Coupled seismic and electromagnetic wave propagation

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Summary

Coupled seismic and electromagnetic wave propagation

Coupled seismic and electromagnetic wave propagation is studied theoretically and experimentally. This coupling arises because of the electrochemical double layer, which exists along the solid-grain/fluid-electrolyte boundaries of porous media. Within the double layer, charge is redistributed, creating an excess electrical charge in the fluid along the boundary. Electrokinetic theory describes coupled seismic and electromagnetic wave propagation. It predicts that seismic waves disturb the fluid excess charge, thereby creating an electric streaming current (seismoelectric effect). Inversely, the theory predicts that electromagnetic waves generate mechanical/seismic signals (electroseismic effect). Electrokinetic conversions can potentially be used as an effective means of detecting hydrocarbon reservoirs: it inherently combines seismic resolution with electromagnetic hydrocarbon sensitivity. Validating electrokinetic wave theory is therefore of paramount importance.

Electrokinetic theory predicts the existence of two seismoelectric effects: (1) a coseismic (electric) field that is coupled to seismic waves, and therefore propagates with seismic wave velocity, and (2) a seismic wave that traverses an interface with a contrast in electrical or mechanical properties and produces electromagnetic (EM) signals that propagate outside the support of the seismic waves with much higher EM-wave speeds. These are called the coseismic and interface response fields, respectively. Electrostatic counterparts of these fields exist as well.

In this thesis, electrokinetic theory is reformulated along the lines sketched by Biot (1956a,b). The reformulation employs effective frequency-dependent densities, in which both viscous and electrokinetic coupling are comprised. The reformulated theory predicts the existence of four wave modes within a fluid-saturated porous medium: fast and slow P-waves, a shear wave and an EM-wave. The electrokinetic dispersion relations, which give wave speed and intrinsic attenuation of each wave, are expressed in terms of generalized elastic coefficients and the effective densities. Each of the wave modes has a specific fluid-solid amplitude displacement ratio. These are derived and expressed also in terms of generalized elastic coefficients and effective densities. Each wave mode also has a specific ratio of electric potential and solid displacement potential. These are expressed in terms of the fluid-solid ratios and the so-called (frequency-dependent) electrokinetic coupling coefficient. This coefficient describes the coupling between electric and mechanical fields. When the

coupling coefficient is zero, the electrokinetic equations decouple into the familiar Biot's poroelastic equations and Maxwell's EM relations.

Subsequently, the wave coupling at a fluid/porous-medium interface is theoretically solved, where the modified theoretical formulation is applied. First a straightforward scattering problem of an incident fluid P-wave into fluid electromagnetic and pressure waves, and porous medium waves is considered. Second, the electrokinetic scattering matrix for a fluid/porous-medium interface is derived. This matrix summarizes all electrokinetic reflection and transmission coefficients applicable to this boundary. These coefficients describe how incident P-waves are converted at electrical/mechanical interfaces to electromagnetic signals, and, vice-versa, how electromagnetic signals are converted into acoustic signals. They enter the full-waveform seismoelectric and electroseismic interface response field models. The seismoelectric model employs the Sommerfeld approach, while the electroseismic model uses wave-field (de)composition techniques. These models provide electrokinetic theoretical predictions.

Laboratory seismoelectric fluid/porous-medium interface response field measurements are performed as a function of time, space and fluid salinity. These measurements are compared against the seismoelectric model predictions. It is found that the seismoelectric model predictions excellently describe the measured interface response fields in terms of waveform, spatial amplitude pattern, and travel times. One scalar amplitude scaling factor is needed to reconcile the amplitudes of the theoretical predictions and the measurements. The factor is shown to depend on electric conductivity, i.e., predicted amplitudes significantly deviate at low pore fluid conductivity ($\sim 10^{-3}$ S/m), while they are close to the actual measurements for higher conductivities ($\sim 10^{-2}$ S/m). The poroelastic, electromagnetic and specific electrokinetic parameters that enter the seismoelectric model are independently measured or known from literature. The seismoelectric Sommerfeld integral model incorporates a so-called directivity function that closely resembles the independently measured spatial distribution pattern of the incident acoustic field. As a check, seismoelectric origin of the measured electric potentials is confirmed. Next, an expanded seismoelectric model that predicts interface response fields at different boundaries, as well as coseismic fields, is presented and compared against measurements. The seismoelectric model predictions excellently describe the measured electric potentials in terms of travel times, waveform, polarity, amplitude and spatial amplitude decay, demonstrating that seismoelectric effects are comprehensively described by theory. Finally, an electroseismic laboratory setup is described and electroseismic measurements are presented.

Samenvatting

Gekoppelde seismische en elektromagnetische golfvoortplanting

Gekoppelde seismische en elektromagnetische golfvoortplanting wordt theoretisch en experimenteel bestudeerd. Deze koppeling bestaat vanwege de elektrochemische dubbellaag, die gevormd wordt aan het grensvlak tussen de korrels en een elektrolytische oplossing in een poreus medium. De lading in de dubbellaag wordt herverdeeld. Dit brengt een overschot aan elektrische lading in de vloeistof in het grensvlak tot stand. Elektrokinetische theorie beschrijft gekoppelde seismische en elektromagnetische golfvoortplanting. Het voorspelt dat seismische golven het ladingsoverschot in de vloeistof beïnvloeden, waardoor een elektrische stroom ontstaat (seismoelectrisch effect). Voor het tegenovergestelde geval voorspelt de theorie dat elektromagnetische golven mechanische/seismische signalen veroorzaken (elektroseismisch effect). Elektrokinetische omzettingen kunnen mogelijk gebruikt worden als een effectief middel bij het vinden van olie- en gasvoorkomens: het verbindt dan de hoge resolutie van de seismiek met de gevoeligheid voor koolwaterstoffen van elektromagnetische metingen. Dientengevolge is het valideren van de elektrokinetische golftheorie van groot belang.

De elektrokinetische theorie voorspelt het bestaan van twee seismoelectrische effecten: (1) een coseismisch (elektrisch) veld dat is gekoppeld aan seismische golven, en zich daardoor voortplant met de seismische golfsnelheid, en (2) elektromagnetische (EM) signalen die zich onafhankelijk van een seismische golf voortplanten met veel grotere EM-golfsnelheden, en die geproduceerd worden door een seismische golf die een plotselinge overgang in elektrische en/of mechanische eigenschappen passeert. Deze golven worden respectievelijk coseismische en grensreactiegolven genoemd. Er bestaan ook elektroseismische tegenhangers van deze velden.

In dit proefschrift wordt de bestaande elektrokinetische theorie opnieuw geformuleerd gebruik makend van Biot's (1956a,b) notatie. De herformulering gebruikt effectieve, frequentieafhankelijke dichtheden, die zowel viskeuze als elektrokinetische koppeling beschrijven. De geherformuleerde theorie voorspelt het bestaan van vier golfmodi voor een met vloeistof verzadigd poreus medium: een snelle en een langzame P-golf, een schuifgolf en een EM-golf. De elektrokinetische dispersierelaties, die golfsnelheid en intrinsieke demping van elke golf voorspellen, worden uitgedrukt in generaliseerde elastische coëfficiënten en effectieve dichtheden. Elke golfmodus

heeft ook een specifieke coëfficiënt die de verhouding beschrijft tussen de verplaatsingsamplitudes van vaste stof en vloeistof. Deze worden afgeleid en ook uitgedrukt in de gegeneraliseerde elastische coëfficiënten en effectieve dichtheden. Elke golfmodus heeft ook een specifieke verhouding tussen de elektrische potentiaal en de verplaatsingspotentiaal van de vaste stof. Deze worden uitgedrukt in de eerder genoemde verplaatsingsverhoudingen tussen vloeistof en vaste stof, en de zogenaamde (frequentieafhankelijke) elektrokinetische koppelingscoëfficiënt. Deze laatste coëfficiënt beschrijft de koppeling tussen elektrische en mechanische velden. Als de koppelingscoëfficiënt nul is, verworden de elektrokinetische vergelijkingen tot de bekende poro-elastische vergelijkingen van Biot en Maxwell's EM-vergelijkingen.

Vervolgens worden de gekoppelde golfvergelijkingen in het geval van een overgang tussen een vloeistof en een poreus medium theoretisch opgelost met behulp van de aangepaste theoretische formulering. Eerst wordt een rechthoekig reflectieprobleem beschouwd, waarbij een P-golf vanuit de vloeistof invalt en weerkaatst (in de vloeistof) en doorgelaten wordt (in het poreuze materiaal) als een combinatie van elektromagnetische- en drukgolven. Vervolgens wordt de totale elektrokinetische verstrooiingsmatrix afgeleid voor deze overgang tussen de vloeistof en het poreuze medium. Deze matrix omvat dus alle elektrokinetische reflectie en transmissiecoëfficiënten die betrekking hebben op deze overgang. Deze individuele coëfficiënten beschrijven hoe invallende P-golven aan een grensvlak waar de elektrische en/of mechanische eigenschappen plotseling veranderen, omgezet worden in elektromagnetische signalen, en, mutatis mutandis, hoe elektromagnetische signalen in akoestische signalen omgezet worden. Deze coëfficiënten zijn integraal onderdeel van de seismoelektrische- en elektrozeismische grensreactiegolfmodellen. Het seismoelektrische model gebruikt de Sommerfeld integraal, terwijl het elektrozeismische model gebruik maakt van golfveld (de)compositietechnieken. Deze modellen voorspellen dan de elektrokinetische signalen.

Laboratoriummetingen van seismoelektrische grensreactievelden aan de overgang tussen een vloeistof en een poreus materiaal worden verricht als functie van tijd, positie en zoutconcentratie van de vloeistof. Deze metingen worden vergeleken met seismoelektrische modelvoorspellingen. De seismoelektrische modelvoorspellingen beschrijven de gemeten reistijden, golfvormen en het ruimtelijk amplitudepatroon van de grensreactievelden uitstekend. Een schalingsfactor is echter wel benodigd om de theoretisch voorspelde amplitudes met de gemeten amplitudes in overeenstemming te brengen. De factor hangt af van het zoutgehalte. Voorspelde amplitudes wijken significant af van de metingen bij een lage geleidbaarheid van de porievloeistof ($\sim 10^{-3}$ S/m), terwijl ze dicht bij de eigenlijke metingen liggen voor hogere geleidbaarheid ($\sim 10^{-2}$ S/m). De poro-elastische, elektromagnetische en specifieke elektrokinetische parameters, die benodigd zijn voor het seismoelektrische model, worden verkregen uit onafhankelijke metingen of vanuit de literatuur. In het seismoelektrische Sommerfeld integraal model wordt een zogenaamde richtingsfunctie opgenomen die nauwkeurig het onafhankelijk gemeten ruimtelijke verdeelpatroon van het invallende akoestische veld weergeeft. Een controle-experiment bevestigt dat de elektrische potentialen inderdaad veroorzaakt worden door het seis-

moelektrische effect. Vervolgens wordt een uitgebreid seismoelektrisch model, dat grensreactievelden en coseismische velden bij verschillende overgangen voorspelt, gepresenteerd en vergeleken met metingen. De seismoelektrische modelvoorspellingen beschrijven de gemeten reistijden, golfvormen, polariteiten, amplitudes en het ruimtelijk amplitudepatroon uitstekend, wat laat zien dat seismoelektrische effecten volledig worden beschreven door de theorie. Ten slotte wordt een laboratoriumopstelling beschreven ten behoeve van elektrozeismische metingen en worden enkele elektrozeismische metingen getoond.

Chapter 1

Introduction

1.1 Geophysical exploration

Land-based and marine seismics are commonly employed to investigate the earth's subsurface. The method is non-destructive, which means that we do not need to excavate or retrieve wellbore (core) samples. The disadvantage is that we have to interpret the seismic data for their structural and lithological content. The basic principle of marine seismics is outlined in Figure 1.1 (left). A source of acoustic energy (typically an airgun), generates sound waves that travel through the sub-seafloor layering of the earth. At layer boundaries, part of the incident seismic energy is reflected. This energy travels upwards to the sea surface where it is recorded by so-called hydrophones. By analyzing and processing these recordings, information about the subsurface layering is obtained. A relative recent method to study the earth's sub-seafloor is by marine controlled source electromagnetics (CSEM). Figure 1.1 (right) shows a boat which tows an antenna. The antenna emits EM-waves that propagate through the sub-seafloor layers of the earth. Similar to the seismic method, information about the subsurface is obtained by recording the EM field (Chave *et al.*, 1991; Eidesmo *et al.*, 2002; Buland *et al.*, 2011). Another application of EM exploration is the ground penetrating radar (GPR) that is used for shallow subsurface interpretation. It is particularly sensitive to soil water content and has a typical penetration range of 10–30 m in sand (Slob *et al.*, 2010).

The seismic method senses impedance contrasts, which are determined by the product of seismic wave velocity and mass density of a layer. The EM method is sensitive to the electrical permittivity and conductivity contrasts of the layers. The size of the structures/layers that can be detected by the seismic and/or EM method is called resolution. The resolution is determined by the wavelength.

A non-destructive method that aims to identify hydrocarbon reservoirs ideally combines high resolution (small wavelength) with a strong sensitivity to hydrocarbons. Hydrocarbon reservoirs occur in earth layers typically at several kilometers depth. The seismic method has good resolution and excellent penetration depth, but the impedance contrasts between hydrocarbon reservoirs and surrounding formations can be small (no sensitivity). For EM methods, electrical conductivity

Figure 1.1: Seismic (left) and electromagnetic (right) subsurface exploration.

contrasts between hydrocarbons deposits and surrounding formations are typically good. However, EM methods have poor resolution, because the EM-waves have large wavelengths. Obviously, a method that combines seismic resolution with EM hydrocarbon sensitivity is desirable.

In fluid-saturated porous rocks, seismic wave energy can convert into electromagnetic energy and vice versa. This phenomenon is called “electrokinetic conversion” and arises because of the presence of an “electrochemical double layer” at all grain-water interfaces. Electrokinetic conversion potentially combines seismic resolution with the conductivity sensitivity of EM methods.

1.2 The electrochemical double layer

Grain surfaces of porous rocks that are in contact with pore fluids typically acquire a chemically bound surface charge (see Figure 1.2) due to surface reactions such as deprotonization and ion adsorption. This bound charge is balanced by mobile counter ions in a thin fluid layer surrounding the grains. The bound charge is immobile, whereas the counter ions are transported when the pore fluid flows. The distribution of mobile ions is determined by a balance between electrostatic forces and thermal diffusivity. At the interface between the immobile and counter ions the so-called zeta-potential is defined. This electric potential varies exponentially when one moves away from the interface. The corresponding characteristic decay length is called the Debye length, which is on the order of a few nanometers for typical grain-electrolyte combinations (Pride, 1994). The ensemble of adsorbed and diffuse charge layers is referred to as the electrochemical double layer. The double layer gives rise to coupled seismic and electromagnetic wave propagation.

Figure 1.2: Sketch of the electrochemical double layer. When the pore fluid flows, mobile charge is transported, while the bound charge remains with the solid. A detailed description of the structure of the double layer is beyond the scope of this thesis.

1.3 Coupled seismic and electromagnetic wave propagation

A seismic wave that travels through a fluid-saturated porous rock creates an electric field within the pulse. This effect is called the “coseismic field”. Figure 1.3 (left) shows a compressional wave that travels through a homogeneous fluid-saturated porous rock. The wave is represented by the vertical lines. Where the lines are closely together pressure peaks exist, and where they are further apart, we have pressure troughs. The resulting fluid flow will transport the counter ions of the double layer relative to the immobile, fixed charge. In this way, counter ions (+) accumulate in pressure troughs and bound charge (−) becomes exposed in pressure peaks, creating an electric coseismic field on the scale of the wavelength. This electric field drives a conduction current that exactly balances the hydraulic current flow. Thus there is no net electric current. The coseismic field has no support outside the wave (Pride & Haartsen, 1996; Pride & Garambois, 2002).

A seismic wave that traverses a boundary with a contrast in electrical and/or mechanical properties, creates an EM signal (Figure 1.3, right). This effect is called the “interface response field”. As the wave crosses the boundary, the charge distribution on one side of the interface will in general be different from that on the other side due to the different medium properties. Thus a current imbalance is pro-

Figure 1.3: Seismoelectric effects. The coseismic electric field (left) is created by the charge separation within the compressional wave. The interface response field (right) is produced when the compressional pulse traverses a boundary with a contrast in electrical and/or mechanical properties.

duced that varies with the time signature of the wave. The resulting EM radiation can typically be characterized as that due to an oscillating electric dipole, oriented perpendicularly to the interface. When the seismic wave has crossed the interface, no EM radiation is emitted anymore (Pride & Haartsen, 1996; Pride & Garambois, 2002).

Inversely, Pride & Haartsen (1996) derive that mechanical fields are created by an EM plane wave that travels through a fluid-saturated porous medium. Also, Thompson *et al.* (2007) report on field studies in which conversions of EM to seismic energy at depth were recorded. In this thesis, we refer to electrical effects caused by seismic waves as “seismoelectricity”, whereas we denote seismic effects caused by EM sources as “electroseismicity”.

1.4 Literature review

In an attempt to explain coseismic field measurements, Frenkel presented a comprehensive electrokinetic wave theory in 1944. This theory correctly predicts that (1) two longitudinal wave modes exist in fluid-saturated media and (2) the coseismic field of these modes is proportional to the solid particle acceleration. Frenkel’s work also provides a complete set of equations for seismic wave propagation in a fluid-saturated porous frame. However, the so-called interaction terms were not fully modelled. Biot (1956a,b, 1962a,b) derived the full set of equations, which is now commonly employed to describe poroelastic wave behaviour. Biot & Willis (1957) related the generalized elastic constants that enter the stress-strain relations of Biot’s poroelastic theory to measurable bulk moduli of the porous aggregate. Neev & Yeatts (1989) incorporated an electric term into the poroelastic equations

by assuming that electrokinetic effects are caused by electroosmotic and streaming potential effects. Thus they did not take the complete set of electromagnetic equations into account. Pride *et al.* (1992) obtained Biot's theory by volume averaging the equations of motion and stress-strain relations that apply in the solid and fluid phase. Pride (1994) extended this approach by incorporating Maxwell's electromagnetic relations into the volume averaging procedure, which resulted in a complete set of electrokinetic governing equations. They have the form of Maxwell's electromagnetic relations coupled to Biot's poroelastic equations by extended Darcy's and Ohm's laws comprising the frequency-dependent electrokinetic coupling coefficient. This coupling coefficient is proportional to the zeta-potential. The frequency-dependency of the electrokinetic coupling coefficient was experimentally validated by Reppert *et al.* (2001) and Schoemaker (2011). Revil & Linde (2006) derived an alternative and frequency-independent expression for the coupling coefficient, in which the amount of diffuse layer excess charge quantifies the coupling of mechanical and electrical fields. Pride & Haartsen (1996) generalized the homogeneous and isotropic governing equations of Pride (1994) to anisotropic and heterogeneous porous media and provided the background theory with which electrokinetic wave propagation problems can be solved. They also predicted the existence of coseismic and interface response fields, which were subsequently numerically simulated in a stack of poroelastic layers by Haartsen & Pride (1997). Other full-waveform seismoelectric numerical simulations were performed by Han & Wang (2001); Garambois & Dietrich (2002); Pain *et al.* (2005); Pride & Garambois (2005); Haines & Pride (2006); Cui *et al.* (2007a,b); Zhu *et al.* (2008b); de Ridder *et al.* (2009); Jardani *et al.* (2010) and Revil & Jardani (2010). Full-waveform electroseismic simulations were performed by White & Zhou (2006); Hu *et al.* (2007); Guan & Hu (2008) and Zyserman *et al.* (2010). A wide range of field and laboratory measurements of the coseismic and interface response fields was presented (see e.g., Martner & Sparks, 1959; Thompson & Gist, 1993; Butler *et al.*, 1996; Mikhailov *et al.*, 1997; Beamish, 1999; Zhu *et al.*, 1999; Mikhailov *et al.*, 2000; Zhu *et al.*, 2000; Garambois & Dietrich, 2001; Zhu & Toksöz, 2003, 2005; Block & Harris, 2006; Bordes *et al.*, 2006; Dupuis & Butler, 2006; Haines *et al.*, 2007; Dupuis *et al.*, 2007; Strahser *et al.*, 2007; Liu *et al.*, 2008; Zhu *et al.*, 2008a,b; Dupuis *et al.*, 2009). Zhu & Toksöz (2005) and Bordes *et al.* (2006, 2008) reported on coseismic magnetic field measurements associated with a Stoneley wave and a shear wave, respectively. The inverse effect was also measured. Electrostatic measurements were performed by Zhu *et al.* (1999, 2008a); Hornbostel & Thompson (2007) and Thompson *et al.* (2007).

Some researches compared full-waveform seismoelectric predictions against measurements. Mikhailov *et al.* (1997) and Haines *et al.* (2007) qualitatively compare seismoelectric numerical predictions with field measurements. Zhu *et al.* (2000) and Zhu *et al.* (2008b) found qualitative agreement between full-waveform seismoelectric predictions and laboratory measurements, while Charara *et al.* (2009) found agreement between modelled and measured seismoelectric signals at a fluid/porous-medium interface in a laboratory setup.

1.5 Thesis aim and outline

Understanding seismoelectric wave propagation is of paramount importance in the development of electrokinetic exploration methods. Despite the wide range of seismoelectric field and laboratory studies, direct comparisons between full-waveform seismoelectric predictions and measurements are scarce. In this thesis, the predictive power of electrokinetic theory for the seismoelectric interface response and coseismic fields is tested. Laboratory measurements of these fields are compared with wave propagation model predictions based on the electrokinetic theory in terms of travel times, waveform, amplitude and spatial amplitude decay.

In chapter 2, Pride's electrokinetic theory is presented and reformulated along the lines sketched by Biot (1956a,b). In chapter 3 the so-called seismoelectric reflection and transmission coefficients at a fluid/porous-medium interface are derived. The seismoelectric reflection coefficient appears in the seismoelectric full-waveform models, which are presented in chapter 4. In chapter 4 also, full-waveform electroseismic models and the complete electrokinetic scattering matrix for a fluid/porous-medium interface are derived. In chapters 5 and 6, the seismoelectric models are compared against laboratory measurements of the interface response and coseismic fields. In chapter 5 also electroseismic measurements are presented. Conclusions are given in chapter 7.

Chapter 2

Electrokinetic theory

2.1 Introduction

In a porous aggregate of a solid frame saturated with a fluid electrolyte, coupling occurs between the mechanical and electromagnetic (EM) fields due to the excess electrical charge created near the solid-fluid interface. The interaction of the mechanical solid and fluid fields is usually attributed to Biot (1956a,b). Pride (1994) extends Biot's theory to an electrokinetic theory that describes the interaction of mechanical fields with EM fields. Pride & Haartsen (1996) show that this theory predicts the existence of four wave modes: two longitudinal waves, the fast P-wave and the slow P-wave, and two transversal waves: an EM-wave and a seismic shear wave.

This chapter starts with the complete set of Pride's electrokinetic governing equations, which are subsequently recast in momentum equations. An electric term is added to the (Biot) momentum equations for the solid and the fluid, while a momentum equation for the electric field closes the system. Employing Helmholtz decomposition, the electrokinetic dispersion relations, yielding phase velocities and attenuations of each wave mode, are derived in section 2.3. It is shown that in the low-frequency range the slow P-wave and EM-wave obey a diffusion equation. The fast longitudinal wave and shear wave are propagatory. In the high-frequency range all waves are propagatory. In section 2.4 so-called fluid-to-solid and electric-to-solid field ratios are derived. The fluid-to-solid ratio describes the fluid-to-solid displacement amplitude ratio, while the electric-to-solid field ratio describes the strength of the electric field with respect to the solid displacement field. In section 2.5 we perform a sensitivity study by analyzing the influence of porous medium parameters on the electric-to-solid field ratio of the fast P-wave. Finally, a reformulation of electrokinetic theory is discussed in section 2.6.

2.2 The Biot-Pride electrokinetic theory

We consider a poroelastic matrix saturated by an electrolyte and adopt an $\exp[i\omega t]$ convention for time varying fields. The governing linearized equations in an isotropic, homogeneous poroelastic medium were given by Pride (1994). We rewrite the equations here in a somewhat different form because we aim to derive the dispersion relations in terms of effective densities. The governing equations read as

$$\begin{aligned} -\nabla \cdot \underline{\hat{\sigma}} - (1 - \phi)\nabla \hat{p} \\ = -\omega^2 \left[\rho_{11}(\omega)\hat{\mathbf{u}} + \rho_{12}(\omega)\hat{\mathbf{U}} \right] + \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \end{aligned} \quad (2.1)$$

$$-\phi\nabla \hat{p} = -\omega^2 \left[\rho_{12}(\omega)\hat{\mathbf{u}} + \rho_{22}(\omega)\hat{\mathbf{U}} \right] - \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \quad (2.2)$$

$$\begin{aligned} \underline{\hat{\sigma}} = -G \left[\nabla \hat{\mathbf{u}} + (\nabla \hat{\mathbf{u}})^T \right] \\ - \left[\left(A - \frac{1-\phi}{\phi}Q \right) \nabla \cdot \hat{\mathbf{u}} + \left(Q - \frac{1-\phi}{\phi}R \right) \nabla \cdot \hat{\mathbf{U}} \right] \underline{\mathbf{I}}, \end{aligned} \quad (2.3)$$

$$\hat{p} = -\frac{1}{\phi} \left(Q \nabla \cdot \hat{\mathbf{u}} + R \nabla \cdot \hat{\mathbf{U}} \right), \quad (2.4)$$

$$\hat{\mathbf{J}} = \sigma(\omega)\hat{\mathbf{E}} + L(\omega) \left(-\nabla \hat{p} + \omega^2 \rho_f \hat{\mathbf{u}} \right), \quad (2.5)$$

$$\nabla \times \hat{\mathbf{H}} = i\omega\varepsilon\hat{\mathbf{E}} + \hat{\mathbf{J}}, \quad (2.6)$$

$$\nabla \times \hat{\mathbf{E}} = -i\omega\mu\hat{\mathbf{H}}. \quad (2.7)$$

Hats over field variables indicate frequency-domain quantities, ω is the angular frequency, η is the pore fluid viscosity, ϕ is the porosity, ρ_f is the pore fluid density, μ is the magnetic permeability, ε is the bulk electrical permittivity given by $\varepsilon = \varepsilon_0 [\phi(\epsilon_f - \epsilon_s)/\alpha_\infty + \epsilon_s]$, with solid and pore fluid relative permittivities ϵ_s and ϵ_f , vacuum permittivity ε_0 , and tortuosity α_∞ . $\underline{\mathbf{I}}$ denotes the identity matrix. The field variables are: solid and (pore) fluid displacements $\hat{\mathbf{u}}$ and $\hat{\mathbf{U}}$, intergranular stress $\underline{\hat{\sigma}}$, pore pressure $\hat{p}$, electric current density $\hat{\mathbf{J}}$, and the electric and magnetic fields $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$. A , Q and R are the generalized elastic coefficients that are related to the bulk modulus of the skeleton grains K_s , the shear modulus G , the bulk modulus of the pore fluid K_f and the bulk modulus of the framework of grains K_b as follows (Biot & Willis, 1957):

$$A = \frac{(1 - \phi)^2 K_f K_s - (1 - \phi) K_f K_b + \phi K_b K_s}{K_f (1 - \phi - K_b/K_s) + \phi K_s} - \frac{2}{3}G, \quad (2.8)$$

$$Q = \frac{\phi K_f [(1 - \phi) K_s - K_b]}{K_f (1 - \phi - K_b/K_s) + \phi K_s}, \quad (2.9)$$

$$R = \frac{\phi^2 K_f K_s}{K_f (1 - \phi - K_b/K_s) + \phi K_s}. \quad (2.10)$$

Note that for $L(\omega) = 0$ (no electrokinetic coupling), mechanical and EM fields are decoupled in equations (2.1)-(2.7). Equations (2.1)-(2.4) then transform into the original Biot equations (Biot, 1956a,b) and equations (2.5)-(2.7) into Ohm's law and Maxwell's equations. Also note that Biot's (1956a; 1956b) and Pride's (1994) theories require the seismic wavelengths to be much larger than the grain sizes. Therefore a typical frequency upper bound for these theories is on the order of 1 MHz for sandstones.

The dynamic permeability $k(\omega)$ describes the transition from viscosity towards inertia-dominated flow and is given by (Johnson *et al.*, 1987)

$$k(\omega) = k_0 \left[\sqrt{1 + i \frac{\omega}{\omega_t} \frac{M}{2}} + i \frac{\omega}{\omega_t} \right]^{-1}, \quad (2.11)$$

where k_0 is permeability. The transition frequency ω_t and the similarity parameter M are given by

$$\omega_t = \frac{\eta \phi}{\alpha_\infty k_0 \rho_f}, \quad (2.12)$$

$$M = \frac{8 \alpha_\infty k_0}{\phi \Lambda^2}, \quad (2.13)$$

where Λ is a weighted pore volume-to-surface ratio. As the dynamic permeability and the fluid and solid density terms always appear together, it is convenient to define effective densities $\rho_{11}(\omega)$, $\rho_{12}(\omega)$ and $\rho_{22}(\omega)$

$$\rho_{11}(\omega) = (1 - \phi) \rho_s - \rho_{12}(\omega), \quad (2.14)$$

$$\rho_{12}(\omega) = \phi \rho_f \left[1 + i \alpha_\infty \frac{\omega_t}{\omega} \frac{k_0}{k(\omega)} \right], \quad (2.15)$$

$$\rho_{22}(\omega) = \phi \rho_f - \rho_{12}(\omega), \quad (2.16)$$

where ρ_s is the solid density. The electrokinetic coupling coefficient $L(\omega)$ couples mechanical fields to EM fields and is expressed as (Pride, 1994)

$$L(\omega) = L_0 \left\{ 1 + 2i \frac{\omega}{\omega_t M} \left[1 - 2 \frac{d}{\Lambda} \right]^2 \left[1 + \frac{d}{\delta(\omega)} (1 + i) \right]^2 \right\}^{-1/2}, \quad (2.17)$$

$$L_0 = -\frac{\phi}{\alpha_\infty} \frac{\varepsilon_0 \epsilon_f \zeta}{\eta} \left(1 - 2 \frac{d}{\Lambda} \right), \quad (2.18)$$

where ζ is the zeta-potential, which is an electrical potential quantifying the mobile excess charge in the electrochemical double layer. If ζ in equation (2.18) is zero, no electrokinetic coupling is induced. The viscous skin depth is defined as $\delta(\omega) = \sqrt{2\eta/\omega \rho_f}$. The Debye length d is a measure for the thickness of the diffuse double layer and is given by

$$d = \left[\sum_{l=1}^N \frac{(e z_l)^2 N_l}{\varepsilon_0 \epsilon_f k_B T} \right]^{-1/2}, \quad (2.19)$$

where e is the electron charge, z_l is the l -species valency, k_B is Boltzmann's constant, T is the temperature, N_l is the l -species ion amount per volume, and N is the amount of ion species (e.g., $N = 2$ for a NaCl salt solution). For the parameters given in Table 2.1, the amplitude and phase of $L(\omega)$ are given as a function of frequency in Figure 2.1 (top). These values are representative for a shallow sandstone (Denneman *et al.*, 2002). It can be seen that low and high-frequency electrokinetic coupling regimes are separated by the transition frequency $\omega_t/2\pi$. This behaviour is similar to that of $k(\omega)$, which is shown in the lower panels of Figure 2.1. The frequency-dependence of $k(\omega)$ is determined by a transition from viscosity dominated pore fluid flow to inertial flow. Electrokinetic coupling arises due to the flow of the pore fluid with respect to the solid and is thus similarly affected by this transition. It must be noted, however, that $L(\omega)$ relaxes at $\omega_t M/2$, while $k(\omega)$ relaxes at ω_t . Finally, we specify the dynamic conductivity of the pore fluid $\sigma(\omega)$, which is expressed as

$$\sigma(\omega) = \frac{\phi\sigma_f}{\alpha_\infty} \left[1 + \frac{2[C_{em} + C_{os}(\omega)]}{\sigma_f\Lambda} \right], \quad (2.20)$$

where σ_f is conductivity of the pore fluid, C_{em} is excess conductance associated with electromigration of excess charge, and $C_{os}(\omega)$ is excess conductance associated with electroosmosis. Expressions for σ_f , C_{em} and $C_{os}(\omega)$ are given in appendix A of this chapter. An overview of all electrokinetic parameters is given in Table 2.1.

