{yz} = \frac{\partial L_y}{\partial z} + \frac{\partial L_z}{\partial y}, \]  
(6.4b)

\[ \gamma_{zx} = \frac{\partial L_z}{\partial x} + \frac{\partial L_x}{\partial z}. \]  
(6.4c)

It is appropriate at this point to note that for very small motions the rotation of an object also can be expressed in terms of derivatives of displacement. The components \( \partial x \), \( \partial y \) and \( \partial z \) of rotation about \( x \), \( y \) and \( z \)-axes, respectively, are as follows:

\[ \omega_x = \frac{1}{2} \left( \frac{\partial L_z}{\partial y} - \frac{\partial L_y}{\partial z} \right), \]  
(6.5a)

\[ \omega_y = \frac{1}{2} \left( \frac{\partial L_x}{\partial z} - \frac{\partial L_z}{\partial x} \right), \]  
(6.5b)

\[ \omega_z = \frac{1}{2} \left( \frac{\partial L_y}{\partial x} - \frac{\partial L_x}{\partial y} \right). \]  
(6.5c)

In Eq. (6.2), \( S \) expresses a complete 3D strain tensor. Using optical methods, however, only in-plane derivatives can be determined. For that reason it is
Section 6.2  Strain analysis

It is convenient to define a local coordinate system at each point, in which the $z'$-axis represents the surface normal. $x'$ - $y'$ will then define the tangential plane, and $L_x'$, $L_y'$ and $L_z'$ the components of the displacement vector onto local coordinate axes. So defined, neither $L_z'$, nor the $\partial / \partial z'$ terms will contribute to the surface strain, which can be represented by the four upper left elements of $S$:  

$$
S' = \begin{bmatrix}
\frac{\partial L_x'}{\partial x'} & \frac{1}{2} \left( \frac{\partial L_x'}{\partial y'} + \frac{\partial L_y'}{\partial x'} \right) \\
\frac{1}{2} \left( \frac{\partial L_y'}{\partial x'} + \frac{\partial L_x'}{\partial y'} \right) & \frac{\partial L_y'}{\partial y'}
\end{bmatrix}.
$$

(6.6)

Fig. 6.6  T-shaped aluminium test objec (thickness 1 cm).

For a flat object surface perpendicular to the viewing direction, the two coordinate systems are approximately the same for each point on the object surface. In that case the in-plane strain components are the normal strains $e_x$ and $e_y$ and the shear strain $\gamma_{xy}$ and they can be expressed as:  

\[ e_x, e_y, \gamma_{xy} \]
\[ \varepsilon_x = \frac{\partial L_x}{\partial x}, \quad \varepsilon_y = \frac{\partial L_y}{\partial y} \quad \text{and} \quad \gamma_{xy} = -\frac{\partial L_x}{\partial y} + \frac{\partial L_y}{\partial x} \] (6.7)

The in-plane strain components can thus be obtained by calculating the first

**Fig. 6.7** Performing two phase change measurements with collimated illumination beams that are symmetrical with regard to the normal of the object surface.

**Fig. 6.8** (a) The phase change in the direction of the sensitivity vector and (b) the in-plane phase change in the x-direction computed by the subtraction of two phase change measurements.
derivatives of the in-plane displacement components with respect to the local coordinates.

6.2.2 Experimental results

A T-shaped aluminium test object was used to investigate the performance of the strain measurements (Fig. 6.6). A deformation was introduced by putting a weight on the tip of the arm of the object. By performing two phase change measurements with collimated illumination beams that were symmetrical with regard to the normal of the object surface, one of the in-plane displacement components could easily be calculated by subtracting the results of each measurement (see Appendix A):

\[(\Delta \phi)_2 - (\Delta \phi)_1 = (K_2 - K_1) \cdot L = (4\pi/\lambda) L_x \sin [\alpha], \quad (6.8)\]

where \(L_x\) is the displacement in the x-direction and \(\alpha\) is the angle between the surface normal and the illumination direction (see Fig. 6.7). In Fig. 6.8 the displacement in the direction of one of the two sensitivity vectors and the in-plane displacement component \(L_x\) is shown and in Fig. 6.9 the displacement components \(L_x\) and \(L_y\) after a 25 x 25 smoothing filter. The normal strain in the x-direction and the shear strain calculated from this displacement

![Fig. 6.9](image)

(a) The in-plane displacement in the x-direction and (b) the y-direction with a contour spacing of 340 nm after a 25 x 25 smoothing filter.
measurement are shown in Fig. 6.10 and Fig. 6.11.

6.2.3 Representation and interpretation of the data

Several techniques have been used to visualize the measured data. The most straightforward technique is to display the data as a grey value coded image, such as those in Fig. 6.9. A disadvantage is the appearance of 'discontinuities' in the data due to the 0-255 grey value range (pixel values out of this range are shown modulo 255). The deformation or strain, however, is better visualized for observers than the grey value image after contrast stretchting such as that in Fig. 6.10b.

To improve the interpretation of an image such as in Fig. 6.10b, contours of equal grey value are drawn in the image. The computation of these lines is performed using the following simple algorithm. First, the grey value image is processed with a maximum filter, that takes the maximum pixel value from a square neighborhood of the central pixel. The size of the window must be equal to the desired distance (in grey values) between the contours (contour-spacing). After the maximum filter the original image is subtracted from the result image. The result of the subtraction are the iso-grey-value contours.

Another technique to visualize data is the use of vectors, drawn on a grid that is placed over the displayed object (called vector plots). Especially in the case of deformation measurements, vector plots can be very useful. The size of the drawn vector equals the amount of deformation of the grid point. The direction of the vector equals the direction of that deformation. In Fig. 6.12 some vector plots of the strain measurements described in this section are shown.

Finally, a kind of quasi three-dimensional plotting has been used. In that case a wire frame is plotted using the the values of the measured data. Especially in the case of shape and vibration measurements, this plotting technique produces nice, illustrative pictures. An example of a quasi three-dimensional plot is shown in Fig. 6.13.

The described display techniques have been used in dynamic interferometry measurements as well as with the static analysis of a single interferogram (see Chapter 9).
6.2.4 Error sources and accuracy considerations

During interferometric measurements, several error sources can influence the result of the measurement. A few of them are:

- *Speckle decorrelation*
- *Phase step errors*
- *Electronic noise*
- *Photon noise*
- *Mechanical vibrations*
- *Temperature changes*
- *Non-linearities of the piezo, causing phase step errors*
- *Non-linearities or geometrical distortions of the detector*
- *Fluctuations of the laser frequency*
- *Incoherence of the laser light beam*

![Image of strain analysis](image)

**Fig. 6.10** (a) The in-plane normal strain in horizontal direction and (b) the same data shown with iso-strain lines (contour spacing 3 μstrain)

The following considerations has to be made about the smoothing technique (fitting procedure), described in Section 5.5. If invalid pixels occur in the neighborhood, the fitting procedure is more accurate than the application of convolution filters, because the inaccurate values of those pixels are excluded from this procedure. Near the object edges a systematic error can occur since most valid data points are located on one side of the center of the rectangular
neighborhood. This error can be reduced by performing a second order LLS fit, but the noise reduction will then be less.

In our present experimental system the accuracy of a phase change measurement is mainly determined by speckle decorrelation and systematic errors. The phase error caused by speckle decorrelation is randomly distributed over the speckles and can therefore be reduced by applying a smoothing filter as discussed in Section 5.5. A systematic error such as the determination of the sensitivity vector from the geometry of the interferometer will directly effect the result. In our laboratory system, that digitizes four phase shifted interference patterns in 160 ms, the measurements could be performed without isolation and shielding of the optical set-up.

The repeatability of the strain measured at each pixel, when using a 45 x 45 pixel filter, amounts to approximately 0.3 μstrain rms. For small loads the estimated inaccuracy approaches the repeatability divided by $\sqrt{2}$, because the speckle decorrelation becomes negligible. For the measurements presented in this chapter an area of 3.5 mm$^2$ around each object point (corresponding to 45 x 45 pixels) was used to calculate the derivatives.
Fig. 6.12  In-plane displacement vector plots for two different scaling factors.

The upper limit of the measuring range of the in-plane strain depends on the size of the object region that is imaged on the detector, the amount of out-of-plane displacement and the maximum number of phase fringes that can still be successfully unwrapped. In our experimental system, using the new phase unwrapping algorithm, about 60 phase fringes across the image could be unwrapped. For a 100 x 100 mm$^2$ object region, this implies 0.6 fringe/mm. The maximum phase change that can be detected, thus, equals (0.6 x 2π)/mm.

