ON TRANSFORMER MODELLING

A physically based three-phase transformer model for the study of low frequency phenomena in electrical power systems

Harold E. Dijk
ON TRANSFORMER MODELLING

A physically based three-phase transformer model for the study of low frequency phenomena in electrical power systems
ON TRANSFORMER MODELLING

A physically based three-phase transformer model for the study of low frequency phenomena in electrical power systems

Proefschrift

Ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus, prof.drs. P.A. Schenck, in het openbaar te verdedigen ten overstaan van een commissie door het College van Dekanen daartoe aangewezen, op maandag 19 september 1988, te 14.00 uur.

door

Harold Edwin Dijk

elektrotechnisch ingenieur

geboren te Aruba
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>GENERAL INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>ON TRANSFORMER MODELLING</td>
<td>7</td>
</tr>
<tr>
<td>2.1</td>
<td>Maxwell's Equations</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Single-Phase Transformer</td>
<td>9</td>
</tr>
<tr>
<td>2.3</td>
<td>Three-Phase Transformer</td>
<td>23</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Tree-Limb Transformer</td>
<td>24</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Five-Limb Transformer</td>
<td>28</td>
</tr>
<tr>
<td>2.4</td>
<td>Model Parameters</td>
<td>32</td>
</tr>
<tr>
<td>2.5</td>
<td>Transformer Voltage Equations</td>
<td>40</td>
</tr>
<tr>
<td>3.</td>
<td>NONLINEAR B–H RELATIONSHIP</td>
<td>45</td>
</tr>
<tr>
<td>3.1</td>
<td>Saturation Curve</td>
<td>46</td>
</tr>
<tr>
<td>3.2</td>
<td>Hysteresis</td>
<td>50</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Dynamic Hysteresis</td>
<td>51</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Static Hysteresis</td>
<td>55</td>
</tr>
<tr>
<td>3.2.3</td>
<td>Determination of the Parameters of the</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>Hysteresis Model</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>TRANSIENT NETWORK ANALYSIS</td>
<td>68</td>
</tr>
<tr>
<td>4.1</td>
<td>Network Equations</td>
<td>68</td>
</tr>
<tr>
<td>4.2</td>
<td>The Modified Nodal Approach (MNA)</td>
<td>73</td>
</tr>
<tr>
<td>4.3</td>
<td>Nonlinear Network Formulation</td>
<td>75</td>
</tr>
<tr>
<td>4.4</td>
<td>Nonlinear Equations</td>
<td>81</td>
</tr>
<tr>
<td>4.5</td>
<td>Numerical Integration</td>
<td>84</td>
</tr>
<tr>
<td>5.</td>
<td>TRANSFORMER RESPONSE</td>
<td>101</td>
</tr>
<tr>
<td>5.1</td>
<td>Hysteresis and Eddy-Current Effect</td>
<td>101</td>
</tr>
<tr>
<td>5.2</td>
<td>Residual Core Fluxes</td>
<td>107</td>
</tr>
</tbody>
</table>
1 GENERAL INTRODUCTION

According to the records of the history of electric power engineering, the first single-phase transformer was constructed in 1885. Five years later the three-phase transformer was introduced. With the invention of the transformer the development of power systems commenced, and they grew to what they now are: highly interconnected networks in which the transformer is one of the most important components.

The fact that many textbooks and treatises on transformers are available more than 100 years after the data of invention can hardly be called a surprise. Indeed, after a cursory survey of this subject, it might be difficult to justify another dissertation on transformers, except that this arises from some specific needs. These needs are (and always will be) directly related to one of the major objectives of electric power engineering, namely, the care of making power systems more and more controllable and manageable. An attractive help in attaining this objective is mathematical modelling.

Since the inception of power engineering, modelling has been recognized as a powerful tool. However, it was not until the maturation of computer engineering that the significance and necessity of modelling has been really fully appreciated. This can be read not alone from the various functions of power systems that already have been revisited, automatized or digitalized, but also from the renewed attention that is being paid to modelling of power system components. In particular, these components are now being reconsidered in order to derive sophisticated models suited for a setting dominated by computers. Highly accurate models are achieved by exploiting the possibilities of the computer to a full extent.

The purpose of this thesis is to contribute to this reconsideration of power system components. More specific, in this thesis some basic concepts of transformer modelling are presented, as well as transformer models emanating from these concepts. By means of these models some topics in the field of transformers – topics concerned with their electromagnetic behaviour – are discussed, that is, computed waveforms are discussed, and in some cases compared with measured waveforms.

In the beginning of the theory of transformers much attention was paid to the concept of leakage field and the determination of leakage inductances. Of fundamental interest for this part of the transformer theory was the work of Kapp [3,4]. He recognized that the difference between the no-load voltage and the voltage at load can be simply obtained from the short-circuit test of the transformer. His phasor (voltage regulation) diagram as well as the trapezoidal distribution of the magnetic field across the transformer window space became well known, and still are the basis for many practical space calculations.
In 1898, ten years after Kapp's revelation, Rogowski developed a method - he forced in opposite direction currents into primary and secondary windings (at a transformer ratio 1:1) - for calculating the leakage inductances [5]. Like Kapp, his study had been set up for a two-winding transformer, and two leakage inductances associated with the two sides of the transformer were determined. These two leakage inductances can be found in the well-known and still used T-network representation, which dates from that time.

The T-network and its underlying assumption that the field of the (two-winding) transformer can be divided into a main field and (two) leakage fields, gave rise to a lengthy discussion, which lasted almost 20 years [6-9]. The enlightening contribution of Bodefeld put an end to this discussion. He showed very clearly that the T-network has no physical base, but originates from pure mathematical manipulations, based on the superposition principle [10,12]. In this thesis the main lines of this discussion are picked up again and related with another representation of the transformer, the M-network.

In contrast with the T-network, no flux division is applied for the derivation of the M-network. The M-network, which has been advocated by Blume [11] (and also is used by other authors [15, 19, 41-43, 48]), is a more realistic representation. Where relevant this M-network will be compared with the other method of representing the transformer, the T-network.

