BTA/PE/12-08 Model for Gas Sweep with Foam

29-06-2012 R.M. de Velde Harsenhorst
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Abstract

In this BSc-thesis a Surfactant-Alternating-Gas (SAG) foam displacement is represented by an idealized model. Shan and Rossen show in the article ‘Optimal Injection Strategies for Foam IOR’ (2004) that this model, though greatly simplified, is a useful representation of a foam displacement in the physical world, where pressure gradient is the most important factor in controlling gravity override. The process of building the model and numerical problems and solutions are discussed. The foam displacement is extended beyond the range computed by Shan and Rossen, to an dimensionless position $X_D$ of 4. The following cases are considered: $k_v = k_h$, $0<k_v<k_h$ and $k_v = 0$. A comparison shows that the smaller the $k_v$, the less convex the foam displacement front is. Ironically, in this case, increasing vertical permeability reduces the extent of gravity segregation of gas and increases vertical sweep.
Introduction

World energy
The world’s energy demand is expected to continue to rise throughout at least the coming twenty to thirty years. To fulfil in this need for energy, resources of all sorts are needed. Not only does the world’s growing population need more renewables, hydro-electric power and nuclear power, it also needs more coal, natural gas and oil (BP Statistical Review, 2012).

A way to express the remaining fossil fuels beneath the earth’s surface is the R/P-ratio. This is a ratio of the proved reserves to the production in that specific year. The outcome is an estimate of the remaining amount years of production of these fuels remaining. In 2010 the R/P value for oil was 46.2, for coal 118 years and for natural gas 58.6 years. This points out that the resources are finite, which is why it is important to produce the remaining oil and gas reserves as efficiently as possible (BP Statistical Review, 2012).

Recovery stages
The importance of enhanced oil recovery can be seen in Fig. 2. It shows that primary recovery retrieves only 10% of the oil from the reservoir. This production is driven by the natural pressure in the reservoir. As pressure declines over time, or if it is low at the start, artificial lift techniques may be used (Stosur et al., 2003).

Secondary recovery is the next step in getting as much oil as possible out of the reservoir. The most common methods are water flooding and gas injection. These methods are meant to keep the pressure in the reservoir at a significant level and to help the hydrocarbons flow to the surface.

The tertiary recovery stage is also called the Enhanced Oil Recovery (EOR) stage. Enhanced oil recovery is defined as ‘one or more of a variety of processes that seek to improve the recovery of hydrocarbons from a reservoir after the secondary production
phase’. The intent of EOR methods are either to improve sweep efficiency, by reducing the mobility ratio between injected and in-place fluids, or to eliminate or reduce capillary and interfacial forces and thus improve displacement efficiency, or do both at the same time (Alvarado and Manrique, 2010).

To put the importance of EOR in perspective, out of the 3 trillion barrels of oil known to exist in conventional reservoirs, so far only one-third have been produced and consumed since the beginning of the oil business. An additional one-third is projected to be produced by the use of non-conventional techniques and advanced, economically viable EOR-processes (Alvarado and Manrique, 2010).

A general overview of a typical ‘Field Development Plan’, in which EOR is considered or implemented, can be seen in Fig. 3. EOR lengthens the lifetime and recovery of the oilfield.
**Enhanced oil recovery (EOR)**

Why is oil trapped in the reservoir after conventional water flooding and gas injection? There are a number of causes, both at pore and reservoir scale.

The trapping of oil at pore scale is generally caused by a number of interfacial effects that are associated with the rock-fluid interactions. In water-wet rock the interfacial tension between the oil and water phase, below a certain (residual) oil saturation, creates a ‘barrier’ in the pore throats which is difficult to overcome. This means that injected water will go around the hydrocarbons trapped in the pores, because for the oil to pass through pore throats the pressure difference must be great enough to overcome the pressure difference across the tightly curved interface at the pore throat. Injecting for example surfactants can reduce this capillary resistance to flow and mobilize the oil. The dimensionless number used to represent this mechanism is the capillary number:

$$N_{ca} = \frac{v\mu}{\sigma \cos \theta}$$

where \(v\) is superficial velocity of the displacing phase, \(\mu\) the viscosity of the displacing phase, \(\sigma\) the interfacial tension between displacing and trapped phases, and \(\theta\) contact angle on the rock. The larger the capillary number, the less oil is trapped and the better the displacement efficiency (Alvarado and Manrique, 2010).

Bypassing of oil at a larger scale is caused by permeability heterogeneities, channelling of flow through thief zones or fracture networks, and an adverse mobility ratio. When a viscous fluid like oil is pushed forward by a less-viscous displacing phase (gas or water), a phenomenon called viscous fingering can occur. This leads to a significant reduction in sweep efficiency. Lowering the mobility ratio can be accomplished by injecting a viscosified fluid, for example water with polymers. Also gravity override, especially by injected gas, can occur, with the gas sweeping only the top of the reservoir, because of density differences between gas and water or oil (Alvarado and Manrique, 2010).

The various existing EOR methods can be subdivided in three different groups:

- **Thermal**: Including steam stimulation (“huff and puff”), steam flooding, steam-assisted gravity drainage and in-situ combustion. The goal is to reduce the viscosity of the oil.
- **Chemical**: There are three types: First, injection of interfacially-active components such as surfactants and alkalis, polymers, and chemical blends. The goal is to reduce the interfacial tension and thereby the capillary number. The second type is use of polymers to viscosify water and improve sweep efficiency. Often these two types are used together. Third, surfactants can form foam with injected gases, thereby providing a low-mobility drive fluid.
- **Miscible or Solvent Injection**: Injection of hydrocarbon gas, carbon dioxide, and nitrogen. This approach relies on the injectant’s miscibility with or mutual solubility with the oil phase. For instance, CO\(_2\) flooding achieves miscibility by swelling the oil and extraction of oil fractions into CO\(_2\).
Also a combination of the methods listed above is possible. For example WAG (Water-alternating-gas) is used in the field, in which water and gas are injected after one another, using the gas as miscible solvent and the water to reduce the mobility of the gas (Alvarado and Manrique, 2010).

