THE LUBRICATING-FILM MODEL
FOR
CORE-ANNULAR FLOW
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The technique based on core-annular flow is attractive for transporting viscous oils in pipelines, not least because the pumping power required is comparable to that for pure water flow. The principle is that a thin water film is present between the oil and the pipe wall. This film acts as a lubricant, giving a pressure gradient reduction.

This thesis starts with an investigation of the characteristics of the core flow technique and a literature review of theoretical models and correlations to determine pressure gradients for stable core flow as well as methods to determine pressures required for restart from a stratified configuration that has formed during shut-down. Stable core-annular flow in pipes occurs for oil velocities greater than a certain critical value, provided the oil viscosity is sufficiently large.

Observations of stable core flow in horizontal transparent pipes indicate that the oil core, flowing at an eccentric position in the pipe, which is a function of buoyancy, invariably has a wavy interface. The older models for stable core flow ignored the presence of the waves and attempted to predict pressure gradients using the water film hold-up and core eccentricity as input parameters. Compared to measured pressure gradients, model predictions were far too low.

The main theme of this theoretical study has been to consider the systematic development of a lubricating film model for the description of core-annular flow of two viscous fluids (oil and water) in a horizontal pipe. In this model the observed existence of interfacial waves with an asymmetric shape is an essential feature. For a known interfacial wave geometry the model is capable of computing both the core eccentricity and the pressure gradient, using oil velocity, density difference, input water fraction and water film hold-up as input parameters. For the latter empirical correlations are used. Ultimately these can be replaced by a relationship between waterfilm thickness and input water fraction determined by the theoretical model as is shown for the simplest model version. Such a relationship will depend on the interfacial wave geometry.
Starting with the laminar version of the lubricating film model, a sensitivity study reveals the importance of the interfacial wave parameters (amplitude, length, asymmetry and shape) for an accurate determination of pressure gradients. The results of two series of measurements have been used to obtain more information on the interfacial waves during core-annular flow, and to critically examine the validity of some simplifying model assumptions. The measurements indicate the waves to have amplitudes that are proportional to the distance of the oil core to the pipe, wavelengths to increase with input water fraction and oil viscosity, and wave asymmetry parameters between 0.4 and 0.5, giving waves with a relatively short front end, that is flattened at the pipe top and a steep tail end i.e. quite different from the saw tooth shape originally assumed. A simple long wavelength model, that considers Couette-Poiseuille flow of two viscous fluids between parallel plates, is only partly capable of predicting the observed wave features. There clearly is a need for a more sophisticated interfacial wave model, that in particular is capable of predicting the wavelengths of stable finite amplitude waves on a liquid/liquid interface close to a rigid wall.

Another interesting observation made during the core flow experiments in the 2-inch pipe is the turbulence in the waterfilm. This has led to the formulation of the turbulent version of the lubricating film model for core-annular flow. Throughout the study experimental results for a 3000 mPa.s oil with a density difference of 20 kg/m^3 flowing in the core flow mode in a 2-in horizontal pipe at an input water fraction of 0.10 have been used as a reference case. The turbulent lubricating film model gives excellent predictions of the pressure gradients, provided the observed wave data are used. For a fixed oil velocity the measured pressure gradients only mildly depend on input water fraction, showing a minimum at a water fraction of 0.10. The model, however, predicts the pressure gradient to increase with input water fraction. It is not clear whether the disagreement between model and reality is due to an incorrect dependence of wave data on input water fraction or to an artefact of the model. A disagreement between model and experiments was also found when the core eccentricities were analysed: according to the model the oil core, although moving towards the pipe centre at an increasing oil velocity, remains at a relatively eccentric position in the pipe, whereas the experiments indicate that as from 0.75 m/s a near-concentric oil core position is reached. This disagreement most likely can be resolved by introducing inertia effects: it is shown that for oil velocities larger than 0.75 m/s the assumption of neglecting inertia forces is no longer justified for the 2-in pipe. Model equations, in which at least the largest inertia terms have been incorporated, suggest that the inertia effects may lead to an increase in load capacity hence a reduction of the core eccentricity. In view of mathematical difficulties this version of the lubricating film model is subject of further study.

When applied to larger pipe sizes, using the wave data measured in the 2-in pipe, the model indicates that because of an increase in buoyancy the core flow technique becomes less attractive. The apparent viscosity for core flow transport increases more or less linearly with pipe size. The limited set of pressure loss data available for larger pipes are reasonably well predicted by the model. It is shown that for an oil velocity of 1 m/s, typical for practical applications, the assumption of neglecting inertia effects is justified for larger pipe sizes, at least at the top of the pipe, where lubrication effects are largest.
Chapter 1
INTRODUCTION

To facilitate the flow of a highly viscous heavy crude oil through a pipeline, it has been customary to reduce the viscosity of the oil either through the addition of a hydrocarbon diluent or through the installation of heating equipment at short intervals along the pipeline. The former method is practicable only in the somewhat unusual case where a supply of light oil is available in the same region as the heavy oil; the latter system is both inconvenient and costly.

Another possibility is the simultaneous transport through the pipe of the highly viscous oil and an immiscible "low-viscosity" liquid such as water. Experiments to examine this possibility (Russell et al., 1959 and Charles et al., 1961) have been carried out for a series of different flow patterns, including water drops in oil, stratified flow, concentric oil-in-water (core-annular) flow, and oil drops in water. The measured pressure drops over the pipe indicated that the addition of water can greatly reduce the pressure gradient.

It was found that of all the flow patterns observed the one most desirable for simultaneous flow was the one with the highly viscous oil as a core and the water only in the annular space between the core and the pipe wall. The experiments showed that with such a core-annular flow the pressure drop over the pipe could be of the same order of magnitude as, or even smaller than, the pressure drop for the flow of water alone at the same mean velocity as for the mixture. The annular film can be very thin (see Figure 1) and thus the required amount of water small; therefore, the pumping power necessary to move this water is negligible.
In the core flow mode of operation, the oil core (immersed in water) is expected to rise to the top of the pipe due to the forces of buoyancy. These forces will become greater as the density difference becomes larger and the oil volume greater. It has been observed that stable core flow in horizontal pipes exists only in a certain range of oil velocities. For practical operation then it is very important to know the velocity boundaries: too large a value for the lower velocity limit for stable core flow will rule out this transportation technique altogether, since it unacceptably reduces the system's flexibility.

Another important practical aspect of this technology is the question of how to guarantee that a certain oil-water system will operate in the core flow pattern. There is some evidence that in particular flow situations a transition to core flow can take place. In practical situations, however, this flow mode is brought about by using special inlet nozzles in combination with physicochemical agents to both facilitate wetting of the pipe wall by water and to improve the stability of the oil-water interface. The oil viscosity seems to be important for this stability. As a rule of thumb, one can say that the core flow technique is attractive for crude oils of the heavy-oil type with viscosity exceeding 500 mPa.s and density above 950 kg/m³.

Finally, shut-down and restart procedures must be known. For the technique to be practical, it should be possible to restart a pipeline from the stratified oil-water situation to the core flow pattern in a reasonable amount of time, without the requirement of excessive pumping power.

In reviewing the state of the art of core flow as part of two-phase liquid-liquid science and technology, we have tried in this thesis to cover the aspects just mentioned. First, core flow is put into perspective against other oil-water flow patterns in horizontal pipes. Attention is then paid to the aspects of hydrodynamic stability. It is quite obvious that a great deal of research still needs to be done in this area. In particular, there is a need to quantify the characteristics of the ripples on the oil core that are observed in practice. Subsequently the important aspect of the velocity range for stable core flow is dealt with. Unfortunately, all that the literature provides is a summary of observations on lower and upper critical velocities, with hardly any attempt at physical modeling or qualitative study of the physical mechanisms involved in maintaining a stable core flow pattern.

The literature has been reviewed for the existing theoretical models and correlation methods of calculating pressure gradients for stable core flow. In our review we make a distinction between concentric and eccentric core flow models with laminar flow both in the oil core.
and in the water film. (The effect of turbulence in the water film on pressure loss has received little attention thus far.) It is quite unsatisfactory that the models have taken for granted the stable core flow pattern; in other words, they have entirely bypassed the question of how the configuration of an oil core surrounded by a water film is established. They do show that pressure gradients depend at least on hold-up and core eccentricity. They also invariably predict pressure gradients that are very much lower than those measured in practice. Clearly, some important physical parameters are missing. Of course, for specific conditions, pressure gradients can be predicted accurately with the aid of empirical correlations. However, these correlations are only of limited help for improving our understanding of the physical mechanisms involved.

Two models are available in the literature for restart from a stratified situation to core flow. The first is for a completely stratified situation, and the other is for a partially stratified situation. The latter gives the lower restart pressures. Comparisons are made with experimental data. Additional research is needed to attempt to bring restart pressures down to practicably acceptable levels.

The literature study shows that there is a need for a stable core flow model that is not only capable of predicting pressure gradients for various pipe sizes but that will also give information on the velocity window within which stable core flow is possible. Not too long ago a first version of such a model, the lubricating-film model was introduced. It is based on a model introduced by Ooms et al. (1972) to answer the central question regarding core-annular flow, namely, how is the buoyancy force on the core counterbalanced to result in a stable core at some eccentric position in the pipe? The oil viscosity in the model is assumed to be so high that any flow in the core, and hence any variation in the shape of the oil-water interface with time, may be neglected. The core surface is assumed to be rippled, in agreement with observation. The shape of the ripple is determined from experiments. Furthermore, the water hold-up has to be specified. This parameter is also calculated from an empirical correlation.

The main objective of this thesis is to critically examine the various model parameters and simplifying assumptions. This investigation starts in chapter 3 with an introduction of the first version of the model: the laminar-lubricating-film model. After a sensitivity study and a comparison of model predictions with existing experimental data, in the remainder of this chapter attention is given to the flow in the water annulus, the wave characteristics and the water hold-up. This leads to a second version of the model in which turbulence in the water film is allowed for and an improved water hold-up calculation method is introduced.

The wave characteristics are so important for the model, that a separate chapter (chapter 4) is devoted to experimental and theoretical information on waves at a water/oil interface. Best estimates for wave amplitude, length, asymmetry and shape for core-annular flow in a 2-in pipe are derived in this chapter.

In particular, because of the poor understanding of wave formation, application of the model to core flow in larger diameter pipes is risky. Still to gain insight into the capabilities of the model the scaling effect is studied in chapter 5. Also a comparison is made there with some limited data for 6-in and 8-in pipes.

It is made clear in the Conclusions that the lubricating-film model raises new questions even as it answers old ones. The model forms a physical framework containing only a number of the characteristics of stable core flow in horizontal pipes. For various other aspects, additional information is required. In particular, the interplay between wave formation and core position, both as a result of forces in the waterlayer is a point of concern. Moreover, some disagreements between model predictions and measurements that arise under certain flow conditions require further attention.
Chapter 2

CORE FLOW CHARACTERISTICS AND LITERATURE REVIEW

Although the emphasis here will be on horizontal core flow, and in particular on a high-viscosity oil core surrounded by a water annulus with small water fractions, it is interesting to see how this core flow mode fits in with other possible oil-water transportation systems.

Figure 2 shows a schematic setup, introduced in this form by Seymour (1968) for problems related to oil-water flow. According to this scheme, a stratified oil-water flow system may, at some critical velocity, transform into either a distributed type of flow or into a core-annular flow (disregarding the possibility of slug flow). Some of the distributed-flow-type systems may, however, also have a core flow character. For instance, the flow of oil-water dispersion may be a regime in which the oil flows as a core of droplets surrounded by a thin annular water film. The flow of water-in-oil emulsions can also be of the core flow type.

Stein (1978) studied the flow of a concentric water-in-oil (W/O) emulsion core surrounded by an oil-in-water (O/W) annulus. These hybrid core-type flow systems are representatives of a general case: a viscous liquid phase surrounded by a less viscous liquid phase. We will concentrate on a core consisting of pure oil and an annulus containing only water.

As indicated in Figure 2, the analysis of core-annular flow involves a number of component problems:

1. Criteria for the existence of core-annular flow,
2. A model for the pressure loss for core-annular flow,
3. Criteria for the breakup of core-annular flow, and
4. A model for the pressure loss during start-up of core flow from a stratified layer.

It is convenient here to treat the questions on the existence and breakup as parts of the general problem of establishing the criteria for the existence of stable core-annular flow.

2.1 Criteria for the Existence of Stable Core-Annular Flow

2.1.1 Practical Observations

Core-type flow can be established by using water that contains additives (e.g., surface-active agents for preferential wetting of the pipe surface by water and/or additives to prevent the formation of water-in-oil emulsions) as proposed, for example, by Clark and Shapiro (1949). It has been found that it is necessary to use suitable additives in the water phase, and also that a special inlet device is beneficial (Netherlands Patents, 1967, 1977).
2.1.2 Hydrodynamic Stability

The problem of hydrodynamic stability is the determination of the conditions (velocity, fluid properties, water fraction) under which a stable core flow configuration can be maintained. An attempt to define the causes of hydrodynamic stability for a horizontal core-annular flow configuration was made by Ooms (1971, 1972). He considered two factors for instability of the core/water interface:

1. A Rayleigh-Taylor-type instability, which is due to the combined effect of the interfacial surface tensions and/or the unfavourable density gradients in the two fluids, and

2. A Kelvin-Helmholtz-type instability, which is due to an imbalance of the interfacial velocities of the fluids and manifests itself by a crinkling of the fluid-fluid interface.

Ooms (1972) concluded that, if the thickness of the annular layer at the wall is much smaller than the radius of the pipe, the pipe wall will have a strong reducing effect on the growth of possible Rayleigh-Taylor and Kelvin-Helmholtz instabilities at the interface between the two liquids. A core-annular flow of two ideal liquids with a smooth interface is hydrodynamically unstable; the interface will become rippled. In practice, a stable form of core-annular flow having a rippled interface has been observed. No information is given on the dimensions (amplitude, wavelength) of the ripple at stable flow conditions. In a later article, Ooms and Beckers (1972) derived that core-annular flow is only possible if the ripple has a nonsymmetrical shape. Qualitative and semiquantitative aspects of the lubricating-film model used for this derivation will be discussed below.

The nature of core-annular flow stability was also studied by Hickox (1971) using a standard perturbation method for two fluids flowing concentrically in a straight, circular, vertical pipe. He also demonstrated the possibility of finite-amplitude waves in axisymmetric pipe flow when interfaces between fluids with different properties are present. The most important single cause of instability is noted to be the difference in viscosity between the fluid regions. In this study, density variation and surface tension have less pronounced effects. A linear stability analysis recently carried out by Joseph, Renardy and Renardy (1984) revealed that the core-annular configuration is stable when the more viscous fluid is located at the core and occupies most of the pipe.

In experimental and theoretical studies, Shertok (1976) examined the stability aspects of core-annular flow in a 2-in, 3-m-long pipe placed vertically to eliminate the Rayleigh-Taylor density instability (which could have also been accomplished in a horizontal setup by carefully matching the fluid densities, as was done by Charles et al., 1961). He concluded that, although core-annular flow is characterized by a fluctuating rippled interface for vertical flow, these interfacial fluctuations do not have measurable effects on the bulk flow velocities. Moreover, no tangential velocity components were detected by the Laser-Doppler velocimeter. It was shown qualitatively that the radial disturbances of the bulk flow of the core disappeared quickly.

2.1.3 Velocity Range for Stable Core Flow

The question of the range of fluid velocities in which stable core flow in pipelines is possible was addressed for concentric flow by Charles et al (1961). For an oil-water flow system in a 1-in horizontal pipe, and using oils with viscosities of 6.3, 16.8, and 65 mPa.s and a density of 998 kg/m³, they determined flow regime maps with superficial oil and water velocities as parameters. They considered a range of superficial water velocities of 0.03–1.07 m/s. The experimental results show that at a fixed water fraction there exists a lower critical oil velocity below which core flow cannot be maintained.

In the concentric flow situation a transition to the oil-slugs-in-water flow regime occurs, while in the more general case of an eccentric flow situation (with a given density difference) a transition to stratified flow may be expected. The position of the boundaries observed for the three different oils was identical. The lower critical velocity
increases with increasing water fraction; from 0.1 m/s at a water fraction of 0.17 to 0.5 m/s at a water fraction of 0.67.

The shear between the water annulus and the oil core places an upper limit, or upper critical velocity, beyond which core flow breaks down. High shearing forces may emulsify the oil and water. Low viscosities and interfacial tensions are expected to favour the emulsification or dispersion, but relations for predicting the conditions under which this will occur are not yet available. The experiments carried out by Charles et al. (1961) also show that at high oil velocities a transition to a water-drops-in-oil flow pattern takes place. The boundary for the 16.8-mPa. s oil is at somewhat higher oil velocity than for the 6.3-mPa. s oil (both having an interfacial oil-water tension of about 0.044 N/m).

The upper critical velocity is an increasing function of the water fraction. For the 6.3-mPa. s oil, it varies from approximately 0.2 m/s for a water fraction of 0.09 to 1 m/s for a water fraction of 0.67. The boundary for the most viscous oil used in these experiments (65 mPa. s) however, is not displaced further but is in approximately the same position as the boundary for the 6.3-mPa. s oil. This may be due to a difference in oil-water interfacial tension (0.030 instead of 0.044 N/m).

Shertok (1976) compared predictions by Hickox's linear stability theory with the above-mentioned experimental range of critical velocities for stable core flow. He reported a reasonable agreement in the region of low superficial fluid velocities in either phase. At higher superficial velocities, however, the results diverged considerably.

Shertok explained this by the fact that the lower superficial velocity region is characterized by instabilities of the long-wavelength type. The Hickox development was derived by assuming that only long-wavelength disturbances are important in core-annular flow stability; hence it is not applicable to higher superficial velocities, characterized by short-wavelength instabilities, where core-annular flow breaks up into droplets.

Further literature data are rather limited on critical velocities for the stable core-annular flows of different crudes and pipe sizes. Glass (1961) carried out experiments in 1.20-m-long, 1-cm-ID tubing, in which the oil viscosity was varied from 10 to 30,000 cSt, the oil gravity from 0.97 to 1.03, the volumetric water fraction from 0.09 to 0.80 (on total fluid), and the superficial velocity of the oil from 0.06 to 1.28 m/s. From visual observation he indicated some trends: First, the less viscous the oil, the more the core would tend to break up into globs of oil of various sizes; second, below a critical viscosity no distinguishable core-annulus structure was retained. With oils lighter than water, the oil was carried slightly high in the tube. Shell Oil Co. (1972) reported that core flow in their 6-in line operated with a high-viscosity oil and 30% water, remained stable, as long as the flow rate in the line did not drop below 1 m/s. The line had a design capacity of about 27,000 barrels/day, corresponding to a superficial oil velocity of 1.9 m/s. Stein (1978) reported that there were critical velocities for 155- and 680-mPa.s oils for the configuration of a W/O emulsion core surrounded by an O/W emulsion. He considered three pipe sizes (1, 1.5, and 2 in), however, the information was too scanty to show significant variation of critical velocities with corresponding variation of the pipe diameter. For the 2-in pipe with 155-mPa.s oil and a water fraction of 0.50, a transition from stratified to eccentric W/O-core-O/W annulus flow is reported to occur at 0.7 m/s. Concentric core flow is only reached for these emulsions at superficial oil velocities beyond 3 m/s. For the 680-mPa.s oil-water mixture, the critical velocity is even higher: 4 m/s. A complication here is that different flow patterns arise, depending on whether the input oil fraction is greater (core flow) or less (dispersed flow) than the phase-inversion concentration (see Figure 2).

It is difficult to derive generalized correlation methods from the scattered information available on the lower and upper critical velocities for stable core flow. The lower boundary appears to be hardly affected by oil viscosity, as suggested by the data of Charles et al. (1961), while the additional data for 2- and 6-in pipes...
suggest, for a given water fraction, the lower critical velocity to be an increasing function of pipe size. However, a systematic study is required to substantiate this, since density effects, for instance, may have to be taken into account as well. For the upper critical velocity, the information is also too restricted to develop a proper correlation method. The experiments of Charles et al. (1961) seem to confirm the expected trends for this upper critical velocity: it increases with oil viscosity and interfacial tension. Moreover, the upper critical velocity is expected to increase with pipe size.

2.2 Existing Theoretical Models and Correlation Methods

In modelling core-annular flow of oil and water through a pipeline once the flow pattern has been established, two aspects must be considered - as indeed with any two-phase flow system, namely, the hold-up of the phases and the pressure loss of the system.

2.2.1 Hold-up

When two fluids flow together in a pipeline, the in situ volumetric ratio is, in general, different from the input volumetric ratio. Differences in density and/or viscosity give rise to an important feature of two-phase flow - the occurrence of the "slip" of one phase past the other, or the "hold-up" of one phase relative to the other (one phase accumulating in the pipe). There are various ways to describe this hold-up effect, as have been reviewed by Govier and Aziz (1972) in their book on the flow of complex mixtures in pipes. Charles et al. (1961) used the concept of a hold-up ratio, which they defined as the ratio of the input oil-water ratio to the in situ oil-water ratio. When the hold-up ratio is greater than unity, water is the accumulating phase. When the hold-up ratio is less than unity, oil is the accumulating phase. The two phases flow as a homogeneous mixture when the hold-up ratio is unity (i.e., a no-slip situation). This definition is equivalent to the more generalized definition by Govier and Aziz (1972), who stated that the hold-up ratio is the ratio, at any cross section, of the in situ volume fraction ratio of the heavier to the lighter phase \( H_W/H_0 \) to the input volume fraction ratio of the heavier to the lighter phase \( C_W/C_0 \). In other words,

\[
\text{Hold-up ratio} = \frac{H_W/H_0}{C_W/C_0}
\]

where \( H \) is the in situ volume fraction or hold-up, \( C \) is the input volume fraction, and the subscripts \( W \) and \( O \) stand for water and oil - the heavier and lighter phases in our case - respectively.

Phase hold-ups are defined as follows

\[
H_W = \frac{A_W}{A_p} \quad (1)
\]

\[
H_O = 1 - H_W = \frac{A_O}{A_p} \quad (2)
\]

where \( A_p \) is the pipe cross-sectional area, and \( A_W \) and \( A_O \) are the cross-sectional areas occupied by water and oil, respectively.

The phase hold-ups for the water and oil (\( H_W \) and \( H_O = 1 - H_W \)) are convenient parameters for describing the slip effect. A phase is an accumulating one when its hold-up is larger than its input volume fraction (or, more generally, greater than the local no-slip volume fraction of the phase). The in situ velocities of the phases at any cross section of the pipe can be obtained by dividing the superficial phase velocities \( V_{SW} \) and \( V_{SO} \) by the phase hold-ups, or

\[
V_W = \frac{V_{SW}}{H_W} \quad (3)
\]

and
Here the superficial velocities $V_{gw}$ and $V_{go}$ are defined as the volumetric flow rates of the respective phases divided by the cross-sectional area of the pipe.

