Hysteresis and defects of spin-polarized edge states in the integer quantum Hall regime

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The magnetic configuration of edge channels near the boundary of a quasi-two-dimensional electron gas in the integer quantum Hall regime is studied in the Hartree-Fock approximation. In earlier work [J. Dempsey, B.Y. Gelfand, and B.I. Halperin, *Phys. Rev. Lett.* **70**, 3639 (1993)] electron-electron interactions were found to cause a spin-polarizing transition of the outermost edge states. These calculations are extended by including a nonequilibrium occupation of spin levels. A sudden reversal of edge-channel positions is predicted at values of the chemical-potential difference much smaller than the enhanced spin splitting at the edges. The crossings, which arise when edge channels switch position in open systems, are identified as solitonlike, one-dimensional magnetic domain walls. The interaction between these magnetic defects is investigated. Possible consequences and verification by experiment of the predictions are discussed.

I. INTRODUCTION

In mesoscopic structures of the quasi-two-dimensional electron gas the concept of edge channels^{1,2} can be used to describe magnetotransport along the boundaries in the integer quantum Hall regime. Recently, the influence of electron-electron interactions on the ground state of edge channels has been studied in the Hartree-Fock approximation,³ in which single-particle wave functions have an integral occupation at zero temperature. The direct interactions were shown to cause a spin-polarizing transition of the outermost edge states, whereas the exchange interaction was believed to stabilize the spin-polarized, integer filling configuration relative to states with a fractional filling. It should be mentioned that the Hartree-Fock approach has been challenged on the grounds that the global electrostatics are not taken into account properly.⁴

In the present paper we extend the work of Dempsey *et al.*³ on the magnetic configuration of edge channels to situations where spin states of a Landau level have a nonequilibrium occupation. The finite width of the quasi-two-dimensional electron gas is taken into account. We find interesting instabilities, which might be experimentally observable. In Sec. II the edge-channel polarization at equilibrium is calculated. The introduction of a chemical-potential difference in Sec. III leads to a bistable reversal of edge-channel positions in infinite or closed systems. Section IV describes a possible realization of the chemical-potential difference in open systems, which leads to solitonlike crossings of edge states. The properties of these defects are investigated in Sec. V, and possible experimental consequences in Sec. VI.

II. HARTREE-FOCK THEORY

We consider a quasi-two-dimensional quantum wire along the y axis, with a uniform magnetic field B in the z direction. The Hartree-Fock wave function of the total system is approximated by a determinant of single-particle Landau level wave functions $\psi_{n\sigma}^{\sigma}(\mathbf{r})$, where σ labels the spin component along the z axis. In the Landau gauge ($\mathbf{A} = Bx\hat{y}$), $\psi_{n\sigma}^{\sigma}(\mathbf{r}) = e^{ixy/\ell^2} \phi_{n\sigma}^{\sigma}(x, z)$, where $\phi_{n\sigma}^{\sigma}(x, z)$ is an eigenfunction of the single-particle Hamiltonian $\mathcal{H} = \mathcal{H}_0 + V_c(x) + V_H(x, z) + \mathcal{H}_{\text{ex}}^{\sigma} + \sigma\epsilon_z + V_z(z)$, with eigenvalue $\epsilon_n^{\sigma}(X)$, where the magnetic length $\ell \equiv \sqrt{\hbar c/e|B|}$. Here $\mathcal{H}_0 = \frac{1}{2m}[p_x^2 + p_z^2 + \hbar^2\ell^{-4}(x - X)^2]$, $V_H(x, z)$ is the Hartree potential, $\mathcal{H}_{\text{ex}}^{\sigma}$ is the exchange operator, $\sigma\epsilon_z$ is the Zeeman splitting (with $\epsilon_z = \frac{1}{2}g\mu_B B$), and $V_z(z)$ is the quantum well potential of the heterojunction. We assume the motion in the plane of the heterojunction to be independent of the motion perpendicular to it, and write $\phi_{n\sigma}^{\sigma}(x, z) = \chi_n(x - X)\zeta(z)$, with $\chi_n(x) = (2^N N! \sqrt{\pi} \ell)^{-\frac{1}{2}} \exp(-x^2/2\ell^2) H_N(x)$, and $H_N(x)$ a Hermite polynomial. The one-electron energy due to magnetic field $(N + \frac{1}{2})\hbar\omega_c$ and size quantization does not depend on x , and is taken as energy origin. The approximations are valid when the bare confining potential as well as the effective confining potential introduced below vary (i) slowly with respect to the magnetic length which allows us to disregard mixing with higher Landau levels, and (ii) sufficiently rapidly in order to keep the edge states incompressible, i.e., a single-determinant ground state wave function.⁵