In the beginning these first single-phase transformer descriptions were used for studying relatively simple transformer behaviour, such as for example the (single-phase) linear behaviour, with the use of phasor diagrams as tool par excellence for representing the (steady state) solution. However, more complex behaviour were not shunned; by introducing simplifications the basics of nonlinear phenomena such as the (single-phase) inrush phenomenon [20-23] and (single-phase) ferroresonance [24-29] could be understood.

Later, as the computer entered the field of power engineering, system studies were carried out more and more, and transformer models suited for system studies were introduced [44-51]. That is, in most of these descriptions the transformer equations were organized in matrix form. Actually, the equations related to the single-phase transformer representation were rearranged in a matrix form [48,49]. In the early three-phase system studies, the three-phase transformer was represented by three interconnected single-phase transformers, (e.g., [44]), and thus, ignoring the magnetic coupling which exists in, for example, the three-limb transformer.

Now that the computer (as tool) has (also) been integrated in the field of power engineering, and simulation (computer) programs have become a quite common and indispensable tool for electrical engineers, there is a great need for revising some of the early component models. A critical look at these programs shows that there still are models which are in fact digitized versions of those from the computerless era. As a rule this means, that assumptions and simplifications made in that pre-computer era - and what is meant here are those assumptions that are mainly made for (hand) computational reasons - needlessly have been transferred to the computer models.
A good example is the use of the method of symmetrical components - a method basically developed to facilitate hand calculation - for describing the transformer behaviour in a three-phase system (e.g. [38-40]). Though the computer has taken over most of the calculation, this method is still used in computer programs (e.g. [77, 78]). One should also bear in mind that essentially the method of symmetrical components can only be applied to cyclic symmetrical linear components of a three-phase system. Generally speaking, a three-phase transformer, with the exception of a transformer bank, does not have this symmetrical property, and besides that, the transformer is nonlinear.

Further it can be noticed that most models of transformers used in existing programs are discretized versions of the T-network equivalent, and as such based on the assumptions made in the early days of the transformer. This thesis gives a system approach of transformer modelling. This implies that transformer descriptions are given, which are suited for being incorporated in transient analysis programs. More precisely, in a network approach it will be shown how magnetic networks (describing the nonlinear magnetic behaviour) are integrated in electric networks. It is not claimed that any new theories are introduced here; Faraday's law and Ampere's law still form the base for the transformer equations. However, a physically based transformer description is given, which tries to describe the flux behaviour of the transformer - not only the single-phase transformer - as realistic as possible. That is, the magnetic configuration of the transformer core is taken into account.

One of the first transformer models which took the magnetic configuration of the core into account, particularly that of the three-limb transformer, was given by Macfadyen et al [60]. In this description the flux has been divided in a main flux, leakage fluxes and interphase fluxes, a division which is similar to the one underlying the T-network. Nakra has used a similar division, except that he used one interphase flux path in stead of three [63]. He also used a general magnetic circuit (network) for describing magnetic configurations of several transformer cores. His transformer model has also been used in three-phase ferroresonance studies [72].

Recently, there is a growing tendency to describe the flux behaviour of the transformer as a whole, rather than with a flux which is being perceived as a composition of a main flux and leakage fluxes [64, 75, 80, 82]. In most cases use has been made of magnetic circuits. However, the step to an electric description is not always clear, especially the occurrence of negative inductances in this circuit [74, 80]. In doing so, an opaque description of the transformer is obtained, meaning that the link with existing fluxes is not directly available. In this thesis it will be shown that the T-network representation (of the single-phase transformer), whose base has been made for pure computational reasons, is related via a star-delta transformation to the И-network representation which has a physical base. Since the transformer is a nonlinear device, this transformation comes into trouble, and the fluxes in the T-network cannot easily be related to physical fluxes. Obviously, this deficiency also occurs in three-phase transformer models which underlie the same principle - a division of fluxes in a main flux and leakage fluxes - of the T-network representation. The aim of this thesis
is to clear up this deficiency and to develop a new physically based three-phase transformer model.

This thesis discusses the fundamentals of transformer modelling. Care has been taken that the constraints imposed by the limited possibilities of hand calculation are now of minor importance, if not irrelevant, to the computer approach followed here. More specific, transformer models are discussed and developed with the emphasis on the physical description. That is, the flux behaviour of the transformer is treated as a whole and described by a magnetic network. Unlike most transformer models (e.g., [80]), no dual network of this magnetic network is constructed for incorporating the transformer model in a transient program. This has been done for two purposes.

First, when the flux behaviour is described by a magnetic network, all fluxes of this network can directly be related to the real fluxes of the transformer. Hence, the use of this network is more powerful than its dual network, the electric network of the transformer. With the approach given in this thesis it will be shown that there is no impelling reason to use an electric network representation for incorporating the transformer model in an electric network analysis program. For this purpose the modified nodal approach is used.

The second reason for using the magnetic network has to do with matters concerned with modelling. It will be seen that the transformer, at least its flux behaviour, can be described in terms of a magnetic system, i.e., a system in which currents are the driving forces. Owing to this, all magnetic characteristics of the transformer can be easily subjected to preliminary considerations. In particular, the magnetic network proves to be very tractable to test the nonlinear behaviour - this is described by algebraic equations -, especially the performance of hysteresis (inner) loops, and residual conditions in a three-phase core configuration.

For studying various transformer connections and network configurations a transient analysis program has been developed, in which the transformer models have been incorporated. No attempt has been made to incorporate the transformer models into existing (transient) analysis programs in order to avoid possible constraints imposed by these programs. For instance, it is known that the well-known transient analysis program, EMTP [66], suffers from numerical oscillations. Moreover, it is rigid in use, since it can handle only models written in the admittance form. Hence, an own (transient) analysis program has been developed of which the most salient aspects are discussed in this thesis. Attention is paid to the insertion of the transformer equations into an algorithm for solving electrical networks, and to the use of a convenient (stable) numerical integration method. In this respect, the material presented in this thesis can be regarded as self-contained.