In their oil-water experiments with a 1-in pipe, Charles et al. (1961) determined the in situ contents, hence overall hold-ups, by using quick-action valves at either end of the test section and a "pig" for removal of the liquids. For different superficial oil velocities, they presented the results as hold-up ratios versus input oil-water ratios. For the 6.3-mPa.s oil, the hold-up ratio curves pass through maxima (larger than unity), the locations of which coincide with the concentric oil-in-water and oil-slugs-in-water flow patterns. This indicates that for these flow patterns the accumulating effect of the water phase is largest. The hold-up ratio is found to be a decreasing function of the superficial oil velocity (at a given input oil/water ratio or water fraction).

Similar trends have been found for the 16.8-mPa.s oil; the hold-up ratio values for the most viscous oil (65mPa.s), however, reflect the different behaviour of this oil: at high oil/water ratios (water fractions below 0.33) the hold-up ratio is less than unity, indicating that instead of water, oil is now the accumulating phase. Also, for a given oil/water ratio (or input water fraction), the hold-up ratio is hardly dependent on the oil velocity.

From these hold-up data, reported by Charles et al. (1961) in a table and a number of figures for the five different flow patterns observed in the 1-in pipe, we have determined the water hold-ups for core flow as a function of the input volume fraction of water. As shown in Figure 3 for concentric core flow, the water hold-ups for the three different oil viscosities can be significantly larger than the input volume fractions of water. The dissimilar behaviour for the most viscous oil at low water fractions is evident from the two measured water hold-up values being smaller than the input water fraction, i.e., with oil as the accumulating phase instead of water. As stated before, this may be due to the different oil-water interface properties of this oil compared with the other two.

For the 16.8-mPa.s oil at water fractions below 0.20, the water hold-up is some 40% larger than the input water fraction. Because of this accumulation of water, a smaller part of the cross-sectional pipe area will be available for the oil flow. Consequently, for these water fractions, the in situ oil velocities will be some 25% larger than the superficial oil velocities (calculated from an average oil hold-up value of 0.80 for input water fractions of 0.09 to 0.17).

Unfortunately, since the publication of the above key article on concentric core flow, not a great deal of data have been published on measured hold-ups for this flow pattern. Stein (1978) mentioned that...
some measurements carried out at Purdue University using two techniques (sampling and electrical conductivity) gave inconsistent results. More hold-up data for core-annular flow are needed to develop a correlation method that is sufficiently generalized to be suitable for scaling to larger pipe sizes and for a variety of oils. A proper hold-up calculation method will make it possible to compute the actual oil core velocities. These velocities can then be used in a model to compute pressure losses for stable core flow and also in a model for the upper critical velocities for this flow pattern.

2.2.2 Pressure Loss for Stable Core Flow

A pressure-loss calculation method for stable core-annular flow must be of a general nature so that predictions can be made for a variety of pipe sizes, crude oils, and water fractions. The beneficial effect in terms of a decrease in pressure drop, which results from the introduction of water in controlled amounts into pipelines carrying heavy crude oil, is indicated by the pressure drop reduction factor. This factor is defined as the ratio of the pressure drop for the oil flowing alone in the pipe (i.e., at the superficial oil velocity) to the pressure drop of stable core flow. We will also use the reciprocal of this quantity, which is the core flow pressure drop expressed as a fraction of the pressure drop for oil flowing alone: \( \frac{\Delta P_{OW}}{\Delta P_{SO}} \) (where \( \Delta P_{SO} \) = the pressure drop for oil flowing at the superficial oil velocity).

Since we are mainly interested in core flow for highly viscous crudes (500 mPa.s and higher), we can disregard a turbulent core and distinguish between two types of model: a laminar core surrounded by either a laminar or a turbulent water film. For either of these flow conditions, there are again two possibilities: concentric and eccentric core flow. As mentioned above, concentric core flow was experimentally obtained in a horizontal pipe by Charles et al. (1961) by carefully matching the densities of the oil and water. In practice, however, there will most likely be a density difference between the phases, leading to an eccentric core position in the pipe (see Figure 1 and Glass (1961)).

Concentric Laminar Oil Core-Laminar Water Film

A model for concentric laminar oil core-laminar water film flow was developed by Russell and Charles (1959). Work on this subject was also reported by Charles (1963) and by Epstein (1963). The liquids were assumed to have equal densities. Equations were derived for the flow rates of the two immiscible, incompressible Newtonian liquids in laminar flow. These flow rates were expressed in terms of pressure loss, fluid viscosity, pipe radius, and radius of the oil core. In this model the position of the interface (for concentric flow) is then determined by the value at which the flow rate of the viscous phase is at its maximum:

\[
k = \frac{R_C}{R} = \left[ \frac{\nu_O}{2\nu_O - \nu_W} \right]^{1/2}
\]

where the parameter \( k \) is known as the diameter ratio (or the ratio of core radius to pipe radius), and \( \nu_O \) and \( \nu_W \) are the oil and water viscosities, respectively.

The diameter ratio \( k \) is directly related to the phase hold-up, e.g., the water hold-up can be calculated from

\[
\nu_W = 1 - k^2
\]

From the maximum flow rate for oil in the form of a concentric core and the expression for oil flowing alone in the pipe, Russell and Charles derived the following equation for the pressure gradient ratio:

\[
\frac{\Delta P_{OW}}{\Delta P_{SO}} = \frac{(2\nu_O - \nu_W)\nu_W}{\nu_O^2}
\]

If \( \nu_O \) is very much greater than \( \nu_W \), this equation simplifies to

\[
\frac{\Delta P_{OW}}{\Delta P_{SO}} \approx \frac{2\nu_W}{\nu_O}
\]
In this approximation the diameter ratio $k$ corresponding to the optimum flow rate [Eq. (5)] is equal to 0.71; using Eq. (6), the optimum water hold-up becomes 0.50.

Russell and Charles also determined the position of the interface (i.e., $k$ value, or value for water hold-up) for the minimum power requirement. Assuming, again, the oil viscosity to be very much larger than the water viscosity, they found the optimum power savings to occur at a larger $k$ value: $k = 0.786$, i.e., a smaller water hold-up $H_w = 0.38$ (as against 0.50). The power requirement cannot be reduced to the same extent as the pressure gradient because of the increase in volumetric flow caused by the addition of the water phase. Russell and Charles found for the power required for core flow as a fraction of the power for oil flowing alone: $2.78 \frac{H_w}{H_o}$ [compare Eq. (8) for the pressure drop fraction].

Russell and Charles compared pressure gradient reduction factors predicted by this model with measured data for three concentric core flow systems. The calculated pressure gradient reduction factors [the reciprocal of Eq. (8)] were very much larger than the measured values, e.g., for a 6-in pipe and a crude oil with an estimated average viscosity of 900 mPa.s, the predicted pressure gradient reduction factor is 450, whereas the measured values for water fractions of 0.07 to 0.24 range from 7.8 to 10.5. The optimum pressure reduction in these tests was measured when 8%-10% water was injected with the crude oil. This optimum for the input water fraction (0.08-0.10) has to be related to the optimum water hold-up of 0.50 calculated by the model. Glass (1961) reported a maximum pressure gradient reduction factor to occur at an input water fraction of about 0.35. For a 200-cSt oil in a 1-cm pipe, a pressure gradient reduction factor of 22 was measured at this water fraction. In this case, too, the Russell-Charles concentric model predicts a larger pressure gradient reduction factor, namely, a value of 100.

Eccentric Laminar Oil Core-Laminar Water Film

The eccentric laminar flow core was studied theoretically by Bentwich et al. (1970). Laminar velocity profiles of Bentwich (1964) were integrated to give the volumetric flow rates in a tube of two immiscible liquids with an eccentric circular interface. Sample plots were given of the pressure drop and power reduction factors for core flow as a function of three parameters: the viscosity ratio $\mu_o/\mu_w$, the diameter ratio $k = R_o/R$, and the eccentricity $E' = E/(1 - k)$. Here, $E = e/R$, where $e$ is the distance between the tube centre and the core centre and $R$ is the pipe radius.

Bentwich et al. (1970) expressed the eccentricity in terms of $E'$ rather than the also dimensionless eccentricity $E = e/R$, because the former parameter, unlike the latter, always ranges from zero (concentric flow) to unity (fully eccentric flow), irrespective of the diameter ratio $k$. The diameter ratio also ranges from zero (single-phase water flow: $H_w = 1$) to unity (pure oil flow: $H_w = 0$). Finally, the viscosity ratio may vary from infinite (solid core, e.g., capsule flow) to 1 (single-phase liquid flow). Of these three parameters for an oil-water system, only the viscosity ratio will be a known input parameter (unless emulsification occurs during transport, affecting the liquid viscosities of both core and annulus). The other two are dependent variables, for which suitable calculation methods have to be used.

In summarizing the results of calculations with the eccentric laminar flow model we will, for the sake of clarity, focus on oil core-water annulus systems comprised of a viscous core surrounded by a less viscous annulus, as done by Bentwich et al. (1970), rather than systems of a more general nature. For the oil-water system an increase of the viscosity ratio will mean an increase of the oil viscosity. Wherever appropriate, we will relate values of the diameter ratio to values of the water hold-up [Eq. (6)].

The first question to be answered is how the oil core flow rate depends on the eccentricity/diameter ratio (i.e., water hold-up) with the oil viscosity as a parameter. The core flow rate used here is based
on the assumption that there is no change in pressure gradient from the situation prevailing when the pipe is transporting oil alone \((k = 1\), or water hold-up equals zero). For all viscosities, at a given diameter ratio \((\text{water hold-up})\) the flow rate of the core liquid decreases as the eccentricity becomes larger. On the other hand, for a given eccentricity \((E' = 0.9\) in the sample calculation by Bentwich et al. \((1970)\)), the core flow rate reaches a maximum at a diameter ratio \((\text{water hold-up})\) that represents the optimum interface location. The optimum \(k\) value decreases with increasing oil viscosity but never to below 0.7. In terms of water hold-up, this means that for higher oil viscosities the optimum water hold-up will also be larger, but it will never exceed a value of 0.50. However, according to the plots presented by Bentwich et al. \((1970)\), the effect of oil viscosity is small only for values of \(\nu O\) beyond 100 mPa.s.

A second point of interest is this: Knowing that the oil flow rate for a particular eccentricity will reach a maximum at an optimum water hold-up value \((\text{or } k\) value\), what is the range of optimum water hold-ups \((k\) values\) when the eccentricity varies from zero to unity? For an oil viscosity of 10 mPa.s, we can derive this range of optimum water hold-ups \((k\) values\) from two plots presented by Bentwich et al. \((1970)\) - one for the pressure gradient reduction factor and one for the power reduction factor. In both cases the optimum \(k\) values increase with increasing eccentricity. The ranges of the \(k\) values are different, however. Since the eccentricity varies from zero to unity, the optimum \(k\) values for the pressure gradient reduction factor vary from 0.71 to 0.84, whereas for the maximum power reduction factor the \(k\) value variation ranges from 0.79 to 0.89.

Figure 4, a plot of water hold-up versus diameter ratio, shows the optimum \(k\) value ranges and the corresponding optimum water hold-up ranges. For maximum pressure gradient reduction, optimum water hold-ups range from 0.50 for concentric flow to 0.29 for fully eccentric flow. For the maximum power reduction factor, the water hold-ups are smaller. They range from 0.38 \((E' = 0)\) to 0.21 \((E' = 1)\). Note that the optimum water hold-ups for the concentric case are equal to the values computed with the Russell-Charles model.

Finally, let us consider more quantitatively the maximum pressure gradient reduction factors computed with the model. As shown in Figure 5 for the 10-mPa.s example, as the eccentricity increases from zero to unity the maximum pressure gradient reduction factor decreases from 5.3 (equal to the value computed with the Russell-Charles model) to 3, i.e., nearly half the initial value. The paper by Bentwich et al. \((1970)\) is purely theoretical in nature and, therefore, presents no experimental verification for the computed results. Only at the end, in a discussion by Kruyer, is it demonstrated that for capsule flow, where the viscosity ratio is infinite, the eccentricities \((\text{or capsule clearances})\) for a known \(k\) value can be selected such that calculated pressure reductions coincide with those measured. However, no measurements of the eccentricities \((\text{or clearances})\) are available as a final check on the model. Since for concentric flow the model reduces to that of Russell and Charles, it suffers from the mismatch between the predictions and measurements of

\[
\begin{align*}
\text{Figure 4: Optimum } k \text{ values and corresponding water hold-ups for maximum pressure gradient or power reduction. Derived from Bentwich et al. (1970) for a 10-mPa.s oil.}
\end{align*}
\]
Figure 5: Pressure drop reduction factors for 10-mPa. soil (dashed line is locus of maxima) from Bentwich et al. (1970).

The pressure gradient reduction factors noted before. Even when it is assumed that the measured data relate to fully eccentric rather than concentric flow, model predictions are still very much higher than measured data (by several factors).

Laminar Oil Core-Turbulent Water Film

There are only a few publications on the problem of a laminar oil core surrounded by a turbulent water film. Only the concentric flow situation has been considered. Using a number of simplifying assumptions Charles (1963) made an analysis of the concentric flow of a capsule (i.e., viscosity ratio infinite). He assumed a 1/7th power velocity distribution law for the turbulent water film region with the core velocity equal to the interfacial velocity. He expressed the core velocity, computed by matching the assumed velocity profile, as a function of the diameter ratio k and the mixture velocity Vm (the sum of the superficial core and annulus velocities). The capsule velocity always exceeds the mixture velocity according to this model. In fact, it is possible to rewrite his expressions as a relationship between the water hold-up (k value) and the input water fraction. This relationship ought to be compared with experimental data for the thickness of the annulus in order to check the validity of both the simplifying assumptions and the velocity profiles selected for turbulent flow.

Stein (1978) also considered the flow of a concentric core surrounded by a turbulent annulus. In his case the core was a W/O emulsion, while in the annulus an O/W emulsion flowed. He assumed a logarithmic velocity profile for the annulus and derived from it an expression for the friction factor as a function of the Reynolds number and two empirical parameters. These parameters depend on the oil volume fraction, the inversion concentration, and the viscosity ratio. Values for the parameters were determined from measurements in 1-, 1.5-, and 2-in pipes with 155- and 680-mPa. soil. No hold-up data were measured. Stein concluded that experiments with larger pipe sizes were needed to extend the range of the Reynolds numbers.

Empirical Correlation Methods for Pressure Loss

A completely empirical correlation for the pressure loss of concentric annular flow was presented by Glass (1961). He based his correlation on measurements in a horizontal, 1.2-m long, 1-cm-ID glass tube. The oil viscosity was varied from 10 to 30,000 cSt and the specific gravity of the oil from 0.97 to 1.03. He considered input water fractions from 0.2- to 0.90 and superficial oil velocities from 0.06 to 1.28 m/s. Glass noticed that as the input water fraction increased, the core pressure gradient dropped at first, then it passed through a minimum at a water fraction of 0.30-0.40, and, finally, it rose again. The pressure gradient at a given water fraction for core-annular flow was found to go up with the 1.8 power of the flow rate. This can be ascribed to losses
due to turbulence in the water annulus. His correlation for the pressure loss ratio was

\[
\frac{\Delta P_{\text{OW}}}{\Delta P_{\text{SO}}} = \frac{\text{Re}_{\text{SO}}}{700}
\]

where

\[
\text{Re}_{\text{SO}} = \frac{\Delta P_{\text{VSO}}}{\nu_o}
\]

the superficial-oil Reynolds number.

The smaller the Reynolds number for oil flowing alone in the tube, the greater the reduction in pressure gradient to be achieved with a water annulus. For an oil flowing alone with a Reynolds number greater than 700, this equation predicts that the addition of water will increase the pressure gradient. A point worth mentioning is that the larger the oil density, the larger the pressure gradient for core flow according to Eq. (9). This contradicts the theoretical findings from the eccentric laminar-laminar model of Bentwich et al. (1970): when the oil density is larger, the core eccentricity will be smaller, giving a larger pressure gradient reduction (see Figure 5), i.e., a smaller pressure gradient for core flow.

In 1969 another empirical correlation for computing core flow pressure losses was published by Sinclair (1969). This correlation was based on measurements in the laboratory on 15-m-long horizontal pipe circuits, 3/4- and 1-in in diameter and on ~1600-m-long, 2 1/2-in tubing in a vertical production well. He considered a 10% water annulus surrounding a W/O dispersion with an oil viscosity of 1000 mPa.s. This correlation is presented in the form of a (Fanning) friction factor for core flow

\[
\epsilon_{\text{OW}} = \frac{2.0}{(\text{Re}_w)^{0.5}}
\]

where

\[
\text{Re}_w = \frac{\Delta P_{\text{Vw}}}{\nu_w}
\]

the Reynolds number of the water.

Although not clearly stated in the paper, we presume that the velocity of the mixture (i.e., oil + water) was used in the Reynolds number of the water. The correlation implies no significant dependence on oil viscosity for the high-viscosity oils considered (a range of 500-5000 mPa.s is mentioned). Sinclair derived this simple correlation from a laminar core-turbulent film model. The merits of this modelistic approach should be evaluated using experimental data for a wider range of pipe sizes.

2.3 Pressure Loss during Restart of Core Flow from a Stratified Layer

2.3.1 Introduction

If during shutdown of a pipeline operated in core flow a stratified oil layer has formed, it is important to know how to restart the system. Supposing a proper procedure has been found to restart the core flow from an initially completely or partially stratified situation, what pressure gradients would have to be applied during such a procedure? To answer this question, we have to consider pressure gradient calculation methods for a stratified flow of oil and water (or, more generally, a viscous and less viscous phase, which also have different densities). In general, stratification due to a density difference is counteracted by the tendency of one of the liquids to wet the pipe’s inner surface more than the other. When the wetting effect is negligible, the flow pattern will be completely stratified with a flat, horizontal oil-water interface. Attention is focused on the more promising case where the less viscous liquid spreads thinly. In that case partially stratified flow will arise with a naturally curved interface.
2.3.2 Completely Stratified Flow

Methods are available for estimating pressure drop reductions and power savings for completely stratified flow (Charles and Redberger, 1962; Yu and Sparrow, 1967; Ranger and Davis, 1979). Yu and Sparrow (1967) reported that for a flat interface and a viscosity ratio of 1000 (i.e., \( \nu_0 = 1000 \text{ mPa.s} \), in our case), the maximum pressure gradient reduction factor is 1.37. The interface location (i.e., water hold-up) corresponding to this maximum is in the lower part of the tube cross section and is insensitive to the viscosity ratio. Charles and Redberger (1962) reported a maximum pressure gradient reduction factor that is 5% lower. The optimum interface position is the same as that reported by Yu and Sparrow: \( h/R = 0.4 \), where \( h \) is the distance of the interface from the bottom of the pipe. This corresponds to a water hold-up of about 0.35 (see Govier and Aziz, 1972, p. 564). They noted that for oil viscosities higher than 100 mPa.s the reduction factor is approximately constant. The method can be used only on a system of practical interest if the interface location \( h/R \) has been determined. This means that to optimize this kind of two-phase oil-water flow the water hold-up has to be known as a function of the input water fraction.

2.3.3 Partially Stratified Flow

A theoretical model for partially stratified flow, i.e., with a naturally curved interface, was developed by Bentwich (1976). This is a technically more advantageous situation, in which the interface is allowed to have a cross section of constant curvature by including surface tension effects. In this way the less viscous fluid can "lubricate" a larger portion of the pipe wall, i.e., the flow pattern can be considered intermediate between completely stratified and core-annular (see Figure 6). The pressure gradient reduction factors for partially stratified flow are then expected to be smaller than those for core flow but not as small as those for the completely stratified flow pattern. Indeed, Bentwich calculated with his model a pressure gradient reduction factor of 2, for a viscosity ratio of 20 (oil viscosity 20 mPa.s), compared with the maximum value of 1.37 for completely stratified flow.

Bentwich assumed that the flow in the pipe is laminar and that the fluid moves only in the axial direction. The two liquids are immiscible and separated by a continuous, well-defined interface. The geometry is determined in terms of the following parameters (see Figure 6):

\[ \gamma = \frac{\pi}{6} \]

- the contact angle: \( \gamma \equiv \cos \gamma = (\sigma_1 - \sigma_2)/T \);
- the stratification parameter: \( S \equiv (\rho_2 - \rho_1)gR^2/T \); and
- the point where the interface intersects the pipe's surface, defined by the angle \( \beta \) (Figure 6).

Here \( \rho_{1,2} \) is the density of liquids 1 and 2, \( \sigma_{1,2} \) is the energy per unit area of the pipe's inner surface wetted by liquid 1 or 2, \( T \) is the surface tension, \( R \) is the pipe radius, and \( g \) is the gravitational acceleration.
The angle $\beta$ is used as a variable in the model and is related to the hold-up of fluid 2 (water in our case). All the other parameters may be assumed to be known a priori, since they are properties of the fluids and the pipe's inner surface.

It was pointed out by Bentwich (1976) that the whole topic of the dynamics of wetting is still the subject of intensive research, so exact values of the contact angles under flow conditions will have to be determined experimentally. See also the discussion by Dusson V. (1983) on static and dynamic contact lines. As shown in Figure 6 for a given contact angle ($\gamma = \pi/6$), the stratification parameter $S$ affects the geometry of the interface. Clearly, there is an advantage in reducing $S$ since a lower $S$-value means that with the same pipe surface area wetted by water a larger part of the cross section will be available for the flow of the more viscous fluid (oil). A reduction of $S$ can be achieved by choosing liquid 2 such that $\mu_2 - \mu_1$ is small. As is apparent from the definition of $S$, an increase in pipe size will lead to larger $S$ values ($S \propto R^2$) and consequently less favorable interface geometries.

Results of calculations with the partially stratified flow model are presented as plots of the dimensionless oil flow rate (equal to the pressure gradient reduction factor) as a function of $\beta$, with $S$ values of 1 and 10 and viscosity ratios of 6, 10, and 20 as parameters, assuming a given contact angle $\gamma = \pi/6$ (see Figure 7). Power reduction factors are also given by Bentwich for these parameter values but now as a function of the ratio between volumetric flow rates. The pressure gradient reduction factor is defined as the ratio between the pressure gradient required to transport a given volume of oil without water addition to that with water present; this ratio reaches a maximum at a certain value of $\beta$. The optimum $\beta$ values decrease as the viscosities become larger. This means that for higher viscosities larger optimum water levels or water hold-ups are calculated (see Figures 6 and 7).

For an oil viscosity of 20 mPa.s, the optimum is $\beta = \pi/3$.

Bentwich remarked that the maximum pressure gradient reduction that can be obtained for a particular selection of variables ($\gamma = \pi/6$, $\mu_1/\mu_2 = 6$) is relatively insensitive to changes in $S$: an increase in $S$ from 1 to 10 gives rise to a relatively moderate decrease in the maximum pressure reduction from 1.53 to 1.3. However, as is clear from Figure 7, the optimum $\beta$ values are quite different: increases from 60° for $S = 1$ to 135° for $S = 10$ are found. In other words, the optimum water hold-up decreases with $S$. Finally, since the geometry is influenced by the wetting properties, with the same values of $\mu_1/\mu_2$ and $S$, a larger pressure reduction is calculated for a lower value of the contact angle $\gamma$.