The Hartree energy of an electron with quantum number X is

$$\varepsilon_H(X) = \int dx |\chi_N(x - X)|^2 \int d\xi' \tilde{n}(\xi') \times \int_{-\infty}^{+\infty} \frac{dq}{2\pi} V(q) e^{iq(\xi - \xi')}, \quad (1)$$

where $\tilde{n}(x/\ell) = \ell^2 \sum_{\sigma N X} \nu_N^\sigma(X) |\chi_N^\sigma(x)|^2$ is the (scaled) two-dimensional electron density, and $\nu_N^\sigma(X)$ is the X -dependent filling factor for each Landau level and spin state; $\nu_N^\sigma(X) = f(\varepsilon_N^\sigma(X))$, where $f(\varepsilon)$ is the Fermi function. The form factor $F(q)$ in the Coulomb interaction $V(q) \equiv 2\pi e^2 (\epsilon\ell)^{-1} F(q)/|q|$,

$$F(q) = \int dz_1 \int dz_2 |\zeta(z_1)|^2 |\zeta(z_2)|^2 e^{-q|z_1 - z_2|/\ell},$$

takes the finite z extent of the charge distribution into account. The logarithmic divergence of Eq. (1) for small q is canceled by the potential of the positive donor charge.

The general expression for the exchange (self-) energy $\varepsilon_{\text{ex}}^{N\sigma}(X)$ of the electrons is given by

$$\varepsilon_{\text{ex}}^{N\sigma}(X) = - \sum_{N'} \int \frac{d\mathbf{q}}{2\pi} V(q) |J_{NN'}(q)|^2 \times f(\varepsilon_{N'}^\sigma(q_y \ell + X) - \mu_\sigma).$$

This is the energy gain due to exchange with electrons with equal spin direction, which are selected by f and the chemical potential μ_σ . In equilibrium $\mu_\uparrow = \mu_\downarrow = \bar{\mu}$. The generalized Fourier transform $J_{NN'}(q)$ reads

$$J_{NN'}(q) = \sqrt{\frac{n!}{m!}} e^{-q^2/4} \left(\frac{q}{\sqrt{2}}\right)^{m-n} L_m^{(m-n)}\left(\frac{q^2}{2}\right),$$

where L_m^α is an associated Laguerre polynomial, $m = \max(N, N')$, and $n = \min(N, N')$.

In order to focus on an edge region, we follow Ref. 3 in separating the Hartree energy into spatially slowly and rapidly varying contributions. The two-dimensional electron density is decomposed into a uniform slab of density $\nu_{\text{bulk}}/2\pi$, and dipolar deviations from this slab, which do not have a net charge: $\tilde{n} = \tilde{n}_{\text{slab}} + \Delta\tilde{n}^L + \Delta\tilde{n}^R$, with

$$\tilde{n}_{\text{slab}}(\xi) \equiv \left(\frac{\nu_{\text{bulk}}}{2\pi}\right) \Theta(\xi - x_L/\ell) \Theta(x_R/\ell - \xi),$$

in which $\Theta(\xi)$ is the unit step function. For the right hand edge, $x_R/\ell \equiv (\nu_{\text{bulk}}/2\pi) \int_0^\infty d\xi \tilde{n}(\xi)$, and $\Delta\tilde{n}^R(\xi) \equiv \Theta(\xi) [\tilde{n}(\xi) - \tilde{n}_{\text{slab}}(\xi)]$. Similar definitions apply to the left hand edge.