This thesis is organized as follows. In Chapter 2 the theoretical background of the transformer is given. The basic steps involved in deriving a flux model of a single-phase transformer via the physical approach are given. A T-network, the result of an integral flux approach, will be compared with the well-known T-network, the result of a divided flux approach. Then, the integral flux approach is used for describing the three-phase transformer. The determination of the model
parameters (of a three-phase, three-limb transformer) is also given. Actually, the model parameters are chosen so as to predict the rated characteristics (steady states) of the transformer. Finally, it is shown, that the formulation of the transformer equations, necessary for the network, can be kept very simple.

Chapter 3 deals with the modelling aspects of the characteristics of the transformer core. An analytical expression is used which is found to give suitable approximations of saturation curves. With the aid of this function the hysteresis phenomenon, dynamic and static hysteresis is described. The description of the hysteresis phenomenon is needed for describing residual core conditions and not so much for modelling transformer losses. It is shown that the description of dynamic hysteresis is closely related to what is called eddy current losses. Special attention is paid to the derivation of a static hysteresis model, which is relatively simple and suitable for transformer studies. On algorithm level it is shown how a Preisach hysteresis model can be described, and how the algorithm can be adjusted as to obtain a limited storage of reversals. Notice at this point, that only in this part of the transformer modelling process the phenomenological approach is used.

In Chapter 4 most of the functions necessary for writing a (generic) transient analysis program is given. A computer program has been developed, since most existing programs are inaccessible, have a rigid frame, or even show numerical instability in some cases. The Modified Nodal Approach (MNA) has been chosen to organize the network equations. Since the nonlinear transformer equations must be incorporated in these equations, it will be shown how the MNA is adjusted for handling nonlinear elements. A method is presented which reduces the nonlinear network problem to a problem of a lower dimension, the solution of a (nonlinear) Thevenin equivalent network. Further, special attention is paid to the application of a numerical integration method. By means of a simple example it is shown that the trapezoidal rule, which has been adopted by almost every program for simulation power system transients, will not always behave well. A linear multi-step differentiation formula (backward differentiation formula), suitable for integration of stiff differential equations and easy to implement, is used here.

The result of some numerical experiments are shown in Chapter 5. The influence of the modelling of the magnetic characteristic of the core on transients are shown. A simple transient simulation is carried out for four cases of modelling of core losses. The aim of this study is to show the damping effect of the hysteresis model (in comparison with the resistance representation).

A case of trapped charge is also given. This case should be viewed as a test case for the three-phase hysteresis description used in the three-phase, three-limb transformer model. However, this case is also meant to show how (consistent) residual core conditions can be obtained in a three-phase system.

The residual core conditions of a three-limb transformer, related to three simple loads, viz, capacitance, inductance and resistance, are calculated. Formulas are derived for the ideal cases in order to understand these residual condition.

It is also shown that two-phase switching of power transformers can lead to ferroresonance. The ferroresonance state can be readily appreciated by considering the single-phase
ferroresonant circuit which can be derived from a wye-connected transformer, whose neutral is ungrounded. In addition, only two of its terminals (from the wye-side) must be connected with the three-phase voltage source. The capacitance in this circuit is the representation of the (cable) capacitance at the transformer terminals.

A voltage jump phenomenon, which has been observed at the transformer terminals, is also explained. This case is almost similar to the ferroresonance case, except that here the capacitance at the transformer terminal is much smaller. To understand the mechanisms of this phenomenon, a classical approach has been used. That is, an analytical explanation (solution) is formulated for a simplified version of the problem. It should be emphasized here that the possibility of taking the flux waveforms into considerations (via the magnetic network) has been conducive to the revelation of this voltage jump phenomenon. At the end, waveforms obtained from a numerical solution are compared with experimentally obtained waveforms.

Finally, the no-load condition of the transformer is considered more closely. In particular, the harmonic contents of voltages and currents are considered. Two transformer models, one originated from an integral flux approach and the other from the divided flux approach, are confronted with experimentally obtained waveforms. It will be seen that the predictive ability of the models are poor as far as prediction of this harmonic content is concerned.
2 ON TRANSFORMER MODELLING

This chapter is concerned with the derivation of transformer models which are suited to be incorporated in an integral description of electrical power systems. More specific, transformer models are derived for analysing the electromagnetic transient behaviour of a transformer interconnected with power systems. Hence, the transformer will be treated as a component which must fit in a larger system description and not as an isolated case.

Contrary to the conventional transformer description, no idealizations are introduced for hand computation reasons. The idealization introduced here, are introduced for other purposes, namely for suitting the transformer description to that of power systems.

The conceptual framework and the mathematical formalism needed for the transformer modelling process is presented in this chapter. In particular, a physically based model for studying the electromagnetic behaviour of the transformer will be discussed. Perhaps given excessively, the Maxwell's equations are considered in Section 2.1 to have a ground to fall back upon. Starting from these well-known electromagnetic laws, a fundamental treatment can be given in discussing the transformer behaviour. That is, the restrictions on phenomena which can be studied with the transformer model can directly be related to these fundamental equations. Hereafter, the modelling of a single-phase transformer will be discussed in Section 2.2, followed by a discussion of the three-phase transformer in Section 2.3. In Section 2.4 the determination of the model parameters is discussed, and finally, in Section 2.5 it is illustrated how the transformer model can be incorporated in the network equations.

2.1 Maxwell's Equations

The Maxwell's equations constitute the base for the description of every electromagnetic device. Since it is our aim to start from scratch in describing the transformer behaviour, we will have recourse to these equations here. This enables us to fundamentally argue which phenomena can satisfactorily be described and which cannot.