![Figure 7: Pressure gradient reduction for partially stratified flow as a function of the intersection angle for various values of $\mu_1/\mu_2$; $\gamma = \pi/6$. From Bentwich (1976).](image)

2.3.4 Comparison with Experimental Data

Restart tests for core flow have been carried out at the Koninklijke/Shell-Laboratorium in Amsterdam with oil viscosities of 2200 - 4600 mPa.s in an 888-m-long, 8-in-diameter pipe. During restart the average value of the required pressure gradient was about one-third to
one-fourth the pressure loss for pure oil. The use of an additive in the water phase was demonstrated to be essential for a successful restart (Netherlands patent, 1977). In Figure 8 we have plotted the optimum pressure gradient ratios for completely and partially stratified flow models. The experimental data also shown in the figure seem to confirm the predictions for the latter flow pattern.

Although pressure losses for the partially stratified situation are substantially smaller than for the completely stratified case, during restart the pressure gradients can become quite high, especially when high-viscosity oils are used. An important aspect for further investigation is assessing whether pressure drop ratios continue decreasing with oil viscosity or level off, as is the case for the completely stratified flow (see Figure 8). However, even if it continues to decrease, the rate of decrease with \( \mu \) will be slow. Of course, this flow mode will prevail only during the restart period when the flow rates are relatively small, and the line will very soon operate partly in the core flow mode for which the pressure losses are very much smaller.

Finally, we should mention that the calculated pressure gradient ratios shown in Figure 8 are the optimum values, i.e., at optimum water hold-ups (or intersection angles). For stable core flow, too, optimum hold-up values exist, which need not be equal to those for stratified flow with a curved interface. It is important to be able to compute from input water fractions the hold-ups for these flow patterns, so that an acceptable procedure for pipeline operation can be determined. The operator may then decide, for instance, to operate stable core flow at a slightly unfavourable water hold-up (requiring more power) in order to always have an optimum hold-up value for restart from stratified flow after an emergency shutdown. Care should be taken not to use too much water. Charles and Lilleleht (1966) showed that when the water phase is in turbulent flow for the completely stratified situation, the pressure gradients will be larger than those for oil alone! This restriction may set an upper limit to the pipe size for which restart can be done at a reasonable pressure gradient.

Figure 8: Optimum pressure loss ratios for completely and partially stratified flow.
Chapter 3

THE LUBRICATING-FILM MODEL

It is clear from the models for stable core flow discussed above that there is a need for a model that correctly predicts the eccentricity of the oil core in the pipe. Moreover, the pressure gradient reductions calculated with the models discussed exaggerate the benefits of the core flow operation mode. It is also important to develop better insight into the possibility of using core flow in larger pipes.

In this section we will introduce a new core flow model - the lubricating-film model - that gives a more realistic prediction of pressure reduction by properly taking into account the effect of eccentricity. This model is a further development of one suggested by Ooms and Beckers (1972). It has been demonstrated with this model by Ooms (1972) that a pure lubrication force can counteract the buoyancy for a core geometry, which is compatible with observation. Here, we will investigate the contribution of pure lubrication forces to the pressure gradient for core flow. We will start with the case of laminar lubrication.

3.1 Laminar Lubrication

3.1.1 Model Description

The oil core in the lubricating-film model is assumed to be solid, and hence the oil-water interface is assumed to be a solid-liquid interface. This assumption simplifies the flow problem considerably. Solving the real flow problem, in which both core and annulus have a finite viscosity, is very difficult; see Ooms (1971).

A frame of reference is chosen, according to which the core is at rest and the pipe wall has a velocity \( W \) in the \( x \) direction (see Figure 9); \( r, \theta, \) and \( x \) are cylindrical coordinates; \( h(\theta,x) \) represents the thickness of the water annulus; and \( R \) is the radius of the pipe. In accordance with the observations, the solid core is assumed to be rippled. In the \( x \) direction (\( \theta = \text{const} \)), \( h \) is assumed to be periodic and of sawtooth shape; the wavelength \( \lambda \) is of the same order of magnitude as \( R \). The ripple shape is assumed to be independent of \( \theta \). This is in conflict with the observations shown in Figure 1, where the ripple shapes at the top and bottom of the pipe are not similar. However, for the contribution of lubrication forces, the important part of the water film is at the top, where the downthrust is generated. The contribution of lubrication forces at the bottom is negligible as shown below. In the \( \theta \) direction (\( x = \text{const} \)), \( h \) is assumed to be symmetric with respect to the line through \( \theta = 0 \) and \( \theta = \pi \) (see Figure 9). For \( 0 \leq \theta \leq \pi/2 \), \( h \) is independent of \( \theta \) (see Figure 9); for \( \pi/2 \leq \theta \leq \pi \), \( h \) increases with \( \theta \) according to an elliptically shaped oil core. This can be written as

\[
\begin{align*}
    h & = h_c - e = h_0 + \frac{a}{2} - a \frac{x}{R} - e \quad \text{for } 0 \leq \theta \leq \pi/2 \text{ and } 0 \leq x \leq L' \\
    h & = h_c - e = h_f - \frac{a}{2} + a \frac{L' - x}{R - L'} - e \quad \text{for } \pi/2 \leq \theta \leq \pi \text{ and } L' \leq x \leq L
\end{align*}
\]

Figure 9: Core-annular flow with a sawtooth-shaped solid core
The equations give minimum and maximum thicknesses at the pipe
top at \( x = t' \) and \( x = 0 \) as \( h_0 - \frac{a}{2} - e \) and \( h_0 + \frac{a}{2} - e \), respectively.

Due to the assumed elliptic shape the lower part of the oil core is
displaced over a distance larger than \( e \) (for \( e = 0 \) the oil core has
a circularly shaped cross-section, the size of which varies with \( x \)).

The parameter \( a \) gives the amplitude of the ripple in the core
surface; \( z' \) is the distance from \( x = 0 \) to the minimum of \( h \) (see
Figure 9).

For a stable situation to arise, a balance is required between
the buoyancy force on the core and the vertical components of the
pressure and viscous forces on the core generated by the water flow
in the annulus. In the calculations the hydrodynamic lubrication
theory will be applied to the water flow in the annulus. This means
that the following conditions are assumed to hold:

\[
\frac{a}{R} < 1
\]

\[
\frac{a}{\ell} < 1
\]

\[
\frac{gwh \ a}{\mu \ z'} < 1
\]

in which \( \rho \) represents the density of the water in the annulus and
\( \mu \) the viscosity of water.

From the observations shown in Figure 1, it can be concluded
that the conditions of Eqs. (15) and (16) are satisfied. To calculate
the condition of Eq. (17) the values of \( h \), \( a \), and \( \ell \) must be known.
From the observations it can be concluded that because of the buoyancy
effect the water film is extremely thin in the upper part of the pipe.
It is, however, very difficult to perform accurate measurements of \( h \)
and \( a \) in the upper part. The important part of the water film is at
the top, where, as mentioned earlier, the downthrust due to lubrication
forces is generated. In the following calculation it is assumed, there­
fore, that the condition in Eq. (17) is also satisfied, so we will
concentrate on the contribution from lubrication forces alone. Of course,
it has to be checked a posteriori that the results of calculations made
with the model do indeed satisfy the condition in Eq. (17). Results
of experiments that attempt to measure \( h \) and \( a \) are discussed below.

The purpose of the calculation is to investigate the possibility
of a steady core-annular flow. Somehow, forces must therefore be built­
up in the annulus, such that in steady flow they neutralize the gravity
force on the core caused by the difference in density between the core
and the annulus. This is impossible if the flow of the water in the
annulus is parallel to the wall of the tube. To counterbalance the
gravity force, there must evidently be secondary flows perpendicular
to the pipe axis. In the following we shall calculate these secondary
flows and we shall require a balance between the pressure-, viscous­
and gravity forces over a total wavelength of a possible ripple on
the interface.

We start from the continuity equation

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( ur \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left( \theta w \right) + \frac{\partial w}{\partial z} = 0
\]

and from the time-independent equations of motion
in which \( u, v \) and \( w \) are of the order of magnitude of \( u, v \) and \( w \) respectively.

Because of (23) it follows from (24), that

\[
U << V \quad \text{and} \quad U << W. \tag{25}
\]

The order of magnitude of the intertial- and viscous terms in the equations of motion (19), (20) and (21) are

\[
\frac{\partial (u \frac{3u}{r} + v \frac{3v}{r} + w \frac{3w}{r})}{\partial r} = \frac{1}{r^2} \frac{\partial^2 (u \frac{3v}{r} + v \frac{3u}{r} + w \frac{3w}{r})}{\partial r^2} - \frac{2}{r^2} \frac{\partial^2 (u \frac{3w}{r} + w \frac{3u}{r} + v \frac{3v}{r})}{\partial r^2} - \frac{2}{r^2} \frac{\partial^2 (u \frac{3u}{r} + v \frac{3v}{r} + w \frac{3w}{r})}{\partial r^2} \tag{26}
\]

\[
\frac{1}{r} \frac{\partial^2 (u \frac{3u}{r} + v \frac{3v}{r} + w \frac{3w}{r})}{\partial r^2} = \frac{1}{r^2} \frac{\partial^2 (u \frac{3v}{r} + v \frac{3u}{r} + w \frac{3w}{r})}{\partial r^2} - \frac{2}{r^2} \frac{\partial^2 (u \frac{3w}{r} + w \frac{3u}{r} + v \frac{3v}{r})}{\partial r^2} - \frac{2}{r^2} \frac{\partial^2 (u \frac{3u}{r} + v \frac{3v}{r} + w \frac{3w}{r})}{\partial r^2} \tag{27}
\]
Using (23) and (25) we find that the order of magnitude of the largest inertial terms is $O(\nu W/h^2)$; the order of magnitude of the largest viscous terms is $O(\nu W/h^2)$. So the ratio of the largest inertial terms and the largest viscous terms is $O(\nu W/h^2)$, which is the Reynolds number of the water flow in the annulus multiplied by $\frac{h}{R}$. Because of $h << 1$ we assume in our further calculations, that this ratio is much smaller than unity. So the inertial terms are neglected with respect to the viscous terms. Restricting ourselves to the largest viscous terms we can then simplify the equations of motion considerably.

\[
\frac{3\phi}{3r} = 0
\]  
\[\frac{3\phi}{r} + \frac{2v}{3r} = \frac{2v}{3r} + \frac{2v}{3x} \]  
\[\frac{3\phi}{3x} = \frac{2w}{3x} + \frac{2w}{3x} \]

The correctness of such simplifications is discussed extensively by Tipei (1962).

We replace $r$ by the following new independent variable $y$

\[y = R - r.\]

So $y$ is the distance from the pipe wall. Because of (32) we can write

\[3r = -3y.\]

Because the thickness of the annulus $h$ is assumed to be very small compared with the radius $R$ of the pipe, the following approximation holds in the annulus

\[r = R.\]

Using (33) and (34) we can write (24), (29), (30) and (31) in the following form

\[-\frac{2}{3y} (\frac{\partial u}{\partial y}) + \frac{2v}{\partial x} + R \frac{3w}{3x} = 0\]

\[\frac{3\phi}{3y} = 0\]

\[\frac{1}{\nu} \frac{\partial^2 \phi}{\partial y^2} + \frac{2v}{3y} = \frac{2v}{3y}\]

\[\frac{1}{\nu} \frac{\partial^2 \phi}{\partial x^2} + \frac{2w}{3y} = \frac{2w}{3y}\]

From (36) follows, that $\phi$ is independent of $y$. Equations (37) and (39) can then be integrated and yield

\[v = \frac{1}{2\nu R} \phi y (y-h)\]

\[w = \frac{1}{2\nu} \frac{\phi}{3x} y (y-h) + W (1 - \frac{y}{h})\]

in which the following boundary conditions for $u$, $v$, and $w$ have been used:

for $y = 0 : u = 0, v = 0$ and $w = W$

for $y = h : u = 0, v = 0$ and $w = 0$.

We have now derived expressions for the velocity components $v$ and $w$ of the water in the annulus. However, we can only calculate $v$ and $w$, if $\phi(\theta, x)$ is known. Let us, therefore, derive an equation for $\phi$. 
To that purpose we integrate (35) between the values \( y = 0 \) and \( y = h \). Then we find

\[
\int_0^h \frac{\partial \phi}{\partial y} \, (uR) \, dy = uR \int_0^h \frac{\partial \phi}{\partial y} \, dy + \int_0^h R \, \frac{\partial u}{\partial y} \, dy.
\]  

(36)

From (39) and (40) we find

\[
\frac{\partial \phi}{\partial y} = \frac{1}{2\mu R} \frac{\partial}{\partial y} \left( \frac{y(y-h)}{y^2} \right) + \frac{wR}{h^2} \frac{\partial h}{\partial x}.
\]  

(41)

So

\[
\int_0^h \frac{\partial \phi}{\partial y} \, dy = \left[ \frac{1}{2\mu R} \frac{\partial}{\partial y} \left( \frac{y^2}{2} - \frac{y^3}{2} \right) + \frac{wR}{h^2} \frac{\partial h}{\partial x} \right]_{y=0}^{y=h} = \frac{h^2}{12\mu R} \frac{\partial h}{\partial x} + \frac{wR}{h^2} \frac{\partial h}{\partial x}.
\]  

(47)

Substitution of (46) and (47) in (43) yields the equation for \( \phi \)

\[
\frac{\partial \phi}{\partial y} = \frac{1}{2\mu R} \frac{\partial}{\partial y} \left( \frac{y^2}{2} - \frac{y^3}{2} \right) - \frac{1}{2\mu R} \frac{\partial}{\partial y} \left( \frac{y^2}{2} - \frac{y^3}{2} \right) + \frac{wR}{h^2} \frac{\partial h}{\partial x}.
\]  

(48)

which is, of course, the Reynolds equation of the hydrodynamic lubrication theory.

If the thickness \( h \) of the annulus, the velocity \( W \), the viscosity \( \mu \), the radius \( R \) and the boundary conditions for \( \phi \) are known, the function \( \phi \) can be determined by solving equation (48). In our calculations we assumed \( h \) to have a sawtooth shape as shown in Figure 9. So \( h \) is a periodical function of \( \theta \) and \( x \); the wavelength in the \( \theta \)-direction is \( 2\pi \) and in the \( x \)-direction \( l \). Moreover, \( h \) is assumed to be symmetrical with respect to the line through \( \theta = 0 \) and \( \theta = \pi \). The core is supposed to be eccentric with respect to the pipe axis; so \( h(\theta = 0; x) < h(\theta = \pi; x) \).

The boundary conditions for \( \phi \) in the \( \theta \)-direction are given by

\[
\text{for } \theta = 0 : \phi = \phi_1 \text{ (is constant)}
\]  

(49)

\[
\text{for } \theta = \pi : \phi = \phi_2 \text{ (is constant)}
\]  

(50)

with respect to the boundary conditions in the \( x \)-direction it is assumed that

\[
\text{for } x = 0 : \phi = \phi_3 \text{ (is constant)}
\]  

(49)

\[
\text{for } x = \ell : \phi = \phi_4 \text{ (is constant)}
\]  

(50)

in which \( (\phi_2 - \phi_1) \ell \) is the pressure gradient over the pipe.

For known values of \( \phi \), \( v \) and \( w \) the forces exerted by the water on the core in horizontal and vertical directions can be calculated. From these expressions can be obtained for the force balance in vertical direction (enabling the determination of the oil core position) and in horizontal direction (giving the pressure gradient).

In steady flow the static pressure and viscous forces over the pipe wall in the vertical direction equilibrate the weight of liquid. Hence

\[
\rho \frac{\partial y}{\partial x} = \int_{x}^{x+l} \int_{0}^{\pi} (p) \, y = 0 \cos \theta \, d\theta \, dx + 2\pi \int_{0}^{\pi} \int_{0}^{\pi} (p) \, y = 0 \sin \theta \, d\theta \, dx + \rho v \frac{\partial y}{\partial x} = 0,
\]  

(53)

in which \( \rho_C \) represents the density of the core; \( V_C \) the volume of the core between \( x \) and \( x+l \), and \( \rho_w \) the volume of the water annulus between \( x \) and \( x+l \). Substitution of (22) and (39) in (53) yields
which is a representation of Archimedes' principle. At lower water percentages, $V_C = \pi R^2 h$ and (54) becomes

$$2R \int_0^L \phi \cos \theta \, d\theta - \int_0^L h \frac{\partial}{\partial \theta} \sin \theta \, d\theta = (\rho - \rho_c) g V_C.$$  \hspace{1cm} (55)

In the horizontal direction the axial pressure drop force just equilibrates the viscous force along the pipe wall. Hence over a wavelength

$$\left( p_1 - p_2 \right) \pi R^2 \int_0^L \frac{\partial \theta}{\partial \gamma} \, d\gamma = 0,$$  \hspace{1cm} (56)

in which $(p_1 - p_2)$ represents the axial pressure drop over one wavelength. Substitution of (22) and (40) in (56) yields

$$\pi R^2 (\phi_2 - \phi_1) = R \int_0^L \int_0^L \frac{2 \phi}{3 \gamma} \, d\theta \, d\gamma + 2 \mu \pi L \int_0^L \frac{1}{H} \, d\theta.$$  \hspace{1cm} (57)

In Appendix I the solution procedure is given for the two basic equations (55) and (57) in dimensionless form. To this end a dimensionless density $\tilde{\rho}$, and a dimensionless pressure are defined as:

$$\tilde{\rho} = \frac{\rho R^2}{6 \mu W},$$  \hspace{1cm} (58)

$$\phi = \frac{\theta R}{6 \mu W}.$$  \hspace{1cm} (59)

Upon introduction of the dimensionless variables $X = x/R$, $H = h/R$, and $L = L/R$, the two basic equations that have to be solved read

$$\tilde{\rho} - \tilde{\rho}_c = \frac{2}{\pi L} \int_0^L \phi \cos \theta \, d\theta - \frac{1}{\pi L} \int_0^L H \frac{2 \phi}{3 \gamma} \sin \theta \, d\theta$$  \hspace{1cm} (60)

and

$$\frac{\phi_2 - \phi_1}{L} = \frac{1}{\pi L} \int_0^L H \frac{2 \phi}{3 \gamma} \, d\theta + \frac{1}{3 \pi L} \int_0^L \frac{1}{H} \, d\theta.$$  \hspace{1cm} (61)

3.1.2 Qualitative Predictions

In this section the dependence of model predictions on wave characteristics (wavelength, amplitude, asymmetry) and in situ water fraction (taken equal to $H_0$) will be considered. We will also verify that the contribution from lubrication forces to the pressure gradient in the lower part of the pipe is small.

3.1.2.1 Sensitivity Study

For studying the sensitivity aspects of model predictions, a particular core flow problem was simulated. For $H_0$, $L$, $A$, and $L'$, the following values were chosen: $H_0 = 0.06$, $L = 1$, $L' = 0.9$, and $A = 0.006$. The dimensionless eccentricity $E = e/R$ was increased step by step from zero to the value at which the core almost touched the pipe wall. Although the real core eccentricity is equal to $2e$, in these calculations we use $E = e/R$ as the dimensionless eccentricity related to the top of the pipe.

For the values of the parameters given above, Eq. (48) was solved with the boundary conditions (49)–(52), and the integrals of Eqs. (60) and (61) were calculated. Equation (60) yields the dimensionless density difference $\tilde{\rho} - \tilde{\rho}_c$ between water and oil that can be counterbalanced by the lubrication forces. Equation (61) gives the dimensionless pressure gradient over the pipe. The results are shown in Figures 10a and b.

The dots in these figures are the calculated data, and the point where the core touches the pipe wall is indicated. In Figure 10a the dimensionless density difference required for stable core-annular flow in the case of a 2-in pipe is also given (assuming $\Delta \rho = 100 \text{ kg/m}^3$ and $W = 0.2 \text{ m/s}$ for this example). The value of $E$ at which the dimensionless density difference that can be counterbalanced is equal to the required value gives the dimensionless eccentricity at which stable core-annular flow for the 2-in pipe experiment is possible. From Figure 10b the dimensionless pressure gradient for stable core-annular flow can then be found. As is evident from
Figure 10a, stable core-annular flow is possible only with a very eccentric core. Only when the annular film in the upper part of the pipe has become very thin are the lubrication forces large enough to counterbalance the buoyancy force on the core. This is qualitatively similar to the flow configuration shown in Figure 1.

The dimensionless density difference required for stable core-annular flow in an 8-in pipe is given in Figure 10a. As in the 2-in-pipe example, it is assumed that $\rho - \rho_c = 100 \text{ kg/m}^3$. It can be concluded from Figure 10a that, for stable core-annular flow in an 8-in pipe, a larger dimensionless eccentricity is required than for a 2-in pipe; it follows from Figure 10b then that the dimensionless gradient is also larger. So the dimensionless pressure gradient increases with the pipe diameter.

Figure 11: Distribution over a wavelength of the ripple for four values of $\theta$. 
In Figure 11 the distribution of $\phi$ over a wavelength for four values of $\theta$ is given for $H_0 = 0.06, L = 1, L' = 0.9, A = 0.006,$ and $E = 0.053$. As can be seen, the variations in $\phi$ (and thus in the pressure $p$) are very large only in the upper part of the pipe, where the water film is very thin. The variations are negligibly small in the lower part.

The azimuthal film flow can be found by substitution of the results for $\phi$ from Figure 11 in Eq. (61). On one side of the ripple the pressure is higher at the top than at the bottom. The pressure gradient drives the water out of the thin upper film around the core to the thick lower film. However, at the other side of the ripple the pressure at the top is lower than at the bottom. By this reverse pressure gradient the water is driven back to the upper film. So, superposed on the flow in the axial direction of the pipe is an oscillatory azimuthal film flow; the azimuthal flow is changing direction from one side of the ripple to the other.

A sensitivity study was carried out to investigate the influence of the ripple parameters on the hydrodynamic lubrication of the core by the water film. The influence of the dimensionless wavelength $L$, the dimensionless amplitude $A$, the dimensionless location $L'$ of the maximum of the ripple amplitude, and the water fraction ($\approx H_0 = h_0/R$) were studied. As a reference case, the following values were selected: $H_0 = 0.02, L = 1, L' = 0.9, A = 0.003$. The choice of this particular reference case was arbitrary, although not in contradiction with the experiments. One of the parameters was varied, while the others were kept constant.