We now recast equations (2.1)-(2.7) into three momentum equations containing the fields $\hat{\mathbf{u}}$, $\hat{\mathbf{U}}$ and $\hat{\mathbf{E}}$. Substituting equations (2.3) and (2.4) into equations (2.1) and (2.2) yields

$$\begin{aligned} G\nabla^2\hat{\mathbf{u}} + (A + G)\nabla\nabla \cdot \hat{\mathbf{u}} + Q\nabla\nabla \cdot \hat{\mathbf{U}} \\ = -\omega^2 \left[\rho_{11}(\omega)\hat{\mathbf{u}} + \rho_{12}(\omega)\hat{\mathbf{U}} \right] + \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \end{aligned} \quad (2.21)$$

$$\begin{aligned} Q\nabla\nabla \cdot \hat{\mathbf{u}} + R\nabla\nabla \cdot \hat{\mathbf{U}} \\ = -\omega^2 \left[\rho_{12}(\omega)\hat{\mathbf{u}} + \rho_{22}(\omega)\hat{\mathbf{U}} \right] - \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}. \end{aligned} \quad (2.22)$$

With the definitions for $\rho_{12}(\omega)$ and $\rho_{22}(\omega)$, equation (2.2) is written as

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta} \left(-\nabla\hat{p} + \omega^2\rho_f\hat{\mathbf{u}} \right), \quad (2.23)$$

Eliminating $(-\nabla\hat{p} + \omega^2\rho_f\hat{\mathbf{u}})$ from equations (2.5) and (2.23) we obtain

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta} \left(\frac{\hat{\mathbf{J}} - \sigma(\omega)\hat{\mathbf{E}}}{L(\omega)} \right). \quad (2.24)$$

Substituting equation (2.6) into equation (2.24) gives

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta L(\omega)} \left(\nabla \times \hat{\mathbf{H}} - [i\omega\varepsilon + \sigma(\omega)] \hat{\mathbf{E}} \right). \quad (2.25)$$

Table 2.1: Electrokinetic parameters. Mechanical parameters from Denneman *et al.* (2002).

K_b	Bulk modulus framework of grains	5.8×10^9	Pa
G	Shear modulus framework of grains	3.4×10^9	Pa
K_s	Bulk modulus skeleton grains	40×10^9	Pa
K_f	Bulk modulus pore fluid	2.22×10^9	Pa
η	Pore fluid viscosity	1×10^{-3}	Pa s
ρ_f	Pore fluid density	1.00×10^3	kg/m ³
ρ_s	Solid density	2.76×10^3	kg/m ³
α_∞	Tortuosity of the porous medium	2.3	-
ϕ	Porosity of the porous medium	0.24	-
k_0	Permeability	0.390×10^{-12}	m ²
T	Temperature	295	K
pH	Hydrogen exponent	7	-
ϵ_f	Relative permittivity of the pore fluid	80	-
ϵ_s	Relative permittivity of the solid	4	-
μ	Magnetic permeability ($=\mu_0$)	$4\pi \times 10^{-7}$	H/m
z_1	Valence of 1-species ion	1	-
z_2	Valence of 2-species ion	-1	-
b_1	Mobility of 1-species ion ^a	3.246×10^{11}	m/(N s)
b_2	Mobility of 2-species ion ^a	4.931×10^{11}	m/(N s)
ζ	Zeta-potential ^b	-65×10^{-3}	V
M	Similarity parameter	1	-
C	Concentration of the electrolyte	1×10^{-3}	M

^aKeller (1988).^bFrom equation (A-5), this chapter.

Finally, substituting equation (2.7) into equation (2.25) leads to

$$\omega^2 \mu \bar{\epsilon}(\omega) \hat{\mathbf{E}} + \omega^2 \frac{\eta \phi \mu L(\omega)}{k(\omega)} (\hat{\mathbf{U}} - \hat{\mathbf{u}}) = \nabla \nabla \cdot \hat{\mathbf{E}} - \nabla^2 \hat{\mathbf{E}}, \quad (2.26)$$

where $\bar{\epsilon}(\omega)$ is the newly defined effective electrical permittivity

$$\bar{\epsilon}(\omega) = \epsilon - i \frac{\sigma(\omega)}{\omega} + i \frac{\eta L^2(\omega)}{\omega k(\omega)}. \quad (2.27)$$

Equations (2.21), (2.22), and (2.26) form a closed set of equations for the fields $\hat{\mathbf{u}}$, $\hat{\mathbf{U}}$, and $\hat{\mathbf{E}}$.

2.3 Electrokinetic wave modes

Electrokinetic theory in homogeneous, isotropic media predicts the existence of uncoupled longitudinal and transversal modes. There are two longitudinal waves, the

Figure 2.1: Amplitudes (left) and phase values (right) of the electrokinetic coupling coefficient $L(\omega)$ and the dynamic permeability $k(\omega)$ as a function of frequency using parameters that are typical for a natural shallow sandstone (see Table 2.1). The transition frequency is given by $\omega_t/2\pi$.

fast P-wave (Pf) and the slow P-wave (Ps), and two transversal waves: an EM-wave and a seismic shear (S-)wave (Pride & Haartsen, 1996). Dispersion relations, yielding wave speed and attenuation as a function of frequency for each mode, are derived here in a procedure similar to that of Allard (1993).

Using equation (2.22) to eliminate the electric field $\hat{\mathbf{E}}$ from equations (2.21) and (2.26), we obtain two modified momentum equations

$$G\nabla^2\hat{\mathbf{u}} + (A + G)\nabla\nabla \cdot \hat{\mathbf{u}} + Q\nabla\nabla \cdot \hat{\mathbf{u}} + Q\nabla\nabla \cdot \hat{\mathbf{U}} + R\nabla\nabla \cdot \hat{\mathbf{U}} = -\omega^2 \left[(\rho_{11}(\omega) + \rho_{12}(\omega))\hat{\mathbf{u}} + (\rho_{12}(\omega) + \rho_{22}(\omega))\hat{\mathbf{U}} \right], \quad (2.28)$$

$$Q\nabla\nabla \cdot \hat{\mathbf{u}} + R\nabla\nabla \cdot \hat{\mathbf{U}} = -\omega^2 \left(\bar{\rho}_{12}(\omega)\hat{\mathbf{u}} + \bar{\rho}_{22}(\omega)\hat{\mathbf{U}} \right) + \frac{\rho_{12}(\omega)}{\mu\bar{\varepsilon}(\omega)} (\nabla\nabla \cdot \hat{\mathbf{u}} - \nabla^2\hat{\mathbf{u}}) + \frac{\rho_{22}(\omega)}{\mu\bar{\varepsilon}(\omega)} (\nabla\nabla \cdot \hat{\mathbf{U}} - \nabla^2\hat{\mathbf{U}}), \quad (2.29)$$

where complex effective densities $\bar{\rho}_{11}(\omega)$, $\bar{\rho}_{12}(\omega)$ and $\bar{\rho}_{22}(\omega)$, containing the elec-

trokinetic coupling factor $E_K(\omega)$, are defined as follows:

$$\bar{\rho}_{11}(\omega) = \rho_{11}(\omega) - E_K(\omega), \quad (2.30)$$

$$\bar{\rho}_{12}(\omega) = \rho_{12}(\omega) + E_K(\omega), \quad (2.31)$$

$$\bar{\rho}_{22}(\omega) = \rho_{22}(\omega) - E_K(\omega), \quad (2.32)$$

$$E_K(\omega) = \frac{\eta^2 \phi^2 L^2(\omega)}{\omega^2 k^2(\omega) \bar{\epsilon}(\omega)}. \quad (2.33)$$

Employing Helmholtz decomposition for the fields $\hat{\mathbf{u}}$ and $\hat{\mathbf{U}}$ leads to

$$\hat{\mathbf{u}} = \nabla \hat{\phi}^s + \nabla \times \hat{\boldsymbol{\Psi}}^s, \quad (2.34)$$

$$\hat{\mathbf{U}} = \nabla \hat{\phi}^f + \nabla \times \hat{\boldsymbol{\Psi}}^f. \quad (2.35)$$

Substituting expressions (2.34)-(2.35) into equations (2.28)-(2.29) yields

$$\begin{aligned} & \nabla \left[((P + Q) \nabla^2 + \omega^2 (1 - \phi) \rho_s) \hat{\phi}^s + ((Q + R) \nabla^2 + \omega^2 \phi \rho_f) \hat{\phi}^f \right] \\ & + \nabla \times \left[(G \nabla^2 + \omega^2 (1 - \phi) \rho_s) \hat{\boldsymbol{\Psi}}^s + \omega^2 \phi \rho_f \hat{\boldsymbol{\Psi}}^f \right] = \mathbf{0}, \end{aligned} \quad (2.36)$$

$$\begin{aligned} & \nabla \left[(Q \nabla^2 + \omega^2 \bar{\rho}_{12}(\omega)) \hat{\phi}^s + (R \nabla^2 + \omega^2 \bar{\rho}_{22}(\omega)) \hat{\phi}^f \right] \\ & + \nabla \times \left[\left(\omega^2 \bar{\rho}_{12}(\omega) + \frac{\rho_{12}(\omega)}{\mu \bar{\epsilon}(\omega)} \nabla^2 \right) \hat{\boldsymbol{\Psi}}^s + \left(\omega^2 \bar{\rho}_{22}(\omega) + \frac{\rho_{22}(\omega)}{\mu \bar{\epsilon}(\omega)} \nabla^2 \right) \hat{\boldsymbol{\Psi}}^f \right] = \mathbf{0}, \end{aligned} \quad (2.37)$$

where $P = A + 2G$. For the longitudinal waves, associated with potentials $\hat{\phi}^s$ and $\hat{\phi}^f$, the first terms in square brackets of equations (2.36)-(2.37) are set equal to zero from which we obtain

$$\begin{pmatrix} P & Q \\ Q & R \end{pmatrix} \nabla^2 \begin{pmatrix} \hat{\phi}^s \\ \hat{\phi}^f \end{pmatrix} = -\omega^2 \begin{pmatrix} \bar{\rho}_{11}(\omega) & \bar{\rho}_{12}(\omega) \\ \bar{\rho}_{12}(\omega) & \bar{\rho}_{22}(\omega) \end{pmatrix} \begin{pmatrix} \hat{\phi}^s \\ \hat{\phi}^f \end{pmatrix}, \quad (2.38)$$

where we used that $(1 - \phi) \rho_s - \bar{\rho}_{12}(\omega) = \bar{\rho}_{11}(\omega)$, and $\phi \rho_f - \bar{\rho}_{12}(\omega) = \bar{\rho}_{22}(\omega)$. Applying a spatial Fourier transformation and recasting equation (2.38) into an eigenvalue problem leads to

$$\frac{1}{PR - Q^2} \begin{pmatrix} \bar{\rho}_{11}(\omega)R - \bar{\rho}_{12}(\omega)Q & \bar{\rho}_{12}(\omega)R - \bar{\rho}_{22}(\omega)Q \\ \bar{\rho}_{12}(\omega)P - \bar{\rho}_{11}(\omega)Q & \bar{\rho}_{22}(\omega)P - \bar{\rho}_{12}(\omega)Q \end{pmatrix} \begin{pmatrix} \tilde{\phi}^s \\ \tilde{\phi}^f \end{pmatrix} = \frac{\mathbf{k} \cdot \mathbf{k}}{\omega^2} \begin{pmatrix} \tilde{\phi}^s \\ \tilde{\phi}^f \end{pmatrix}, \quad (2.39)$$

where $\mathbf{k}$ is the wavenumber vector and tildes over a potential indicate frequency-wavenumber domain quantities. The complex eigenvalues correspond with the squared slowness of the fast (Pf) and slow (Ps) longitudinal waves $s_l^2(\omega)$, $l = Pf$ or Ps

$$s_l^2(\omega) = \frac{-d_1(\omega)}{2d_2} \mp \frac{d_1(\omega)}{2d_2} \sqrt{1 - 4 \frac{d_0(\omega)d_2}{[d_1(\omega)]^2}}, \quad (2.40)$$

where

$$\begin{aligned} d_0(\omega) &= \bar{\rho}_{11}(\omega)\bar{\rho}_{22}(\omega) - [\bar{\rho}_{12}(\omega)]^2, \\ d_1(\omega) &= -[\bar{\rho}_{22}(\omega)P + \bar{\rho}_{11}(\omega)R - 2\bar{\rho}_{12}(\omega)Q], \\ d_2 &= PR - Q^2. \end{aligned} \quad (2.41)$$

For the transversal waves, associated with potentials $\tilde{\Psi}^s$ and $\tilde{\Psi}^f$, the second terms in square brackets of equations (2.36)-(2.37) are set equal to the zero vector which gives

$$\begin{pmatrix} \left[-G \frac{\mathbf{k} \cdot \mathbf{k}}{\omega^2} + (1 - \phi) \rho_s \right] \underline{\mathbf{I}} & \phi \rho_f \underline{\mathbf{I}} \\ \left[-\frac{\rho_{12}(\omega)}{\mu \bar{\epsilon}(\omega)} \frac{\mathbf{k} \cdot \mathbf{k}}{\omega^2} + \bar{\rho}_{12}(\omega) \right] \underline{\mathbf{I}} & \left[-\frac{\rho_{22}(\omega)}{\mu \bar{\epsilon}(\omega)} \frac{\mathbf{k} \cdot \mathbf{k}}{\omega^2} + \bar{\rho}_{22}(\omega) \right] \underline{\mathbf{I}} \end{pmatrix} \begin{pmatrix} \tilde{\Psi}^s \\ \tilde{\Psi}^f \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \quad (2.42)$$

where a spatial Fourier transformation is applied. Nontrivial solutions for $\mathbf{k} \cdot \mathbf{k}/\omega^2$ are obtained by requiring the determinant of the matrix in equation (2.42) to be equal to zero. The solutions correspond with the squared complex slowness of the electromagnetic (*EM*) and seismic shear (*S*) transversal waves. The dispersion relations are given in equation (2.40) for $l = EM$ or *S* where

$$\begin{aligned} d_0(\omega) &= \mu \bar{\epsilon}(\omega) \frac{\bar{\rho}_{11}(\omega)\bar{\rho}_{22}(\omega) - [\bar{\rho}_{12}(\omega)]^2}{G}, \\ d_1(\omega) &= -\mu \bar{\epsilon}(\omega) \bar{\rho}_{22}(\omega) - \frac{\rho_{11}(\omega)\rho_{22}(\omega) - [\rho_{12}(\omega)]^2}{G}, \\ d_2(\omega) &= \rho_{22}(\omega), \end{aligned} \quad (2.43)$$

and where we used that $(1 - \phi)\rho_s\rho_{22}(\omega) - \phi\rho_f\rho_{12}(\omega) = \rho_{11}(\omega)\rho_{22}(\omega) - [\rho_{12}(\omega)]^2$ and $(1 - \phi)\rho_s\bar{\rho}_{22}(\omega) - \phi\rho_f\bar{\rho}_{12}(\omega) = \bar{\rho}_{11}(\omega)\bar{\rho}_{22}(\omega) - [\bar{\rho}_{12}(\omega)]^2$. Note that d_2 in equation (2.40) is now frequency dependent. Dispersion relations given by equations (2.40), (2.41) and (2.43) are equal to the expressions given by Pride & Haartsen (1996).

Wave speed v_l and attenuation a_l of wave l are calculated from equation (2.40) as follows (Pride & Haartsen, 1996):

$$v_l = \frac{1}{\text{Re}[s_l]}, \quad (2.44)$$

$$a_l = -\omega \text{Im}[s_l], \quad (2.45)$$

where we require $\text{Im}[s_l] \leq 0$.

For the parameters given in Table 2.1, wave speed and attenuation of each electrokinetic wave mode are given as a function of frequency in Figures 2.2 and 2.3, respectively.

Figure 2.2: Electrokinetic wave speeds of the fast P-wave (v_{Pf}), the slow P-wave (v_{Ps}), the EM-wave (v_{EM}) and the shear wave (v_S) as a function of frequency using parameters of a typical natural water-saturated shallow sandstone (see Table 2.1). Dashed lines indicate low and high-frequency asymptotes. The transition frequency is given by $\omega_t/2\pi$.

2.3.1 Low and high-frequency behaviour of electrokinetic wave velocities

Investigating the dispersion relations (2.40) for low and high frequencies yields significant insight into the behaviour of the electrokinetic wave modes. In the low-frequency range both the Ps and EM-wave obey a diffusion equation. The Pf and S-wave are propagatory. In the high-frequency range, all electrokinetic waves are propagatory.

Low-frequency behaviour

We seek for a binomial series expansion, i.e., $\sqrt{1 + f(\omega)} = 1 + f(\omega)/2 - \dots$, of the square root in equation (2.40) for longitudinal waves. In the low-frequency limit, electrokinetic densities [equations (2.30)-(2.33)] are dominated by the viscous and electric terms, which can be seen from equations (2.11), (2.17), (2.20), (2.27) and the expressions given in appendix A of this chapter. We have that

$$\lim_{\omega \rightarrow 0} 4 \frac{d_0(\omega)d_2}{[d_1(\omega)]^2} = 0, \quad (2.46)$$

because $\lim_{\omega \rightarrow 0} d_0(\omega) = \lim_{\omega \rightarrow 0} d_1(\omega) = \lim_{\omega \rightarrow 0} O(\omega^{-1})$. Expanding the square root in relations (2.40) up to the second term leads to the following low-frequency

Figure 2.3: Electrokinetic wave attenuations of the fast P-wave (a_{Pf}), the slow P-wave (a_{Ps}), the EM-wave (a_{EM}) and the shear wave (a_S) as a function of frequency using parameters of a typical natural water-saturated shallow sandstone (see Table 2.1). Low-frequency asymptotes are indicated with dashed lines. The transition frequency is given by $\omega_t/2\pi$.

behaviour for the fast longitudinal wave ($\omega \ll \omega_t$):

$$s_{Pf0}^2 = \frac{\rho_b}{P + R + 2Q} \equiv \frac{\rho_b}{H}, \quad (2.47)$$

where $\rho_b = (1 - \phi)\rho_s + \phi\rho_f$. Equation (2.47) gives the same low-frequency behaviour as obtained from Biot's poroelastic theory. Equation (2.47) is known as the Biot-Gassmann result (e.g., Wisse, 1999). The fast longitudinal wave has no intrinsic loss for low frequencies (see also Figure 2.3). For the slow longitudinal wave we obtain ($\omega \ll \omega_t$)

$$s_{Ps0}^2 = -\frac{i}{\omega} \frac{\eta\phi^2}{k_0} \frac{H}{PR - Q^2} \times \left\{ 1 + \frac{\eta L_0^2}{k_0} \left(\frac{\phi\sigma_f}{\alpha_\infty} \left[1 + \frac{2(C_{em} + C_{os0})}{\sigma_f\Lambda} \right] - \frac{\eta L_0^2}{k_0} \right)^{-1} \right\}, \quad (2.48)$$

where $C_{os0} = (\epsilon_0\epsilon_f)^2\zeta^2 P_C/2\eta d$ is the low-frequency limit of $C_{os}(\omega)$, with P_C defined in appendix A of this chapter. The first factor on the RHS is recognized as the squared slowness predicted by poroelastic theory (see e.g., Wisse, 1999). The last term is real-valued and represents electrokinetic effects on the Ps-wave slowness. For the parameters given in Table 2.1, at a frequency of 100 Hz, $\eta L_0^2/k_0$ is on the order of

10^{-8} S/m whereas $(\phi\sigma_f/\alpha_\infty) [1 + 2(C_{em} + C_{os0})/\sigma_f\Lambda]$ is on the order of 10^{-3} S/m. Hence, for low frequencies, electrokinetic effects result in a slightly smaller Ps-wave speed and larger attenuation. Pride & Haartsen (1996) also found that the intrinsic attenuation of the slow longitudinal wave is enhanced due to $L(\omega)$ by at most a few percent for a sandstone in which a 100 Hz Ps-wave propagates.

The low-frequency behaviour of the Ps-wave is described by the diffusion equation

$$D_{Ps}\nabla^2\Upsilon = i\omega\Upsilon, \quad (2.49)$$

for some scalar variable Υ . The diffusivity of the slow wave D_{Ps} (m^2/s) is given by

$$D_{Ps} = \frac{k_0}{\eta\phi^2} \frac{PR - Q^2}{H} \left\{ 1 + \frac{\eta L_0^2}{k_0} \left(\frac{\phi\sigma_f}{\alpha_\infty} \left[1 + \frac{2(C_{em} + C_{os0})}{\sigma_f\Lambda} \right] - \frac{\eta L_0^2}{k_0} \right)^{-1} \right\}^{-1}. \quad (2.50)$$

For transversal waves we follow a similar approach as for the longitudinal waves. We have [equations (2.43)]

$$\lim_{\omega \rightarrow 0} 4 \frac{d_0(\omega)d_2(\omega)}{[d_1(\omega)]^2} = 0, \quad (2.51)$$

because $\lim_{\omega \rightarrow 0} d_0(\omega)/d_1(\omega) = -\rho_b/G$ and $\lim_{\omega \rightarrow 0} d_2(\omega)/d_1(\omega) = 0$. We obtain the following low-frequency behaviour for the EM-wave ($\omega \ll \omega_t$):

$$s_{EM0}^2 = -\frac{i}{\omega} \frac{\phi\mu\sigma_f}{\alpha_\infty} \left[1 + \frac{2(C_{em} + C_{os0})}{\sigma_f\Lambda} \right], \quad (2.52)$$

which is recognized as the EM solution in rigid porous media and no fluid displacement with surface conduction taken into account. Note that this low-frequency expression satisfies the diffusion equation (2.49) where the diffusivity of the EM-wave D_{EM} (m^2/s) is given by

$$D_{EM} = \frac{\alpha_\infty}{\phi\mu\sigma_f} \left[1 + \frac{2(C_{em} + C_{os0})}{\sigma_f\Lambda} \right]^{-1}. \quad (2.53)$$

From equations (2.20) and (2.27) we conclude that the EM diffusive process is determined by the imaginary part of complex electrokinetic permittivity. For the shear wave, the binomial series expansion for $\omega \ll \omega_t$ leads to

$$s_{S0}^2 = \frac{\rho_b}{G}, \quad (2.54)$$

which is the same low-frequency behaviour as obtained from Biot's poroelastic theory (e.g., Wisse, 1999). The shear wave is propagatory for low frequencies.

High-frequency behaviour

In the high-frequency limit the viscous term is real valued, i.e., $\lim_{\omega \rightarrow \infty} \rho_{12}(\omega) = -\phi \rho_f(\alpha_\infty - 1)$, and the electrokinetic coupling is zero, i.e., $\lim_{\omega \rightarrow \infty} E_K(\omega) = 0$. Therefore the dispersion relations become real-valued [see equations (2.40), (2.41), and (2.43)] so that all waves are propagatory. For the transversal wave behaviour in the high-frequency limit, we need to show that

$$\lim_{\omega \rightarrow \infty} \left| 4 \frac{d_0(\omega)d_2(\omega)}{[d_1(\omega)]^2} \right| < 1. \quad (2.55)$$

This is the case if

$$\frac{(1 - \phi)\rho_s + \phi\rho_f(1 - 1/\alpha_\infty)}{G} > \varepsilon\mu. \quad (2.56)$$

The LHS of equation (2.56) corresponds with the high-frequency limit of the squared shear wave slowness. The RHS of equation (2.56) is recognized as the high-frequency limit of an EM-wave travelling through a porous material. Obviously, the condition in equation (2.56) is satisfied for a typical porous material. We now make a binomial expansion, i.e., $\sqrt{1 + f(\omega)} = 1 + f(\omega)/2 - \dots$, of the square root in equation (2.40). We obtain the following high-frequency behaviour for the EM-wave ($\omega \gg \omega_t$):

$$s_{EM\infty}^2 = \left[\frac{G}{(1 - \phi)\rho_s + \phi\rho_f(1 - 1/\alpha_\infty)} + \frac{1}{\varepsilon\mu} \right]^{-1}. \quad (2.57)$$

The first term between brackets is the mechanical effect. The second term represents the EM effect.

We obtain the following high-frequency behaviour for the shear wave ($\omega \gg \omega_t$):

$$s_{S\infty}^2 = \varepsilon\mu + \frac{(1 - \phi)\rho_s + \phi\rho_f(1 - 1/\alpha_\infty)}{G} - \left[\frac{G}{(1 - \phi)\rho_s + \phi\rho_f(1 - 1/\alpha_\infty)} + \frac{1}{\varepsilon\mu} \right]^{-1}. \quad (2.58)$$

For the parameters given in Table 2.1, the high-frequency behaviour of the shear wave slowness is dominated by the second purely mechanical term on the RHS of equation (2.58).

Wave speeds and attenuations are shown in Figures 2.2 and 2.3 respectively, together with their asymptotes. The high-frequency asymptotes of the longitudinal waves are obtained by inserting the real-valued electrokinetic densities into the dispersion relations (2.40) and (2.41).

The Pf and S-wave are slightly dispersive, whereas the Ps and EM-wave show strong frequency dependence due to the transition from a diffusive to a propagatory wave. The frequency-dependence in the dispersion relations is determined by the dynamic coefficients $k(\omega)$, $L(\omega)$ and $\bar{\varepsilon}(\omega)$ [equations (2.11), (2.17) and (2.27)] which enter the effective densities [equations (2.14)-(2.16) and (2.30)-(2.33)]. The

mechanical waves show a transition around $\omega_t/2\pi$, which is the relaxation frequency that separates viscosity from inertia-dominated flow. Dynamic permeability relaxes at this frequency, while the electrokinetic coupling factor relaxes at $\omega_t M/4\pi$ (Pride, 1994). The transition of the EM-wave occurs at a higher frequency due to the transition from diffusive towards propagatory behaviour at 2 MHz, i.e., ε becomes dominant over $\sigma(\omega)/\omega$ in $\bar{\varepsilon}(\omega)$ [equation (2.27)].

2.4 Fluid-to-solid and electric-to-solid ratios

The dispersion relations given in equation (2.40) not only predict the phase velocities and attenuation coefficients, but also the fluid-to-solid amplitude ratios $\beta_m(\omega)$, $\beta_n(\omega)$, $m = Pf$ or Ps , and $n = EM$ or S and the ratios $\alpha_m(\omega)$, $\alpha_n(\omega)$ (V/m^2) that describe the strength of the electric field with respect to the solid displacement amplitude, for each wave. This section shows their behaviour in the seismic frequency range.

2.4.1 Fluid-to-solid ratios

Equation (2.38) (apply a spatial Fourier transformation) and (2.42) (write vector potentials as, e.g., $\Psi_n^s = (0, \psi_n^s, 0)^T$) yield for the longitudinal and transversal amplitude ratios

$$\beta_m(\omega) = \frac{\tilde{\varphi}_m^f}{\tilde{\varphi}_m^s} = \frac{\bar{\rho}_{11}(\omega) - Ps_m^2(\omega)}{Qs_m^2(\omega) - \bar{\rho}_{12}(\omega)}, \quad (2.59)$$

$$\beta_n(\omega) = \frac{\tilde{\psi}_n^f}{\tilde{\psi}_n^s} = \frac{Gs_n^2(\omega) - (1 - \phi)\rho_s}{\phi\rho_f}. \quad (2.60)$$

The longitudinal expressions are written in the same form as those given by Allard (1993). Note that the electrokinetic effects are comprised in the effective electrokinetic density terms.

Low and high-frequency behaviour of fluid-to-solid ratios

From equations (2.30), (2.31), (2.40), (2.41), (2.47) and (2.48) we obtain that for low frequencies $\beta_{Pf0} = 1$ and $\beta_{Ps0} = -(P + Q)/(Q + R)$. These expressions are the same as in Biot's poroelastic theory and indicate that wave propagation is not affected by the electric double layer for low frequencies. A fluid-solid ratio of one means that the fluid displacement is locked-on to the solid's (Wisse, 1999). For high frequencies, all fluid-solid ratios are real-valued.

From equations (2.40), (2.43) and (2.54) we obtain for low frequencies $\beta_{S0} = 1$, while for high frequencies, real-valued transversal ratios are obtained. In Figure 2.4 we plot the real parts of fluid-solid ratios, and low and high-frequency asymptotes, using the parameters given in Table 2.1. The imaginary parts of the ratios are negligible. The mechanical fluid-solid ratios show a transition from low-frequency to high-frequency behaviour.

Figure 2.4: Variation of the fluid-to-solid ratio with frequency for the Pf-wave (β_{Pf}), the Ps-wave (β_{Ps}), the EM-wave (β_{EM}) and the S-wave (β_S). Dashed lines indicate low and high-frequency asymptotes. For β_{EM} only the high-frequency asymptote is shown. The transition frequency is given by $\omega_t/2\pi$.