Another combined method is SAG (Surfactant-Alternating-Gas) injection, which is considered in this bachelor’s thesis. In this method alternating slugs of surfactant and gas are injected. The gas can be any gas listed above for miscible injection, i.e. hydrocarbon gas, carbon dioxide or nitrogen. SAG injection creates foam in the formation, as further discussed in the next chapter.
Foams

History
Since the early 1900’s gases, such as steam, carbon dioxide, nitrogen and hydrocarbon gases have been used to drive the oil out of the underground reservoirs in improved oil recovery (IOR) processes (Shan and Rossen, 2004).
The first application of foam to mobility control was proposed in 1958 by Bond and Holbrook. The first research was done in the 1960’s and in 1980 the first experiments on the performance of foam for mobility control in surfactant-alternating-gas flooding processes were performed by Lawson and Reisberg (Namdar Zanganeh, 2011). Actual production tests were done in the late 1980’s, throughout the 1990’s and still are being performed. Therefore many advances have been made in understanding the injection of foam in reservoirs (Namdar Zanganeh, 2011).

Sweep efficiency
As already discussed, the biggest problem with conventional gas injection is that the sweep efficiency of the reservoir is very poor. Only a small part of the reservoir and its oil is actually contacted and swept by the gas injected. Three major causes are the high mobility and low density of injected gases, and geological differences (heterogeneities) between reservoir layers.

Low gas density causes gas to rise to the top of the reservoir and override the oil-rich zones. Also the high mobility of the gas leads directly to viscous instability in the reservoir and increases the tendency for the gas to flow through zones with the highest permeability in the reservoir once these zones are saturated with gas. (Rossen, 1996) As a result oil is left behind in the reservoir and gas soon shoots towards the production wells following high mobility zones and also ‘overriding’ the reservoir. This leads to high production rates of injected gases and poor oil recovery (Shan and Rossen, 2004).

Foam can improve the sweep efficiency by decreasing gas mobility. The foam can trap a large fraction of gas in place, reducing gas mobility. In addition, for the gas inside the foam bubbles to move forward, it has to overcome the capillary resistance to movement caused by the curvature of the foam films, called lamellae. Foam in porous media is defined as “a dispersion of gas in a liquid such that the liquid phase is continuous, and at least some part of the gas is made discontinuous by thin liquid films called lamellae” (Namdar Zanganeh, 2011). Surfactants that adsorb at the gas-liquid interface stabilize the lamellae, which are of the order 10 – 100 nm thick (Rossen, 1996). Furthermore the gas also has to overcome the drag between moving lamellae and the pore walls.
Foam texture (bubble size) is key in determining gas mobility in the presence of foam. Smaller bubbles reduce gas mobility more than larger bubbles. Lower capillary pressure $P_c$ favors foam generation and stability. Higher capillary pressure causes foam collapse. When capillary pressure increases, the lamellae start to thin and, at a single value of $P_c$, $P_c^*$, the foam abruptly collapses. This ensures that $P_c$ is close to $P_c^*$ and water saturation $S_w$ close to $S_w^*$ over a large range of flow rates of gas and water (Shan and Rossen, 2004).

**SAG processes**

There are two ways to place foam in a reservoir. The first is by injecting liquid and gas simultaneously at a fixed ratio. The second method is alternating injection of surfactant and gas slugs, also called surfactant-alternating-gas (SAG) injection.

The coinjection of liquid and gas is useful if one wants to create and maintain foam in the near-well region (injection well), or to ensure that all layers in a heterogeneous reservoir are invaded by foam. The disadvantage of continuous foam injection is that to improve sweep efficiency the injection-well pressure must be raised, which might cause the formation to fracture. Shan and Rossen (2004) have shown that foam processes employing SAG injection at fixed injection pressure control gravity override better than continuous foam injection or SAG at fixed injection rate. Based on numerical simulations, the best process employs one sufficiently large slug of surfactant solution and one large slug of gas. If injection pressure is limited, in a homogenous reservoir where gravity segregation is the problem, SAG processes have a better effect on sweep efficiency than continuous foam injection.

Shan and Rossen (2004) show with fractional-flow analysis that most of the well-to-well pressure drop is focused near the displacement front of the foam, where sweep efficiency is determined. An idealized model, discussed in the next chapter, shows why the process works and why it is relatively insensitive to the details of behavior of the foam formulations. For simplicity, the assumption is made in the model that oil is absent, or does not significantly affect the flow of gas and water.

The assumptions made in fractional-flow theory include:

- Incompressible phases
- Newtonian mobilities
- One-dimensional displacement
- No dispersion
- No gradients of capillary pressures
- No viscous fingering
- Immediate attainment of local steady state

In fractional-flow theory one first plots fractional-flow curves from the mobilities of the individual phases and then time/distance diagrams for a given displacement. In Fig. 5, foam fractional-flow curves for two different concentrations of surfactant in the water are shown. During gas injection in a SAG displacement, there is a shock front from the high liquid saturation initially present to a point on the fractional-flow curves at extremely low water fractional flow $f_w$ (high foam quality). This shock front is represented by a dashed line in Fig. 5. In this case the shock is drawn to the fractional-flow curve for 0.02% surfactant in the foam (Shan and Rossen, 2004).
Fig. 6 and 7 are two time-distance diagrams of two different foam models for gas injection into a large slug of surfactant solution in a 1D reservoir initially at 100% liquid saturation. In Fig. 6 the foam collapses completely at the shock front at the leading edge of the gas bank. In Fig. 7 the foam collapses more gradually through the reservoir. The numbers in the boxes represent the total relative mobility in (Pa*s)^{-1} in regions of constant states, bounded by solid lines. The numbers in ellipses represent total relative mobilities for specific locations (dotted lines) within spreading waves (Shan and Rossen, 2004).

Figure 5: Fractional flow curves (Shan and Rossen, 2004).

Figure 6: Time/distance diagram with complete collapse at the shock front (Shan and Rossen, 2004).
The lines drawn through the origin in Fig. 6 and 7 are waves moving downstream from the injection point and the slope of the wave indicates the velocity of the spreading wave in the reservoir. The width of the wave in a plot represents the width of the wave in a porous medium (Shan and Rossen, 2004). Fig. 7 illustrates the situation assumed below in the model of gas sweep, in which we simplify the low-mobility zone as a narrow front, very small on the scale of the reservoir, with low mobility. In a displacement, any wave or region of constant state increases in thickness as the displacement proceeds.

**Gravity override**

A competition between gravity and horizontal pressure gradient can lead to gravity override, especially when gravity differences are large and injection pressures are not sufficiently high. In a process of continuous foam injection, high velocity and low foam mobility cause high injection pressures, risking formation damage. Velocity and pressure gradient at the displacement front decrease as the front moves away from the well, which allows gravity override (Shan and Rossen, 2004).