Fig. 6.13  Quasi-three-dimensional plot of the shape of the test object (cube).
Determination of deformation, strain and shape

If the amount of out-of-plane displacement equals zero, Eq. (A.2) can be rewritten as (see Appendix A with $\beta = \pi/2$):

$$\Delta \phi_1 = (2\pi/\lambda) L_x \sin [\alpha].$$  \hspace{1cm} (6.9)

If $\alpha$ equals $\pi/6$, this implies an upper limit for $L_x$ of $(0.6\lambda / \sin [\pi/6]) = (2 \times 0.6 \times 514.5) \equiv 600 \mu$ strain.

6.3 Shape measurement

In general, the sensitivity vector $\vec{K}(x,y)$ depends on the position on the surface of the object and thus on the position in the image that contains the phase data. For a complete surface strain analysis of an object, the shape of the object has to be known or measured in order to determine the in-plane strain components from the measured 3D displacement data. A modification of the well-known two-illumination-source technique (applied in holographic interferometry by HILDEBRAND and HAINES [1966]) was used to measure the shape of objects. The same phase-measuring techniques and image-processing algorithms (described in Chapter 3 and 5) were used for these shape measurements. It is

Fig. 6.14  (a) The phase change data after positioning a small cube in the the interferometer and the use of a two-source-illuminating contouring technique. (b) After a grey level scaling, an artefact caused by the phase unwrapping is clearly visible.
Section 6.3  Shape measurement

beyond the scope of this thesis to go into detail about shape measurements. The reader is referred to MAAS [1991]. In this section one example of a shape measurement is shown in Fig. 6.14. A small cube was placed in the interferometric configuration. Several methods (discussed in Section 6.2.3) are used to visualize the shape of the object.
Part III

The Static Approach
Single interferogram analysis

7.1 Introduction

As we have seen in Chapter 2, the general expression for the intensity distribution of an interference pattern due to two interfering waves with equal frequency, not considering noise and non-linear effects, is:

\[ I(x,y) = A(x,y) + B(x,y) \cos[\phi(x,y)] , \]  

(7.1)

where, \( A(x,y) \) is the background intensity, \( B(x,y) \) is the modulation intensity and \( \phi(x,y) \) is the phase difference between the two interfering waves. Normally \( \phi(x,y) \) is related to the physical quantity to be measured. A registration of the interference pattern on for example a photograph is called an interferogram. This chapter deals with the quantitative analysis of interferograms. The analysis is called static interferometry, because the interferometric set-up is not required during the analysis. The analysis of an interferogram with an intensity distribution given by Eq. (7.1) can be performed in several ways. Normally an interferogram contains a number of sinusoidal intensity variations, which are called 'fringes'. The analysis presented here is based on the determination of those positions in the interferogram where the intensity \( I(x,y) \) has a given value. For these positions the fringe extrema (maxima or minima of the cosine in Eq. (7.1) are taken. At these positions the \( I(x,y) \) equals \( 2k\pi \), where \( k \) is the fringe order number. The
Single interferogram analysis

analysis is based on the position of the fringe extrema rather than the "zero"-crossings through the mean value, because a non-linear behavior of the photographic material and of the H-D characteristic of the camera does not cause a displacement of the extrema. DORST [1986c] pointed out that if:

$$I_{measured}(x,y) = f_d(I_{real}(x,y)),$$

(7.2)

where $f_d$ is a (usually strictly monotonic) distortion, the location of the extrema in $I_{real}(x,y)$ are preserved:

$$\frac{\partial I_{measured}}{\partial x} = 0 \iff \frac{\partial f_d}{\partial I_{real}} \frac{\partial I_{real}}{\partial x} = 0$$

$$\iff \begin{cases} \frac{\partial f_d}{\partial I_{real}} = 0 \\ \frac{\partial I_{real}}{\partial x} = 0 \end{cases} \quad \text{or} \quad \frac{\partial I_{real}}{\partial x} = 0$$

$$\iff \frac{\partial I_{real}}{\partial x} \quad (f_d \text{ is strictly monotonic}).$$

(7.3)

The proposed analysis of an interferogram consists of the following steps:

• **noise reduction**
  If the interferogram is very noisy, the determination of the fringe extrema can be difficult. A pre-processing step may be needed.

• **normalization**
  The image has to be normalized, i.e. transformed into an 'ideal' image, in which the background intensity and the modulation intensity amplitude are constant over the image. To perform the normalization, the background intensity $A(x,y)$ and the intensity modulation amplitude $B(x,y)$ have to be estimated. In fact the normalization consists of two steps, shading correction and local contrast stretching.

• ***masking***
  Areas with low modulation (low fringe visibility) have to be detected and masked, because they might disturb subsequent processing steps.

• **fringe extrema detection in the presence of speckle noise**
  In order to determine the points where the phase $\phi(x,y)$ equals $2k\pi$, where $k$ is the fringe order number, the fringe extrema have to be detected. Processing is only allowed if it does not influence the position
of the fringe extrema (this is also required in the normalization procedure).

- **fringe extrema ordering**
  The fringe extrema have to be ordered. It appears to be very difficult to fully automate this processing step successfully for an arbitrary interferogram. One of the reasons is that the sign of the slope of the phase surface cannot be extracted from a single interferogram.

- **phase value interpolation**
  After the ordering an interpolation technique has to be used to estimate the local phase \( \phi(x,y) \) between fringe extrema.

- **graphical representation of the computed phase**
  Finally the data must be represented in a format that is useful for evaluation.

![Flowchart of the interferogram analysis.](image)

In Fig. 7.1 a general flowchart of the processing of an interference pattern is shown. In this chapter a more detailed description of the processing steps mentioned above will be presented. In Section 7.6 a simple theoretical one-dimensional model will be given to gain more insight in the displacement of
the fringe extrema during the analysis. In Chapter 8 several image-processing algorithms for the analysis of interferograms are discussed.

### 7.2 Some aspects about noise in interferograms

It is not easy to quantify the signal-to-noise ratio in a single interferogram. Some knowledge about the different noise sources must be available to estimate the signal-to-noise ratio. Possible noise sources are:

- **electronic noise**
- **quantization noise due to digitization**

<table>
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<tr>
<th>Selected ROI number</th>
<th>Mean grey value</th>
<th>Standard deviation</th>
<th>Signal-to-noise ratio</th>
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<td>5.8</td>
</tr>
<tr>
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</tr>
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</tr>
</tbody>
</table>

**Table 7.1** Estimating the signal-to-noise ratio an interferogram.

- photon noise due to the detector
- speckle noise due to the interferometric recording technique
- other noise sources during the interferometric recording technique
Section 7.2  Some aspects about noise in interferograms

One method to measure the noise in an interferogram is the measurement of the mean and the standard deviation of the grey values in certain areas (selected by the user) in the interferogram. For the interferogram, used for the evaluation of the algorithms described in Chapter 7 and 8, the measurement of the signal-to-noise-ratio has been performed in this way. The areas chosen were: rectangular areas in the background of the image with a size of about 1000 pixels, linear regions of interest following the fringe maxima in the image and linear regions of interest following the fringe minima in the image. The areas and contours were drawn with the use of a mouse interface. In Table 7.1. the results of the measurements are shown.

The signal-to-noise ratio in the interferogram is clearly data dependent. It increases at higher intensities. To produce a theoretical model of an interferogram a multiplicative noise component should therefore be taken into account. One remark has to be made about the interferogram used. The signal-to-noise ratio was high compared to most interferograms produced in industry. Also the selection of background areas and other regions of interest was easy due to the relatively good quality of the interference pattern (see Section 2.2.5)

7.3 Normalization

If thresholding and binary skeletonization are used to detect the fringe extrema (Section 7.4), the image has to be normalized first, i.e. transformed into an 'ideal' image, in which the background intensity and the modulation amplitude are constant over the image. Hence, the normalized intensity \( I_n \) is only a function of the phase:

\[
I_n(x,y) = A + B \cos [\phi(x,y)],
\] (7.4)

To perform this normalization procedure, the background intensity \( A(x,y) \) and the intensity modulation amplitude \( B(x,y) \) have to be estimated.