After integrating the contribution by the slab over ξ , we can separate the Hartree potential into two terms, one with coordinates relative to the right hand edge

$$V_H^R(x) = -\frac{2}{\pi} \frac{e^2}{\epsilon\ell} \int \frac{dq}{q^2} F(q) \sin q(\xi - \xi_R) + \int_{-\infty}^{+\infty} d\xi' \Delta\tilde{n}^R(\xi') \int_{-\infty}^{+\infty} \frac{dq}{2\pi} V(q) e^{iq(\xi - \xi_R)}, \quad (2)$$

and another with coordinates relative to the left hand edge. The logarithmic divergence of the first (slab-related) term on the right hand side of Eq. (2) is due

to the absence of a positive background in the present treatment. Inclusion of the background eliminates the singularity for small q .

In a strictly two-dimensional system [$F(q) = 1$] the q integrals can be carried out analytically, yielding a Hartree potential in real space of the form $V_H^{2D}(x) = -2(e^2/\epsilon\ell) \int d\xi' \tilde{n}(\xi') \ln|x/\ell - \xi'|$ as used, e.g., in Ref. 3. We have not been able to derive an analogous analytic expression for the quasi-two-dimensional system, and proceed as follows. We assume that the small- q contributions of the slab-related terms in (2) are canceled by the background exactly up to a cutoff value $q = \delta$, whereas the background potential is assumed to vanish for $q > \delta$. The linear term of the remaining large, but slowly varying, contributions from small q is approximated by the strictly two-dimensional expression, which has a $\ln|\delta|$ dependence on the cutoff value δ . Any deviations from the linear correction correspond to long-range variations of the potential in real space due to the background charge distribution, which are not of present interest. The contributions from the terms with coordinates relative to the left hand side also vary slowly near x_R , although they may be large. Following Ref. 3, these contributions are combined with the bare confining potential to form an effective confining potential $V_c^{\text{eff}}(x)$ near x_R . In the absence of microscopic information on sample geometry, gate potential, and the distribution of screening charges, the external confinement $V_c^{\text{eff}}(x)$ is treated as an adjustable parametric potential, and Eq. (2) as the effective Hartree potential. Both potentials are shown schematically in Fig. 1.

The electron density can minimize the potential energy near the edge (shown in the inset) by either a gradual, fractional filling of states, or a spin-polarized, integral

FIG. 1. Density $n(x)$ and potential profiles in a quantum wire. The sum $V_c + V_H$ of the bare confining potential V_c and the Hartree potential V_H is shown as a thin line. The enlargement shows the separation of the sum into V_H^R and the effective confining potential $V_c^{\text{eff}} \equiv V_c + V_H^L$.

filling configuration. Assuming that the exchange sufficiently favors integral filling, we concentrate on spin polarization of the lowest Landau level, and solve the $\nu_{\text{bulk}} = 2$ case explicitly for zero temperature. We shift the origin to x_R and write the filling factor as $\nu(X; \Delta X) = \Theta(-X - \sigma \Delta X/2)$, where the width parameter ΔX is determined by minimizing the total energy of the system. The single-particle energies depend on ΔX :

$$\varepsilon_0^\sigma(X; \Delta X) = \varepsilon_c^{\text{eff}}(X) + \varepsilon_H(X; \Delta X) + \varepsilon_{\text{ex}}^\sigma(X; \Delta X) + \sigma \varepsilon_Z,$$

where⁶

$$\varepsilon_c^{\text{eff}}(X) = \frac{1}{\sqrt{\pi}} \int d\xi e^{-(\xi - \frac{X}{\ell})^2} V_c^{\text{eff}}(\ell\xi), \quad (3)$$

$$\begin{aligned} \varepsilon_{\text{ex}}^\sigma(X; \Delta X) = & -\frac{e^2}{4\pi\epsilon\ell} \\ & \times \int_0^{(X + \frac{1}{2}\sigma\Delta X)/\ell} d\xi e^{-\frac{1}{2}\xi^2} \\ & \times \int dq \frac{1}{\sqrt{q^2 + 1}} F\left(\xi\sqrt{q^2 + 1}\right) e^{-\frac{1}{2}\xi^2 q^2}, \end{aligned} \quad (4)$$