Figures 12a and b present the results for $L$. Figure 12a shows the dimensionless density difference $\tilde{\rho} - \tilde{\rho}_C$ that can be counter-balanced by the lubrication force as a function of the dimensionless eccentricity $E$ for three values of $L$. Here, too, the values of the dimensionless density differences that are required for stable core-annular flow with $\tilde{\rho} - \tilde{\rho}_C = 100 \text{ kg/m}^3$, $h_0 = 0.001 \text{ kg/m s}$, $W = 0.2 \text{ m/s}$, and $R = 0.025 \text{ m or } R = 0.1 \text{ m}$, are indicated in the figure, as is the required density difference $R = 0.1 \text{ kg/m}$ for $R = 0.025 \text{ m}$ or $R = 0.1 \text{ m}$.

Figure 12: (a) Dimensionless density difference and (b) dimensionless pressure gradient as a function of the dimensionless eccentricity for three values of $L$. 
eccentricity at which the core touches the pipe wall. Figure 12b shows the dimensionless pressure gradient \((\Phi_2 - \Phi_1)/L\) over the pipe as a function of \(E\) for the three values of \(L\). It can be concluded that when \(L\) decreases, the eccentricity at which the dimensionless density difference is equal to the required value increases, as does the dimensionless pressure gradient. However, the effect is not very pronounced in the region of parameter values where the calculations were performed.

Figures 13a and b present the results for \(A\). When \(A\) decreases, the eccentricity at which the dimensionless density difference is equal to the required value increases, as does the dimensionless pressure gradient. When the ripple disappears completely \((A = 0)\), the eccentricity increases until the core touches the upper part of the pipe wall. In that case the dimensionless pressure gradient becomes very large.

Figures 14a and b present the results for \(L'\). When \(L'\) decreases, the eccentricity at which the dimensionless density difference is equal to the required value increases, and so does the dimensionless pressure gradient. When the ripple is symmetrical \((L' = 0.5)\), the eccentricity increases until the core touches the pipe wall. In such a case, too, the dimensionless pressure gradient becomes very large.

Figures 15a and b give the results for \(H_o\). When \(H_o\) decreases, the eccentricity at which the dimensionless density difference is equal to the required value decreases, and the dimensionless pressure gradient increases. The thickness of the water film in the upper part of the pipe increases only slightly. When more water is supplied, the greater portion of it goes to the lower part of the pipe. This is qualitatively similar to the visual observations (Figure 1).

Figure 13: (a) Dimensionless density difference and (b) dimensionless pressure gradient as a function of the dimensionless eccentricity for three values of \(A\).
Figure 14: (a) Dimensionless density difference and (b) dimensionless pressure gradient as a function of the dimensionless eccentricity for three values of $L'$. 

Figure 15: (a) Dimensionless density difference and (b) dimensionless pressure gradient as a function of the dimensionless eccentricity for three values of $H_0$. 

- $\tilde{L}' = 0.9$  
- $\tilde{L}' = 0.7$  
- $\tilde{L}' = 0.5$
3.1.2.2 Conclusions

It can be concluded from the results for the lubricating-film model that, for certain water fractions and ripple parameters, an eccentricity can be calculated at which the dimensionless density difference that can be counterbalanced by the lubrication force alone, can be made equal to the required value \((\rho - \rho_C)gR^2/6\mu W\).

If the velocity \(W\) becomes zero, the calculated eccentricity will be that at which the core will touch the pipe wall, except when the density difference \(\rho - \rho_C\) is also zero. So for a nonzero density difference to be counterbalanced, the core must move with respect to the pipe.

The movement of the rippled core with respect to the pipe wall induces pressure variations in the water film, which can exert a force on the core in the vertical direction. This force can be so large that it counterbalances the buoyancy force on the core, allowing stable core-annular flow to arise. The ripples in the core are essential: when the amplitude of the ripple becomes zero, there will no longer be a force on the core to counteract the buoyancy force. In such a case the core will rise in the pipe until it touches the pipe wall. The magnitude of the force is strongly dependent on the shape of the ripple; when the ripple is symmetrical, again a counteracting force will not be present. Adding more water to the flow does not change the flow pattern very much. The additional water goes largely to the lower part of the pipe.

Pressure gradients calculated for stable core flow increase as the ripple becomes smaller (i.e., decreasing amplitude or wave-length) or its shape becomes less asymmetric. Larger pressure gradients are also computed when the water fraction becomes smaller (at least for water fractions of 0.03 and below, as considered in the sensitivity study).

3.1.3 Experimental

3.1.3.1 Experiments in 2- and 8-in pipes

To investigate whether realistic pressure gradient predictions can be made with the lubricating-film model for stable core flow in larger pipes, a base set of reliable experimental data is required. To this end, experiments were carried out with high-viscosity oil (500 cP and larger) in a number of pipes with sizes ranging from 1 to 8 in. By way of illustration we will use here some of the core flow results for 2- and 8-in pipes.

The first set of experiments were performed in a 9-m-long Perspex pipe of 2-in diameter. The difference in density between water and oil was about 30 kg/m³. The amount of water was varied between 3% and 20%. The oil viscosities varied from 2300 to 3300 mPa.s. Oil and water were introduced into the pipe via an inlet device that consisted of a central tube surrounded by an annular slit. The oil was supplied via the tube, the water via the slit.

Although the oil core was pumped into the pipe concentrically, it positioned itself eccentrically, due to the density difference. As long as the oil core was supplied at a velocity above a certain critical value (about 0.1 m/s), there remained a water film between the oil core and the pipe wall, not only in the lower part of the pipe but also in the upper part. However, in the upper part the film was much thinner. Some photographs of the experiments are shown in Figures 1a, b, and c. Only when the oil core was supplied at a velocity below the critical value did the oil core touch (and foul) the upper part of the pipe. In that case the flow pattern changed from eccentric core-annular to stratified, with all the oil in the upper part of the pipe and only the water in the lower part.

Above the critical velocity, the eccentric core-annular flow was stable after traveling a few diameters. Changing the amount of water did not have a great influence on the flow pattern. The thin water film in the upper part remained almost the same; only the thickness of the water film in the lower part changed (see...
Figures 1a-c). During some of the experiments, a rectangular roughness element was placed against the pipe wall at a certain location inside the pipe to study the reaction of the flow. The eccentric core-annular flow passed this impediment without difficulty; a few diameters downstream it was stable again.

Immediately after the inlet device a ripple appeared at the oil-water interface. The growth velocity of the amplitude of this ripple quickly damped out to leave a ripple with a finite amplitude at the interface. As can be seen from Figures 1a-c, the ripple had a sawtoothlike shape. Its wavelength was of the same order of magnitude as the radius of the pipe. For the theoretical model, the ripple shape was chosen in accordance with this observation.

The second set of experiments was carried out in an 888-m-long, horizontal 8-in pipe circuit comprising 22 right-angle bends of 2.5–1.5 diameter radius, which did not pose any problems for the stable core flow operation. The difference in density between water and oil was about 45 kg/m³. The oil viscosity for the pressure gradient tests considered here varied from 1200 to 2200 mPa.s at a superficial oil velocity of 1 m/s. The input water fraction ranged from 0.10 to 0.01.

We also established the stable/unstable flow pattern for this circuit. Two different oils were used, with average densities of 950 and 960 kg/m³, respectively. The lower critical velocity for stable core flow in the 8-in pipe turned out to be approximately 0.5 m/s, i.e., appreciably larger than for the 2-in pipe. Figure 16 shows a flow pattern map with the oil viscosity and input water fraction as variables. As shown, stable core flow is possible with only 2% water provided the viscosity exceeds 2200 cSt, and the oil velocity is greater than 0.5 m/s. Especially at these low water fractions, the use of an additive in the water phase (sodium silicate in these tests) was found to be necessary as a core stabilizer and cleansing agent. After the stable/nonstable regions had been established, accurate pressure gradient measurements were carried out for high-viscosity core flow, especially at low water contents.

The pressure gradient ratios ($\frac{\Delta P_{\text{cor}}}{\Delta P_{\text{eq}}}$) measured in the 2- and 8-in pipes at a superficial velocity of approximately 1 m/s are summarized in Figure 17. It is quite remarkable that at these low water contents the pressure gradient ratio hardly changes with the water fraction. It is also evident that the benefit of core flow operation is smaller for the 8-in than for the 2-in pipe: the average pressure loss ratio is 0.10 as against to 0.01 (the oil pressure drop in 10% versus 1%).

3.1.3.2 Comparison between the theoretical model and experiments

For known interfacial-wave characteristics (the parameters $\lambda$, $L$, and $L'$), and for a known dimensionless thickness of the water annulus ($H_0 = 1 - k$), calculations with the lubricating-film model for a particular pipe size are carried out as follows:

1. Determine plots for the dimensionless density difference, $\bar{\rho} - \bar{\rho}_o$ and the dimensionless pressure gradient per unit length $\frac{\Delta P}{\Delta L}$ as functions of core eccentricity $E$ (e.g., as shown in Figure 10),
Figure 17: Pressure gradient ratio measured in 2- and 8-in pipes for various input water fractions.

2. Find the eccentricity corresponding to the dimensionless density difference for the particular pipeline problem.

3. With this eccentricity, calculate the dimensionless pressure gradient.

The pressure gradient ratio $\Delta P_{OW}/\Delta P_{SO}$ can then be found from

$$\frac{\Delta P_{OW}}{\Delta P_{SO}} = \frac{\mu W}{\mu_0} \frac{H_0}{1 - H_W} L$$

in which the hold-up term arises from the relationship between the core velocity and the superficial oil velocity: $W = V_{SO}/(1 - H_W)$.

The dimensionless density difference, being a function of the core velocity, also depends on the hold-up

$$\tilde{\rho} = \frac{\rho - \rho_0}{\rho_0} = \frac{\mu_0}{\mu} \frac{V_{SO}}{V_{SO}/(1 - H_W)}$$

As in the sensitivity study, in the calculations below we will put the dimensionless film thickness $H_0$ equal to the input water fraction $C_W$. In fact, this means that for the water hold-up we use the following expression:

$$H_W = C_W (2 - C_W)$$

This can easily be derived from Eq. (6) using the relationship $H_0 = 1 - k$ for the dimensionless film thickness and the core/pipe diameter ratio. For low water fractions, Eq. (64) gives water hold-ups that are twice as large as the input water fraction. According to Eq. (62), larger hold-up values will lead to larger pressure-loss ratios, which are to some extent counteracted by a reduction of the dimensionless lubrication pressure loss, because the core will be in a less eccentric position due to its actual velocity being higher ($\tilde{\rho}_W$ is smaller; see also Figure 10). In Figure 18 water hold-ups calculated with the simple relationship in Eq. (64) are compared with data of Charles et al. (1961) for oils with viscosities from 6 to 65 mPa.s. Clearly, there is scope for improvement of the hold-up calculation method, especially at larger water fractions.

The remaining problem is determining what values to choose for the parameters that describe the interfacial wave. To this end we used the photographs taken during the tests of the 2-in pipe (see Figure 1) and the measured pressure losses. For tests with a water fraction of 0.06, our best estimates for these parameters were $A = 0.003$, $L = 0.8$, and $L' = 0.8$ ($L$ and $L'$ were determined from the photographs, while the value chosen for $A$ gives best agreement with the measured pressure loss). This choice corresponds to an average wave amplitude of 0.075 mm and a wavelength of 20 mm.

i.e., nearly equal to the pipe radius. Since we used the same or similar crude oils in the 8-in tests and no measurements on wave characteristics are available for this pipe, we used the same absolute values for the wave parameters. The dimensionless parameters \( A \) and \( L \), being inversely proportional to \( R \), are then smaller than for the 2-in pipe. As can easily be verified, this guarantees that for a core velocity of \( 1 \text{ m/s} \) all three conditions \([\text{Eqs. (15)-(17)}]\) for the applicability of the lubrication theory are satisfied for all pipe sizes of 2 in and larger. Only when we consider larger core velocities with less eccentric flow will the third condition \([\text{Eq. (17)}]\) ultimately no longer be satisfied at some critical velocity, which depends on the pipe size.

In Figure 19 we have plotted pressure gradient ratios measured in the 2-in pipe with average oil viscosities of 2300, 3200, and 3300 mPa.s; superficial oil velocities of about 1 m/s; and results of calculations with the lubricating-film model. The calculated data scatter around the measured values; the average deviations do not exceed 7%. It is interesting to make a comparison with predictions by other models (e.g., the concentric and eccentric laminar-laminar models discussed in Sec. 2.2.2). For the 2300-mPa.s oil, for example, an average pressure gradient reduction factor of 82 is calculated with the lubricating-film model, while the measured value is 83; with the Russell-Charles model for concentric flow the calculated reduction factor would be as large as 1150, and with the eccentric flow model of Bentwich et al. (1970) a value not below 640 would be found (i.e., assuming the measurements are for optimum \( k \) value (cf. Figure 5)). The simplifying assumptions of the model need to be critically examined using a more extensive set of data with differentviscosity ranges.
oils at different velocities. In particular, attention should be paid to improving the hold-up determination, as already mentioned. It is even more important to obtain satisfactory information on the interfacial-wave characteristics, especially the wave amplitude. Although readings from photographs cannot be made reliably, they seem to suggest A-values larger than the value of 0.003 that gives the best agreement with measured pressure losses. This would mean that apart from the lubrication forces the effect of inertia forces should be considered as well. For the determination of interfacial-wave amplitudes, both theoretical and experimental work is required.

The problem of a more accurate determination of interfacial wave characteristics (shape, amplitude and wavelength) will be addressed below by making use of dedicated core flow experiments and a simple theoretical model (for wave shape and amplitude). In the remainder of this chapter we will concentrate on the effect of turbulence in the water film (section 3.2) and on an improved calculation method for the water film hold-up (section 3.3). Since the experiments that were carried out to assess whether or not to incorporate turbulence in our model, also provided new insight into the wave amplitudes and lengths, the latter data were also used in the turbulent version of the model. Wave characteristics dealt with in chapter 4 are a further refinement of these preliminary data.

3.2 Turbulent Lubrication

In the present section core flow tests with a 3000 mPa.s fuel oil in a 2-inch test facility are presented. The experiments supply important information on the amplitudes and lengths of waves at the oil/water interface. Moreover, the pressure-gradient data obtained suggest that the effect of turbulence in the water film is most likely not restricted to the lower part of the pipe, so the lubricating film model has to be adapted accordingly. The set of model equations, extended to include the effect of turbulence, is then given. Finally, the generalized lubricating-film-model predictions are compared with core-flow pressure gradients measured in the 2-inch test facility.

3.2.1 Experimental investigation

A 2-inch horizontal pipe loop with a total length of 16 m was used for a new series of core-flow measurements. Not only were pressure gradients measured, as in the past; the shape, amplitude and length of the waves also received attention, together with film thickness variation around the pipe circumference and water hold-up. The interfacial wave and water annulus data, recorded with the aid of specially developed instrumentation, are discussed in chapter 4 and section 3.3. Here, for a range of oil velocities and water fractions, pressure-loss data and some interesting information derived from photographs taken during the core-flow tests will be presented.

Photographs were taken during core-flow operation with a 3000 mPa.s fuel oil, which has a density 20 kg/m³ smaller than that of water. For this purpose a transparent section was installed in the pipe loop, consisting of a Perspex tube immersed in a plane rectangular trough of the same material filled with glycerin. The combination of the liquid and the Perspex precluded the problem of optical distortion due to diffraction. With a horizontally aligned camera, the upper and lower wave profiles of the core could be photographed. Figure 20 shows the oil flow from right to left, with superficial velocities \( V_{n} \) and input water fractions \( C_{w} \) of 0.5 m/s and 0.20 (A), 1 m/s and 0.03 (B), and 1 m/s and 0.20 (C).

When the photographed profiles were copied and analysed, the following points were noted:

- The autocorrelation functions of most of the profiles were weak, and the profiles were not regular; in each case, though, a dominant wavelength could be found.
- There was no interrelation between an upper profile and its corresponding lower profile because the two could not be cross-correlated.
- For input water fractions less than 0.15, the difference in water-layer thickness between the upper and lower sides was less than 5% of the pipe diameter.
A: $V_{so} = 0.5 \text{ m/s}, C_w = 0.20$

B: $V_{so} = 1.0 \text{ m/s}, C_w = 0.03$

C: $V_{so} = 1.0 \text{ m/s}, C_w = 0.20$

**Figure 20**: Core flow of a 3000 nPa.s fuel oil in the 2-inch test loop.

**Figure 21**: Plot of wavelength ($\lambda$) for different input water fractions ($C_w$) measured in the 2-inch test loop.
The correlation for the wavelength shown in Figure 22 can be expressed as:

$$\lambda = 13.6 \times R \left[1 - \sqrt{1 - C_w}\right] \times V_{SO}^{0.594}$$  (65)

where $\lambda$ is the wavelength, $R$ the pipe radius, $C_w$ the input water fraction and $V_{SO}$ the superficial oil velocity.

Water hold-ups ($H_w$) could only be determined from the photographs. The values obtained, however, were all smaller than those calculated according to the expression:

$$\text{PRESSURE GRADIENT, Pa/m}$$

Figure 23: Pressure gradient measured in the 2-inch test loop for various water fractions and oil velocities.

<table>
<thead>
<tr>
<th>$V_{SO}$ (m/s)</th>
<th>Core flow pressure gradient (Pa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_w = 5%$</td>
</tr>
<tr>
<td>1.95</td>
<td>1570</td>
</tr>
<tr>
<td>1.75</td>
<td>1450</td>
</tr>
<tr>
<td>1.50</td>
<td>950</td>
</tr>
<tr>
<td>1.25</td>
<td>700</td>
</tr>
<tr>
<td>0.93</td>
<td>400</td>
</tr>
<tr>
<td>0.75</td>
<td>265</td>
</tr>
<tr>
<td>0.5</td>
<td>135</td>
</tr>
<tr>
<td>0.29</td>
<td>93</td>
</tr>
</tbody>
</table>
Core-flow pressure gradients measured for water fractions ranging from \( C = 0.05 \) to \( C = 0.20 \) (Table I) are plotted against mixture velocity in Figure 23. For comparison, a curve for pressure gradients calculated for pure water flow is also given. At velocities greater than 0.5 m/s the core-flow pressure gradient varies with the mixture velocity to the power 1.8, i.e. typical for turbulent flow. There is only a slight dependence on water fraction.

3.2.2 Adapted model equations

The experimental observation that turbulence in the water film is of importance forced us to reconsider the theoretical model for steady core flow in horizontal pipes. A study was initiated of the possibility of extending the core-flow model, which is based on the hydrodynamic lubrication theory, (Ooms et al. 1984) to a model in which turbulence in the water film is fully taken into account. The approach considered here is one of generalizing the flow equations for the water film to those for the turbulent lubrication theory.

The starting-point for the turbulent-lubricating-film model for steady core flow in a pipe is the following set of equations derived from the continuity equation (eq. (18)) and the time-independent equations of motion eqs. (19) to (21) given above (in cylindrical coordinates \( x, y \) and \( \theta \) for the axial, radial and azimuthal directions, respectively):

\[
\begin{align*}
\frac{\partial}{\partial y} (\rho u R^2) + \frac{\partial}{\partial \theta} (\rho v R^2) + \frac{\partial}{\partial x} (\rho w R) &= 0 \\
\frac{\partial}{\partial y} (\rho u) &= 0 \\
\frac{\partial}{\partial y} \left( \rho \left( \frac{\partial u}{\partial y} \right) \right) &= \frac{1}{R} \frac{\partial}{\partial \theta} \left( \frac{\partial R}{\partial \theta} \right)
\end{align*}
\]  

(67)  

(68)  

(69)

where: \( u, v, w \) = velocity components of water in \( y, \theta \) and \( x \) directions respectively, m/s  
\( \phi = p + \rho g r \cos \theta + 2/3 \rho k \); pressure variable, Pa  
\( \rho \) = density, kg/m\(^3\)  
\( k \) = turbulent kinetic energy per unit mass, J/kg  
\( r = R - y \); radial length, m  
\( g \) = acceleration due to gravity, m/s\(^2\).

In equations (69) and (70) the viscosity \( \mu^* \) consists of a turbulent and a molecular part: \( \mu^* = \mu_t^* + \mu_m^* \), where \( \mu_t^* \) is a function of \( x, y \) and \( \theta \). The above equations for \( u, v, w \) and \( \phi \) have been derived from the continuity equation and the Navier-Stokes equations on the same assumption as in our previous work (Ooms et al., 1984), i.e. that not only is the thickness \( h \) of the water film much smaller than both the pipe radius \( R \) and the length \( L \) of the wave on the oil core, but also the group \( (Wh/\mu^*).h/\ell \) is less than 1 (here \( W \) is the velocity of the oil core). As before, the assumption is made that the oil core is rigid (with a "frozen" ripple at its surface).

The basic equation for \( \phi \), a function of \( x \) and \( \theta \) only (according to equation (68)), is derived in the same way as the first version of the Reynolds equation (eq. (48)). In equation (67), expressions are substituted for the velocities \( v \) and \( w \) obtained from integration of equations (69) and (70). For computational convenience, the viscosity \( \mu^* \) has been replaced by its average \( \bar{\mu} \) over the radial distance, \( y \):

\[
\bar{\mu} = \frac{\partial}{\partial \theta} \left( \int_0^h \mu^* dy \right)
\]

(71)

Thus, the viscosity \( \bar{\mu} \), like \( \phi \), is now a function of \( x \) and \( \theta \) only. The expression for the turbulent Reynolds equation - the partial differential equation describing the pressure generated in a turbulent thin-film flow - then becomes:
This is a generalization of the equation used in the earlier version of the core flow model (see eq. (48) and Ooms et al., 1984, eq. (8)).

For the description of turbulence via the viscosity $\nu_t$ in $\mu^*$, a simple turbulence model, Prandtl's mixing length model, has been chosen:

$$\nu_t = \frac{\mu_t}{\rho} = 4 \cdot \frac{4}{m} \left. \frac{\partial \nu^*}{\partial y} \right|_y$$

(73)

Here the expression for the mixing length $l_m$ is the one according to Nikuradse for channel flow with Van Driest's hypothesis for the effect of the wall:

$$l_m = h \left( 0.14 - 0.08(1-y/h)^2 - 0.06(1-y/h)^4 \right) \left( 1 - \exp \left( \frac{-y^4}{A} \right) \right)$$

(74)

where $y^* = \frac{y \cdot Re^*}{h}$

(75)

with $Re^* = 0.11$ ($Re$) 0.911

$Re = Wh/\nu_m$

$W$ = maximum water velocity (= oil core velocity)

$A = 26$ (Van Driest constant).