2.4.2 Electric-to-solid ratios

We apply Helmholtz decomposition to the fields $\hat{\mathbf{u}}$, $\hat{\mathbf{U}}$ [equations (2.34), (2.35)] and $\hat{\mathbf{E}}$: $\hat{\mathbf{E}} = \nabla \hat{\phi}^E + \nabla \times \hat{\Psi}^E$, and substitute these expressions in equation (2.26) which gives

$$\begin{aligned} & \nabla \left[\omega^2 \mu \bar{\varepsilon}(\omega) \hat{\phi}^E + \omega^2 \frac{\eta \phi \mu L(\omega)}{k(\omega)} (\hat{\phi}^f - \hat{\phi}^s) \right] \\ & + \nabla \times \left[(\omega^2 \mu \bar{\varepsilon}(\omega) + \nabla^2) \hat{\Psi}^E + \omega^2 \frac{\eta \phi \mu L(\omega)}{k(\omega)} (\hat{\Psi}^f - \hat{\Psi}^s) \right] = \mathbf{0}. \end{aligned} \quad (2.61)$$

The scalar potentials are associated with longitudinal wave behaviour and the vector potentials with transversal wave behaviour. Applying a spatial Fourier transformation to equation (2.61) yields for the longitudinal and transversal ratios

$$\alpha_m(\omega) = \frac{\tilde{\varphi}_m^E}{\tilde{\varphi}_m^s} = \frac{\eta \phi L(\omega)}{k(\omega) \bar{\varepsilon}(\omega)} [1 - \beta_m(\omega)], \quad (2.62)$$

$$\alpha_n(\omega) = \frac{\tilde{\psi}_n^E}{\tilde{\psi}_n^s} = \frac{\eta \phi \mu L(\omega)}{k(\omega) [\mu \bar{\varepsilon}(\omega) - s_n^2(\omega)]} [1 - \beta_n(\omega)]. \quad (2.63)$$

Figure 2.5: Variation of the electric-to-solid ratio with frequency for the Pf-wave (α_{Pf}), the Ps-wave (α_{Ps}), the EM-wave (α_{EM}) and the S-wave (α_S). The transition frequency is given by $\omega_t/2\pi$.

Low and high-frequency behaviour of electric-to-solid ratios

In the low-frequency range, $\alpha_{Pf}(\omega)$, $\alpha_{Ps}(\omega)$ and $\alpha_S(\omega)$ tend to zero. We plot magnitudes of electric-solid ratios in Figure 2.5, using the parameters given in Table 2.1. The electric-solid ratio magnitudes show an increase with frequency. We note that in the 10^6 – 10^7 Hz frequency range, absolute values of real and imaginary parts of each electric-solid ratio are approximately equal.

2.5 Sensitivity study

When the fast compressional wave travels through a porous medium, an electric coseismic field $\mathbf{E}_{Pf}$ at the scale of the wavelength is created (e.g., Haines *et al.*, 2007). This field was measured both in the laboratory (e.g., Zhu *et al.*, 2000; Bordes *et al.*, 2006; Block & Harris, 2006) and in the field (e.g., Garambois & Dietrich, 2001; Haines *et al.*, 2007). $\mathbf{E}_{Pf}$ is related to the solid displacement due to the Pf-wave $\mathbf{u}_{Pf}$ by $\mathbf{E}_{Pf} = \alpha_{Pf}(\omega)\mathbf{u}_{Pf}$. Electrokinetic conversion in general is sensitive to electrolyte concentration (C), pore fluid viscosity (η) and permeability (k_0), while tortuosity (α_∞), poroelastic parameters (G , K_f , K_b and K_s) and porosity (ϕ) also have an influence (e.g., Haartsen & Pride, 1997; Mikhailov *et al.*, 2000; Garambois & Dietrich, 2001).

We show the influence of C , η , k_0 and α_∞ on $\alpha_{Pf}(\omega)$ at $\omega = 10^6$ rad/s in

Figure 2.6. $\text{Re}(\alpha_{Pf})$ and $\text{Im}(\alpha_{Pf})$ decrease with increasing electrolyte concentration because the absolute value of the zeta-potential decreases [see equations (2.17) and (A-5)], which decreases $L(\omega)$. On the other hand, $L(\omega)$ peaks at around $C = 10^{-6}$ M, because the Debye length d approaches the (weighted) pore volume-to-surface ratio [see equations (2.13), (2.17), and (2.19)]. When d is on the order of the pore volume-to-surface ratio, mobile excess charge is reduced, reducing electrokinetic coupling.

$|\text{Re}(\alpha_{Pf})|$ and $\text{Im}(\alpha_{Pf})$ decrease with increasing pore fluid viscosity. This is intuitively explained because a low viscosity will favor EM-wave generation as fluid circulation is enhanced, as was noted by Garambois & Dietrich (2002). The extremum close to $\eta = 10^{-6}$ is the combined result of the pore fluid viscosity influence on $k(\omega)$, $L(\omega)$ and $\bar{\varepsilon}(\omega)$. The first increases with η , while the latter two decrease with η . We note that surface conductivity due to electroosmotic conductance $C_{os}(\omega)$ becomes significant for η -values smaller than 10^{-4} Pa.s.

$\text{Re}(\alpha_{Pf})$ and $\text{Im}(\alpha_{Pf})$ vary over several orders of magnitude as a function of permeability. This behaviour is attributed to the influence of k_0 on the electrokinetic coupling coefficient $L(\omega)$. From equation (2.13) we see that by increasing permeability we also increase the (weighted) pore volume-to-surface ratio Λ , which has two competing effects on $L(\omega)$. Initially Λ is on the order of d so that $L(\omega)$ increases with increasing Λ . For permeabilities larger than 10^{-14} m² the influence of Λ^2 in the denominator of $L(\omega)$ becomes dominant. Pores become larger while the porosity remains fixed, reducing the amount of double layer excess charge. Tortuosity also influences $L(\omega)$, but its effect is smaller than that of k_0 because of its smaller variability. Increasing tortuosity effectively increases Λ . Here, d/Λ is always negligible so that $|\text{Re}(\alpha_{Pf})|$ and $\text{Im}(\alpha_{Pf})$ decrease with increasing tortuosity.

We show the variation of $\text{Re}(\alpha_{Pf})$ and $\text{Im}(\alpha_{Pf})$ as a function of elastic moduli (K_s , K_b , K_f and G) at $\omega = 10^6$ rad/s in Figure 2.7. The effects are entirely due to changes in β_{Pf} [see equation (2.59)]. Obviously, varying the fluid-to-solid ratio has an influence on the electric-to-solid ratios, as electrokinetic effects occur when excess charge is displaced by the pore fluid along the solid wall. The sign changes shown in Figure 2.7 are due to the term $[1 - \beta_{Pf}(\omega)]$ in equation (2.62), which changes sign when the real part of β_{Pf} is one. We note that $\alpha_{Pf} = 0$, i.e., there are no electrokinetic effects, when $\beta_{Pf} = 1$ (see also, Hu *et al.*, 2002; Hu & Gao, 2009).

The influence of porosity ϕ , pH, solid density ρ_s and pore fluid density ρ_f on $\text{Re}(\alpha_{Pf})$ and $\text{Im}(\alpha_{Pf})$ at $\omega = 10^6$ rad/s is given in Figure 2.8. For ϕ , ρ_s and ρ_f , $\text{Re}(\alpha_{Pf})$ and $\text{Im}(\alpha_{Pf})$ change sign at $\text{Re}(\beta_{Pf}) = 1$. The observed approximate linear trend with increasing porosity is explained by the fact that $L(\omega)$ depends approximately linearly on ϕ [see equation (2.17)], implying that an increase in surface area increases electrokinetic coupling. $|\text{Re}(\alpha_{Pf})|$ and $\text{Im}(\alpha_{Pf})$ increase approximately linearly with pH, which is a result of the linear relation used for the zeta-potential [see equation (A-5)]. Also $L(\omega)$ is directly proportional to ζ [see equation (2.17)]. The variation of α_{Pf} with ρ_s is entirely caused by changes in β_{Pf} . The variation of α_{Pf} with ρ_f is dominated by changes in β_{Pf} , but also influenced through variations in the transition frequency [equation (2.12)] and viscous skin depth $\delta(\omega) = \sqrt{2\eta/\omega\rho_f}$.

Figure 2.6: Real (solid) and imaginary (dashed) parts of the electric-to-solid ratio associated with the Pf-wave α_{Pf} , as a function of electrolyte concentration (C), pore fluid viscosity (η), permeability (k_0) and tortuosity (α_∞) at $\omega = 10^6$ rad/s.

2.6 Electrokinetic theory formulated in terms of excess electrical charge

Revil & Linde (2006) obtained electrokinetic relations where the coupling between mechanical and EM fields arises due to the excess electrical charge in the pore volume Q_V (C/m³). These relations read as (upon neglecting osmotic pressure and gravitational forces)

$$\hat{\mathbf{J}} = \sigma \hat{\mathbf{E}} + \frac{k_0}{\eta} Q_V (-\nabla \hat{p} + \omega^2 \rho_f \hat{\mathbf{u}}), \quad (2.64)$$

$$i\omega\phi (\hat{\mathbf{U}} - \hat{\mathbf{u}}) = \frac{k_0}{\eta} Q_V \hat{\mathbf{E}} + \frac{k_0}{\eta} (-\nabla \hat{p} + \omega^2 \rho_f \hat{\mathbf{u}}), \quad (2.65)$$

where σ is the electrical conductivity of the porous medium. The maximum pore-size considered is on the order of 100 nm, which implies that for the frequencies of interest (below 1 MHz), the pore fluid flow is in the viscous, low-frequency, regime. The coefficients in equations (2.64) and (2.65) are therefore frequency-independent. When the substitution $k_0 Q_V / \eta = L_0$ is applied, equations (2.64)-(2.65) are seen to be similar to equations (2.5) and (2.23).

Figure 2.7: Real (solid) and imaginary (dashed) parts of the electric-to-solid ratio associated with the Pf-wave α_{Pf} , as a function of the bulk moduli of (1) the skeleton grains (K_s), (2) the framework of grains (K_b), and (3) the pore fluid (K_f) and as a function of the shear modulus of the framework of grains (G) at $\omega = 10^6$ rad/s.

Appendix A: Dynamic conductivity terms

The expressions for σ_f , C_{em} and $C_{os}(\omega)$ are (Pride, 1994)

$$\sigma_f = \sum_{l=1}^N (ez_l)^2 b_l N_l, \quad (\text{A-1})$$

$$C_{em} = 2d \sum_{l=1}^N (ez_l)^2 b_l N_l \left[e^{-ez_l \zeta / 2k_B T} - 1 \right], \quad (\text{A-2})$$

$$C_{os}(\omega) = \frac{(\varepsilon_0 \epsilon_f)^2 \zeta^2}{2\eta d} P_C \left[1 + \frac{2d}{P_C \delta(\omega)} (1 + i) \right]^{-1}, \quad (\text{A-3})$$

$$P_C = \frac{8k_B T d^2}{\varepsilon_0 \epsilon_f \zeta^2} \sum_{l=1}^N N_l \left[e^{-ez_l \zeta / 2k_B T} - 1 \right], \quad (\text{A-4})$$

In this chapter we consider a binary symmetric electrolyte ($N = 2$; $z_1 = -z_2 = 1$; $N_1 = N_2$) with mobilities b_1 and b_2 (see Table 2.1). A semi-empirical relationship between the zeta-potential, electrolyte concentration (C), and pH is used (Pride &

Figure 2.8: Real (solid) and imaginary (dashed) parts of the electric-to-solid ratio associated with the Pf-wave α_{Pf} , as a function of porosity (ϕ), pore fluid viscosity (η), permeability (k_0) and tortuosity (α_∞) at $\omega = 10^6$ rad/s.

Garambois, 2005) as follows:

$$\zeta = [0.010 + 0.025 \log_{10}(C)] \frac{\text{pH} - 2}{5}. \quad (\text{A-5})$$

Chapter 3

Seismoelectric reflection and transmission at a fluid/porous-medium interface¹

Abstract

The dispersion relation for seismoelectric wave propagation in poroelastic media is formulated in terms of effective densities comprising all viscous and electrokinetic coupling effects. Using Helmholtz decomposition, two seismoelectric conversion coefficients are derived for an incident P-wave upon an interface between a compressible fluid and a poroelastic medium. These coefficients relate the incident P-wave to a reflected electromagnetic wave in the fluid, and a transmitted electromagnetic wave in the porous medium. The dependency on angle of incidence and frequency is computed. Using orthodox and interference fluxes, it is shown that energy conservation is satisfied. A sensitivity analysis indicates that electrolyte concentration, viscosity, and permeability highly influence seismoelectric conversion.

3.1 Introduction

When the grain surfaces of soils and rocks are in contact with a fluid electrolyte, they typically acquire a chemically bound surface charge that is balanced by mobile counter ions in a thin fluid layer surrounding the grains. The bound charge is immobile, whereas the counter ions can move. The distribution of mobile ions is determined by a balance between electrostatic forces and thermal diffusivity. At the interface between the immobile and counter ions the so-called zeta-potential is defined. The potential varies exponentially when one moves away from the interface. The corresponding characteristic length scale is called the Debye length, which is on the order of some tens of nanometers for typical grain-electrolyte combinations

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(Pride, 1994). The ensemble of bound and mobile charge layers is referred to as the electrochemical double layer.

A compressional wave in fluid-saturated porous media will create pressure peaks and troughs on the scale of the wavelength. The resulting hydraulic flow will transport the counter ions relative to the immobile, bound charge. In this way, counter ions accumulate in pressure troughs and bound charge becomes exposed in pressure peaks, creating an electric coseismic field at the scale of the wavelength (Haines *et al.*, 2007). This electric field drives a conduction current that exactly balances the hydraulic current flow. Thus there is no net electric current for a propagating compressional wave within a homogeneous material. For interfaces, however, this is no longer the case. When a compressional or shear wave traverses an interface with a contrast in electrical or mechanical properties, an electric current imbalance is produced resulting in electromagnetic (EM-)waves that can propagate outside the support of the seismic waves (Haartsen & Pride, 1997). The generation of these waves at interfaces can thus be associated with new, electrokinetic, reflection, and transmission coefficients.

Several studies measured the conversion of seismic to electromagnetic energy in the field (Thompson & Gist, 1993; Butler *et al.*, 1996; Mikhailov *et al.*, 1997; Garambois & Dietrich, 2001; Haines *et al.*, 2007). The inverse electroseismic conversion was also measured (Thompson *et al.*, 2007; Zhu *et al.*, 2008a). So-called electroseismograms in layered media were presented by several authors (e.g., Haartsen & Pride, 1997; Han & Wang, 2001; White & Zhou, 2006; Haines & Pride, 2006). In order to obtain the electrokinetic reflection and transmission coefficients at a fluid/porous-medium interface Block & Harris (2006) approximated Pride's (1994) electrokinetic theory by solving the Biot reflection problem only, from which the relative pore fluid flow is obtained. This flow subsequently acts as a source term in the electromagnetic boundary conditions of Pride's (1994) theory. In this paper, we solve the complete boundary value problem in a procedure similar to Denneman *et al.* (2002), where an acoustic wave from a compressible fluid impinges on a poroelastic medium. Two new parameters are defined, describing the conversion from seismic to EM-waves. The incident fluid wave is transmitted into fast and slow compressional waves, a shear wave, and into an EM-wave. Part of the incident wave is reflected as a fluid wave and a fluid EM-wave. In section 3.2 we introduce the governing equations and we present the electrokinetic dispersion relations. The dependency of the seismo-electric reflection and transmission coefficients on angle of incidence and frequency is derived in section 3.3. In section 3.4 we recast the reflection and transmission coefficients in vertical energy flux coefficients and perform a sensitivity analysis.

3.2 Governing equations

We consider a poroelastic matrix saturated by an electrolyte and adopt an $\exp[i\omega t]$ convention for time varying fields. The governing linearized equations in an isotropic, homogeneous poroelastic medium were given by Pride & Haartsen (1996).

We rewrite the equations here in a somewhat different form because we aim to derive the dispersion relations in terms of effective densities. We will confirm full consistency with the original Pride equations in the forthcoming. The governing equations read as

$$\begin{aligned} -\nabla \cdot \underline{\hat{\boldsymbol{\sigma}}} - (1 - \phi)\nabla \hat{p} \\ = -\omega^2 \left[\rho_{11}(\omega)\hat{\mathbf{u}} + \rho_{12}(\omega)\hat{\mathbf{U}} \right] + \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \end{aligned} \quad (3.1)$$

$$-\phi\nabla \hat{p} = -\omega^2 \left[\rho_{12}(\omega)\hat{\mathbf{u}} + \rho_{22}(\omega)\hat{\mathbf{U}} \right] - \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \quad (3.2)$$

$$\begin{aligned} \underline{\hat{\boldsymbol{\sigma}}} = & -G \left[\nabla \hat{\mathbf{u}} + (\nabla \hat{\mathbf{u}})^T \right] \\ & - \left[\left(A - \frac{1-\phi}{\phi}Q \right) \nabla \cdot \hat{\mathbf{u}} + \left(Q - \frac{1-\phi}{\phi}R \right) \nabla \cdot \hat{\mathbf{U}} \right] \underline{\mathbf{I}}, \end{aligned} \quad (3.3)$$

$$\hat{p} = -\frac{1}{\phi} \left(Q \nabla \cdot \hat{\mathbf{u}} + R \nabla \cdot \hat{\mathbf{U}} \right), \quad (3.4)$$

$$\hat{\mathbf{J}} = \sigma(\omega)\hat{\mathbf{E}} + L(\omega) \left(-\nabla \hat{p} + \omega^2 \rho_f \hat{\mathbf{u}} \right), \quad (3.5)$$

$$\nabla \times \hat{\mathbf{H}} = i\omega\varepsilon\hat{\mathbf{E}} + \hat{\mathbf{J}}, \quad (3.6)$$

$$\nabla \times \hat{\mathbf{E}} = -i\omega\mu\hat{\mathbf{H}}. \quad (3.7)$$

Hats over field variables indicate small frequency-domain quantities, ω is the angular frequency, η is the pore fluid viscosity, ϕ is the porosity, ρ_f is the pore fluid density, μ is the magnetic permeability, ε is the bulk electrical permittivity given by $\varepsilon = \varepsilon_0 [\phi(\epsilon_f - \epsilon_s)/\alpha_\infty + \epsilon_s]$, with solid and pore fluid relative permittivities ϵ_s and ϵ_f , and vacuum permittivity ε_0 . $\underline{\mathbf{I}}$ denotes the identity matrix. The field variables are solid and (pore) fluid displacements $\hat{\mathbf{u}}$ and $\hat{\mathbf{U}}$, intergranular stress $\underline{\hat{\boldsymbol{\sigma}}}$, pore pressure $\hat{p}$, electric current density $\hat{\mathbf{J}}$, and the electric and magnetic fields $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$. A , Q and R are the generalized elastic coefficients that are related to the bulk modulus of the skeleton grains K_s , the shear modulus G , the bulk modulus of the pore fluid K_f and the bulk modulus of the framework of grains K_b as follows (Biot & Willis, 1957):

$$A = \frac{(1-\phi)^2 K_f K_s - (1-\phi) K_f K_b + \phi K_b K_s}{K_f (1-\phi - K_b/K_s) + \phi K_s} - \frac{2}{3}G, \quad (3.8)$$

$$Q = \frac{\phi K_f [(1-\phi) K_s - K_b]}{K_f (1-\phi - K_b/K_s) + \phi K_s}, \quad (3.9)$$

$$R = \frac{\phi^2 K_f K_s}{K_f (1-\phi - K_b/K_s) + \phi K_s}. \quad (3.10)$$

The dynamic permeability $k(\omega)$ describes the transition from viscosity towards inertia-dominated flow. The electrokinetic coupling coefficient $L(\omega)$ couples mechanical and EM-wave behaviour. Note that for $L(\omega) = 0$, mechanical and EM-fields

are decoupled in equations (3.1)-(3.7). Equations (3.1)-(3.4) then transform into the original Biot equations (Biot, 1956a,b) and equations (3.5)-(3.7) into Ohm's law and Maxwell's equations. The conductivity of the pore fluid is denoted as $\sigma(\omega)$. Expressions for $k(\omega)$, $L(\omega)$ and $\sigma(\omega)$ are given in appendix A of this chapter. As the dynamic permeability and the fluid and solid density terms always appear together, it is convenient to define effective densities $\rho_{11}(\omega)$, $\rho_{12}(\omega)$ and $\rho_{22}(\omega)$ as follows:

$$\rho_{11}(\omega) = (1 - \phi)\rho_s - \rho_{12}(\omega), \quad (3.11)$$

$$\rho_{12}(\omega) = \phi\rho_f \left[1 + i\alpha_\infty \frac{\omega_t}{\omega} \frac{k_0}{k(\omega)} \right], \quad (3.12)$$

$$\rho_{22}(\omega) = \phi\rho_f - \rho_{12}(\omega), \quad (3.13)$$

$$\omega_t = \frac{\eta\phi}{\alpha_\infty k_0 \rho_f}, \quad (3.14)$$

where ρ_s is the solid density, k_0 is the permeability, and α_∞ is the tortuosity. Adding equations (3.1) and (3.2), we obtain the original dynamic equation (3) by Pride & Haartsen (1996) (no source term). With the definitions for $\rho_{12}(\omega)$ and $\rho_{22}(\omega)$, equation (2.2) is written as

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta} \left(-\nabla\hat{p} + \omega^2\rho_f\hat{\mathbf{u}} \right), \quad (3.15)$$

which is equation (5) from Pride & Haartsen (1996) (no source term). By adding equations (3.3) and (3.4), we obtain an expression for total stress $\hat{\underline{\underline{\tau}}} = -\hat{\underline{\underline{\sigma}}} - \hat{p}\underline{\underline{\mathbf{I}}}$, which corresponds to equation (6) by Pride & Haartsen (1996). Full consistency with the original Pride equations is thus confirmed.

Substituting equations (3.3) and (3.4) into equations (3.1) and (3.2) yields

$$\begin{aligned} G\nabla^2\hat{\mathbf{u}} + (A + G)\nabla\nabla \cdot \hat{\mathbf{u}} + Q\nabla\nabla \cdot \hat{\mathbf{U}} \\ = -\omega^2 \left[\rho_{11}(\omega)\hat{\mathbf{u}} + \rho_{12}(\omega)\hat{\mathbf{U}} \right] + \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}, \end{aligned} \quad (3.16)$$

$$\begin{aligned} Q\nabla\nabla \cdot \hat{\mathbf{u}} + R\nabla\nabla \cdot \hat{\mathbf{U}} \\ = -\omega^2 \left[\rho_{12}(\omega)\hat{\mathbf{u}} + \rho_{22}(\omega)\hat{\mathbf{U}} \right] - \frac{\eta\phi L(\omega)}{k(\omega)}\hat{\mathbf{E}}. \end{aligned} \quad (3.17)$$

Eliminating $(-\nabla\hat{p} + \omega^2\rho_f\hat{\mathbf{u}})$ from equations (3.5) and (3.15) we obtain

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta} \left(\frac{\hat{\mathbf{J}} - \sigma(\omega)\hat{\mathbf{E}}}{L(\omega)} \right). \quad (3.18)$$

Substituting equation (3.6) into equation (3.18) we obtain

$$i\omega\phi \left(\hat{\mathbf{U}} - \hat{\mathbf{u}} \right) = L(\omega)\hat{\mathbf{E}} + \frac{k(\omega)}{\eta L(\omega)} \left(\nabla \times \hat{\mathbf{H}} - [i\omega\varepsilon + \sigma(\omega)]\hat{\mathbf{E}} \right). \quad (3.19)$$

Finally, substituting equation (3.7) into equation (3.19) we obtain

$$\omega^2 \mu \bar{\varepsilon}(\omega) \hat{\mathbf{E}} + \omega^2 \frac{\eta \phi \mu L(\omega)}{k(\omega)} (\hat{\mathbf{U}} - \hat{\mathbf{u}}) = \nabla \nabla \cdot \hat{\mathbf{E}} - \nabla^2 \hat{\mathbf{E}}, \quad (3.20)$$

where $\bar{\varepsilon}(\omega)$ is the newly defined effective electrical permittivity

$$\bar{\varepsilon}(\omega) = \varepsilon - i \frac{\sigma(\omega)}{\omega} + i \frac{\eta L^2(\omega)}{\omega k(\omega)}. \quad (3.21)$$

Equations (3.16), (3.17), and (3.20) form a closed set of equations for the fields $\hat{\mathbf{u}}$, $\hat{\mathbf{U}}$, and $\hat{\mathbf{E}}$. A summary of electrokinetic symbols is given in Table 3.1.

3.2.1 Electrokinetic wave velocities

Electrokinetic theory in isotropic, homogeneous media predicts the existence of uncoupled longitudinal and transversal modes. There are two longitudinal waves, the fast P-wave and the slow P-wave, and two transversal waves: an electromagnetic wave and a seismic shear wave (Pride & Haartsen, 1996). We derive complex electrokinetic wave velocities in a procedure similar to that of Allard (1993). First, we define the following Helmholtz decompositions:

$$\hat{\mathbf{u}} = \nabla \hat{\varphi}^s + \nabla \times \hat{\boldsymbol{\Psi}}^s, \quad (3.22)$$

$$\hat{\mathbf{U}} = \nabla \hat{\varphi}^f + \nabla \times \hat{\boldsymbol{\Psi}}^f, \quad (3.23)$$

$$\hat{\mathbf{E}} = \nabla \hat{\varphi}^E + \nabla \times \hat{\boldsymbol{\Psi}}^E. \quad (3.24)$$

These are substituted in equations (3.16), (3.17) and (3.20). This yields two separate sets, one for the longitudinal waves and one for the transversal waves. Subsequently, plane wave propagation is introduced in the $(x_1, 0, x_3)$ plane (see Figure 3.1), e.g., $\hat{\varphi}^s = \tilde{\varphi}^s \exp[-i\mathbf{k} \cdot \mathbf{x}]$ and $\hat{\boldsymbol{\Psi}}^s = (0, \tilde{\psi}^s \exp[-i\mathbf{k} \cdot \mathbf{x}], 0)^T$, where $\mathbf{k}$ is the wavenumber vector and $\mathbf{x}$ the position vector in that plane. Through elimination of $\tilde{\varphi}^E$ in the longitudinal wave equations a system of equations for $\tilde{\varphi}^s$ and $\tilde{\varphi}^f$ is obtained. The nontrivial solutions yield the squared complex slowness (i.e., inverse complex-valued wave velocity) of the longitudinal waves $s_l^2(\omega)$

$$s_l^2(\omega) = \frac{-d_1(\omega) \mp \frac{d_1(\omega)}{2d_2}}{2d_2} \sqrt{1 - 4 \frac{d_0(\omega)d_2}{[d_1(\omega)]^2}}, \quad (3.25)$$

where $l = Pf$ or Ps refers to the fast and slow longitudinal waves, respectively, and

$$\begin{aligned} d_0(\omega) &= \bar{\rho}_{11}(\omega)\bar{\rho}_{22}(\omega) - [\bar{\rho}_{12}(\omega)]^2, \\ d_1(\omega) &= -[\bar{\rho}_{22}(\omega)P + \bar{\rho}_{11}(\omega)R - 2\bar{\rho}_{12}(\omega)Q], \\ d_2 &= PR - Q^2, \end{aligned} \quad (3.26)$$

Table 3.1: Parameters of the poroelastic medium and fluid. Parameters of poroelastic medium from Denneman *et al.* (2002).

Parameters poroelastic medium			
K_b	Bulk modulus framework of grains	5.8×10^9	Pa
G	Shear modulus framework of grains	3.4×10^9	Pa
K_s	Bulk modulus skeleton grains	40×10^9	Pa
K_f	Bulk modulus pore fluid	2.22×10^9	Pa
η	Pore fluid viscosity	1×10^{-3}	Pa s
ρ_f	Pore fluid density	1.00×10^3	kg/m ³
ρ_s	Solid density	2.76×10^3	kg/m ³
α_∞	Tortuosity of the porous medium	2.3	-
ϕ	Porosity of the porous medium	0.24	-
k_0	Permeability	0.390×10^{-12}	m ²
T	Temperature	295	K
pH	Hydrogen exponent	7	-
ϵ_f	Relative permittivity of the pore fluid	80	-
ϵ_s	Relative permittivity of the solid	4	-
μ	Magnetic permeability ($=\mu_0$)	$4\pi \times 10^{-7}$	H/m
z_1	Valence of 1-species ion	1	-
z_2	Valence of 2-species ion	-1	-
b_1	Mobility of 1-species ion ^a	3.246×10^{11}	m/(N s)
b_2	Mobility of 2-species ion ^a	4.931×10^{11}	m/(N s)
ζ	Zeta-potential ^b	-65×10^{-3}	V
M	Similarity parameter	1	-
C	Concentration of the electrolyte	1×10^{-3}	M
Parameters fluid			
K_{fl}	Bulk modulus fluid	2.22×10^9	Pa
ρ_{fl}	Fluid density	1.00×10^3	kg/m ³
ϵ_{fl}	Fluid relative permittivity	80	-
μ_{fl}	Fluid magnetic permeability ($=\mu_0$)	$4\pi \times 10^{-7}$	H/m
σ_{fl}	Fluid conductivity	5.0×10^{-3}	S/m

^aKeller (1988).

^bFrom equation (A-5), this chapter.

where $P = A + 2G$. As the effective densities and the electrokinetic coupling factor always appear together, it is convenient to define new effective densities $\bar{\rho}_{11}(\omega)$, $\bar{\rho}_{12}(\omega)$, and $\bar{\rho}_{22}(\omega)$ as follows:

$$\bar{\rho}_{11}(\omega) = \rho_{11}(\omega) - E_K(\omega), \quad (3.27)$$

$$\bar{\rho}_{12}(\omega) = \rho_{12}(\omega) + E_K(\omega), \quad (3.28)$$

$$\bar{\rho}_{22}(\omega) = \rho_{22}(\omega) - E_K(\omega), \quad (3.29)$$

$$E_K(\omega) = \frac{\eta^2 \phi^2 L^2(\omega)}{\omega^2 k^2(\omega) \bar{\epsilon}(\omega)}. \quad (3.30)$$

We note that both viscous and electrokinetic coupling effects are now elegantly comprised in effective frequency-dependent densities.

A similar approach is used for the transversal waves. Through elimination of $\tilde{\Psi}^E$ in the transversal wave equations, a system of equations for $\tilde{\Psi}^s$ and $\tilde{\Psi}^f$ is obtained. The nontrivial solution yields the squared complex slowness of the transversal waves $s_l^2(\omega)$ according to equation (3.25), where now $l = EM$ or S , refers to the EM and seismic shear waves, respectively, and

$$\begin{aligned} d_0(\omega) &= \mu \bar{\epsilon}(\omega) \frac{\bar{\rho}_{11}(\omega) \bar{\rho}_{22}(\omega) - [\bar{\rho}_{12}(\omega)]^2}{G}, \\ d_1(\omega) &= -\mu \bar{\epsilon}(\omega) \bar{\rho}_{22}(\omega) - \frac{\rho_{11}(\omega) \rho_{22}(\omega) - [\rho_{12}(\omega)]^2}{G}, \\ d_2(\omega) &= \rho_{22}(\omega). \end{aligned} \quad (3.31)$$

From the complex-valued slowness, phase velocities and attenuation coefficients are readily obtained.

3.3 Reflection and transmission at a plane fluid/porous-medium interface

A compressible fluid overlays an isotropic, homogeneous porous medium. The horizontal interface is at $x_3 = 0$ (see Figure 3.1). We adopt a right-hand (x_1, x_2, x_3) coordinate system with the x_3 -axis pointing downward. A plane fluid wave (scalar potential $\hat{\varphi}_I^{fl}$) impinges on the interface where it reflects as a P-wave (scalar potential $\hat{\varphi}^{fl}$) and a transverse magnetic (TM) EM-wave (vector potential $\hat{\Psi}^{fl}$). It transmits as a TM-wave (vector potential $\hat{\Psi}_{TM}^s$), a fast longitudinal wave (scalar potential $\hat{\varphi}_{Pf}^s$), a slow longitudinal wave (scalar potential $\hat{\varphi}_{Ps}^s$) and a vertical shear (SV)-wave (vector potential $\hat{\Psi}_{SV}^s$). The reflection and transmission coefficients are obtained from the (open-pore) boundary conditions, which are continuity of fluid volume displacement, fluid pressure, the horizontal magnetic and electric fields, and vanishing vertical and horizontal intergranular stresses (Deresiewicz & Skalak, 1963;

Figure 3.1: Seismoelectric reflection and transmission potentials due to an incident fluid wave.