$$\begin{aligned} \varepsilon_H(X; \Delta X) = & -\int_\delta \frac{dq}{\pi^2} \frac{V(q)}{q} e^{-\frac{1}{4}q^2} \sin\left(q\frac{X}{\ell}\right) \\ & + \int_{-\infty}^{+\infty} \frac{dq}{2\pi} V(q) e^{-\frac{1}{4}q^2} e^{iq\frac{X}{\ell}} \\ & \times \int d\xi' \Delta\tilde{n}^R(\xi') e^{-iq\xi'}, \end{aligned} \quad (5)$$

with

$$\begin{aligned} \Delta\tilde{n}^R(\xi; \Delta X) = & \frac{1}{4\pi} \sum_\sigma \left[1 - \text{sgn}\left(\xi + \frac{\sigma}{2\ell}\Delta X\right) \right. \\ & \left. \times \text{erf}\left[\xi + \frac{\sigma}{2\ell}\Delta X\right] \right] - \frac{1}{\pi} \Theta(-\xi), \end{aligned}$$

and in which the integration $\int_\delta dq$ is to be understood to run from δ to infinity and to include the linear correction term. $\text{erf}(\xi)$ is the error function. The single-particle exchange energy $\varepsilon_{\text{ex}}^\sigma(X; \Delta X)$ is defined such that $\varepsilon_{\text{ex}}^\sigma(0; 0) = 0$. The form factor $F(q)$ has been calculated for a Gaussian envelope function which minimizes the Hartree energy of a modulation-doped rectangular quantum well.⁷ This is a sufficiently accurate representation of the heterojunction for the present purposes. The explicit functional dependence of $F(q)$ reads

$$F(q) = \exp\left(\frac{q^2}{2\kappa}\right) \text{erfc}\left(\frac{q}{\sqrt{2\kappa}}\right),$$

where erfc denotes the complementary error function, and κ is a variational parameter which minimizes the Hartree energy of a 100 Å wide quantum well.

The single-particle energies must satisfy the self-consistency requirement $\varepsilon_0^\sigma(X; \Delta X^*) < \varepsilon_F$ for $X \leq -\sigma\Delta X^*/2$, where ΔX^* is the value of the separation which minimizes the total energy,

$$\begin{aligned} E(\Delta X) = & \sum_{\sigma X} \nu_0^\sigma(X; \Delta X) \{ \varepsilon_0^\sigma(X; \Delta X) - \frac{1}{2} [\varepsilon_H(X; \Delta X) + \varepsilon_{\text{ex}}^\sigma(X; \Delta X)] \} \\ = & E(0) + \frac{1}{2\pi\ell^2} \int_0^{\frac{1}{2}\Delta X} dX [\varepsilon_c^{\text{eff}}(X) - \varepsilon_c^{\text{eff}}(-X) + \varepsilon_H(2X; 0) - 2\varepsilon_Z]. \end{aligned} \quad (6)$$

The contribution of the Hartree energy to the integrand in (6) has been evaluated numerically, and is plotted in Fig. 2. As a function of ΔX , the energy decreases up to the value of ΔX where $\varepsilon_H(\Delta X; 0)$ is zero. The actual position ΔX^* of the total energy minimum is determined by $\varepsilon_c^{\text{eff}}(X)$, which is smoothly varying near the edge. It may be expanded in the series $\varepsilon_c^{\text{eff}}(X) = a_1 X + a_2 X^2 + \dots$ [taking $\varepsilon_c^{\text{eff}}(0) = 0$], in which the coefficient a_1 corresponds to an electric field confining the electrons at the Fermi energy. We retain the first two terms and minimize $E(\Delta X)$ by solving

$$a_1 \Delta X + \varepsilon_H(\Delta X; 0) - 2\varepsilon_Z = 0 \quad (7)$$

self-consistently for positive values of ΔX . This equation reflects compensation of the Zeeman energy difference of electrons at the Fermi energy by the sum of confining and Hartree energies. The exchange energy does not enter the expression, since the total exchange energy of the system is the same for any solution with integer filling.

The numerical solution of Eq. (7) as a function of

FIG. 2. Solid line: one-electron Hartree energy $\varepsilon_H(\Delta X; 0)$ (units $\frac{e^2}{\epsilon\ell}$) of quasi-two-dimensional electrons versus $\Delta X/\ell$, for an inversion layer width of about 100 Å. Dashed line: two-dimensional limit [$F(q) = 1$].