The velocity gradient $dw/dy$ in equation (73) can be computed according to:

$$\frac{dw}{dy} = \frac{\nu^*}{y}$$

(76)

with $\kappa$ (Von Kármán constant) = 0.35 and $\nu^* = \nu_m Re^*/h$

The next steps in the turbulent-lubricating-film model for steady core flow are the determinations of the position of the oil core (its eccentricity) and the corresponding pressure gradient. The equation stating that in steady core-annular flow the buoyancy force on the core is counterbalanced by hydrodynamic lubrication forces (with due account taken of turbulence in the water annulus) reads as follows:

$$2 \pi \int_0^\pi \int_0^\infty \phi \cos \theta \, d\theta \, dx - 2 \pi \int_0^\pi \frac{\phi}{a} \frac{d\theta}{\theta} \sin \theta \, dx = (\rho - \rho_c) g \pi \frac{R^2}{2}$$

(77)

where $\rho_c$ = core density

$$a = \int_0^h \frac{y}{y^2-1} \, dy \int_0^h \frac{1}{y^2} \, dy$$

(78)

This equation differs from the one used in the earlier version of the lubricating-film model (eq. (55) and Ooms et al. (1984), eq. (22)) in that the film thickness $h$ in the integrand of the second integral on the left-hand side is replaced by $2a$, a function of the turbulent viscosity (note that when $\nu_t$ equals zero, i.e. $\mu^* = \nu_m = constant$, $2a$ equals $h$). The equation for the pressure gradient in steady core flow is affected in a similar manner. It differs from the expression used earlier in that $h$ and $\nu_m/h$ are replaced by the turbulent-viscosity-dependent functions $2a$ and $1/\beta$, respectively:

$$2 \pi \int_0^\pi \int_0^\infty \phi \cos \theta \, d\theta \, dx + 2 \pi \int_0^\pi \frac{\phi}{\beta} \frac{d\theta}{\theta} \sin \theta \, dx = \pi R^2 (\rho_c - \rho)$$

(79)

(80)

The way in which turbulence effects are introduced into the lubricating-film model for core-annular flow is a very simple one, not very different from the mixing-length approach to turbulent lubrication proposed by Constantinescu in 1959. However, it may well be sufficient for our needs, since Launder & Leschziner (1978) have shown that, for finite-width thrust bearings, the much more sophisticated $k$-$\epsilon$ turbulence
model leads to effects of turbulence in lubricating films similar to those predicted by Constantinescu.

An aspect that warrants critical examination is the validity of the assumption that the group \((\rho V^2 / \mu) \cdot (h/k')\) is less than 1, for the purposes of which inertial effects are disregarded. Should this assumption prove invalid, allowance for fluid inertia will have to be incorporated in the turbulent-lubricating-film model. This problem has already been addressed by King & Taylor (1977) and Launder & Leschziner (1978) for a plane inclined slider thrust bearing operating with a turbulent film. They conclude that the effect of fluid inertia relative to non-inertial flow conditions is an increase in load capacity and a relatively small increase in frictional traction on the moving surface. Lubrication with comparable viscous and inertia forces has been studied by Tuck & Bentwich (1983).

Only comparison with measured core flow data, however, will reveal whether or not refinements (a more sophisticated turbulence model or allowance for fluid inertia) have to be incorporated in the turbulent-lubricating-film model for core-flow applications.

3.2.3 Comparison of model predictions with experiment

For steady core flow of a 3000 mPa.s oil in the 2-inch test facility with an input water fraction of 0.10 calculations have been performed with two versions of the lubricating-film model, one without turbulence in the water film and one including turbulence effects, as described above. In these calculations the original default values were used for wave amplitude (0.075 mm) and wavelength (20 mm). The results of the calculations are shown in Figure 24 as dashed and dashed-dotted lines, respectively. At a superficial oil velocity of nearly 2 m/s, the effect of turbulence can lead to a pressure gradient increase of some 30%. This value is in line with results obtained with an intermediate model with turbulence effects restricted to the lower part of the water annulus; at this particular wave amplitude/wavelength combination the position of the oil core in the pipe is highly eccentric, resulting in a very thin water film with hardly any turbulence in the upper part of the annulus.

Compared to measured pressure gradients, also shown in Figure 24, neither calculation gives a fair prediction. Subsequently, we performed calculations with the two versions of the lubricating-film model, applying wave data that had been observed during the tests. At a water fraction of 0.10, the observed wave amplitude was 2 mm, while the wavelengths varied from 35 mm at a superficial oil velocity of 0.3 m/s to 11 mm at 1.95 m/s. For these wave amplitude/wavelength combinations the position of the oil core in the pipe is much less eccentric than in the previous case. As a consequence, the effect of turbulence on pressure gradients is much stronger, since it plays a role in a larger part of the water annulus. The results of the calculations are indicated in Figure 24 as solid (with turbulence) and dotted (without turbulence).
lines. As illustrated by the solid curve in Figure 24, the use of the observed wave data in the turbulent-lubricating-film model brings the pressure gradient predictions much more closely into line with measurements. The results of calculations with the original model (i.e. without turbulence) when the observed wave data for amplitude and length are used give pressure gradients that are very much lower.

The pressure build-up in the water film is quite different for the laminar and turbulent lubrication as illustrated in Figures 25 and 26, where the distribution of the dimensionless pressure (ψ) is plotted over a wavelength for four values of the angle θ. The examples shown are for the case with a superficial oil velocity of 1.95 m/s (and an input water fraction of 0.10) with a wave amplitude of 2 mm and a wavelength of 12 mm. For turbulent lubrication not only is the pressure level much larger than for the laminar case, but it is also a much weaker function of the angle θ. As opposed to the earlier situation when still the default wave amplitude/wavelength values were used (Figure 11) the variation of θ is no longer negligibly small in the lower part of the pipe, and for the turbulent case even varies over the ripple there.

The promising results with the new model for core flow reveal the importance of turbulence in the water film and of a proper description of the waves on the oil core. Still, as shown in Figure 24, even with the turbulent-lubricating-film model pressure gradients are systematically underpredicted, although the dependence on oil velocity is correctly represented. This underprediction could be caused by the fact that the model neglects the contribution of inertial effects. Other reasons for the underprediction can be too high a water film hold-up value or a not quite correct wave geometry. For a better distinction between the effects of fluid inertia and interfacial waves.
on calculated pressure gradients, interfacial wave data more accurate
than those derived from photographs are needed. The analysis of
dedicated measurements of film thickness variation around the pipe
circumference will lead to improved water film hold-up values as
discussed in Section 3.3 and improve the quality of the wave data
(presented in Chapter 4). In particular, the wave shape (asymmetry,
for which here a value of 0.8 was used, corresponding to a wave with
a long gradual front and a steep tail) will be reconsidered.

3.3 Water film hold-up

The water film hold-up is an important parameter of the
lubricating film model, since it affects the dimensionless density
difference (see eq. (63)), hence the oil core eccentricity and the
corresponding pressure gradient. In the initial version of the model
we used for the water hold-up calculation a simple expression that
formed an upper bound for the scanty experimental data available at
the time (see Figure 18):

\[ H_w = C_w (2 - C_w) \tag{64} \]

As mentioned before this relationship means that the input water
fraction \( C_w \) is equal to the dimensionless film thickness for
concentric flow, \( H_0 \). In principle the model enables us to compute
the input water fraction (from the total water flow) for a given
choice of \( H_0 \) at a particular wave geometry and dimensionless
density difference as follows: the water flow rate \( Q_w \) is obtained
by integrating the velocity profile \( w \) (eq. 40) over the annulus:

\[
Q_w = 2 \int_0^{\pi} w R \, d\theta \, dy = 2 \int_0^{\pi} w(R-y) \, d\theta \, dy \tag{81}
\]

The ratio of the water and oil flow rates is:

\[
\frac{Q_w}{Q_o} = \frac{C_w}{1-C_w} = \frac{h \pi}{\pi(R-h)^2} \tag{82}
\]

which, upon introduction of dimensionless variables \( Y = y/R, H = h/R \)
and \( \phi = 6R/\pi w \) (see eq. (59)) becomes, after integration over \( Y \)
averaged over a wavelength:

\[
\frac{C_w}{1-C_w} = \frac{2}{\pi L (1-H_w)^2} \int_0^L \int_0^{\pi} \left[ \frac{H}{2} \left( \frac{H}{2} - \frac{H^2}{H_c} \right) + \frac{H^3}{H_c^2} - \frac{H^4}{H_c^3} \right] \, d\phi \, dX \tag{83}
\]
When the laminar model is used with the wave parameters of the sensitivity study ($a = 0.075 \text{ mm}$, $\ell = 20 \text{ mm}$, $\ell' = 16 \text{ mm}$) the above formula gives for $\Delta \rho = 35 \text{ kg/m}^3$ and $V_{\text{so}} = 1 \text{ m/s}$ that the input water fraction $C_w$ equals $H_0$ within 2%. A change in input parameters does not affect this result, except for some influence of wave amplitude (a larger amplitude gives rise to an increase in slip).

In the remainder of this section we will concentrate on measured hold-up values, bearing in mind that a model check like the one above can be carried out, provided reliable wave amplitudes are available.

Photographs taken during the 2-in core flow tests for a range of oil velocities and input water fractions with a 3000 mPa.s oil provided the following expression as an upperbound for the observed hold-ups:

$$H_w = C_w (1 + (1-C_w)^5) \tag{66}$$

Water hold-ups calculated with eq. (66) are much smaller than those obtained from the earlier expression (eq. (64)).

After completion of the 3000 mPa.s tests, a series of tests have been performed in the same horizontal 2-inch test facility with a specially developed interfacial wave detector. This time three different oil viscosities (1500, 3000 and 8000 mPa.s) were considered for approximately the same ranges of oil velocities and input water fractions. The film thickness data obtained with the wave detector, based on the reflection of ultrasonic signals provided water hold-up data much more accurate than thus far available. The measured data for different oil viscosities and superficial oil velocities are plotted in Figure 27 as function of input water fraction. For comparison several curves have been plotted in Figure 27, representing different correlations by which the water hold-up can be computed: the two upper curves are determined from eq. (64) and eq. (66) respectively, while the lower curve represents the no-slip situation ($H_w = C_w =$ input water fraction). Note that the test data are not far removed from the latter curve, and in most cases measured hold-ups are very much smaller than the hold-ups calculated with eqs. (64) and (66). A reasonable upperbound for the measured data is given by the upper dash-dotted curve in the Figure, which represents the following equation:

$$H_w = C_w (1 + 0.5 (1-C_w)^5) \tag{84}$$
It equals the average of the water hold-up calculated by eq. (66)
and the no-slip hold-up. The average measured hold-up (for different
oil viscosity and oil velocity conditions) can be fitted by
\[ H = C_n + 0.2 \left(1-C_n\right) \]
and is presented by the lower dash-dotted curve in Figure 27.
In the remainder of the thesis we will use expression (84) for the
water hold-up, bearing in mind it will have to be replaced by a more
accurate expression which properly accounts for the effect of oil
viscosity and velocity changes.

Figure 28 shows the effect of using the more realistic (lower)
water hold-up given by eq. (84) on the computation of core flow
pressure gradients with the turbulent lubricating film model.
Calculated pressure gradients are larger than those computed with
the larger water hold-ups (eq. (66)). The relative core eccentricity
for the 2-inch 3000 mPa.s calculation is shown in Figure 29. Here the
relative core eccentricity is defined as
\[ E_{rel} = \frac{h_o - a/2 - h_{top}}{h_o - a/2} \]
where \( h_o \) = film thickness for concentric flow (in mm), \( a \) = wave amplitude (mm)
and \( h_{top} \) is the distance of the oil core to the top of the pipe.
This parameter can vary from 0 (concentric flow) to 1 (completely
eccentric flow). For superficial oil velocities ranging from 0.3 to
1.95 m/s it varies from 0.82 to 0.18 i.e. even at a superficial oil
velocity of nearly 2 m/s the oil core is not yet in a concentric
position. At that velocity the distance of the core to the top of
the pipe, also indicated in Figure 29 as a dashed curve, is 0.57 mm,
while at 0.3 m/s it is as low as 0.13 mm.

---

**Figure 28**: Effect of water hold-up on pressure gradients
(3000 mPa.s oil with a water fraction of 0.10 in
the 2" loop).

**Figure 29**: Relative eccentricity and distance to top of pipe
for oil core in 2 inch tests.
Chapter 4
INTERFACIAL WAVE CHARACTERISTICS

In the lubricating-film model for core-annular flow the oil viscosity is assumed to be so high that any flow in the core, and hence any variation in the oil-water interface form with time, can be neglected. So the core is assumed to be solid and the interface to be a solid-liquid interface. However, the magnitude of the lubrication force depends to a large extent on the shape of the waves (see e.g. section 3.2). For an application of the core flow model knowledge is required about the amplitude and shape of the interfacial waves. Thus far information on amplitudes and lengths of interfacial waves was derived from photographs taken during 2-inch core flow tests with a 3000 mPa.s oil. In Section 4.1 of this chapter, again using the photographic material, the wave shape (its form and asymmetry) will be considered more closely and the effect of a change in wave form and asymmetry on model predictions will be investigated. Subsequently, in Section 4.2, a simple model for long waves of finite amplitude will be introduced and applied to the 3000 mPa.s core flow problem. The model in principle is capable of predicting the wave form, its asymmetry and the wave amplitude. Finally in the last Section of this chapter on interfacial waves, some results on wave characteristics measured by an ultrasonic wave detector will be discussed. The experimental data will be used as input for the core flow model to compute pressure gradients and core eccentricities for the 2-inch situation used as a test case in this thesis.

4.1 Observed wave shapes

The wave shape that is used in the model is a point of concern. In all calculations presented in Sections 2 and 3 of the last chapter we invariably have used a value of 0.8 for the asymmetry parameter, which means a wave with a large gradual front and a steep tail. The photographs suggest, however, that interfacial waves have a steep front and a long gradual tail (see Figure 20 a and c). Also the wave form is a point worth considering. A closer inspection of the photographs taken during the 2-inch core flow tests shows that the waves most likely may be better described by the drawn curve in Figure 30 instead of the saw-tooth form also given in the Figure. The asymmetry parameter value of 0.6 selected here is considered more appropriate than the value of 0.8 hitherto used. The photographs even suggest that values around 0.5 are not unlikely (still with an asymmetrically shaped wave).

When we keep the asymmetry parameter at 0.8 and merely change the form of the wave from a saw-tooth into a curved form similar to the one shown for $L' = 0.6$ in Figure 30, the pressure gradient calculated for the 1.95 m/s oil velocity condition in the 2-inch pipe ($C_w = 0.10$, and further conditions similar to the calculations in Section 3.3) increases from 1277 Pa/m to 1406 Pa/m, while the distance...
to the top of the pipe increases by 9% from 0.57 mm to 0.62 mm.

Keeping the curved wave form, but now decreasing the asymmetry parameter value from $L' = 0.8$ to 0.6 leads for the above test condition to a further increase of the calculated pressure gradient (to 1339 Pa/m), but now with a more eccentric position of the oil core than we had with the saw-tooth form and $L' = 0.8$: a distance to the top of the pipe of 0.49 mm. Plots for the dimensionless pressure distribution over the wavelength for the curved wave forms with $L' = 0.8$ and $L' = 0.6$ are given in Figures 31 and 32. As compared to the saw-tooth interface situation the maximum pressure for the $L' = 0.8$ case in Figure 31 occurs at $X = 0.5$ in stead of $X = 0.6$.

The pressure gradients calculated for the complete range of oil velocities using the curved wave form and $L' = 0.6$ assumption are compared in Figure 33. The agreement with measured pressure gradients is excellent. As shown in Figure 34 the calculated relative eccentricities for the oil core range from 0.77 for 0.3 m/s to 0.3 for 1.95 m/s i.e. are somewhat different from those given in Figure 29. The distance of the oil core to the top of the pipe is calculated to vary from 0.16 mm to 0.5 mm.
4.2 Modelling of long-wavelength waves at the interface between two viscous fluids

Models for the description of waves at the interface between two viscous fluids have not yet been developed to such an extent that all wave data required for the core flow model (wave amplitude, length and shape) can be predicted for this flow mode. The calculation of finite amplitude waves for a core-annular flow with an eccentric core through a horizontal pipe is very complicated. Therefore, as a crude first approximation to the real flow problem, the finite amplitude waves are considered for the much simpler flow problem of plane Couette-Poiseuille flow of two superposed layers of fluids with different viscosity between two horizontal plates.

Yih (1967) has shown that such a system is unstable to a long wavelength disturbance which persists at arbitrary small values of Reynolds number. The existence of the instability depends upon three parameters: the viscosity ratio, the depth ratio and the density ratio of the fluids. Even with a stabilizing effect of density, an instability can still be found that is associated with the jump in viscosity across the interface. Similar results were found by Li (1969) for a three-layer viscous stratified flow between two horizontal plates and by Hickox (1971) for the flow of two fluids flowing concentrically in a straight circular tube. Hooper & Boyd (1983) studied the stability of a cocurrent flow of two fluids of different viscosity in an unbounded region; they found that the interface may be unstable to a small-wave-length perturbation. At large Reynolds number, one would expect turbulence to result from the instability. However, at moderate Reynolds number the system may find a finite amplitude configuration. In an extension of Yih’s approach Hooper & Grimshaw (1985) show that the interface can either return to its original undisturbed state or evolve to some finite amplitude steady state. They claim that surface tension stabilizes the linear long wave instability and that convective nonlinearity will cause a sinusoidal disturbance to deform into a wave with a steep front face.
and a long gradual tail. In the approach of Ooms, Segal, Cheung and Oliemans (1985) wave shapes with a different asymmetry (long front, short steep tail) are predicted to occur at the interface of two viscous fluids (inertia and surface tension effects are neglected). Here, we will introduce this approach and by applying it to a core flow situation assess to what extent it is capable of reproducing observed wave data.

As opposed to Yih and Hooper & Grimshaw in our calculation the amplitude of the disturbance is not assumed to be small compared to the distance between the plates (the observed wave amplitude is of the order of the film thickness in core flow). So, instead of the Orr-Sommerfeld equations the Navier-Stokes equations will be used. However, like Yih these equations will be simplified by using a perturbation calculation with the ratio of the distance between the plates and the disturbance wave length as the small perturbation parameter \((h/L << 1)\) i.e. the long wave length approximation is maintained.

4.2.1 Model equations

Figure 35 gives a sketch of the flow problem. At the interface between fluid 1 and fluid 2 long waves of finite amplitude are supposed to be present. The waves are assumed to be two-dimensional. As reference system a system is chosen according to which the lower plate is at rest. The upper plate has a velocity \(w_w\) in the \(x\)-direction.

The flow of the two fluids can be calculated with the aid of the continuity equations

\[
\begin{align*}
\frac{3w_1}{3x} + \frac{\partial u_1}{\partial y} &= 0 \\
\frac{3w_2}{3x} + \frac{\partial u_2}{\partial y} &= 0
\end{align*}
\]

the Navier-Stokes equations

\[
\begin{align*}
\frac{\partial w_1}{\partial t} + \rho_1 \frac{\partial w_1}{\partial x} + \rho_1 u_1 \frac{\partial w_1}{\partial y} &= -\frac{\partial p_1}{\partial x} + \frac{\mu_1}{2} \left( \frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} \right) \\
\frac{\partial w_2}{\partial t} + \rho_2 \frac{\partial w_2}{\partial x} + \rho_2 u_2 \frac{\partial w_2}{\partial y} &= -\frac{\partial p_2}{\partial x} + \frac{\mu_2}{2} \left( \frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} \right)
\end{align*}
\]

supplemented with appropriate boundary conditions. \(w_1\) and \(u_1\) are the velocity components of fluid 1; \(w_2\) and \(u_2\) those of fluid 2. \(t\) represents the time; the \(x\)- and \(y\)-coordinates are chosen as shown in Figure 35. The pressure variables \(\phi_1\) and \(\phi_2\) are given by
where \( \phi_1 = p_1 + \rho_1 g y \) \( (93) \)
\[ \phi_2 = p_2 + \rho_2 g y , \] \( (94) \)
in which \( p_1 \) and \( p_2 \) are the pressures in the fluids and \( g \) is the acceleration due to gravity.

The boundary conditions are no-slip at the walls and continuity of velocity at the interface, the kinematic interface condition and a force balance at the interface (dynamic boundary condition).

To simplify the set of equations for the flow problem the following assumptions are made:

\[ \frac{h}{L} \ll 1 \] \( (95) \)
\[ R_j \equiv \frac{h \nu_j}{v_j} = \frac{h}{L} \ll 1 \] \( (96) \)
\[ S_j \equiv \frac{\nu_j}{\nu_j \frac{h^2}{L}} = \left( \frac{h}{L} \right)^3 \ll 1 \] \( (97) \)

where \( j=1 \) or 2 for the lower and upper fluid, respectively.

In the derivation of the approximate flow equations for our problem the order of magnitude of the terms in equations (87) to (92) is determined. As a length scale in the \( x \)-direction \( L \) is used, and as a length scale in the \( y \)-direction \( h \). As velocity scale in the \( x \)-direction \( w \) is used and in the \( y \)-direction \( u \). The order of magnitude of the terms in (87) are

\[ \frac{3w_1}{\nu_j} + \frac{3u_1}{\nu_j} = 0 \]
\[ \frac{\partial w_1}{\partial y} = \frac{\partial u_1}{\partial y} \]
\[ \frac{\nu_j}{L} \]
\[ 0 \left( \frac{h}{L} \right) \]
\[ 0 \left( \frac{h}{L} \right) \]

This yields

\[ u = 0 \left( \frac{h}{L} \right) . \]

\[ (99) \]

The time scales in the \( x \)-direction \( \frac{L}{w} \) and in the \( y \)-direction \( \frac{L}{u} \) are of the same order of magnitude. With the aid of (99) the order of magnitude of the inertial terms and viscous terms of (89) and (90) can be written as

\[ \rho_1 \frac{3w_1}{\nu_j} + \rho_1 \frac{3u_1}{\nu_j} = \frac{\partial w_1}{\partial x} + \frac{\partial u_1}{\partial y} \]
\[ 0 \left( \frac{L}{w} \right) \]
\[ 0 \left( \frac{L}{u} \right) \]
\[ 0 \left( \frac{L}{w} \right) \]
\[ 0 \left( \frac{L}{u} \right) \]

This study is restricted to waves with a wavelength large compared with the distance between the plates; so

\[ \frac{h}{L} \ll 1 . \]

Following Yih, in first approximation, all terms which are a factor \( \frac{h}{L} \) smaller than other terms in the differential equations, are ignored. This means that the inertial and viscous terms of (101) are ignored, as these terms are a factor \( \frac{h}{L} \) smaller than the corresponding terms in (100).

As the term

\[ \frac{\mu_1}{L^2} \frac{3w_1}{\nu_j} \]

is a factor of \( \frac{h^2}{L} \) smaller than the term \( \frac{\mu_1}{L^2} \frac{3w_1}{\nu_j} \),
it can of course also be omitted. The ratio of the inertial terms
in (100) and the viscous term \( \frac{\partial^2 \omega}{\partial y^2} \) of that equation is 0\( \frac{\rho \cdot \omega \cdot h}{\mu_1} \).