Pride & Haartsen, 1996)

$$(1 - \phi)\hat{u}_3 + \phi\hat{U}_3 = \hat{U}_3^{fl}, \quad (3.32)$$

$$\hat{p} = \hat{p}^{fl}, \quad (3.33)$$

$$\hat{\sigma}_{13} = \hat{\sigma}_{33} = 0, \quad (3.34)$$

$$\hat{H}_2 = \hat{H}_2^{fl}, \quad (3.35)$$

$$\hat{E}_1 = \hat{E}_1^{fl}. \quad (3.36)$$

The pressure in the upper fluid layer is denoted by $\hat{p}^{fl}$. Magnetic and electric fields in the upper fluid layer are indicated by $\hat{\mathbf{H}}^{fl}$ and $\hat{\mathbf{E}}^{fl}$, respectively. We relate $\hat{p}^{fl}$ to upper fluid layer displacement $\hat{\mathbf{U}}^{fl}$ by $\hat{p}^{fl} = -K_{fl}\nabla \cdot \hat{\mathbf{U}}^{fl}$, where K_{fl} is the bulk modulus of the upper fluid layer. The upper fluid layer wave slowness s_P is related to the upper layer fluid density ρ_{fl} by $s_P = \sqrt{\rho_{fl}/K_{fl}}$. Viscosity does not play a role in the overlying fluid because the ratio of viscous and inertia terms is $\omega\eta s_P^2/\rho_{fl}$, which is on the order $10^{-11} - 10^{-6}$ for the frequencies considered in this paper.

Scalar potentials $\hat{\phi}^s$, $\hat{\phi}^f$, and $\hat{\phi}^E$, see equations (3.22)-(3.24), are composed of contributions of the longitudinal waves, e.g., $\hat{\phi}^s = \hat{\phi}_{Pf}^s + \hat{\phi}_{Ps}^s$, and, similarly, vector potentials $\hat{\Psi}^s$, $\hat{\Psi}^f$, and $\hat{\Psi}^E$ are composed of contributions of the transversal waves, e.g., $\hat{\Psi}^s = \hat{\Psi}_{TM}^s + \hat{\Psi}_{SV}^s = (0, \hat{\psi}_{TM}^s, 0)^T + (0, \hat{\psi}_{SV}^s, 0)^T$. The dispersion relations given in equation (3.25) not only predict the phase velocities and attenuation coefficients, but also the fluid-to-solid amplitude ratios $\beta_m(\omega)$ and $\beta_n(\omega)$ and the amplitude ratios $\alpha_m(\omega)$ and $\alpha_n(\omega)$ (V/m²) that describe the strength of the electric field with respect to the solid displacement amplitude, for each wave

$$\beta_m(\omega) = \frac{\hat{\phi}_m^f}{\hat{\phi}_m^s} = \frac{\bar{\rho}_{11}(\omega) - P s_m^2(\omega)}{Q s_m^2(\omega) - \bar{\rho}_{12}(\omega)}, \quad (3.37)$$

$$\beta_n(\omega) = \frac{\hat{\psi}_n^f}{\hat{\psi}_n^s} = \frac{G s_n^2(\omega) - (1 - \phi)\rho_s}{\phi\rho_f}, \quad (3.38)$$

$$\alpha_m(\omega) = \frac{\hat{\phi}_m^E}{\hat{\phi}_m^s} = \frac{\eta\phi L(\omega)}{k(\omega)\bar{\varepsilon}(\omega)} [1 - \beta_m(\omega)], \quad (3.39)$$

$$\alpha_n(\omega) = \frac{\hat{\psi}_n^E}{\hat{\psi}_n^s} = \frac{\eta\phi\mu L(\omega)}{k(\omega) [\mu\bar{\varepsilon}(\omega) - s_n^2(\omega)]} [1 - \beta_n(\omega)], \quad (3.40)$$

for $m = Pf$ or Ps , and $n = TM$ or SV . We now specify the following plane wave propagation expressions

$$\hat{\phi}_I^{fl} = \tilde{\phi}_I^{fl} e^{-i(k_1 x_1 + k_3^{fl} x_3)}, \quad (3.41)$$

$$\hat{\phi}^{fl} = \tilde{\phi}^{fl} e^{-i(k_1 x_1 - k_3^{fl} x_3)}, \quad (3.42)$$

$$\hat{\Psi}^{fl} = (0, \tilde{\psi}^{fl} e^{-i(k_1 x_1 - k_3^{EM} x_3)}, 0)^T, \quad (3.43)$$

$$\hat{\phi}_m^s = \tilde{\phi}_m^s e^{-i(k_1 x_1 + k_3^m x_3)}, \quad (3.44)$$

$$\hat{\Psi}_n^s = (0, \tilde{\psi}_n^s e^{-i(k_1 x_1 + k_3^n x_3)}, 0)^T, \quad (3.45)$$

where k_3^{fl} and k_3^{EM} are the vertical wavenumbers of the incident fluid wave and reflected EM-wave respectively. The reflection and transmission coefficients are

$$\begin{aligned} r_{EM,P} &= \frac{\tilde{\psi}_I^{fl}}{\tilde{\phi}_I^{fl}}, & r_{P,P} &= \frac{\tilde{\phi}^{fl}}{\tilde{\phi}_I^{fl}}, & t_{Pf,P} &= \frac{\tilde{\phi}_{Pf}^s}{\tilde{\phi}_I^{fl}}, \\ t_{Ps,P} &= \frac{\tilde{\phi}_{Ps}^s}{\tilde{\phi}_I^{fl}}, & t_{TM,P} &= \frac{\tilde{\psi}_{TM}^s}{\tilde{\phi}_I^{fl}}, & t_{SV,P} &= \frac{\tilde{\psi}_{SV}^s}{\tilde{\phi}_I^{fl}}, \end{aligned} \quad (3.46)$$

where $r_{P,P}$ indicates the mechanical reflection coefficient and $r_{EM,P}$ indicates the seismoelectric reflection coefficient. Applying boundary conditions (3.32)-(3.36) yields a linear system of six equations and six unknowns

$$\underline{\underline{A}} \cdot (r_{EM,P}, r_{P,P}, t_{Pf,P}, t_{Ps,P}, t_{TM,P}, t_{SV,P})^T = (k_3^{fl}, \phi_{\rho fl}, 0, 0, 0, 0)^T, \quad (3.47)$$

where elements of matrix $\underline{\underline{A}}$ are given in appendix B of this chapter. We solve equation (3.47) analytically by Cramer's rule. The solution is numerically implemented and shown for $r_{EM,P}$ and $t_{TM,P}$ as a function of incidence angle θ at a fixed frequency of 10 Hz in Figure 3.2. The parameter values, representative for a water/shallow-sandstone interface, are given in Table 3.1. We use these values in all computations of the reflection and transmission coefficients. We choose to plot $\sin \theta$ from zero to two. Discontinuities that fall within this range are associated with critical reflection of poroelastic waves. For $\sin \theta$ between ~ 0 and ~ 0.6 , $|r_{EM,P}|$ is approximately constant. Note that for $\sin \theta = 0$, no reflected EM-wave is generated due to the prescribed boundary conditions. For the parameters of Table 3.1, $|r_{EM,P}|$ is approximately 3 V/m², which is equivalent to $\sim 8 \times 10^{-7}$ V/Pa at 10 Hz. At $\sin \theta \approx 0.57$, $|r_{EM,P}|$ has a local minimum associated with critical reflection of the fast P-wave. Obviously, $\sin \theta = 1$ corresponds to grazing angle of incidence and there is no seismoelectric conversion ($|r_{EM,P}| = 0$). We associate $\sin \theta \approx 1.24$ with critical reflection of the SV-wave and we associate the large (finite) value at

Figure 3.2: Amplitudes (left) and phase values (right) of seismoelectric reflection ($r_{EM,P}$) and transmission ($t_{TM,P}$) coefficients at a fluid/porous-medium boundary.

$\sin \theta \approx 1.44$ with the Stoneley wave. The above mentioned behaviour is also visible in the phase behaviour of $r_{EM,P}$, and the behaviour of $t_{TM,P}$. At $\sin \theta = 0$ and $\sin \theta = 1$, $r_{EM,P}$ and $t_{TM,P}$ are identically zero. We note that generally $|t_{TM,P}|$ is on the order 10^{-10} .

Next we show in Figure 3.3 the frequency-dependence of $|r_{EM,P}|$ and $|t_{TM,P}|$ for 45° angle of incidence. Both seismoelectric coefficients increase strongly with frequency, which corresponds to increasing EM-wave velocities in the diffusive regime for both the fluid and poroelastic medium. We note that the increase in $|t_{TM,P}|$ sharply diminishes at $\omega = \omega_t$, i.e., for the frequency that separates viscosity-dominated flow from inertia-dominated flow.

3.4 Sensitivity analysis

A natural way to perform a sensitivity analysis is to analyze the ratio of reflected/transmitted wave energy fluxes to the incident flux; see e.g., Dutta & Odé (1983), and Santos *et al.* (1992).

3.4.1 Vertical electrokinetic energy fluxes

The electrokinetic vertical energy flux $Y_3(\mathbf{x}, t)$ of the porous medium, defined by the dot product of the Poynting vector $\mathbf{Y}$ and the unit normal to the interface

Figure 3.3: Magnitude of seismoelectric coefficients $r_{EM,P}$ and $t_{TM,P}$ as a function of normalized frequency at $\theta = 45^\circ$ angle of incidence.

$\mathbf{n} = (0, 0, 1)^T$, is given by Haartsen (1995)

$$\begin{aligned}
 Y_3(\mathbf{x}, t) &= \left[\mathbf{E}(\mathbf{x}, t) \times \mathbf{H}(\mathbf{x}, t) + \underline{\boldsymbol{\tau}}(\mathbf{x}, t) \cdot \dot{\mathbf{u}}(\mathbf{x}, t) \right. \\
 &\quad \left. - \phi p(\mathbf{x}, t) \left(\dot{\mathbf{U}}(\mathbf{x}, t) - \dot{\mathbf{u}}(\mathbf{x}, t) \right) \right] \cdot \mathbf{n} \\
 &= E_1(\mathbf{x}, t) H_2(\mathbf{x}, t) + \tau_{31}(\mathbf{x}, t) \dot{u}_1(\mathbf{x}, t) + \tau_{33}(\mathbf{x}, t) \dot{u}_3(\mathbf{x}, t) \\
 &\quad - \phi p(\mathbf{x}, t) \left(\dot{U}_3(\mathbf{x}, t) - \dot{u}_3(\mathbf{x}, t) \right). \tag{3.48}
 \end{aligned}$$

Similar expressions to equation (3.48) were provided by Pride & Haartsen (1996); Dutta & Odé (1983), and Santos *et al.* (1992). We note that in our geometry (see Figure 3.1) $H_1(\mathbf{x}, t) = E_2(\mathbf{x}, t) = \tau_{32}(\mathbf{x}, t) = u_2(\mathbf{x}, t) = 0$. We now write real parts of all frequency-domain variables and calculate a time-average over one cycle in time so that $\langle Y_3(\mathbf{x}, t) \rangle$ becomes

$$\begin{aligned}
 \langle Y_3(\mathbf{x}, t) \rangle &= \frac{1}{2\pi} \int_0^{2\pi} \left(\text{Re} \left[\hat{E}_1 e^{i\omega t} \right] \text{Re} \left[\hat{H}_2 e^{i\omega t} \right] + \text{Re} \left[\hat{\tau}_{31} e^{i\omega t} \right] \text{Re} \left[i\omega \hat{u}_1 e^{i\omega t} \right] \right. \\
 &\quad \left. + \text{Re} \left[\hat{\tau}_{33} e^{i\omega t} \right] \text{Re} \left[i\omega \hat{u}_3 e^{i\omega t} \right] - \text{Re} \left[\hat{p} e^{i\omega t} \right] \text{Re} \left[i\omega \phi (\hat{U}_3 - \hat{u}_3) e^{i\omega t} \right] \right) d(\omega t) \\
 &= \frac{1}{2} [\hat{E}_1' \hat{H}_2' + \hat{E}_1'' \hat{H}_2''] + \frac{\omega}{2} [\hat{\tau}_{31}'' \hat{u}_1' - \hat{\tau}_{31}' \hat{u}_1''] + \frac{\omega}{2} [\hat{\tau}_{33}'' \hat{u}_3' - \hat{\tau}_{33}' \hat{u}_3''] \\
 &\quad - \frac{\omega}{2} [\phi \hat{p}'' (\hat{U}_3' - \hat{u}_3') - \phi \hat{p}' (\hat{U}_3'' - \hat{u}_3'')], \tag{3.49}
 \end{aligned}$$

where single and double primes denote real and imaginary parts of complex quantities, respectively. Denoting complex conjugation by an asterisk, we find that

$$\begin{aligned}
 \langle Y_3(\mathbf{x}, t) \rangle &= \frac{1}{4} \left\{ \left(\hat{E}_1 \hat{H}_2^* + \hat{E}_1^* \hat{H}_2 \right) - i\omega \left[\left(\hat{\tau}_{31} \hat{u}_1^* - \hat{\tau}_{31}^* \hat{u}_1 \right) + \left(\hat{\tau}_{33} \hat{u}_3^* - \hat{\tau}_{33}^* \hat{u}_3 \right) \right. \right. \\
 &\quad \left. \left. - \left(\phi \hat{p} (\hat{U}_3^* - \hat{u}_3^*) - \phi \hat{p}^* (\hat{U}_3 - \hat{u}_3) \right) \right] \right\}. \tag{3.50}
 \end{aligned}$$

Each of the components in equation (3.50) is composed of contributions from the four electrokinetic waves, e.g., $\hat{E}_1 = \hat{E}_1^{Pf} + \hat{E}_1^{Ps} + \hat{E}_1^{TM} + \hat{E}_1^{SV}$. Writing all terms in equation (3.50) in separate contributions from each of the waves, we will have four orthodox fluxes, resulting from wave amplitudes multiplied by themselves, and six interference fluxes, resulting from cross-terms (Dutta & Odé, 1983; Santos *et al.*, 1992). In the non-dissipative fluid layer there is only an orthodox reflected EM-flux $\langle Y_3^{fl}(\mathbf{x}, t) \rangle_{EM,EM}$, and incident and reflected orthodox acoustic fluxes $\langle Y_3^{fl}(\mathbf{x}, t) \rangle_{Pi,Pi}$ and $\langle Y_3^{fl}(\mathbf{x}, t) \rangle_{Pr,Pr}$. Vertical energy flux conservation requires

$$\langle Y_3(\mathbf{x}, t) \rangle - \langle Y_3^{fl}(\mathbf{x}, t) \rangle_{EM,EM} - \langle Y_3^{fl}(\mathbf{x}, t) \rangle_{Pr,Pr} = \langle Y_3^{fl}(\mathbf{x}, t) \rangle_{Pi,Pi}. \quad (3.51)$$

Introducing energy transmission coefficients, e.g., $t^E = \langle Y_3(\mathbf{x}, t) \rangle / \langle Y_3^{fl}(\mathbf{x}, t) \rangle_{Pi,Pi}$, we write

$$t^E - r_{EM,EM}^E - r_{Pr,Pr}^E = 1. \quad (3.52)$$

We note, again, that t^E consists of four orthodox flux transmission coefficients and six interference flux transmission coefficients. Numerical examples are given in Tables 3.2 and 3.3. In Table 3.2, the angle of incidence θ is fixed at 30° and frequencies f of 10 Hz (left column) and 500 kHz (right column) are chosen. Higher frequencies favor seismoelectric conversion. In Table 3.3, the angular frequency is fixed at 10^6 rad/s, and angles of incidence $\theta = 30^\circ$ (left column) and $\theta = 45^\circ$ (right column) are chosen. Equation (3.52) is satisfied indeed. The coefficients $t_{Pf,Ps}^E$ and $t_{Ps,SV}^E$ are negative, because of the out-of-phase behaviour of the slow P-wave. The coefficient $r_{Pr,Pr}^E$ is negative, because the reflected acoustic pressure and fluid displacement have opposite phase. The coefficient $t_{Pf,Pf}^E$ approaches zero for $\theta = 45^\circ$ because this angle is beyond the critical angle of incidence of the fast P-wave.

3.4.2 Parameter variations

Seismoelectric conversions are generated at contrasts in (i) elastic properties (e.g., elastic moduli), (ii) fluid chemistry (pH and electrolyte concentration), and (iii) transport properties such as permeability (Haartsen and Pride, 1997). Electrolyte concentration (C), pore fluid viscosity (η), and permeability (k_0) are dominant seismoelectric conversion parameters while pH, pore fluid bulk modulus (K_f), porosity (ϕ), and tortuosity (α_∞) influence the seismoelectric conversion to a smaller extent (see also, Haartsen & Pride, 1997; Mikhailov *et al.*, 2000; Garambois & Dietrich, 2002; Bordes *et al.*, 2006). We investigate the variation of $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of these parameters in Figures 3.4–3.7 at fixed angular frequency and angle of incidence: $\omega = 10^6$ rad/s and $\theta = 45^\circ$.

In Figure 3.4, $r_{EM,EM}^E$ decreases with increasing electrolyte concentration because the absolute value of the zeta-potential decreases, which reduces the electrokinetic coupling, as can be seen from equation (A-4). The coefficient $t_{TM,TM}^E$ peaks around $C = 10^{-4}$ M. This behaviour is related to the vertical electrokinetic

Table 3.2: Seismoelectric reflection and transmission coefficients for $\theta = 30^\circ$ at $f = 10$ Hz and $f = 500$ kHz.

	$f = 10$ Hz	$f = 500$ kHz
$t_{Pf,Pf}^E$	4.6700×10^{-1}	4.7334×10^{-1}
$t_{Ps,Ps}^E$	1.8083×10^{-3}	1.3026×10^{-1}
$t_{TM,TM}^E$	-2.6500×10^{-10}	-4.9895×10^{-7}
$t_{SV,SV}^E$	1.3084×10^{-1}	7.0295×10^{-2}
$t_{Pf,Ps}^E$	-2.0028×10^{-10}	-8.0868×10^{-7}
$t_{Pf,TM}^E$	-2.7470×10^{-12}	-2.5059×10^{-9}
$t_{Pf,SV}^E$	7.4887×10^{-11}	3.0585×10^{-7}
$t_{Ps,TM}^E$	6.7059×10^{-10}	5.2570×10^{-7}
$t_{Ps,SV}^E$	-1.0420×10^{-8}	-3.7939×10^{-5}
$t_{TM,SV}^E$	4.3970×10^{-14}	1.0814×10^{-7}
$r_{EM,EM}^E$	7.4497×10^{-11}	1.1871×10^{-7}
$r_{Pr,Pr}^E$	-4.0035×10^{-1}	-3.2615×10^{-1}
$t^E - r_{EM,EM}^E - r_{Pr,Pr}^E$	1.0000×10^0	1.0000×10^0

energy flux, which depends on the TM-wave velocity. We found that the corresponding maximum attenuation of this wave is at $C = 10^{-4}$ M.

Both coefficients strongly decrease with increasing fluid viscosity. This is intuitively explained because a low viscosity will favor EM-wave generation as fluid circulation is enhanced, as was noted by Garambois & Dietrich (2002).

In the upper two graphs of Figure 3.5 we see that seismoelectric coefficients vary over several orders of magnitude as a function of the permeability k_0 . This behaviour is attributed to the influence of k_0 on the electrokinetic coupling coefficient $L(\omega)$. From equation (A-3) we see that by increasing permeability we also increase the pore volume-to-surface ratio Λ , which has two competing effects on $L(\omega)$. Initially the length scale of Λ approaches the characteristic length of the electric double layer d so that $L(\omega)$ increases with increasing Λ . For permeabilities larger than 10^{-14} m² the influence of Λ^2 in the denominator of $L(\omega)$ becomes dominant so that the seismoelectric coefficients decrease. The effect of tortuosity on the seismoelectric coefficients is shown in the lower two graphs of Figure 3.5. Tortuosity also influences $L(\omega)$, but its effect is smaller than that of k_0 because of its smaller variability. Again, increasing tortuosity effectively increases Λ . For these parameter value combinations, d/Λ is always negligible so that the seismoelectric coefficients decrease with increasing tortuosity.

The coefficients $r_{EM,EM}^E$ and $t_{TM,TM}^E$ increase approximately linearly with pH

Table 3.3: Seismoelectric reflection and transmission coefficients for $\omega = 10^6$ rad/s at $\theta = 30^\circ$ and $\theta = 45^\circ$.

	$\theta = 30^\circ$	$\theta = 45^\circ$
$t_{Pf,Pf}^E$	4.7389×10^{-1}	1.6454×10^{-7}
$t_{Ps,Ps}^E$	1.2478×10^{-1}	2.7971×10^{-1}
$t_{TM,TM}^E$	-5.0880×10^{-7}	-1.6581×10^{-6}
$t_{SV,SV}^E$	7.2674×10^{-2}	6.6128×10^{-1}
$t_{Pf,Ps}^E$	-2.6036×10^{-6}	3.2114×10^{-5}
$t_{Pf,TM}^E$	-3.3667×10^{-9}	4.4496×10^{-9}
$t_{Pf,SV}^E$	9.8734×10^{-7}	5.2354×10^{-5}
$t_{Ps,TM}^E$	7.1950×10^{-7}	2.4385×10^{-6}
$t_{Ps,SV}^E$	-1.2373×10^{-4}	1.9413×10^{-3}
$t_{TM,SV}^E$	1.3113×10^{-7}	8.4593×10^{-7}
$r_{EM,EM}^E$	1.3503×10^{-7}	5.4312×10^{-7}
$r_{Pr,Pr}^E$	-3.2877×10^{-1}	-5.6982×10^{-2}
$t^E - r_{EM,EM}^E - r_{Pr,Pr}^E$	1.0000×10^0	1.0000×10^0

Figure 3.4: Variation of $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of electrolyte concentration (C) and pore fluid viscosity (η) at $\omega = 10^6$ rad/s and $\theta = 45^\circ$.

Figure 3.5: Variation of $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of permeability (k_0) and tortuosity (α_∞) at $\omega = 10^6$ rad/s and $\theta = 45^\circ$.

(upper two graphs of Figure 3.6), which is a result of the linear relation used for the zeta-potential [see equation (A-5)]. Also $L(\omega)$ is directly proportional to ζ [see equation (A-4)].

In the lower two graphs of Figure 3.6, we notice that as the pore fluid becomes stiffer, seismoelectric coefficients $r_{EM,EM}^E$ and $t_{TM,TM}^E$ increase toward a limiting value for large pore fluid bulk moduli. This increase corresponds with a larger contrast in seismoelectric properties across the interface as the compressional wave velocities increase with increasing incompressibility of the pore fluid.

The coefficients $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of porosity ϕ are shown in Figure 3.7. The increase with porosity of both seismoelectric coefficients is explained by the fact that $L(\omega)$ depends approximately linearly on ϕ [see equation (A-4)], implying that an increase in surface area increases electrokinetic coupling.

This sensitivity analysis indicates that for a water-saturated natural sandstone, electrolyte concentration, pore fluid viscosity, and permeability should be sufficiently low ($C \leq 10^{-3}$ M, $\eta \leq 10^{-3}$ Pa s, and $k_0 \leq 10^{-12}$ m²), and the pH sufficiently high (say pH = 7 or more) in order to have appreciable seismoelectric conversion.

3.5 Conclusions

The dispersion relations for seismoelectric wave propagation in poroelastic media were solved in terms of effective densities. This allows an elegant incorporation of both viscous and electrokinetic coupling in the effective densities. We solved the seis-

Figure 3.6: Variation of $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of pH and bulk modulus of the pore fluid (K_f) at $\omega = 10^6$ rad/s and $\theta = 45^\circ$.

moelectric reflection and transmission coefficients at a fluid/porous-medium boundary. The dependence of the reflected and transmitted EM-waves due to an incident fluid wave on angle of incidence and frequency was shown. Recasting seismoelectric reflection and transmission coefficients to vertical energy fluxes, while accounting for orthodox and interference fluxes, it was asserted that energy conservation is obeyed. For a water/shallow-sandstone interface, seismoelectric conversion at 10 Hz is on the order 10^{-10} and at 500 kHz it is on the order 10^{-7} for energy flux coefficients. A sensitivity analysis was performed. Electrolyte concentration, pore fluid viscosity, and permeability were varied within realistic ranges and it was shown that this influenced seismoelectric conversions over several orders of magnitude. This sensitivity analysis allows the possibility to devise experiments in which the seismoelectric effect can be detected. Moreover, it allows us to appreciate the use of the electrokinetic effect as an exploration technique that is complementary to conventional seismic surveys for exploring hydrocarbon reserves.

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Figure 3.7: Variation of $r_{EM,EM}^E$ and $t_{TM,TM}^E$ as a function of porosity (ϕ) at $\omega = 10^6$ rad/s and $\theta = 45^\circ$.

Appendix A: Dynamic coefficients

The dynamic permeability $k(\omega)$ is given by (Johnson *et al.*, 1987)

$$k(\omega) = k_0 \left[\sqrt{1 + i \frac{\omega}{\omega_t} \frac{M}{2}} + i \frac{\omega}{\omega_t} \right]^{-1}, \quad (\text{A-1})$$

where k_0 is permeability. The transition frequency ω_t and the similarity parameter M are given by

$$\omega_t = \frac{\eta \phi}{\alpha_\infty k_0 \rho_f}, \quad (\text{A-2})$$

$$M = \frac{8 \alpha_\infty k_0}{\phi \Lambda^2}, \quad (\text{A-3})$$

where α_∞ is the tortuosity and Λ is a weighted pore volume-to-surface ratio. The electrokinetic coupling coefficient $L(\omega)$ is obtained from Pride's (1994) theory

$$L(\omega) = -\frac{\phi}{\alpha_\infty} \frac{\varepsilon_0 \epsilon_f \zeta}{\eta} \left(1 - 2 \frac{d}{\Lambda} \right) \left\{ 1 + 2i \frac{\omega}{\omega_t M} \left[1 - 2 \frac{d}{\Lambda} \right]^2 \left[1 + \frac{d}{\delta(\omega)} (1 + i) \right]^2 \right\}^{-1/2}, \quad (\text{A-4})$$

where d is the Debye length (defined below), ζ is the zeta-potential, and $\delta(\omega) = \sqrt{2\eta/\omega\rho_f}$ is the viscous skin depth. A semi-empirical relationship between the zeta-potential, electrolyte concentration (C), and pH is used (Pride & Garambois, 2005) as follows:

$$\zeta = [0.010 + 0.025 \log_{10}(C)] \frac{\text{pH} - 2}{5}. \quad (\text{A-5})$$

The dynamic conductivity $\sigma(\omega)$ is given by

$$\sigma(\omega) = \frac{\phi \sigma_f}{\alpha_\infty} \left[1 + \frac{2 [C_{em} + C_{os}(\omega)]}{\sigma_f \Lambda} \right], \quad (\text{A-6})$$

where σ_f is conductivity of the pore fluid, C_{em} is excess conductance associated with electromigration of excess charge, and $C_{os}(\omega)$ is excess conductance associated with electroosmosis. Expressions for σ_f , C_{em} and $C_{os}(\omega)$ are

$$\sigma_f = \sum_{l=1}^N (ez_l)^2 b_l N_l, \quad (\text{A-7})$$

$$C_{em} = 2d \sum_{l=1}^N (ez_l)^2 b_l N_l \left[e^{-ez_l \zeta / 2k_B T} - 1 \right], \quad (\text{A-8})$$

$$C_{os}(\omega) = \frac{(\varepsilon_0 \epsilon_f)^2 \zeta^2}{2\eta d} P_C \left[1 + \frac{2d}{P_C \delta(\omega)} (1 + i) \right]^{-1}, \quad (\text{A-9})$$

$$P_C = \frac{8k_B T d^2}{\varepsilon_0 \epsilon_f \zeta^2} \sum_{l=1}^N N_l \left[e^{-ez_l \zeta / 2k_B T} - 1 \right], \quad (\text{A-10})$$

$$d = \left[\sum_{l=1}^N \frac{(ez_l)^2 N_l}{\varepsilon_0 \epsilon_f k_B T} \right]^{-1/2}, \quad (\text{A-11})$$

where e is the electron charge, k_B is Boltzmann's constant, T is the temperature, and N_l is l -species ion amount per volume. In this article we consider a binary symmetric electrolyte ($N = 2$; $z_1 = -z_2 = 1$; $N_1 = N_2$) with mobilities b_1 and b_2 (see Table 3.1).

