Electron tunneling in one-dimensional channels: Effects of electromagnetic interaction

A. A. Odintsov

*Department of Applied Physics, Delft University of Technology, 2628 CJ Delft, The Netherlands
and Nuclear Physics Institute, Moscow State University, Moscow 119 899 GSP, Russia*

(Received 2 December 1991)

We show that electromagnetic interaction suppresses the rate of electron tunneling through the barrier in a one-dimensional (1D) electron channel at low voltages and temperatures. For narrow channels, the Fermi velocity may be comparable to the phase velocity of electromagnetic excitations (1D plasmons). In this case, tunneling electrons experience a quantum effective impedance $Z_{\text{eff}} = h/e^2$, which results in a universal behavior of the zero-temperature I - V characteristic ($I \propto V^3$) and of the temperature dependence of the conductance ($G \propto T^2$).

In the last few years striking progress has been achieved in the study of semiconductor nanostructures,¹ including the discovery of a number of interesting physical effects. One group of such effects can be understood in the framework of noninteracting electrons. Due to the small size $w \sim \lambda_F \approx 50$ nm of the system [e.g., narrow constriction in two-dimensional (2D) electron gas] quantization of transversal motion of electrons occurs. As a result, adiabatic quantum transport channels arise and the conductance G of a point contact shows quantization at multiples of $2e^2/h$ (Refs. 2–4). If there are transmitting channels in the constriction ($G \geq 2e^2/h$) the conductance is of a metallic type and the electromagnetic interaction does not drastically affect the transport properties.

On the other hand, single-charge effects^{5,6} caused by the Coulomb interaction of electrons were observed^{7,8} in semiconductor nanostructures with two barriers separating a small island of electron gas from two broad electron gas regions (reservoirs). The electron transport through the barriers occurs by means of tunneling ($G \ll e^2/h$) and the electric charge of the island is quantized in units of e . Due to the small capacitance of the island, the addition of one electron to it significantly changes the conditions of tunneling for other electrons.

Electromagnetic interaction manifests itself differently for the tunneling through a single barrier. Now we do not have a well-defined region whose charge is quantized in units of e . The electron-tunneling process is accompanied by the spread of the charge-density excitation away from the point of tunneling. In turn, the tunneling electron interacts with the electromagnetic field created by this excitation. This sort of single-charge effect occurs in small tunnel junctions.^{9–12}

One can assume that similar effects could also take place in semiconductor nanostructures. Specifically, we consider electron tunneling through a barrier in a long 1D electron channel (Fig. 1). There are at least two important features of this system: (1) electrons in the channel move ballistically in one dimension; (2) the Fermi velocity (for narrow channels) may be comparable to the ve-

locity of electromagnetic excitations. In contrast, in tunnel junctions electrons spread diffusely; their velocity is much less than the velocity of electromagnetic field propagation.

In this paper we consider the case of large electron concentration n in the channel $n > a_B^{-1}$, where a_B is the Bohr radius. In the opposite limit Wigner crystallization occurs. The transport of the 1D Wigner crystal through the barrier (i.e., pinning center) was studied in Ref. 13.

To describe the effects of electromagnetic interaction on electron tunneling we adapt the real-time version¹⁰ of the approach developed by Nazarov for the tunnel junctions. When a voltage V is applied to the channel, the tunneling current $I = e(\Gamma_{l,r} - \Gamma_{r,l})$ is given by the difference of the rates of tunneling from left to right $\Gamma_{l,r}$ and from right to left $\Gamma_{r,l}$ [$\Gamma_{l,r}(V) = \Gamma_{r,l}(-V)$],

$$\Gamma_{l,r}(V) = \frac{1}{\pi\hbar} \sum_m T_m \int d\varepsilon \int d\varepsilon' f_l^{(m)}(\varepsilon - eV/2) \times [1 - f_r^{(m)}(\varepsilon' + eV/2)] \times P_m(\varepsilon - \varepsilon'). \quad (1)$$

Here $f_l^{(m)}$ and $f_r^{(m)}$ are Fermi functions for the electrons in the m th subband and T_m is the transparency of the m th transport channel at the Fermi level. We will consider the case of small voltages and temperatures $\max(eV, T) \ll \hbar/\tau_{tr}^{(m)}$, when the transparency T_m does not change substantially at the relevant energy scale ε , $\varepsilon' \sim \max(eV, T)$ (here $\tau_{tr}^{(m)}$ is the traversal time for the barrier in the m th transport channel). We also neglect inter-channel scattering in Eq. (1).