It is supposed that the value of the Reynolds number \( \frac{\rho \cdot \omega \cdot h}{\mu_1} \) is such, that

\[
\frac{\rho \cdot \omega \cdot h}{\mu_1} \cdot \frac{h}{\Delta} \ll 1.
\]

This means, that as in Yih's first approximation, also the inertial terms of (100) can be ignored. In this way (100) and (101) simplify to

\[
0 = -\frac{3 \phi_1}{\Delta x} + \frac{3 \omega_1}{\Delta y^2}
\]

\[
0 = -\frac{3 \phi_1}{\Delta y}
\]

Equation (103) shows that \( \phi_1 \) is a function of \( x \) and \( t \) only.

Integration of (102) then gives

\[
u_1 = \frac{1}{2 \rho_1} \frac{3 \phi_1}{\Delta x} y^2 + c_1 y + c_2,
\]

in which \( c_1 \) and \( c_2 \) are integration constants. With the aid of the boundary conditions

for \( y = 0 \) : \( \omega_1 = 0 \)

for \( y = h \): \( \omega_1 = \omega_1 \)

it is found that

\[
\omega_1 = \frac{1}{2 \rho_1} \frac{3 \phi_1}{\Delta x} (y - h) + \omega_1 \frac{y}{h_1}.
\]

From the continuity equation (87) follows

\[
\frac{\partial \omega_1}{\partial y} = -\frac{3 \omega_1}{\Delta x}
\]

Or

\[
\frac{\partial \omega_1}{\partial y} + \frac{h_1}{\Delta x} \frac{\partial \omega_1}{\partial y} = \frac{1}{\Delta x} \frac{\partial \omega_1}{\partial y} = -\omega_1 \frac{\partial \omega_1}{\partial x}
\]

Substitution of (107) in (109) gives

\[
\omega_1 = \frac{h_1}{2 \rho_1} \frac{3 \phi_1}{\Delta x} + \frac{h_1^2}{4 \rho_1} \frac{3 \phi_1}{\Delta x} + \frac{\omega_1}{2} \frac{\partial \omega_1}{\partial x} - \frac{h_1}{2} \frac{3 \omega_1}{\Delta x}
\]

If also \( \frac{\rho \cdot \omega \cdot h}{\mu_2} \cdot \frac{h}{\Delta} \ll 1 \) it can be shown in the same way, that

\[
\frac{3 \phi_2}{\Delta x} \left( \frac{3 \phi_2}{\Delta x} \right) = 6 \mu_2 \left( \frac{3 \phi_2}{\Delta x} \right) + 6 \mu_2 \frac{\partial \omega_1}{\partial x} + \frac{12 \mu_2 \omega_1}{\Delta x}
\]

For \( \omega_2 \) an expression similar to (107) for \( \omega_1 \) can be derived, namely

\[
\omega_2 = \frac{1}{2 \rho_2} \frac{3 \phi_2}{\Delta x} y'(y' - h_2) + \omega_2 \frac{y'}{h_2} + \omega_2 (1 - \frac{y'}{h_2})
\]

in which

\[
y' = h - y
\]
Kinematic and dynamic boundary conditions.

At the interface between the two fluids the kinematic boundary condition holds. This condition can be written as

\[
\frac{\partial F}{\partial t} = 0 ,
\]

in which \( \frac{\partial}{\partial t} \) represents the material derivative and \( F \) the equation of the interface

\[
F = y - h_1(x,t) = 0 .
\]

Substitution of (116) in (115) gives

\[
u = w_i \frac{\partial h_1}{\partial x} + \frac{\partial h_1}{\partial t} .
\]

The order of magnitude of the terms of (117) are given by

\[
u = w_i \frac{\partial h_1}{\partial x} + \frac{\partial h_1}{\partial t} ,
\]

From (99) it can be concluded that the terms are of the same order of magnitude. So the kinematic boundary condition cannot be simplified. It gives a relation, equation (117), between the shape \( h_1 \) of the interface, and the velocity components \( w_i \) and \( u_i \) of the fluids at the interface.

At the interface of the fluids also the dynamic boundary condition holds. This condition can be written as

\[
n_i \sigma_{2,ik} - n_i \sigma_{1,ik} = \gamma \frac{\partial^2 h_1}{\partial x^2} n_i \quad \text{for} \quad y = h_1 ,
\]

in which \( n_i \) are the components of the unit normal to the interface, \( \gamma \) the surface tension coefficient, and \( \sigma_{1,ik} \) and \( \sigma_{2,ik} \) the stress tensors for fluid 1 and fluid 2 respectively. \( \sigma_{1,ik} \) is given by

\[
\sigma_{1,ik} = \begin{pmatrix}
-\pi_1 + 2w_i \frac{\partial h_1}{\partial x} & w_i \frac{\partial^2 h_1}{\partial x \partial y} & w_i \frac{\partial h_1}{\partial y} \\
0 & \frac{\partial w_i}{\partial y} & 0 \\
0 & 0 & -\pi_1 + 2w_i \frac{\partial h_1}{\partial y}
\end{pmatrix}
\]

The order of magnitude of the terms in (120) will be determined.

From (93), (100) and (102) follows

\[
\pi_1 = O\left(-\frac{\partial^2 h_1}{x^2}\right).
\]

The other terms can be estimated with the aid of (99); it is then found that

\[
2w_i \frac{\partial^2 h_1}{\partial x^2} = O\left(-\frac{\partial^2 h_1}{x^2} \cdot \frac{\partial^2 h_1}{y^2}\right).
\]

For \( \sigma_{2,ik} \) a similar estimate can be made.
The ratio of the two components $n_1$ and $n_2$ of the unit normal can be estimated as
\[ \frac{n_2}{n_1} = O(\frac{h^2}{z^2}). \]  

(126)

Substitution of (120) to (126) in (119) gives in first approximation
\[ -(p_1 - p_2)n_1 + (\frac{\partial n_1}{\partial y} - \frac{\partial n_2}{\partial y})n_2 = \frac{\partial^2 h}{\partial x^2} \text{ for } y = h_1 \]  

(127)

\[ (u_1 \frac{\partial n_1}{\partial y} - u_2 \frac{\partial n_2}{\partial y})n_1 - (p_1 - p_2)n_2 = \frac{\partial^2 h}{\partial x^2} \text{ for } y = h_1. \]  

(128)

These relations must be satisfied for any combination $(n_1, n_2)$. That is only possible, if
\[ -(p_1 - p_2) = \frac{\partial^2 h}{\partial x^2} \text{ for } y = h_1 \]  

(129)

\[ u_1 \frac{\partial n_1}{\partial y} - u_2 \frac{\partial n_2}{\partial y} = 0 \text{ for } y = h_1. \]  

(130)

The ratio of the surface tension term and pressure terms in (129) is respectively of the following order of magnitude
\[ 0(\frac{\gamma}{v_1} \frac{h^3}{z^3}) \text{ and } 0(\frac{\gamma}{v_2} \frac{h^3}{z^3}). \]

If it is assumed that
\[ \frac{\gamma}{v_j} \frac{h^3}{z^3} << 1 \text{ with } j = 1,2 \]  

(97)
equation (129) reduces to
\[ (p_1)_y = (p_2)_y = \phi_1. \]  

(131)

Substitution of (93) and (94) gives
\[ \phi_1 = \phi_2 + (\rho_1 - \rho_2)gh_1. \]  

(132)

Substitution of expressions for $u_1$ (eq. (107) and $u_2$ (eq. (113)) in (130) gives
\[ \frac{h_1}{2} \frac{\partial \phi_1}{\partial y} + \frac{1}{2} \frac{u_1}{h_1} = - \frac{h_2}{2} \frac{\partial \phi_2}{\partial y} + \frac{u_2}{h_2} \left( \frac{u_1}{h_1} - \frac{u_2}{h_2} \right). \]  

This can be written as
\[ \frac{u_1}{h_1} = \frac{1}{\frac{h_1}{h_2}} \left( - \frac{h_2}{2} \frac{\partial \phi_1}{\partial y} - \frac{h_1}{2} \frac{\partial \phi_2}{\partial y} + \frac{u_2}{h_2} \right). \]  

(134)

(132) and (134) are the dynamic boundary conditions.

Summarizing, the flow problem is described by the following set of 6 equations in $x$ and $t$:
\[ \frac{\partial}{\partial x} (h_1 \frac{\partial \phi_1}{\partial x}) = -6u_1 \frac{\partial w}{\partial x} + 6u_1 h_1 \frac{\partial w}{\partial x} + 12u_1 \frac{u_1}{h_1} \]  

(111)

\[ \frac{\partial}{\partial x} (h_2 \frac{\partial \phi_2}{\partial x}) = 6u_2 (w - w_1) + 6u_2 h_2 \frac{\partial w}{\partial x} - 12u_2 \frac{u_2}{h_2} \]  

(112)

\[ \frac{u_1}{h_1} = \frac{\partial h_1}{\partial x} + \frac{\partial \phi_1}{\partial t} \]  

(117)

\[ \phi_1 = \phi_2 + (\rho_1 - \rho_2)gh_1 \]  

(132)

\[ \frac{u_1}{h_1} = \frac{1}{\frac{h_1}{h_2}} \left( - \frac{h_2}{2} \frac{\partial \phi_1}{\partial y} - \frac{h_1}{2} \frac{\partial \phi_2}{\partial y} + \frac{u_2}{h_2} \right). \]  

(134)

\[ h_1 + h_2 = h. \]  

(135)
Here eqs. (111) and (112) follow from eqs. (87) - (92) by using the long wavelength assumption and the assumption that viscous forces are dominant (eqs. (95) and (96)). Eq. (117) is the kinematic boundary condition and Eqs. (132) and (134) are the dynamic boundary conditions simplified by using the assumptions (95) and (97): long wavelength and neglect of surface tension.

From this set of equations a single equation for the variable \( h \) is derived by Ooms, Segal, Cheung and Oliemans (1985). This amplitude evolution equation is a function of the following dimensionless parameters:

\[
C_1 = \frac{u_2}{u_1} \tag{136}
\]

\[
C_2 = \frac{(\rho_1 - \rho_2)gh^2}{(u_1 u_\nu)} \tag{137}
\]

\[
C_3 = h \left[ \phi_1(x,t) - \phi_2(x+L,t) \right] / (u_1 u_\nu) \tag{135}
\]

\[
\tilde{H}_1 = \frac{\tilde{H}_1}{h} \tag{129}
\]

\[
A(0) = \frac{a(0)}{h} \tag{142}
\]

\[
L = \frac{L}{h} \tag{141}
\]

Here \( C_1, C_2 \) and \( C_3 \) are the viscosity ratio, and the effect of gravity and pressure gradient, while \( \tilde{H}_1 \) is the average dimensionless thickness of the lower fluid (fluid 1), \( A(0) \) represents the dimensionless initial amplitude of the wave and \( L \) the dimensionless wavelength.

A simulation is started by assuming the shape of a sine function

\[
\tilde{H}_1(0) = \tilde{H}_1 + A(0) \sin \frac{2\pi x}{L} \tag{142}
\]

where \( X = x/h \), the dimensionless distance.

The equation for the shape of the interface, \( h_1(x,t) \), was solved for a number of values for the above dimensionless parameters.

4.2.2 Application to the 2-inch core flow example

In the abovementioned article the equation for the shape of the interface, \( h_1(x,t) \), was solved for particular values of \( \tilde{H}_1 \), the average dimensionless thickness of the lower fluid and \( L \), the dimensionless wavelength. It was shown that an imposed symmetric wave disturbance develops in time into a stable asymmetric finite amplitude wave, provided the initial amplitude \( A(0) \) exceeds a certain value \( A(0) > 0.07 \). The equilibrium shape of the wave is a function of the viscosity ratio of the fluids \( (C_1) \), the density difference between the fluids \( (C_2) \) and of the applied pressure gradient \( (C_3) \). Values selected for these parameters for the examples presented in the paper are given in Table II.

In that table we have also indicated the parameter values we obtain for the waves at the top of the oil core in the 2-inch core flow situation considered in this thesis for model development. For

<table>
<thead>
<tr>
<th>Parameter</th>
<th>base case in paper</th>
<th>variation in paper</th>
<th>2 inch core flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( \frac{u_2}{u_1} )</td>
<td>10^{-4}</td>
<td>10^{-4}, 10^{-4}, 3.3 \times 10^{-4}</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( \frac{(\rho_1 - \rho_2)gh^2}{(u_1 u_\nu)} )</td>
<td>0, 0.5</td>
<td>-0.8 \times 10^{-3}</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( h \left[ \phi_1(x,t) = \phi_2(x+L,t) \right] / (u_1 u_\nu) )</td>
<td>0, -0.1</td>
<td>8 \times 10^{-3}</td>
</tr>
<tr>
<td>( \tilde{H}_1 )</td>
<td>( \frac{\tilde{H}_1}{h} )</td>
<td>0.8</td>
<td>0.5, 0.68</td>
</tr>
<tr>
<td>( A(0) )</td>
<td>( \frac{a(0)}{h} )</td>
<td>0.1</td>
<td>0.02-0.15, 0.3</td>
</tr>
<tr>
<td>( L )</td>
<td>( \frac{L}{h} )</td>
<td>6</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Here, \( C_1, C_2, C_3 \) are the viscosity ratio, and the effect of gravity and pressure gradient, while \( \tilde{H}_1 \) is the average dimensionless thickness of the lower fluid (fluid 1). \( A(0) \) represents the dimensionless initial amplitude of the wave and \( L \) the dimensionless wavelength.
This example the dimensionless parameters are determined, based on the following: \( u_1 = 0 = 3000 \text{ mPa.s}, \rho_1 = \rho_0 = 975 \text{ kg/m}^3, \rho_2 = \rho_0 = 995 \text{ kg/m}^3, C_m = 0.10, \) giving \( h_0 = 1.7 \text{ mm}, \) \( \nu = \frac{V}{t} = 1 \text{ m/s} \) (the upper fluid being water, the lower oil). According to the photographs the amplitude of the asymmetric wave is 2 mm, so \( \alpha = 1 \text{ mm} \) and its wavelength 18.5 mm. The measured pressure gradient is 385 Pa/m.

It is not at all clear what value to select for \( h, \) the distance between the plates in the long wavelength model applied to the core flow system. Of course, \( h \) must remain very much smaller than the wavelength \( \lambda = 18.5 \text{ mm} \) to comply with (95) and moreover, condition (96) should be met (which for fluid 2, the water film, is not possible as will be shown below). We have, rather arbitrarily, chosen \( h = 2 h_0 = 3.4 \text{ mm}. \) Another point of concern is the value of the thickness of the upper fluid, \( h. \) Here we have considered two cases: an extreme eccentric position at which the oil core is at a distance of only 0.1 mm from the pipe wall \( (h_2 = 0.1 + \alpha = 1.1 \text{ mm}). \) and the concentric situation \( (h_2 = h_0 = 1.7 \text{ mm}). \) The position of the oil core is a result of a balance between lubricating and buoyancy forces: any change in magnitude of a force in vertical direction in the fluids will tend to change the position of the oil core. Consequently, for different oil core positions, there is a complicated interplay between forces deforming the interface and forces moving the oil core away from the pipe wall. One would expect only for large oil viscosities the interface shape to be effectively 'frozen' (as assumed in the lubricating-film model) so that the forces will mainly act upon the oil core position.

When we calculate for the above flow and fluid property values the parameters \( h/\lambda, R_{1,2}, \) and \( S_{1,2} \) (see eqs. (95) to (97)) they all have values smaller than 1 as assumed, except for \( R_2 \) which is very much larger than 1 (it reaches a value of 600). This means that for the flow in the water film (the upper fluid) the assumption that inertia forces are very much smaller than viscous forces is not valid.

Figure 36 Wave shapes predicted by long wave model for concentric core flow situation at different dimensionless time steps.
Figure 36 presents the wave shape development with time for the concentric case. Starting from a sine form wave, the wave reaches an asymmetrically shaped equilibrium form. The front end shows a change in shape with a flattened part close to the pipe wall. Contrary to the photographic information and Hooper & Grimshaw’s (1985) prediction, the wave that is formed has a long front face and a steep tail. The asymmetry parameter is 0.7, the wave amplitude 1.7 mm.

For the eccentric core situation with a starting distance to the pipe wall of 1.1 mm (instead of 1.7 mm) the model predicts that due to the proximity of the wall the wave form is modified: the flattening of the front-end part is more pronounced (see Figure 37), the wave amplitude is further reduced ($a = 1.6$ mm) and the asymmetry parameter is larger ($L' = 0.9$).

 Apparently the simple long wavelength model for flow between flat plates is capable of predicting the wave form that corresponds to the photographs (flattening) but not the location of the wave crest ($L' = 0.7 - 0.9$ instead of $L' = 0.6$ or smaller). Points for further study are the effect of turbulence and inertia in the upper fluid on wave formation. Moreover, the uncertainty in $h$-selection referred to above has to be resolved. When for the distance between the plates a smaller value is chosen ($h = 1.5 h_0$), the predictions for $a$ and $L'$ are not affected, but the wave form in the concentric core case more closely resembles that of the previous eccentric situation. A possible route in studying the effect of the thickness of the oil layer could be to apply a modification of Hooper’s (1985) method for a bounded and unbounded two-fluid system (in his approach the lower fluid is bounded and more viscous than the unbounded fluid).

4.3 Core flow calculations using measured wave data

4.3.1 Ultrasonic measurements of interfacial waves

Experimentally it is difficult to obtain all information for waves at an oil/water interface that is needed in the lubricating-film model for core flow in horizontal pipes. Attempts have been made to collect some information on the interfacial waves by using an ultrasonic wave detection device. At five different locations around the 2-inch pipe ultrasonic signals reflected by the oil core are detected. In that manner the variation of the water film thickness around the pipe can be determined (Wu and Duijvestijn (1985)). Unfortunately, the instrument only detects distances to the crests and troughs of the waves, so that no information is obtained on the form of the wave. Still, the measurements make it possible to determine the distance of the oil core to the top of the pipe, wave amplitude, asymmetry and wavelength, and the water film hold-up (which was previously used in Section 3.3). The measurements, in particular those for the wave data suffer from serious scatter. The ultrasonic wave tests have been made for three oil viscosities of about 1500, 4000 and 8000 mPa.s with densities of ca. 960 kg/m$^3$ (i.e. smaller than
in the 3000 mPa.s tests). In the 2-inch test facility the same range of oil velocities and input water fractions was used as considered in the 3000 mPa.s tests (see Section 3.2.1). The results for wave amplitudes, wavelengths and minimum distance to the top of the pipe as measured by Wu & Duijvestijn (1985) for an input water fraction of \( C_w = 0.11 \) are displayed in Figure 38 (results for the other input water fractions are similar). The measurements indicate that only the wavelengths vary with oil viscosity. As a function of superficial oil velocity, the three parameters plotted in Figure 38 roughly show a similar behaviour, at least up to a velocity of 2 m/s. In particular, the reduced values for a superficial oil velocity of 0.5 m/s are noteworthy. Wu & Duijvestijn (1985) show that the measured wave amplitudes and wavelengths for all values of \( V_{bo} \) and \( C_w \) are proportional to the measured local mean water layer thickness. The wavelength data for the three oil viscosities scatter around three parallel lines, when plotted as a function of mean water layer thickness, whereby the wavelength is smaller for less viscous oil cores.

Of importance for the lubricating-film model is to consider the data for the wave amplitude, wavelength and asymmetry parameter separately as a function of input parameters instead of the mean water film thickness, which is an output parameter of that model.

Measured wave amplitudes increase for increasing water fraction, are only slightly dependent on oil viscosity and virtually independent of oil velocity (at least for velocities larger than 0.75 m/s). Measured wave amplitudes plotted in Figure 39 are compared with water film thickness values \( h_0 \) for a concentric oil core calculated according to:

\[
h_0 = R \times (1 - \sqrt{1 - H_w})
\]  

(66)

where \( R \) is the pipe radius in mm, and \( H_w \) is the measured water hold-up. As shown in Figure 39 the measured wave amplitudes correlate quite well with this film thickness and can as a first approximation be calculated by:

\[
a = 1.1 \times h_0
\]  

(67)

The observed wavelengths only very slightly decrease with oil velocity, as opposed to the relationship derived from the photographs (Figure 21).
Wavelengths increase with input water fraction ($C_w$) and oil viscosity as shown in Figure 40, in which the data measured at a superficial oil velocity of 1 m/s have been plotted. The experimental point for the 1500 cSt test with a water fraction of 0.04 is suspect, since it lies close to the lower detection limit of 10 mm of the ultrasonic measuring device (note that extrapolation of the other data at this viscosity would provide a wavelength of about 7-8 mm for $C_w = 0.04$). To obtain a better insight into the oil viscosity dependence of the wavelengths we have replotted the data in Figure 41. Although the wavelengths increase with increasing oil viscosity, the curves in Figure 41 suggest that at higher oil viscosities the wave lengths may become independent of the oil viscosity. This is a point for further study.

Finally, the ultrasonic wave data confirmed the asymmetry already observed in the photographs and provided values for the asymmetry parameter (maximum wave height location) ranging from 0.38 to 0.51. The waves tend to become more asymmetrical (lower parameter values) for increasing superficial oil velocity. No clear dependence on either input water fraction or oil viscosity could be discerned. It should be admitted that exact wave shapes are difficult to detect with the ultrasonic equipment due to signal losses by scattering on curved interfaces.
4.3.2 Predictions of core flow pressure gradient and core eccentricity

For an input water fraction of 0.10 the measured wave amplitudes as given in Figure 38 and 39 have an average value of 1.6 mm and a maximum of 1.75 mm. Measured wavelengths for a 3000 mPa.s oil at this input water fraction may be estimated from Figures 39 and 40 to have a value of about 23 mm. A reasonable estimate for the asymmetry parameter seems to be $L' = 0.5$. These wave data have been used in the lubricating-film model for the curved wave form shown in Figure 42. (Note that the wave crest is located such that ultrasonic measurements would give an asymmetry parameter variation of $L' = 0.4$ to $L' = 0.5$.) As indicated in Figure 43 for the case of a superficial oil velocity of 1.95 m/s the dimensionless pressure now reaches higher values than for the $L' = 0.6$ case with somewhat different $a, b$ values (Figure 32). Nevertheless, a similar good fit of the pressure gradients is obtained as shown in Figure 44. Compared to the previous computer simulations the oil core is now predicted to be in a slightly more eccentric position (compare Figures 45 and 34). At the lowest oil velocity investigated here (0.3 m/s) the oil core is now only a distance of 0.09 mm away from the pipe wall.