Appendix B: Elements of $\underline{\underline{\mathbf{A}}}$

Applying the boundary conditions for the seismoelectric reflection and transmission problem yields a linear system

$$\underline{\underline{\mathbf{A}}} \cdot (r_{EM,P}, r_{P,P}, t_{Pf,P}, t_{Ps,P}, t_{TM,P}, t_{SV,P})^T = (k_3^{fl}, \phi \rho_{fl}, 0, 0, 0, 0)^T, \quad (\text{B-1})$$

where elements of matrix $\underline{\underline{\mathbf{A}}}$ are

$$a_{11} = 0, \quad (\text{B-2})$$

$$a_{12} = k_3^{fl}, \quad (\text{B-3})$$

$$a_{13} = k_3^{Pf} [1 - \phi + \phi \beta_{Pf}(\omega)], \quad (\text{B-4})$$

$$a_{14} = k_3^{Ps} [1 - \phi + \phi \beta_{Ps}(\omega)], \quad (\text{B-5})$$

$$a_{15} = k_1 [1 - \phi + \phi \beta_{TM}(\omega)], \quad (\text{B-6})$$

$$a_{16} = k_1 [1 - \phi + \phi \beta_{SV}(\omega)], \quad (\text{B-7})$$

$$a_{21} = 0, \quad (\text{B-8})$$

$$a_{22} = -\phi \rho_{fl}, \quad (\text{B-9})$$

$$a_{23} = [Q + R\beta_{Pf}(\omega)] s_{Pf}^2(\omega), \quad (\text{B-10})$$

$$a_{24} = [Q + R\beta_{Ps}(\omega)] s_{Ps}^2(\omega), \quad (\text{B-11})$$

$$a_{25} = 0, \quad (\text{B-12})$$

$$a_{26} = 0, \quad (\text{B-13})$$

$$a_{31} = 0, \quad (\text{B-14})$$

$$a_{32} = 0, \quad (\text{B-15})$$

$$a_{33} = k_1 k_3^{Pf}, \quad (\text{B-16})$$

$$a_{34} = k_1 k_3^{Ps}, \quad (\text{B-17})$$

$$a_{35} = k_1^2 - \omega^2 s_{TM}^2(\omega)/2, \quad (\text{B-18})$$

$$a_{36} = k_1^2 - \omega^2 s_{SV}^2(\omega)/2, \quad (\text{B-19})$$

$$a_{41} = 0, \quad (\text{B-20})$$

$$a_{42} = 0, \quad (\text{B-21})$$

$$a_{43} = k_1^2 - \omega^2 \frac{N_{Pf}(\omega)}{2G} s_{Pf}^2(\omega), \quad (\text{B-22})$$

$$a_{44} = k_1^2 - \omega^2 \frac{N_{Ps}(\omega)}{2G} s_{Ps}^2(\omega), \quad (\text{B-23})$$

$$a_{45} = -k_1 k_3^{TM}, \quad (\text{B-24})$$

$$a_{46} = -k_1 k_3^{SV}, \quad (\text{B-25})$$

$$a_{51} = - \left[s_{EM}^{fl}(\omega) \right]^2 / \mu_{fl}, \quad (\text{B-26})$$

$$a_{52} = 0, \quad (\text{B-27})$$

$$a_{53} = 0, \quad (\text{B-28})$$

$$a_{54} = 0, \quad (\text{B-29})$$

$$a_{55} = \alpha_{TM}(\omega) s_{TM}^2(\omega) / \mu, \quad (\text{B-30})$$

$$a_{56} = \alpha_{SV}(\omega) s_{SV}^2(\omega) / \mu, \quad (\text{B-31})$$

$$a_{61} = -k_3^{EM}, \quad (\text{B-32})$$

$$a_{62} = 0, \quad (\text{B-33})$$

$$a_{63} = k_1 \alpha_{Pf}(\omega), \quad (\text{B-34})$$

$$a_{64} = k_1 \alpha_{Ps}(\omega), \quad (\text{B-35})$$

$$a_{65} = -k_3^{TM} \alpha_{TM}(\omega), \quad (\text{B-36})$$

$$a_{66} = -k_3^{SV} \alpha_{SV}(\omega), \quad (\text{B-37})$$

where $N_{Pf}(\omega) = P - (1 - \phi) Q / \phi + [Q - (1 - \phi) R / \phi] \beta_{Pf}(\omega)$, $N_{Ps}(\omega) = P - (1 - \phi) Q / \phi + [Q - (1 - \phi) R / \phi] \beta_{Ps}(\omega)$, and $s_{EM}^{fl}(\omega)$ is the slowness of the electromagnetic wave in the fluid, which can be calculated using $\left[s_{EM}^{fl}(\omega) \right]^2 = (\omega^2 \epsilon_0 \epsilon_{fl} \mu_{fl} - i \omega \mu_{fl} \sigma_{fl}) / \omega^2$ (Keller, 1988) with upper fluid layer relative permittivity, permeability,

and conductivity denoted by ϵ_{fl} , μ_{fl} , and σ_{fl} respectively.

Chapter 4

Seismoelectric and electroseismic modelling at a fluid/porous-medium interface

4.1 Introduction

When a wave encounters an interface it scatters to reflected and transmitted waves. The extent to which a wave is reflected or transmitted is described by reflection and transmission coefficients, respectively. Here we derive all electrokinetic reflection and transmission coefficients applicable to a fluid/porous-medium boundary. The results are summarized in the so-called scattering matrix. One column of this matrix coincides with the solutions obtained in the previous chapter, where only one incident wave is considered. The solutions given by the scattering matrix are part of the full-waveform seismoelectric and electroseismic models described in sections 4.3 and 4.4, respectively. Full-waveform models are employed to compare electrokinetic theory against experimental results in chapters 5 and 6.

4.2 Electrokinetic scattering matrix at a fluid/porous-medium interface

A compressible fluid is overlying an isotropic, homogeneous poroelastic medium. The horizontal interface is at $x_3 = 0$ (see Figure 4.1). We adopt a right-hand (x_1, x_2, x_3) coordinate system with the x_3 -axis pointing downward. In the fluid, a compressional wave (scalar potentials $\tilde{\varphi}_I^{fl}$ and $\tilde{\varphi}_O^{fl}$) and a transverse magnetic (TM) electromagnetic (EM-)wave propagate (potentials $\tilde{\psi}_I^{fl}$ and $\tilde{\psi}_O^{fl}$). In the poroelastic medium a TM-wave (potentials $\tilde{\psi}_{TM,I}^s$ and $\tilde{\psi}_{TM,O}^s$), a fast longitudinal wave (potentials $\tilde{\varphi}_{Pf,I}^s$ and $\tilde{\varphi}_{Pf,O}^s$), a slow longitudinal wave (potentials $\tilde{\varphi}_{Ps,I}^s$ and $\tilde{\varphi}_{Ps,O}^s$) and a vertical shear (SV)-wave (potentials $\tilde{\psi}_{SV,I}^s$ and $\tilde{\psi}_{SV,O}^s$) propagate. Note that solid displacement potentials (as in chapter 3) and the so-called SV-TM polarization (e.g., Pride & Haartsen, 1996) are considered. Each of the incident potentials (denoted with I)

Figure 4.1: Seismoelectric and electroseismic reflection and transmission potentials at a fluid/porous-medium interface.

depicted in Figure 4.1 scatters to all six outgoing potentials (denoted with O). The scattering matrix is derived by rewriting the governing equations for the fluid and porous medium in terms of a first order matrix-vector differential equation in displacement, stress and electromagnetic field (see e.g., Aki & Richards, 2002). The matrices that appear are subsequently manipulated into the scattering matrix. In appendix A of this chapter it is shown that the governing equations of the fluid layer can be written as

$$\frac{d\tilde{\mathbf{f}}^{fl}(x_3)}{dx_3} = \mathbf{A}^{fl}\tilde{\mathbf{f}}^{fl}(x_3), \quad (4.1)$$

where

$$\tilde{\mathbf{f}}^{fl}(x_3) = \left(\tilde{U}_3^{fl}, \tilde{p}^{fl}, \tilde{H}_2^{fl}, \tilde{E}_1^{fl} \right)^T, \quad (4.2)$$

and the matrix $\mathbf{A}^{fl}$ contains properties of the fluid independent of x_3 . All matrices appearing in this section are specified in appendix A of this chapter. Checks denote transformation of field variables to the (k_1, x_3, ω) -domain. Similar to the fluid case, we can rewrite the governing equations of the porous medium (see appendix A of this chapter) as

$$\frac{d\tilde{\mathbf{f}}^p(x_3)}{dx_3} = \mathbf{A}^p\tilde{\mathbf{f}}^p(x_3), \quad (4.3)$$

with

$$\tilde{\mathbf{f}}^p(x_3) = \left(\tilde{u}_1, \tilde{u}_3, \phi(\tilde{U}_3 - \tilde{u}_3), \tilde{p}, \tilde{\sigma}_{13}, \tilde{\sigma}_{33}, \tilde{H}_2, \tilde{E}_1 \right)^T. \quad (4.4)$$

Solutions to equations (4.1) and (4.3) are

$$\tilde{\mathbf{f}}^{fl}(x_3) = \mathbf{L}^{fl} \text{diag} \left(e^{-ik_3^{fl}x_3}, e^{-ik_3^{EM}x_3}, e^{ik_3^{fl}x_3}, e^{ik_3^{EM}x_3} \right) \tilde{\mathbf{w}}^{fl}, \quad (4.5)$$

and

$$\check{\mathbf{f}}^p(x_3) = \mathbf{L}^p \text{diag} \left(e^{-ik_3^{Pf} x_3}, e^{-ik_3^{Ps} x_3}, e^{-ik_3^{TM} x_3}, e^{-ik_3^{SV} x_3}, \dots \right. \\ \left. e^{ik_3^{Pf} x_3}, e^{ik_3^{Ps} x_3}, e^{ik_3^{TM} x_3}, e^{ik_3^{SV} x_3} \right) \tilde{\mathbf{w}}^p, \quad (4.6)$$

where the columns of matrices $\mathbf{L}^{fl}$ and $\mathbf{L}^p$ are the eigenvectors of $\mathbf{A}^{fl}$ and $\mathbf{A}^p$, respectively. The plane-wave amplitudes (in terms of potentials) $\tilde{\mathbf{w}}^{fl}$ and $\tilde{\mathbf{w}}^p$ are given by

$$\tilde{\mathbf{w}}^{fl} = (\tilde{\varphi}_I^{fl}, \tilde{\psi}_I^{fl}, \tilde{\varphi}_O^{fl}, \tilde{\psi}_O^{fl})^T, \quad (4.7)$$

$$\tilde{\mathbf{w}}^p = (\tilde{\varphi}_{Pf,O}^s, \tilde{\varphi}_{Ps,O}^s, \tilde{\psi}_{TM,O}^s, \tilde{\psi}_{SV,O}^s, \tilde{\varphi}_{Pf,I}^s, \tilde{\varphi}_{Ps,I}^s, \tilde{\psi}_{TM,I}^s, \tilde{\psi}_{SV,I}^s)^T. \quad (4.8)$$

At the interface, the (open-pore) boundary conditions are continuity of fluid volume displacement, fluid pressure, the horizontal magnetic and electric fields, and vanishing vertical and horizontal intergranular stresses (Deresiewicz & Skalak, 1963; Pride & Haartsen, 1996). We define the boundary condition vectors of the fluid $\hat{\mathbf{v}}^{fl}$ and of the porous medium $\hat{\mathbf{v}}^p$ as follows:

$$\hat{\mathbf{v}}^{fl} = \left(\hat{U}_3^{fl}, \hat{p}^{fl}, 0, 0, \hat{H}_2^{fl}, \hat{E}_1^{fl} \right)^T, \quad (4.9)$$

$$\hat{\mathbf{v}}^p = \left(\left[(1 - \phi) \hat{u}_3 + \phi \hat{U}_3 \right], \hat{p}, \hat{\sigma}_{13}, \hat{\sigma}_{33}, \hat{H}_2, \hat{E}_1 \right)^T. \quad (4.10)$$

In order to solve the scattering boundary value problem, the eigenvector-columns of $\mathbf{L}^{fl}$ and $\mathbf{L}^p$ must be expressed in terms of the fields contained in $\hat{\mathbf{v}}^{fl}$ and $\hat{\mathbf{v}}^p$. This yields matrices $\mathbf{M}^{fl}$ and $\mathbf{M}^p$. At $x_3 = 0$ we obtain

$$\mathbf{M}^{fl} \tilde{\mathbf{w}}^{fl} = \mathbf{M}^p \tilde{\mathbf{w}}^p. \quad (4.11)$$

Equations (4.11) are reordered such that incident plane wave amplitudes $\tilde{\mathbf{w}}_I$ are related to outgoing plane wave amplitudes $\tilde{\mathbf{w}}_O$.

$$\mathbf{S}_I \tilde{\mathbf{w}}_I = \mathbf{S}_O \tilde{\mathbf{w}}_O, \quad (4.12)$$

where $\tilde{\mathbf{w}}_I$ and $\tilde{\mathbf{w}}_O$ are given by

$$\tilde{\mathbf{w}}_I = (\tilde{\varphi}_I^{fl}, \tilde{\psi}_I^{fl}, \tilde{\varphi}_{Pf,I}^s, \tilde{\varphi}_{Ps,I}^s, \tilde{\psi}_{TM,I}^s, \tilde{\psi}_{SV,I}^s)^T, \quad (4.13)$$

$$\tilde{\mathbf{w}}_O = (\tilde{\varphi}_O^{fl}, \tilde{\psi}_O^{fl}, \tilde{\varphi}_{Pf,O}^s, \tilde{\varphi}_{Ps,O}^s, \tilde{\psi}_{TM,O}^s, \tilde{\psi}_{SV,O}^s)^T. \quad (4.14)$$

The scattering matrix $\mathbf{S}$ relates the incident waves to the outgoing waves and is given by

$$\mathbf{S} = \mathbf{S}_O^{-1} \mathbf{S}_I. \quad (4.15)$$

Schematically $\mathbf{S}$ is written as

$$\mathbf{S} = \begin{pmatrix} r_{P,P} & r_{P,EM} & t_{P,Pf} & t_{P,Ps} & t_{P,TM} & t_{P,SV} \\ r_{EM,P} & r_{EM,EM} & t_{EM,Pf} & t_{EM,Ps} & t_{EM,TM} & t_{EM,SV} \\ t_{Pf,P} & t_{Pf,EM} & r_{Pf,Pf} & r_{Pf,Ps} & r_{Pf,TM} & r_{Pf,SV} \\ t_{Ps,P} & t_{Ps,EM} & r_{Ps,Pf} & r_{Ps,Ps} & r_{Ps,TM} & r_{Ps,SV} \\ t_{TM,P} & t_{TM,EM} & r_{TM,Pf} & r_{TM,Ps} & r_{TM,TM} & r_{TM,SV} \\ t_{SV,P} & t_{SV,EM} & r_{SV,Pf} & r_{SV,Ps} & r_{SV,TM} & r_{SV,SV} \end{pmatrix}. \quad (4.16)$$

The seismoelectric coefficient $r_{EM,P}$ describes the reflection of an EM-wave due to an incident P-wave from the fluid. The electroseismic coefficient $t_{P,TM}$ describes the transmission of a P-wave in the fluid due to an incident TM-wave from the poroelastic medium. The first column of $\mathbf{S}$ contains the same solutions as derived in chapter 3.

Using the Schur complement technique, we obtained a full analytical solution for $\mathbf{S}_O^{-1}$. Hence an analytic expression for $\mathbf{S}$ is obtained. We show electroseismic coefficients $r_{P,EM}$ and $t_{P,TM}$ as a function of the angle θ and at a fixed frequency of 10 Hz in Figure 4.2. The parameter values are given in Table 3.1. The angle θ corresponds with the fluid P-wave angle of reflection/transmission. In chapter 3, where seismoelectric conversion is considered (instead of electroseismic conversion), the angle θ corresponds with the angle of the incident fluid P-wave. For $\sin \theta$ between ~ 0 and ~ 0.6 , $|r_{P,EM}|$ increases and has a typical value of $1 \times 10^{-11} \text{ m}^2/\text{V}$, which is equivalent to $\sim 4 \times 10^{-5} \text{ Pa/V}$ at 10 Hz. Obviously, $\sin \theta = 1$ corresponds to critical reflection/transmission of the P-wave. The peak at $\sin \theta \approx 1.44$ is associated with the Stoneley wave. At $\sin \theta \approx 0.57$ and at $\sin \theta \approx 1.24$ minima are observed. These are associated with critical reflection of the Pf and SV-wave, respectively, for an incident P-wave from the fluid. The above mentioned behaviour is also visible in the phase behaviour of $r_{P,EM}$, and the behaviour of $t_{P,TM}$. At $\sin \theta = 0$, $r_{P,EM}$ and $t_{P,TM}$ are identically zero, i.e., no reflected/transmitted P-wave is generated for normally incident plane EM-waves. We note that between $\sin \theta = 0$ and $\sin \theta = 0.6$, $|t_{P,TM}|$ is typically 1.

Next we show in Figure 4.3 the frequency-dependence of $|r_{P,EM}|$ and $|t_{P,TM}|$ for a 45° angle of reflection/transmission. The decrease of $|r_{P,EM}|$ corresponds to the decreasing fluid EM-wave slowness (inverse velocity) in the diffusive regime. For $|t_{P,TM}|$ the effect of the decreasing EM-wave slowness of the porous medium is compensated for by an increase in $\alpha_{TM}(\omega)$.

4.3 Full-waveform seismoelectric modelling

A compressible fluid is overlying an isotropic, homogeneous poroelastic medium (see Figure 4.4). The horizontal interface is at $x_3 = 0$. An acoustic monopole source (e.g., Hall, 1987) at $(r, x_3) = (r^S, x_3^S)$ emits P-waves that travel towards the interface where they are converted to reflected EM signal. The electric potential is recorded by the electric receiver at $(r, x_3) = (r^R, x_3^R)$. The acoustic source pressure $\hat{p}^S(\omega, R^S)$

Figure 4.2: Amplitudes (left) and phase values (right) of electroseismic reflection ($r_{P,EM}$) and transmission ($t_{P,TM}$) coefficients at a fluid/porous-medium interface.

Figure 4.3: Magnitude of electroseismic coefficients $r_{P,EM}$ and $t_{P,TM}$ as a function of normalized frequency at $\theta = 45^\circ$ angle of reflection/transmission. The transition frequency $\omega_t/2\pi$ separates viscosity-dominated flow from inertia-dominated flow.

Figure 4.4: Geometry of seismoelectric model.

is modelled by

$$\hat{p}^S(\omega, R^S) = \frac{W(\omega)}{R^S} e^{-ik^{fl} R^S}, \quad (4.17)$$

where ω is the radial frequency, $W(\omega)$ the amplitude spectrum and the distance to the source R^S is given by

$$R^S = \sqrt{(r - r^S)^2 + (x_3 - x_3^S)^2}. \quad (4.18)$$

The source pressure wavefield is expanded into conical waves by means of the Sommerfeld integral (see e.g., Brekhovskikh, 1960; Aki & Richards, 2002). The total reflected electric potential $\hat{\psi}^{fl}(\omega, r^R, x_3^R)$ can then be expressed as follows ($r^S = 0$; $x_3^S, x_3^R < 0$):

$$\hat{\psi}^{fl}(\omega, r^R, x_3^R) = -iW(\omega) \int_0^\infty \frac{k_r}{k_3^{fl}} J_0(k_r r^R) e^{ik_3^{fl} x_3^S} r_{EM,P}(k_r) e^{ik_3^{EM} x_3^R} dk_r, \quad (4.19)$$

where $k_r = k^{fl} \sin \theta$ and $k_3^{fl} = k^{fl} \cos \theta$ are the radial and vertical components of k^{fl} , respectively, and J_0 is the Bessel function of the first kind and zeroth order. Expressing k_r and k_3^{fl} in terms of k^{fl} and θ , equation (4.19) is written as

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -iW(\omega) \int_0^{\pi/2+i\infty} k^{fl} \sin \theta J_0(k^{fl} r^R \sin \theta) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta, \end{aligned} \quad (4.20)$$

where $k_3^{EM}(\theta) = \omega \sqrt{[s_{EM}^{fl}(\omega)]^2 - \sin^2 \theta s_P^2}$ and $\text{Im}[k_3^{EM}(\theta)] < 0$, with $s_{EM}^{fl}(\omega)$ and s_P the fluid EM and P-wave slowness, respectively. The path of integration is along straight lines from 0 to $\pi/2$ and from $\pi/2$ to $\pi/2 + i\infty$ in the complex θ -plane. The

angle θ is indicated in Figure 4.4. The second integral over complex θ is simplified further employing the substitution $\theta = \pi/2 + i \ln \left[\sqrt{\gamma^2 + 1} + \gamma \right]$

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -iW(\omega) \int_0^{\pi/2} k^{fl} \sin \theta J_0 \left(k^{fl} r^R \sin \theta \right) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta \\ & + W(\omega) \int_0^\infty k^{fl} J_0 \left(k^{fl} r^R \sqrt{\gamma^2 + 1} \right) \\ & \times e^{k^{fl} x_3^S \gamma} r_{EM,P}(\gamma) e^{ik_3^{EM}(\gamma) x_3^R} d\gamma. \end{aligned} \quad (4.21)$$

Equation (4.21) can readily be evaluated numerically and yields the total reflected EM-wavefield.

We now consider an acoustic source and an electrical receiver located at $(r^S, x_3^S) = (r^R, x_3^R) = (0, -5)$ cm. The source emits an acoustic Ricker wavelet $W(t)$ defined as (e.g., Ricker, 1977; van Dalen *et al.*, 2010)

$$W(t) = \left[\frac{\omega_0^2}{2} \tau^2 - 1 \right] e^{-\omega_0^2 \tau^2 / 4}, \quad (4.22)$$

where $\omega_0 = 2\pi f_0$, with center frequency $f_0 = 500$ kHz, and where $\tau = t - t_s$, with time shift $t_s = 2 \mu\text{s}$. $W(t)$ and its spectrum are shown in Figure 4.5. Equation (4.21) is evaluated for each frequency by a recursive adaptive Simpson quadrature algorithm implemented in Matlab. The parameters of Table 3.1 are used. Figure 4.6 (left) shows the resulting modelled reflected electric potential. An arrival is recorded around $34 \mu\text{s}$ which corresponds with the acoustic travel time from the source to the fluid/porous-medium interface.

Figure 4.6 (right) shows the modelled pulse where now $(r^S, x_3^S) = (0, -0.3)$ cm and $(r^R, x_3^R) = (5, -0.3)$ cm. Both source and receiver are close to the interface in order to study surface wave phenomena. Indeed, additional wave modes are computed. The different travel times of the pulses allow the arrivals to be characterized. The first pulse originates from acoustic waves that are converted to EM-waves at near-vertical incidence. The arrival around $20 \mu\text{s}$ corresponds to the Pf-head-wave, while the pulse arriving around $34 \mu\text{s}$ originates from acoustic waves that are converted to EM-waves close to the receiver position (i.e., around $(r, x_3) = (5, 0)$ cm). The last pulse is due to the pseudo-Stoneley wave (Feng & Johnson, 1983).

4.4 Full-waveform electroseismic modelling

A compressible fluid is overlying an isotropic, homogeneous poroelastic medium (see Figure 4.7). The horizontal interface is at $x_3 = 0$. An electromagnetic source at $(x_1, x_3) = (x_1^S, x_3^S)$ emits EM-waves that travel towards the interface where they are converted to reflected P-wave signal. The acoustic potential is recorded by the acoustic receiver at $(x_1, x_3) = (x_1^R, x_3^R)$. We express the electromagnetic source as an

Figure 4.5: Ricker wavelet $W(t)$ (left) and its spectrum $W(\omega)$ (right).

Figure 4.6: Modelled reflected electric potential at $(r^R, x_3^R) = (0, -5)$ cm due to acoustic source at $(r^S, x_3^S) = (0, -5)$ cm (left) and at $(r^R, x_3^R) = (5, -0.3)$ cm due to source at $(r^S, x_3^S) = (0, -0.3)$ cm (right).

electric current density $\hat{\mathbf{C}}^{fl}$ which is incorporated into Ampere's law [viz. equations (A-3) and (A-5), appendix A, this chapter] as follows:

$$\nabla \times \hat{\mathbf{H}}^{fl} - i\omega\epsilon_0\epsilon_{fl}\hat{\mathbf{E}}^{fl} = \sigma_{fl}\hat{\mathbf{E}}^{fl} + \hat{\mathbf{C}}^{fl}, \quad (4.23)$$

$$\hat{\mathbf{C}}^{fl} = \left(0, 0, \hat{C}_3\delta(x_3 - x_3^S)\delta(x_1 - x_1^S)\right)^T, \quad (4.24)$$

where $\delta(x)$ is the delta function. Hence we consider a line source of vertical current density (see the box in Figure 4.7).

The first order matrix-vector differential equation of the fluid layer [see equation (4.1)] with incorporation of the source now becomes

$$\frac{d\check{\mathbf{f}}^{fl}(x_3)}{dx_3} = \mathbf{A}^{fl}\check{\mathbf{f}}^{fl}(x_3) + \check{\mathbf{S}}^{fl}\delta(x_3 - x_3^S), \quad (4.25)$$

Figure 4.7: Geometry of electroseismic model. The 3D orientation of the line source is indicated in the upper left box.

where $\check{\mathbf{S}}^{fl}$ is given by

$$\check{\mathbf{S}}^{fl} = \left(\mathbf{0}, \check{\mathbf{S}}_C^{fl} \right)^T, \quad (4.26)$$

$$\check{\mathbf{S}}_C^{fl} = \left(0, i \frac{k_1 e^{ik_1 x_1^S}}{i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl}} \hat{C}_3 \right)^T. \quad (4.27)$$

Equation (4.25) is solved by defining the one-way wave vector $\check{\mathbf{f}}_o^{fl}(x_3)$ (see e.g., Wapenaar & Berkhout, 1989) as follows:

$$\check{\mathbf{f}}_o^{fl}(x_3) = \left(\mathbf{L}^{fl} \right)^{-1} \check{\mathbf{f}}^{fl}(x_3), \quad (4.28)$$

or

$$\check{\mathbf{f}}^{fl}(x_3) = \mathbf{L}^{fl} \check{\mathbf{f}}_o^{fl}(x_3). \quad (4.29)$$

$\mathbf{L}^{fl}$, given in appendix A of this chapter, and $\left(\mathbf{L}^{fl} \right)^{-1}$ are the composition and decomposition matrices, respectively. The composition matrix composes the two-way wavefields $\check{\mathbf{f}}^{fl}(x_3)$ from the one-way (downgoing and upgoing) plane waves contained in $\check{\mathbf{f}}_o^{fl}(x_3)$ and, vice versa, the decomposition matrix decomposes the two-way wavefields in $\check{\mathbf{f}}^{fl}(x_3)$ into one-way waves. The one-way downgoing waves propagate in the positive x_3 -direction, while the upgoing plane waves propagate in the negative x_3 -direction (see also Figure 4.7). The solution for $\check{\mathbf{f}}_o^{fl}(x_3)$ is ($x_3 > x_3^S$)

$$\check{\mathbf{f}}_o^{fl}(x_3) = \mathbf{c} + \mathbf{W}(x_3, x_3^S) \left(\mathbf{L}^{fl} \right)^{-1} \check{\mathbf{S}}^{fl}, \quad (4.30)$$

where the boundary condition vector $\mathbf{c}$ is given by

$$\mathbf{c} = -i \frac{e^{ik_3^{EM}(x_3 - x_3^S)}}{2k_3^{EM}} \check{\mathbf{S}}^{fl}, \quad (4.31)$$

and where the one-way extrapolation operator $\mathbf{W}(x_3, x_3^S)$ is given by

$$\mathbf{W}(x_3, x_3^S) = \begin{pmatrix} \mathbf{W}_d(x_3, x_3^S) & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_u(x_3, x_3^S) \end{pmatrix}, \quad (4.32)$$

$$\mathbf{W}_d(x_3, x_3^S) = \begin{pmatrix} e^{-ik_3^{fl}(x_3 - x_3^S)} & 0 \\ 0 & e^{-ik_3^{EM}(x_3 - x_3^S)} \end{pmatrix}, \quad (4.33)$$

and $\mathbf{W}_u(x_3, x_3^S) = (\mathbf{W}_d(x_3, x_3^S))^{-1}$. We now simulate the reflected fields

$$\check{\mathbf{f}}_s^{fl}(x_3^R) = \left(\check{p}^{fl}(x_3^R), \check{E}_1^{fl}(x_3^R) \right)^T, \quad (4.34)$$

for the geometry of Figure 4.7. Downgoing waves emitted from the source that reflect as upgoing waves and subsequently travel towards the receiver position at $(x_1, x_3) = (x_1^R, x_3^R)$ are considered. Hence the electroseismic model does not consider waves that are reflected from above the source ($x_3 < x_3^S$), i.e., there are no interfaces above the source. The following electroseismic model for the reflected fields ($x_3^R < 0$) is obtained:

$$\check{\mathbf{f}}_s^{fl}(x_3^R) = \frac{1}{2} \mathbf{L}_u^{fl} \mathbf{W}_u(x_3^R, 0) \mathbf{R}^{fp} \mathbf{W}_d(0, x_3^S) \left(\mathbf{L}_d^{fl} \right)^{-1} \check{\mathbf{S}}_C^{fl}, \quad (4.35)$$

where

$$\mathbf{L}_u^{fl} = \begin{pmatrix} \omega^2 \rho_{fl} & 0 \\ 0 & -ik_3^E \end{pmatrix}, \quad (4.36)$$

$$\mathbf{R}^{fp} = \begin{pmatrix} r_{P,P} & r_{P,EM} \\ r_{EM,P} & r_{EM,EM} \end{pmatrix}, \quad (4.37)$$

$$\mathbf{L}_d^{fl} = \begin{pmatrix} \omega^2 \rho_{fl} & 0 \\ 0 & ik_3^E \end{pmatrix}. \quad (4.38)$$

The fluid downgoing wave decomposition operator $\left(\mathbf{L}_d^{fl} \right)^{-1}$ transforms source vector $\check{\mathbf{S}}_C^{fl}$ into a one-way source vector. The one-way source fields are subsequently propagated downward from the source (at $x_3 = x_3^S$) to the interface (at $x_3 = 0$) by $\mathbf{W}_d(0, x_3^S)$. Then, incident potential is related to reflected potential by $\mathbf{R}^{fp}$ and propagated upwards to the receiver position x_3^R by $\mathbf{W}_u(x_3^R, 0)$. Finally two-way wavefields $\check{\mathbf{f}}_s^{fl}(x_3^R)$ are composed from the one-way upward travelling wavefields by $\mathbf{L}_u^{fl}$. From equations (4.27) and (4.32)-(4.38) we obtain ($x_1^S = 0$)

$$\begin{pmatrix} \check{p}^{fl}(x_3^R) \\ \check{E}_1^{fl}(x_3^R) \end{pmatrix} = \begin{pmatrix} \frac{\omega^2 k_1 \rho_{fl} \hat{C}_3}{2k_3^{EM} (i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl})} r_{P,EM} e^{ik_3^{fl} x_3^R} e^{ik_3^{EM} x_3^S} \\ -i \frac{k_1 \hat{C}_3}{2(i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl})} r_{EM,EM} e^{ik_3^{EM} (x_3^R + x_3^S)} \end{pmatrix}. \quad (4.39)$$

Figure 4.8: Modelled reflected pressure at $(x_1^R, x_3^R) = (0, -5)$ cm due to a vertical electric current density line-source at $(x_1^S, x_3^S) = (0, -5)$ cm (left) and at $(x_1^R, x_3^R) = (5, -0.3)$ cm due to a source at $(x_1^S, x_3^S) = (0, -0.3)$ cm (right).

Equation (4.39) can readily be evaluated numerically and yields the total reflected pressure wavefield through the inverse fast Fourier transformation.

We now consider that the vertical current density line-source is at $(x_1^S, x_3^S) = (0, -5)$ cm and that a pressure receiver is located at $(x_1^R, x_3^R) = (0, -5)$ cm. The source emits the Ricker wavelet depicted in Figure 4.5. Equation (4.39) is evaluated for each frequency by a recursive adaptive Simpson quadrature algorithm implemented in Matlab. The parameters of Table 3.1 are used. Figure 4.8 (left) shows the resulting modelled reflected pressure. An arrival is recorded at around $34 \mu\text{s}$ which corresponds with the acoustic travel time from the fluid/porous-medium interface to the receiver.

Figure 4.8 (right) shows the modelled pulse where now $(x_1^S, x_3^S) = (0, -0.3)$ cm and $(x_1^R, x_3^R) = (5, -0.3)$ cm. Again, as in section 4.3, the pulses are characterized by their travel times. The first pulse originates from electromagnetic waves that are converted to acoustic waves close to the receiver position (i.e., around $(x_1, x_3) = (5, 0)$ cm). Besides reflected acoustic waves, the electromagnetic source also generates interface waves (e.g., the Pf-head-wave). The arrival around $20 \mu\text{s}$ corresponds to the Pf-head-wave, while the pulse arriving at around $34 \mu\text{s}$ originates from EM-waves that are converted to acoustic waves at near-vertical incidence. The last pulse is due to the pseudo-Stoneley wave.