The relationship between the electric field intensity \( \mathbf{E} \) and the magnetic field intensity \( \mathbf{H} \), and their associated free current and charge distribution \( \mathbf{J}_f \) and \( \rho_f \), respectively, are provided by the well known Maxwell’s equations and can be found in every textbook (e.g., [1,2]) on electromagnetic fields. For material regions and in the presence of electric charge density, these basic field laws can be expressed in terms of the magnetic flux density \( \mathbf{B} \) and the electric flux density \( \mathbf{D} \). These equations can be formulated in the following integral form:
\[ \oint_{S} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_{S} \mathbf{B} \cdot d\mathbf{s} \quad (2.1.1) \]
\[ \oint_{S} \mathbf{H} \cdot d\mathbf{l} = \int_{S} \mathbf{J}_{f} \cdot d\mathbf{s} + \frac{d}{dt} \int_{V} \mathbf{D} \cdot d\mathbf{v} \quad (2.1.2) \]
\[ \oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0 \quad (2.1.3) \]
\[ \int_{V} \mathbf{D} \cdot d\mathbf{v} = \int_{V} \rho_{f} \, d\mathbf{v} \quad (2.1.4) \]
\[ \int_{V} \mathbf{J} \cdot d\mathbf{v} = -\frac{d}{dt} \int_{V} \rho_{f} \, d\mathbf{v} \quad (2.1.5) \]

where in Eqs. (2.1.1) and (2.1.2) \( S \) is any two sided simply-connected surface bounded by a closed contour \( C \); \( d\mathbf{l} \) is a vector differential element of \( C \), and \( d\mathbf{s} \) is a vector differential element of \( S \) normal to \( S \) and directed relative to \( d\mathbf{l} \) according to the usual right hand sense. In Eqs. (2.1.3) - (2.1.5) \( V \) is an arbitrary volume bounded by a closed surface \( S \), and \( d\mathbf{v} \) is a vector differential element of \( S \) normal to \( S \) and outward directed.

This set of equations is not complete in that the properties of the material we are concerned with, have not yet been given. These properties can be expressed in the following constituent relations

\[ \mathbf{B} = \mu \mathbf{H} \quad (2.1.6) \]
\[ \mathbf{D} = \varepsilon \mathbf{E} \quad (2.1.7) \]
\[ \mathbf{J}_{f} = \sigma \mathbf{E} \quad (2.1.8) \]

where \( \varepsilon, \mu \) and \( \sigma \) respectively are the susceptibility, the permeability and the conductivity.

The Maxwell's equations (2.1.1) through (2.1.5) must simultaneously be satisfied by the field solutions of \( H \) and \( E \) for all possible closed paths \( C \) and surfaces \( S \) in the region occupied by these fields. Needless to say that in the case of describing the transformer behaviour and most other cases this would be a monumental task. Moreover, in trying to solve the electromagnetic field problem for the purpose of a transformer model, we defeat our own object. Our main concern in this study is to find a suitable model for describing the behaviour of transformers connected in electric power systems. That is, a lumped rather than a distributed description is required, and moreover, this description must be in terms of the electric quantities currents and voltages. For this purpose, above fundamental laws will be reduced, and from these reduced equations the transformer equations will emerge. However, before this reduction is made a simplification will be introduced in the field equations.

One of the main characteristics of Maxwell's equations is the coupling between electric and magnetic field through the time variation of \( B \) and \( D \) in Eqs. (2.1.1) and (2.1.2). By omitting the source term of the displacement currents - the contribution of the time variation of \( D \) in Eq. (2.1.2) -, the magnetic field can
be determined independently from the electric field. In particular, it is seen that in the resulting equation, the currents act as source of the H-field. Generally speaking, this equation, known as Ampere's circuit law, is used whenever we are dealing with slowly time varying, low frequency currents, which give rise to slowly varying fields. This approach, termed the quasi-static approach, can be considered as a first order approach, since the field solutions do not exactly satisfy the Maxwell's equations. Notice at this point, that a system is amenable to a quasi-static method of attack, if the dimension of the electromagnetic system is smaller than the wave length corresponding to the highest frequency of operation [2]. Consequently, it is important to recognize at this stage of the modelling process that the transformer model will not be suited for studying the response of transformer windings to the impact of lightning surges. Indeed, in these studies we are not only interested in the behaviour at the transformer terminals, but also in the field distribution along the windings.

With the conceptual steps, taken above, the fundamental electromagnetic laws (2.1.1) - (2.1.6) are now be re-grouped so as to suit our purposes.

\[
\frac{\phi}{C} \mathbf{E} \cdot d\mathbf{l} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s} \tag{2.1.9}
\]

\[
\frac{\phi}{C} \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{s} \tag{2.1.10}
\]

\[
\frac{\phi}{S} \mathbf{B} \cdot d\mathbf{s} = 0 \tag{2.1.11}
\]

\[
\frac{\phi}{S} \mathbf{J} \cdot d\mathbf{s} = 0 \tag{2.1.12}
\]

Observe at this point, that static magnetic fields are required to satisfy (2.1.10) through (2.1.12). Equation (2.1.10) states that the sources of this field are steady state currents of density \( \mathbf{J} \), while Eq. (2.1.11) - the divergenceless property - specifies that flux lines are always closed. The static (dc) character of the current is assured by (2.1.12). In the next section these equations will be used for describing the magnetic behaviour of the transformer.

### 2.2 Single-Phase Transformer

The major purpose of this section is to introduce the conceptual framework and the mathematical formalism needed in describing the magnetic behaviour of the transformer. It will be seen later that, having found the magnetic description of the transformer, its electric behaviour is easily obtained.

In this section the fundamental equations for static magnetic fields is taken as starting point. A two-winding, single-phase transformer is taken to follow the steps from concept to mathematical description. Two approaches will be considered, a divided flux approach and an integral flux approach, resulting in a T-network and a H-network respectively.
It will be shown that the fluxes in the \( W \)-network are physically based, in contrast with those in the \( T \)-network. The discussion on modelling the single-phase transformer is held extensively, since it serve as a base for modelling three-phase transformers. It is noticed that the single-phase transformer used here, has been taken for didactic reasons.

Since it is our intention to study only phenomena whose time variations are relatively slow, it suffices to describe the transformer behaviour with the quasi-static field equations given in the previous section. In other words, a lumped description of the transformer behaviour will be given, that is, ordinary differential equations will be used for describing the transformer behaviour. Consequently, it is now of direct concern to convert the (quasi-static) field equations into a form which can directly be related to the transformer terminal quantities.

If, for conceptual reasons, the fundamental Eqs. (2.1.10) - (2.1.12) are considered, then it is seen that there is no coupling from the electric field to the magnetic field. Particularly, if the current density \( (J) \) is compelled to be the driving force of the electromagnetic system (the transformer), the static magnetic field will be well-defined. That means that the magnetic nature of the transformer can be discussed without being too anxious about its electric behaviour. It is this idea that will be next worked out.