In view of the dependence of both the wave amplitude and the wavelength on the input water fraction (Figures 39, 40) it is of interest to perform a series of calculations with the lubricating-film model for a fixed oil velocity and a range of input water fractions and compare calculated and measured pressure gradients. For a superficial oil velocity of 0.93 m/s calculations were performed for input water fractions ranging from 0.05 to 0.20. Wave amplitudes and wavelengths estimated from Figures 38, 41 for input water fractions of 0.05, 0.10,
Figure 43: Distribution of dimensionless pressure over a wavelength with shape shown in Figure 42 (symbols and flow parameters as in Figure 31).

\[ \Delta p = 20 \text{ kg/m}^3 \]
\[ \mu_o = 3000 \text{ mPa.s} \]
\[ C_w = 0.10 \quad V_{so} = 95 \text{ m/s} \]
\[ a = 16 \text{ mm} \quad L = 23 \text{ mm} \quad L' = 0.5 \]

Table III
Input wave data as a function of input water fraction at \( V_{so} = 0.93 \text{ m/s} \)

<table>
<thead>
<tr>
<th>( C_w )</th>
<th>( a )</th>
<th>( L )</th>
<th>( L' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.8</td>
<td>15</td>
<td>0.5</td>
</tr>
<tr>
<td>0.10</td>
<td>1.6</td>
<td>23</td>
<td>0.5</td>
</tr>
<tr>
<td>0.15</td>
<td>2.25</td>
<td>28</td>
<td>0.5</td>
</tr>
<tr>
<td>0.20</td>
<td>3.1</td>
<td>30</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 44: Measured and calculated pressure gradient in 2" loop for 3000 cP oil with a water fraction of 0.10

Figure 45: Calculated relative eccentricity and distance to top of 2" pipe for oil core using ultrasonic wave data and shape from photographs.
0.15 and 0.20 are shown in Table III. Figure 46 shows that the calculated pressure gradients do not correspond very well with the measurements. While measured pressure gradients hardly vary with input water fraction, the calculated data show a distinct increase with input water fraction. The calculations indicate a rather gradual increase of the core eccentricity with input water fraction from 0.70 to about 0.76; the distance to the top of the pipe increases quite strongly with input water fraction (see Figure 47) from 0.15 mm to about 0.4 mm. The reasons for the disagreement between model and measurement shown in Figure 46 are not well understood. Of course the wave data are those for oils with a different density difference. To establish the correctness of the values for wave amplitudes and lengths and their dependence on \( C_w \), a critical examination of the wave data with the help of a proper interfacial wave model is necessary. It should be noted that when constant \( a \) and \( \lambda \) values are used the dependence of pressure gradient on \( C_w \) is correctly predicted as with the earlier version of the lubricating film model (Figure 19).

Thus far the only check on the prediction ability of the lubricating-film model was to compare calculated results with measured pressure gradients. However, the ultrasonic measurements have provided data on another output parameter of the model: the core eccentricity expressed as the distance of the oil core to the top of the pipe. Figure 38 shows that for the 0.75-1.5 m/s velocity range the minimum distance to the top of the pipe as measured by the ultrasonic technique has a value of nearly 0.8 mm. Bearing in mind that the measured wave amplitude at these conditions is about 1.6 mm and the water film thickness for concentric flow is 1.7 mm, the measurements indicate the oil core to be in a nearly concentric position. Clearly this is not what the lubricating-film model predicts: Figure 45 shows that the distance of the oil core to the top of the pipe, \( h_{\text{top}} \) as calculated with the model varies from about 0.2 mm at 0.75 m/s to a value of 0.35 mm at a velocity of 1.5 m/s i.e. far smaller than the measured distance. Even at a velocity as high as...
1.95 m/s the calculated distance still is only 0.44 mm, while in practice it reaches a value of 1.1 mm. At a velocity of 0.5 m/s the measured distance is smaller than for the intermediate velocity range: 0.7 mm and even reaches a value as low as 0.3 mm for the 8000 mPa.s oil. Calculations indicate here a distance of 0.13 mm for the 3000 mPa.s oil. The disagreement between calculated and measured core to wall distances (or eccentricities) is the more disturbing if one realizes that wave measurements are for an oil with a larger buoyancy ($\Delta \rho = 40$ kg/m$^3$) than the oil for which the core flow calculations have been performed ($\Delta \rho = 20$ kg/m$^3$).

The key to the solution of the problem of the mismatch between measured and calculated core eccentricities may be found by looking at the values of the group $R_1 = \left(\frac{\rho W h}{u}\right) \left(\frac{a}{h}\right)^2$ for the range of oil velocities from 0.3 m/s to 1.95 m/s. It turns out that $R_1$ for this velocity range varies from a value of 3 for 0.3 m/s to a value as high as 44 for an oil velocity of 1.95 m/s (see Table IV). Consequently for the determination of the velocity profiles the assumption of neglecting inertia effects is not justified. On the other hand for the lubrication effects and the corresponding pressure gradient not $R_1$ is the most important parameter but $R_1' = \left(\frac{\rho W h}{5}\right) \left(\frac{\partial h}{\partial x}\right)_t$, in which $\partial h/\partial x$, determined at the flat front-end part of the wave (see Figure 42), represents the tangent of the angle between the two lubricating surfaces (Batchelor (1967)). For the wave profile used in our calculations (Figure 42) $(\partial h/\partial x)_t = 0.002$. In that case the group $R_1' = \left(\frac{\rho W h}{5}\right) \left(\frac{\partial h}{\partial x}\right)_t$ is less than 1 for oil velocities up to 0.75 m/s and somewhat greater than 1 (in the range 1.3 to 1.5) for the higher velocities considered (up to 1.95 m/s). If indeed the main effect of the introduction of inertia is an increase in load capacity resulting in a less eccentric oil core position, the above could explain the observed minimum distances shown in Figure 38. In Appendix B an approximate way of incorporating at least the largest inertia terms in our model equations qualitatively supports the load capacity increase mentioned in the literature.

---

**Table IV**

Data relevant for inertia effects in 2-in pipe ($Re = \rho Wh/5$)

<table>
<thead>
<tr>
<th>$V_{SO}$ (m/s)</th>
<th>0.29</th>
<th>0.5</th>
<th>0.75</th>
<th>0.93</th>
<th>1.25</th>
<th>1.5</th>
<th>1.75</th>
<th>1.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1 = Re \times (a/L)$</td>
<td>3</td>
<td>12</td>
<td>39</td>
<td>53</td>
<td>54</td>
<td>51</td>
<td>49</td>
<td>44</td>
</tr>
<tr>
<td>$R_1' = Re \times (\partial h/\partial x)_t$</td>
<td>0.08</td>
<td>0.3</td>
<td>1.1</td>
<td>1.5</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td></td>
</tr>
</tbody>
</table>
Without resolving the question of how to compute the interfacial wave geometry we are not in a good position to use the lubricating-film model for larger pipe sizes. As discussed in Section 4.3.1 the ultrasonic measurements in the 2-inch test facility indicate the wave amplitudes and wavelengths at the top of the pipe to be proportional to the local mean water layer thickness, \( h_{top} \) (Wu & Duijvestijn (1985)). For increasing pipe size on one hand the water film thickness for a concentric oil core (\( h_0 \)) will increase (proportional to \( R \)), on the other the core eccentricity will increase due to an increase in buoyancy (the dimensionless density difference being proportional to \( R^2 \), ref. eq. (58) and e.g. Figure 10a). The expected net result will be that the distance of the oil core to the pipe top, \( h_{top} \), will either remain the same or become smaller than that for the 2-inch situation as the pipe diameter becomes larger.

For lack of a useful wave formation model, we will have to await the outcome of measurements for waves and film thicknesses in larger pipes to establish the proper scaling rules. Pending such experiments we will here investigate the scaling potential of the lubricating-film model by using the wave data for the 2-inch pipe as used in Sections 4.1 (from photographs) and 4.3 (from ultrasonic measurements) i.e. ignore any change in wave data with pipe size or tacitly assume they are mainly determined by oil viscosity. For the wave form we use the curved shape introduced in Section 4.1 instead of the initial saw-tooth interface form.

In the study of the diameter effect we will present the results of the calculations with the lubricating-film model by plotting the apparent viscosity next to the core flow pressure gradient. The apparent viscosity (see Oliemans & Ooms (1985)) is obtained from the dimensionless pressure gradient \( \Delta \rho/L \) as:

\[
\mu_{app} = \frac{3 \rho v}{4 L} \frac{\Delta \rho}{1-H}
\]

(it can also be found by multiplying the pressure gradient ratio (eq. (62)) by the oil viscosity). In this equation, as in the lubricating-film model, the water film hold-up, \( H_0 \), is calculated according to eq. (84), which for some flow or oil viscosity conditions may give too large a value (see Figure 27) leading to a conservative estimate for \( \mu_{app} \); this effect will to some extent be compensated by too small a value for the pressure gradient \( \Delta \rho/L \) in eq. (143).

First of all the photographic wave information was used to assess both with the turbulent and the laminar version of the lubricating-film model the effect of an increase in pipe size on calculated apparent viscosity and pressure gradient. Calculations were performed for the 3000 mPa.s oil, that has been used as our test case throughout this thesis. The turbulent-lubricating-film model with the photographic wave data very well predicted the core flow pressure gradient as a function of superficial oil velocity in the 2-inch pipe (Figure 33). As shown earlier the laminar version of the lubricating-film model severely underpredicted the pressure gradients, when the observed wave data are inserted in the model (Figure 24). The results of the diameter scaling at a superficial oil velocity of 2 m/s for the two versions of the model are displayed in Figure 48. There remains a large difference between the laminar and turbulent versions of the model for the full range of pipe sizes from 2 to 16 inches. The apparent viscosity for the core flow mode increases with pipe size from 60 mPa.s for a 2-inch pipe to nearly 500 mPa.s for a 16-inch pipe (as computed by the turbulent-lubricating film model). The corresponding pressure gradients decrease from 1600 Pa/m to 200 Pa/m. This illustrates that the core flow technique for viscous crude oils becomes less attractive for larger pipes (for a long distance pipeline a value of 100 Pa/m = 1 bar/km would be acceptable). The decrease of \( dp/dx \) as a linear function of pipe diameter \( D \) can be understood when we realize that in the apparent viscosity concept (with laminar flow) \( dp/dx \approx 12 \mu_{app} v_{avg} D^2 \) and \( \mu_{app} \) increases linearly with \( D \).
As a second step in the assessment of the scaling potential of the turbulent-lubricating film model we have repeated the same calculations, but now with wave amplitude, wavelength and asymmetry taken from the ultrasonic data. The most important difference being a shift in asymmetry from \( L' = 0.6 \) to \( L' = 0.5 \) (while the amplitude 'a' changes from 2 mm to 1.6 mm and the wavelength 'L' from 18.5 mm to 23 mm). As expected the results shown in Figure 50 are quite similar to the previous ones. The decrease in pressure gradient (and apparent viscosity) due to the smaller amplitude and larger wavelength is slightly overruled by the increase caused by a more asymmetric wave (\( L' = 0.5 \) instead of \( L' = 0.6 \)).

For a 16-inch pipe the resulting apparent viscosity is now nearly 550 mPa.s instead of 500 mPa.s. As shown in Figure 51 the larger pressure gradients (and apparent viscosities) are a consequence of the more eccentric position of the oil core in the pipe. In the 16-inch pipe the minimum film thickness is now only 0.1 mm i.e. nearly two times smaller than when the photographic wave data are used.
A check for the ability to make reasonable predictions - using the parameter values for the interfacial waves determined by the measurements in the 2-inch pipe - for pipe sizes other than 2-inches is a crucial test of the model. To verify this we have repeated calculations for different pipe sizes with density differences of 30 kg/m$^3$ and 45 kg/m$^3$ and a water fraction of 0.06 corresponding to those of the measurements in 2-in and 8-in pipes presented earlier (Figure 17 of Section 3.1.3). The results for a superficial oil velocity of 1 m/s shown in Figure 52 indicate that the experimental data (expressed as an apparent viscosity) are quite well predicted. The figure also contains experimental data points for a 6-in line published by Clark & Shapiro (1949) and used by Russell and Charles (1959) for comparison with predictions of their concentric flow model. Apparent viscosities have been computed from the experimental pressure gradient reduction factors, and an average oil viscosity of 900 mPa.s as quoted by them. Density difference and velocity data have been worked out from the original patent description by Clark and Shapiro (1949). The 8-in test result that is not well predicted and is also significantly higher than the other two 8-in test results relates to a test with a 1200 mPa.s oil that was operated at the borderline between stable and unstable core flow according to the empirical flow pattern map (Figure 16). Note that for a 45 kg/m$^3$ density difference operation at an oil velocity of 1 m/s and $C_w = 0.06$ in a 16-in pipe in the core flow mode occurs at an apparent viscosity of 450 mPa.s, which is considerably larger than predicted by the first version of the model (Oliemans and Ooms (1985)).

Finally, we have verified for the case with a density difference of 30 kg/m$^3$ and a superficial oil velocity of 1 m/s whether or not the group $(pWh/G), (sH/3x)$ is less than 1. At the top of the pipe the values
for this group range from 0.21 for the 2-in pipe to a value as low as 0.04 for the 16-in pipe (see Table V). It suggests that the assumption of neglecting inertia effects is better justified the larger the pipe (at least in that part of the pipe where lubricating forces are largest).

Figure 52 Calculated and measured apparent viscosities.

TABLE V

Data relevant for inertia effects
(a = 1.0 mm, \( \delta = 14 \) mm, hence \( a/\delta = 0.07 \))

<table>
<thead>
<tr>
<th>Diameter (in)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re. (a/\theta)</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>Re. (\delta h/\delta x)_{top}</td>
<td>0.2</td>
<td>0.13</td>
<td>0.10</td>
<td>0.08</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Chapter 6

DISCUSSION AND CONCLUSIONS

A number of models have been suggested to describe the phenomenon of core-annular flow in pipes. In particular, the eccentric laminar-laminar model of Bentwich has greatly enhanced the understanding of the qualitative effects of various parameters: viscosity ratio, core/pipe diameter ratio (or hold-up), and core eccentricity. Unfortunately, this model, as indeed all models that use a smooth core, greatly exaggerates the benefits of operation in the core flow mode: the predicted pressure loss ratios are extremely low, or alternatively, the pressure gradient reduction factors are very large. More realistic pressure loss ratios can be predicted by the lubricating-film model, in which the ripples at the interface play an essential role in the very existence of the core flow. For known values of density and velocity of the oil core, the model gives the eccentricity in the pipe at which buoyancy forces are counterbalanced by the lubrication force. With this eccentricity the pressure gradient is determined.

Key parameters in the lubricating-film model are the interfacial wave characteristics (amplitude, length, asymmetry and shape), the flow in the water annulus (laminar, turbulent with/without inertial effects) and the water film hold-up. For the laminar version of the model the sensitivity of pressure gradient predictions to changes in wave geometry and water hold-up were discussed in Section 3.1. Ripple parameter values were estimated from photographs and pressure losses found in core flow tests with very viscous oil in a 2-in pipe. Because water-oil hold-up data for core flow were scanty, a simple correlation had to be used.

Since the formulation of the initial version of the model additional dedicated measurements have become available, that can be used to critically examine the model assumptions and methods of determining the above mentioned three parameters. Of these, the flow in the water annulus and the calculation method for the water hold-up have been treated in the remainder of chapter 3, while chapter 4 is completely devoted to the wave characteristics. Model implications of changes in parameter values or model assumptions are assessed by using a set of core flow measurements for a 3000 mPa.s oil at a fixed input water fraction ($C_w = 0.10$) and a range of oil velocities as a reference. Two sets of experimental results in the 2-in test facility were used to derive model parameter values: a set of core flow measurements with the 3000 mPa.s oil during which a series of photographs were taken for wave geometry and hold-up determination and a number of additional tests with, unfortunately, a different oil ($\delta p = 40$ kg/m$^3$ and $\mu_0 = 1500-8000$ mPa.s) during which the same parameters were determined using dedicated instrumentation.

The first series of measurements in the 2-in pipe (for the oil with $\delta p = 20$ kg/m$^3$ and $\mu_0 = 3000$ mPa.s) have led to the following set of conclusions:

(i) The flow in the water annulus is turbulent rather than laminar.

(ii) The wave amplitudes derived from photographs are of the same order of magnitude as the water layer thickness i.e. much larger than the value of 0.08 mm originally taken as default in the model.

(iii) The wavelengths vary with water fraction and oil velocity.

(iv) The waveshape is curved, most likely with a flattened part close to the pipe wall in stead of having a saw-tooth shape.

(v) The wave asymmetry parameter is about 0.6 in stead of 0.8.

(vi) Observed water hold-up values are much smaller than those obtained with the correlation based on literature data (i.e. there is less slip between the fluids).

Based on these conclusions a turbulent version of the lubricating-film model was formulated. Using the wave data and the water hold-up observed during the tests the model slightly underpredicts the increase of core flow pressure gradient with oil velocity (for a fixed input water fraction $C_w = 0.10$), although the trend is correctly reproduced (Figure 24). Replacement of the water hold-up by the still smaller values that were
derived from ultrasonic measurements (in the second series of tests) and using a curved wave shape with asymmetry \( L' = 0.6 \) completely removes the pressure gradient underprediction by the model (Figure 33).

More detailed information on wave characteristics became available during the second series of measurements in the 2-in pipe (using an oil with \( \Delta \rho = 40 \text{ kg/m}^3 \) and \( \nu_0 = 1500-8000 \text{ mPa.s} \)), thanks to the use of dedicated instrumentation. With respect to waves on the oil/water interface, a preliminary analysis of the data revealed the following:

(i) Wave amplitudes and wavelengths vary proportional to the measured average water layer thickness; they are increasing functions of \( \omega_w \) and, over a broad oil velocity range, not depending on oil velocity (in contrast to the photographic information on wavelengths).

(ii) The wave asymmetry parameter values vary from 0.4 to 0.5, without a clear dependence on fluid or flow parameters.

For the reference set of test conditions (the pressure gradients for the 3000 mPa.s oil with \( \Delta \rho = 20 \text{ kg/m}^3 \), \( \omega_w = 0.10 \) and \( V_{so} = 0.3-1.95 \text{ m/s} \)) the wave amplitude and length derived from the ultrasonic measurements (ignoring possible effects due to a difference in buoyancy) are 1.6 mm and 23 mm, respectively. When a value of 0.5 is chosen for the asymmetry parameter and the curved wave shape derived from the photographs is used, pressure gradients are predicted by the model with the same degree of accuracy as before.

The second set of detailed experiments also provided wave data as function of the input water fraction (\( \omega_w \)) and data on the core eccentricity, which enable additional model checks. First, the model predictions with regard to pressure gradient as a function of \( \omega_w \) (for a fixed value of \( V_{so} \)) was addressed. It turned out that the model results, using the observed wave amplitude and length relationships with \( \omega_w \), disagree with measured pressure gradients: calculated pressure gradients increase with \( \omega_w \) while the measured data lie on a shallow curve with a minimum at \( \omega_w = 0.12 \). It is not clear whether this disagreement is due to an artefact of the measurements (the preliminary data show large standard deviations) or of the model. Secondly, the calculated core eccentricities were compared with measurements. Again a serious disagreement between model predictions and experiments came to light: the oil core is predicted to be in a relatively eccentric position, while the measurements indicate a near-concentric situation. It is expected that this disagreement may be resolved by introducing inertial effects in the model, which according to the literature on lubrication theory will mainly increase the load capacity i.e. shift the oil core to a more concentric position. The necessity of extending the turbulent lubricating-film model with inertial effects becomes apparent if one realizes that for the reference set of conditions the group \( (p\omega_w/\nu).C (h/\delta) \) is no longer less than 1 for oil velocities larger than 0.75 m/s (for velocities up to \( V_{so} = 1.95 \text{ m/s} \) the group has values in the range 1.3 to 1.5, while at 0.3 m/s its value is 0.08 i.e. much smaller than 1).

Wave amplitude, shape and asymmetry were investigated by applying a simple interfacial wave model for two viscous fluids between parallel plates, using the long wavelength approximation. Applied to the core flow situation at the top of the pipe calculations with this model give a waveshape and amplitude that agree with measurements, but an asymmetry parameter value that is larger than observed. These results should be treated with care, since they depend to some extent on the distance between the plates, which, for the core flow application, is difficult to define. The model results further indicate that the wave amplitude becomes smaller, the wave shape more flattened and the asymmetry parameter larger as the oil core approaches the pipe wall (or, in terms of the parallel plate model, the thickness of the upper fluid (water) becomes smaller). The reduction in wave amplitude with reducing water layer thickness is in line with the measurements. In the current version of the lubricating-film model changes in wave geometry as the core approaches the pipe wall are completely ignored. In practice, forces acting upon the oil core may partly deform the interfacial wave, partly push the core to a certain (eccentric) position. One would expect for large oil viscosities the assumption of a 'frozen' wave to be better justified. However, this has to be further investigated. Finally, necessary extensions of the interfacial wave parallel plate model are
the introduction of turbulence and inertial effects in the water layer (according to Hooper (1985) the latter may contribute to a short wave/long tail configuration i.e. a form more in line with observations). Ultimately, quite a different interfacial wave model for water/oil is required in which no longer the long wavelength approximation is made, so that also wavelengths can be computed as function of flow and fluid parameters.

As long as a proper interfacial wave model is lacking and the core eccentricity is not correctly predicted, scaling to larger pipe sizes remains problematic (certainly if one realizes the shape to depend on the distance to the pipe wall). Nevertheless, some attempt was made to gain insight into the potential for core flow in large size pipelines by performing calculations with the turbulent-lubricating-film model using wave parameter values from the 2-inch conditions as input. The rationale is that the fluid properties are the same, while the flow conditions are such that water layer thicknesses at the top of the pipe are not very different from those encountered in the 2-inch reference case. That, due to an increasing buoyancy, the core flow technique for larger size pipes becomes less attractive is illustrated by calculating the apparent viscosities for a range of pipe diameters using the data from the 2-inch reference case at an oil velocity of 2 m/s ($\mu_0 = 3000 \text{ mPa.s}, \Delta \rho = 20 \text{ kg/m}^3, C_W = 0.10$, and for the interfacial wave: $a = 1.6 \text{ mm}, \epsilon = 23 \text{ mm}, L' = 0.5$; ref. Figure 48). The apparent viscosity for core flow transportation increases from a value of 55 mPa.s for the 2-inch pipe to a value of 550 mPa.s for a 16-inch pipe. According to the current version of the model this diameter increase reduces the water layer thickness at the top of the pipe from 0.45 mm to 0.10 mm. At the flat wave top the group $\left(\frac{\pi h}{U}\right)\left(\frac{\Omega h}{2x}\right)$ is less than 1 for all pipe sizes, justifying the neglect of inertia there. The limited set of large diameter core flow data that is available (only pressure gradient data for an oil velocity of about 1 m/s, no information on waves or core eccentricity) are reasonably well predicted by the lubricating-film model (ref. Figure 52). A more extensive set of large diameter data is required (on pressure gradient, wave data and core eccentricity) to establish the reliability of model predictions.