Appendix A: Matrices of chapter 4

The (source-free) governing equations for the fluid are

$$\nabla \hat{p}^{fl} = \omega^2 \rho_{fl} \hat{\mathbf{U}}^{fl}, \quad (\text{A-1})$$

$$\hat{p}^{fl} = -K_{fl} \nabla \cdot \hat{\mathbf{U}}^{fl}, \quad (\text{A-2})$$

$$-\nabla \times \hat{\mathbf{H}}^{fl} + i\omega \varepsilon_0 \epsilon_{fl} \hat{\mathbf{E}}^{fl} = -\hat{\mathbf{J}}^{fl}, \quad (\text{A-3})$$

$$\nabla \times \hat{\mathbf{E}}^{fl} + i\omega \mu_{fl} \hat{\mathbf{H}}^{fl} = \mathbf{0}, \quad (\text{A-4})$$

$$\hat{\mathbf{J}}^{fl} = \sigma_{fl} \hat{\mathbf{E}}^{fl}, \quad (\text{A-5})$$

where $\hat{\mathbf{J}}^{fl}$ is the electric current density in the fluid. Transforming equations (A-1)-(A-5) to the (k_1, x_3, ω) -domain, indicated with a check, and considering the SV-TM polarization in the (x_1, x_3) plane (see Figure 4.1), yields

$$ik_1 \check{p}^{fl} = -\omega^2 \rho_{fl} \check{U}_1^{fl}, \quad (\text{A-6})$$

$$\frac{d\check{p}^{fl}}{dx_3} = \omega^2 \rho_{fl} \check{U}_3^{fl}, \quad (\text{A-7})$$

$$\check{p}^{fl} = ik_1 K_{fl} \check{U}_1^{fl} - K_{fl} \frac{d\check{U}_3^{fl}}{dx_3}, \quad (\text{A-8})$$

$$\frac{d\check{H}_2^{fl}}{dx_3} + i\omega \varepsilon_0 \epsilon_{fl} \check{E}_1^{fl} = -\sigma_{fl} \check{E}_1^{fl}, \quad (\text{A-9})$$

$$ik_1 \check{H}_2^{fl} + i\omega \varepsilon_0 \epsilon_{fl} \check{E}_3^{fl} = -\sigma_{fl} \check{E}_3^{fl}, \quad (\text{A-10})$$

$$ik_1 \check{E}_3^{fl} + \frac{d\check{E}_1^{fl}}{dx_3} + i\omega \mu_{fl} \check{H}_2^{fl} = 0. \quad (\text{A-11})$$

We now rewrite equations (A-6)-(A-11) in terms of x_3 -derivatives of $\check{U}_3^{fl}$, $\check{p}^{fl}$, $\check{H}_2^{fl}$ and $\check{E}_1^{fl}$. Solving equation (A-8) for $\check{U}_1^{fl}$ and substituting the result into equation (A-6) gives

$$\frac{d\check{U}_3^{fl}}{dx_3} = \left(\frac{k_1^2}{\omega^2 \rho_{fl}} - \frac{1}{K_{fl}} \right) \check{p}^{fl}. \quad (\text{A-12})$$

Obviously, equation (A-9) gives

$$\frac{d\check{H}_2^{fl}}{dx_3} = -(i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl}) \check{E}_1^{fl}. \quad (\text{A-13})$$

Solving equation (A-10) for $\check{E}_3^{fl}$ and substituting the result into equation (A-11) yields

$$\frac{d\check{E}_1^{fl}}{dx_3} = \left(\frac{\omega^2 \varepsilon_0 \epsilon_{fl} \mu_{fl} - i\omega \mu_{fl} \sigma_{fl} - k_1^2}{i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl}} \right) \check{H}_2^{fl}. \quad (\text{A-14})$$

Equations (A-7), (A-12)-(A-14) are written as

$$\frac{d\check{\mathbf{f}}^{fl}}{dx_3} = \mathbf{A}^{fl}\check{\mathbf{f}}^{fl}, \quad (\text{A-15})$$

where $\check{\mathbf{f}}^{fl}$ and $\mathbf{A}^{fl}$ are given by

$$\check{\mathbf{f}}^{fl} = \left(\check{U}_3^{fl}, \check{p}^{fl}, \check{H}_2^{fl}, \check{E}_1^{fl} \right)^T, \quad (\text{A-16})$$

$$\mathbf{A}^{fl} = \begin{pmatrix} 0 & \frac{k_1^2}{\omega^2 \rho_{fl}} - \frac{1}{K_{fl}} & 0 & 0 \\ \omega^2 \rho_{fl} & 0 & 0 & 0 \\ 0 & 0 & 0 & -(i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl}) \\ 0 & 0 & \frac{\omega^2 \varepsilon_0 \epsilon_{fl} \mu_{fl} - i\omega \mu_{fl} \sigma_{fl} - k_1^2}{i\omega \varepsilon_0 \epsilon_{fl} + \sigma_{fl}} & 0 \end{pmatrix}. \quad (\text{A-17})$$

Eigenvectors of $\mathbf{A}^{fl}$ can be obtained by direct calculation, however, here we obtain them from plane wave expressions of the fields (see e.g., Haartsen, 1995; Pride & Haartsen, 1996). Substituting the following plane wave expressions:

$$\hat{\mathbf{U}}_k^{fl} = \nabla \tilde{\varphi}_k^{fl} e^{-i(k_1 x_1 \pm k_3^{fl} x_3)}, \quad (\text{A-18})$$

$$\hat{\mathbf{E}}_k^{fl} = \nabla \times \left(0, \tilde{\psi}_k^{fl} e^{-i(k_1 x_1 \pm k_3^{EM} x_3)}, 0 \right)^T, \quad (\text{A-19})$$

where $k = I$ or O , into equations (A-2) and (A-4) respectively, leads to the following four eigenvectors for the fluid which are sorted in the columns of matrix $\mathbf{L}^{fl} = (\mathbf{b}_{P,I}, \mathbf{b}_{EM,I}, \mathbf{b}_{P,O}, \mathbf{b}_{EM,O})$

$$\mathbf{b}_{P,k} = \left(\mp i k_3^{fl}, \omega^2 \rho_{fl}, 0, 0 \right)^T, \quad (\text{A-20})$$

$$\mathbf{b}_{EM,k} = \left(0, 0, i \frac{\omega}{\mu_{fl}} \left[s_{EM}^{fl}(\omega) \right]^2, \pm i k_3^{EM} \right)^T, \quad (\text{A-21})$$

where $s_{EM}^{fl}(\omega)$ is the fluid EM-wave slowness. The upper sign corresponds with $k = I$ and the lower sign with $k = O$ (see Figure 4.1). Hence the first columns correspond with downward travelling waves. In terms of the boundary condition vector of the fluid $\hat{\mathbf{v}}^{fl}$

$$\hat{\mathbf{v}}^{fl} = \left(\hat{U}_3^{fl}, \hat{p}^{fl}, 0, 0, \hat{H}_2^{fl}, \hat{E}_1^{fl} \right)^T, \quad (\text{A-22})$$

the eigenvector-matrix $\mathbf{M}^{fl} = (\mathbf{a}_{P,I}, \mathbf{a}_{EM,I}, \mathbf{a}_{P,O}, \mathbf{a}_{EM,O})$ becomes

$$\mathbf{a}_{P,k} = \left(\mp i k_3^{fl}, \omega^2 \rho_{fl}, 0, 0, 0, 0 \right)^T, \quad (\text{A-23})$$

$$\mathbf{a}_{EM,k} = \left(0, 0, 0, 0, i \frac{\omega}{\mu_{fl}} \left[s_{EM}^{fl}(\omega) \right]^2, \pm i k_3^{EM} \right)^T. \quad (\text{A-24})$$

The (source-free) governing equations for the porous medium are given in equations (2.1)-(2.7). Transforming to the (k_1, x_3, ω) -domain, and considering the SV-TM polarization in the (x_1, x_3) plane (see Figure 4.1), yields

$$\frac{d\check{\sigma}_{13}}{dx_3} - ik_1\check{\sigma}_{11} - ik_1(1-\phi)\check{p} = \omega^2\rho_{11}(\omega)\check{u}_1 + \omega^2\rho_{12}(\omega)\check{U}_1 - \frac{\eta\phi L(\omega)}{k(\omega)}\check{E}_1, \quad (\text{A-25})$$

$$\frac{d\check{\sigma}_{33}}{dx_3} - ik_1\check{\sigma}_{13} + (1-\phi)\frac{d\check{p}}{dx_3} = \omega^2\rho_{11}(\omega)\check{u}_3 + \omega^2\rho_{12}(\omega)\check{U}_3 - \frac{\eta\phi L(\omega)}{k(\omega)}\check{E}_3, \quad (\text{A-26})$$

$$-ik_1\phi\check{p} = \omega^2\rho_{12}(\omega)\check{u}_1 + \omega^2\rho_{22}(\omega)\check{U}_1 + \frac{\eta\phi L(\omega)}{k(\omega)}\check{E}_1, \quad (\text{A-27})$$

$$\phi\frac{d\check{p}}{dx_3} = \omega^2\rho_{12}(\omega)\check{u}_3 + \omega^2\rho_{22}(\omega)\check{U}_3 + \frac{\eta\phi L(\omega)}{k(\omega)}\check{E}_3, \quad (\text{A-28})$$

$$\check{\sigma}_{11} = 2ik_1G\check{u}_1 + \left(A - \frac{1-\phi}{\phi}Q\right)\left(ik_1\check{u}_1 - \frac{d\check{u}_3}{dx_3}\right) + \left(Q - \frac{1-\phi}{\phi}R\right)\left(ik_1\check{U}_1 - \frac{d\check{U}_3}{dx_3}\right), \quad (\text{A-29})$$

$$G\frac{d\check{u}_1}{dx_3} + \check{\sigma}_{13} = ik_1G\check{u}_3, \quad (\text{A-30})$$

$$2G\frac{d\check{u}_3}{dx_3} + \check{\sigma}_{33} = \left(A - \frac{1-\phi}{\phi}Q\right)\left(ik_1\check{u}_1 - \frac{d\check{u}_3}{dx_3}\right) + \left(Q - \frac{1-\phi}{\phi}R\right)\left(ik_1\check{U}_1 - \frac{d\check{U}_3}{dx_3}\right), \quad (\text{A-31})$$

$$\phi\check{p} = Q\left(ik_1\check{u}_1 - \frac{d\check{u}_3}{dx_3}\right) + R\left(ik_1\check{U}_1 - \frac{d\check{U}_3}{dx_3}\right), \quad (\text{A-32})$$

$$\check{J}_1 = \sigma(\omega)\check{E}_1 + ik_1L(\omega)\check{p} + \omega^2\rho_fL(\omega)\check{u}_1, \quad (\text{A-33})$$

$$L(\omega)\frac{d\check{p}}{dx_3} + \check{J}_3 = \sigma(\omega)\check{E}_3 + \omega^2\rho_fL(\omega)\check{u}_3, \quad (\text{A-34})$$

$$-\frac{d\check{H}_2}{dx_3} = i\omega\varepsilon\check{E}_1 + \check{J}_1, \quad (\text{A-35})$$

$$-ik_1\check{H}_2 = i\omega\varepsilon\check{E}_3 + \check{J}_3, \quad (\text{A-36})$$

$$\frac{d\check{E}_1}{dx_3} + ik_1\check{E}_3 = -i\omega\mu\check{H}_2. \quad (\text{A-37})$$

We now rewrite equations (A-25)-(A-37) in terms of x_3 -derivatives of $\check{u}_1$, $\check{u}_3$, $\phi(\check{U}_3 - \check{u}_3)$, $\check{p}$, $\check{\sigma}_{13}$, $\check{\sigma}_{33}$, $\check{H}_2$ and $\check{E}_1$. Equation (A-30) is rewritten as

$$\frac{d\check{u}_1}{dx_3} = ik_1\check{u}_3 - \check{\sigma}_{13}/G. \quad (\text{A-38})$$

Eliminating $\left(ik_1\check{U}_1 - \frac{d\check{U}_3}{dx_3}\right)$ in equations (A-31)-(A-32) and rearranging for the x_3 -derivative of $\check{u}_3$ yields

$$\frac{d\check{u}_3}{dx_3} = ik_1 \frac{AR - Q^2}{d_2} \check{u}_1 + \frac{\phi Q - (1 - \phi)R}{d_2} \check{p} - \frac{R}{d_2} \check{\sigma}_{33}, \quad (\text{A-39})$$

where $d_2 = PR - Q^2$ has been used. Equation (A-27) is solved for $\check{U}_1$ yielding

$$\check{U}_1 = -\frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \check{u}_1 - i \frac{k_1 \phi}{\omega^2 \rho_{22}(\omega)} \check{p} - \frac{\eta \phi L(\omega)}{\omega^2 k(\omega) \rho_{22}(\omega)} \check{E}_1. \quad (\text{A-40})$$

Substituting equations (A-39)-(A-40) into equation (A-32) gives

$$\begin{aligned} \frac{d\check{U}_3}{dx_3} = ik_1 \left[\frac{2GQ}{d_2} - \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right] \check{u}_1 - \left[\frac{\phi P - (1 - \phi)Q}{d_2} - \frac{k_1^2 \phi}{\omega^2 \rho_{22}(\omega)} \right] \check{p} \\ + \frac{Q}{d_2} \check{\sigma}_{33} - i \frac{k_1 \eta \phi L(\omega)}{\omega^2 k(\omega) \rho_{22}(\omega)} \check{E}_1. \end{aligned} \quad (\text{A-41})$$

From equations (A-39) and (A-41) we obtain

$$\begin{aligned} \frac{d}{dx_3} [\phi (\check{U}_3 - \check{u}_3)] = ik_1 \phi \left[\frac{2GQ - (AR - Q^2)}{d_2} - \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right] \check{u}_1 \\ - \phi \left[\frac{\phi H - (Q + R)}{d_2} - \frac{k_1^2 \phi}{\omega^2 \rho_{22}(\omega)} \right] \check{p} + \phi \frac{Q + R}{d_2} \check{\sigma}_{33} - i \frac{k_1 \eta \phi^2 L(\omega)}{\omega^2 k(\omega) \rho_{22}(\omega)} \check{E}_1, \end{aligned} \quad (\text{A-42})$$

where $H = P + R + 2Q$ has been used. Eliminating $\check{J}_3$ in equations (A-34) and (A-36) gives

$$\frac{d\check{p}}{dx_3} = \omega^2 \rho_f \check{u}_3 + \frac{i\omega\varepsilon + \sigma(\omega)}{L(\omega)} \check{E}_3 + i \frac{k_1}{L(\omega)} \check{H}_2. \quad (\text{A-43})$$

Substituting equation (A-43) into equation (A-28) and solving for $\check{E}_3$ gives

$$\check{E}_3 = -i\omega \frac{L(\omega) \rho_{22}(\omega)}{\phi^2 \bar{\varepsilon}(\omega)} [\phi (\check{U}_3 - \check{u}_3)] - \frac{k_1}{\omega \bar{\varepsilon}(\omega)} \check{H}_2, \quad (\text{A-44})$$

where we note that $\rho_{22}(\omega) = \phi \rho_f - \rho_{12}(\omega)$ and $\bar{\varepsilon}(\omega) = \varepsilon - i\sigma(\omega)/\omega + i\eta L^2(\omega)/\omega k(\omega)$. Back substitution of equation (A-44) into equation (A-28) yields

$$\frac{d\check{p}}{dx_3} = \omega^2 \rho_f \check{u}_3 + \frac{\omega^2 \rho_{22}(\omega)}{\phi^2} \left(1 - i \frac{\eta L^2(\omega)}{\omega k(\omega) \bar{\varepsilon}(\omega)} \right) [\phi (\check{U}_3 - \check{u}_3)] - \frac{k_1 \eta L(\omega)}{\omega k(\omega) \bar{\varepsilon}(\omega)} \check{H}_2. \quad (\text{A-45})$$

Substituting equation (A-29) into equation (A-25) gives

$$\begin{aligned} \frac{d\check{\sigma}_{13}}{dx_3} = \left[\omega^2 \rho_{11}(\omega) - k_1^2 \left(P - \frac{1 - \phi}{\phi} Q \right) \right] \check{u}_1 + ik_1 (1 - \phi) \check{p} - \frac{\eta \phi L(\omega)}{k(\omega)} \check{E}_1 \\ + \left[\omega^2 \rho_{12}(\omega) - k_1^2 \left(Q - \frac{1 - \phi}{\phi} R \right) \right] \check{U}_1 - ik_1 \left(A - \frac{1 - \phi}{\phi} Q \right) \frac{d\check{u}_3}{dx_3} \\ - ik_1 \left(Q - \frac{1 - \phi}{\phi} R \right) \frac{d\check{U}_3}{dx_3}. \end{aligned} \quad (\text{A-46})$$

Substituting equations (A-39)-(A-41) into equation (A-46) yields

$$\begin{aligned} \frac{d\check{\sigma}_{13}}{dx_3} = & \left[\omega^2 \left(\rho_{11}(\omega) - \frac{[\rho_{12}(\omega)]^2}{\rho_{22}(\omega)} \right) - 4k_1^2 G \left(1 - \frac{GR}{d_2} \right) \right] \check{u}_1 \\ & + ik_1 \left[1 - \phi \left(1 + \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right) + \frac{2G}{d_2} (\phi Q - (1 - \phi)R) \right] \check{p} + ik_1 \frac{AR - Q^2}{d_2} \check{\sigma}_{33} \\ & - \frac{\eta \phi L(\omega)}{k(\omega)} \left(1 + \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right) \check{E}_1. \end{aligned} \quad (\text{A-47})$$

Using $\rho_{11}(\omega) = (1 - \phi) \rho_s - \rho_{12}(\omega)$ in equation (A-26) gives

$$\begin{aligned} \frac{d\check{\sigma}_{33}}{dx_3} = & \omega^2 (1 - \phi) \rho_s \check{u}_3 + \frac{\omega^2 \rho_{12}(\omega)}{\phi} [\phi (\check{U}_3 - \check{u}_3)] + ik_1 \check{\sigma}_{13} \\ & - \frac{\eta \phi L(\omega)}{k(\omega)} \check{E}_3 - (1 - \phi) \frac{d\check{p}}{dx_3}. \end{aligned} \quad (\text{A-48})$$

Substitution of equations (A-44)-(A-45) into equation (A-48) leads to

$$\begin{aligned} \frac{d\check{\sigma}_{33}}{dx_3} = & \omega^2 (1 - \phi) (\rho_s - \rho_f) \check{u}_3 \\ & + \omega^2 \left[\rho_f - \frac{\rho_{22}(\omega)}{\phi^2} \left(1 - i \frac{\eta L^2(\omega)}{\omega k(\omega) \bar{\epsilon}(\omega)} \right) \right] [\phi (\check{U}_3 - \check{u}_3)] \\ & + ik_1 \check{\sigma}_{13} + \frac{k_1 \eta L(\omega)}{\omega k(\omega) \bar{\epsilon}(\omega)} \check{H}_2, \end{aligned} \quad (\text{A-49})$$

where we note again that $\rho_{22}(\omega) = \phi \rho_f - \rho_{12}(\omega)$. Eliminating $\check{J}_1$ in equations (A-33) and (A-35) gives

$$\frac{d\check{H}_2}{dx_3} = -\omega^2 \rho_f L(\omega) \check{u}_1 - ik_1 L(\omega) \check{p} - [i\omega \epsilon + \sigma(\omega)] \check{E}_1. \quad (\text{A-50})$$

Substituting equation (A-44) into equation (A-37) leads to

$$\frac{d\check{E}_1}{dx_3} = -\omega \frac{k_1 L(\omega) \rho_{22}(\omega)}{\phi^2 \bar{\epsilon}(\omega)} [\phi (\check{U}_3 - \check{u}_3)] + i \left[\frac{k_1^2}{\omega \bar{\epsilon}(\omega)} - \omega \mu \right] \check{H}_2. \quad (\text{A-51})$$

Equations (A-38)-(A-39), (A-42), (A-45), (A-47) and (A-49)-(A-51) are written as

$$\frac{d\check{\mathbf{f}}^p}{dx_3} = \mathbf{A}^p \check{\mathbf{f}}^p, \quad (\text{A-52})$$

where $\check{\mathbf{f}}^p$ and $\mathbf{A}^p$ are given by

$$\check{\mathbf{f}}^p = (\check{u}_1, \check{u}_3, \phi(\check{U}_3 - \check{u}_3), \check{p}, \check{\sigma}_{13}, \check{\sigma}_{33}, \check{H}_2, \check{E}_1)^T, \quad (\text{A-53})$$

$$\mathbf{A}^p = \begin{pmatrix} 0 & a_{12} & 0 & 0 & a_{15} & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} & 0 & a_{26} & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} & 0 & a_{36} & 0 & a_{38} \\ 0 & a_{42} & a_{43} & 0 & 0 & 0 & a_{47} & 0 \\ a_{51} & 0 & 0 & a_{54} & 0 & a_{56} & 0 & a_{58} \\ 0 & a_{62} & a_{63} & 0 & a_{65} & 0 & a_{67} & 0 \\ a_{71} & 0 & 0 & a_{74} & 0 & 0 & 0 & a_{78} \\ 0 & 0 & a_{83} & 0 & 0 & 0 & a_{87} & 0 \end{pmatrix}, \quad (\text{A-54})$$

and where

$$a_{12} = ik_1, \quad (\text{A-55})$$

$$a_{15} = -1/G, \quad (\text{A-56})$$

$$a_{21} = ik_1 \frac{AR - Q^2}{d_2}, \quad (\text{A-57})$$

$$a_{24} = \frac{\phi Q - (1 - \phi)R}{d_2}, \quad (\text{A-58})$$

$$a_{26} = -R/d_2, \quad (\text{A-59})$$

$$a_{31} = ik_1 \phi \left[\frac{2GQ - (AR - Q^2)}{d_2} - \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right], \quad (\text{A-60})$$

$$a_{34} = -\phi \left[\frac{\phi H - (Q + R)}{d_2} - \frac{k_1^2 \phi}{\omega^2 \rho_{22}(\omega)} \right], \quad (\text{A-61})$$

$$a_{36} = \phi \frac{Q + R}{d_2}, \quad (\text{A-62})$$

$$a_{38} = -i \frac{k_1 \eta \phi^2 L(\omega)}{\omega^2 k(\omega) \rho_{22}(\omega)}, \quad (\text{A-63})$$

$$a_{42} = \omega^2 \rho_f, \quad (\text{A-64})$$

$$a_{43} = \frac{\omega^2 \rho_{22}(\omega)}{\phi^2} \left(1 - i \frac{\eta L^2(\omega)}{\omega k(\omega) \bar{\varepsilon}(\omega)} \right), \quad (\text{A-65})$$

$$a_{47} = -\frac{k_1 \eta L(\omega)}{\omega k(\omega) \bar{\varepsilon}(\omega)}, \quad (\text{A-66})$$

$$a_{51} = \omega^2 \left(\rho_{11}(\omega) - \frac{[\rho_{12}(\omega)]^2}{\rho_{22}(\omega)} \right) - 4k_1^2 G \left(1 - \frac{GR}{d_2} \right), \quad (\text{A-67})$$

$$a_{54} = ik_1 \left[1 - \phi \left(1 + \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right) + \frac{2G}{d_2} (\phi Q - (1 - \phi)R) \right], \quad (\text{A-68})$$

$$a_{56} = ik_1 \frac{AR - Q^2}{d_2}, \quad (\text{A-69})$$

$$a_{58} = -\frac{\eta\phi L(\omega)}{k(\omega)} \left(1 + \frac{\rho_{12}(\omega)}{\rho_{22}(\omega)} \right), \quad (\text{A-70})$$

$$a_{62} = \omega^2 (1 - \phi) (\rho_s - \rho_f), \quad (\text{A-71})$$

$$a_{63} = \omega^2 \left[\rho_f - \frac{\rho_{22}(\omega)}{\phi^2} \left(1 - i \frac{\eta L^2(\omega)}{\omega k(\omega) \bar{\epsilon}(\omega)} \right) \right], \quad (\text{A-72})$$

$$a_{65} = ik_1, \quad (\text{A-73})$$

$$a_{67} = \frac{k_1 \eta L(\omega)}{\omega k(\omega) \bar{\epsilon}(\omega)}, \quad (\text{A-74})$$

$$a_{71} = -\omega^2 \rho_f L(\omega), \quad (\text{A-75})$$

$$a_{74} = -ik_1 L(\omega), \quad (\text{A-76})$$

$$a_{78} = -[i\omega\epsilon + \sigma(\omega)], \quad (\text{A-77})$$

$$a_{83} = -\omega \frac{k_1 L(\omega) \rho_{22}(\omega)}{\phi^2 \bar{\epsilon}(\omega)}, \quad (\text{A-78})$$

$$a_{87} = i \left[\frac{k_1^2}{\omega \bar{\epsilon}(\omega)} - \omega \mu \right]. \quad (\text{A-79})$$

We obtain the eigenvectors of $\mathbf{A}^p$ from the following plane wave expressions:

$$\hat{\varphi}_{m,k}^s = \tilde{\varphi}_{m,k}^s e^{-i(k_1 x_1 \pm k_3^m x_3)}, \quad (\text{A-80})$$

$$\hat{\Psi}_{n,k}^s = \left(0, \tilde{\psi}_{n,k}^s e^{-i(k_1 x_1 \pm k_3^n x_3)}, 0 \right)^T, \quad (\text{A-81})$$

where $m = Pf$ or Ps , $n = TM$ or SV , $k = I$ or O and

$$\hat{\mathbf{u}}_{m,k} = \nabla \hat{\varphi}_{m,k}^s, \quad (\text{A-82})$$

$$\hat{\mathbf{u}}_{n,k} = \nabla \times \hat{\Psi}_{n,k}^s. \quad (\text{A-83})$$

For the geometry of Figure 4.1, outgoing potentials ($k = O$) correspond with the plus sign in equations (A-80)-(A-81), while incident potentials ($k = I$) correspond with the minus sign. The fluid displacement and electric fields are related to solid displacement through the fluid-to-solid and electric-to-solid ratios, e.g., $\hat{\mathbf{U}}_{Pf,I} = \beta_{Pf}(\omega) \hat{\mathbf{u}}_{Pf,I}$ and $\hat{\mathbf{E}}_{Pf,I} = \alpha_{Pf}(\omega) \hat{\mathbf{u}}_{Pf,I}$. Intergranular stress, pore pressure and the magnetic field are calculated from, respectively, equations (2.3), (2.4) and (2.7). The

eigenvectors are sorted in the columns of $\mathbf{L}^p = (\mathbf{L}_O^p, \mathbf{L}_I^p)$, where

$$\mathbf{L}_O^p = (\mathbf{b}_{Pf,O}, \mathbf{b}_{Ps,O}, \mathbf{b}_{TM,O}, \mathbf{b}_{SV,O}), \quad (\text{A-84})$$

$$\mathbf{L}_I^p = (\mathbf{b}_{Pf,I}, \mathbf{b}_{Ps,I}, \mathbf{b}_{TM,I}, \mathbf{b}_{SV,I}), \quad (\text{A-85})$$

$$\begin{aligned} \mathbf{b}_{m,k} = & \left(-ik_1, \mp ik_3^m, \mp i\phi k_3^m [\beta_m(\omega) - 1], \frac{\omega^2}{\phi} [Q + R\beta_m(\omega)] s_m^2(\omega), \dots \right. \\ & \left. \pm 2k_1 k_3^m G, [-2k_1^2 G + \omega^2 N_m(\omega) s_m^2(\omega)], 0, -ik_1 \alpha_m(\omega) \right)^T, \end{aligned} \quad (\text{A-86})$$

$$\begin{aligned} \mathbf{b}_{n,k} = & \left(\pm ik_3^n, -ik_1, -ik_1 \phi [\beta_n(\omega) - 1], 0, \dots \right. \\ & \left. G [2k_1^2 - \omega^2 s_n^2(\omega)], \pm 2k_1 k_3^n G, i \frac{\omega}{\mu} \alpha_n(\omega) s_n^2(\omega), \pm ik_3^n \alpha_n(\omega) \right)^T, \end{aligned} \quad (\text{A-87})$$

where $N_m(\omega) = P - (1 - \phi) Q / \phi + [Q - (1 - \phi) R / \phi] \beta_m(\omega)$ and where the upper sign corresponds with $k = O$ and the lower sign with $k = I$. These eigenvectors agree with those given by Pride & Haartsen (1996) who adopt an opposite sign convention in the plane wave expressions and use a different formulation of electrokinetic theory (see chapter 3). In terms of the boundary condition vector of the poroelastic medium $\hat{\mathbf{v}}^p$,

$$\hat{\mathbf{v}}^p = \left([(1 - \phi) \hat{u}_3 + \phi \hat{U}_3], \hat{p}, \hat{\sigma}_{13}, \hat{\sigma}_{33}, \hat{H}_2, \hat{E}_1 \right)^T, \quad (\text{A-88})$$

the eigenvector-matrix becomes $\mathbf{M}^p = (\mathbf{M}_O^p, \mathbf{M}_I^p)$, where

$$\mathbf{M}_O^p = (\mathbf{a}_{Pf,O}, \mathbf{a}_{Ps,O}, \mathbf{a}_{TM,O}, \mathbf{a}_{SV,O}), \quad (\text{A-89})$$

$$\mathbf{M}_I^p = (\mathbf{a}_{Pf,I}, \mathbf{a}_{Ps,I}, \mathbf{a}_{TM,I}, \mathbf{a}_{SV,I}), \quad (\text{A-90})$$

$$\begin{aligned} \mathbf{a}_{m,k} = & \left(\mp ik_3^m [1 - \phi + \phi \beta_m(\omega)], \frac{\omega^2}{\phi} [Q + R\beta_m(\omega)] s_m^2(\omega), \pm 2k_1 k_3^m G, \dots \right. \\ & \left. [-2k_1^2 G + \omega^2 N_m(\omega) s_m^2(\omega)], 0, -ik_1 \alpha_m(\omega) \right)^T, \end{aligned} \quad (\text{A-91})$$

$$\begin{aligned} \mathbf{a}_{n,k} = & \left(-ik_1 [1 - \phi + \phi \beta_n(\omega)], 0, G [2k_1^2 - \omega^2 s_n^2(\omega)], \dots \right. \\ & \left. \pm 2k_1 k_3^n G, i \frac{\omega}{\mu} \alpha_n(\omega) s_n^2(\omega), \pm ik_3^n \alpha_n(\omega) \right)^T. \end{aligned} \quad (\text{A-92})$$

Applying the boundary conditions, $\hat{\mathbf{v}}^{fl} = \hat{\mathbf{v}}^p$, at $x_3 = 0$ leads to $\mathbf{M}^{fl} \tilde{\mathbf{w}}^{fl} = \mathbf{M}^p \tilde{\mathbf{w}}^p$, where $\tilde{\mathbf{w}}^{fl}$ and $\tilde{\mathbf{w}}^p$ are given by

$$\tilde{\mathbf{w}}^{fl} = (\tilde{\varphi}_I^{fl}, \tilde{\psi}_I^{fl}, \tilde{\varphi}_O^{fl}, \tilde{\psi}_O^{fl})^T, \quad (\text{A-93})$$

$$\tilde{\mathbf{w}}^p = (\tilde{\varphi}_{Pf,O}^s, \tilde{\varphi}_{Ps,O}^s, \tilde{\psi}_{TM,O}^s, \tilde{\psi}_{SV,O}^s, \tilde{\varphi}_{Pf,I}^s, \tilde{\varphi}_{Ps,I}^s, \tilde{\psi}_{TM,I}^s, \tilde{\psi}_{SV,I}^s)^T. \quad (\text{A-94})$$

In order to obtain the scattering matrix, outgoing potentials are related to ingoing potentials

$$\mathbf{S}_I \tilde{\mathbf{w}}_I = \mathbf{S}_O \tilde{\mathbf{w}}_O, \quad (\text{A-95})$$

where

$$\mathbf{S}_I = (-\mathbf{a}_{P,I}, -\mathbf{a}_{EM,I}, \mathbf{a}_{Pf,I}, \mathbf{a}_{Ps,I}, \mathbf{a}_{TM,I}, \mathbf{a}_{SV,I}), \quad (\text{A-96})$$

$$\mathbf{S}_O = (\mathbf{a}_{P,O}, \mathbf{a}_{EM,O}, -\mathbf{a}_{Pf,O}, -\mathbf{a}_{Ps,O}, -\mathbf{a}_{TM,O}, -\mathbf{a}_{SV,O}), \quad (\text{A-97})$$

and where

$$\tilde{\mathbf{w}}_I = (\tilde{\varphi}_I^{fl}, \tilde{\psi}_I^{fl}, \tilde{\varphi}_{Pf,I}^s, \tilde{\varphi}_{Ps,I}^s, \tilde{\psi}_{TM,I}^s, \tilde{\psi}_{SV,I}^s)^T, \quad (\text{A-98})$$

$$\tilde{\mathbf{w}}_O = (\tilde{\varphi}_O^{fl}, \tilde{\psi}_O^{fl}, \tilde{\varphi}_{Pf,O}^s, \tilde{\varphi}_{Ps,O}^s, \tilde{\psi}_{TM,O}^s, \tilde{\psi}_{SV,O}^s)^T. \quad (\text{A-99})$$

The scattering matrix $\mathbf{S}$ is given by $\mathbf{S} = \mathbf{S}_O^{-1} \mathbf{S}_I$.