In a process of SAG injection, water saturation declines during the gas injection and mobility increases near the injection well. Fig. 8 shows total relative mobility in one SAG process (Shan and Rossen, 2004). Because of high mobilities near the injection well in a SAG-process, higher injection rates are possible without fracturing the formation, thus helping prevent gravity override. As in the model in the next section, the most of the pressure drop is concentrated near the displacement front (Shan and Rossen, 2004).
**Best foam injection strategy**

According to Shan and Rossen, the overall best injection strategy is to inject using a SAG method with very large slugs of surfactant and gas. Injection pressure should be constantly maintained at its maximum, just below fracturing pressure. Shan and Rossen show that the performance of a fixed-pressure SAG process changes remarkably little with differences between foam models and foam properties; pressure gradient is more important in controlling gravity override. This fact helps to motivate and justify the simplified model in the next section.

**Objectives of this thesis**

The intent of this bachelor thesis is to extend the model of a foam displacement presented by Shan and Rossen (2004) to larger dimensionless time than shown there and to extend the model for differences between horizontal and vertical permeability. The next section presents the model of Shan and Rossen, including my adjustments to the model, the problems faced and the results from the model. Three cases are considered: $k_v = k_h$, $0 < k_v < k_h$ and $k_v = 0$. The focus is on the extreme cases and therefore calculations have been done for $k_v = 0$ and $k_v = k_h$. 

---

*Figure 8: Total relative mobility after 0.2PV gas injection into a large slug of surfactant (Shan and Rossen, 2004).*
Model

Concept
The model of Shan and Rossen, described below, has several limitations. First of all, the solution was not extended far enough into the horizontal distance (only to a dimensionless distance of 1, which is not necessarily the well-to-well distance). Second, the assumption was made that horizontal and vertical permeability are equal, i.e. \( k_v = k_h \), which is generally not true in oil reservoirs (Clark, 1969).

In the model several assumptions are made. Fig. 9 gives a schematic overview of the model and assumptions. Fractional-flow theory (see, e.g., Figs. 6 and 7 above) shows that the ideal SAG-injection process creates a spreading wave with a very low mobility at the front and a very high mobility near the injection well. Shan and Rossen simplify this description as follows: all resistance to flow is concentrated in a narrow front (see, e.g., Fig. 10 below). On both sides of the gas-displacement front the mobility is assumed to be (in comparison to the low-mobility zone) infinite (see, e.g., Fig. 11). Although always narrow on the scale of the reservoir, the low-mobility front increases in width with distance travelled as do the waves in Figs. 7 and 10 (Shan and Rossen, 2004).

![Figure 9: Schematic of the idealized model for SAG displacements (from Shan and Rossen, 2004). The model does not actually require the gas density to be zero; the difference in density between gas and water is assumed to be constant, and is a parameter of the model.](image)

Water saturation ahead of the front is assumed to be uniform and at \( S_w = 1 \). The pressure in the water bank is governed by simple hydrostatics, because of infinite mobility there. In the gas bank, the \( S_w \) is uniform at \( S_{wf} \) and the pressure is governed by hydrostatics and gas density, which is assumed to be uniform and constant. This means the whole pressure drop in the reservoir is focused on the narrow front at the leading edge of the gas bank. This front has width \( \tau \), which, consistent with fractional-flow theory, we assume increases with increasing distance the front has traveled, \( s \) (Shan and Rossen, 2004).
Figs. 10 and 11 show versions of Figs. 7 and 8, modified to the assumptions in the model. Fig. 10 is the time/distance diagram of the idealized model, in which mobility is infinite in front of and behind the narrow front. In the spreading wave, which runs through the origin, mobility is very low.

Figure 10: Time/distance diagram, based on model assumptions.

Figure 11 displays how mobility changes through the formation. Only near the displacement front mobility is very low. Both in the gas (foam) and water banks mobility is (in comparison) infinitely large.

Figure 11: Total relative mobility in the reservoir, based on model assumptions.

Fig. 11 displays how mobility changes through the formation. Only near the displacement front mobility is very low. Both in the gas (foam) and water banks mobility is (in comparison) infinitely large.
Though the model is greatly simplified, it fits reality and offers insight in the following respects (Shan and Rossen, 2004):

- In a simplified way, it represents the essence of a successful SAG foam process for overcoming gravity override: high injectivity, low mobility near the foam front, and most of the well-to-well pressure difference focused near the foam front, where gravity override is determined.
- It fits simulation results well for extremely strong foams, and qualitatively weak foams.
- It predicts that foam properties are not as important as injection pressure in preventing gravity override, a point supported by simulations.
- In one case it gave a rapid prediction of feasibility of foam for preventing gravity override over long distances between wells in the North Sea, a prediction that was later supported by simulations (WR Rossen, personal communication).
- It explains why numerical dispersion can give the appearance of successful mobility control even in a process where foam collapses immediately behind the shock font in a SAG process.
- It shows that part of the value of foam is in driving gas downward where the front tips forward.
**Mathematical derivation**

The mathematical derivation of the model in Fig. 9. by Shan and Rossen (2004) starts by applying Darcy’s law to the low-mobility zone. The Darcy velocity of gas inside the low-mobility zone is:

\[ u_g = k \lambda_g |\nabla P| \]

where \( \lambda_g \) is gas relative permeability and \( |\nabla P| \) is the magnitude of pressure gradient in the low-mobility zone.

The pressure gradient across the front is determined by the width of the front, \( \tau \), which itself is proportional to \( s \), the distance the front has traveled. In the front thickness equation, \( \tau^* \) and \( L^* \) are model parameters.

\[ |\nabla P| = \frac{\Delta P(z)}{\tau} \text{ (pressure gradient),} \quad \tau = \frac{\tau^*}{L^*} \text{ (front thickness).} \]

\[ \Delta P(z) = \Delta P^0 - \Delta \rho g z \]

The interstitial velocity \( v \) of the displacement front is as follows:

\[ v = \frac{k \lambda_g |\nabla P|}{(1 - S_{wf}) \phi} \]

where \( \phi \) is porosity of the reservoir.

