Packing Optimization for Digital Fabrication

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Abstract. We present a design-computation method of design-to-production automation and optimization in digital fabrication; an algorithmic process minimizing material use, reducing fabrication time and improving production costs of complex architectural form. Our system compacts structural elements of variable dimensions within fixed-size sheets of stock material, revisiting a classical challenge known as the two-dimensional bin-packing problem. We demonstrate improvements in performance using our heuristic metric, an approach with potential for a wider range of architectural and engineering design-built digital fabrication applications, and discuss the challenges of constructing free-form design efficiently using operational research methodologies.

Keywords. Design computation; digital fabrication; automation; optimization.

INTRODUCTION

Contemporary computer aided design systems became popular during the past decade and offered architects the unprecedented opportunity to experiment with digital free-form geometry and explore its material implications (Kolarevic, 2003; Schodek et al, 2004; Kolarevic and Klinger, 2008; Iwamoto, 2009; Glynn and Sheil, 2012). Free or complex architectural form, underpins the departure from rectilinear morphology supported by industrial methods of production towards the investigation of irregular or curved geometries and arrangements. Such profound liberalization of expressive capacity enabled by digital media was not met without technical as well as economic challenges emerging from the requirement for advanced manufacturing methods such as the demand for sophisticated information modeling and management protocols and numerical control fabrication. Certain of those barriers to entry as well as architectural considerations of feasibility have subsided by the reduction of costs associated with digital fabrication and the development of literacy in computation within architecture, engineering and the construction industry thanks to the adoption of parametric and building information modeling technologies.

We made great strides in addressing the problem of design description: expressing complex form using modern computer aided design systems and we resolved certain issues of materialization: manufacturing complex form with advanced computer aided manufacturing and numerical control digital fabrication technologies. However, the challenges of realizing digital design are still great and offer the ground for design research. Parametric and BIM methods, derived from workflow principles of drawing or visual modeling techniques, offer some assistance in controlling design information complexity but often fall short in operating within large volumes of algorithmically generated information exactly because of their descriptive foundation. Problems such as controlling variation among irregular building components, approximating smooth
building envelopes with discrete elements, automating the layout and scheduling of building components for production, and optimizing material and cost resources with respect to visual and technical constraints are examples of the challenges that cannot be practically addressed using conventional CAD/CAM systems. Instead they call for a procedural, imperative rather than descriptive, vocabulary of advanced design-computation techniques to meet the increasing levels of design information density in digital design-built processes.

This body of knowledge already exists as many recent problems (re)discovered in the light of digital design praxis have been tackled in the past, albeit for different ends and industries. Methods of automation and optimization in manufacturing have been studied within the domains of applied mathematics, computer science and operations research. Our interests are situated at the intersection of those fields and architecture which we may attribute as operations research for creative design processes that is low-volume, highly complex, customized and particularly context-aware design using contemporary modes of production. We study methods of automation for facilitating the translation of design to construction and focus on optimization strategies using design-computation to improve design-built performance. In this paper we explore the process of optimizing digital fabrication of complex architectural form using packing methods. Our problem statement is quite simple: given fixed-size sheets of material stock, how can one automate the layout of variably sized building components and optimize material use, machining time and the cost of production (Figure 1)? This is a common problem that has a broad range of architectural, engineering and construction applications such as sizing and cutting glass panels for building envelope cladding systems, steel plate or standard section laser/plasma fabrication for structural elements and even building component packing/palletizing for transport and on-site storage.

**PROJECT DESCRIPTION**

The research is part of the design-development of the library extension outdoor study area of the Singapore University of Technology and Design created by the City Form Lab (Figure 2). The free-form funicular canopy is comprised of approximately four thousand timber structural beam members machined in 12mm, 1.2m x 2.4m, outdoor grade film plywood. Timber beams, grouped into triangles, are joint using steel hinges that take some of the structure’s sheer load and assist the assembly and instal-
lation by modularization. In addition, the structure’s load bearing capacity is augmented by the introduction of 25mm marine grade plywood between modules, from the same stock dimensions, and steel plates at the base triangles as well as some supporting spine arches. All wood shapes were manufactured using 3-axis CNC direct cutting while steel plates were laser cut. Our method was employed in both scenarios.