7.3.1 Shading correction and local contrast stretching

The determination of the normalized intensity is done by applying the operators \( U \) and \( L \) yielding the upper and lower envelope of \( I(x,y) \) (see DORST [1986c]):
Single interferogram analysis

\[ U \{ I(x,y) \} = A(x,y) + B(x,y) \]  \hspace{1cm} (7.5)

and

\[ L \{ I(x,y) \} = A(x,y) - B(x,y). \]  \hspace{1cm} (7.6)

Thus \( A(x,y) \) and \( B(x,y) \) can be retrieved using a linear combination of the upper and the lower envelope:

\[ A(x,y) = \left( \frac{U + L}{2} \right) \{ I(x,y) \} \]  \hspace{1cm} (7.7)

and

\[ B(x,y) = \left( \frac{U - L}{2} \right) \{ I(x,y) \}. \]  \hspace{1cm} (7.8)

\[ \text{Fig. 7.2} \quad \text{One-dimensional section of an image and its upper and lower envelope.} \]

So, by defining the normalization operator \( N \) as

\[ N = \left( \frac{O - L}{U - L} \right), \]  \hspace{1cm} (7.9)

where \( O \) is the unit operator, and applying \( N \) to \( \text{Eq. (7.1)} \)
\[ N\{I(x,y)\} = \frac{1 + \cos[\phi(x,y)]}{2}, \] (7.10)

\(A(x,y)\) and \(B(x,y)\) are eliminated.

The normalization procedure consists, in fact, of two steps. First the background shading is removed by subtracting the lower envelope \(L\) from the original signal. Then the modulation is set to the maximum value at each point by dividing the result by \((U - L)\). This corresponds to a division by the local contrast. The whole normalization procedure can be seen as a position dependent contrast stretch (a contrast stretch between the upper and lower envelope of the signal).

The local contrast stretch described above is not really necessary. Because the normalization is followed by a thresholding, subtracting the background shading will be sufficient. If, however, the intensity data is needed for other processing steps (e.g. noise characterization), the local contrast stretch can be useful. If only a qualitative inspection of the interferogram is required, for example in the case of artefact detection, a contrast stretch will definitely lighten the task.

In Fig. 7.2 an example of the upper and lower envelopes is shown. A one-dimensional section of the interferogram is shown.

### 7.3.2 Masking of noisy areas

The operators \(U\) and \(L\) are also used to produce a binary mask that indicates areas without fringes. This is done by thresholding \(B(x,y)\) in Eq. (7.8). The threshold defines the lowest modulation amplitude allowed. In principle, the masking separates areas with fringes (sufficient modulation) from areas with noise (low modulation). A practical choice for the threshold will be about \(5\sigma_n\) (the standard deviation of the noise in the interferogram). If \(\sigma_n\) is not known, a value for the threshold has to be found experimentally.

A special advantage of the masking is the fact that areas where the upper envelope \(U\{I(x,y)\}\) almost equals the lower envelope \(L\{I(x,y)\}\) are avoided during further processing, otherwise \(U = L\) would give \(N = \infty\) (see Eq. (7.9)).
After the detection of the region of interest containing the relevant interferometric data, subsequent processing steps have been performed to compute the $\phi(x,y)$. In Fig. 7.3 the processing steps are shown, using a computer generated interference pattern. First, the image is thresholded and skeletonized to detect the fringe maxima. Then, the fringe maxima are ordered to solve the problem of ambiguity. Information has to be inserted by the user to add numbers to the detected fringe extrema. After the ordering, an interpolation has to be performed to compute the phase values in between the fringe extrema. Finally, different display techniques are used to emphasize the
computed, physical entity. In the next chapter a detailed description will be given about the algorithms used to perform the mentioned processing steps

### 7.5 Displacement of fringe extrema

A problem that arises during the interferogram analysis as described in this chapter is the displacement of the fringe extrema due to several processing steps. It is not easy to compute the amount of displacement analytically. If, however, a very simple one-dimensional model is used, the following calculations can be made for the displacement of fringe extrema due to shading. Suppose that we have the following intensity distribution:

\[
I(x) = C_1x + C_2 \cos[\omega x],
\]  

(7.11)

representing a fringe pattern with a frequency \(\omega\), a constant modulation \(C_2\) and a linear shading component \(C_1x\). If \(C_1\) equals zero (no shading) the positions of the fringe extrema are:

\[
x_{e1} = \frac{k\pi}{\omega}.
\]  

(7.12)

If we set the first derivative of the intensity to zero, the positions of the fringe extrema can be computed:

\[
x_{e2} = \frac{1}{\omega} \arcsin\left[\frac{C_1}{C_2\omega}\right] + \frac{k\pi}{\omega}.
\]  

(7.13)

So, in the presence of shading, the displacement of the fringe extrema becomes:

\[
\delta_e = x_{e2} - x_{e1} = \frac{1}{\omega} \arcsin\left[\frac{C_1}{C_2\omega}\right].
\]  

(7.14)

It is important to compute the parameter values for the case that the displacement exceeds 0.5 pixel, because in that case the displacement influences the accuracy of the interferogram analysis. Near \(x = k\pi/\omega\) we may approximate the arcsin function, yielding

\[
\delta_e = - (-1)^k \frac{C_1}{C_2\omega^2},
\]  

(7.15)
The absolute value of the displacement $|\delta_x|$ seems to be independent of the order number $k$, thus independent of the position $x$. A numerical example is:

$$I(x) = x + 16 \cos \left[ \frac{16 \pi x}{255} \right], \quad x = 0 \text{ to } 255,$$

(7.16)

representing a one-dimensional signal with 8 'fringes' and a linear shading. The displacement of the extrema will then become $|\delta_x| = 1.6$ pixels. Now suppose that we have the following intensity distribution:

$$I(x) = C_1 x^2 + C_2 x + C_3 \cos \left[ \alpha x \right],$$

(7.17)

representing a fringe pattern with a frequency $\omega$, a constant modulation $C_3$ and a quadratic shading component $C_1 x^2 + C_2 x$. If we now approximate the sine term when computing the derivatives (we get a transcendental equation), the positions of the fringe extrema become:

$$x_{e2} = \frac{-C_2 - C_3 \omega k \pi (-1)^k}{2C_1 - C_3 \omega^2 (-1)^k}.$$

(7.18)

In the presence of this quadratic shading, the displacement of the fringe extrema becomes:

$$\delta_e = \frac{-C_2 \omega - 2k \pi C_1}{2C_1 \omega - C_3 \omega^3 (-1)^k}.$$

(7.19)

A numerical example is:

$$I(x) = -\frac{x^2}{255} + x + 16 \cos \left[ \frac{16 \pi x}{255} \right], \quad x = 0 \text{ to } 255,$$

(7.20)

representing a one-dimensional signal with 8 'fringes' and a quadratic shading. The displacement of the extrema $|\delta_x|$ will then become about 1.6 pixels at locations $x = 0$ and $x = 255$ and 0 pixels at location $x = 127$. In fact the displacement is related to the first derivative of the shading function. The numerical examples, described above, indicate that the extrema shift will only cause problems in interferograms with 'low-amplitude' fringes and substantial shading.

By removing the shading with the normalization procedure, described in Section 7.3, the shift of the fringe extrema can be reduced. By introducing
other filter techniques, however, such as averaging or median filtering, new
shifts can be introduced. Special care should be taken, therefore, during the
(static) processing of the interferograms (for example with techniques used for
noise reduction or image enhancement). During the measurements, described
in this thesis, as little filtering as possible has been used, to avoid fringe
extrema shift problems. Also during the speckle interferometric
measurements, described in Part II, the influence of the filter techniques on the
raw, detected data has been kept to a minimum to achieve more accurate
measurements.
Chapter 8

Algorithms for interferogram analysis

In this chapter a number of image processing techniques to analyse a single interferogram are discussed. Several problems involved in the analysis of a single interferogram are discussed in detail and a number of algorithms to solve those problems are described.

8.1 Noise reduction

Depending on the kind of noise in the image, several techniques can be used to suppress the noise.

8.1.1 Linear Filtering

A linear filter can be applied to perform a uniform or gaussian averaging of the neighborhood. In the case of uniform filtering the standard deviation of the noise after filtering will be:

\[ \sigma_{n2} = \frac{\sigma_{n1}}{\sqrt{N}} \]  

(8.1)

where \( \sigma_{n1} \) is the standard deviation before filtering, \( \sigma_{n2} \) the standard deviation after filtering and \( N \) the number of points taken into account. One way to express the signal-to-noise ratio in an image is:
\[ \text{snr} = \frac{I_{\text{max}} - I_{\text{min}}}{\sigma_n}, \]

(8.2)

where \( I_{\text{max}} \) and \( I_{\text{min}} \) are the maximum and minimum intensity of the signal in the image and \( \sigma_n \) is the standard deviation of the noise in the image. A small uniform averaging filter can therefore be used to increase the signal-to-noise ratio.

