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Identification of Time-Varying Manual-Control Adaptations with Recursive ARX Models

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Time-varying human-operator (HO) adaptation in basic manual-control tasks is barely understood, as most HO identification methods do not explicitly take into account time variations. An identification procedure based on both batch and recursive autoregressive exogenous (ARX) models is used in this paper for capturing HO adaptation to time-varying changes in controlled-element dynamics in compensatory tracking tasks. Conditions with constant controlled-element dynamics, matching recent experimental work, and gradual and sudden transitions in the controlled-element dynamics were compared by means of Monte Carlo simulations with different remnant noise intensities. Overall, with batch ARX model identification results for conditions with constant dynamics, it is shown that differences between remnant noise and HO dynamics, both of which are directly coupled in the ARX model structure, will bias the identified ARX model parameters. Nevertheless, recursive ARX model identification results were still found to provide a reasonably estimate of time-varying HO model parameters for the tested time-varying scenarios. A direct comparison of using a constant scalar forgetting factor λ and a constant forgetting matrix Λ , containing separate forgetting factors for each ARX model parameter, showed that a forgetting matrix did not result in a significant improvement for the considered HO identification problem. A forgetting factor of $\lambda = 0.99609$, corresponding to a memory horizon of 256 samples at 100 Hz, is found to be optimal for all tested conditions. An evaluation of real experimental manual-control data shows that, with straightforward online implementation, the method has potential as a tool for investigating, modeling, and measuring operators' adaptive control characteristics.

Nomenclature

$A(q)$	ARX model's output polynomial	$H_n(t)$	Time-varying remnant filter
A_t	Sinusoid amplitude, deg	$H_{nm}(s)$	Neuromuscular dynamics
a_i	Coefficient i of $A(q)$	I	Identity matrix
$B(q)$	ARX model's input polynomial	$K(t)$	Gain vector
b_j	Coefficient j of $B(q)$	$K_c(t)$	Control gain
d	Decimation factor	$K_e(t)$	Error gain
$e(t)$	Error signal, deg	$K_{\dot{e}}(t)$	Error-rate gain, s
$\dot{e}(t)$	Error-rate signal, deg/s	$K_n(t)$	Remnant gain
f_s	Sampling frequency, Hz	k_c	$= \max(n_a + 1, n_b + n_k)$, critical instant
$f_t(t)$	Target forcing function signal, deg	M	Monte Carlo replications
G	Maximum rate of change, s ⁻¹	m	Order of remnant filter
$G(z)$	Discrete-time transfer function	N	Number of samples
$H_{CE}(s, t)$	Time-varying CE dynamics	N_0	Memory horizon in samples
$H_{eq}(s, t)$	Time-varying equalization dynamics	$n(t)$	Remnant signal, deg
$H_{HO}(s, t)$	Time-varying HO dynamics	n_a	Number of coefficients in $A(q)$
$H_{HO_e}(s, t)$	Time-varying HO's linear response to $e(t)$	n_b	Number of coefficients in $B(q)$

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n'_b	$= n_b + n_k - 1$, highest order of $B(q)$	$\hat{\theta}_0$	Initial parameter estimate
n_k	ARX model's integer delay	Λ	Forgetting matrix
n_k^*	Estimated ARX model's integer delay	λ	Forgetting factor (scalar)
n_k^0	True ARX model's integer delay	σ^2	Variance
n_t	Integer multiple	τ_e	Time delay, s
$n_w(t)$	Zero-mean Gaussian white-noise signal with unit variance, deg	Φ	Regression matrix
$P(t)$	Scaled covariance matrix	ϕ_t	Sinusoid phase shift, rad
P_0	Initial scaled covariance matrix	φ_m	Phase margin, deg
P_n	Remnant-intensity level	$\varphi(t)$	Regression vector
$P_\theta(t)$	Actual covariance matrix of parameters θ	ω	Frequency, rad/s
$p(t)$	Parameter function	$\omega_b(t)$	Break frequency, rad/s
$Q(t)$	Matrix obtained by RLS criterion minimization	ω_c	Crossover frequency, rad/s
q	Discrete-time shift operator	ω_m	Measurement-base frequency, rad/s
s	Laplace variable	ω_{nm}	Neuromuscular frequency, rad/s
T_0	Memory horizon, s	ω_t	Sinusoid frequency, rad/s
$T_L(t)$	Lead-time constant, s	<i>Other</i>	
T_m	Measurement time/window, s	$\hat{}$	Estimated; Predicted; Simulated
T_n	Remnant-time constant, s	$\sim$	Preprocessed
T_s	Sampling time/interval, s	$\mathcal{M}_{\text{ARX}}$	ARX model
t	Time, s	$\mathcal{M}_{\text{HO}}$	HO model
t_M	Time of maximum rate of change, s	<i>Subscripts</i>	
$u(t)$	Control-output signal, deg	i	Initial
V	Loss function	e	Estimation
VOF	Variance accounted for, %	f	Final
$v(t)$	Disturbance signal, deg	m	Measurement
$x(t)$	System-output signal, deg	p	One-step-ahead prediction
z	z-transform variable	s	Simulation
δ	Initial parameter covariance setting	v	Validation; Verification
$\epsilon(t)$	White noise, deg	<i>Superscripts</i>	
$\varepsilon(t)$	Prediction error, deg	0	True
ζ_{nm}	Neuromuscular damping ratio	c	Continuous time
θ	Parameter vector		

I. Introduction

HUMAN manual-control behavior inherently varies over time, across different situations, and between operators. Identification methods have been developed that are able to describe the dynamic characteristics of human operators (HOs) in skill-based manual-control tasks.¹⁻⁷ These are, however, mostly restricted in their use to cases where the control behavior is assumed to be time invariant.^{8,9} A long-standing desire is to have control-theoretic models available that are capable of capturing both the adaptive and learning characteristics of manual-control behavior.⁹⁻¹² State-of-the-art cybernetics cannot explain, in depth, how HOs adapt themselves to cope with control-task transitions. Rapid advancement of understanding how humans truly interact with dynamic control systems requires the further development of time-varying identification methods.⁹

Most research on identifying time-varying manual-control behavior focuses on induced changes in the task variables, in particular, on transitions in the controlled-element (CE) dynamics.^{2,10,12-14} For any change in these variables, substantial and highly noticeable adaptation changes are expected. Many studies start with investigating single-axis compensatory control tasks and are based on the well-known crossover model,² bearing in mind that extensions to multiple axes of control or even the usage of other classes of inputs to the HO should not pose fundamental difficulties.¹² Previous investigations make clear that, before using a novel identification method in laboratory experiments or real-life tasks, its accuracy should be evaluated thoroughly by means of Monte Carlo simulations.^{6,7,15} In addition to task variables, manual-control behavior depends on a myriad of other factors, e.g., suffering fatigue or alterations in motivation are directly related to the human operator. Usually, these factors are lumped together as remnant, taking into account different remnant intensities.

Literature shows that different approaches have been taken to modeling time-varying manual-control behavior. Wavelet-based identification can detect time-varying control adaptations, but is very sensitive to HO remnant.^{16,17} The identification method based on genetic maximum-likelihood estimation^{7,17} is less sensitive to remnant and can provide accurate estimations, but requires restrictive definitions about how certain parameters will change.¹³ Explicit linear parameter-varying predictor-based subspace identification methods require the definition of precise scheduling functions, which is challenging for our human-in-the-loop problem.¹⁸ Finally, recursive identification methods are beneficial for their inherent ability to track the evolution of model parameters over time, and even in real time (i.e., online). Methods based on Kalman filters^{14,19–21} and recursive least-squares (RLS) algorithms with a forgetting factor²² thus are highly promising candidate to consider. For the latter, to the authors' knowledge, no previous study has thoroughly investigated the tuning (e.g., forgetting) of RLS algorithms for the identification of manual control.

In manual-control cybernetics, very few studies have yet attempted to make use of recursive autoregressive exogenous (ARX) models. In contrast to autoregressive moving average exogenous (ARMAX), output-error (OE), or Box-Jenkins (BJ) model structures, the structure of ARX models allows direct linear regression estimation.²³ This convenience is particularly relevant in identifying manual-control behavior as the real process is of infinite order. This study aims to extend the work of Refs. 6 and 15 with recursive estimation, i.e., to take the ARX model structure and to recursively estimate the model parameters using RLS. Thus, this study aims to develop an identification method able to capture and characterize time variations in manual-control behavior without making explicit *a priori* assumptions on how HO parameters will evolve over time.

This paper investigates the use and feasibility of a recursive ARX model identification method for capturing time-varying manual-control adaptations. RLS algorithms equipped with a constant scalar forgetting factor and a constant forgetting matrix are considered. To analyze the estimation performance and the effect of the process and noise model coupling in the ARX model structure, a Monte Carlo simulation analysis with $M = 100$ replications is performed. In this analysis, different remnant-filter orders ($m = 1–4$) as well as remnant intensity levels (0–30%, in increments of 5%) are tested. The emphasis is placed on identifying the operator's adaptive responses in a compensatory tracking task when faced with gradual and sudden transitions in the CE dynamics, as studied by Zaal.¹³ For reference, the quality of the obtained results is explicitly compared with batch (i.e., non-recursive) ARX model estimation of the operator's responses to constant CE dynamics.