Figure 2.1 shows an idealized picture of a three-limb, single-phase, two-winding transformer. As can be seen, the two outer limbs are unwound, whereas the centre limb is enclosed by two windings. For explanatory reasons, the windings are assumed to have a cylindrical-concentric coil arrangement.

The next step will reveal how transformers - in this case

---

**Figure 2.1** Single-phase two-winding transformer and corresponding magnetic field.

a) three-limb single-phase transformer  
b) magnetic field, front view  
c) magnetic field, side view
of the core type given in Fig. 2.1 - can be simplified so as to obtain a mathematical description which suits our purpose. First we will concentrate on the magnetic field of the transformer generated by the currents in the windings. Recognize that, with the picture given in Fig 2.1a, it is assumed that from our point of view the transformer solely consists of an iron core and windings. Hence, it can be stated that the field lines are supposed to occupy the air, windings and core. It should also be recognized that the magnetic field produced is spatial and that the exact evolution is almost impossible to evaluate. Consequently, this distributed nature of the magnetic field is tackled first. This is done by considering an idealized space independent field.

The result of the idealization is given in Figs. 2.1b and 2.1c. These idealized pictures can be viewed as a reflection of our physical notion of the transformer magnetic behaviour. As can be seen, the problem of dealing with a spatial flux distribution has been overcome by considering the longitudinal and cross sections of the transformer. With the flux distribution given in these sections the transformer flux behaviour can be characterized satisfactorily. Notice further, that by assuming that the flux is intercepted by well-defined regions - transformer core and air paths -, also known as flux tubes, it is always possible to speak of a transformer magnetic system (and its behaviour) with reference to these well-defined paths.

Of course we are aware of the crudeness of this idealization of the flux distribution in the transformer. For instance, one may notice that the extension of the windings is not taken into account, and that there exist flux lines which do not effectively link the whole winding, but partially or only some of its turns. We also did not consider the effect of gaps, corners, bolt holes and other impurities of the core. Though all these aspects are very interesting, they are not considered here, since they needlessly would have enlarged the scope of study; their effect lies within the accuracy usually taken into account for system studies. Also notice, that in defining well-defined regions occupied by the fluxes, we avoid getting entangled in a rigorous solution of integral equations. With this idealized picture of the magnetic field, also used in many textbooks on transformers (e.g. [18,19]), the magnetic behaviour will be further discussed.

The physical significance of the depicted flux paths can be conceived by considering two characteristic extreme states of the transformer, namely the no-load and the short-circuit condition. In the no-load condition of the transformer, the flux in the air paths can be neglected, due to the high magnetic resistance of air compared with that of the iron core. This can readily be appreciated by realizing that the ratio of the permeability of iron (silicon-steel) to that of air is of the order of thousand. Under these circumstances it can safely be stated that the core is responsible for the magnetic behaviour of the transformer. On the other hand, if the transformer is short-circuited, the flux is forced into the air paths. Thus, for producing this state of the transformer, the air paths cannot be neglected. A detailed pictorial view of the short-circuited transformer - the resistance of the windings is neglected here and in the further discussion, unless otherwise stated - is given in the Figs. 2.2a - 2.2d. From these figures it is evident that, magnetically seen,
the transformer has two different kinds of short-circuited conditions. The magnetic picture of a transformer with short-circuited inner winding (seen from the core) as illustrated in Figs. 2.2a and 2.2b, differs from that related with short-circuited outer winding (cf Figs. 2.2c and 2.2d).

The question now is: how can these magnetic features of the transformer be described in a mathematically valid and systematic way, while in the mean time the description of its terminal behaviour is being anticipated? Notice, that it is tacitly assumed that an adequate representation of these extreme states also results in an adequate description of the intervening states.

In order to describe the magnetic nature of the transformer (and for the sake of convenience), the previously discussed equations of the static magnetic fields are re-considered now:
\[ \phi_{H.dl} = \int_{S} J.ds \]  
(2.2.1)

\[ \phi_{B.ds} = 0 \]  
(2.2.2)

\[ B = \mu H \]  
(2.2.3)

For the sake of simplicity, eddy currents effects are ignored here, assuming a zero conductivity of the iron core. In Chapter 3, the eddy currents effects will be discussed.

When deriving the mathematical model of the transformer, evaluating of integral equation (2.2.1) plays a major role. It will be seen that the above mentioned equations can easily be reduced to a tractable set of equations with the following definitions:

the magnetic flux

\[ \psi_{q} = \int B_{q}.ds = B_{av,q} S_{q} \]  
(2.2.4)

the (current corresponding to the) magnetomotive force (m.m.f.)

\[ \theta = \int H_{dl} = \sum_{q} H_{av,q} l_{q} = \sum_{q} \theta_{q} \]  
(2.2.5)

and the magnetic reluctance

\[ R_{q} = \theta_{q}/\psi_{q} \]  
(2.2.6)

Here \( S_{q} \) is a cross-section area of flux tube \( q \) representing cross-sections related to core and air, \( B_{av,q} \) is the average flux density over \( S_{q} \), assuming \( B_{av,q} \) perpendicular to \( S_{q} \). In Eq. (2.2.5) the relationship \( B_{av,q} = \mu H_{av,q} \) has been used, assuming a linear constituent relationship, with \( \mu_{q} \) the permeability associated with flux tube \( q \), and \( l_{q} \) the median path of a flux tube \( q \).

With the above defined quantities (\( \psi, \theta \) and \( R \)) and the physical interpretation of Eqs. (2.2.1) through (2.2.3), the construction of the mathematical model will be carried out. In particular, the magnetic behaviour of the transformer will be represented with a network. It is worth knowing, that once the magnetic behaviour of the transformer has been captured in a network, this behaviour can be described quantitatively, making use of algorithms of the network theory.