8.1.2 Non-Linear Filtering

In some cases percentile filtering may lead to better results. For example a median filter or a combination of maximum and minimum filters can be used to suppress the noise. The reader is referred to VERBEEK, VROOMAN and V. VLIET [1988] for a detailed description of non-linear filter techniques used for

Fig. 8.1  Normalization with max / min filters: (a) The application of maximum and minimum filters with the same window size (window sizes are 63) leads to a flattening of the 'fringe' extrema (b).

edge detection in noisy images. V. VLIET, YOUNG and BECKERS [1989] also present an adequate non-linear filter technique for edge detection.
Section 8.1  Noise reduction

8.1.3  Spin filter

An averaging filter that preserves edges nicely is described by YU [1988]. Although there are no 'sharp' edges in an interferogram (fast intensity variations), this filter technique is useful because it blurs (averages) the data along the fringe extrema and not perpendicular to the fringe extrema (avoiding a decrease of the fringe visibility). For each pixel this filter takes a set of one-dimensional neighborhoods in eight different directions. For each neighborhood the mean of the grey values is computed. Then, the sum of the absolute differences between the grey levels of every point and the average value is computed. This sum expresses the variation gradient of grey levels in the specific direction. In the third step, the direction with the minimal variation is determined. For this direction the average value is computed again, discarding the very pixel which maximizes the difference between grey value and average value in that direction. Finally, this new computed average value is substituted at the central pixel.

This technique is closely related to the edge preserving filter technique of KUWAHARA [1976]. The spin filter can be seen as a one-dimensional version of these techniques. The name 'spin filter' refers to the spinning of the one-dimensional neighborhood around the central pixel.

8.2  Normalization

8.2.1  Max/min filtering to estimate the envelopes

It remains to provide expressions for U and L. We have used non-linear filters namely a local maximum filter $M_n$ and a local minimum filter $m_n$. Both filters replace the grey value of a pixel at position $(x,y)$ by the maximum / minimum grey value in a local neighborhood. Here, the neighborhood is an $n \times n$ square. The use of 'circular' neighborhoods gives slightly better results (see VERBEEK, VROOMAN and V. VLIEET [1988]). Expressed in these filters, U and L are determined by:

$$U = m_{n-2}M_n \quad \text{and} \quad L = M_{n-2}m_n \quad (8.3)$$

A cascade of operators a and b means that first operator b is performed and secondly operator a. The neighborhood size $n$ should be somewhat larger
than the largest fringe separation in the image. The \((n-2)\)'s in Eq. (8.3) are needed to prevent a flattening of the extrema. Consider a one-dimensional intensity distribution as drawn in Fig. 8.1. If a maximum filter is performed followed by a minimum filter with the same window size, the resulting upper envelope equals the original signal at a number of locations. Because the normalization operator \(N\) is defined as (Chapter 7):

\[
N = \left( \frac{O - L}{U - L} \right)
\] (8.4)

the upper envelope will equal 'one' at those locations (representing a flattening of the extremum). In the same way the lower envelope will result in a line segment with value 'zero'. If the window size of the second operator is smaller than the window size of the first operator, the upper and lower envelopes are equal to the original signal only at the maxima and minima of the intensity distribution (see Fig. 8.2).

![Fig. 8.2](image)

(a) The application of maximum and minimum filters with different window sizes yields better normalization results. Here, window size 63 was used for the first operator and size 61 for the second operator. The upper and lower envelope only equals the original at the fringe extrema. (b) This leads to a slightly better normalization result.
8.2.2 Average filtering to correct background shading

The removal of the background shading can also be done with linear average filtering. Taking the average in a neighborhood with a size equal to the maximum fringe period yields an estimation for the background intensity. The filter size should be odd to prevent a shift of the fringes. By subtracting the result from the original image the background shading is removed.

![Graphs](image_url)

**Fig. 8.3** (a) Uniform filtering to normalize the one-dimensional interferogram. First the average value in a square neighborhood is computed (window size 63). (b) The average image is subtracted from the original image and a contrast stretch is applied.

A few remarks have to be made about maximum, minimum and average filtering.

- **overshoot**
  With an averaging filter, overshoots can be introduced in the signal. These overshoots can lead to inaccurate results, compared to maximum and minimum filters. Maximum and minimum filters ‘follow’ the signal more accurate and provides a better estimation of the background intensity.

- **influence of noise**
  Maximum and minimum filters are more sensitive to noise. Extreme high or low values on the slopes of the fringes will lead to unwanted artefacts.
An averaging filter, however, has an inherent noise smoothing that will smooth these extreme noise values.

- **processing times**
  With all filters, the processing time is independent of the chosen filter sizes, when square neighborhoods are used. In the case of circular neighborhoods, the processing times are linearly proportional to the chosen filter size, because circular filters can not be separated into two one-dimensional filters (see VERBEEK, VROOMAN and V. VLIEET [1988]).

### 8.2.3 The rolling ball algorithm

To compute a ‘better’ estimate of the upper and lower envelope of the signal, the ‘rolling ball algorithm’ was implemented. With this algorithm, well known in the grey scale morphology area (see SERRA [1982, 1986] and STERNBERG [1986]), grey scale dilations, erosions, openings and closings are performed with a sphere as structuring element. Imagine the input image as a landscape of peaks and valleys and visualize a sphere which is free to move above the terrain surface but whose downward mobility is constrained by the surface. Through such a visualization we describe the grey scale closing of the

![Diagram](image_url)  
*Fig. 8.4 One line out of a 128 x 128 image that is processed by the rolling ball algorithm. The structuring element was a sphere with radius 15. From top to bottom the grey scale dilation, the grey scale closing, the original, the grey scale opening and the grey scale erosion are shown.*
grey level surface by a sphere. The output is formed by the union of all translations of the sphere above and touching the grey level surface. Those that touch the surface are said to be rolling on the surface, hence the name ‘rolling ball algorithm’. Closing a grey level surface by dilating and then eroding by a spherical structuring element rolls the ball. The rolling ball exactly traces the smoothly varying contours of the landscape. When the surface dips sharply downward in narrow pits, the rolling ball smooths the surface, because the ball can not enter the pits. Opening a grey scale image by a spherical structuring element is visualized as a rolling ball along the underside of the grey level surface.

This algorithm is used to estimate the upper and lower envelope of an image. The ‘curviness’ of the envelopes is related to the diameter of the spherical structuring element. In fact ‘the exact upper envelope’ is not defined in the three dimensional case. In Fig. 8.4 a one-dimensional example of the upper and lower envelope, computed with the rolling ball algorithm, is shown.

8.3 Fringe extrema detection

In order to determine the points where the phase $\phi(x,y)$ is a multiple of $2\pi$, the fringe maxima have to be detected. To determine the location of the fringe maxima several techniques can be used. A straightforward method is the thresholding of the normalized image producing a binary image, followed by a

![Real fringe maximum](image)

**Fig. 8.5** In the case of different slopes on each side of a fringe, thresholding followed by skeletonization will lead to an inaccurate detection of the fringe maximum.
skeletonization. Another possibility for fringe extrema detection is the grey value skeleton (see SERRA [1982]). Both algorithms will be discussed in this section. Small visible artefacts of the computed skeleton (1 or 2 missing pixels, or small branches of pixels) can be removed by binary operations such as opening, closing and majority voting (see SERRA [1982], YOUNG [1981], HARALICK [1987]).

8.3.1 Thresholding and skeleton

After normalization, the interferogram is thresholded, i.e. binarized. Each pixel with a grey value greater than or equal to a certain threshold is set to 'one' and all other pixels are set to 'zero'. The threshold can be computed automatically by different algorithms (see WESZKA [1978]) or a fixed threshold can be used. After thresholding, the image is skeletonized to obtain the fringe maxima. Skeletonizing the logical inverse of the binary

![Fig. 8.6](image_url) *(a) Original interference pattern and (b) its grey value skeleton.*

image produces the fringe minima. There are many skeleton (thinning) algorithms and each algorithm has its advantages and disadvantages (see VERWER [1988], HILDITCH [1969], ROSENFELD [1969, p. 133], PAVLIDIS [1977, p. 217], PRATT [1978, p. 517], DORST [1986], LOBREGT [1980]).

A problem occurs if the fringes in the image have different slopes on each side. If the slope on one side of the fringe is steeper than the slope on the
other side, thresholding and skeletonization will not lead to the correct position of the fringe extrema (Fig. 8.5). The detected extremum will be shifted from the steeper slope.