The paper is structured as follows. The compensatory manual-control task is described in Section II. Section III outlines the ARX model identification approach. The setup of the Monte Carlo simulation analysis is given in Section IV. Section V provides example results. An evaluation of the identification method on real experimental manual-control data is presented in Section VI. Section VII discusses the identification scheme and its limitations. Conclusions are drawn in Section VIII.

II. Compensatory Manual-Control Task

In continuation of research by Zaal,¹³ this study concentrates on the single-axis compensatory manual-control task shown in Fig. 1. The HO, modeled as $H_{HO}(s, t)$, observes and acts, with a control output $u(t)$, upon the error $e(t)$ between a target $f_t(t)$ (e.g., a multisine) and the output $x(t)$ of the CE dynamics $H_{CE}(s, t)$. The HO dynamics consist of deterministic responses $H_{HO_e}(s, t)$ and remnant $n(t)$ accounting for control nonlinearities.² Changes over time in the CE dynamics require control-strategy adaptations in the HO.¹⁰

Figure 1. Single-axis compensatory manual-control task with time-varying dynamics. (a) Compensatory display: $e(t)$ acts as stimulus. (b) Block diagram.

II.A. Controlled-Element Dynamics

In Ref. 13, the following second-order CE dynamics, which represent a generic low-order approximation of typical vehicle dynamics,² were considered:

$$H_{CE}(s, t) = \frac{K_c(t)}{s(s + \omega_b(t))} \quad (1)$$

The CE parameters are the control gain $K_c(t)$ and the break frequency $\omega_b(t)$, which both can vary in time. Around $\omega_b(t)$ the CE dynamics transition from single- to double-integrator dynamics (i.e., $1/s \leftrightarrow 1/s^2$). The time variation in the CE parameters in Eq. (1) was defined using a sigmoid function,¹³ i.e., for a generic parameter $p(t)$:

$$p(t) = p_i + \frac{p_f - p_i}{1 + e^{-G(t-t_M)}} \quad (2)$$

In Eq. (2), the initial and final parameter values are set by p_i and p_f , respectively. Fig. 2 clarifies the tuning of the maximum rate of transition G and the time t_M at which this change happens. For all G , $p(t_M)$ equals $p_i + \frac{p_f - p_i}{2}$. Gradual transitions are obtained for low values of G . Around $G = 10 \text{ s}^{-1}$, the sigmoid already approaches a step function.

Figure 2. Sigmoid-function variation with the maximum rate of change parameter G .

II.B. Human-Operator Dynamics

A suitable HO model for the time-varying CE dynamics to be controlled^{2,3,13} is defined by:

$$H_{HO_e}(s, t) = \overbrace{K_e(t) [T_L(t)s + 1]}^{\text{Equalization}} \overbrace{e^{-s\tau_e} H_{nm}(s)}^{\text{Limitations}}, \quad (3)$$

with:

$$H_{nm}(s) = \frac{\omega_{nm}^2}{s^2 + 2\zeta_{nm}\omega_{nm}s + \omega_{nm}^2} \quad (4)$$

The HO model in Eq. (3) consists of equalization dynamics $H_{eq}(s, t)$ and operator limitations that include the neuromuscular dynamics $H_{nm}(s)$. The equalization parameters are the error gain $K_e(t)$ and the lead-time constant $T_L(t)$. The limitation parameters are the time delay τ_e , the natural frequency ω_{nm} , and the damping ratio ζ_{nm} , which are assumed to be time invariant.¹³ If the HO's equalization-model structure in Eq. (3) is used and the response to the error rate $\dot{e}(t)$ is dominant, a strong (negative) parameter correlation arises between $K_e(t)$ and $T_L(t)$. An equivalent alternative parametrization for $H_{eq}(s, t)$, easing identification,¹⁸ can be obtained with the error-rate gain $K_{\dot{e}}(t) = K_e(t) T_L(t)$:

$$H_{eq}(s, t) = K_{\dot{e}}(t)s + K_e(t) \quad (5)$$

III. ARX Model Identification

III.A. ARX Model Structure

Transfer functions models with a rational polynomial structure, such as ARX models, provide high flexibility in describing the HO dynamics. Computationally efficient polynomial estimation is offered by the ARX model structure.

Fig. 3 represents this structure, which is equivalent to the HO model structure in Fig. 1. The process model $\frac{B(q)}{A(q)}$ relates $e(t)$ to $u(t)$. The disturbance signal $v(t)$, obtained by filtering the white-noise signal $\epsilon(t)$ using the noise model $\frac{1}{A(q)}$, is similar to the remnant signal $n(t)$, see Fig. 1(b). The coupling of the process and noise model is the main, critical assumption of the ARX model structure.

Figure 3. The ARX model structure.

Denoting q as the discrete-time shift operator, the ARX model's difference equation is:

$$A(q)u(t) = B(q)e(t) + \epsilon(t) \quad (6)$$

with the output and input polynomials $A(q)$ and $B(q)$, having orders n_a and $n_b - 1$, respectively:

$$A(q) = 1 + a_1q^{-1} + \dots + a_{n_a}q^{-n_a} \quad (7)$$

$$B(q) = (b_0 + b_1q^{-1} + \dots + b_{n_b}q^{-n_b+1})q^{-n_k} \quad (8)$$

The integers n_a and n_b set the number of parameters a_i for $i \geq 1$ and b_j for $j \geq 0$, respectively. The parameter vector θ , containing the adjustable parameters, is thus:

$$\theta = [a_1 \ a_2 \ \dots \ a_{n_a} \ b_0 \ b_1 \ \dots \ b_{n_b}]^T \quad (9)$$

The integer n_k models a discrete input delay, leading to exponent shifts in $B(q)$. To ease notation, $n'_b = n_b + n_k - 1$ can be used. If n_k is zero, parameter b_0 provides direct transmission.

III.B. OLS and RLS Estimation

As the ARX model structure allows the application of linear regression,²³ ordinary least-squares (OLS) and recursive least-squares (RLS) estimation of the parameter vector θ in Eq. (9) is straightforward. OLS estimation acts in batch mode (i.e., uses all samples in a single least-squares estimation step), while RLS estimation updates the parameter estimate with each new measurement sample that becomes available, and is, therefore, able to track time variations with appropriate use of a forgetting profile. In both types of least-squares estimations, the regression vector $\varphi(t)$ plays a central role. For the input-output data set of our HO model estimation problem $\{e(t), u(t)\}$, it is defined as:

$$\varphi(t) = [-u(t-1) \ -u(t-2) \ \dots \ -u(t-n_a) \ e(t-n_k) \ e(t-n_k-1) \ \dots \ e(t-n'_b)]^T \quad (10)$$

From an identification perspective, time variations in the input-output data are best captured if the sampling time T_s is as low as possible. It might be, however, that the ARX model transfer function from $e(t)$ to $u(t)$ cannot properly explain the HO remnant characteristics. Typically, there are noise disturbances in the data, above the frequencies of interest. Then, a remedy is to decimate the input-output data.^{6,23} To circumvent the issue of decreased resolution in capturing the integer time delay, the input signal $e(t)$ can be shifted with the assumed interval steps n_k^* of time delay before decimating.

Furthermore, it should be noted that the closed-loop identification problem considered here will be tackled by means of the direct identification approach. In Ref. 24, it was found that the direct identification approach with ARX models is able to provide more accurate estimates in the frequency range of interest for manual control than the indirect approach. Hence, closed-loop identification issues are not explicitly taken into account.

The OLS and RLS methods and their implementations are described below.

III.B.1. OLS Estimation

From the input-output data set consisting of N samples, a regression matrix Φ and output vector $\mathbf{u}$ should be constructed from $\varphi(t)$ and $u(t)$, respectively. In order to consider only known input-output data, regressors $\varphi(t)$ cannot be fully constructed at discrete time instants earlier than $k_c = \max(n_a + 1, n_b + n_k)$, unless assumptions (e.g., zero-padding) are made. OLS estimates $\hat{\theta}^{\text{OLS}}$ can be obtained using QR factorization provided that Φ is full rank.²³

III.B.2. RLS Estimation

The generic RLS algorithm is provided by Eqs. (11) to (16).²³ By adjusting the gain vector $K(t)$, the prediction error $\varepsilon(t)$ between $u(t)$, i.e., the observed output at time t , and $\hat{u}(t)$, i.e., the prediction of $u(t)$ based on observations up to time $t - 1$, can be minimized. The computation of $Q(t)$ makes use of the scaled covariance matrix $P(t)$.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t) \varepsilon(t) \quad (11)$$

$$\varepsilon(t) = u(t) - \hat{u}(t) \quad (12)$$

$$\hat{u}(t) = \varphi^T(t) \hat{\theta}(t-1) \quad (13)$$

$$K(t) = Q(t) \varphi(t) \quad (14)$$

$$Q(t) = \frac{P(t-1)}{\lambda(t) + \varphi^T(t) P(t-1) \varphi(t)} \quad (15)$$

$$P(t) = \frac{P(t-1)}{\lambda(t)} (I - K(t) \varphi^T(t)) \quad (16)$$

The critical design variable of an RLS algorithm is the forgetting factor $\lambda(t) \in (0, 1)$, which assigns less weight to older measurements and, more importantly, controls the variance (smoothing) of $\hat{\theta}(t)$. In essence, $\lambda(t)$ sets the trade-off between noise sensitivity (i.e., $\lambda(t)$ close to 1, slow adaptation, and low variance in $\hat{\theta}(t)$) and tracking ability (i.e., lowered $\lambda(t)$, fast adaptation, and high variance in $\hat{\theta}(t)$). Numerous strategies exist for implementing and tuning the forgetting profile. Particularly for manual-control identification, it is essential “to look for the simplest possibility of making estimators reliably adaptive, within a rigorous framework”.²⁵ Hence, in this paper, a conservative approach is taken here where only constant (time-invariant) forgetting factors $\lambda(t) \equiv \lambda$ are used. However, for our application, both a scalar forgetting factor λ (i.e., the same forgetting profile for all parameters) and a forgetting matrix Λ are considered. The latter allows for different λ values for different parameters, with possibly different timescales of variations.²⁶ For the parameter vector θ in Eq. (9), Λ is defined as:

$$\Lambda = \text{diag}(\lambda_{a_1}, \lambda_{a_2}, \dots, \lambda_{a_{n_a}}, \lambda_{b_0}, \lambda_{b_1}, \dots, \lambda_{b_{n_b}})^{-1/2} \quad (17)$$