FIG. 3. Equilibrium separation $\Delta X^*/\ell$ of edge states as a function of the scaled slope $\alpha = a_1\ell/(\epsilon_c^2)$ of the confining potential V_c^{eff} , with and without Zeeman energy for $B = 7.2$ T. Results are shown for a strictly two-dimensional (dashed lines), and a quasi-two-dimensional (solid lines) system. The upper curves of each set contain the Zeeman energy. Inset: self-consistent single-particle energies $\epsilon_0^\sigma(X; \Delta X^*)$ for $\Delta X^* \approx 2\ell$ (solid lines); Fermi level (dashed line).

the scaled electric field $\alpha = a_1\ell/(\epsilon_c^2)$ of ϵ_c^{eff} is displayed in Fig. 3 for $F(q) = 1$ [strictly two-dimensional (2D)], and $F(q) \neq 1$ (quasi-2D), both with and without Zeeman splitting. The values have been calculated for $B = 7.2$ T, corresponding to a zero field density of $3.5 \times 10^{11} \text{ cm}^{-2}$. In GaAs-Al_xGa_{1-x}As the static dielectric constant $\epsilon = 12.5$, and the bare Landé factor $|g| = 0.44$. The inset depicts the self-consistent single-particle energies $\epsilon_0^\sigma(X; \Delta X^*)$ for a value of α corresponding to $\Delta X^* \approx 2\ell$. It is noted that the enhancement of the energy difference between spin-up and spin-down electrons is caused by exchange, whereas the separation ΔX^* is determined by direct interactions. Quantitatively, the results for the quasi-2D system differ significantly from those of the strictly 2D system obtained earlier by Dempsey *et al.*³ The Coulomb interaction form factor significantly weakens the effect of the interaction. Still, the enhancement of the spin splitting at the edge is about a factor of 50. This appears to be too large, which means that exchange effects are overestimated in the Hartree-Fock approximation.

The values of α are restricted by the self-consistency requirement for a model with only one filled, spin-split bulk Landau level: Neglecting the bare spin splitting, the deviations of the sum $\epsilon_c(X) + \epsilon_{\text{ex}}(X)$ from the absolute value of the Hartree energy must be less than $\hbar\omega_c$, or higher Landau levels become occupied. In wide wires this restricts the slope of a slowly varying effective confining potential to very small values. On the other hand, the present treatment is not necessarily limited to the $\nu = 2$ situation, as long as the edges due to higher Landau levels are sufficiently separated from that of the first Landau level. Of course the presence of additional electrons has an effect on the effective confining potential.⁸

III. HYSTERESIS

When the spin-up and spin-down levels are populated up to different chemical potentials $\mu_\uparrow = \bar{\mu} - \Delta\mu$, and

$\mu_\downarrow = \bar{\mu} + \Delta\mu$, the value of ΔX^* will change. Instead of having the same Fermi energy, the outermost electrons of each spin level differ in energy by $2\Delta\mu$. Generalizing Eq. (7) to include this difference, the equation for ΔX^* becomes

$$a_1\Delta X + \epsilon_H(\Delta X; 0) = 2\epsilon_z - 2\Delta\mu. \quad (8)$$

Solving this equation for positive ΔX imposes an upper bound on $\Delta\mu$; if the value of $2(\epsilon_z - \Delta\mu)$ is lower than the (local) minimum value of $a_1\Delta X + \epsilon_H(\Delta X; 0)$, Eq. (8) does not have a real solution for $\Delta X^* \geq 0$. However, solutions do exist for *negative* values of ΔX , corresponding to *reversed channel positions*. They satisfy

$$a_1|\Delta X| + \epsilon_H(|\Delta X|; 0) = 2(\Delta\mu - \epsilon_z), \quad (9)$$

which also permits solutions for smaller $|\Delta X|$ ($\Delta X < 0$). In fact, these solutions correspond to local minima of the energy, if the value of $\Delta\mu$ is reduced again. Similarly to Eq. (8), Eq. (9) imposes a lower bound on $\Delta\mu$ for negative values of ΔX .