---

LIST OF SYMBOLS

- $a$ wave amplitude (m)
- $A$ dimensionless wave amplitude ($= a/R$) (-); van Driest constant in eq. (74)
- $A_{0,W}$ part of cross-sectional area occupied by oil-water ($m^2$)
- $A_F$ cross-sectional area of pipe ($m^2$)
- $C_{0,W}$ oil-water input, as volume fraction (-)
- $D_{1,2,3}$ dimensionless parameters of long wavelength model
- $e$ eccentricity (m)
- $E$ dimensionless eccentricity (-)
- $f$ friction factor (-)
- $g$ acceleration due to gravity ($m/s^2$)
- $h$ film thickness (m)
- $h_0$ film thickness for concentric flow (m)
- $H_{H_0}$ dimensionless film thickness for eccentric-concentric flow (-)
- $H_1$ dimensionless height
- $H_{0,W}$ oil-water hold-up (-)
- $k$ core/pipe diameter ratio (-); turb. kinetic energy per unit mass ($J/kg$)
- $L$ wavelength (m)
- $l_1$ mixing length (m)
- $l''$ location of maximum wave height (m)
- $L$ dimensionless wavelength ($= 1/R$) (-)
- $L'$ dimensionless location of maximum wavelength (-)
- $n_{1,2}$ unit normals on interface
- $P$ pressure ($N/m^2$)
- $r$ radial coordinate (m)
- $R$ pipe radius (m)
- $R_c$ core radius (m)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>Reynolds number (-)</td>
</tr>
<tr>
<td>(Re^\kappa)</td>
<td>Reynolds number related to (u^\kappa) (-)</td>
</tr>
<tr>
<td>(S)</td>
<td>stratification parameter (-)</td>
</tr>
<tr>
<td>(T)</td>
<td>surface tension (N/m)</td>
</tr>
<tr>
<td>(u, v, w)</td>
<td>velocity components of water in y, (\delta), x direction (m/s)</td>
</tr>
<tr>
<td>(V_{\text{HC}})</td>
<td>water-core volume (m³)</td>
</tr>
<tr>
<td>(V_{\text{SO}})</td>
<td>superficial oil velocity, m/s</td>
</tr>
<tr>
<td>(W)</td>
<td>core velocity (m/s)</td>
</tr>
<tr>
<td>(u^*)</td>
<td>friction velocity (m/s)</td>
</tr>
<tr>
<td>(x)</td>
<td>axial coordinate</td>
</tr>
<tr>
<td>(y)</td>
<td>distance from pipe wall (- (R - r)) (m)</td>
</tr>
<tr>
<td>(y^+)</td>
<td>dimensionless distance from the wall (-)</td>
</tr>
<tr>
<td>(X)</td>
<td>dimensionless axial coordinate (= (x/R)) (-)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Greek Letters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>length scale parameter in force balance defined in eq. (78) (m)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>fluids intersection angle (-); parameter defined in eq. (80) (m²/ (Pa.s))</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>contact angle (-); surface tension (N/m)</td>
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<tr>
<td>(\theta)</td>
<td>azimuthal coordinate</td>
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<tr>
<td>(\kappa)</td>
<td>Von Kármán constant</td>
</tr>
<tr>
<td>(\mu)</td>
<td>viscosity (Pa.s)</td>
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<tr>
<td>(\mu_m)</td>
<td>molecular dynamic viscosity (Pa.s)</td>
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<tr>
<td>(\mu_t)</td>
<td>turbulent dynamic viscosity (Pa.s)</td>
</tr>
<tr>
<td>(\mu^*)</td>
<td>(u_\tau + \mu_t = \text{total fluid viscosity, Pa.s})</td>
</tr>
<tr>
<td>(\bar{\mu})</td>
<td>total viscosity, averaged over radial distance (y), Pa.s</td>
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<tr>
<td>(\nu_t)</td>
<td>turbulent kinematic viscosity, (m^2/s)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density, kg/m³</td>
</tr>
<tr>
<td>(\rho^*)</td>
<td>dimensionless density, (-)</td>
</tr>
<tr>
<td>(\rho_C)</td>
<td>density of oil core, kg/m³</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>energy per unit area, J/m²</td>
</tr>
<tr>
<td>(\phi)</td>
<td>pressure variable, Pa</td>
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<tr>
<td>(\phi^*)</td>
<td>dimensionless pressure (-)</td>
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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>C</td>
<td>core</td>
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<tr>
<td>(i)</td>
<td>interfacial</td>
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<tr>
<td>M</td>
<td>mixture</td>
</tr>
<tr>
<td>OW</td>
<td>oil-water</td>
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<tr>
<td>O</td>
<td>oil</td>
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<tr>
<td>P</td>
<td>pipe</td>
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<tr>
<td>SO</td>
<td>superficial oil</td>
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<td>SW</td>
<td>superficial water</td>
</tr>
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<td>water</td>
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<tr>
<td>1, 2</td>
<td>lower/upper fluid in long wavelength model</td>
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Appendix A

SOLUTION PROCEDURE FOR EQUATIONS IN DIMENSIONLESS FORM

Dimensionless variables $X = x/R$, $H = h/R$, $L = E/R$, $\phi = \phi R/6u$, $\delta = gR^2/6u$, and $p = gR^2/6y$ are introduced. Equation (48) becomes, in dimensionless form,

$$
\frac{3}{6} \left( H^3 \frac{3}{3H} \frac{3}{3X} \right) + \frac{3}{6} \left( \frac{3}{3X} \frac{3}{3X} \frac{3}{3X} \right) = 3H
$$

(A-1)

while the boundary conditions become

for $\phi = 0$: $\frac{3}{6} = 0$

(A-2)

for $\phi = \pi$: $\frac{3}{6} = 0$

(A-3)

for $X = 0$: $\phi = \phi_1$

(A-4)

for $X = nL$: $\phi = \phi_2$

(A-5)

in which $n$ is an integer and $(\phi_2 - \phi_1)/nL$ is the pressure gradient over the pipe. The objective of considering a number of wavelengths $n$ is to find the periodic solution. (An easier alternative is to impose periodicity and to compute over one wavelength only.) Near-periodic conditions are attained for $n$ equal to at least 7.

On introducing the dimensionless variables and dividing by $nL$, the equation for the dimensionless density can be written as

$$
\frac{2}{nL} \int_0^L \phi \cos \theta \ d\theta \ dx - \frac{1}{nL} \int_0^L H \frac{3}{3H} \sin \theta \ d\theta \ dx = \delta - \delta_C
$$

(A-6)

Originally the solution procedure was as follows. First $H(0, X)$ and a certain value for $(\phi_2 - \phi_1)$ were chosen. For $H$ the sawtooth function of Figure 9 defined by Eqs. (11)-(14) was chosen. Then Eq. (A-1), with the boundary conditions in Eqs. (A-2)-(A-5), was solved. The calculated value of $H$ was then substituted in Eq. (A-7), and it was usually found that this condition was not satisfied. So a new value for $(\phi_2 - \phi_1)$ was chosen and the calculation repeated; this iteration was continued until Eq. (A-7) was satisfied. Substitution of the final solution for $H$ in Eq. (A-6) yielded the dimensionless density difference $(\delta - \delta_C)$ between annulus and core, which could be counterbalanced.

To avoid the iteration, the following solution procedure was adopted. Without loss of generality, take $\phi_1 = 0$. Let $\phi^C$ be the solution of Eq. (A-1) and Eqs. (A-2)-(A-5) with $\phi_2 = 1$ and the right-hand side of Eq. (A-1) replaced by zero. Let $\phi^C$ be the solution of Eq. (A-1) and Eqs. (A-2)-(A-5) with $\phi_2 = 0$. The general solution then is $\phi = \phi^C + \phi^0$. Let the first and second terms on the left side of Eq. (A-7) be indicated as $D_1$ and $D_2$, i.e.,

$$
D_1 = \frac{1}{nL} \int_0^L \frac{1}{3H} \sin \theta \ d\theta \ dx
$$

(A-8)

$$
D_2 = \frac{1}{3nL} \int_0^L \frac{1}{3H} \sin \theta \ d\theta \ dx
$$

(A-9)

$D_1$ is linear in $\phi$ and $D_2$ is independent of $\phi$. Let $D^C = D_1(\phi^C) + D_2$. Then Eq. (A-7) becomes

$$
\phi_2 D_1(\phi^C) + D^C = \frac{\delta_2 - \delta_1}{nL}
$$

(A-10)
Hence the value of $\phi_2$ is given by

$$\phi_2 = \frac{D^C}{nL} - D^U(\phi^U)$$  \hspace{1cm} (A-11)

This solution procedure requires only two calculations; first, for $\phi^C$, from which $D^C$ is calculated, and then for $\phi^U$, from which $D^U(\phi^U)$ follows. $\phi_2$ can then be calculated from Eq. (A-11), after which the solution $\phi = \phi^U + \phi^C$ is known. Substitution of this solution in Eq. (A-6) yields the dimensionless density difference.

The solution of Eq. (A-1) with the boundary conditions of Eqs. (A-2)-(A-5) is performed with the aid of the finite-element method.

Appendix B
TURBULENT LUBRICATION INCLUDING INERTIA EFFECTS

For the 2-inch core flow example starting at an oil velocity of 0.75 m/s the group $R_1 = (\rho \cdot \mu h)/(\mu^2 h^3/3x)$ no longer is smaller than 1, necessitating the incorporation of inertia effects in the turbulent-lubricating-film model. Lubrication with inertia forces has been studied in the past by Constantinescu (1970), Leschziner (1976), King & Taylor (1977), Launder & Leschziner (1978) and, more recently, by Tuck & Bentwich (1983). For a plane inclined slider thrust bearing operating with a turbulent film the effect of convective fluid inertia relative to non-inertia flow conditions is summarized by King & Taylor (1977) as:

(i) The load capacity may be significantly increased.
(ii) An increase in frictional traction on the moving surface results. However, this is not as substantial an increase as with the load capacity.
(iii) There is a marginal movement of the centre of pressure of the film towards the pad leading edge.

It is tempting to make an estimate of the effect of inertia on the core flow results. To this end we rederive the Reynolds equation for the pressure variable $\phi$ by using the equations of motion (19) to (21) as a starting point. Inspection of the order of magnitude of the various terms in these equations (ref. eqs. (26)-(28)) reveals that the leading inertia terms are $\mu \frac{\partial v}{\partial x}$ and $\mu \frac{\partial w}{\partial x}$ in equations (27) and (28), respectively. When we maintain these inertia terms, but still use the usual simplifying assumptions (eq. (23),(25)) the set of continuity and momentum equations for turbulent lubrication with inertia becomes:

$$-\frac{2}{3y}(u\phi) + \frac{3v}{3x} + R \frac{3v}{3x} = 0$$  \hspace{1cm} (B-1)
In order to derive the expressions for the velocities $v$ and $w$ that are needed in eq. (B-1) to determine the Reynolds equation for the pressure $\Psi$, the equations (B-3) and (B-4) have to be integrated. For computational convenience the LHS of these equations are made independent of $y$ by replacing the inertial terms by their average over the film:

$$\begin{align}
\frac{1}{\mu R} \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial v}{\partial x} \right) &= \frac{\partial^2 v}{\partial y^2} \tag{B-5} \\
\frac{1}{\mu} \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} \right) &= \frac{\partial^2 w}{\partial y^2} \tag{B-6}
\end{align}$$

In calculating the averaged inertia terms we have used the equations (39) and (40) for the velocities $v$ and $w$ (with the viscosity $\mu$ replaced by $\nu$) i.e. assumed that the shape of the velocity profiles remains unchanged, following the practice by Constantinescu (1970) and Leschziner (1976). Moreover, terms of order $h^2$ after integration have been neglected.

When we substitute eqs. (B-5) and (B-6) in eqs. (B-3) and (B-4), the equations of motion taken the following form:

$$\begin{align}
\frac{\partial^2 v}{\partial y^2} &= \frac{1}{\mu R} \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial y} \right) - \frac{\nu h}{12\nu} \frac{\partial^2 v}{\partial x^2} \tag{B-7} \\
\frac{\partial^2 w}{\partial y^2} &= \frac{1}{\mu} \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial y} \right) - \frac{\nu h}{12\mu} \frac{\partial^2 w}{\partial x^2} \tag{B-8}
\end{align}$$

Integration of these equations and substitution of the resulting expressions for $\partial v/\partial y$ and $\partial w/\partial x$ in the continuity equation (B-1) in the now familiar manner leads to the Reynolds equation for the pressure variable $\Psi$ for turbulent lubrication including inertia:

$$\begin{align}
\frac{3}{30} \left[ h^3 \frac{\partial^2 \Psi}{\partial y^2} \left( 1 - \frac{\nu h}{12\nu} \frac{\partial^2 \Psi}{\partial x^2} \right) \right] + \frac{2}{3x} \left[ \frac{h^3}{v} \frac{\partial \Psi}{\partial y} \left( 1 - \frac{\nu h}{12\nu} \frac{\partial \Psi}{\partial x} \right) \right] + \\
\frac{\rho \mu^2}{6h} \frac{\partial h}{\partial x} \left( \frac{\partial h}{\partial y} \right) = 6W \frac{\partial h}{\partial x} \tag{B-9}
\end{align}$$

To assess the mathematical implications the inertia correction in the second term on the LHS, that does not contain the pressure gradient $\partial \Psi/\partial x$ is transferred to the RHS of eq. (B-9) as a part of the source term:

$$\begin{align}
\frac{3}{30} \left[ h^3 \frac{\partial^2 \Psi}{\partial y^2} \left( 1 - \frac{\nu h}{12\nu} \frac{\partial^2 \Psi}{\partial x^2} \right) \right] + \frac{2}{3x} \left[ \frac{h^3}{v} \frac{\partial \Psi}{\partial y} \left( 1 - \frac{\nu h}{12\nu} \frac{\partial \Psi}{\partial x} \right) \right] = \\
= 6W \frac{\partial h}{\partial x} \left[ 1 - \frac{\nu h}{18\nu} \frac{\partial \Psi}{\partial x} \left( 1 - \frac{h}{2\nu} \frac{\partial \Psi}{\partial x} \right) \right] \tag{B-10}
\end{align}$$

(In deriving the modified source term we have neglected a contribution $-\frac{\rho \nu^2 h^2}{36\nu} \frac{\partial^2 h}{\partial x^2}$ in the expression between square brackets).

The equation for the density difference (a generalization of eq. (77)) that is used to determine the core eccentricity, becomes:

$$\begin{align}
2\pi \int_0^\pi \int_0^{\pi/2} \rho \left( \frac{\partial \Psi}{\partial y} \right) \sin \theta \, d\theta \, dx - \int_0^\pi \int_0^{\pi/2} 2a \frac{\partial \Psi}{\partial y} \left( 1 - \frac{\nu h}{12\nu} \frac{\partial \Psi}{\partial x} \right) \sin \theta \, d\theta \, dx = \\
= (\sigma - \rho) g \frac{v^2 \pi \kappa R^2}{2} \tag{B-11}
\end{align}$$
Finally, the equation for the pressure gradient in steady core flow is affected by inertia effects (compare eq. (79)):

$$\frac{\pi R}{2a} \int_0^0 \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x} \left( 1 - \frac{\rho_W h^2}{\rho_f h} + \frac{\rho_U h^2}{h} \right) \right] \, dx \quad + \quad + \quad 2 \frac{\rho_W}{\rho_f} \int_0^0 \frac{\partial}{\partial x} \, dx = \alpha R^2 (\beta - \psi)$$

in which $\alpha$ and $\beta$ depend on the turbulent viscosity as defined in eqs. (78) and (80).

Mathematically, the Reynolds equation (B-10) poses a difficulty: when the factors containing inertia effects by which $\partial^2/\partial x$ and $\partial^2/\partial x$ are multiplied have a different sign the problem no longer is elliptic but hyperbolic. This occurs in the range $8 < \rho_W h^2 < 12$.

As long as the group $\frac{\rho_W h^2}{\rho_f} < 6$ qualitatively the effect of incorporating inertia is to multiply the gradients of $\phi$ in the LHS of the Reynolds equation and the source term in the RHS by a factor less than 1. At the front end part of the wave, the net effect is lubrication with a larger viscosity of the lubricant (water) and a smaller angle between the surfaces that are lubricated. One might expect, as observed in the literature, a larger load capacity as a result.

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**SAMENVATTING**

De techniek gebaseerd op kernringstroming is vooral aantrekkelijk voor het pijpleidingtransport van hoog-viskeuze olie, omdat het benodigde pompvermogen vergelijkbaar is met dat voor waterstroming. Bij het kernstromingstype wordt een dunne waterfilm aangebracht tussen de olie en de pijpwand. Deze film heeft een smeerende werking en reduceert daardoor het drukverlies.

Dit proefschrift begint met een onderzoek naar de kenmerkende eigenschappen van de kernringstromingstechniek en met een literatuuroverzicht van enerzijds theoretisch modellen en correlaties voor het bepalen van de drukval bij stabiele kernringstroming, en anderzijds methoden om de drukken te bepalen die benodigd zijn voor het opnieuw starten vanuit een gelaagde water/olieconfiguratie die zich heeft ingesteld gedurende een periode van stilstand. Stabiele kernringstroming in pijpen treedt op bij oliesnelheden boven een kritische waarde, mits de olieviscositeit hoog genoeg is. Uit waarnemingen van dit stromingspatroon in doorzichtige, horizontale buizen blijkt dat de oliekrakens, die excentrisch in de pijp stromen ten gevolge van de opwaartse kracht veroorzaakt door het dichthedoverschil tussen olie en water, een golfvormig oppervlak hebben. In de oudere modellen voor stabiele kernringstroming werd het bestaan van de golven niet meegenomen en werd gepoogd om drukvallen te voorspellen door slechts de hold-up in de waterfilm en de excentriciteit van de oliekansen als modelparameters te gebruiken. Vergeleken met metingen waren de modelvoorspellingen van de drukgradiënten veel te laag.

Het hoofdthema van onze theoretische studie betreft de systematische ontwikkeling van een smeerfilmmodel voor de kernringstroming van twee viskeuze fluida (hier olie en water) in een horizontale pijp. In dit model spelen de waargenomen asymmetrische golven op het olie/waterverdelingsvlak een essentiële rol. Bij een bekende golfgeometrie is het in principe mogelijk om met het model zowel de excentrische positie
van de olikern als de drukgradient te berekenen, waarbij de snelheid van de olie, het dichtheidsverschil, de volumefractie water en de film hold-up als bekende grootheden worden beschouwd. Voor de laatstgenoemde parameter worden empirische correlaties gebruikt. In feite kunnen deze correlaties worden vervangen door een relatie met de volumefractie water bepaald met het model zelf, zoals wordt aangetoond met de eenvoudigste versie van het model. Een dergelijke relatie zal ook van de golfgeometrie afhangen.

Voor de lambinaire versie van het smeerfilmmodel toont een gevoeligheidsanalyse het belang aan van de golfparameters (amplitude, lengte, asymmetrie en vorm) voor een nauwkeurige bepaling van drukgradienten. De resultaten van twee series metingen in een 2-inch pijp worden gebruikt om meer informatie over de golven op het olie/waterscheidingsvlak te verkrijgen en om de geldigheid van enkele vereenvoudigende modelaannames kritisch te onderzoeken. De metingen tonen aan dat de golfamplitudes evenredig zijn met de afstand van de oliekern tot de pijpwand, de golf- lengtes groter worden naarmate de volumefractie water of de olieviscositeit toeneemt, en de parameter voor de golfasymmetrie een waarde heeft tussen 0,4 en 0,5, hetgeen resulteert in een golf met een relatief korte top, en een lange staart, d.w.z. een vorm die duidelijk verschillt van de zagen tandvorm die oorspronkelijk werd aangenomen. Met een eenvoudig lange-golfmodel, dat Couette-Poiseuille-stroming van twee viskeuze fluida tussen evenwijdige vlakken beschrijft, kunnen de waargenomen golfverschijnselen maar ten dele worden voorspeld. Er is behoefte aan een vollediger golfmodel dat in het bijzonder in staat is om de golflengte te voorspellen van stabiele golven met eindige amplitude op een vloeistof/vloeistofoppervlak nabij een vaste wand.

De kernringstromingsexperimenten geven voorts aan dat de stroming in de waterfilm turbulent is. Dit heeft geleid tot het formuleren van de turbulente versie van het smeerfilmmodel. Als referentie wordt in deze studie een serie kernringstromingsmetingen in de 2-inch horizontale pijp gebruikt voor een 3000-mPa.s-olie met een dichtheidsverschil van 20 kg/m² bij een volumefractie van 0,10. Met de turbulente versie van het smeerfilmmodel kan het verloop van de drukgradient met de oliesnelheid vrij goed worden voorspeld, mits de bij deze waterfractie waargenomen golfgeometrie wordt gebruikt in het model. Voor een constante oliesnelheid voorspelt het model echter dat de drukgradient een toenemende functie van de waterfractie is, terwijl de metingen aangeven dat de drukgradient nagenoeg constant blijft. Het is niet duidelijk of deze discrepantie te wijten is aan een onjuiste afhankelijkheid van golflengte en amplitude van de waterfractie of aan een tekortkoming van het model.

Analyse van de kernexcentriciteiten brengt een ander verschil tussen model en experiment aan het licht: het model geeft aan dat, hoewel de olikern zich als functie van de oliesnelheid naar het centrum van de pijp toe beweegt, deze zich vrij excentrisch in de pijp blijft bewegen, terwijl de experimenten aangeven dat voor oliesnelheden groter dan 0,75 m/s de olikern zich in een bijna-concentrische positie bevindt. Vermoedelijk kan dit verschil tussen model en experiment worden opgeheven door het model uit te breiden met traagheids-effecten: voor oliesnelheden groter dan 0,75 m/s is, zelfs voor het afgeplatte deel van de golf, de dichtheid bij de top van de pijp, de aannemer dat traagheids-effecten te verwaarlozen zijn niet langer gerechtvaardigd. De modelvergelijkingen, waarin tenminste de geringste traagheidsterm zijn meegenomen, suggereren dat het effect van deze termen een toename van de verticale neerwaartse kracht is en daardoor een reductie van de kern-excentriciteit. Het lijkt de moeite waard aandacht te besteden aan de wiskundige oplossingstechniek voor deze versie van het smeerfilmmodel.

Bij toepassing van het turbulente-smeerfilmmodel op horizontale pijpen met een grote diameter, met gebruikmaking van de golfgegevens uit de 2-inch-pijp-metingen, blijkt dat ten gevolge van een toenemende opwaartse kracht de kernringstromingtechniek minder aantrekkelijk wordt. De schijnbare viscositeit voor het kernringstromingstransport neemt ongeveer lineair toe met de pijdpijndiameter. De in beperkte mate beschikbare drukvalmetingen in pijpen met grotere diameter worden door het model redelijk goed voorspeld. Het blijkt dat bij een oliesnelheid
van 1 m/s, welke representatief is voor praktische toepassingen, in een situatie met een dichtheidsverschil van 30 kg/m³, de aanname dat traagheidseffecten kunnen worden verwaarloosd in pijpen met diameters groter dan 2 inch gerechtvaardigd is (d.w.z. bovenin de pijp voor het afgeplatte golfdeel, waar de smeringseffecten het grootst zijn).

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LEVENSBERICHT

Hij is getrouwd met Gloria le Roy en vader van twee zonen.