It is a diagonal matrix with inverse square roots of forgetting factors matching specific parameters. When using Λ , Eqs. (15) and (16) are changed, respectively, into:

$$Q(t) = \frac{\Lambda P(t-1) \Lambda}{1 + \varphi^T(t) \Lambda P(t-1) \Lambda \varphi(t)} \quad (18)$$

$$P(t) = \Lambda P(t-1) \Lambda (I - K(t) \varphi^T(t)) \quad (19)$$

This ensures that all entries in $P(t-1)$ are weighted properly, e.g., if $n_a = n_b = 2$ (omitting $(t-1)$ for readability):

$$\Lambda P \Lambda = \begin{bmatrix} \frac{P_{a_1}}{\lambda_{a_1}} & \frac{P_{a_1, a_2}}{\sqrt{\lambda_{a_1} \lambda_{a_2}}} & \frac{P_{a_1, b_0}}{\sqrt{\lambda_{a_1} \lambda_{b_0}}} & \frac{P_{a_1, b_1}}{\sqrt{\lambda_{a_1} \lambda_{b_1}}} \\ \frac{P_{a_1, a_2}}{\sqrt{\lambda_{a_1} \lambda_{a_2}}} & \frac{P_{a_2}}{\lambda_{a_2}} & \frac{P_{a_2, b_0}}{\sqrt{\lambda_{a_2} \lambda_{b_0}}} & \frac{P_{a_2, b_1}}{\sqrt{\lambda_{a_2} \lambda_{b_1}}} \\ \frac{P_{a_1, b_0}}{\sqrt{\lambda_{a_1} \lambda_{b_0}}} & \frac{P_{a_2, b_0}}{\sqrt{\lambda_{a_2} \lambda_{b_0}}} & \frac{P_{b_0}}{\lambda_{b_0}} & \frac{P_{b_0, b_1}}{\sqrt{\lambda_{b_0} \lambda_{b_1}}} \\ \frac{P_{a_1, b_1}}{\sqrt{\lambda_{a_1} \lambda_{b_1}}} & \frac{P_{a_2, b_1}}{\sqrt{\lambda_{a_2} \lambda_{b_1}}} & \frac{P_{b_0, b_1}}{\sqrt{\lambda_{b_0} \lambda_{b_1}}} & \frac{P_{b_1}}{\lambda_{b_1}} \end{bmatrix} \quad (20)$$

As only constant forgetting factors λ are considered in this paper, old measurements are discounted exponentially.²³ A measurement that is $N_0 = 1/(1 - \lambda)$ samples old has a weight equal to $\lambda^{N_0} \approx e^{-1} \approx 37\%$ times the weight of the most recent observation. The memory horizon N_0 is easier to interpret than the value of λ , e.g., if $\lambda = 0.9375$, N_0 equals 16 samples. Alternatively, denoting T_s as the sampling time in seconds, the memory horizon can be expressed in seconds: $T_0 = T_s/(1 - \lambda)$, e.g., if $T_s = 0.01$ s and $\lambda = 0.9375$, T_0 equals 0.16 s.

The RLS algorithm is completed by specifying the initial parameter estimate $\hat{\theta}_0$, and the initial scaled covariance matrix P_0 (typically set equal to δI , with δ a positive constant). The actual covariance matrix of the parameters, i.e., $P_\theta(t)$, can be obtained by multiplying $P(t)$ with $\sigma_\varepsilon^2(t)/2$, where $\sigma_\varepsilon^2(t)$ denotes the variance of $\varepsilon(t)$, see Ref. 23 [pp. 381–382]. Similar as described in Section III.B.1, if only measured input-output data is considered, RLS estimation starts at instant k_c .

III.C. HO Model and ARX Model

As also shown by Hess and Mnich,²⁷ the discrete-time transfer function $G(z)$ of the continuous-time HO model structure in Eq. (3), without considering τ_e , and assuming zero-order hold (ZOH) discretization would be:

$$G(z) = z^{-1} \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (21)$$

Due to the unit-sample delay z^{-1} caused by ZOH discretization, this function corresponds to an ARX model transfer function from $e(t)$ to $u(t)$ with $n_a = n_b = 2$ and $n_k = 1$.

For easier interpretation, ARX models can also be converted back to the HO-model structure under consideration. If $n_a = n_b = 2$, this conversion to the rational continuous-time ZOH equivalent model is straightforward:

$$H(s) = \frac{b_1^c s + b_0^c}{s^2 + a_1^c s + a_0^c} \quad (22)$$

The HO model parameters are then retrieved as:

$$\hat{K}_e = \frac{b_0^c}{a_0^c}, \quad \hat{T}_L = \frac{b_1^c}{b_0^c}, \quad \hat{K}_{\dot{e}} = \frac{b_1^c}{a_0^c}, \quad \hat{\omega}_{nm} = \sqrt{a_0^c}, \quad \hat{\zeta}_{nm} = \frac{a_1^c}{2\sqrt{a_0^c}} \quad (23)$$

Matching the ARX and HO models becomes notably more complex if n_a and n_b are not both equal to 2, as then model order reduction techniques should be applied. It means that a nonlinear optimization problem based on a certain frequency-domain criterion should be solved. The poles of Eqs. (21) and (22) are mapped according to $z = e^{sT_s}$. For the mapping of zeros, such an expression does not exist. From Eq. (23), it can be observed that the HO's neuromuscular parameters are only related to the ARX model's output polynomial $A(q)$. Inspecting again Fig. 3, the ARX model estimation thus compromises between estimating the neuromuscular parameters and the remnant characteristics using $A(q)$.

Methods are available to capture τ_e as a rational (i.e., not a discrete integer sample shift) delay,²⁰ however, this would mean that extra parameters a_i and b_i are needed. The study aims to identify time-varying CE adaptations in the HO, as unambiguously as possible. Therefore, n_a and n_b are both fixed to 2 and τ_e is modeled as an integer multiple of the sampling interval T_s , taking into account the additional ZOH unit-sample delay.

III.D. Identification Approach

The general identification approach taken in this study is shown in Fig. 4. The gray blocks indicate the main tuning options.

Figure 4. Flow chart of the ARX model identification method.

In order to fine-tune the procedure, two input-output data sets are considered, one for estimation $\{e_e(t), u_e(t)\}$, and one for validation $\{e_v(t), u_v(t)\}$, both sampled with frequency f_s . The procedure has 9 steps.

1. *Discard run-in time data.* Discard the transients present in the beginning of the recordings $e_e(t)$, $u_e(t)$, $u_v(t)$ and obtain the measurement signals $e_{e,m}(t)$, $u_{e,m}(t)$, $u_{v,m}(t)$, respectively, all of length T_m and containing $N_m = f_s T_m$ samples.

2. *ARX model setup.* Select the integers n_a and n_b , and the range of integers $n_{k,\min}^*$ and $n_{k,\max}^*$ to be tested for estimating the time delay $\hat{\tau}_e$ with n_k^*/f_s . The initial choice is to use physical insight, and, by applying the principle of parsimony, select n_a and n_b as low as possible. Insights could also be gained from nonparametric analysis, e.g., using Fourier coefficients,⁵ for cases where the HO is time invariant.

3. *Input-delay estimation.* Simultaneously estimate the selected ARX models, all with different n_k^* , using OLS. Pick out the n_k^* that results in the lowest loss function V of the one-step-ahead prediction $\hat{u}_{e,p}(t)$. This V is defined as:

$$V = \frac{1}{N} \sum_{t=1}^N (u_{e,m}(t) - \hat{u}_{e,p}(t)) \quad (24)$$

4. *Input-delay compensation.* Shift the recordings $e_e(t)$ and $e_v(t)$ with n_k^* samples, and obtain the signals $e_e(t - \hat{\tau}_e)$ and $e_v(t - \hat{\tau}_e)$, respectively.

5. *Signal selection.* Select the number of samples N_e for estimation, $N_{e,v}$ for verification, and $N_{v,v}$ for validation. Obtain from $e_e(t - \hat{\tau}_e)$ and $u_{e,m}(t)$ the signals $\tilde{e}_e(t)$ and $\tilde{u}_e(t)$, respectively, for the actual ARX model estimation; from $e_e(t - \hat{\tau}_e)$ and $e_v(t - \hat{\tau}_e)$ the signals $\tilde{e}_{e,s}(t)$ and $\tilde{e}_{v,s}(t)$, respectively, for simulation; from $u_{e,m}(t)$ the signal $\tilde{u}_e(t)$ for verification; and from $u_{v,m}(t)$ the signal $\tilde{u}_v(t)$ for validation.