Speaking in terms of network theory, it is convenient to recognize Eq. (2.2.1) as a mesh equation. Having conceived this, the construction of the network is evident. The constructing procedure consists of evaluating the line integral of \( H \) round the given closed paths of Fig. 2.1. Notice, that the property of paths being closed, is confined in the divergenceless property given by Eq. (2.2.2). Furthermore, one must realize, that as the constitutive relationship (Eq. (2.2.3)) has been taken linear, the branch elements of the network must also be linear. Armed
with this information, the idealized magnetic field given in Fig. 2.1 is ready to be attacked.

Figure 2.3a shows the (idealized) field of the cross section and Fig. 2.3b that of the longitudinal section of the transformer. These two fields cannot be considered separately. In fact, there is only one field; its flux lines are given in Fig. 2.3c. The separated fields (Figs. 2.3a and 2.3b) have only been given for the sake of clarity. A quick check will show that the constructed network must contain eight ($= 4 + 4$) meshes and two distinct sources. The sources of this magnetic network can be found by evaluating the surface integral of the right-hand side of Ampere's law (Eq. (2.2.1)). Actually, these sources are equal to the net current of the enclosed area. Eventually, the magnetic network given in Fig. 2.3d is the result of equating (2.2.1), using the magnetic quantities defined in (2.2.4) through (2.2.6).

Figure 2.3

Idealized magnetic field and corresponding network of the single-phase transformer

a) magnetic field, side view
b) magnetic field, front view
c) magnetic field, transformer omitted
d) magnetic network
The branch elements of the network simply follow from the corresponding paths of the idealized magnetic field, which is reconsidered in Figs. 2.3a and 2.3b. The paths of this field are related to the network through the given flux specification.

Given these two representations of the transformer, the idealized magnetic field and the network, some features of the modelling procedure can now be better explained. One of the major steps is the assumption that the magnetic behaviour of the transformer can be characterized by introducing decisive flux paths only. Recognize further, that we also have introduced nodes to cope with junctions of the flux paths. In doing so, we are able to describe (approximate), for instance, the flux behaviour at the top of the windings. As can be verified, this is simply achieved by letting the flux at the top of the centre limb divide into two parts. One part, flux \( (\psi_+ + \psi_-) \), is passing through the air and is composed of \( \psi_+ \) and \( \psi_- \) (see Fig. 2.3a). The other part of the flux \( \psi_x = 1/2 \psi_+ - \psi_- \) is proceeding its way through the yoke until another node is detected, and again, a likewise flux division occurs (see Fig. 2.3b). It is also possible now, to verify the cases of \( \psi_+ = 0 \) and \( \psi_- = 0 \), being the two short-circuited conditions of the transformer. If \( \psi_x = 0 \), that means, the inner winding is short-circuited, it can be shown (quantitatively), that the flux will be forced through the air space between the two windings, represented by the branch elements \( R_a \) and \( R_b \) in the network. In this case the flux has to complete its path mainly through the outer unwound limbs. It can also easily be shown, that in the case of the outer winding being short-circuited, \( \psi_+ = 0 \), the similar holds, except that the flux will complete its path through the centre limb.

Thus, conceptually seen, the magnetic behaviour of a transformer can be characterized by defining flux paths and nodes. The network resulting from this characterization, must be seen as a quantitative representation of the magnetic behaviour of the transformer. It may be next to redundancy to state that in this context, the magnetic network of Fig. 2.3 is nothing else than a convenient way of describing the static magnetic field (of the transformer) governed by Eqs. (2.2.1) through (2.2.3).

Having derived the magnetic network of the transformer, the magnetic behaviour can now readily be analyzed. Since it is our intention to compare this model of the transformer with the commonly used T-equivalent, some simplifications are introduced which enable us to give a comparison as transparent as possible.

First, the influence of the air fluxes, as shown in Fig. 2.3a is neglected, and it is assumed that the magnetic behaviour of the single phase transformer can satisfactorily be characterized by the field as specified in Fig. 2.3b. This can be appreciated by realizing that (in the unsaturated state) the flux will give preference to iron over air. (In the next section it will be seen that the fluxes which have been omitted here for the single-phase transformer, will be needed for describing a particular three-phase behaviour known as the zero-sequence behaviour.) Consequently, the flux which is thought to occupy the interwinding space of the single-phase transformer, will from now on be represented by the flux path between the winding as shown in Fig. 2.3b. As far as the magnetic network is concerned, this assumption is substantiated by omitting the corresponding branches (see Fig. 2.4a). If furthermore, advantage is taken of the symmetry of the transformer, the magnetic network can be
reduced to the network shown in Fig. 2.4b.

In this reduced network the magnetic reluctance $R_2$ accounts for the centre limb and the adjacent part of the yokes (see Fig. 2.4a), whereas the unwound outer limbs and their adjacent parts of the yoke are responsible for $R_1$. The magnetic resistance of the air path is represented by $R_3$. It is easy to verify that this magnetic network can be described by

$$\begin{vmatrix} n_1 & i_1 \\ n_2 & i_2 \end{vmatrix} = \begin{vmatrix} R + R & -R \\ -R & R + R \end{vmatrix} \begin{vmatrix} \Psi_1 \\ \Psi_2 \end{vmatrix}$$

(2.2.7)

or

$$\begin{vmatrix} \Psi_1 \\ \Psi_2 \end{vmatrix} = \begin{vmatrix} 1 \\ R_1 + R_2 + R_3 \end{vmatrix} \begin{vmatrix} R + R_3 \\ R + R_3 \end{vmatrix} \begin{vmatrix} n_1 \\ n_2 \end{vmatrix}$$

(2.2.8)

Above set of equations will later be used when this model is compared with the T-equivalent.

In most textbooks on transformers (e.g. [18]) the magnetic field given in Fig. 2.3b is also used for the derivation of the transformer equations. The ultimate model however, the T-equivalent, cannot be seen as the depiction of the real magnetic field, not even an idealized scheme of the field. Actually, the T-equivalent does not represent the idealized magnetic field of
the transformer (given in Fig. 2.1), but rather is a substitute of the magnetic behaviour, introduced for calculation purposes. In the following this matter will briefly be discussed.