The Darcy velocity \( u_g \), interstitial velocity \( v \) and the pressure gradient across the displacement front (low-mobility zone) are perpendicular to the interface between the gas and water zones. We track the movement of the front by tracking the advance of individual points \( i \) in the front at positions \([x_i(t), z_i(t)]\). The distance traveled for each point \( i \) is \( s_i(t) \). The angle \( \alpha \) indicates the direction the front is advancing, perpendicular to the current front at time \( t \):

\[ \alpha_i = \arctan \left( -\frac{\partial x_i}{\partial z_i}(t) \right) \]

where \(-\frac{\partial x_i}{\partial z_i}(t)\) is the slope of the displacement front at point \( i \).

Using the angle \( \alpha \) in the equation for interstitial velocity, in other words splitting the interstitial velocity in an \( x \) and \( z \) components, gives the following equations:

\[ v_{x,i} = \frac{dx_i}{dt} = \frac{k \lambda_g |\nabla P|}{(1 - S_{wf}) \phi} \cos(\alpha_i) \]

\[ v_{z,i} = \frac{dz_i}{dt} = \frac{k \lambda_g |\nabla P|}{(1 - S_{wf}) \phi} \sin(\alpha_i) \]
\[
\frac{ds_{i}}{dt} = \sqrt{v_{x,i}^2 + v_{z,i}^2}
\]

\[
|\nabla P| = \frac{(\Delta P^0 - \Delta \rho g z_i) L^*}{\tau_i}
\]

Thus \(v_{x,i}\) and \(v_{z,i}\) are the interstitial velocities in the x and z directions, \((ds_i/dt)\) is the derivative of the distance traveled (in effect, the velocity \(v_{s,i}\)) and \(\Delta \rho\) is the density difference between phases, which we assume is independent of depth.

The initial and boundary conditions are:

IC: at \(t = 0\), \(x_i = 0\)
IC: at \(t = 0\), \(s_i = 0\)
BC: at \(z = 0\), \(\alpha_i = 0\)

**Dimensionless differential equations**

To make the modelling process easier, the differential equations are converted to dimensionless differential equations:

\[
\frac{dX_{D,i}}{dt_D} = \frac{(1-Z_{D,i}) \ast \cos(\alpha_i)}{s_{D,i}} \quad \frac{dZ_{D,i}}{dt_D} = \frac{(1-Z_{D,i}) \ast \sin(\alpha_i)}{s_{D,i}}
\]

\[
\frac{ds_{D,i}}{dt_D} = \sqrt{\left(\frac{dx_{D,i}}{dt_D}\right)^2 + \left(\frac{dz_{D,i}}{dt_D}\right)^2}
\]

where:

\[
\alpha_i = \arctan\left( -\frac{X_{D,i} - X_{D,i-1}}{Z_{D,i} - Z_{D,i-1}} \right)
\]

\[
X_{D,i} = \frac{\Delta \rho g x_i}{\Delta P^0}
\]

\[
Z_{D,i} = \frac{\Delta \rho g z_i}{\Delta P^0}
\]

\[
t_D = \frac{k \lambda_{sf} \Delta \rho g}{(1-S_{ef}) \phi \tau} t
\]

\[
s_{D,i} = \frac{\Delta \rho g s_i}{\Delta P^0}
\]

The dimensionless boundary and initial conditions are:

IC: at \(t_D = 0\), \(X_{D,i} = 0\)
IC: at \(t_D = 0\), \(s_{D,i} = 0\)
BC: at \(Z_D = 0\), \(\alpha_i = 0\)

The equations above are sufficient to create the simplified model to describe the foam penetration into the reservoir. Shan and Rossen show good agreement between even this greatly simplified model and simulation of SAG processes with a very strong foam; even in cases where it is not quantitatively correct, it provides insights into SAG processes and its simplicity makes it a quick way to do preliminary estimates of gravity override with foam.
**Building the model**

The equations are embedded in an MS Excel spreadsheet. From the initial conditions one can calculate the \((dX_{D,i}/dt_D, dZ_{D,i}/dt_D)\), and \((ds_{D,i}/dt_D)\) at the start, and at any time step one can calculate those values for that time step. If one multiplies the calculated derivatives by the chosen time increment, one can obtain the values of \(X_{D,i}, Z_{D,i}\) and \(s_{D,i}\) for the new time step. Angle \(\alpha_i\) can then be calculated from the differences between values of \(X_{D,i}\) and \(Z_{D,i}\) for adjacent points. This process can be iterated until one reaches the desired dimensionless distance \(X_D\).

Time steps must be very small at the beginning of the iterations (note \((dX_{D,i}/dt_D) \to \infty\) at \(t_D = 0\)) and can be larger later on. To avoid infinite velocity \((dX_{D,i}/dt_D)\) in the first time step we take \(s_D\) as a very small number at \(t_D=0\), for example \(s_D = 1E-7\). For later times this difference does not matter significantly. (A better solution for the first time step was discovered later, as described below, but this is the method we used in our calculations.)

Fig. 12 shows a schematic overview of the calculations of the foam front for the first time step.

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Figure 12: Example of calculations in MS Excel.

The numerical method used here is the Euler Method. In this method the derivative at a point is used to calculate the next point. The smaller the time step, the more accurate the method is. The calculations that are demonstrated in Fig. 12 can then be continued until the desired dimensionless distance is reached.

Because of the angle \(\alpha\) in the model, every point except the points at the bottom and top progress towards the bottom right over time. This means that a gap opens up between the first and second points near the top, as can be seen in Fig. 13.
Adding in points

The problem of the gap opening can be partially solved by adding in new points near the top every few iterations. The gap is created by data points moving to the bottom right in the model. Fig. 14 shows an expanded view of the top of the front, with points added in at the top of the model. Unfortunately the adding in of points leads to an extra instability in the model, as can be seen in Fig. 14. This error is on top of the error that also exists in the model without adding in points, shown at later dimensionless times in Fig. 13.

We have added in points in several different ways:

1) The first method is adding in 9 points at certain dimensionless times, where it seemed necessary. The points are added between the first and second point of a curve and are linearly interpolated between these two points. Sometimes 9 points were also added in between the second and the third original data points. The results of this method can be seen in Figs. 14 and 15. The method seems to be working fine, but errors grow.

2) The second method is adding in 1 point at each time step, also between the first and the second point and using linear interpolation. The results of this method are worse than the first method and can be seen in Fig. 16.