6. *Estimation setup.* This includes the choice between OLS and RLS estimation. The latter requires specifications for $\hat{\theta}_0$, P_0 , and the forgetting strategy, i.e., λ or Λ .

7. *ARX model estimation.* Estimate the ARX model $\mathcal{M}_{\text{ARX}}$. Note that here the structure is defined with integers n_a and n_b .

8. *Verification, validation, and optional revision.* Simulate using the ARX model equations with zero initial output condition. Obtain the verification output $\hat{u}_{e,v}(t)$ and validation output $\hat{u}_{v,v}(t)$. Verify and validate $\mathcal{M}_{\text{ARX}}$ using the variance accounted for metrics VAF_e and VAF_v , respectively. The VAF measures the quality of fit and requires the (preprocessed) measured output $\tilde{u}(t)$ and (preprocessed) simulated output $\hat{u}(t)$:

$$\text{VAF} = \max \left(0, \left(1 - \frac{\sum_{t=1}^N |\tilde{u}(t) - \hat{u}(t)|^2}{\sum_{t=1}^N |\tilde{u}(t)|^2} \right) \cdot 100\% \right) \quad (25)$$

If $\text{VAF}_e/\text{VAF}_v \gg 1$, overfitting occurs,¹⁵ as this implies that $\mathcal{M}_{\text{ARX}}$ provides a high model quality on the estimation data, but it cannot be generalized to other data. The user should specify the acceptable range of VAF ratios, e.g., accept a VAF ratio of 1.1 for all remnant intensities. A revision of $\mathcal{M}_{\text{ARX}}$ is made possible by the following options.

(a) *Data decimation.* Return to Step 5 and select a decimation factor d , for convenience a power of two.

(b) *Estimation setup adjustment.* Return to Step 6, actually only applicable for RLS estimation.

(c) *ARX model setup adjustment.* Return to Step 2.

9. *ARX model conversion.* Obtain the continuous-time HO model $\mathcal{M}_{\text{HO}}$ by converting $\mathcal{M}_{\text{ARX}}$ and n_k^* . Retrieve the HO model parameters as described in Section III.C.

IV. Method

To study the functionality and feasibility of the recursive ARX model identification method, a Monte Carlo simulation analysis with $M = 100$ replications is set up. The compensatory manual-control model in Fig. 1 is simulated for different conditions. In particular, the sensitivity to remnant is investigated. The simulations are run with $f_s = 100$ Hz. The measurement window is set to $T_m = 81.92$ s. The total recording time is set to a multiple of eight times T_m to reduce the effects of transients.

IV.A. Target Forcing Function

The closed-loop compensatory manual-control system is driven by the same target forcing function as in Ref. 13:

$$f_t(t) = \sum_{k=1}^{N_t} A_t(k) \sin[\omega_t(k)t + \phi_t(k)], \quad (26)$$

for which a sum of $N_t = 10$ sinusoids is considered. Table 1 lists the amplitude $A_t(k)$, excitation frequency $\omega_t(k)$, and phase shift $\phi_t(k)$ of the k th sinusoid. All $\omega_t(k)$ are integer multiples n_t of the measurement-base frequency $\omega_m = 2\pi/T_m = 0.0767$ rad/s. Fig. 5 shows the time trace of $f_t(t)$. The amplitude distribution is varied and scaled to attain a variance of $\sigma_{f_t}^2 = 2.2490$ deg² in T_m .

Table 1. Target forcing function properties.

$k_t, -$	$n_t, -$	$\omega_t, \text{rad/s}$	A_t, deg	ϕ_t, rad
1	3	0.230	1.186	-0.753
2	5	0.384	1.121	1.564
3	8	0.614	0.991	0.588
4	13	0.997	0.756	-0.546
5	22	1.687	0.447	0.674
6	34	2.608	0.245	-1.724
7	53	4.065	0.123	-1.963
8	86	6.596	0.061	-2.189
9	139	10.661	0.036	0.875
10	229	17.564	0.025	0.604

Figure 5. Target forcing function time trace.

IV.B. Parameter Variations and Conditions

Table 2 lists the two sets of parameter values that are considered for the CE dynamics and the assumed HO dynamics. For completeness, the corresponding open-loop crossover frequency ω_c and phase margin φ_m are included. The values are based on experimental data. Bode plots of the corresponding pre- and post-transition states are shown in Fig. 6. State 1 approximates $1/s$ CE dynamics, while State 2 is close to control of pure $1/s^2$. For State 2, a strong increase in lead generation by the HO is required.¹³

Table 2. CE and HO parameters for pre- and post-transition states based on Ref. 13.

State	CE		HO						HO×CE	
	K_c	ω_b	K_e	T_L	$K_{\dot{e}}$	τ_e	ω_{nm}	ζ_{nm}	ω_c	φ_m
	–	rad/s	–	s	s	s	rad/s	–	rad/s	deg
1	90	6	0.09	0.4	0.036	0.28	11.25	0.35	1.5	77.0
2	30	0.2	0.07	1.2	0.084				2.8	22.7

Table 3 presents the six different conditions that are evaluated. The same conditions were also studied in Ref. 13. Conditions C1 and C2 have constant dynamics and are associated with States 1 and 2, respectively. Such dynamics were also considered in Ref. 6, in which HO dynamics were successfully identified using batch ARX models. Conditions C1 and C2 are thus used as reference conditions for the identification with recursive ARX models.

Table 3. Tested conditions.

Condition	$H_{CE}(s)$	G, s^{-1}
C1	$H_{CE,1}(s)$	n/a
C2	$H_{CE,2}(s)$	n/a
C3	$H_{CE,1}(s) \rightarrow H_{CE,2}(s)$	0.5
C4	$H_{CE,1}(s) \rightarrow H_{CE,2}(s)$	100
C5	$H_{CE,2}(s) \rightarrow H_{CE,1}(s)$	0.5
C6	$H_{CE,2}(s) \rightarrow H_{CE,1}(s)$	100

Figure 6. Bode plots corresponding to the two states listed in Table 2. (a) Set of CE dynamics. (b) Set of assumed HO dynamics.

Conditions C3–C6 include time-varying dynamics. Here, the sigmoid’s t_M value is set to $T_m/2$. Both gradual and sudden transitions (i.e., $G = 0.5$ and 100 s^{-1} , respectively) are considered from condition C1 to condition C2, and vice versa. The time variations introduced in the HO’s equalization parameters are equivalent to the sigmoid function used to vary the CE dynamics. The parameter functions for these equalization parameters can be expanded with deterministic perturbations.¹⁸ This is more realistic than assuming that the HO’s equalization parameters are prescribed by the same sigmoid function introduced for the variation in the CE parameters. This study, however, focuses on algorithm performance and the question is whether it could capture such transitions at all. Example remnant-free input-output measurement-time traces for conditions C3 and C4 are shown in Fig. 7. The gray line indicates the time instant t_M of the maximum transition rate G . Around t_M , small but noticeable differences between the time traces of the two conditions can be seen.

ZOH equivalent parameter values for the two assumed HO models listed in Table 2 (without considering the delay) are presented in Table 4. These values are considered as the true ARX model parameters, $\theta^0 = [a_1^0 \ a_2^0 \ b_0^0 \ b_1^0]^T$. A range of these true parameters at each time instant (i.e., $\theta^0(t)$) is equivalently obtained for conditions C3–C6.

Table 4. ARX model parameter values for ZOH discretization of assumed HO dynamics.

T_s, s	States 1 and 2		State 1		State 2	
	a_1^0	a_2^0	b_0^0	b_1^0	b_0^0	b_1^0
0.01	-1.9121	0.9243	0.0443	-0.0432	0.1024	-0.1016

IV.C. Remnant Model

HO remnant $n(t)$ is not yet fully characterized and in literature there is no clear agreement on how this remnant should be modeled and accounted for in Monte Carlo simulations.^{6,7,28–30} Here, remnant is modeled as a zero-mean Gaussian white-noise signal $n_w(t)$, with unit variance, passed through a low-pass filter. The following m th-order remnant-filter structure is considered:

$$H_n^m(s, t) = \frac{K_n(t)}{(T_n s + 1)^m}, \quad (27)$$

where the remnant gain $K_n(t)$ may vary in time. The remnant-time constant T_n is set to 0.06 s ,³¹ resulting in a cutoff frequency of 16.67 rad/s . A key issue of the ARX model structure is that the noise model $\frac{1}{A(q)}$ (see again Fig. 3) is

Figure 7. Remnant-free input-output measurement-time traces. (a) and (b) Condition C3. (c) and (d) Condition C4.

directly linked to the process model $\frac{B(q)}{A(q)}$. Hence, the remnant characteristics cannot be parametrized independently and the estimation of the parameters in the linear HO dynamics becomes biased if the remnant characteristics do not match $\frac{1}{A(q)}$. This aspect was not fully investigated in earlier work on HO identification with ARX models.^{6,15} Therefore, this study explicitly investigates this by considering different remnant models in our Monte Carlo simulations. Integer orders of $m = 1, 2, 3$, and 4 are tested. In addition, a special remnant filter, similar to $H_{nm}(s)$, is considered ($m = nm$), with the values of the neuromuscular parameters listed in Table 2:

$$H_n^{nm}(s, t) = K_n(t) H_{nm}(s) \quad (28)$$

It is expected that the $H_n^{nm}(s, t)$ filter will show the most accurate ARX model identification results, due to the perfect match of the noise and process dynamics.