Chapter 5

Seismoelectric and electroseismic measurements and seismoelectric modelling

5.1 Introduction

In section 5.2, we describe a seismoelectric laboratory setup and present seismoelectric measurements. We also develop a full-waveform electrokinetic theoretical model, and compare theoretical predictions against the seismoelectric measurements. In section 5.3, we describe an electroseismic laboratory setup and present electroseismic measurements.

5.2 Seismoelectric interface response: experimental results and forward model¹

Abstract

Understanding the seismoelectric interface response is important for developing seismoelectric field methods for oil exploration and in environmental/engineering geophysics. The existing seismoelectric theory has never been validated systematically by controlled experiments. We have designed and developed an experimental setup in which acoustic-to-electromagnetic wave conversions at interfaces are measured. An acoustic source emits a pressure wave that impinges upon a porous sample. The reflected electric wave potential is recorded by a wire electrode. We have also developed a full-waveform electrokinetic theoretical model based on the Sommerfeld approach and have compared it with measurements at positions perpendicular and parallel to the fluid/porous-medium interface. We performed experiments at several

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salinities. For 10^{-3} and 10^{-2} M sodium chloride (NaCl) solutions, both waveforms and amplitudes agree. For 10^{-4} M NaCl, however, amplitude deviations occur. We found that a single amplitude field scaling factor describes these discrepancies. We also checked the repeatability of the experiments. The amplitudes are constant for the duration of an experiment (1-4 hours) but decrease on longer time scales (~ 24 hours). However, the waveforms and spatial amplitude pattern of the electric wavefield are preserved over time. Our results validate electrokinetic theory for the seismic-to-electromagnetic-wave conversion at interfaces for subsurface exploration purposes.

5.2.1 Introduction

When the grain surfaces of soils and rocks are in contact with a fluid electrolyte, they typically acquire a chemically bound surface charge that is balanced by mobile counter ions in a thin fluid layer surrounding the grains. The bound charge is immobile, whereas the counter ions can move with the fluid. At the interface between the immobile and counter ions the so-called zeta-potential is defined. A comprehensive theory for coupled seismic and electromagnetic (EM) wave propagation with the zeta-potential as a key parameter is derived by Pride (1994). From research based on this electrokinetic theory, one can distinguish two seismoelectric effects: (1) a coseismic (electric) field is coupled to seismic waves and therefore propagates with seismic wave velocity (e.g., Pride & Haartsen, 1996) and (2) a seismic wave that traverses an interface with a contrast in electrical or mechanical properties produces EM-waves that propagate outside the support of the seismic waves with much higher EM-wave speeds (e.g., Pride & Haartsen, 1996; Haartsen & Pride, 1997). These effects have been verified experimentally in the laboratory (e.g., Zhu *et al.*, 1999, 2000; Zhu & Toksöz, 2003; Block & Harris, 2006; Bordes *et al.*, 2006) and in the field (e.g., Long & Rivers, 1975; Butler *et al.*, 1996; Mikhailov *et al.*, 1997; Beamish, 1999; Garambois & Dietrich, 2001; Dupuis *et al.*, 2007; Haines *et al.*, 2007). Also, the coseismic magnetic fields associated with a Stoneley wave (Zhu & Toksöz, 2005) and with a shear wave have been measured (Bordes *et al.*, 2006, 2008).

Electrokinetic theory combines poroelastic and EM theory through coupling in flux/force relations. The so-called dynamic electrokinetic coupling coefficient that occurs in the flux/force relations has been validated experimentally (Reppert *et al.*, 2001). Seismoelectric exploration can combine seismic resolution and the sensitivity to pore fluids of EM methods (Haines *et al.*, 2007). Garambois & Dietrich (2001); Block & Harris (2006) and Dupuis & Butler (2006) compare measured and theoretically predicted coseismic electric field amplitudes. The numerically predicted amplitude decay pattern of the seismoelectric conversion at an interface (e.g., Haartsen & Pride, 1997; Garambois & Dietrich, 2002; Haines & Pride, 2006) has been observed by several workers (e.g., Butler *et al.*, 1996; Mikhailov *et al.*, 1997; Dupuis *et al.*, 2007). Mikhailov *et al.* (1997) compare field measurements of this pattern as a function of horizontal distance with a numerical model and find reasonable agreement, and Charara *et al.* (2009) model and measure seismoelectric signals at a

fluid/porous-medium interface in a laboratory experiment. However, Charara *et al.* do not consider the spatial variation of reflected waveform and amplitude.

In this article, we test the predictive power of electrokinetic theory for the seismoelectric interface response. Laboratory measurements and modelling on the basis of the Sommerfeld approach and Pride's electrokinetic theory are presented. Measurements are conducted at several positions in a water tank in which a fluid-saturated porous sample is immersed. Excellent agreement between theory and experiments for waveforms and spatial amplitude pattern is found. Measured amplitudes are shown to be dependent on salinity. Only a single amplitude field scaling factor is necessary to find absolute agreement. Our research validates electrokinetic theory for the seismoelectric interface response.

5.2.2 Seismoelectric experimental setup

The experimental setup consists of a $58 \times 39 \times 28$ cm water tank in which a 1.125 inch diameter P-wave Panametrics source transducer (model V3638, 500 kHz center frequency), a $380 \mu\text{m}$ diameter A-M Systems silver/silver chloride (Ag/AgCl) electrode, and a Technoglas porous sample (P3 glass filter) are mounted (see Figure 5.1). The sample is made of sintered 40–100 μm glass particles that are carefully saturated with and immersed in a 10^{-3} M sodium chloride (NaCl) solution. A 500 kHz single sine pulse with a 500 mV peak-to-peak amplitude from an Agilent waveform generator (model 33220A) is fed into an ENI amplifier (model 2100L RF Power Amplifier) set at 50 dB gain. The output signal is coupled into a piezoelectric source transducer. The electrode is clamped in a computer-controlled mounting bracket so it can be positioned in the water layer with high accuracy. An Analogic preamplifier (model D1000) and a Yokogawa oscilloscope (model DL4200) are used for recording. Parameter values are given in Table 5.1.

The cylindrical coordinate system (r, x_3) is also indicated in Figure 5.1. The x_3 -axis connects the centers of the sample and the transducer. At several positions along the x_3 -axis (Figure 5.1), electric potential measurements are performed. These on-axis recordings, averaged over 8192 source pulses, are shown in Figure 5.2 (top). A 144–896 kHz numerical band-pass filter is applied in all measurements. The amplifier is calibrated with respect to amplitude and waveform using typical time signals. An electric pulse arrives at approximately 0.1 ms at each position, corresponding to the travel time of the acoustic wave from the source to the sample surface. Because of the large EM-wave speed (approximately 1.1×10^7 m/s), the electric pulses arrive almost simultaneously at each position. Note that the pulse decays in amplitude as the distance from the interface increases. The measured (true) peak-to-peak amplitude at $(r, x_3) = (0, -0.3)$ cm is approximately 0.25 mV. In a separate experiment, acoustic pressure was measured along the x_3 -axis. In Figure 5.2 (bottom) we show the acoustic waveforms measured with a Specialty Engineering Associates needle hydrophone (model PZT-Z44-1000-S/N), calibrated over the frequency range of interest. From these measurements, we can extrapolate that the acoustic pulse will indeed arrive at the interface around 0.1 ms.

Figure 5.1: Schematic of the experimental setup. The electrode is clamped in a computer-controlled mounting bracket. Electrode recordings are with respect to ground level. The shielding of the wire electrode is also connected to ground level. The r - and x_3 -axes coincide with those of Figure 5.3.

Figure 5.2: Electric potential (top) and acoustic pressure (bottom) as a function of vertical distance x_3 at $r = 0$.

Table 5.1: Parameters of the porous medium. The values for K_s , $K_f = K_{fl}$, η , $\rho_f = \rho_{fl}$, Λ , $\epsilon_f = \epsilon_{fl}$, ϵ_s and $\mu = \mu_{fl}$ are taken from the literature (see below). Values for K_b and G are derived from experiments, and ϕ , ρ_s , k_0 , and α_∞ are measured directly. Details of the measurements can be found in appendix A of this thesis. When applicable, fluid parameters are the same as for the porous medium. The temperature is 293.15 K.

K_s	Bulk modulus skeleton grains ^a	49.9×10^9	Pa
K_f, K_{fl}	Bulk modulus (pore) fluid ^b	2.2×10^9	Pa
η	Pore fluid viscosity ^b	1×10^{-3}	Pa s
ρ_f, ρ_{fl}	(pore) Fluid density ^b	998	kg/m ³
$\epsilon_f, \epsilon_{fl}$	Relative permittivity of the (pore) fluid ^b	80.1	—
ϵ_s	Relative permittivity of the solid ^b	4	—
Λ	Weighted pore volume-to-surface ratio ^c	1.229×10^{-5}	m
μ, μ_{fl}	(fluid) Magnetic permeability ($=\mu_0$)	$4\pi \times 10^{-7}$	H/m
K_b	Bulk modulus framework of grains	6.6×10^9	Pa
G	Shear modulus framework of grains	5.5×10^9	Pa
ϕ	Porosity of the porous medium	0.345	—
ρ_s	Solid density	2.212×10^3	kg/m ³
k_0	Permeability	3.1×10^{-12}	m ²
α_∞	Tortuosity of the porous medium	2.1	—

^a Johnson & Plona (1982).

^b Lide (2009-2010).

^c Pride (1994); Johnson *et al.* (1994); Jocker & Smeulders (2009).

5.2.3 Forward modelling of the seismoelectric interface response

The acoustic source is modelled by (e.g., Hall, 1987)

$$\hat{p}^S(\omega, R^S, \theta) = \frac{W(\omega)}{R^S} e^{-ik^{fl}R^S} D(\theta), \quad (5.1)$$

where ω is the radial frequency, $R^S = \sqrt{(r - r^S)^2 + (x_3 - x_3^S)^2}$ is the distance to the source, θ is the angle of incidence (see Figure 5.3), $W(\omega)$ is the amplitude spectrum, and k^{fl} is the fluid wavenumber. The directivity function $D(\theta)$ is given by

$$D(\theta) = \frac{J_1(k^{fl}a \sin \theta)}{k^{fl}a \sin \theta}. \quad (5.2)$$

Here, J_1 is the Bessel function of the first kind and first order, and a is the radius of the transducer. At $(r, x_3) = (0, 0)$, $W(\omega)$ is determined directly from the measurement (see Figure 5.4). As an additional test, the source response as a function of r is measured, for a 15.3 cm on-axis distance from the transducer. A 500 kHz single sine source pulse with a 500 mV peak-to-peak amplitude is used to excite the source transducer. Results are shown by the asterisks in Figure 5.5. The dashed line is the theoretical value obtained from equation (5.1). We notice that the wavefield of

Figure 5.3: Geometry of seismoelectric model and experiment. A pressure (P-) wave is converted to an EM-wave at the interface.

the transducer is well described by equations (5.1) and (5.2), although the theory predicts interference patterns that are not recorded in the experiment.

The modelled source pressure wavefield is now expanded into conical waves by means of the Sommerfeld integral (see e.g., Brekhovskikh, 1960; Aki & Richards, 2002). The total reflected electric potential $\hat{\psi}^{fl}(\omega, r^R, x_3^R)$ can then be expressed in terms of a Sommerfeld integral ($r^S = 0$; $x_3^S, x_3^R < 0$)

$$\hat{\psi}^{fl}(\omega, r^R, x_3^R) = -iW(\omega) \int_0^\infty \frac{k_r}{k_3^{fl}} D(k_r) J_0(k_r r^R) e^{ik_3^{fl} x_3^S} r_{EM,P}^p(k_r) e^{ik_3^{EM} x_3^R} dk_r, \quad (5.3)$$

where $k_r = k^{fl} \sin \theta$ and $k_3^{fl} = k^{fl} \cos \theta$ are the radial and vertical components of k^{fl} , respectively, and k_3^{EM} is the vertical component of the fluid EM-wavenumber. The seismoelectric reflection coefficient $r_{EM,P}^p(k_r)$ relates the incident pressure wavefront to a reflected EM signal as derived in chapter 3, adopting full electrokinetic theory (Pride, 1994). Note that the coefficient in chapter 3 is displacement-normalized, whereas $r_{EM,P}^p$ is pressure-normalized (in V/Pa).

Figure 5.4: Measured pressure pulse (top) and its frequency spectrum (bottom) at $(r, x_3) = (0, 0)$.

Figure 5.5: Measured (asterisks) and modelled (dashed line) pressure as a function of r . On-axis distance from the transducer is 15.3 cm ($R^S = 15.3$ cm when $\theta = 0$).

Expressing k_r and k_3^{fl} in terms of k^{fl} and θ , equation (5.3) is written as

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -iW(\omega) \int_0^{\pi/2+i\infty} D(\theta) k^{fl} \sin \theta J_0(k^{fl} r^R \sin \theta) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}^p(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta, \end{aligned} \quad (5.4)$$

where $k_3^{EM}(\theta) = \omega \sqrt{[s_{EM}^{fl}(\omega)]^2 - \sin^2 \theta s_P^2}$ and $\text{Im}[k_3^{EM}(\theta)] < 0$. Fluid EM and pressure wave slownesses are given by, respectively, $s_{EM}^{fl}(\omega) = \sqrt{\mu \epsilon_0 \epsilon_{fl} - i \mu \sigma_{fl} / \omega}$ and $s_P = \sqrt{\rho_{fl} / K_{fl}}$ (see Table 3.1). The path of integration is along straight lines from 0 to $\pi/2$ and from $\pi/2$ to $\pi/2 + i\infty$ in the complex θ -plane.

The second integral over complex θ is simplified further employing the substitution $\theta = \pi/2 + i \ln[\sqrt{\gamma^2 + 1} + \gamma]$

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -i \frac{W(\omega)}{a} \int_0^{\pi/2} J_0(k^{fl} r^R \sin \theta) J_1(k^{fl} a \sin \theta) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}^p(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta \\ & + \frac{W(\omega)}{a} \int_0^\infty J_0(k^{fl} r^R \sqrt{\gamma^2 + 1}) \frac{J_1(k^{fl} a \sqrt{\gamma^2 + 1})}{\sqrt{\gamma^2 + 1}} \\ & \times e^{k^{fl} x_3^S \gamma} r_{EM,P}^p(\gamma) e^{ik_3^{EM}(\gamma) x_3^R} d\gamma, \end{aligned} \quad (5.5)$$

where equation (5.2) is used for $D(\theta)$. Equation (5.5) can be readily evaluated numerically and yields the total reflected EM-wavefield through the inverse fast Fourier transform. The source is located at $(r^S, x_3^S) = (0, -15)$ cm (see Figure 5.3). For each frequency, we use a recursive adaptive Simpson quadrature algorithm implemented in Matlab. A 144–896 kHz numerical band-pass filter is applied, and the input parameters of Tables 5.1 and 5.2 are used. Figure 5.6 shows the resulting modelled reflected electric potential (top) and its frequency spectrum (bottom) for a 10^{-3} M NaCl salinity.

5.2.4 Comparison between experiment and forward model

Figure 5.7 compares measured waveforms (solid lines) at various positions in the fluid along the negative x_3 -axis (see Figure 5.3) with modelled waveforms (dashed lines). The agreement between measured and modelled waveforms is excellent. The modelled amplitudes are scaled to the measured amplitudes by the amplitude field scaling factor $A_C = 0.19$, which is obtained by comparing the theoretical Fourier spectrum maxima of the entire field to the measured maxima. Thus, $A_C < 1$ means that the theory overpredicts the actual measured electric potentials.

In Figure 5.8, the scaled theoretical Fourier spectra are compared with the experiments. Note that the measured pulses have their energy distributed over slightly smaller frequencies than predicted by theory. Figure 5.9 compares measured waveforms (solid lines) at various positions in the fluid along the r -axis at $x_3 = -1.3$ cm (see Figure 5.3) with modelled waveforms (dashed lines). Again, $A_C = 0.19$ is used. As in the measurements along the x_3 -axis, good agreement in the waveforms as well as in the spatial amplitude pattern is observed. We notice a slight asymmetry in the measured data. This is most likely caused by a small misalignment in the setup. In Figure 5.10 the corresponding frequency spectra are shown. We recall that the $A_C = 0.19$ value is obtained from Fourier spectra of the entire field shown in Figures 5.8 and 5.10.

To compare predicted amplitudes with measurements more directly, peak-to-peak amplitudes (V_{pp}) are plotted in Figure 5.11 and compared with our model (dashed lines). Again, the spatial amplitude pattern is excellently predicted by the-

Table 5.2: Electrochemical parameters and amplitude field scaling factors A_C . Conductivities were directly measured in the fluid surrounding the porous medium, whereas zeta-potentials (ζ) were obtained from Zetasizer Nano ZS measurements on crushed porous sample material suspensions in the corresponding salt solutions (see appendix A of this thesis for details).

Salinity (M NaCl)	Conductivity (S/m)	ζ (mV)	A_C
10^{-4}	1.27×10^{-3}	-51.7	0.03
10^{-3}	1.20×10^{-2}	-61.5	0.19
10^{-2}	1.01×10^{-1}	-58.1	0.41

Figure 5.6: Modelled reflected electric potential for an acoustic source at $(r^S, x_3^S) = (0, -15)$ cm (top) and its frequency spectrum (bottom).

ory for the measurements along the negative x_3 -axis (top) and parallel to the r -axis at $x_3 = -1.3$ cm (bottom). We also check the repeatability of the experiment. There is a small amplitude decrease over two days. The waveform is preserved on these longer timescales. In Figures 5.11 (grey lines), we also plot the vertical electric dipole approximation $A_D x_3 e^{-ik^{EM}d} / (r^2 + x_3^2)^{3/2}$, which is commonly used to describe the spatial amplitude pattern of seismoelectric interface response conversions (see e.g., Dupuis *et al.*, 2007; Haines *et al.*, 2007). The coefficient d is the distance from the origin and the coefficient A_D is obtained from a best-fit procedure. The vertical electric dipole approximation overestimates the spatial amplitude decay. This means that the full model [equation (5.5)] is needed to explain the amplitude measurements.

Figures 5.12 and 5.13 show measured and full-waveform modelled V_{pp} -values where $C = 10^{-4}$ and $C = 10^{-2}$ M NaCl solutions are used, respectively. We determine the A_C -values in the same way as for the 10^{-3} M NaCl solution. Resulting values are given in Table 5.2. The spatial amplitude pattern is also excellently predicted for these salt solutions.

For higher salinities, the A_C -values are closer to (or approach) unity. This observation is in agreement with Block & Harris (2006), who find that the discrepancy between electrokinetic theoretical predictions and measurements decreases with increasing salinity. Block & Harris (2006) also suggest that when low salinity electrolytes are used to saturate the pores of sandstones, surface conductivity effects other than those predicted by Pride's electrokinetic theory may become dominant. The

Figure 5.7: Measured (solid lines) and modelled (dashed lines) electric potential waveforms at various positions in the fluid along the x_3 -axis at $r = 0$ for $C = 10^{-3}$ M NaCl.

Figure 5.8: Measured (solid lines) and modelled (dashed lines) electric potential spectra at various positions in the fluid along the x_3 -axis at $r = 0$ for $C = 10^{-3}$ M NaCl.

Figure 5.9: Measured (solid lines) and modelled (dashed lines) electric potential waveforms at various positions in the fluid parallel to the r -axis at $x_3 = -1.3$ cm for $C = 10^{-3}$ M NaCl.

Figure 5.10: Measured (solid lines) and modelled (dashed lines) electric potential spectra at various positions in the fluid parallel to the r -axis at $x_3 = -1.3$ cm for $C = 10^{-3}$ M NaCl. The black and grey solid lines correspond with measured spectra for negative and positive r -coordinates, respectively.

Pride theory used here incorporates diffuse layer excess conduction associated with electromigration and electroosmosis. When the pore width is much larger than the Debye length, which is a measure of the diffuse layer thickness, surface conductivity effects become negligible with increasing salinity. Our sample has a 16–40 μm pore width, while the Debye length is on the order 3–30 nm.

We also observe that for all salinities, the A_C -values differ from unity. Several possible reasons can be identified. First, the zeta-potentials are obtained from crushed sample material suspensions. A Zetasizer is used for this (see appendix A of this thesis). Probably, the zeta-potential measured on ground-up material has a much higher value than that measured on intact (aged) surface material. This observation alone could be responsible for A_C being different from unity. We also performed zeta-potential calibration measurements on 99.8% silicon dioxide (SiO_2) powder suspensions. The measured values were different by a factor of approximately two to three from streaming potential measurements reported in literature (see e.g., Li & De Bruyn, 1966; Pride & Morgan, 1991), which indicates the uncertainty in our zeta-potential determination. Second, the fluid conductivities given in Table 5.2 are measured in the fluid outside the pores; therefore, the actual pore fluid conductivity may be different. Finally, the seismoelectric interface response amplitudes decrease on timescales that are longer (approximately 24 hours) than

Figure 5.11: Measured (symbols) and modelled (dashed lines) peak-to-peak amplitudes V_{pp} as a function of x_3 at $r = 0$ (top) and r at $x_3 = -1.3$ cm (bottom) for $C = 10^{-3}$ M NaCl. Experiments were repeated several times to check repeatability. Here, $A_C = 0.19$. Also, the electric dipole approximation is plotted (grey lines).

Figure 5.12: Measured (symbols) and modelled (dashed lines) peak-to-peak amplitudes V_{pp} as a function of x_3 at $r = 0$ (top) and r at $x_3 = -1.3$ cm (bottom) for $C = 10^{-4}$ M NaCl. Experiments were repeated several times to check repeatability. Here, $A_C = 0.03$.

Figure 5.13: Measured (symbols) and modelled (dashed lines) peak-to-peak amplitudes V_{pp} as a function of x_3 at $r = 0$ (top) and r at $x_3 = -1.3$ cm (bottom) for $C = 10^{-2}$ M NaCl. Experiments were repeated several times to check repeatability. Here, $A_C = 0.41$.

the duration of a single measurement series (1–4 hours). For example, the measured amplitudes in Figure 5.13 from the last measurement series (143–144 hours) are, on average, 1.9 times smaller than those of the first series (0–1 hours). This observation could be related to increasing bulk conductivity (Block & Harris, 2006) and/or incomplete equilibration of the electric double layer. For silica, the electric double layer may take several days to become stable (Morgan *et al.*, 1989), and it may take hours at soil interfaces (Liu *et al.*, 2008). One could also speculate on the electrodes developing a voltage drop at their surface, thus reducing the measured amplitudes over time. However, replacing the electrodes under stable experimental conditions does not change potential readings significantly.

5.2.5 Conclusions

The electrokinetic conversion of an acoustic wave to an EM-wave at an interface was systematically investigated in a laboratory setup. We independently determined the mechanical and electrical parameters. It was demonstrated that this effect is well predicted by electrokinetic theory in terms of waveform and spatial amplitude pattern. At $C = 10^{-3}$ and $C = 10^{-2}$ M NaCl, amplitude field scaling factors of 0.19 and 0.41 were used. At $C = 10^{-4}$ M NaCl, however, the value was only 0.03, which may be related to surface conductivity effects other than those present in electrokinetic theory. The magnitude of acoustic to EM-conversion was shown to depend on the salinity of the saturating fluid, which is promising for applications in the oil and gas industry because the zones of interest are saturated with (resistive) hydrocarbons instead of water. A careful characterization of the source radiation pattern was part of the forward model, indicating that electrokinetic interface response predictions require proper modelling of the source. The results of this study indicate that the acoustic-to-EM wave responses of these rock/fluid combinations can be predicted by a full-waveform model for general subsurface exploration purposes.

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5.3 Electro seismic measurements

In section 5.2, a seismoelectric laboratory setup and a seismoelectric forward model are described. Now we describe an electro seismic laboratory setup and present electro seismic measurements.

5.3.1 Electro seismic laboratory setup

The experimental setup at the Earth Resources Laboratory of the Massachusetts Institute of Technology (see Figure 5.14) consists of a $100 \times 60 \times 50$ cm water tank in which a positive and a negative 5.5×6.0 cm copper mesh electrode are on both sides of a fluid-saturated porous sample. A receiver hydrophone (Brüel and Kjær 8103) is located 19 cm at the right side of the negative electrode (see Figure 5.14). A 100 kHz single sine electric pulse, with an amplitude of 10 V, from a waveform generator (Hewlett Packard 3314A) is fed into an amplifier (AE Techtron 3620 Linear Amplifier), which amplifies the signal 15 times. The resulting 150 V source pulse is coupled into the positive and negative electrodes. The negative electrode is connected to ground level. The pressure is recorded by the calibrated receiver hydrophone, which is connected to a 60 dB preamplifier (Olympus NDT 5660C) and a high-pass analogue filter (Krohn-Hite 3202R) set at 40 kHz. A computer-controlled oscilloscope (Analogic Data Precision 6000) is used for the recordings. A numerical 196 kHz low-pass filter is applied to the recordings. The sintered glass porous sample (parameters listed in Table 5.1) has a width of 2.2 cm. Water conductivity σ_{fl} was 2.6×10^{-2} S/m and the source electrodes are placed ~ 4 cm away from the sample. An average recording over 1024 source pulses is shown in Figure 5.15 (top). Pressure waves arrive at around 0.14 ms. When the porous sample is removed, the pressure measurement shown in Figure 5.15 (bottom) is obtained. Pressure waves arrive at around 0.14 ms and 0.20 ms. These are due to thermal dissipation of the source electric pulse energy around the positive and negative electrodes, which creates propagating acoustic waves (Zhu *et al.*, 2008a). Indeed, the acoustic travel times correspond with the distances to the electrodes. These thermodilatation pulses are also present in the porous sample recordings. The larger amplitude pressure pulses in between the thermodilatation waves are due to electro seismic conversion.

5.3.2 Frequency-dependent electro seismic conversions

Waveform generator frequencies are set in the range 50–150 kHz. Figure 5.16 shows hydrophone receiver recordings at the 50, 100, and 150 kHz waveform generator frequencies. We observe electro seismic arrivals for each frequency. Note that the amplitude of the 150 kHz recording is smaller than the others. Observed electro seismic amplitudes as a function of frequency are shown in Figure 5.17. A maximum is observed at 70 kHz.

Figure 5.14: Schematic of the MIT Earth Resources Laboratory electroseismic experimental setup. The negative electrode is connected to ground level. Clamps attached to support stands suspend the electrodes, which are 6.0 cm in width and 5.5 cm in height.

Figure 5.15: Hydrophone receiver recording in electro seismic laboratory setup (top) and measurement without a porous sample (bottom). Pressure signals associated with the source electrodes are indicated with “Thermodilatation”.

Figure 5.16: Hydrophone receiver recordings as a function of waveform generator frequency. Pressure signals associated with the source electrodes are indicated with “Thermodilatation”.

Figure 5.17: Measured pressure amplitudes as a function of waveform generator frequency.

5.3.3 Conclusions

Oscillating AC electric fields over a porous sample are unambiguously shown to generate acoustic pressure waves in the fluid surrounding the sample. As the wavelength of the fluid pressure field (approximately 3 cm at 50 kHz and 1 cm at 150 kHz) is much smaller than that of the fluid electric field (approximately 44 m at 50 kHz and 26 m at 150 kHz) a compressional wave is detected by the pressure sensor. The water conductivity was 2.6×10^{-2} S/m, while that of the sintered glass porous sample was approximately 4.3×10^{-3} S/m (σ_{fl}/F , where $F = \alpha_{\infty}/\phi$). Note, however, that also a mechanical contrast is created over the water/porous-sample interfaces, so that the recorded electroseismic conversion cannot be attributed to the conductivity contrast alone.

Chapter 6

Laboratory measurements and theoretical modelling of seismoelectric interface response and coseismic wave fields¹

Abstract

A full-waveform seismoelectric numerical model incorporating the directivity pattern of a pressure source is developed. This model provides predictions of coseismic electric fields and the electromagnetic waves that originate from a fluid/porous-medium interface. An experimental setup in which coseismic electric fields and interface responses are measured is constructed. The seismoelectric origin of the signals is confirmed. The numerically predicted polarity reversal of the interfacial signal and seismoelectric effects due to multiple scattering are detected in the measurements. Both the simulated coseismic electric fields and the electromagnetic waves originating from interfaces agree with the measurements in terms of travel times, waveform, polarity, amplitude and spatial amplitude decay, demonstrating that seismoelectric effects are comprehensively described by theory.

6.1 Introduction

Grain surfaces in contact with an electrolyte acquire a chemically bound surface charge that is balanced by mobile counter ions in a thin fluid layer surrounding the grains. The distribution of mobile ions is determined by a balance between electrostatic forces and thermal diffusivity. At the interface between the immobile and counter ions the so-called zeta-potential is defined. This electric potential decreases exponentially when one moves away from the interface. The characteristic decay

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length (the Debye length) is on the order of a few nanometers for typical grain-electrolyte combinations (Pride, 1994). The ensemble of bound and mobile charge layers is referred to as the electric double layer.