3) The third way was adding in one point every time step, using Lagrange interpolation. The value of $Z_{D,i}$ is determined by using the two next values of $Z_D$ ($Z_{D,i+1}$ and $Z_{D,i+2}$) and their values of $X_D$ ($X_{D,i+1}$ and $X_{D,i+2}$) and the previous value of $Z_D$ ($Z_{D,i-1}$) and its value of $X_D$ ($X_{D,i-1}$). Furthermore the value of $X_{D,i}$ of the interpolated point is linearly interpolated between the value above and below it ($X_{D,i-1}$ and $X_{D,i+1}$). This resulted in up going points at the top of the reservoir, something that is not supposed to happen. Fig. 17 shows the results of this method.
Fig. 15 shows the extension of Fig. 14 until a dimensionless time of 1. If you compare Fig. 15 to Fig. 13 you can see that the errors until a dimensionless time of 1 have been suppressed by the adding in of points. Later on though, the errors become a severe disturbance as in Fig. 14.
Nonetheless, the errors in Figs. 16 and 17 are substantially larger than in Figs. 14 and 15. Based on this, we have chosen to add in points according to method 1. All the adding-in mentioned below in this thesis is done using method 1.
**The instability problem**
Probably the source of the instability lies in the method used to build the model. A small error at one of the earlier time steps may be only in the order of $10^{-8}$ or even less, but as the points in the model proceed over time, the errors grow and grow, until they become significant and points start to interfere with each other. There is no natural corrective in the calculation method; a slightly wrong position leads to a wrong direction, and then a bigger error in position in the next time step. The errors are most likely caused by an incorrect angle of $\alpha$ at some points, leading to a wrong trajectory of those points. The points can for example cross each other, which should not happen. To illustrate the problem, we put deliberate errors into the model at different time steps to see the result.

Fig. 18 has been made in exactly the same way as Fig. 13, except that in Fig. 18 a very small random error is inserted at a dimensionless time $t_D = 4E-3$, which is around a dimensionless position $X_D$ of 0.1 at the top of the model $(Z_D = 0)$. To the value of every $\alpha_i$ at this time step a value is added which is normally distributed around zero with standard deviation 0.00001.

The command used in Excel is NORMINV(RAND();0;0.00001). This function returns the inverse of the normal cumulative distribution for the specified mean and standard deviation. In this case the specified mean is $\mu = 0$, the standard deviation is $\sigma = 0.00001$.

The error term is added to the calculation of $\alpha$ at that time step. This leads to the errors that can be seen in Fig. 18. The error is different at the different depths $Z_D$ and one can see that just after adding the ‘disturbance’ nothing significant seems to be happening. But the curve at $t_D = 3E-1$ shows differently. Compare Fig. 13 and 18 to see the significant difference.

![Figure 18: Foam front through dimensionless time. Inserted error with a standard deviation of 0.00001 (1E-5) at $t_D = 4E-3$.]
Fig. 19 shows the same case, but with a standard deviation of $\sigma = 0.000001$ for the initial error. Fig. 19 shows that the error is still present at larger dimensionless times, though smaller than in Fig. 18.

![Figure 19](image1.png)

**Figure 19:** Foam front through dimensionless time. Inserted error with a standard deviation of $0.000001 \times 10^{-6}$ at $tD = 4 \times 10^{-3}$.

**Smoothing $\alpha$ using less $zD$-points**

Fig. 20 shows angle $\alpha$ in the original model, from Fig. 13, i.e. without the adding in of points. In Fig. 20, the value of $\alpha$ is plotted against depth. One would expect values to be smoothly increasing from zero at the top to a certain value at the bottom.

![Figure 20](image2.png)

**Figure 20:** Alpha plotted against $zD$ at $tD = 3.6 \times 10^{-1}$, using the original model of Fig. 13.
As one can see in Fig. 20, the values of $\alpha$ are neatly increasing, except in the area between $Z_D = 0.15$ and $Z_D = 0.3$. In this area the values of $\alpha$ are wildly fluctuating, causing the distortions in the foam front in Fig. 13.

The first option to solve this problem is to decrease the number of data points in the z-direction. This may reduce error in the numerical evaluation of $\alpha$, because the points are further apart from each other. A disadvantage is that the gap between the first and second data point increases over time. Fig. 21 shows the front using fewer data points. It is possible now to get further in dimensionless time, without numerical issues, but at $t_D = 2$, there are no data points left in the top part of the reservoir, apart from the very top point.

![Figure 21: Foam front through dimensionless time, without adding in points and with fewer zD points.](image)

Also the first numerical issues are evident by this time. Adding in of points could further improve Fig. 21 at the top, but it would be better to try and smooth Fig. 21, in order that no numerical issues will occur at the bottom of the front and to be sure that the added in values also remain stable.

The stabilized version of Fig. 21 is shown in Fig. 23, which is displayed below. The stabilizing was done by checking each data point $(X_D, Z_D, \alpha)$ with an if-statement, if the trend in either $\alpha$, $X_D$ or $Z_D$ is not monotonic. Specifically, at each time step we ask whether the given values of $\alpha$ and $X_D$ are smaller than the point above and $Z_D$ is larger than the point above it. If the if-statement returns a negative answer, the given parameter that is out of sequence ($\alpha$, $Z_D$, $X_D$) is reset to the average of the values above and below it, to create a smooth trend.

The statement for $X_D$ looks like this: $=IF(AND(I12 < I11; I12 > I13); I12; (I11 + I13)/2)$. This makes sure that outliers are corrected early on. Fig. 22 shows an example of the smoothing algorithm, in this case where the value in of $Z_D$ in cell J12 is altered to the value in M12. This is because this value of $Z_D$ is not following the sequence of increasing values moving down the column, as you can see by looking at the red value.
(J12). The grid points are treated individually, so if a point in a row differs from the regular sequence, only the point itself is corrected.

<table>
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<th>smooth</th>
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<td>0.16</td>
<td>0.3</td>
<td>-0.03</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Figure 22: Illustration of smoothing, value in red is smoothed, the other values remain the same.

Figure 23: Foam front through dimensionless time, without adding in points and with fewer zD points. Smoothed values of X_D, Z_D and α.
**Result \( k_v = k_h \) case**

Using the first method of adding in points on this stabilized model of Fig. 23 leads to even better results. Key in this method is waiting sufficiently long between places where points are added in, in other words waiting for a sufficient gap between the first and the second data points. This together with the smoothing makes sure that points do not start to interfere with each other. The results of this model can be seen in Fig. 24, which continues up to a dimensionless time \( t_D \) of 4.