The system therefore shows a bistable behavior when $\Delta\mu$ is varied. Starting from $\Delta\mu = 0$, the separation ΔX^* continuously decreases, when $\Delta\mu$ is increased. At a critical value $\Delta\mu_{\text{cr}}^+ > 0$, the edge channels switch position and the absolute value of the separation increases. When the applied bias is reduced again, an energy barrier between the edge channels prohibits the relaxation of the system until a value $\Delta\mu_{\text{cr}}^-$ is reached.

This is illustrated in Fig. 4, where the separation ΔX^* is plotted versus $2\Delta\mu$, for $B = 7.2$ T and $\alpha = 0.05$. The value of $\Delta\mu_{\text{cr}}^+$ is of the order of $\sim 8\epsilon_z$. The edge channels switch position at values $2\Delta\mu$ of about $\frac{1}{6}$ of the enhanced energy difference, which is again forced by the direct and not the exchange interaction.

FIG. 4. Equilibrium separation $\Delta X^*/\ell$ of edge states as a function of a chemical-potential difference $2\Delta\mu$ between the spin-up and spin-down levels (units $\epsilon_c^2/\epsilon\ell$) for $B = 7.2$ T and $\alpha = 0.05$. Negative ΔX^* correspond to reversed channel positions.

IV. REALIZATION OF $\Delta\mu$

We now want to address the question of how the chemical-potential difference can be realized experimentally. The calculations of the previous section have been carried out for a large system with periodic boundary conditions. With slight modifications the results also hold for a dot with a radius much larger than the magnetic length. In a dot the chemical-potential difference can be realized by irradiation with polarized light (see Sec. VI). However, a chemical-potential difference is most easily realized by applying a voltage difference on the contacts of an open system.⁹ A nonequilibrium population of adjacent spin levels can be obtained by a spin-selective potential barrier created by a gate across the Hall bar,¹⁰ as shown in Fig. 5(a). When the gate potential raises a Landau level above the Fermi energy, the corresponding edge channel is reflected. Spin selectivity¹¹ of the barrier is facilitated by the exchange-induced enhancement of the g factor when spin-split Landau levels have a different occupation.¹²

The enhancement is essential to retain spin selectivity when a chemical-potential difference $2\Delta\mu$ is present between the reservoirs on either side of the quantum wire. It can be seen from Fig. 5(b) that strict channel selection is impossible when the energy difference $2\epsilon_z^{\text{enh}}$ between the bottoms of the reflected and transmitted Landau levels is less than $2\Delta\mu$ above the barrier. Besides this, a chemical-potential difference induces a redistribution of electronic charge in the wire, which imposes an additional, but not very stringent, restriction on the values of $\Delta\mu$: channel selection remains possible if

FIG. 5. Creation of a chemical-potential difference $2\Delta\mu$ between adjacent edge channels. (a) Top view: selective spin transmission of edge channels (thin lines) by a reflecting gate (hatched) across the Hall bar. Arrows indicate electron motion along the boundaries (thin lines). (b) The maximum enhancement of the spin splitting $\Delta\epsilon_z^{\text{bare}}$ to $\Delta\epsilon_z^{\text{enh}}$ above the barrier (V_G) restricts the values of $\Delta\mu$ for which spin selection is possible to $2\Delta\mu < \Delta\epsilon_z^{\text{enh}}$. (c) Cross section of the single-particle energy profiles (thick lines) along the dashed line in (a). Of the two departing edge channels ($\otimes$) the reflected one differs by $2\Delta\mu$ in chemical potential from the arriving, lower-chemical-potential channel ($\odot$) at the left hand side of the wire.

FIG. 6. Ratio of $\Delta\mu_{\text{cr}}^+$ and $\Delta\epsilon_z^{\text{bare}}$ as a function of the scaled slope α of the confining potential V_c^{eff} and the magnetic field B . At a chemical-potential difference $2\Delta\mu_{\text{cr}}^+$ edge channels reverse position.