We have introduced in Eq. (1) the conditional probability $P_m(E)$ of an electron propagation through the channel with the change of energy E due to the electromagnetic interaction [in the absence of interaction the tunneling is elastic, $P_m(E) = \delta(E)$]. The probability $P_m(E)$ can be obtained by considering the modification of the electron propagation amplitude by the electromagnetic field.¹⁰ In the nonrelativistic limit one obtains (we will omit the subscript m)

$$P(E) \propto \langle \hat{K}^+(E) \hat{K}(E) \rangle, \quad (2)$$

$$\hat{K}(E) = \lim_{x_1 T_1 \rightarrow -\infty; x_2 T_2 \rightarrow \infty} \int dt \exp \left[\frac{i}{\hbar} Et \right] \exp \left[\frac{i}{\hbar} \int_{t+T_1}^{t+T_2} d\tau \int_{x_1}^{x_2} dx e\rho(x, \tau - t) \hat{\phi}(x, \tau) \right]. \quad (3)$$

FIG. 1. An example of the system under consideration: gates 1 and 2 on top of the (Al,Ga)As heterostructure define the 1D electron channel interrupted by constriction.

Here $\rho(x,t)$ is the conditional probability density, describing the motion of the electron, which tunnels at $t=0$; the operator $\hat{\phi}(x,\tau)$ describes fluctuations of the electric potential along the electron trajectory.¹⁴ It can be considered as a sum of a number of Bose operators which, in our case, correspond to the excitations of the electron density in the channel. Hence, it is possible to express the averages (2) of exponential operators (3) via the exponents of the Green functions $\langle \hat{\phi}(x',\tau)\hat{\phi}(x,0) \rangle$. Using a Fourier representation for the Green functions, we obtain the result¹⁰ for the probability $P(E)$,

$$P(E) = \int \frac{dt}{2\pi} \exp \left[\frac{i}{\hbar} Et - S(t) \right], \quad (4)$$

$$S(t) = \frac{e^2}{\hbar} \int \frac{d\omega}{2\pi} (1 - e^{-i\omega t}) \times \left[1 + \coth \left[\frac{\hbar\omega}{2T} \right] \right] \frac{\text{Re} Z_{\text{eff}}(\omega)}{\omega}, \quad (5)$$

$$Z_{\text{eff}}(\omega) = \frac{i\omega}{e} \int dx \rho_{-\omega}(x) \Phi_{\omega}(x). \quad (6)$$

Here, the potential

$$\Phi_{\omega}(x) = e \int D_{\omega}^{(r)}(x, x') \rho_{\omega}(x') dx'$$

is a response to the charge-density excitation $e\rho_{\omega}(x)$, and $D_{\omega}^{(r)}(x, x')$ is the retarded Green function

$$D_{\omega}^{(r)}(x, x') = -\frac{i}{\hbar} \int dt \Theta(t) \langle [\hat{\phi}(x, t), \hat{\phi}(x', 0)] \rangle e^{i\omega t}. \quad (7)$$