The adding-in of points does lead to a slightly unequal distribution of data points, but the curvature of the front is fairly smooth, meaning this is still likely a good representation of the actual situation.

![Figure 24: Foam front through dimensionless time, with adding in points and with fewer \( zD \) points. Smoothed values of \( X_D \), \( Z_D \) and \( \alpha \).](image)

**Comparison of different methods in the \( k_v = k_h \) case**

To make sure the smoothing and adding in of points hasn’t made a substantial difference to the displacement front, it is good to compare the different methods with the standard model in which no points are added in nor the data is smoothed. This can be done at a dimensionless time of \( t_D \) is 1 and some later time, for example \( t_D \) is 2.5.

The first comparison is between the model without adding in points and without smoothing (original in Fig. 13) and the model with only adding in (original in Fig. 14) at \( t_D = 5E-1 \). This comparison is shown in Fig. 25. As you can see the two lines overlap completely, except for the numerical errors in the blue line.
The second comparison is between the model without adding in points and without smoothing (original in Fig. 13) and the model with fewer points in the ZD-direction without adding in and without smoothing (original in Fig. 21) at tD = 5E-1. This comparison is shown in Fig. 26. As you can see the lines overlap completely again, except for the error region of the blue line.

The third comparison is between the model with fewer points in the ZD-direction without adding in points and without smoothing (original in Fig. 21) and the same model with smoothing (original in Fig. 23) at tD = 3. This comparison is shown in Fig.
27. As you can see the two lines overlap, except for the top part, where the smoothed line is a little bit further in the $X_D$-direction. The difference is very small though.

![Figure 27: Foam front at $t_D = 3$. The blue line: less $Z_D$-points, no adding in, no smoothing; the red line: less $Z_D$-points, no adding in, smoothing.](image)

The fourth and last comparison is between the model with fewer points in the $Z_D$-direction without adding in points and without smoothing (original in Fig. 21) and the model with fewer points in the $Z_D$-direction with adding in points and with smoothing (original in Fig. 24) at $t_D = 3$. This comparison is shown in Fig. 26. The front is more filled-in near the top in the latter case, because of the adding-in of points. Over the rest of the front, the fronts overlap, suggesting a correct representation of the displacement front.

![Figure 28: Foam front at $t_D = 3$. The blue line: less $Z_D$-points, no adding in, no smoothing; the red line: less $Z_D$-points, both adding in and smoothing.](image)
Horizontal permeability unequal to the vertical permeability ($0 < k_v < k_h$)

In the preceding model, one assumes that the horizontal permeability is equal to the vertical permeability. In reservoirs this is generally not true, because layers are deposited horizontally through time. Therefore, one expects the horizontal permeability to be larger than the vertical permeability (Clark, 1969).

The derivation for this case ($0 < k_v < k_h$) is the same as the $k_v = k_h$ case, except that the velocity is split into $x$ and $z$ components. The definition of $\tau$ is also adapted. Because the direction that individual points in the front are moving is not perpendicular to the front (direction of $\alpha$), we define an angle $\beta$ as the direction the given point is moving (relative to the horizontal). This affects the definition of $\tau$ and the other equations. Fig. 29 illustrates this case. The given point on the front travels in the direction of $s$, which at its current location is at an angle $\beta$ to the horizontal. As before, $\alpha$ is the angle perpendicular to the front, as determined from the positions of nearby points at the given time (solid diagonal line at upper left of figure). The low-mobility front is the region between the two parallel diagonal lines. The key to this derivation is that the front width $\tau$, which as before is proportional to distance traveled, $s$, is in the direction of the streamline, $\beta$.

![Figure 29: Geometry behind the derivation for $0 < k_v < k_h$.](image)

Now $\tau$, $\tau_\alpha$, $\tau_x$ and $\tau_z$ can be defined as follows. As before,

$$\tau = \frac{\tau\cdot s}{L}$$

The thickness of the front in other directions is given by

$$\tau_\alpha = \tau \cos(\alpha - \beta) \quad \tau_x = \frac{\tau \cos(\alpha - \beta)}{\cos(\alpha)} \quad \tau_z = \frac{\tau \cos(\alpha - \beta)}{\sin(\alpha)}$$
The pressure gradient is defined as before as

$$| \nabla P | = \frac{\Delta P(z)}{\tau} \quad \text{(pressure gradient)}$$

where $\Delta P(z) = \Delta P^0 - \Delta \rho g z$

The pressure gradients in the x and z direction can be defined as follows:

$$| \nabla P |_x = \frac{\Delta P(z)}{\tau_{x,i}} = \frac{(\Delta P^0 - \Delta \rho g z_i)}{\tau} \frac{\cos(\alpha_i)}{\cos(\alpha_i - \beta_i)} = \frac{(\Delta P^0 - \Delta \rho g z_i) L_x}{\tau s_i} \frac{\cos(\alpha_i)}{\cos(\alpha_i - \beta_i)}$$

$$| \nabla P |_z = \frac{\Delta P(z)}{\tau_{z,i}} = \frac{(\Delta P^0 - \Delta \rho g z_i)}{\tau} \frac{\sin(\alpha_i)}{\cos(\alpha_i - \beta_i)} = \frac{(\Delta P^0 - \Delta \rho g z_i) L_z}{\tau s_i} \frac{\sin(\alpha_i)}{\cos(\alpha_i - \beta_i)}$$

The angle $\alpha$ is defined in the same way as in the previous derivation; it's calculated from the differences in $X_{D,i}$ and $Z_{D,i}$ values of adjacent points i at one time step. The angle $\beta$ is determined by $(dZ_{D,i}/dt_{D,i})$ and $(dX_{D,i}/dt_{D,i})$ for point i at the previous time step:

$$\alpha_i = \arctan \left( \frac{x_i - x_{i-1}}{z_i - z_{i-1}} \right)$$

$$\beta_i = \arctan \left( \frac{dz_i}{dx_i} \right)$$

The components of velocity $v_{x,i}$ and $v_{z,i}$ depend on the pressure gradient in x and z directions and also on the differing vertical and horizontal permeabilities:

$$v_{x,i} = \frac{dx_i}{dt} = \frac{k_h \lambda_{rf}}{1 - S_{w,i}} | \nabla P |_{x,i}$$

$$v_{z,i} = \frac{dz_i}{dt} = \frac{k_h \lambda_{rf}}{k_h} | \nabla P |_{z,i} (1 - S_{w,i}) \phi$$

$$\frac{ds_i}{dt} = \sqrt{v_{x,i}^2 + v_{z,i}^2}$$

The initial and boundary conditions are:

IC: at $t = 0$, $x_i = 0$

IC: at $t = 0$, $s_i = 0$

BC: at $z = 0$, $q_i = 0$
Then making the equations dimensionless gives:

\[
\frac{dX_{Di}}{dt_D} = \frac{(1-Z_{Di})}{s_{Di}} \frac{\cos(\alpha_i)}{\cos(\alpha_i - \beta_i)} \\
\frac{dZ_{Di}}{dt_D} = \frac{k_v}{k_h} \frac{(1-Z_{Di})}{s_{Di}} \frac{\sin(\alpha_i)}{\cos(\alpha_i - \beta_i)} \\
\frac{ds_{Di}}{dt_D} = \sqrt{\left(\frac{dx_{Di}}{dt_D}\right)^2 + \left(\frac{dz_{Di}}{dt_D}\right)^2}
\]

where:

\[
\alpha_i = \arctan\left(\frac{-X_{Di} - X_{Di-1}}{Z_{Di} - Z_{Di-1}}\right) \\
\beta_i = \arctan\left(\frac{-\frac{dt_D}{dx_{Di}}}{\frac{dt_D}{dz_{Di}}}\right)_{t_{Dp}}^{-1} \\
X_{Di} = \frac{\Delta \rho g x_i}{\Delta P^0} \\
Z_{Di} = \frac{\Delta \rho g z_i}{\Delta P^0} \\
I_D = \frac{k_h \lambda_i \Delta \rho g}{(1 - S_{wf}) \phi \tau^2} t \\
s_{Di} = \frac{\Delta \rho g s_i}{\Delta P^0}
\]

The dimensionless boundary and initial conditions are:

- IC: at \( t_D = 0 \), \( X_{Di} = 0 \)
- IC: at \( t_D = 0 \), \( s_{Di} = 0 \)
- BC: at \( Z_D = 0 \), \( \alpha_i = 0 \)

The equations above are sufficient to create a second simplified model, which describes the foam penetration into the reservoir, with \( 0 < k_v < k_h \). The calculations can be done in exactly the same way as in Fig. 24. The only difference is the \( (k_v/k_h) \) factor in the \( (dZ_{Di}/dt_D) \) equation, which can be adjusted to several different ratios, and the additional factor of \( [1/\cos(\alpha - \beta)] \) in equations for both \( (dX_{Di}/dt_D) \) and \( (dZ_{Di}/dt_D) \). We have not done the actual calculations, because the focus in this thesis was on the extreme case, i.e. \( k_v = 0 \) and \( k_h = k_v \).
**Vertical permeability is zero ($k_v = 0$)**

In the extreme case, $k_v = 0$, $\beta = 0$ and $(k_v/k_h) = 0$. Thus $v_{z,i} = 0$ and the equations can be defined as follows:

$$v_{x,i} = \frac{dx_i}{dt} = \frac{k_h \lambda_{of} \left| \nabla P \right|}{(1 - S_{nf})\phi}, \quad v_{z,i} = 0$$

$$\frac{ds_i}{dt} = v_{x,i}$$

The pressure gradient in the low mobility zone is as follows:

$$\left| \nabla P \right| = \frac{\Delta P_0 - \Delta \rho g z_i}{\tau_i} = \frac{(\Delta P_0 - \Delta \rho g z_i)L^*}{\tau^* s_i}$$

The initial and boundary conditions also remain the same:

**IC:** at $t = 0$, $x_i = 0$

**IC:** at $t = 0$, $s_i = 0$

**BC:** at $z = 0$, $\alpha_i = 0$

Then making the equations dimensionless gives:

$$\frac{dX_{D,i}}{dt_D} = \frac{(1-Z_{D,i})}{s_{D,i}} \quad \frac{dZ_{D,j}}{dt_D} = 0 \quad \frac{ds_{D,j}}{dt_D} = \frac{dX_{D,j}}{dt_D}.$$

where:

$$X_{D,i} = \frac{\Delta \rho g x_i}{\Delta P_0^0}$$

$$Z_{D,j} = \frac{\Delta \rho g z_i}{\Delta P_0^0} \quad t_D = \frac{k_h \lambda_{of} \Delta \rho g}{(1-S_{nf})\phi \tau^* t}$$

$$s_{D,i} = \frac{\Delta \rho g s_i}{\Delta P_0^0} \quad L^* = \frac{\Delta P_0^0}{\Delta \rho g}$$

The dimensionless boundary and initial conditions also remain the same:

**IC:** at $t_D = 0$, $X_{D,i} = 0$

**IC:** at $t_D = 0$, $s_{D,i} = 0$

**BC:** at $Z_D = 0$, $\alpha_i = 0$

The angle $\alpha$ is irrelevant to the motion of the front, because as $\beta = 0$, the term $[\cos(\alpha-\beta)/\cos(\alpha)] = [\cos(\alpha)/\cos(\alpha)] = 1$. The calculations are done in exactly the same way as in the $k_v = k_h$ model, only lacking the columns of $\alpha$ and $dZ_{D,j}/dt_D$. 

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In Fig. 30, you can see the schematic overview of the calculations done in MS Excel. The number of columns is smaller than in the previous model.

Without numerical issues to deal with this time, the results were easier to obtain and can be seen in Fig. 31. The solution is extended until a dimensionless position $X_D$ of 4.

In Fig. 30, you can see the schematic overview of the calculations done in MS Excel. The number of columns is smaller than in the previous model.