For the different $H_n^m(s, t)$, $K_n(t)$ can be set to induce remnant-intensity levels $P_n = \sigma_n^2 / \sigma_u^2$ in the measurement window. Seven levels of P_n are selected: 0 (i.e., no remnant), 0.05, 0.10, 0.15, 0.20, 0.25, and 0.30. By means of the nonlinear programming solver `fmincon`,³² a constant K_n is tuned for the constant dynamics conditions C1 and C2, based on ten $n_w(t)$ realizations. For conditions C3–C6, $K_n(t)$ can then be varied according a sigmoid analogous to the HO equalization and CE parameters. After tuning K_n , another 110 independent $n_w(t)$ are generated. A hundred of these were used to simulate estimation data sets. The other ten were used to create validation data sets. So, different remnant signals $n(t)$ are obtained by filtering exactly the same realizations $n_w(t)$ through the different $H_{n,t}^m(s)$. This approach was used to ensure the fairest comparison between different remnant characteristics.

Fig. 8 shows an overview of the different types of constant $H_n^m(s)$ filters with different tuned K_n to induce a P_n of 0.15 for condition C1. Equivalent Bode plots can be obtained for the other P_n and for condition C2. Over a wide frequency range, $H_n^1(s)$ has the lowest magnitude and $H_n^4(s)$ has the highest magnitude. The opposite can be seen at the highest frequencies. For all $H_n^m(s)$, the phase angle becomes more negative as the magnitude drops.

IV.D. Estimation Setup

As shown in Figs. 2 and 7, if t_M is set around $T_m/2$, the initial and final 30.72 s of the measurement traces for conditions C3–C6 can be assumed to include only constant dynamics. In essence, if the dynamics to be identified are constant, the RLS algorithm will recursively approximate the OLS estimate. Therefore, in these particular regions ARX models can be estimated with OLS serving as a reference for the RLS estimation.

The main interest is concentrated on the recursive estimation in the transition region of 20.48 s around t_M (see again Fig. 2). The implementation with the scalar forgetting factor λ and the forgetting matrix Λ and corresponding

Figure 8. Bode plots of the different remnant-filter models with different tuned K_n to induce a P_n of 0.15 for condition C1.

Table 5. Tested forgetting factors based on N_0 .

$N_0, -$	$\lambda, -$	$N_0, -$	$\lambda, -$
2	0.5	256	0.99609
4	0.75	512	0.99805
8	0.875	1024	0.99902
16	0.9375	2048	0.99951
32	0.96875	4096	0.99976
64	0.98438	8192	0.99988
128	0.99219	∞	1

results is explored. All conditions listed in Table 3 are assessed with a range of forgetting factors, that correspond to a matching range of memory horizons N_0 of integer powers of two (see Table 5). Also, the case of no-forgetting is considered, i.e., $\lambda = 1$ so $N_0 = \infty$. For each recursive estimation, the RLS algorithm is initialized with $\hat{\theta}_0 = \hat{\theta}^{\text{OLS}}$ estimated on the initial 30.72 s, and with $P_0 = I$.

The ARX model's output parameters $a_1(t)$ and $a_2(t)$ remain constant in all conditions with time-varying dynamics, as these parameters are directly linked to the HO's neuromuscular parameters ω_{nm} and ζ_{nm} which were not varied in the offline simulations. Therefore, $a_1(t)$ and $a_2(t)$ are in fact time invariant throughout the whole estimation. On the contrary, the ARX model's input parameters $b_0(t)$ and $b_1(t)$ are not constant. These parameters are closely linked to the HO's equalization parameters $K_e(t)$ and $K_{\dot{e}}(t)$, which were varied according the same sigmoid function. Hence, the forgetting matrix can provide more accurate results as it is able to discount distinct information for each ARX model parameter. The forgetting factors λ_{a_1} and λ_{a_2} in the forgetting matrix Λ are both set equal to 1, while the range of forgetting factors listed in Table 5 is tested for $\lambda_{b_0} = \lambda_{b_1}$.

IV.E. Identification Quality Metrics

Prior to commencing the analysis of the recursive ARX model estimation, the effects of the order of the remnant filter on the resulting estimation accuracy with the selected ARX model structure, i.e., $n_a = n_b = 2$, is assessed for conditions C1 and C2 using batch ARX model estimation. First, the estimation of n_k^* is compared to n_k^0 , i.e., the true integer value of 29 samples based on $\tau_e f_s$ plus a unit-sample delay from the ZOH discretization. The use of n_k^* can be justified by calculating the difference in validation data VAFs for n_k^0 and n_k^* :

$$\Delta \text{VAF}_v^{n_k^0} = \text{VAF}_v^{n_k^0} - \text{VAF}_v^{n_k^*} \quad (29)$$

If $\Delta \text{VAF}_v^{n_k^0}$ is close to zero, the model qualities are practically the same. Second, the batch ARX model parameter estimates will be compared to the ZOH parameter values listed in Table 4. For this, the metric of relative bias B_r is used, which accounts for the sign of the bias:

$$B_r = \frac{\hat{\theta} - \theta^0}{\theta^0} \quad (30)$$

An objective metric to trade off the tracking ability and remnant-level sensitivity is introduced by calculating the difference between the validation data VAFs for $\lambda = 1$ (highest robustness against noise) and for $\lambda < 1$ for a

range of λ (ability to capture time variations). The metrics for the scalar forgetting factor λ and forgetting matrix Λ implementation are defined, respectively, as:

$$\Delta \text{VAF}_v^{\lambda=1} = \text{VAF}_v^\lambda - \text{VAF}_v^{\lambda=1} \quad (31)$$

$$\Delta \text{VAF}_v^{\Lambda=I} = \text{VAF}_v^\Lambda - \text{VAF}_v^{\Lambda=I} \quad (32)$$

As indicated in Section IV.D, the evaluation of the above VAF metrics in the transition region is of particular importance. Note that $\text{VAF}_v^{\lambda=1}$ equals $\text{VAF}_v^{\Lambda=I}$.

V. Results

This section presents the results of the Monte Carlo simulations. The results consist of two parts. First, using OLS ARX model identification, the effects of different remnant filters $H_n^m(s)$ on the estimation accuracy are verified, as well as the effectiveness of the input-delay estimation step. Then, the results of the RLS estimation procedure will be used to compare different forgetting implementations (λ and Λ) and determine an optimal forgetting factor value for our application.

V.A. Influence of Remnant-Filter Type and Delay Estimation

Fig. 9 shows, for condition C1, a box plot of the variable input-delay estimation n_k^* for the different types of $H_n^m(s)$ and intensity levels P_n , for $M = 100$ replications. For condition C2, equivalent results were obtained. In Fig. 9, the dotted gray line represents the true fixed integer delay n_k^0 of 29 samples. In the remnant-free case, i.e., $P_n = 0$, $n_k^* = n_k^0$ is indeed found. A clear distinction can be made between remnant-filter orders 1 and 2 that overestimate n_k^0 , and orders 3 and 4 that underestimate n_k^0 . The n_k^* for the $H_n^{nm}(s)$ filter are centered around n_k^0 . Furthermore, the n_k^* for the $H_n^1(s)$ filter are seen to deviate from n_k^0 , on average by around seven samples, for all P_n . Focusing for a moment on the higher frequency range in the Bode phase-angle plots in Fig. 8, a clear trend can be seen with the results in Fig. 9. The ARX model estimations on data simulated with the $H_{nm}(s)$ filter result in the best estimates of n_k^0 . An increase in delay is needed for the orders 1 and 2, while less delay is needed for the orders 3 and 4.

Figure 9. Variable n_k^* estimation, for condition C1, for different $H_n^m(s)$.

To further assess how the ARX model estimates vary with the different remnant-filter types, it is important to look at the relative bias B_r . Fig. 10 shows, again for condition C1, box plots for all ARX model parameters a_1 , a_2 , b_0 , and b_1 , estimated with the true delay integer n_k^0 and the variable delay integer n_k^* . Equivalent results were found for condition C2. The B_r of all parameters in the remnant-free case is zero. It stands out that the B_r in all parameters for $H_n^1(s)$ is more than ten times larger compared to the other $H_n^m(s)$. Similar to the results in Fig. 9, a clear difference can be seen between orders 1 and 2, and the other remnant-filter types. The output parameters a_1 and a_2 are underestimated for $H_n^1(s)$ and $H_n^2(s)$, while an overestimation of the input parameters b_0 and b_1 occurs. The opposite can be seen for the other $H_n^m(s)$. The lowest relative bias is seen for $H_n^{nm}(s)$. Except for $H_n^1(s)$, the difference between B_r when estimating with n_k^0 and n_k^* is negligible.