From expressions (4) and (5) one sees that the influence of the electromagnetic field on the propagating electron can be described by an *effective impedance* $Z_{\text{eff}}(\omega)$. The sense of the “microscopic” definition (6) of the impedance can be seen from the expression for the power dissipated by the tunneling electron

$$\begin{aligned} W &= \frac{1}{2} \text{Re} \int j_{-\omega}(x) Z_{\omega}(x) j_{\omega}(x) dx \\ &= \frac{1}{2} \text{Re} \int i\omega e \rho_{-\omega}(x) \Phi_{\omega}(x) dx, \end{aligned} \quad (8)$$

where we have defined the impedance $Z_{\omega}(x)$ at point x as follows:

$$-\partial \Phi_{\omega}(x) / \partial x = Z_{\omega}(x) j_{\omega}(x).$$

Since the 1D current density $j_{\omega}(x)$ has the dimensions of charge, and in one-electron processes a natural scale of $j_{\omega}(x)$ is e , we arrive at expression (6) for

$$Z_{\text{eff}}(\omega) = \int Z_{\omega}(x) dx.$$

In order to estimate an impedance $Z_{\text{eff}}(\omega)$ we should determine first the conditional probability density $\rho(x,t)$. We assume that the length l_b of the barrier (located at $x=0$) is much smaller than both the electron mean free path l_e and the length of the channel l_{ch} . For an electron moving from left to right we can write

$$\rho(x,t) = \delta(x - v_F t), \quad (9)$$

for $\tau_{\text{tr}} < t < \tau_l$,

$$\tau_{\text{tr}} \sim l_b / v_F, \quad \tau_l = \min(l_e, l_{\text{ch}}) / v_F. \quad (10)$$

Equivalently, one can consider the probability packet

$$\rho(x,t) = [\delta(x - v_F t) - \delta(x + v_F t)] \Theta(t), \quad (11)$$

describing the motion of an electron to the right from the barrier and one of a hole (formed after electron tunneling) to the left. This description has a simple interpretation. After tunneling, two identical (apart from the sign) charge-density excitations spread to the right and to the left from the barrier interacting with the tunneling electron and hole, changing the energies of these carriers. The sum of the electron and hole energy changes far enough from the barrier corresponds to the shift of energy E of the tunneling electron [see Eq. (2)].

Let us consider now the electrodynamics of the system. On a length scale larger than the electron channel width w , the problem is one dimensional. The Fourier components of the electron density $\rho_{\omega,k} + \tilde{\rho}_{\omega,k}$ [where $\rho_{\omega,k}$ corresponds to the packet (11) and $\tilde{\rho}_{\omega,k}$ refers to all the other electrons] are related to the electric potential $\Phi_{\omega,k}$ by the equation

$$C_{\omega,k} \Phi_{\omega,k} = e(\tilde{\rho}_{\omega,k} + \rho_{\omega,k}), \quad (12)$$

where $C_{\omega,k}$ is the effective capacitance of the channel per unit length. If the distance D from the channel to the external conductors (e.g., gates and conducting substrate) is large $D > w$ the capacitance can be estimated as

$$C_{\omega,k} \simeq \frac{\kappa}{2 \ln[\min(D, k^{-1})/w]}, \quad (13)$$

where κ is the dielectric constant of the semiconductor. In the long-wavelength limit ($kD < 1$) the value of $C_{\omega,k}$ does not depend on k and ω ($C_{\omega,k} = C$).

The current density $\tilde{j}_{\omega,k}$ is related to the electric field $E_{\omega,k} = -ik\Phi_{\omega,k}$ via the complex conductivity $\sigma_{\omega,k}$,

$$\tilde{j}_{\omega,k} = -ik\sigma_{\omega,k}\Phi_{\omega,k}, \quad (14)$$

and satisfies the continuity equation

$$\omega e \tilde{\rho}_{\omega,k} - k \tilde{j}_{\omega,k} = 0. \quad (15)$$

The presence of the barrier provides the boundary condition $\tilde{j}_{\omega}(x = \pm 0) = 0$. We now estimate the conductivity $\sigma_{\omega,k}$ and solve the system of electrodynamic equations (12), (14), (15) with the source (11) for wide and narrow channels.