When the pore fluid is in motion, the mobile counter ions in the double layer are transported by the flow with respect to the charge bound to the grain surface. Thus a seismic wave that travels through a porous medium generates an electric wave current on the scale of the wavelength. A comprehensive theory for coupled seismic and electromagnetic wave propagation identifying the zeta-potential as the key parameter was derived by Pride (1994). Two seismoelectric effects can be distinguished: (1) a coseismic electric field is confined to seismic waves and therefore propagates with seismic wave velocity (e.g., Pride & Haartsen, 1996) and (2) a seismic wave that traverses a boundary between two distinct porous media produces electromagnetic (EM)-waves that propagate outside the support of the seismic waves with much higher EM-wave speeds (e.g., Pride & Haartsen, 1996; Haartsen & Pride, 1997). Following the terminology in Haines & Pride (2006) we refer to the latter as the interface response. Both coseismic and interface response effects are visible in full-waveform numerical simulations (e.g., Haartsen & Pride, 1997; Garambois & Dietrich, 2002; Haines & Pride, 2006; Jardani *et al.*, 2010; Revil & Jardani, 2010). Furthermore, a wide range of field and laboratory validations of the coseismic field and the interface response was presented in the literature (e.g., Long & Rivers, 1975; Thompson & Gist, 1993; Butler *et al.*, 1996; Mikhailov *et al.*, 1997; Beamish, 1999; Zhu *et al.*, 1999, 2000; Garambois & Dietrich, 2001; Zhu & Toksöz, 2003; Block & Harris, 2006; Bordes *et al.*, 2006; Haines *et al.*, 2007; Dupuis *et al.*, 2007, 2009). Several of these works compare either field measurements (i.e., Mikhailov *et al.*, 1997; Haines *et al.*, 2007) or laboratory measurements (i.e., Zhu *et al.*, 2000) with a seismoelectric wave propagation model, corroborating that the coseismic and interface response fields are predicted by the theory.

In this article we complement the study presented in Zhu *et al.* (2000) by developing a full-waveform seismoelectric model for the coseismic and interface response fields based on the Sommerfeld integral, and comparing the numerical predictions with laboratory measurements. We demonstrate the seismoelectric nature of the measured electric potentials in a control experiment. The results are distinct from previous studies in the sense that measured seismoelectric multiples as well as a polarity reversal across an interface are predicted by our model and a typical directivity pattern of the acoustic source wavefield is incorporated in the simulation.

6.2 Modelling of coseismic and interface response fields

We describe our modelling approach in the context of numerical simulations of our laboratory experiments. An isotropic, homogeneous fluid-saturated porous medium layer is within a compressible fluid (see Figure 6.1). The acoustic source pressure is

Figure 6.1: Geometry of the seismoelectric model and measurements. The total electric potential wavefield is given by an integral over complex-valued θ [equations (6.5) and (6.8)]. Positions 1-5 are separated by 0.5 cm with position 1 at $(r, x_3) = (0, -2.3)$ cm and position 5 at $(r, x_3) = (0, -0.3)$ cm. Position 6 is at $(r, x_3) = (0, 1)$ cm.

modelled as a modified spherical wave source (e.g., Hall, 1987)

$$\hat{p}^S(\omega, R^S, \theta) = \frac{W(\omega)}{R^S} e^{-ik^{fl}R^S} D(\theta), \quad (6.1)$$

where ω is the radial frequency, $R^S = \sqrt{(r - r^S)^2 + (x_3 - x_3^S)^2}$ is the distance to the source, θ is the angle of incidence, $W(\omega)$ is the amplitude spectrum, and k^{fl} is the fluid wavenumber. The directivity function $D(\theta)$, that characterizes the radiation pattern of the source, is given by

$$D(\theta) = \frac{J_1(k^{fl}a \sin \theta)}{k^{fl}a \sin \theta}. \quad (6.2)$$

Here, J_1 is the Bessel function of the first kind and first order and a is the radius of the transducer. The modelled source pressure wavefield is expanded into conical waves by means of the Sommerfeld integral (see e.g., Brekhovskikh, 1960; Aki & Richards, 2002). The total reflected electric potential in the fluid $\hat{\psi}^{fl}(\omega, r^R, x_3^R)$, received at position (r^R, x_3^R) , can then be expressed as ($r^S = 0$; $x_3^S, x_3^R < 0$)

$$\hat{\psi}^{fl}(\omega, r^R, x_3^R) = -iW(\omega) \int_0^\infty \frac{k_r}{k_3^{fl}} D(k_r) J_0(k_r r^R) e^{ik_3^{fl} x_3^S} r_{EM,P}^p(k_r) e^{ik_3^{EM} x_3^R} dk_r, \quad (6.3)$$

where $k_r = k^{fl} \sin \theta$ and $k_3^{fl} = k^{fl} \cos \theta$ are the radial and vertical components of k^{fl} , respectively, and k_3^{EM} is the vertical component of the fluid EM-wavenumber.

The seismoelectric conversion coefficient, $r_{EM,P}^p(k_r)$, relates the incident pressure wavefront to an EM signal in the fluid. This conversion coefficient was derived in chapter 3, adopting full seismoelectric theory. Note that this coefficient is distinct from an ordinary reflection coefficient as it describes the conversion of mechanical energy into EM energy. Also note that the notation in chapter 3 is slightly different (see Table 6.1). Expressing k_r and k_3^{fl} in terms of k^{fl} and θ , equation (6.3) is written as,

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -iW(\omega) \int_0^{\pi/2+i\infty} D(\theta) k^{fl} \sin \theta J_0(k^{fl} r^R \sin \theta) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}^p(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta, \end{aligned} \quad (6.4)$$

where $k_3^{EM}(\theta) = \omega \sqrt{[s_{EM}^{fl}(\omega)]^2 - \sin^2 \theta s_P^2}$ and $\text{Im}[k_3^{EM}(\theta) < 0]$, with $s_{EM}^{fl}(\omega)$ and s_P the fluid EM and pressure wave slownesses, respectively. The path of integration is along straight lines from 0 to $\pi/2$ and from $\pi/2$ to $\pi/2 + i\infty$ in the complex θ -plane. The second integral over complex θ is simplified further employing the substitution $\theta = \pi/2 + i \ln[\sqrt{\gamma^2 + 1} + \gamma]$

$$\begin{aligned} \hat{\psi}^{fl}(\omega, r^R, x_3^R) = & -i \frac{W(\omega)}{a} \int_0^{\pi/2} J_0(k^{fl} r^R \sin \theta) J_1(k^{fl} a \sin \theta) \\ & \times e^{ik^{fl} x_3^S \cos \theta} r_{EM,P}^p(\theta) e^{ik_3^{EM}(\theta) x_3^R} d\theta \\ & + \frac{W(\omega)}{a} \int_0^\infty J_0(k^{fl} r^R \sqrt{\gamma^2 + 1}) \frac{J_1(k^{fl} a \sqrt{\gamma^2 + 1})}{\sqrt{\gamma^2 + 1}} \\ & \times e^{k^{fl} x_3^S \gamma} r_{EM,P}^p(\gamma) e^{ik_3^{EM}(\gamma) x_3^R} d\gamma, \end{aligned} \quad (6.5)$$

where equation (6.2) was used for $D(\theta)$. Equation (6.5) can readily be numerically evaluated and yields the total reflected EM-wavefield in the fluid.

Table 6.1: Relation between seismoelectric conversion coefficients in this chapter and those given in chapter 3. The fluid density is denoted by ρ_{fl} .

This chapter	Chapter 3
$r_{EM,P}^p$	$r_{EM,P}/\omega^2 \rho_{fl}$
$t_{TM,P}^f$	$\alpha_{TM}(\omega) t_{TM,P}/\omega^2 \rho_{fl}$
$\alpha_{Pf}^p(\omega) t_{Pf,P}^f$	$\alpha_{Pf}(\omega) t_{Pf,P}/\omega^2 \rho_{fl}$

In the porous medium, pressure waves propagate accompanied by their coseismic fields. We take into account the coseismic field belonging to the pressure wave transmitted from the fluid to the porous medium, as well as the one belonging to the pressure wave that is subsequently reflected from the back of the sample. Only the (fast) pressure (Pf)-wave is considered as it carries virtually all of the acoustic energy. The acoustic waves generate freely propagating transverse magnetic (TM) EM-waves when they encounter interfaces. We take three subsequently generated interface responses into account. We incorporate these coseismic and interface response fields into the Sommerfeld integral. Analogously to equation (6.3) we arrive at the following expression for the electric potential within the porous medium ($r^S = 0$; $x_3^S < 0$ and $x_3^R > 0$):

$$\begin{aligned} \hat{\varphi}^p = & -iW \int_0^\infty \frac{k_r}{k_3^{fl}} D J_0 e^{ik_3^{fl} x_3^S} \\ & \times \left[t_{TM,P}^f e^{-ik_3^{TM} x_3^R} + t_{Pf,P}^f \left(\alpha_{Pf}^p e^{-ik_3^{Pf} x_3^R} + e^{-ik_3^{Pf} w_s} S^E \right) \right] dk_r, \end{aligned} \quad (6.6)$$

where the functional dependencies are not explicitly written. The width of the porous sample is denoted as w_s . The term $t_{TM,P}^f \exp[-ik_3^{TM} x_3^R]$ describes the propagation of the first TM-wave to the receiver at x_3^R . This wave is generated at $x_3 = 0$ by the incident acoustic wave from the fluid. The coefficient $t_{TM,P}^f$ represents the transmission of converted electric potential due to incident acoustic potential at the front of the sample. The term $\alpha_{Pf}^p t_{Pf,P}^f \exp[-ik_3^{Pf} x_3^R]$ describes the coseismic electric field travelling along with the Pf-wave in the porous medium. The amplitude ratio between the electric and the pressure fields is described by α_{Pf}^p , and $t_{Pf,P}^f$ is the acoustic transmission coefficient. The Pf-wave travels to the back of the porous sample where it is reflected upward. The associated seismoelectric effects are described by S^E

$$S^E = r_{TM,Pf}^b e^{-ik_3^{TM}(w_s - x_3^R)} + r_{Pf,Pf}^b \left[\alpha_{Pf}^p e^{-ik_3^{Pf}(w_s - x_3^R)} + e^{-ik_3^{Pf} w_s} r_{TM,Pf}^f e^{-ik_3^{TM} x_3^R} \right]. \quad (6.7)$$

The first two terms in equation (6.7) describe the upwardly propagating second TM and coseismic waves, respectively. The last term describes the third interface response that is generated at $x_3 = 0$ by the incident Pf-wave from below and now propagates downwards as a TM-wave to the receiver. The seismoelectric conversion coefficients at the back and the front of the sample are denoted as $r_{TM,Pf}^b$ and $r_{TM,Pf}^f$, respectively. The reflection coefficient for the Pf-wave at the back of the sample is denoted $r_{Pf,Pf}^b$. These coefficients at a fluid/porous-medium interface due to an incident Pf-wave are derived in a procedure similar to the one described in chapter 3. The notations are slightly different from those in chapter 3 (see Table

6.1). In a similar way as for equation (6.5), we find from equation (6.6)

$$\begin{aligned} \hat{\varphi}^p = & -i \frac{W}{a} \int_0^{\pi/2} J_0 J_1 e^{ik^{fl} x_3^S \cos \theta} \\ & \times \left[t_{TM,P}^f e^{-ik_3^{TM} x_3^R} + t_{Pf,P}^f \left(\alpha_{Pf}^p e^{-ik_3^{Pf} x_3^R} + e^{-ik_3^{Pf} w_s} S^E \right) \right] d\theta \\ & + \frac{W}{a} \int_0^\infty J_0 \frac{J_1}{\sqrt{\gamma^2 + 1}} e^{k^{fl} x_3^S \gamma} \\ & \times \left[t_{TM,P}^f e^{-ik_3^{TM} x_3^R} + t_{Pf,P}^f \left(\alpha_{Pf}^p e^{-ik_3^{Pf} x_3^R} + e^{-ik_3^{Pf} w_s} S^E \right) \right] d\gamma. \end{aligned} \quad (6.8)$$

where equation (6.2) was used for $D(\theta)$. Equation (6.8) describes the electric potential within the porous medium. Equations (6.5) and (6.8) are numerically evaluated through the inverse fast Fourier transform. For each frequency we use a recursive adaptive Simpson quadrature algorithm implemented in Matlab. An experimentally recorded pressure waveform at $(r, x_3) = (0, 0)$ is used for $W(\omega)$ and a 144–896 kHz numerical band-pass filter is applied. The input parameters of Table 6.2 are used. Figure 6.2 (left) shows the resulting modelled reflected electric potentials at positions 1-6 along the x_3 -axis (Figure 6.1).

An electric pulse is computed around 0.10 ms at each position, which corresponds to the travel time of the acoustic wave from the source to the porous medium surface, where it generates an EM signal. Due to the large fluid EM-wave speed ($\sim 1.0 \times 10^7$ m/s), the electric pulses arrive almost simultaneously at each position. Note that the pulse decays in amplitude as the distance from the interface increases (positions 1-5 in Figure 6.1). Position 6 is inside the porous medium. The first pulse arriving around 0.10 ms at this position has a reversed polarity with respect to the fluid computations. This observation is in agreement with the description of the interface response modelled as the radiation from an oscillating electric dipole oriented vertically with respect to the interface (Haartsen & Pride, 1997; Haines *et al.*, 2007). The second pulse at position 6 arrives 5.7 μ s later. It travels along with the Pf-wave at approximately 1900 m/s from the front of the sample to position 6. Therefore it is due to the coseismic electric field supported by the Pf-wave. The pulse arriving around 0.12 ms corresponds with the Pf-wave travel time to the back of the porous sample, where it generates a reflected EM-wave. The fourth and fifth pulses are, respectively, coseismic and interface responses of the compressional wave multiple.

6.3 Seismoelectric experiments

6.3.1 Seismoelectric experimental setup and measurements

The experimental setup consists of a $58 \times 39 \times 28$ cm water tank in which an acoustic wave source transducer (Panametrics V3638), grounded electrodes (A-M

Table 6.2: Parameters of the porous medium. The temperature is 293.15 K.

K_s	Bulk modulus skeleton grains ^a	49.9×10^9	Pa
K_f, K_{fl}	Bulk modulus (pore) fluid ^b	2.2×10^9	Pa
η	Pore fluid viscosity ^b	1×10^{-3}	Pa s
ρ_f, ρ_{fl}	(pore) Fluid density ^b	998	kg/m ³
$\epsilon_f, \epsilon_{fl}$	Relative permittivity of the (pore) fluid ^b	80.1	—
ϵ_s	Relative permittivity of the solid ^b	4	—
Λ	Weighted pore volume-to-surface ratio ^c	9.43×10^{-6}	m
μ, μ_{fl}	(fluid) Magnetic permeability ($=\mu_0$)	$4\pi \times 10^{-7}$	H/m
K_b	Bulk modulus framework of grains ^d	0.93×10^9	Pa
G	Shear modulus framework of grains ^d	0.88×10^9	Pa
ϕ	Porosity of the porous medium ^d	0.52	—
ρ_s	Solid density ^d	2.57×10^3	kg/m ³
k_0	Permeability ^d	3.4×10^{-12}	m ²
α_∞	Tortuosity of the porous medium ^d	1.7	—
σ_f, σ_{fl}	(pore) Fluid conductivity ^e	4.8×10^{-2}	S/m
ζ	Zeta-potential ^f	-40×10^{-3}	V
w_s	Sample width ^e	3.21×10^{-2}	m

^aJohnson & Plona (1982).

^bLide (2009-2010). We take the value of Pyrex 7070 glass for the solid permittivity.

^cPride (1994); Johnson *et al.* (1987, 1994); Jocker & Smeulders (2009).

^dSee N5B in Wisse (1999).

^eMeasured values.

^fSee Pride & Garambois (2005). We assume that conductivity is due to a NaCl salt solution and pH = 6.

systems, Inc., Ag/AgCl diameter 0.015 in.) and a porous sample are installed. The geometry of the experiment is depicted in Figure 6.1. The sample is made of sintered crushed glass (parameters listed in Table 6.2), carefully saturated, and immersed in tap water. The electrode at position 6 is installed in a 1.4 mm diameter cylindrical hole, which was drilled in the porous sample. A 500 kHz single sine pulse (with a 500 mV peak-to-peak amplitude) from a waveform generator (Agilent Technologies 33220A) is fed into an amplifier (ENI 2100L RF Power Amplifier) set at 50 dB gain. The output signal is coupled into a piezoelectric source transducer. A 144–896 kHz band-pass filter is applied in all measurements. Electric potential measurements were performed at positions 1-6. These on-axis measurements are averaged over 8192 source pulses and shown in Figure 6.2 (right). Indeed, the measured seismoelectric pulse patterns are predicted by the model.

The predicted electric potentials (Figure 6.2, left) agree with the measurements (Figure 6.2, right) in terms of travel times, waveform, polarity and the spatial amplitude decay. Note that the panels on the right are scaled down by a factor 2.5.

Figure 6.2: Numerical electric potential waveform predictions (left) and measurements (right) within the fluid (positions 1-5) and within the porous sample (position 6).

This means that the measured amplitudes are lower than predicted, which can be explained by uncertainties in the parameters given in Table 6.2. Notably, our assumption on the zeta-potential may not be fully appropriate. Identical amplitude scaling is present in all the reflected interface response signals at positions 1-5 and the transmitted signal at position 6 that contains coseismic and interface response effects. This means that, by introducing a scaling factor, all coseismic and interface response effects are correctly described by theory. The small remaining differences in waveform are probably related to minor geometric misalignment in the experiments (the transducer-porous sample-electrode geometry deviates slightly from model geometry) and to the ruggedness of the porous sample surface. Also, the predicted polarity reversal across the fluid/porous-medium interface (positions 1-5 versus position 6) is clearly visible in the measurements. In Dupuis *et al.* (2009), a polarity reversal was also reported in a borehole penetrating the water table. Faint arrivals after the first interface response signal are observed in the recordings at positions 4-5. These do not appear in the computed signals, because only the first interface response is modelled for the fluid positions.

6.3.2 Seismoelectric control experiment

We also performed a control experiment to identify seismoelectric conversion as the single mechanism for the electric potential generation. A sample was dried and jacketed before it was immersed in water. A vacuum pump continuously evacuated the sample. Figure 6.3 (left) shows the measurement at position 5. No appreciable signal can be detected by the electric receiver. Subsequently, the jacketed sample was carefully saturated with de-ionized water and the seismoelectric experiment was repeated. Figure 6.3 (right) shows the result. Now, a clear pulse arrives around 0.1 ms. This experiment unambiguously identifies seismoelectric conversion as the mechanism for electric signal generation. In Zhu *et al.* (2000) and Bordes *et al.* (2006) similar conclusions are drawn from different seismoelectric experiments. It is well-known that solids may generate electric effects when under strain, which is referred to as piezoelectricity. This effect however, cannot explain the observations in Figure 6.3, as any piezoelectricity would be screened by the electrolyte (Pride & Haartsen, 1996).

Figure 6.3: Electric potential measurement at position 5 on evacuated (left) and fluid-saturated (right) porous sample.

6.4 Conclusions

A seismoelectric numerical model incorporating a typical directivity pattern of the acoustic source wavefield provided full-waveform numerical predictions for both the coseismic and interface response effects. The existence of the seismoelectric coupling phenomenon was validated in our experimental setup. Measurements of the coseismic electric field and the fluid/porous-medium interface response were performed. Agreement between the predicted and measured wavefields was shown in terms of travel times, waveform, polarity and the spatial amplitude decay. Furthermore, the predicted polarity reversal and Pf-wave multiple scattering effects were validated by the measurements. The unresolved scaling issue is probably primarily caused by variations in pore fluid conductivity and the zeta-potential that are inaccessible to direct independent measurements. Yet, the overall agreement between the measurements and numerical predictions convincingly validates the existing seismoelectric theory. As both the numerical and experimental techniques can be generalized to field situations, this work allows us to appreciate the use of seismoelectric effects as exploration techniques that are complementary to conventional seismic surveys for exploring the subsurface.

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Chapter 7

Conclusions

In this thesis, coupled seismic and electromagnetic wave propagation was studied both numerically and experimentally. Several conclusions are drawn. These are listed below.

7.1 Electrokinetic theory and wave modelling conclusions

1. The dispersion relations of the four electrokinetic wave modes can be expressed in terms of generalized elastic coefficients and effective frequency-dependent densities. These effective densities incorporate viscous and electrokinetic coupling effects. Binomial series expansions of the dispersion relations show that: (1) the Pf and S-wave are propagatory at low frequencies, while the Ps and EM-wave are diffusive at low frequencies; (2) all electrokinetic wave modes are propagatory at high frequencies; (3) electrokinetic effects on wave speed and attenuation are typically small for real rock samples. The fluid-to-solid amplitude ratios can also be expressed in terms of the generalized elastic coefficients and effective frequency-dependent densities. The ratios of electric and displacement potentials are in turn a function of the fluid-to-solid ratios and the electrokinetic coupling coefficient.
2. Using a simple substitution that relates the zeta-potential to excess electrical charge in the pore volume, the low-frequency transport equations of Pride (1994) can be recast into the electrokinetic relations of Revil & Linde (2006).
3. The theory of Pride & Haartsen (1996) can be used to solve the electrokinetic scattering problem at a fluid/porous-medium boundary. Similar to the poroelastic case (Dutta & Odé, 1983), energy conservation for electrokinetic scattering problems needs to take both orthodox and interference fluxes into account.
4. Electrokinetic theory predicts that the seismoelectric conversion amplitude varies over orders of magnitude as a function of fluid salinity, pore fluid viscos-

ity and permeability for realistic ranges of these parameters. Other poroelastic parameters (bulk moduli, tortuosity, porosity) also influence seismoelectric conversion.

5. By combining electrokinetic theory with standard seismological techniques (e.g., the Sommerfeld integral and wavefield (de)composition), full-waveform seismoelectric and electroseismic models can be obtained.

7.2 Experimental conclusions

1. In a setup in which a porous sample is immersed in a fluid, both seismoelectric and electroseismic effects can be measured using pressure transducers and electrodes.
2. The seismoelectric interface response and coseismic wave fields can be investigated as a function of time, space and salinity in a seismoelectric laboratory setup. The amplitude of these fields depends on fluid salinity.
3. Seismoelectric nature can be identified as the prime mechanism in our experiments by comparing measurements on an evacuated-pore porous sample with those on the same porous sample, where the pores are filled with a fluid electrolyte. The measurements are caused by the seismoelectric mechanism when the electric potential energy is significantly larger for the fluid-saturated sample. In this case, piezoelectric effects are disqualified as the dominant source of electric potential generation.

7.3 Conclusions inferred from comparisons between electrokinetic wave models and measurements

1. The seismoelectric interface response and coseismic wave fields are well predicted in terms of waveform, spatial amplitude pattern, and travel times, by the electrokinetic theory of Pride (1994) and Pride & Haartsen (1996). This conclusion also holds for interface response and coseismic wave fields generated by seismic multiples.
2. At small conductivity ($\sim 10^{-3}$ S/m) of the saturating fluid, measured seismoelectric amplitudes are overestimated by the electrokinetic theory of Pride (1994) and Pride & Haartsen (1996).
3. The seismoelectric Sommerfeld integral predictions excellently describe the measured spatial amplitude pattern of the seismoelectric interface response. For the experimental situation described in chapter 5, the dipole approximation theoretical pattern significantly deviates from the measured pattern.

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4. Using mesh electrodes to set up an AC electric field, conversion from electric energy into acoustic energy was unambiguously recorded. Electrostatic wave effects can be modelled and measured, allowing for further quantitative comparison between theory and experiment.

Appendix A

Porous medium parameter measurements

Here we describe the experimental procedures for performing independent poroelastic and electrokinetic measurements on porous samples. The following parameters are considered: solid density, porosity, permeability, tortuosity, frame modulus, shear modulus and zeta-potential. The results for the sintered glass porous sample of chapter 5 are listed in Tables A.1 and A.2.

A.1 Solid density and porosity

Solid density ρ_s and porosity ϕ are measured by determining the dry mass m_d and fluid-saturated, submerged mass m_{fl} of a (cylindrical) core of the porous sample. From a momentum analysis it follows that

$$\rho_s = \frac{m_d}{m_d - m_{fl}} \rho_{fl}, \quad (\text{A.1})$$

$$\phi = 1 - \frac{m_d - m_{fl}}{\rho_{fl} V_b}, \quad (\text{A.2})$$

where ρ_{fl} is the fluid density and V_b is the bulk volume.

A.2 Permeability

The permeability k_0 is measured by performing a one-dimensional flow experiment (Figure A.1). A cylinder (2 cm internal diameter) is suspended above a silicon oil (Bluestar Silicones, Rhodorsil Oils 47V5; density $\rho_o = 915 \text{ kg/m}^3$, viscosity $\eta \sim 5 \times 10^{-3} \text{ Pa s}$) reservoir. A vacuum pump (KNF Laboport N 820.3 FT.18) is connected to the top of the cylinder and an oil-saturated cylindrical porous sample is attached at the bottom end of the cylinder using a tight rubber sleeve. The fluid is carefully pulled up by evacuating the cylinder. At the start of the experiment, atmospheric pressure is created above the fluid level $h(t)$, which is monitored from time $t = 0$

Figure A.1: Permeability experimental setup.

onwards. The one-dimensional Darcy's law applies

$$Q_f = -\frac{k_0 A_s}{\eta} \frac{p_d}{L_s}, \quad (\text{A.3})$$

where Q_f is the fluid volume flow rate, A_s is the cross-sectional area of the sample and p_d is the differential pressure applied across the sample of length L_s . Equation (A.3) is rewritten by expressing Q_f in terms of the cross-sectional area of the cylinder A_c and $h(t)$, and by expressing p_d in terms of the gravitational constant g , $h(t)$ and ρ_o . We arrive at

$$\ln \left[\frac{h_i}{h(t)} \right] / c_0 = k_0 t, \quad (\text{A.4})$$

where h_i is the initial height and $c_0 = A_s \rho_o g / A_c \eta L_s$. Figure A.2 plots $\ln [h_i / h(t)] / c_0$ against t for repeated experiments on two cores ($A_s = 4.86 \text{ cm}^2$; $L_s = 2 \text{ cm}$) drilled from the sintered glass porous sample (see chapter 5). Obviously, the permeability is determined from the slope. The linear trend verifies the applicability of Darcy's law as is expected from the maximum Reynolds number estimate, which is 3.2×10^{-3} (Mavko *et al.*, 2003).

Figure A.2: LHS equation (A.4) against t . Slopes correspond with the permeability measurements, which are indicated. Two runs are shown per porous sample core.

A.3 Tortuosity

The tortuosity α_∞ is determined from the ratio of fluid conductivity σ_{fl} to porous sample conductivity σ_p . For a nonconducting rigid porous frame saturated with an inviscid, incompressible fluid, the electrical ratio σ_{fl}/σ_p is related to tortuosity as follows (Brown, 1980; Johnson *et al.*, 1982):

$$\frac{\sigma_{fl}}{\sigma_p} = \frac{\alpha_\infty}{\phi}. \quad (\text{A.5})$$

The experimental setup is depicted in Figure A.3. A cylindrical porous sample is jacketed in a tight (nonconducting) rubber sleeve and high grade stainless steel cylindrical electrodes are attached at its end faces. The aggregate is placed in a core holder and 2–3 bar confining pressure is applied. Fluid reservoirs are installed at either side of the setup. The fluid can flow through the electrodes. The sample is carefully saturated with reservoir fluid of high salinity ($\sigma_{fl} = 1.16$ S/m). Note that surface conductivity effects become negligible with increasing fluid conductivity (see e.g., Morgan *et al.*, 1989). Frequency-dependent porous sample resistance (Wayne Kerr component analyzer 6440A) is measured directly after saturation. The value at 100 kHz is used to calculate σ_p . A conductivity measurement (WTW LF 340) in the fluid reservoir yields σ_{fl} .

Figure A.3: Tortuosity experimental setup.

A.4 Frame and shear modulus

The (fast) compressional and shear wave speeds, respectively v_{Pf} and v_S , of air saturated ($\rho_{fl}/\rho_s \sim 0$) porous samples can be related to the frame modulus K_b and shear modulus G as follows (Visser, 1988; Wisse, 1999):

$$v_{Pf} = \sqrt{\frac{K_b + 4G/3}{(1 - \phi) \rho_s}}, \quad (\text{A.6})$$

$$v_S = \sqrt{\frac{G}{(1 - \phi) \rho_s}}. \quad (\text{A.7})$$

These velocities are obtained by comparing arrival times in transmission experiments in a submerged experimental setup. A source and receiver transducer span a rotatable air-saturated porous sample that is enclosed in a thin impermeable jacket. The values in Table A.1 are taken from van der Lee (2003).

Table A.1: Poroelastic parameters of the sintered glass porous sample (see chapter 5): solid density ρ_s , porosity ϕ , permeability k_0 , tortuosity α_∞ , frame modulus K_b and shear modulus G .

Sample	ρ_s (kg/m ³)	ϕ	k_0 (m ²)	α_∞	K_b (Pa)	G (Pa)
Intact sample					6.6×10^9	5.5×10^9
Core 1	2.211×10^3	0.331		2.3		
Core 2	2.213×10^3	0.359		1.9		
Core 3	2.212×10^3	0.344		2.1		
Core 4			3.4×10^{-12}			
Core 5			2.8×10^{-12}			

A.5 Zeta-potential

Zeta-potentials (ζ) were obtained from light scattering (Malvern Instruments Ltd., Zetasizer Nano ZS) measurements on particle suspensions in 10^{-4} , 10^{-3} and 10^{-2} M NaCl salt solutions (Table A.2). Grains were scraped off from the sintered glass porous sample (see chapter 5) and crushed. An independent average particle size measurement yielded 831 nm, which is sufficiently small for the Zetasizer apparatus. The suspensions were prepared approximately 24 hours before the measurements. Zeta-potentials obtained from ~ 170 hours old suspensions yielded values that were, on average, 7% smaller. Table A.2 also shows zeta-potential values obtained from 99.8% SiO₂ $\sim 1\mu\text{m}$ particle suspensions. As a reference, values from the empirical relationship given by Pride & Garambois (2005), for pH = 6, are shown.

Table A.2: Zeta-potentials (ζ) obtained from light scattering (Malvern Instruments Ltd., Zetasizer Nano ZS) measurements on sintered glass (see chapter 5) and 99.8% SiO₂ $\sim 1\mu\text{m}$ particles in 10^{-4} , 10^{-3} and 10^{-2} M NaCl salt solutions. Empirical literature values are also shown (Pride & Garambois, 2005).

Salinity (M NaCl)	Sintered glass ζ (mV)	SiO ₂ ζ (mV)	Empirical ζ (mV)
10^{-4}	-51.7	-35.8	-72
10^{-3}	-61.5	-44.9	-52
10^{-2}	-58.1	-52.5	-32

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Curriculum Vitae

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Journal publications

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