To gain a better understanding of the differences and commonalities between the tested remnant filters, Figs. 11(a) and (b) show, for conditions C1 and C2, respectively, Bode plots of the batch ARX models, estimated with the true integer delay n_k^0 , for a single remnant realization with $P_n = 0.15$. Equivalent Bode plots are obtained for the other P_n . Figs. 11(a) and (b) also provide Bode plots of the ZOH equivalent dynamics, for conditions C1 and C2, respectively,

Figure 10. Relative bias in the ARX model parameters, for condition C1, for the different $H_n^m(s)$ when using n_k^0 and n_k^* .

for the values listed in Table 4 and n_k^0 . For both conditions, the batch ARX model estimations on data sampled with $f_s = 100$ Hz and generated with $H_n^1(s)$ cannot provide a good description of the HO dynamics in the frequency range of interest for manual control (0.2–18 rad/s). There is a large misfit at the high frequencies and the magnitude peak is not even captured. The above results clearly express the ARX model's compromise between minimizing the bias in the parameters of the process dynamics and explaining the remnant noise dynamics. In fact, this bias minimization is weighted with $|A(e^{-j\omega})|^2 = |1 + a_1 e^{-j\omega} + a_2 e^{-2j\omega}|^2$.²³ Figs. 11(c) and (d) present Bode magnitude plots of the weighting functions, for conditions C1 and C2, respectively, in correspondence with the Bode plots of the HO dynamics. For $H_n^1(s)$, the high-frequency misfits are penalized much less than for the other $H_n^m(s)$, e.g., for $H_n^2(s)$, the weight at 100 rad/s is 10^3 times higher than the weight at 20 rad/s, whereas for $H_n^1(s)$ this is just 15 times higher.

The results of Figs. 10 and 11 indicate that, as expected, for $H_n^3(s)$, $H_n^4(s)$, and $H_n^{nm}(s)$ ARX model estimations closest to the ZOH discretized HO models are obtained. While the use of $H_n^{nm}(s)$ led to the best estimation of n_k^0 , the lowest bias, and the best fit in the Bode plots, it is not realistic to assume that the HO's remnant dynamics are equivalent to the HO's neuromuscular dynamics—these were only considered as a reference. The second and third best candidates serving as appropriate remnant models for the further analysis are then $H_n^3(s)$ and $H_n^4(s)$. However, to better assess the potential feasibility of the ARX model estimation, the further analysis is based on the second-order remnant filter $H_n^2(s)$. The use of this filter is considered as the compromise limit between bias in the ARX model parameters and the fitting of the remnant by the ARX model.

To give a final justification of using n_k^* and the selection of $H_n^2(s)$, Fig. 12 shows box plots of the metric $\Delta \text{VAF}_v^{n_k^0}$, for conditions C1 and C2. These results indicate that negligible differences in model quality are obtained when using n_k^* compared to n_k^0 , for all remnant intensities P_n .

V.B. Forgetting-Factor Tuning

This section includes the results of recursive ARX model estimation. Fig. 13 visualizes, for all conditions listed in Table 3 and tested remnant intensities P_n , the calculations of the metrics defined in Eqs. (31) and (32), based on the 100 simulations. Data for the scalar forgetting factor λ and the forgetting matrix Λ are shown side by side. The validation data VAFs are calculated on the transition region window of 20.48 s around t_M . For both implementations, the range of memory horizons N_0 listed in Table 5 are compared with the case of no-forgetting, i.e., $\lambda = 1$ and $\Lambda = I$. It stands

Figure 11. Bode plots of the batch ARX model estimations for different $H_n^m(s)$ for a single realization with $P_n = 0.15$ and using n_k^0 . (a) HO dynamics for condition C1. (b) HO dynamics for condition C2. (c) Weighting function for condition C1. (d) Weighting function for condition C2.

Figure 12. Difference between the validation data VAFs when using n_k^0 and n_k^* and selecting $H_n^2(s)$. (a) Condition C1. (b) Condition C2.

out in Fig. 13 that no significant differences can be seen between the two different forgetting-factor implementations. Apparently, the quality of fit in the transition region data is not improved by changing the forgetting strategy. More interesting, per P_n and condition, an optimal N_0 can be observed from Fig. 13. Data with lower P_n require the selection of lower N_0 . In particular, this can be seen for the remnant-free cases. Higher forgetting factors for the constant dynamics conditions C1 and C2 should be selected, as the corresponding data are less variable compared to the data of the conditions with time-varying dynamics. For the different P_n , the values of $\Delta VAF_v^{\lambda=1}$ and $\Delta VAF_v^{\Lambda=I}$ differ more for condition C2 compared to condition C1. The variability in the data increases more with higher P_n for condition C2 than for condition C1. Furthermore, a comparison between conditions C3 and C4 (i.e., $1/s \rightarrow 1/s^2$) and conditions C5 and C6 (i.e., $1/s^2 \rightarrow 1/s$) reveals that differences between no-forgetting and forgetting are notably smaller for the latter. In conditions C5 and C6, the estimations are influenced more by the data based on $1/s^2$ dynamics than $1/s$ dynamics. Hence, the effect of variability in the data is flattened. Overall, the one-optimal forgetting factor, for all conditions, for this data, is observed for $N_0 = 256$ samples, i.e., $\lambda = 0.99609$. So, for data sampled with $f_s = 100$ Hz, the RLS algorithm requires 2.56 s to come up with accurate parameter estimates.

Figure 13. Overview for all conditions to trade off between tracking ability and noise sensitivity for the two forgetting strategies λ and Λ , with P_n ranging between 0 and 0.30 in increments of 0.05. (a) to (d) Conditions with constant dynamics. (e) to (l) Conditions with time-varying dynamics.

While Fig. 13 did not show significant differences between the implementation of the scalar λ and the matrix Λ , the matching recursive estimates of $a_1(t)$, $a_2(t)$, $b_0(t)$, and $b_1(t)$ show that there is in fact a benefit of making use of Λ . Fig. 14 shows in black the time evolutions of the recursive ARX model parameters for condition C4, based on the 100 different estimation data sets, for a scalar forgetting factor λ and for a forgetting matrix Λ . The results for three different remnant-intensity levels are shown, i.e., $P_n = 0$ (no remnant), 0.15 and 0.30. A similar pattern was observed for P_n between 0.05 and 0.30. For $P_n = 0.15$ and 0.30, standard deviations are included. The red lines in Fig. 14 represent the true ZOH parameter values. The green lines, at the beginning and the end, indicate batch ARX model estimations (mean and standard deviations). The gray lines indicate the sigmoid's time of maximum rate of change, t_M . For all P_n , the black lines clearly show that $a_1(t)$ and $b_0(t)$ are negatively correlated with $a_2(t)$ and $b_1(t)$, respectively. In accordance with the results shown in Section V.A, for $P_n > 0$, a high bias with respect to the true parameters is obtained for both the batch and recursive ARX models as the simulation was set up with $H_n^2(s, t)$. For $P_n = 0$, it can be noted that $N_0 = 256$ samples is not the best choice. In this case, the estimation is not driven

by remnant and the tracking ability can be increased by decreasing N_0 . From Fig. 14(b), a significant reduction in the variability of the ARX model parameters is observed for all P_n . This becomes clear, in particular, when looking at the ARX model's output parameters. Better fits are obtained by using the matrix Λ .

Turning now to $\mathcal{M}_{\text{HO}}$ by converting $\mathcal{M}_{\text{ARX}}$ with integers $n_a = n_b = 2$, the HO model parameters can be retrieved using Eq. (23). Only instants of $\mathcal{M}_{\text{ARX}}$ with pairs of complex-conjugate poles are considered. In accordance with Fig. 14, Fig. 15 is presented for condition C4 for both forgetting-factor implementations. The black lines represent the HO model parameters based on the recursive ARX models, and the green lines at the beginning and end are based on batch ARX model estimations. Standard deviations are included for $P_n = 0.15$ and 0.30 . The red lines are the true HO model parameters, of which the equalization parameters vary according the sigmoid function. The gray lines represent t_M . Similar patterns as in Fig. 14 can be seen in Fig. 15 for the two different forgetting-factor implementations. While for $K_e(t)$ and $K_v(t)$ acceptable estimation results are obtained, the estimation of $T_L(t)$ is seen to be problematic in the $1/s^2$ dynamics region. This can be explained by examining the definition of $T_L(t)$ in Eq. (23) and drawing the analogy with the negative correlation between the ARX model parameters $b_0(t)$ and $b_1(t)$. If the continuous-time coefficient $b_1^c(t)$ increases, $b_0^c(t)$ decreases, and vice versa. The variability in the data related to $1/s^2$ dynamics increases, hence the negative correlation effect is stronger. Therefore, no stable estimates for $T_L(t)$ are obtained. High deviations from the true values are observed in $\hat{\omega}_{nm}$ and $\hat{\zeta}_{nm}$. This clearly demonstrates that the ARX model's capacity for capturing the dynamics of the system is indeed strongly biased by differences between the remnant and neuromuscular dynamics. As a result, for $H_n^2(s, t)$ both $\omega_{nm}(t)$ and $\zeta_{nm}(t)$ are consistently overestimated.

VI. Experimental Evaluation

VI.A. Method

An experiment was conducted in the SIMONA Research Simulator at Delft University of Technology, in fixed-base setting. Three subjects, with an average age of 24 years, performed the single-axis compensatory manual-control task (see Fig. 1) for all the conditions listed in Table 3. Seven runs were performed in each condition. The length of the runs was set to 90.00 s. A measurement time T_m of 81.92 s was used for the identification. The maximum rate of transition t_M in conditions C3–C6 was set at 42.93 s in the measurement window (i.e., at 50.00 s in the recording window). For the actual identification, only the five last runs were considered.