*Fig. 8.7* (a) Artefacts of the grey value skeleton and (b) a solution by introducing ‘zerobands’ between the fringe maxima. In (c) the result of the grey value skeleton is shown after a 9 x 9 average filter performed on the original.
8.3.2 Grey value skeleton

Instead of a binary skeleton a grey value skeleton can be used to locate the fringe extrema. The grey value skeleton is a grey value morphological operation. There are different algorithms, leading to different grey value skeletons (see SERRA [1982]). Roughly, one can speak of the upper grey value skeleton and the lower grey value skeleton.

Some disadvantages of this technique are:

- *extrema shifted by shading are not corrected*
- *the grey value skeleton is very noise sensitive*
- *artefacts*

The algorithm introduces a lot of unwanted lines connecting neighboring fringe extrema (see Fig. 8.7). A solution to this problem is a pre-smoothing or a threshold (to create 'zero-bands' between the fringe extrema). Another artefact of the grey value skeleton is the number of different skeletons found due to noise. Especially in the case of 'broad' fringes (more than about 30 pixels), the grey value skeleton does not produce the proper skeleton. A large smoothing filter reduces the number of artefacts. The computed data, however, will be less accurate.

Some advantages of this technique are:

- *no normalization needed*
- *a direct way to detect extrema*

8.4 Fringe extrema ordering

The fringe maxima have to be ordered and an interpolation is needed to compute a continuous phase map. Because the order numbers of the fringe maxima in the interferogram can not be unambiguously computed, one has to add information about the conditions during the digitization of the interference pattern. To add this information a semi-automatic ordering is used.

Starting with a binary image, containing line pieces and closed contours, a track is drawn across several fringes (Fig. 8.8). The start (S) and end (E) location of this track are passed over to the ordering algorithm. Also the start order number has to be specified and if the order numbers increase or decrease. This depends on 'mountains' or 'valleys' being present in the data.
After a track is drawn the ordering can begin. The track is followed from start to end and each time a fringe is detected, the algorithm tries to label the detected fringe, i.e. all pixels on the fringe are given a grey value equal to the current fringe order number. Each time a fringe is labeled the order number increases (or decreases). If a detected fringe is already labeled, no labeling is performed. If the fringe is not a closed contour, the fringe is followed starting at both sides of the track. After the labeling of the fringes is completed, another track can be drawn in a different part of the image. A new start order number and direction (increase or decrease) can be given. In this way all the parts of the image where relevant data is present can be processed.

A few remarks have to be made here about this technique. To detect the fringes, the walking along the track has to be 4-connected, because fringes can be ‘missed’ if it is performed 8-connected (Fig. 8.9). Another problem is the presence of branchpoints in the fringe skeletons. Due to the contour following only one path is followed. And this can be the ‘wrong’ path. One solution to this problem is a pre-processing step to remove small branches and erroneous points from the binary image. Another solution is a contour following technique that has a kind of back-tracking procedure. Although it would be nice to have an automatic ordering algorithm, no algorithm is available at the moment that performs the ordering for an arbitrary interferogram automatically.
8.5 Computation of the phase map

After the ordering of the fringe extrema, the value of the phase \( \phi(x,y) \) is exactly known on the positions of the fringe extrema in the image. This section describes a technique to estimate the phase values \( \phi(x,y) \) between the labeled fringes. To compute the phase at each point of the image, an interpolation has to be carried out. In fact, a phase surface has to be fit to a set of data points. Usually it is not necessary to compute the local phase \( \phi(x,y) \) at each point. Here, the phase is evaluated on a square grid (e.g. 32 x 32). The algorithm is based on the following steps, performed at each grid point. First the nearest fringe extremum is searched for, using track and search algorithms. Then, by scanning perpendicularly to this extremum, the intersections with two neighboring fringe maxima are detected. Finally a zero, first or second order polynomial is fit locally to the three points of intersection. In a similar way the fringe density can be computed on a grid. The results can be interpolated to a larger image if necessary. Let us now discuss these steps in more detail.

8.5.1 Computation of the maximum order number

First the maximum order number in the complete image is computed. This maximum value is used to rescale the final result to get grey values in the range 0-255.
Section 8.5  Computation of the phase map

![Diagram showing interpolation of the phase value between fringe extrema.](image)

**Fig. 8.10**  Interpolation of the phase value between the fringe extrema: 3 fringe extrema are taken into account to estimate the value for \( \phi(p,q) \).

### 8.5.2 Searching for fringe extrema around a grid point

Because interpolation is time consuming, an \( n \times n \) grid is defined on which the phase values will be evaluated. A practical value for \( n \) is 32. Then the distance is calculated from the current gridpoint \( (p,q) \) to the nearest labeled fringes in four directions (Fig. 8.10). The nearest fringe with the shortest distance to the gridpoint \( (p,q) \) is taken as the central fringe. The point where the fringe is detected is called \( (p_0,q_0) \). If no fringe is found at all a flag is set. If a central fringe is found two neighboring fringes are searched for. The slope of the central fringe in \( (p_0,q_0) \) is computed using a line piece of 5 pixels with \( (p_0,q_0) \) as the central pixel. Perpendicular to the central fringe a track is computed through \( (p_0,q_0) \) from one border of the image to another. This track is used to search for two neighboring fringes. The locations where the neighboring fringes are found are called \( (p_1,q_1) \) and \( (p_2,q_2) \). The order numbers at the locations \( (p_0,q_0), (p_1,q_1) \) and \( (p_2,q_2) \) are used in the final interpolation formula.. If one of the neighboring fringes is not found, the corresponding order number is set to 'zero'.

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8.5.3 Computing the phase value at the grid point

First the distances are computed between the points considered:

\[
\begin{align*}
d_1 &= \sqrt{(p_1-p_0)^2+(q_1-q_0)^2} \\
d_2 &= \sqrt{(p_2-p_0)^2+(q_2-q_0)^2}.
\end{align*}
\]  \hspace{1cm} (8.5)

The shortest distance from \((p,q)\) to the central fringe is computed with:

\[
\begin{align*}
d &= \frac{(p-p_0)(p_1-p_0)+(q-q_0)(q_1-q_0)}{d_1}
\end{align*}
\]

or

\[
\begin{align*}
d &= \frac{(p-p_0)(p_2-p_0)+(q-q_0)(q_2-q_0)}{d_2}.
\end{align*}
\]  \hspace{1cm} (8.6)

Fig. 8.11 The interpolation technique can be seen as a 'filing procedure'.

depending on which neighboring fringe is found. To estimate the phase value at the current position \((p,q)\) a zero-, first- or second-order Lagrange interpolation is used (see ALMERING [1977, p. 98]) depending on the number of found fringes. If only the central pixel is found a zero-order interpolation is used:

\[
\phi_{est} = \phi(p_0,q_0). \hspace{1cm} (8.7)
\]
If the central fringe plus one neighboring fringe is found a first-order interpolation is used:

\[
\phi_{\text{est}} = \frac{d}{d_1} \left( \phi(p_1,q_1) - \phi(p_0,q_0) \right) + \phi(p_0,q_0)
\]

or

\[
\phi_{\text{est}} = \frac{d}{d_2} \left( \phi(p_2,q_2) - \phi(p_0,q_0) \right) + \phi(p_0,q_0)
\]

(8.8)

Fig. 8.12 Problems occurring during phase interpolation: Due to the patterns (a) and (b) a very inaccurate value for the estimation of the phase is computed.

If both neighboring fringes are found a second-order interpolation is used:

\[
a = d_2 \left( \phi(p_1,q_1) - \phi(p_0,q_0) \right) + d_1 \left( \phi(p_2,q_2) - \phi(p_0,q_0) \right)
\]

\[
b = -d_2^2 \left( \phi(p_1,q_1) - \phi(p_0,q_0) \right) + d_1^2 \left( \phi(p_2,q_2) - \phi(p_0,q_0) \right)
\]

\[
\phi_{\text{est}} = \frac{a d_2 + b d_1}{d_1 d_2 + d_1 d_2^2} + \phi(p_0,q_0)
\]

(8.9)
After the interpolation the phase value is rescaled to use the full dynamic range:

\[
\phi(p,q) = \frac{255 \phi_{\text{est}}}{\text{Maximum order number} + 1}
\]  

(8.10)

One remark has to be made about the second order interpolation. In fact only the interpolation perpendicular to the fringes is quadratic. The interpolation along the fringes is linear. This follows from the expression for the shortest distance \(d\) in the above calculations. In Fig. 8.11 a schematic picture of the second order interpolation (in fact a tile covering procedure) is shown.