$\Delta\mu \leq 0.9\epsilon_z^{\text{enh}}$. For values of $\Delta\mu$ in this range, a chemical-potential difference between adjacent edge channels is realized at the side of the wire where the reflected, higher-chemical-potential channel meets the transmitted, lower-chemical-potential one. This is shown in Fig. 5(c). Here it is assumed that the small amount of scattering between the spin channels leaves the value of $\Delta\mu$ unaltered over the length of their trajectories through the wire. This is not unreasonable, considering the long spin-flip relaxation times.¹¹

The critical chemical-potential difference $\Delta\mu_{\text{cr}}^+$ at which the edge channels switch position depends on the slope α of the effective confining potential. The ratio $G \equiv \Delta\mu_{\text{cr}}^+ / \epsilon_z^{\text{bare}}$ of $\Delta\mu_{\text{cr}}^+$ and the bare Zeeman energy is plotted versus α in Fig. 6 for different magnetic fields. The maximum g -factor enhancement, for which a nonequilibrium occupation of spin channels can be realized experimentally, restricts the values of α . This in turn restricts the maximum permissible wire width W . Assuming a typical enhancement¹³ of g by a factor of 10 and modeling the effective confining potential by a parabola, one finds $W = 40\ell$ (≈ 3800 Å) for $B = 7.2$ T, which is an attainable wire width. In relatively narrow quantum wires, it should therefore be possible to realize channel switching by means of a reflecting barrier.

V. CHANNEL CROSSING

Channel switching in an open system causes the topological problem of how to connect the channels to the contacts. In a semi-infinite realization of channel switching, edge states have to cross each other to reverse position. Relative to the original configuration, a strip with a reversed spin direction is formed. However, the contacts themselves are supposed to be at thermal equilibrium with normal ordering of spin states, which means that the channels have to switch again somewhere. The magnetic regions are separated by nondispersive defects, corresponding to solitonlike “Bloch points” between one-dimensional, magnetic domains. The defect and antide-

fects repel each other as long as the chemical-potential difference is maintained to be larger than the critical one. When the bias is removed again, the Zeeman energy difference between the domains exerts an attractive force between the defect-antidefect pair. Between these extremes the interaction can be freely tuned. The annihilation of the defects in the attractive regime can be prevented by impurity pinning or the short-range electrostatic repulsion due to the dipolar charges (see below).

We have investigated the short-range interaction by a simple electrostatic model of the defects. Neglecting exchange effects, which favor convex (as seen from the bulk) over concave bending of edge channels, a lower bound for the repulsive energy is obtained. The system strives to minimize any extra charge, and therefore bends the edge channels at the crossing as much as possible. The increase in kinetic energy of an edge state upon bending can be modeled by a hard-core interaction: up to a minimum radius of curvature $r_c^{\min}$ of an edge channel the kinetic energy remains approximately unchanged whereas further bending is possible only at the high energy cost of mixing in higher Landau levels.

We have calculated the electrostatic energy of the dipolar charges defined by the crossing edge channels for a strictly two-dimensional system.¹⁴ The intersecting paths which constitute a defect are approximated by trigonometric functions in the bending regime. Without a chemical potential difference, the total energy of two interacting defects is the sum of electrostatic and Zeeman energies. In Fig. 7 this energy is plotted as function of the distance $\Delta\xi$ between the defects, for an equilibrium separation of edge channels $\Delta X^* = 2\ell$ and a minimum radius of curvature $r_c^{\min} = 2\ell$. The results depend only weakly on the precise values of these parameters. The local minimum in the energy corresponds to a distance $\Delta\xi$, in which the edge channels traverse ΔX^* and return immediately. At this point, either the charge in the dipolar distribution has to increase to allow smaller values of

FIG. 7. Energy (units $\frac{e^2}{\epsilon l}$) of interacting defects as a function of their mutual distance $\Delta\xi$ for an equilibrium separation of edge channels $\Delta X^* = 2\ell$. The energy E_{tot} consists of electrostatic (E_{el} , dashed) and Zeeman (E_Z , dotted) energy ($B = 7.2$ T). The inset shows the dipolar charge at a crossing.