The geometric complexity of the envelope, due to varying curvature, results in a fabrication program where every element is dimensionally unique while they all share the same quadrilateral shape. We investigated the design-built rationalization of the envelope from a post-rational perspective introducing methods of optimization targeting the fabrication process domain rather than reinterpreting geometry using rule-based methods. We began with the translation of design geometry into fabrication files and focused on the search for effective modes of automating the organization of non-standard elements into standard sheets of material. The study was motivated by (a) the difficulty of performing element layout manually, an extremely repetitive and laborious process even for small scale envelopes, which hinders design improvement by iteration; and (b) the simplicity of implementation and flexibility gained, given our problem characteristics, compared to integrating commercial layout applications in the parametric design process. Optimization targeted the reduction of the drafting and fabrication time but collateral we minimized the material used, due to the abrasive milling process, and in consequence overall cost.

RELATED WORK
Optimizing the arrangement of parts towards manufacturing production requirements is a class of problems with a long history of study in operations research. Problems of maximizing layout density, or reciprocally minimizing residual waste, are commonly encountered in industrial applications involving abrasive material operations such as cutting textiles, wood, glass and steel as well as additive operations such as packing, palletizing, transportation and storage. Formally, they belong to a family known as Packing & Cutting Assignment problems, which also includes the knapsack, cutting stock, trim loss and nesting problems (Dyckhoff, 1990; Washer et al, 2007). They are combinatorial optimization problems the solution of which is NP-hard (Garey and Johnson, 1979); it is thus speculated that they don’t have an efficient, polynomial time deterministic solution. The fabrication processes used here classifies our problem as a two-dimensional bin-packing problem (2BPP) which is expressed as a
search: Given a sequence of planar parts of known dimensions find the minimum number of sheets of also known size containing every part provided that there are no overlaps. For a few parts packing can be solved exactly using branch and bound methods (Kolesar, 1967) but for large volumes an approximation can only be reached within reasonable time and a factor from the theoretically optimal solution based on a search heuristic; a search hint, or rule of the thumb, derived from intuition and empirical first principles.

In its classical form the BPP is considered with rectangular parts organized within rectangular bins. Sub-categories are based on: (a) Dimensionality: linear material such as stock sections, planar sheets such as plywood and volume materials such as those in rapid prototyping. (b) Orientation: oriented packing when rotation is not allowed, rectangular packing, when only 90 degrees rotation is possible and free packing when any rotation is admissible. (c) Pattern: packing is regular if it adheres to a lattice type of repetitive configuration or irregular where no layout symmetry exists among parts. (d) Cutting Constraints: a layout is guillotineable when a sheet can be cut in two by a straight line without severing parts, such as by a table-saw, or free when there are no cutting constraints. (e) Prior Knowledge: offline packing, where part sizes are known in advance or online when parts arrive at random and must be placed immediately into bins. (f) Shape: various versions of packing algorithms exist dependent on the shape, size and multiplicity characteristics of both parts and bins. Generally convex shapes are often examined, such as rectangles, but there are irregular shape packing / nesting algorithms that examine concave shapes with or without holes.

We examine a version of the problem where all part shapes are quadrilateral with their long span typically being considerably larger than their short direction and additionally the short span dimensions being fairly similar (Figure 3). Packing almost rectangular shapes within material sheets can be seen as classical rectangular 2BPP or even 1BPP if we assume same width uniformly. However, we explored the potential of achieving improvements in performance by accounting for the irregularity of our shapes, allow rotation and reflection transformation, and use the classical algorithm as a benchmark to evaluate results. In addition, our method is informed by structural, material, fabrication and workflow considerations and constraints typical in CNC digital fabrication. Thus it is a highly specific, context-aware method with potential for general application principally given that the triangulated shell tectonic strategy employed here is fairly common in materializing complex building envelopes.

**METHODOLOGY**

Two relevant types of packing algorithms of interest have been studied in research literature (Chrestofides and Whitlock, 1976; Albano and Orsini, 1980; Martello and Toth, 1990). Shelf-packing is a two-phase strategy where parts are initially organized into shelves/strips along the long dominant direction and later the shelves are packed along the short direction into sheets. Partition-packing is a single phase strategy where parts are placed directly into sheets while a hierarchy of residual areas is retained for guiding subsequent placements. Both algorithms are local optimization methods that operate incrementally by positioning one element at a time, selecting each from a list of pre-sorted elements in
descending order (Figure 4). Augmented packing (Lodi et al, 1999), irregular shape packing (Albano and Sapuppo, 1980; Okano, 2001; Nielsen 2007), polynomial time approximation scheme and meta-heuristic processes such as simulated annealing and genetic algorithms are also common in relevant literature (De La Vega and Lueker, 1981; Goodman et al, 1994).