Wide channel. We consider an electron channel, whose

width w is large compared to the Bohr radius $a_B = \kappa \hbar^2 / me^2$. Since the distance between electrons is less than a_B , their linear concentration $n > w / a_B^2$, as well as the number of filled subbands N is large ($N \simeq [(2/\pi)nw]^{1/2}$ for a hardwall confinement potential). In this case the conductivity $\sigma_{\omega,k} = i/\omega L$, is determined by the electron kinetic inductance $L = m/ne^2$ which is much larger than geometric inductance. One sees that the electrodynamics of a wide channel is equivalent to that of an LC line (apart from weak logarithmic dispersion of the capacitance for $w \ll k^{-1} \ll D$). The phase velocity of 1D plasmons is $c_0 = (LC)^{-1/2}$.¹⁵ Comparing c_0 to the Fermi velocity in the lowest subband

$$v_F = (2\pi \hbar^2 n / m^2 w)^{1/2}$$

(for the hardwall confinement potential), one has

$$v_F / c_0 \simeq (a_B / w)^{1/2} \ll 1.$$

Solving the electrodynamic equations with the source (11) we obtain

$$Z_{\text{eff}}(\omega) = 2Z_0 \left[\frac{c_0}{c_0 - v_F} \right]^2 \frac{c_0}{c_0 + v_F}, \quad (16)$$

where

$$Z_0 = (L/C)^{1/2} = (Cc_0)^{-1}$$

is the impedance of an equivalent LC line. Since $v_F \ll c_0$, the impedance $Z_{\text{eff}}(\omega) \approx 2Z_0$ does not depend on the character of electron motion [which was assumed to be ballistic in Eq. (11)]. Due to the dependence of the inductance L on electron concentration, the impedance varies as $Z_{\text{eff}} \propto n^{-1/2}$.

Narrow channel. In a narrow channel only a few electron subbands are populated. Hence, it is instructive to consider the case of one filled subband. Provided that before the filling of the second subband the electron concentration n is larger than a_B^{-1} , we can estimate the channel width $w < \alpha a_B$, $\alpha \sim 3$ when our consideration is valid.

To determine the conductivity $\sigma_{\omega,k}$, we consider the electron gas in the channel as a collisionless one-dimensional plasma. Using the linearized Boltzmann equation (without collision integral) for the perturbation of the equilibrium Fermi distribution we get

$$\sigma_{\omega,k} = \frac{2ie^2 v_F}{\pi \hbar} \frac{\omega}{\omega^2 - (kv_F)^2} \quad \text{for } T=0, \quad (17)$$

where $v_F = \pi \hbar n / 2m$ is the Fermi velocity.

From Eqs. (12), (14), (15), and (17) we find the phase velocity $c = (c_0^2 + v_F^2)^{1/2}$ of electromagnetic excitations. At low concentrations $n \ll \bar{a}_B^{-1}$ [$\bar{a}_B = (\pi^2 C / 4\kappa) a_B \sim a_B$] this result formally describes the “classical” 1D plasmons ($c \approx c_0 \gg v_F$) considered above. An increase of the electron concentration ($n \gg \bar{a}_B^{-1}$) effectively reduces the Coulomb interaction. As a result the velocity of plasmonlike excitations tends toward, but remains larger than, the Fermi velocity $c \approx v_F \gg c_0$. These excitations correspond to the zero sound in the 1D Fermi liquid.

An expression for the effective impedance reads

$$Z_{\text{eff}}(\omega) = \frac{2}{C(\sqrt{c_0^2 + v_F^2} - v_F)}. \quad (18)$$

In the low-concentration limit ($n \ll \bar{a}_B^{-1}$) the impedance (formally) tends to the value $Z_{\text{eff}} = 2Z_0$ which decreases with increasing n [see text below Eq. (16)]. For $n \gg \bar{a}_B^{-1}$ we obtain a universal value $Z_{\text{eff}} = \hbar / e^2$ (see expressions for v_F , c_0 , and L). As a naive interpretation of this result one can say that the electron and the hole [see Eq. (11)] each feel a quantum resistance $\hbar / 2e^2$ of the channel. We stress, however, that the result is strongly related to the electromagnetic interaction of the tunneling electron with other electrons in the channel, which are considered as a collisionless 1D Fermi plasma.¹⁶