**Fig. 8.13  Bilinear interpolation.**

8.5.4 Problems during interpolation

A problem occurs when patterns similar to that shown in Fig. 8.12 are processed. Because the search for fringes is only performed in 4 directions, the estimation for the 'local' distances to the neighboring fringes will be very inaccurate. A solution to the problem would be a search for fringes in more directions (for example 8). Another suggestion for future research is the use of distance transforms to compute the distance to the nearest fringe.

8.5.5 Bilinear interpolation

Sometimes the phase value at each location of the image has to be estimated. In that case a bilinear interpolation from the \(n \times n\) grid to, for example, a 256 \(\times\) 256 image can be performed. A schematic picture of the bilinear interpolation is shown in Fig. 8.13.
Experimental results

In this chapter an application of the image processing techniques, described in Chapter 6 will be discussed.

9.1 Vibration measurement

Fig. 9.1a shows an interferogram of a vibrating disk, recorded with a double-pulse holographic technique. The algorithms, described in Chapter 8, were used to analyse this interferogram. The goal was a quasi three-dimensional plot of the amplitude of vibration. The image shown in Fig. 9.1a is already filtered with a 3 x 3 uniform average filter. In Fig. 9.1b the lower envelope of the image from Fig. 9.1a is shown. The lower envelope is computed using min/max filters with a filter size of 31 and 29 (see Section 8.2.1). In the same way the upper envelope is computed (Fig. 9.2a) and, using the upper and lower envelope, the image is normalized (Fig. 9.2b). The data is masked to separate areas with fringes from areas with 'noise' (low modulation). If \((U(x,y) - L(x,y)) < 75\), the corresponding pixel at position \((x,y)\) is set to 'zero' (Fig. 9.3a). In this case the value '75' is found empirically. After masking, the image is thresholded at the grey value 128 (because the image was normalized between 0 and 255, 128 is a proper threshold). All pixels with a grey value > 128 are set to 'zero' and the other pixels are set to 'one' (Fig. 9.3b). The fringe maxima were chosen here to be locations
Experimental results

Fig. 9.1 (a) Original interference pattern generated by a vibrating disk using a double-pulse holographic technique and (b) its lower envelope computed with a minimum filter with size 31 followed by a maximum filter with size 29.

Fig. 9.2 (a) The upper envelope of the image from Fig. 9.1a computed with a maximum filter with size 31 followed by a minimum filter with size 29 and (b) the normalized result by applying the normalization operator $N = (O - L) / (U - L)$. 

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Fig. 9.3  (a) The masking and (b) thresholding of the image from Fig. 9.2b. After thresholding the fringe maxima can be computed by skeletonization of the black parts in the right-hand image.

Fig. 9.4  (a) Skeletonization of the fringes, yielding the fringe maxima. (b) After ordering the fringe maxima (by drawing two tracks) the phase data can be computed on a $n \times n$ grid ($n = 32$).

where the phase has a given value. After the skeletonization (Fig. 9.4a), the fringe maxima are ordered and the phase values between the fringe maxima
Experimental results

are estimated using the phase interpolation technique (see Section 8.5). The

![Image](image.png)

**Fig. 9.5**  (a) A bilinear interpolation of the data from Fig. 9.4b to estimate the phase on a 256 x 256 grid. (b) The quasi-3D plot gives a nice impression of the disk vibrating with four lobs (two going down and two coming up).

phase values were computed on a 32 x 32 grid (Fig. 9.4b, blown to 256 x 256 by pixel-copying). Using a bilinear interpolation the image is interpolated to a 256 x 256 image (Fig. 9.5a) and a quasi three-dimensional plot is computed (Fig. 9.5b) to produce a better view of the amplitude of vibration. At this stage of the analysis a continuous phase map is available and further processing (e.g. differentiation, calculation of r.m.s. phase error, detection of phase anomalies) can be performed.

9.2 Accuracy considerations and processing times

As described in Section 7.5, the different processing steps involved in the analysis of interferograms can cause a displacement of the fringe extrema. For example noise reduction and shading removal (normalization) will lead to inaccuracy in the results. Also a ‘flattening’ of the extrema, due the the thresholding and skeletonization techniques (see Section 8.3.1) introduces a displacement of the fringe extrema.
Section 9.2  Accuracy considerations and processing times

The total accuracy of the interferogram analysis depends on the location of the detected fringe extrema, the number of fringes in the image, the number of pixels in the image and the interpolation technique described in Section 8.5. If care has been taken during pre-processing and an accurate skeleton algorithm has been used, the error in the location of the fringe extrema is less than one pixel (for sinusoidal fringes). For an image with \( P \) pixels per fringe, the error in the phase measurement will be about \( 2\pi/P \) radians. For the interferogram discussed in this chapter \( P \approx 20 \), yielding a phase error of approximately 0.12 radians. This corresponds to a displacement error of about 25 nm (\( \lambda \approx 5 \times 10^{-7} \)). For smaller fringes \( P \) decreases, but often the fringe extrema can be located with a higher accuracy if the fringes are smaller.

<table>
<thead>
<tr>
<th>Operation</th>
<th>time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>read data from disk</td>
<td>0.58</td>
</tr>
<tr>
<td>averaging filter (3 x 3)</td>
<td>1.78</td>
</tr>
<tr>
<td>minimum filter (31 x 31)</td>
<td>3.25</td>
</tr>
<tr>
<td>maximum filter (29 x 29)</td>
<td>3.62</td>
</tr>
<tr>
<td>maximum filter (31 x 31)</td>
<td>3.48</td>
</tr>
<tr>
<td>minimum filter (29 x 29)</td>
<td>3.33</td>
</tr>
<tr>
<td>subtract and divide</td>
<td>1.45</td>
</tr>
<tr>
<td>total normalization</td>
<td>15.13</td>
</tr>
<tr>
<td>masking of the minima</td>
<td>4.13</td>
</tr>
<tr>
<td>skeletonization</td>
<td>4.70</td>
</tr>
<tr>
<td>fringe ordering</td>
<td>1.00*</td>
</tr>
<tr>
<td>interpolation to 32 x 32</td>
<td>5.32</td>
</tr>
<tr>
<td>interpolation to 256 x 256</td>
<td>5.42</td>
</tr>
<tr>
<td>computation 3D plot</td>
<td>1.47</td>
</tr>
<tr>
<td>total processing time</td>
<td>39.58</td>
</tr>
</tbody>
</table>

Table 8.1  Processing times for the complete analysis of an interferogram.  
*The fringe ordering is an semi-automatic step: Computation time for an Apple Macintosh IIx without user interaction is given here.

The error introduced by the interpolation technique depends on the kind of vibration of the object surface. During the interferogram analysis, a quadratic
interpolation (Lagrange interpolation) perpendicular to the fringe direction has been used. The bilinear interpolation has been used for visualization purposes only. The number of pixels for which the phase value is computed (the grid size used in the interpolation technique) determines the spatial resolution of the measurement.

Table 8.1 shows the processing times of a complete analysis of a 256 x 256 interferogram (the same interferogram as used in Chapter 9), to get an

<table>
<thead>
<tr>
<th>Step size</th>
<th>Time</th>
<th>Time per pixel (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.58</td>
<td>900</td>
</tr>
<tr>
<td>16</td>
<td>1.53</td>
<td>600</td>
</tr>
<tr>
<td>8</td>
<td>5.33</td>
<td>520</td>
</tr>
<tr>
<td>4</td>
<td>20.55</td>
<td>510</td>
</tr>
<tr>
<td>2</td>
<td>81.85</td>
<td>505</td>
</tr>
<tr>
<td>1</td>
<td>326.03</td>
<td>500</td>
</tr>
</tbody>
</table>

Table 8.2  Processing times for the interpolation technique described in Section 8.5

estimate of the times involved in different processing steps. The times were measured on an Apple Macintosh IIx system. On the Macintosh IIx the complete procedure takes about 40 seconds. Table 8.2 shows different processing times in seconds of the interpolation technique, described in Section 8.5 for different step sizes, applied to the ordered fringes of the interferogram.
Part IV

Conclusions and discussion
Conclusions and discussion

10.1 Conclusions

A measuring method to determine the in-plane displacement components and the in-plane strain components of a deformed flat object has been presented.

Application of a special gradient filter enables accurate determination of the in-plane strain components at each point of the object surface. The application of this filter causes a noise reduction that is proportional to the filter size. The spatial resolution is approximately inversely proportional to the filter size. At edges of the object the accuracy decreases, because less pixels are taken into account.