$\Delta\xi$, or the edge channels have to be bent more strongly than $r_c^{\min}$. Both effects increase the energy, and the minimum will survive a more general treatment. An order of magnitude estimate of the lifetime τ of a double-defect ("bipolaron") system without a chemical-potential difference indicates $\tau \sim 10^{-10}$ s. The electrostatic repulsion is not strong enough to suppress the recombination of the two defects for an appreciable time. The reader is reminded, however, that an applied bias does provide for a long-range repulsion, which can be tuned to create a system of two defects with only a short-range repulsive interaction and a long recombination lifetime.

VI. POSSIBLE EXPERIMENTS

At a crossing, the increased edge-state overlap and spin-flip probability, due to the change in wave vector, produce scattering between spin channels.¹¹ Although this forward scattering will not influence the total conductance,¹⁵ the amount of current per edge channel will be affected. The relaxation of a nonequilibrium current distribution depends exponentially on the distance between the channels. In contrast to the noninteracting picture, the present results show that the separation in typical samples is always of appreciable magnitude, except at the singular defects. This implies a low scattering rate at all applied voltages, which is suddenly enhanced when a crossing appears. Selective detection of the contribution per edge state to the conductance is possible by the use of quantum point contacts as voltage probes,¹⁶ as shown in Fig. 8. An electron spin which is flipped at one defect may be flipped back at the antidefect again, interfering with the path without spin flips. Although calculation of the path length and phase difference is difficult, oscillatory interferences in the conductance of a single spin channel proportional to the defect separation cannot be excluded.

An intriguing possibility is the spin blockade of transport through a quantum dot connected to the leads by tunnel barriers. In a system which contains two Landau levels, tunneling occurs selectively from the reservoirs to the outer spin level.¹⁷ When the polarization of the spin levels inside the dot is reversed, electrons have to undergo a spin flip in addition to the tunneling process, in

FIG. 8. Selective channel detection by an adjustable barrier (shaded) in front of a voltage probe (crossed bar). Of the two edge channels shown ($\nu = 2$), one is reflected by a selective gate (hatched), resulting in a double defect between gate and probe.

order to be able to pass the barrier, which blocks transport. In a quantum dot containing 300 electrons, reversal of the polarization requires a spin flip of about six electrons, which may be achieved by a pulse of polarized microwave radiation. Relaxation of the system out of the metastable spin-blockade state is an example of “macroscopic quantum tunneling of spin,” which is expected to be slow.

VII. CONCLUSIONS

We have used the Hartree-Fock approximation to study the effects of electron-electron interactions on quantum Hall edge states. Although the model has been criticized for its description of screening effects,⁴ it could be valid in relatively narrow quantum wires, due to exchange effects at the edges, which favor integer filling. The pres-

ence of compressible edge channels does not preclude the predicted nonlinearities, but additional investigations are necessary for that situation. We have shown that in the integer filling model a nonequilibrium population of the spin states of a Landau level causes a sudden reversal of the corresponding, spin-polarized edge channels. Realization of the chemical-potential difference by a spin-selective gate across a narrow Hall bar induces a soliton-like crossing of edge states. A sudden increase in inter-edge-channel scattering and spin blockade of transport through a quantum dot are predicted.

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FIG. 5. Creation of a chemical-potential difference $2\Delta\mu$ between adjacent edge channels. (a) Top view: selective spin transmission of edge channels (thin lines) by a reflecting gate (hatched) across the Hall bar. Arrows indicate electron motion along the boundaries (thick lines). (b) The maximum enhancement of the spin splitting $\Delta\epsilon_z^{\text{bare}}$ to $\Delta\epsilon_z^{\text{enh}}$ above the barrier (V_G) restricts the values of $\Delta\mu$ for which spin selection is possible to $2\Delta\mu < \Delta\epsilon_z^{\text{enh}}$. (c) Cross section of the single-particle energy profiles (thick lines) along the dashed line in (a). Of the two departing edge channels ($\otimes$) the reflected one differs by $2\Delta\mu$ in chemical potential from the arriving, lower-chemical-potential channel ($\odot$) at the left hand side of the wire.

FIG. 8. Selective channel detection by an adjustable barrier (shaded) in front of a voltage probe (crossed bar). Of the two edge channels shown ($\nu = 2$), one is reflected by a selective gate (hatched), resulting in a double defect between gate and probe.