Structural engineering recommendation, to orient beam members into sheets along the external plywood grain direction that is to take advantage of plywood’s improved material performance typically manufactured in odd number of plies, constrained our system to 180 degree rotations. Moreover, initially we did not distinguish between the front and back facing sides of the material which allowed us to also permit reflection transformations. However, introduction of serial numbers for identification engraved into parts during the machining, instead of using stickers, further constrained packing to 180 degrees rotations exclusively (Figure 5).

The parameters of our problem hinted the development a shelf-packing rather than partition-packing algorithm which rather more appropriate when rectangular rotations are permitted. Several versions of shelf-packing algorithms exists based on the part selection criteria such as first-fit, next-fit, best-fit and worst-fit (Lodi, 1999). Our algorithm is a variation of the heuristic known as best-fit which selects elements that improve the layout, minimizing strip/bin waste, by evaluating all pending candidates rather than selecting the first available. Within the best-fit class of heuristics there are sub-categories based on the metric used to select the best candidate. We experimented with minimizing various metrics (Figure 6) and developed a composite heuristic that performed exceptionally well for our problem. In detail we examined:

**Block Area**: the area of the rectangle remaining after placing a new part in a strip, as long as the sheet and as wide as the maximum of its current width and the new parts’ width. The first part is placed unconditionally into an empty strip and determines its initial width. This is the classic rectangular best-fit packing metric which we use as benchmark. The obvious drawback is that it produces gaps between parts, especially noticeable when the side edges of two consecutive parts have similar slope. This is due to the bounding box positioning strategy which disregards shape characteristics.

**Residual Length**: the length remaining after placing a new element in a strip as long as the sheet’s length. This metric, as all following, is shape-aware using point of contact instead of bounding
box positioning. It performs marginally better than the rectangular area but the use of length instead of area results in suboptimal selection because it ignores part and strip widths. For instance the residual length can be zero but the discrepancy between contained part-widths may be so wide that the serrated edge produced can waste significant amount of material.

**Slope Misalignment**: measures area of wasted material, a small triangle or quad, between parts due to their edge slopes. It is a metric we developed empirically which compacts parts by eliminating their intermediate gaps. It selects the first part that best aligns to the sheet’s edge thus minimizes the initial placement but its major drawback appears at the end of each strip where material is wasted because the metric does not recognize the boundary condition but only inter-element interfaces.

**Gross Area**: is the area of a strip remaining after placing a new element. It is thus equivalent to the rectangular area of a strip as long as the sheet’s length and as wide as the maximum width of all contained parts, less the sum of areas of all parts. Net area is similar in spirit to the block area metric but due to point of contact positioning it performs better. The last part placed tends to neatly align with the sheet’s edge which is a natural collateral effect of area minimization rather than a specially encoded rule.

**Quad Area**: equals the net residual area, plus the slope misalignment area, plus the part-side area (the material waste due to uneven part widths). Quad area composes characteristics from previous metrics disaggregating gross area into components and using only adjacent part features which are relevant. The insight is derived from the observation that other metrics assume either too much or too little about the local neighborhood. Its initial element is selected by best alignment of part-to-sheet edge; it prefers parts that minimize inter-element gaps like the slope metric; it prioritizes parts with similar widths than waste less material based on area and in excess using the side error; and aligns best with the far end of the strip similar to gross area.