The underlying principle of the T-equivalent is the assumption that the flux is separable in a main (or mutual) flux and leakage fluxes. The main flux is linked with all windings, whereas the leakage fluxes are linked with their corresponding windings. From the foregoing however, it is seen that as far as the simplicity of the description is concerned, this separation of fluxes is not necessary. Since the T-equivalent leads to accurate prediction of the transformer behaviour under most operating conditions, no need is felt to review this concept. However, considering our purposes, a close examination of the premises underlying the T-equivalent will be useful. For not getting choked up in details of (former) interpretations and discussions, we will confine ourselves to the main lines.

According to this concept, when modelling magnetic devices, their effective operation must be reflected in the definition of the main flux. In other words, with the given field lines, the main field, the main operation or behaviour of a magnetic device can be explained. Having defined the main flux, the definition of the leakage flux is evident: the flux not belonging to the main flux is referred to as leakage flux [12]. It may be noticed here, that this approach was not readily conceived, considering the development of the discussions held on this subject [5-10]. Briefly stated: it is felt that many authors have been unsettled in discovering the discrepancy between model and their notion of physical reality. This will be very clearly illustrated by the following example, adopted from Bödefeld.

Figure 2.5 shows the field of a coreless two-turn

![Diagram](image)

Figure 2.5 Magnetic field of a coreless two-turn transformer corresponding with

- a) \( h = -0.7 \) A and \( h_2 = -0.3 \) A
- b) \( h = -2.0 \) A and \( h_2 = 1.0 \) A
transformer for two cases. The current through the turns are $i_1 = -0.7$ A and $i_2 = -0.3$ A in Fig. 2.5a, while in Fig. 2.5b $i_1 = -2$ A and $i_2 = 1$ A. Incidentally, recognize that this field - a static magnetic field - can be viewed as a "snapshot" of the quasi-static approach we are aiming at. The field of the coreless transformer has been obtained by numerically working out Biot Savart's law.

According to above concept - henceforth referred to as the divided flux concept, since the flux is being divided in a main flux and leakage fluxes -, only the field pattern of Fig. 2.5a would have been expected. Since the net Ampere turns related to Fig. 2.5b does not vanish ($i_1 + i_2 = -1$A), this may suggest that also in this case a main flux can be built which links both turns. However, inspection of Fig. 2.5b shows that in this case, no such field lines can be detected: there exists no field lines linking both turns.
Figure 2.6 shows the equivalent field and its associated network obtained with the divided flux concept. As can be verified, this equivalent is in full agreement with this concept, but shows hardly if any agreement with the real field. In this respect, we underline the conclusion of Bödefeld, as he observed that for coreless transformers the fictitious magnetic representation does not correspond at all with the magnetic reality [10,12].

If the divided flux approach is abandoned, and the field is idealized like before, that is, the field is considered as a whole, we arrive at the scheme given in Fig. 2.7. It can be clearly seen that, in contrast with the earlier representation, the idealized field and its associated network are closer to the real field. We are now able to point out fluxes which correspond with fluxes of the real system. Also observe, that even the node (point a in Fig. 2.5b) - mostly not recognized as such - has been converted to the network. In contrast with the divided flux approach, this is an integral flux approach which might have changed our perception of the magnetic field. It is not necessary to look for a main field and leakage fields anymore.

How the divided flux approach is applied to the single-phase transformer and how the corresponding T-equivalent can emerge from this approach, will now be illustrated. Once the T-equivalent has been derived a quantitative comparison is possible between the divided flux approach and the integral flux approach.

For this purpose the idealized magnetic field of the transformer is reconsidered as shown in Fig. 2.8. As can be seen, according to the divided flux concept, the flux behaviour has been characterized by the separation of the fluxes in a main flux and leakage fluxes. The main flux links all turns of both windings, whilst the leakage flux links one winding or the other, but not both [12,15,18]. The reason for splitting up the field into these three parts is, that in doing so, three magnetic systems can be defined. As will be seen, in applying this procedure of separating the field (or the flux), the construction of a network, the T-network, consisting of three inductances becomes straightforward. However, it will also be seen, that this flux division is not necessary for the construction of a network.
Figure 2.9  

a) Magnetic network of the single-phase transformer resulting from the divided flux approach 
b) Dual magnetic network of the network given in Fig. 2.9a

Owing to the flux division, two leakage inductances, each associated with one of the two windings, and one mutual inductance, associated with both windings, can be defined. It can now easily be appreciated that the magnetic network, shown in Fig. 2.9a is related to the idealized magnetic field shown in Fig. 2.8. For this purpose, the branches of the network must be

Figure 2.10  

a) Magnetic network of the single-phase transformer resulting from the integral flux approach 
b) Dual magnetic network of the network given in Fig. 2.10a
related with the corresponding flux paths.

From a theorem of the network-theory, it is known that a dual network produces the same behavior as its original [41]. Hence, it is assumed that, in applying the dual theorem the resulting T-network shown in Fig. 2.9 is a correct interpreter of the original flux equations. It is important to recognize here, that the electric network equations of the transformer (the resistance of the windings is not taken into account) can be found by taking the time derivative of the fluxes in this magnetic T-network.

From the foregoing it is seen that the same departure, as used for the earlier magnetic description (see Fig. 2.4), has been used for the derivation of the T-network, namely the idealized magnetic field. As may be noticed, this magnetic field has been described by two networks. Since it is our purpose to compare the T-network with the previously discussed network of Fig. 2.4, the latter is now considered together with its dual network (Fig. 2.10a) and 2.10b). Recognize, that the elements in the branches of the dual networks (Figs. 2.9b and 2.10b) can be seen as inductances (more precisely as, permeances) and are the reciprocals of the corresponding reluctances of the respective networks (Figs. 2.9a and 2.9b).