The ARX model identification approach, as outlined in Fig. 4, was adopted. Input-output data were recorded with $f_s = 100$ Hz, and $f_t(t)$ was set up with the properties listed in Table 1. Similar to the Monte Carlo analysis, the ARX model structure was set to $n_a = n_b = 2$. The time delay was assumed to be constant and estimated using n_k^* . Batch ARX models were fit on the initial and final 30.72 s of the measurement window. The recursive ARX model estimations were initialized with the initial $\hat{\theta}^{\text{OLS}}$ and P_0 set to I . In accordance with the results from the Monte Carlo analysis in Section V, N_0 was set to 256 samples. Both the scalar forgetting factor λ and the forgetting matrix Λ (with $\lambda_{a_1} = \lambda_{a_2} = 1$) were considered.

VI.B. Results

Example identification results are presented for condition C4, for all three subjects. Equivalent results are obtained for the other conditions. Condition C4 is taken as this allows a comparison with the results of Monte Carlo simulation analysis in Section V. In accordance with Figs. 14 and 15, Figs. 16 and 17 are presented, respectively. The latter two figures include ensemble averages based on batch and recursive ARX model estimation. These batch ARX model estimates fit on the initial and final 30.72 s are indicated with thick green lines, and these recursive ARX model estimates are shown with thick black lines. The sigmoid's instant t_M is indicated with the gray line. Figs. 16 and 17 also include the five recursive single-run ARX model estimates (thin colored lines). No extreme outliers are observed in these single-run estimates.

The ARX model identification results in Fig. 16 show that Subject 3 strongly adapts to the step transition in the CE dynamics. In particular, this becomes clear when focusing on the evolution of $b_0(t)$, and $b_1(t)$. Such strong adaptations are not seen in the results for Subjects 1 and 2. In all cases, the ensemble averages of the recursive ARX model estimates approach the initial and final ensemble averages of the batch ARX model estimates. Furthermore, a comparison between the initial and final batch ARX model estimates and the recursive ARX model estimates in the transition region gives an indication if some ARX model parameters have distinct rates of changes, and if the use of the forgetting matrix Λ is reasonable. For Subject 3, a low variability in $a_1(t)$ and $a_2(t)$ is seen. Hence, for this specific task, there seems to be a justification for using Λ .

The matching conversion of the ARX model parameters to HO-model parameters is presented in Fig. 17. Focusing

Figure 14. ARX model parameters for condition C4 for three different P_n . (a) λ with $N_0 = 256$ samples. (b) Λ with $N_0^{a1} = N_0^{a2} = \infty$ and $N_0^{b0} = N_0^{b1} = 256$ samples.

Figure 15. HO model parameters for condition C4 for three different P_n . (a) λ with $N_0 = 256$ samples. (b) Λ with $N_0^{a1} = N_0^{a2} = \infty$ and $N_0^{b0} = N_0^{b1} = 256$ samples.

Figure 16. ARX model parameters for condition C4 for three different subjects. (a) λ with $N_0 = 256$ samples. (b) Λ with $N_0^{a1} = N_0^{a2} = \infty$ and $N_0^{b0} = N_0^{b1} = 256$ samples.

Figure 17. HO model parameters for condition C4 for three different subjects. (a) λ with $N_0 = 256$ samples. (b) Λ with $N_0^{a1} = N_0^{a2} = \infty$ and $N_0^{b0} = N_0^{b1} = 256$ samples.

on the evolutions of $K_e(t)$, a generation of lead is seen for Subjects 2 and 3. This is not the case for Subject 1. Furthermore, it can be noted that the values of $\omega_{nm}(t)$ for Subject 1 remain low compared with Subjects 2 and 3. It is questionable whether the selected ARX model structure $n_a = n_b = 2$ is appropriate for Subject 1. Similar as observed in the Monte Carlo simulation analysis, the estimation of $T_L(t)$ is problematic in the region with $1/s^2$ dynamics. However, clear patterns are observed in the evolution of $K_e(t)$ and $K_e(t)$. In general, the results are consistent with the findings from the Monte Carlo simulations in Section V.

VII. Discussion

An identification method based on recursive ARX models is described that requires the user to specify the orders of the ARX model and to set up a forgetting strategy. The orders are defined using the integers n_a and n_b , and the sum of these integers sets the number of parameters to be estimated. The choice of the forgetting strategy in the recursive parameter estimation affects the trade-off between tracking ability and noise sensitivity. This study considered the implementation of a constant scalar forgetting factor λ , and a constant forgetting matrix Λ comprising forgetting factors for each ARX model parameter. Through testing on simulation data, an understanding is gained of the identification method's capability to quantitatively describe time-varying compensatory manual-control adaptations if the operator is faced with transitions in the CE dynamics matching Zaal's experiment.¹³

Assuming that the considered HO model, consisting of a visual gain, a lead term, a time delay and neuromuscular dynamics, is the suitable representation for the control of second-order CE dynamics, the results indicate that ARX models can only be used, directly, if the remnant dynamics are sufficiently similar to the (second-order) neuromuscular dynamics. The results presented in this paper show that the bias in the HO parameters becomes very high if the remnant is first-order low-pass filtered Gaussian white noise. Therefore, direct ARX model estimation on measured input-output data cannot be used to identify the specific type of HO model if the remnant dynamics characterizes to first order. However, it was found that the remnant dynamics of orders 2, 3 or 4 are suitable.

For manual-control tasks, where the HO is better represented by for example an extra lead or lag term, the ARX model defined by the integers n_a and n_b can be easily adapted.²⁷ Furthermore, it is straightforward to convert the resulting ARX model to the (easier to interpret) HO model parameters. The computational burden increases, however, if the conversion relationship is inapplicable, e.g., if extra ARX model parameters are added to capture time delay as a fraction. Then, model-order reduction approximations should be carried out, or nonlinear optimization problems should be solved to retrieve the physical HO model parameters.

This study demonstrated, by means of simulated estimation and validation data, a tuning procedure for an optimal forgetting strategy for different conditions and different remnant intensities. These findings are not trivial given the high levels of remnant present in manual-control data. On average, for data sampled with 100 Hz, it can be said that the use of the scalar forgetting factor $\lambda = 0.99609$, corresponding to a memory horizon of 256 samples, is a satisfactory point of departure for the identification of time-varying compensatory manual-control behavior. In general, higher levels of remnant are present in single experimental runs. It means that for this application the value of λ should be increased. For ensemble-averaged experimental runs, the remnant levels will be lower, so lower values of λ can be taken.

In addition to the constant scalar forgetting factor λ , the constant forgetting matrix Λ was considered as forgetting strategy for capturing time-varying compensatory manual-control behavior. It was found that the use of a forgetting matrix did not increase the model quality. However, the variance in the parameter estimates over time will be notably lower. The use of a constant forgetting matrix should be considered, only if there is a strong evidence that HO model parameters vary with different temporal scales, or that certain parameters are constant. To further improve the obtained results, further research could concentrate on time-varying forgetting strategies.^{23,33} Instead of using a constant forgetting strategy, time variations in the forgetting factors can be introduced to locally adapt the forgetting profile. This means that the scalar forgetting factor turns into a vector of forgetting factors. Instead of a constant forgetting matrix, a time-varying forgetting matrix can be used. This can be very useful for identification purposes if the remnant-intensity levels vary during a measurement run.

The ARX model delay estimation method based on the loss function of the one-step-ahead prediction seems to be useful. However, a drawback of the current identification approach is that it does not work for capturing time-varying delay. This issue can be circumvented by means of a sliding window approach, and by updating the input entries of the regression vector $\varphi(t)$ at different time instants. In essence, this results in a switched regression problem, which can be approached by means of piecewise ARX models.³⁴

Further investigations are needed to explore whether the recursive ARX model identification method is also capable to detect time-varying manual-control adaptation due to changes in other task variables. The properties of the

forcing function can be altered over time, e.g., to represent different command inputs to the HO or time-varying disturbances on the CE dynamics.¹⁰ In Refs. 6 and 15, batch ARX model identification was also used to determine the operator's vestibular motion and feedforward responses, respectively. Future work is needed to determine if the recursive ARX model identification method is also able to capture, in addition to the visual response, these two multimodal characteristics of manual-control behavior.

The identification method is computationally efficient. Therefore, online implementation of the proposed approach is relatively straightforward. Furthermore, an experimental evaluation of real manual-control data indicates that the identification method is ready to use in future laboratory-control experiments.

VIII. Conclusion

A recursive ARX model identification method was developed that is able to capture time-varying CE adaptation of the HO in compensatory tracking tasks. The method is practical since no stringent assumptions are required on how manual-control behavior will vary over time. A procedure was shown for the determination of the optimal forgetting factor for different conditions and different remnant intensities. For the considered application, the advice is to select a forgetting factor λ of 0.99609, if input-output data is sampled with 100 Hz. A Monte Carlo simulation analysis with remnant-intensity levels of up to 30% shows that the method is then accurate in tracking time variations in adaptive manual-control behavior. The λ value can be slightly decreased for lower remnant intensities. If parameters have distinct rates of variations, the variance in parameter estimates can be lowered by using a forgetting matrix Λ with different forgetting factors.