The application of special purpose hardware in a VME-bus computer system allows a complete measurement of the phase change modulo $2\pi$ on a 512 x 512 grid every 240 ms.

We have developed new image processing algorithms for phase shifted speckle interference patterns, such as phase unwrapping, phase restoration and phase fitting, that have proven to be very robust with respect to both temporal and decorrelation noise. The ability to distinguish the object from the background, has improved the flexibility of the processing of phase shifted speckle interference patterns. The accuracy of our strain measurements indicates that this method is a potential alternative for strain gauges.

Image processing techniques for quantitative analysis of single interferograms have been developed. Some of the problems that occur during the analysis
have been solved. Maximum and minimum filters have proved to be very useful. The advantages of these filters are: easy integer calculations, relatively small filter sizes, small influence of noise and relative insensitivity to the exact filter sizes. A number of problems remains to be solved.

On the VME/MC68000 system (without a floating point co-processor) the analysis of a single interferogram (256 x 256) takes about 5 minutes. On a SUN workstation it takes about 15 seconds. In the near future the software will be implemented on an Apple Macintosh II system.

A severe problem occurs if the fringe density varies strongly over the image. We are studying the use of data-dependent filter sizes and resolution pyramids to solve this problem.

Other techniques such as the grey value skeleton have yet to be investigated. If normalization is still needed in that case has to be investigated. A different approach is the use of differentiation techniques and edge tracking and detection algorithms. These techniques are, however, very noise sensitive.

The analysis of a single interferogram (static approach) is much more complex than the dynamic approach described in part II of this thesis. Furthermore, the accuracy of the static analysis is much less than the accuracy of phase stepping techniques.

10.2 Comparison of different techniques

Different methods for quantitative evaluation of interferograms have been compared by KREIS [1987]. A comparison of the methods for interference phase determination is given in table 10.1.

The disadvantage of fringe tracking methods (static analysis) is the low accuracy as compared to other methods, since there is no interference phase directly determined at points between fringe extrema.

Heterodyne techniques suffer from the mechanical scanning of the image and the high positioning requirements due to the twin reference waves. Nevertheless, its use would be advisable, when highest resolution is demanded.

Fourier techniques have inherent noise suppression capabilities and offer high flexibility to the user for elimination of unwanted sub-patterns.
Section 10.2  Comparison of different techniques

Phase stepping methods offer high accuracy and the evaluation of the interferograms can be fully automated. The experimental requirements are high, since constant phase shifting or stepping must be performed and multiple interferograms are necessary.

<table>
<thead>
<tr>
<th>Methods for phase determination</th>
<th>Fringe tracking</th>
<th>Heterodyne techniques</th>
<th>Phase stepping</th>
<th>Fourier without carrier frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of interferograms required</td>
<td>1</td>
<td>1</td>
<td>&gt;= 3</td>
<td>1</td>
</tr>
<tr>
<td>Real time possible</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Inherent image enhancement</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Inherent phase interpolation</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Automatic sign detection</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Partly</td>
</tr>
<tr>
<td>Achievable accuracy</td>
<td>Low</td>
<td>Very High</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Sensitivity to experimental errors</td>
<td>Low</td>
<td>Very High</td>
<td>Medium</td>
<td>Low</td>
</tr>
<tr>
<td>Experimental requirements</td>
<td>Low</td>
<td>Very High</td>
<td>High</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table. 10.1  Comparing different interferometric measuring techniques (adapted from KREIS [1987]).

10.3  Suggestions for future research

The phase unwrapping procedure, described in Chapter 5, has proved to be very robust and the influence of noise or background areas has been diminished in an acceptable way. There are still interferograms, however, for which the phase unwrapping procedure does not produce the correct result. I would like to give a few suggestions to improve the unwrapping procedure:
Conclusions and discussion

(Pre- and post-processing steps combined with the phase unwrapping algorithm)

The use of preprocessing steps has to be studied. For example, Kuwahara filtering or other edge preserving smoothing techniques can be very useful as a preprocessing steps. Special care must be taken, however, that the pixel data does not suffer to much from filtering. Also post-processing techniques are promising. In the case of separated unwrapped areas in the image, techniques such as ‘split and merge (see HOROWITZ and PAVLIDIS [1976])’, can be of interest. These techniques can also be useful in the case of incorrect phase unwrapping, where certain areas in the image have grey values that are a multiple of $2\pi$ too low or too high.

(Phase unwrapping using a tiling method)

A completely different phase unwrapping technique, but still very promising, has been suggested by TOWERS [1989]. The image is divided into sub-images (tiles). Each tile is unwrapped with some kind of phase unwrapping algorithm. After the unwrapping, faulty tiles are detected and deleted. Correct tiles are connected to each other using graph matching techniques. The tiles are represented as nodes in the graph and weight factors are used to define the graph branches; a minimum weight spanning tree is used to connect (adding offsets) the tiles in a proper way. The big advantage of this technique is that phase unwrapping errors are detected and that errors do not accumulate over the image.

(Pixels of interest, a new approach)

The use of binary masks during the image processing techniques, described in Chapter 5, has proved to be very useful. It would be advisable to study these ‘regions (pixels) of interest’ for other image processing techniques. A lot of images in practical applications have areas with inaccurate pixel values. It would be nice to see if ‘the pixel of interest approach’ produces useful results in those cases.

(The varying fringe density problem)

A severe problem during the static analysis of interferograms (see part III of this thesis) occurs if the fringe density varies strongly over the image. No proper solution has been found yet for this problem. A few suggestions, however, are given here.
Section 10.3  Suggestions for future research

An adaptive or iterative thresholding is a possible solution. In that case the image is thresholded with a start threshold to produce the first estimate of binary fringes. After the thresholding, some measure for the ‘instantaneous’ frequency could be computed (using a Hilbert transform or simple run count algorithms). Then, the chosen threshold can be adapted dependent upon this ‘frequency measure’.

Another technique, that should be studied, is the use of data dependent filter sizes or resolution pyramids.

(*Auto ordering of fringe extrema*)

The ordering of the fringe extrema makes the static analysis of interferograms a semi-automatic one. I would recommend future research and a development of auto-ordering algorithms to solve this problem.
determination of the in-plane strain component

Suppose that two phase change measurements are performed with two different illumination directions as shown in Fig. A.1. $L$ is the displacement vector of the considered object point. The phase change, measurement with illumination direction 1 is:

$$\Delta \phi_1 = \frac{2\pi}{\lambda} (\vec{i}_y + \vec{i}_i) \cdot \vec{L},\quad (A.1)$$

where $\vec{i}_y$ and $\vec{i}_i$ are unit vectors in the illumination and viewing direction.

![Fig. A.1 Performing two phase change measurements with collimated illumination beams that are symmetrical with regard to the normal of the object surface.](image)

Eq. (A.1) can be rewritten as:

$$\Delta \phi_1 = KL \cos [\beta - \alpha/2]$$

$$= (2\pi/\lambda) 2 \cos [\alpha/2] L \cos [\beta - \alpha/2],\quad (A.2)$$
where $K$ is the magnitude of the sensitivity vector and $L$ is the magnitude of the displacement vector. For the measurement with illumination direction $2$ the phase change becomes:

$$(\Delta \phi)_2 = (2\pi/\lambda) \ 2 \cos [\alpha/2] \ L \cos [\beta + \alpha/2], \quad (A.3)$$

Subtraction of the two measured phase changes yields:

$$(\Delta \phi)_2 - (\Delta \phi)_1 = (2\pi/\lambda) \ 2 \cos [\alpha/2] \ L \left( \cos [\beta - \alpha/2] - \cos [\beta + \alpha/2] \right)$$

$$= (2\pi/\lambda) \ 2 \cos [\alpha/2] \ L \left( 2 \sin [\beta] \sin [\alpha/2] \right)$$

$$= (4\pi/\lambda) \ 2 \cos [\alpha/2] \sin [\alpha/2] \ L \sin [\beta]$$

$$= (4\pi/\lambda) \sin [\alpha] \ L \sin [\beta]$$

$$= (4\pi/\lambda) \sin [\alpha] L_x \quad (A.4)$$
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Summary

This thesis describes the development of interferometric measuring methods and their application in the field of deformation, strain, shape and vibration analysis of solid objects. New image processing algorithms have been developed and dedicated image processing hardware modules have been implemented to build an interferometric measuring system.