Without numerical issues to deal with this time, the results were easier to obtain and can be seen in Fig. 31. The solution is extended until a dimensionless position $X_D$ of 4.
Analytical solution for the $k_v = 0$ case

It’s also possible to solve for $k_v = 0$ analytically. This is because the total traveled distance $s_D$ of a data point equals the distance traveled in the x direction $X_D$.

\[
\frac{dX_D}{dt_D} = \frac{(1-Z_D)}{s_D} = \frac{(1-Z_D)}{X_D}
\]

\[X_D \frac{dX_D}{dt_D} = 1-Z_D\]

\[\frac{1}{2} \frac{dX_D^2}{dt_D} = 1-Z_D\]

\[\frac{1}{2} X_D^2 = (1-Z_D) t_D + C\]

The constant $C$ is zero, because at $t_D = 0$, $X_D = 0$. Multiplying the entire equation by two and taking the square root leads to:

\[X_D = \sqrt{2(1-Z_D) t_D}\]

The solution of this last equation is valid for $k_v = 0$. It could also be used to generate the first set of $X_D$ and $Z_D$ points at the first time step for cases where $k_v = k_h$ or $0 < k_v < k_h$. At $t = 0$, $\beta = \alpha = 0$ and, as above $(dZ_D/dt_D) = 0$ and the $\cos(\alpha)$ term drops out from the equation for $(dX_D/dt_D)$. This avoids the awkward expedient used before of setting a small, finite value of $s_D$ for all points at $t = 0$. Fig. 31 and Fig. 32 are almost exactly the same. The small differences between them reflect small errors in the numerical method.

In Fig. 32 the analytical solution is displayed of the $k_v = 0$ case.

Figure 32: Foam front through dimensionless time, $k_v = 0$, analytical solution.
Comparing the models

Fig. 33 shows foam displacements of the different models, $k_v = 0$ and $k_v = k_h$, at a dimensionless time $t_D$ of 8. It shows that the larger the vertical permeability, the more convex the curve gets. In other words, the more the pressure is able to push points downwards and create a larger bulge in the curve. Ironically, greater vertical permeability in this case leads not to greater vertical migration of gas, but better vertical sweep of the reservoir.

Fig. 31 and Fig. 32 (red and blue line respectively in Fig. 33) are almost exactly similar. These are the figures which show respectively the normal solution and the analytical solution of $k_v = 0$. It is expected that the curves of the case in which $0 < k_v < k_h$ lie in between the orange and red lines in Fig. 33. As the focus lies on the extreme cases, these curves are not calculated.

Figure 33: Foam front at $t_D = 8$ for different vertical permeabilities ($k_v$)
Conclusion and recommendations

The results show that with an increasing $k_v$, the foam displacement is more convex. This is within the range of $0<k_v<k_h$. Instabilities can be smoothed out, which is very favorable for the final result. Using less points in the $Z_D$ direction together with smoothing appears to be a fair solution to avoid numerical instability. We have also found that the best way of adding in points is by linear interpolation at certain dimensionless times, when it seems necessary (method 1).

The fact that the foam displacement is more convex at higher vertical permeabilities is in correspondence with what you would intuitively expect in the model, because the larger pressure difference at the top causes the points to move downward as well. Ironically, increasing vertical permeability reduces the extent of gravity segregation of gas and increases vertical sweep of the reservoir.

To further improve results, one could build a model in Matlab, which automatically inserts points at the top, when, for example, the second data point falls below $Z_D = 0.05$. This is arbitrary. A small time step is favorable. Key is finding a good way of inserting points, without creating numerical problems. To build the first time step for the $0<k_v<k_h$ and $k_v = k_h$ cases, one can use the analytical model of $k_v = 0$ as a good approximation. Another option is to abandon the restriction that the top point remains at the $Z_D = 0$ line, in other words to abandon the idea that at $Z_D = 0$, $\alpha = 0$. One could then add in a new top point every time step or every few time steps. We would recommend to calculate the $0<k_v<k_h$ cases as well, regardless of the method used to do so.
Nomenclature

In order of appearance:

\( \nu \)  \hspace{1cm} \text{kinematic viscosity} \ [m^2/s]  \\
\( \mu \)  \hspace{1cm} \text{dynamic viscosity} \ [kg/ms]  \\
\( \sigma \)  \hspace{1cm} \text{surface tension} \ [kg/s^2]  \\
\( P_c (*) \)  \hspace{1cm} \text{(limiting) capillary pressure} \ [kg/ms^2]  \\
\( S_w (*) \)  \hspace{1cm} \text{water saturation (at limiting capillary pressure)} [-]  \\
\( f_w \)  \hspace{1cm} \text{water fractional flow} [-]  \\
\( k \)  \hspace{1cm} \text{permeability} \ [m^2]  \\
\( \tau \)  \hspace{1cm} \text{front width} \ [m]  \\
\( s \)  \hspace{1cm} \text{traveled distance of a point} \ [m]  \\
\( u_g \)  \hspace{1cm} \text{Darcy velocity} \ [m/s]  \\
\( \lambda \)  \hspace{1cm} \text{mobility} \ (k/\mu) \ [m^3 s/kg]  \\
\( \lambda_{rf} \)  \hspace{1cm} \text{gas relative mobility} \ [m^3 s/kg]  \\
\( L^* \)  \hspace{1cm} \text{traveled distance (model parameter)} \ [m]  \\
\( \tau^* \)  \hspace{1cm} \text{width of front at traveled distance} \ L^* \ [m]  \\
\( |\nabla P| \)  \hspace{1cm} \text{pressure difference in low-mobility zone} \ [kg/ms^2]  \\
\( \Delta P \)  \hspace{1cm} \text{pressure difference} \ [kg/ms^2]  \\
\( \Phi \)  \hspace{1cm} \text{porosity} [-]  \\
\( v \)  \hspace{1cm} \text{velocity} \ [m/s]  \\
\( \alpha \)  \hspace{1cm} \text{front angle} [-]  \\
\( x \)  \hspace{1cm} \text{position in x-direction} \ [m]  \\
\( z \)  \hspace{1cm} \text{position in z-direction} \ [m]  \\
\( t \)  \hspace{1cm} \text{time} \ [s]  \\
\( X_D \)  \hspace{1cm} \text{dimensionless position in x-direction} [-]  \\
\( Z_D \)  \hspace{1cm} \text{dimensionless position in z-direction} [-]  \\
\( s_D \)  \hspace{1cm} \text{dimensionless traveled distance} [-]  \\
\( \rho \)  \hspace{1cm} \text{density} \ [kg/m^3]  \\
\( t_D \)  \hspace{1cm} \text{dimensionless time} [-]  \\
\( \mu \)  \hspace{1cm} \text{mean} [-]  \\
\( \sigma \)  \hspace{1cm} \text{standard deviation} [-]
Acknowledgements

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Sources

(1) Clark, N. J., “Elements of Petroleum Reservoirs”, (Revised Ed) Soc. petrol; Eng., 1969 Dallas, TX, Henry L. Dohery series, p. 19-30