Our analysis shows that an effective impedance is nearly Ohmic, $Z_{\text{eff}}(\omega) \approx Z = \text{const}$ in the frequency range

$$\max(\tau_{\text{em}}^{-1}, \tau_l^{-1}) < \omega < \tau_{\text{tr}}^{-1}, \quad (19)$$

where $\tau_{\text{em}} = l_{\text{ch}} / c$ is the time of electromagnetic excitation propagation along the channel and τ_l , τ_{tr} are determined in Eq. (10). This type of impedance was considered in the theory of Coulomb blockade in small-tunnel junctions (see, e.g., Refs. 6 and 11). We introduce an exponential cutoff of the impedance $Z_{\text{eff}}(\omega) = Z \exp(-\omega / \omega_c)$ at high frequencies ($\omega_c \sim \tau_{\text{tr}}^{-1}$) and find the probability $P(E)$ for zero temperature,

$$P(E) = \frac{1}{\Gamma(z)E} \left[\frac{E}{\hbar\omega_c} \right]^z \exp \left[-\frac{E}{\hbar\omega_c} \right] \Theta(E), \quad (20)$$

where $z = 2e^2 Z / \hbar$ is the dimensionless impedance and Γ is the gamma function. Substituting this expression into Eq. (1) we obtain the tunnel current

$$I = \frac{1}{\Gamma(z+2)} \left[\frac{e|V|}{\hbar\omega_c} \right]^z G_T V, \quad (21)$$

where $G_T = (2e^2 / \hbar) \sum T_m$ is the conductance of the barrier in the absence of electromagnetic effects. [For a wide-electron channel we can simply sum the transparencies T_m of the barriers for different subbands because their effective impedances (16) and functions $P_m(E)$ are virtually the same.] The result (21) is correct for finite temperatures $T \ll eV$. For higher temperatures $T \gg eV$ the tunneling current is proportional to the voltage, $I = GV$, with conductance

$$G = \frac{\pi^{1/2}}{2} \frac{\Gamma(1+z/2)}{\Gamma(3/2+z/2)} \left[\frac{\pi T}{\hbar\omega_c} \right]^z G_T. \quad (22)$$

Equations (21) and (22) are valid if the characteristic frequency $\max(eV, T) / \hbar$ is in the range of (19).

The electromagnetic interaction manifests itself in the power-law nonlinearity of the I - V characteristic (21) and the power-law dependence of the conductance (22) on temperature. The magnitude of electromagnetic effects crucially depends on the ratio z of the effective impedance to the quantum resistance $\hbar / 2e^2$. For a relatively wide channel ($w = 300$ nm) the effective impedance (16) can be estimated as ≈ 3 k Ω (for electron concentration $n = 5 \times 10^6$ cm $^{-1}$). Due to the rather low impedance

($z \simeq 0.23$) the electromagnetic effects are weak. With decreasing channel width, the impedance increases ($Z \propto n^{-1/2}$) and the effects become more pronounced. For narrow channels with only one filled subband, the universal value of the normalized impedance $z=2$ results in the universal behavior of the I - V characteristic ($I \propto V^3$) and of the conductance-temperature dependence ($G \propto T^2$).

We discuss now the conditions for observation of the effects. To separate electromagnetic effects from ones related to the change of the barrier transparency, the channel must be much longer than the barrier $l_{\text{ch}}/l_b \gg c/v_F$ [see Eq. (19)]. The channel must be narrow enough so that the impedance is not much less than $h/2e^2$. These

conditions are fulfilled for the parameters $w \approx 50$ nm, $l_b \approx 50$ nm, $l_{\text{ch}} \approx 1 \mu\text{m}$. The energies eV , T should be in the range $0.15 \text{ meV} < \max(eV, T) < 1.5 \text{ meV}$ (estimated for a hardwall potential). Although the fabrication of a long and narrow uniform electron channel may be a serious technological problem, we believe that manifestations of electromagnetic interaction will be detectable in experiment in the near future.

This work was supported by the Dutch science foundation FOM. The author is grateful to D. V. Averin, F. W. J. Hekking, A. T. Johnson, M. Jonson, L. P. Kouwenhoven, and especially to Yu. V. Nazarov and Gerd Schön for fruitful discussions and encouragement.

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