Two approaches have been investigated. With the dynamic approach the computer system was coupled to the developed interferometer. During the analysis of the interferograms, using digital phase stepping speckle interferometry, the computer system controlled the position of the mirror in the reference beam of the interferometer and the interferograms were recorded with a CCD-camera and stored in computer memory. Part II of this thesis describes this dynamic approach.

The static approach deals with the analysis of single interferograms, e.g. taken from photographs. The analysis of single interferograms is computationally more complex than the phase stepping method described in part II. Several existing algorithms have been used and new algorithms have been developed to compute the phase distribution in the interferogram. Part III of this thesis describes the static approach.

In Chapter 1 a general introduction and some historical background information about interferometry and digital image processing is given. The last section of this chapter describes the combination of both fields.

In Chapter 2 the basic principles of interferometric measurement techniques are described and a number of different techniques are discussed in more detail.

In Chapter 3 a number of phase measuring algorithms are discussed. The basic statistics and some properties of speckle in interference patterns are discussed.

In Chapter 4 the complete system that has been used for the interferometric measurements is described. The interferometer was coupled to an Ironics VME-bus computer system, containing several Datacube image processing modules. The interference patterns were recorded with a CCD-camera and
pixel-synchronous 'grabbing' was used to decrease the amount of video jitter. The application of the special purpose hardware allows a complete measurement of the phase change modulo $2\pi$ on a 512 x 512 grid every 240 ms. Also the possibility of a system based on an Apple Macintosh is investigated in this chapter.

In Chapter 5 the image processing algorithms developed for the computation of the phase map are described. A four-bucket algorithm has been used to compute the phase map, starting with four recorded interferograms. A new phase unwrapping technique based on a pixel queue has been developed to remove $2\pi$-discontinuities in the computed phase. Furthermore, algorithms for phase masking, phase smoothing and computation of derivatives of the phase are developed. The ability to distinguish the object from the background, has improved the flexibility of the processing of phase shifted speckle interference patterns.

In Chapter 6 several applications of the digital phase stepping technique are described. A qualitative measurement of the deformation of a bottle under pressure is described. A measuring method to determine the in-plane displacement components and the in-plane strain components of a deformed flat object is presented. Application of a special gradient filter enables accurate determination of the in-plane strain components at each point of the object surface. The application of this filter causes a noise reduction that is proportional to the filter size.

In Chapter 7 the static approach, i.e. the analysis of a single interferogram, is described. A global scheme of possible processing steps is given. Some problems, such as noise in interferograms and the displacement of the 'fringe' extrema are discussed.

In Chapter 8 a sequel of algorithms to compute the phase map, starting with a single interferogram, are described. Algorithms for noise reduction, interferogram normalization, fringe extrema detection, fringe ordering and phase interpolation are discussed in detail. Maximum and minimum filters have proved to be very useful. The advantages of these filters are: easy integer calculations, relatively small filter sizes, small influence of noise and relative insensitivity to the exact filter sizes. A number of problems remains to be solved.
In Chapter 9 the application of the developed image processing software has been investigated. The interferogram of a vibrating disk has been used to evaluate the developed algorithms. Several representations of the data and some accuracy considerations are described. On the VME/MC68000 system (without a floating point co-processor) the analysis of a single interferogram (256 x 256) takes about 5 minutes. On a SUN/MC68020 workstation it takes about 15 seconds.

In Chapter 10 some concluding remarks are made. Furthermore, a comparison between several interferometric measurement techniques and some suggestions for future research are given.
Dit proefschrift beschrijft de ontwikkeling van enkele interferometrische meetmethoden en hun toepassingen op het gebied van metingen aan vorm, vervorming, rek en trillingen van objecten. Nieuwe beeldbewerkings algoritmes en speciale beeldbewerkings hardware zijn ontwikkeld en geïmplementeerd in een interferometrisch meetsysteem.

Twee verschillende benaderingen zijn onderzocht. Enerzijds is een ‘dynamische’ meetmethode bestudeerd, waarbij het computersysteem direct gekoppeld is aan de ontwikkelde interferometer. Gedurende de analyse van de interferogrammen bestuurde de computer de positie van een spiegel in de referentiebundel van de interferometer en werden de interferogrammen opgenomen met behulp van een CCD-camera en opgeslagen in het computergeheugen. Deel II van dit proefschrift beschrijft deze dynamische benadering.

Anderzijds is gekozen naar de statische analyse van interferogrammen, bijvoorbeeld de analyse van interferogram foto’s. Deze methode is in het algemeen ingewikkelder dan de fase stap methode, beschreven in deel II. Bestaande en nieuwe algoritmes zijn gebruikt om de fase distributie te berekenen uitgaande van een enkel interferogram. Deel III van dit proefschrift beschrijft de statische analyse methode.

In Hoofdstuk 1 wordt een algemene introductie en enige historische achtergronden van interferometrie en digitale beeldbewerking gegeven. De laatste sectie van dit hoofdstuk gaat in op de combinatie van beide onderzoeks gebieden.

In Hoofdstuk 2 worden de basis principes en de theoretische achtergronden van interferometrische meetmethoden behandeld. Verschillende technieken komen aan de orde.

In Hoofdstuk 3 worden enkele fase stap algoritmes besproken. Eveneens wordt ingegaan op de eigenschappen van spikkel in interferentie patronen.

In Hoofdstuk 4 komt het complete interferometrische meetsysteem, dat gedurende het onderzoek is ontwikkeld en gebruikt werd voor de beschreven
metingen, aan de orde. De interferometer is gekoppeld aan een VME-bus computersysteem, waarin enkele Datacube beeldbewerkingssubmodules waren geplaatst. De interferentiapatronen zijn pixel-synchroon opgenomen met behulp van een CCD-camera. De pixel-synchrone digitalisatie geeft een verminderde van ‘video-jitter’. De toepassing van hardware modules resulteert in een volledige fase meting met een resolutie van 512 x 512 meetpunten in ca. 240 ms. Naast bovengenoemd systeem is onderzoek verricht aan de mogelijkheid interferometrische metingen te verrichten met behulp van een Apple Macintosh Ilx computer.

In Hoofdstuk 5 worden de beeldbewerkingen algoritmes behandeld, die ontwikkeld zijn ten behoeve van de fase metingen. Uitgaande van 4 interferogrammen is met behulp van een ‘4-emmer’ algoritme de fase distributie in het beeld berekend. Nieuwe ‘fase-ontrafeling’ technieken zijn gebruikt om de 2π-discontinuïteiten in de berekende fase te verwijderen. Verder zijn er algoritmes ontwikkeld voor fase maskering, fase effening en het berekenen van de afgeleiden van de fase. De mogelijkheid om met behulp van de beeldbewerkingen operaties onderscheid te maken tussen ‘object’ en ‘achtergrond’, is een duidelijk verbetering van de flexibiliteit van de meetmethode.

In Hoofdstuk 6 worden verschillende toepassingen beschreven van de ontwikkelde meettechniek. Een kwalitatieve meting van de vervorming van glazen flessen wordt behandeld. Een kwantitatieve meting van de ‘in-vlak’ vervorming en rek in een T-vormig test object komt eveneens aan de orde. De toepassing van speciale gradiënt filters maakt een nauwkeurige meting van de ‘in-vlak’ rek op elk punt van het object mogelijk. De toepassing van deze filters geeft een ruisvermindering in het beeld, die evenredig is met de filtergrootte.

In Hoofdstuk 7 wordt ingegaan op de ‘statische’ analyse van een enkel interferogram. Een globale reeks van mogelijke beeldbewerkings stappen wordt aangegeven. Enkele problemen, zoals ruis in het interferogram en de verplaatsing van ‘fringe’ extrema worden behandeld.

van deze filters zijn: simpele berekeningen met gehele getallen, relatief kleine filtergroottes, weinig invloed van ruis en relatief ongevoelig voor de exacte filtergrootte.

In Hoofdstuk 9 wordt een toepassing van de ontwikkelde 'statische' methode beschreven. Een interferogram, gegenereerd door de trilling van een ronde plaat, is gebruikt om de algoritmes te evalueren. Verschillende data representaties komen aan de orde. Op het VME/MC68000 systeem (zonder floating point coprocessor) duurt de analyse van een interferogram (256 x 256) circa 5 minuten. Op een SUN/MC68020 'workstation' duurt het ongeveer 15 seconden.

Hoofdstuk 10 bevat enkele conclusies. Verder wordt er een vergelijking gemaakt tussen verschillende interferometrische meettechnieken en worden er suggesties gegeven voor toekomstig onderzoek.
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Curriculum Vitae