As was shown by the experimental evaluation, the method has the potential to become a valuable tool for further analysis of time-varying HO behavior. In addition, the recursive identification algorithm can also directly contribute to the development of novel human-machine systems, such as HO monitoring and adaptive support systems.

References

- ¹McRuer, D. T., Graham, D., Krendel, E. S., and Reisener, W., Jr., "Human Pilot Dynamics in Compensatory Systems: Theory, Models, and Experiments with Controlled Element and Forcing Function Variations," Tech. Rep. AFFDL-TR-65-15, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, OH, 1965.
- ²McRuer, D. T. and Jex, H. R., "A Review of Quasi-Linear Pilot Models," *IEEE Transactions on Human Factors in Electronics*, Vol. HFE-8, No. 3, 1967, pp. 231–249.
- ³McRuer, D. T., Graham, D., and Krendel, E. S., "Manual Control of Single-Loop Systems: Part I & II," *Journal of the Franklin Institute*, Vol. 283, No. 1, 1967, pp. 1–29; Vol. 283, No. 2, 1967, pp. 145–168.
- ⁴Stapleford, R. L., McRuer, D. T., and Magdaleno, R. E., "Pilot Describing Function Measurements in a Multiloop Task," *IEEE Transactions on Human Factors in Electronics*, Vol. HFE-8, No. 2, 1967, pp. 113–125.
- ⁵van Paassen, M. M. and Mulder, M., "Identification of Human Operator Control Behaviour in Multiple-Loop Tracking Tasks," *Proceedings of the 7th IFAC Symposium on Analysis, Design and Evaluation of Man Machine Systems, Kyoto, Japan*, Vol. 31, No. 26 of *IFAC Proceedings Volumes*, 1998, pp. 455–460.
- ⁶Nieuwenhuizen, F. M., Zaal, P. M. T., Mulder, M., van Paassen, M. M., and Mulder, J. A., "Modeling Human Multichannel Perception and Control Using Linear Time-Invariant Models," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 4, 2008, pp. 999–1013.
- ⁷Zaal, P. M. T., Pool, D. M., Chu, Q. P., Mulder, M., van Paassen, M. M., and Mulder, J. A., "Modeling Human Multimodal Perception and Control Using Genetic Maximum Likelihood Estimation," *Journal of Guidance, Control, and Dynamics*, Vol. 32, No. 4, 2009, pp. 1089–1099.
- ⁸Mulder, M., Zaal, P. M. T., Pool, D. M., Damveld, H. J., and van Paassen, M. M., "A Cybernetic Approach to Assess Simulator Fidelity: Looking back and looking forward," *Proceedings of the AIAA Modeling and Simulation Technologies Conference, Boston, MA*, No. AIAA 2013-5225 in *Guidance, Navigation, and Control and Co-located Conferences*, 2013.
- ⁹Mulder, M., Pool, D. M., Abbink, D. A., Boer, E. R., Zaal, P. M. T., Drop, F. M., van der El, K., and van Paassen, M. M., "Manual Control Cybernetics: State-of-the-Art and Current Trends," *IEEE Transactions on Human-Machine Systems*, published online 27 Oct. 2017.
- ¹⁰Young, L. R., "On Adaptive Manual Control," *IEEE Transactions on Man-Machine Systems*, Vol. 10, No. 4, 1969, pp. 292–331.
- ¹¹Hess, R. A., "Modeling Pilot Control Behavior with Sudden Changes in Vehicle Dynamics," *Journal of Aircraft*, Vol. 46, No. 5, 2009, pp. 1584–1592.
- ¹²Hess, R. A., "Modeling Human Pilot Adaptation to Flight Control Anomalies and Changing Task Demands," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 3, 2016, pp. 655–666.
- ¹³Zaal, P. M. T., "Manual Control Adaptation to Changing Vehicle Dynamics in Roll-Pitch Control Tasks," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 5, 2016, pp. 1046–1058.
- ¹⁴Popovici, A., Zaal, P. M. T., and Pool, D. M., "Dual Extended Kalman Filter for the Identification of Time-Varying Human Manual Control Behavior," *Proceedings of the AIAA Modeling and Simulation Technologies Conference, Denver, CO*, No. AIAA 2017-3666 in *AIAA Aviation*, 2017.
- ¹⁵Drop, F. M., Pool, D. M., van Paassen, M. M., Mulder, M., and Bühlhoff, H. H., "Objective Model Selection for Identifying the Human Feedforward Response in Manual Control," *IEEE Transactions on Cybernetics*, published online 19 Sep. 2016.
- ¹⁶Thompson, P. M., Klyde, D. H., and Brenner, M. J., "Wavelet-Based Time-Varying Human Operator Models," *Proceedings of the AIAA*

Atmospheric Flight Mechanics Conference and Exhibit, Montreal, Canada, No. AIAA 2001-4009 in Guidance, Navigation, and Control and Co-located Conferences, 2001.

¹⁷Zaal, P. M. T. and Sweet, B. T., "Estimation of Time-Varying Pilot Model Parameters," *Proceedings of the AIAA Modeling and Simulation Technologies Conference, Portland, OR*, No. AIAA 2011-6474 in Guidance, Navigation, and Control and Co-located Conferences, 2011.

¹⁸Duarte, R. F. M., Pool, D. M., van Paassen, M. M., and Mulder, M., "Experimental Scheduling Functions for Global LPV Human Controller Modeling," *Proceedings of the 20th IFAC World Congress, Toulouse, France*, Vol. 50, No. 1 of *IFAC-PapersOnLine*, 2017, pp. 15853–15858.

¹⁹Schiess, J. R. and Roland, V. R., "Kalman Filter Estimation of Human Pilot-Model Parameters," Tech. Rep. NASA TN D-8024, NASA Langley Research Center, Hampton VA, 1975.

²⁰Boer, E. R. and Kenyon, R. V., "Estimation of Time-Varying Delay Time in Nonstationary Linear Systems: An Approach to Monitor Human Operator Adaptation in Manual Tracking Tasks," *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, Vol. 28, No. 1, 1998, pp. 89–99.

²¹Mandal, T. K. and Gu, Y., "Online Pilot Model Parameter Estimation Using Sub-Scale Aircraft Flight Data," *Proceedings of the AIAA Guidance, Navigation, and Control Conference, San Diego, CA*, No. AIAA 2016-0636 in AIAA SciTech Forum, 2016.

²²Olivari, M., Venrooij, J., Nieuwenhuizen, F. M., Pollini, L., and Bülthoff, H. H., "Identifying Time-Varying Pilot Responses: a Regularized Recursive Least-Squares Algorithm," *Proceedings of the AIAA Modeling and Simulation Technologies Conference, San Diego, CA*, No. AIAA 2016-1182 in AIAA SciTech Forum, 2016.

²³Ljung, L., *System Identification: Theory for the User*, Prentice Hall PTR, Upper Saddle River, NJ, 2nd ed., 1999.

²⁴Drop, F. M., Pool, D. M., Mulder, M., and Bülthoff, H. H., "Constraints in Identification of Multi-Loop Feedforward Human Control Models," *Proceedings of the 13th IFAC Symposium on Analysis, Design, and Evaluation of Human-Machine Systems, Kyoto, Japan*, Vol. 49, No. 19 of *IFAC-PapersOnLine*, 2016, pp. 7–12.

²⁵Kulhavý, R. and Zarrop, M. B., "On a general concept of forgetting," *International Journal of Control*, Vol. 58, No. 4, 1993, pp. 905–924.

²⁶Sælid, S., Egeland, O., and Foss, B. A., "A solution to the blow-up problem in adaptive controllers," *Modeling, Identification and Control*, Vol. 6, No. 1, 1985, pp. 39–56.

²⁷Hess, R. A. and Mnich, M. A., "Identification of Pilot-Vehicle Dynamics from In-Flight Tracking Data," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 4, 1986, pp. 433–440.

²⁸Lone, M. M. and Cooke, A. K., "Review of pilot models used in aircraft flight dynamics," *Aerospace Science and Technology*, 2014.

²⁹Levison, W. H., Baron, S., and Kleinman, D. L., "A Model for Human Controller Remnant," *IEEE Transactions on Man-Machine Systems*, Vol. 10, No. 4, 1969, pp. 101–108.

³⁰Metz, L. D., "A Time-Varying Approach to the Modeling of Human Control Remnant," *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 12, No. 1, 1982, pp. 24–35.

³¹Pool, D. M., Valente Pais, A. R., de Vroome, A. M., van Paassen, M. M., and Mulder, M., "Identification of Nonlinear Motion Perception Dynamics Using Time-Domain Pilot Modeling," *Journal of Guidance, Control, and Dynamics*, Vol. 35, No. 3, 2012, pp. 749–763.

³²The MathWorks, Inc., *Optimization Toolbox User's Guide*, 2016, R2016b.

³³Fortescue, T. R., Kershenbaum, L. S., and Ydstie, B. E., "Implementation of Self-Tuning Regulators with Variable Forgetting Factors," *Automatica*, Vol. 17, No. 6, 1981, pp. 831–835.

³⁴Ohlsson, H. and Ljung, L., "Identification of switched linear regression models using sum-of-norms regularization," *Automatica*, Vol. 49, No. 4, 2013, pp. 1045–1050.