**WORKFLOW**

The design to fabrication workflow was organized into segments to assist teamwork collaboration (Figure 7). The project model, generated in python, produced the design geometry as well as data tables containing part numbers and quadrilateral coordinates. Those were imported into the packing engine, an extension application in C#, which produced layout geometry as well as an associative schedule containing part, strip and sheet numbers, positioning coordinates and transformation flags. Layout data would then pass through a drawing generation sub-system, in python, to produce fabrication ready files containing cutting profiles, hole-center locations and annotation engraving lines organized according to specification defined in collaboration with the fabricator. For design-development and process-prototyping we implemented a sub-system post-processing fabrication files, in C#, producing standard 3-axis CNC g-code which we employed for verifying the fabrication logic, identify the best
routing speed and feed-rate subject to visual and performance criteria as well as to estimate total production time. Additional, details of the layout and packing engine include: the determination of best transformation, subject to constraints, using a symmetry encoding in a look up table which classified parts by their absolute difference of edge slopes, that is the smaller the difference the better the match; and automatic machining compensations such as accounting for sheet margins for handling, router bit diameter padding and any arbitrary preferred spacing by introducing parametric offset into the closest point of incidence between parts layout procedure.

CONCLUSIONS

Our method achieved 4-5% reduction in number of sheets, time and cost which yields 40-50% reduction of waste in comparison to the block area benchmark metric. In addition, we evaluated the process using synthetic data generated using uniform, normal and skew normal distributions, variable number of parts from a hundreds to tens of thousands, variable slope angles as well as width/length aspect ratios to access the performance behavior in general use cases (Figure 8). Overall the results are verifying the intuition that large number of small parts compact better than same total length of large or medium size parts in the same sheet (see distributions chart). The slope of the short span edge contributes significantly to compaction rate (circa 2% per 10 degrees) verifying the original hypothesis that shape may be a critical factor. The algorithm is fast (requires 10sec for the project data of 3912 parts, including drawings and spreadsheet generation) and exhibits quadratic running time behavior which can be further optimized scale-wise with pre-computation of part incidence offsets. The aspect ratio, that is the proportion between part widths and lengths, has also very large contribution to compaction and the quad metric outperforms gross area while residual length and slope are practically counterproductive. Using 15mm margin between parts to account for the machining drill bit and handling produces an in-
crease of waste by approximately 20% which we see as a constant overhead independent of the packing method. However, we note that these may be gross generalizations as the overheads and compaction behavior is very sensitive to part dimensional distribution as well as the material sheet’s dimensions and only perhaps at very large number of parts these differences may average out and form a trend.

We did not perform the layout process manually, due to time constraints, so we can thus only hypothesize that our method may perform better in material use/waste. We are certain though that due to the large number of structural parts the process certainly outperforms any attempt to manually produce the same results in terms of time, accuracy and flexibility by quite a few orders of magnitude. In addition, the integration of optimization in the parametric process simplified the workflow from concept design to manufacturing specification eliminating documentation overheads as a result of design iteration and automated the evaluation of scheduling protocols for segmenting the job into blocks for the purpose of optimizing, albeit empirically, the throughput of material from the factory floor to the site assembly and installation.

Computational heuristics for classical combinatorial problems, already studied extensively from as far back as the 1960s, such as information clustering, bin packing, and the traveling salesman problems can achieve near optimal solutions with very low implementation complexity. These problems are important for addressing design-built issues such as building part simplification, part configuration/layout and machine path optimization, respectively, resulting in efficiencies in materials, fabrication labor-time and cost. The bespoke design and
production regime in architecture, engineering and the construction industry and recent trends of mass customization, using digital design and production processes, makes those methods extremely relevant today. Adaptation to project requirements rather than direct application of their theoretical versions may assist in achieving agility from design to production and address the large volumes, high density and complexity of digital design information such as those highlighted but exclusive to free-form architectural design. We suggest that there is a threshold between general purpose computation: methods of addressing problems in their most generic/generalizable form; and the other extreme of very narrowly tailored solutions: systemic processes solving only one instance or a very narrow band of one particular problem. Perhaps, at that particular point simplicity: the ease of implementation or configuration; flexibility: the ability to experiment, extend or completely change a strategy altogether; and specificity: the ability to take advantage of context awareness can all be reached. Therefore bespoke design manufacturing processes supported by digital fabrication can compete with the efficiencies of standard industrial production. In terms of design research this perspective offers an approach in revisiting classical computation in a highly context-aware mode of contemporary design and production. In conclusion, our research work demonstrates that with relatively benign increase of programmatic overhead one may achieve measurable benefits which we note that in aggregate, if every sub-process is similarly optimized or the project size is sufficiently large, they may yield significant improvements in design-built performance.

REFERENCES