Close examination of the networks shown in Figs. 2.9 and (2.10) suggests that we are dealing with the well known star-delta-transformation. In order to prove this, we fall back on the Eqs. (2.2.7) and (2.2.8), and introduce the flux division as follows

\[
\begin{align*}
\psi_1 &= \frac{1}{R_2 + R_3} \begin{vmatrix} R + R_3 & n_1 \\ R_3 & n_2 \end{vmatrix} \\
\psi_2 &= \frac{1}{R_1 R_2 + R_1 R_3 + R_2 R_3} \begin{vmatrix} R_2 & R_3 & n_1 \\ 0 & R_1 & n_2 \\ R_3 & R_3 & n_2 \end{vmatrix} \\
\psi_m &= \psi_1 + \psi_2
\end{align*}
\]

\[
\psi_{11} = \frac{R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3} n_{11} = \rho_{11} n_{11} \quad (2.2.9.a)
\]

\[
\psi_{12} = \frac{R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3} n_{12} = \rho_{12} n_{12} \quad (2.2.9.b)
\]

\[
\psi_m = \frac{R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} (n_{11} + n_{12}) = \rho_m (n_{11} + n_{12}) \quad (2.2.9.c)
\]
with
\[ p_{11} = \frac{R_2}{R_1 + R_2 + R_3} = \frac{P_1 P_2 P_3}{P_1 + P_2 + P_3} \]  \hspace{1cm} (2.2.10.a)

\[ p_{12} = \frac{R_1}{R_1 + R_2 + R_3} = \frac{P_2 P_3}{P_1 + P_2 + P_3} \]  \hspace{1cm} (2.2.10.b)

\[ p_m = \frac{R_3}{R_1 + R_2 + R_3} = \frac{P_1 P_2}{P_1 + P_2 + P_3} \]  \hspace{1cm} (2.2.10.c)

and
\[ p_i = 1/R_i \quad (i = 1, 2, 3) \]

Equations (2.2.10.a) - (2.2.10.c) are known as the star-delta transformation, and this definitely proves that the flux division effectively results in this transformation. Also notice, that Eqs. (2.2.9.a) through (2.2.9.c) explicitly show that this division of the flux produces three uncoupled magnetic systems; three fluxes \( \psi_{11} \), \( \psi_{12} \) and \( \psi_m \) are considered which all three are caused by their respective ampere-turns \( n_{i1} \), \( n_{i2} \) and \( n_{i1} + n_{i2} \).

Recognize, that the main flux as defined here, is not only determined by the iron paths (the transformer core) but also by the air paths. This can be readily seen by considering the reluctance associated with the main flux in Eq. (2.2.10c). In this expression it is seen that this reluctance is an equivalent reluctance, and that it depends on reluctances related to the iron core and the air paths. It can also be verified that similar arguments hold for the reluctances associated with the leakage fluxes \( \psi_{11} \) and \( \psi_{12} \) (see Eqs. 2.2.10a) and (2.2.10b)).

Above quantitative evaluation shows that from an integral flux standpoint of view, it is not automatically true to say that the main flux solely is determined by the core, and the leakage fluxes by the air characteristics. However, a closer inspection of the Eqs. (2.2.9) and (2.2.10) reveals that the leakage fluxes depend largely upon the air paths and the main flux upon the iron core. This is easily appreciated by realizing that \( p_3 \ll p_1, p_2 \) (or \( R_3 \gg R_1, R_2 \)). Taking this into account, the fluxes can be approximated as follows

\[ \psi_{11} = ((R_1/R_2 + 1)R_3)^{-1} n_{i1} = \frac{p_1}{p_1 + p_2 + p_3} n_{i1} \]  \hspace{1cm} (2.2.11.a)

\[ \psi_{12} = ((R_1/R_2 + 1)R_3)^{-1} n_{i2} = \frac{p_2}{p_1 + p_2 + p_3} n_{i2} \]  \hspace{1cm} (2.2.11.b)

\[ \psi_m = ((R_1/R_2 + 1)R_3)^{-1} (n_{i1} + n_{i2}) = \frac{p_1 p_2}{p_1 + p_2 + p_3} (n_{i1} + n_{i2}) \]  \hspace{1cm} (2.2.11.c)
From the expression of the leakage fluxes (Eqs. (2.2.11.a) and (2.2.11.b)), it can be seen that the co-efficients of leakage induction are not necessarily equal to each other. These co-efficients of self inductance are equal (except for the number of turns of the windings) if $p_1 = p_2$ (or $R_1 = R_2$).

Above exertions are meant to show the difference between the T-network and the Π-network representation, resulting from the divided flux approach and the integral flux approach respectively. It has been clearly shown that the T-network results from an equivalent field which is not a characterization of the real field. As a result, the fluxes in the T-network cannot directly be related to physical fluxes, but through a transformation. However, since the transformer is a nonlinear device, this is not easily obtained. Notice, that the star-delta relationship (Eqs. (2.2.10.b)) only holds for linear networks. Having revealed the difference between the two approaches and knowing that for a nonlinear treatment the T-network representation is not appropriate, the integral flux approach will be used for modelling three-phase transformers.

Although the T-equivalent is more or less accepted, the Π-network can also be found in the literature [11,16,19]. However, a consistent elaboration and formalized continuation of an integral flux approach has not been given for the three-phase transformer. This is illustrated in the often awkward treatment of the three-phase transformer. Actually, in most cases, the behaviour of three-phase transformers is unnecessarily reduced to that of a single-phase transformer.

The next section deals with the modelling of three-phase transformers. In a consistent continuation of the integral flux approach, the magnetic behaviour of the three-phase transformer will be treated.

2.3 Three-phase Transformer

In this section the modelling of the three-phase transformer will be discussed, using the insight obtained in the previous section. In particular, the integral flux approach will be used for describing the flux behaviour of the three-phase transformer. The reason for using an integral flux approach and not a divided flux approach has been explained in previous section; the divided flux approach is not suitable for describing the nonlinear behaviour of the transformer. This implies that in this section the divided flux approach will be abandoned.

When using the integral flux approach, it will be seen that all types of transformer conditions, for instance, the short-circuit condition or the zero sequence behaviour, are automatically included in this integral flux description. The benefit of this method of describing the transformer flux behaviour lies in its ability of supplying the researcher with additional physically based information of the transformer flux behaviour. Notice, that the possibility of falling back on the flux behaviour runs through this thesis like a continuous thread.
2.3.1 Three-limb Transformer

In Fig. 2.11, an idealized picture of the magnetic field of a three-phase, three-limb transformer is given. As may be noticed, this idealization is a consistent continuation of the idealized field of the single-phase transformer. It can be verified that a field similar to that of a single-phase transformer (cf Fig. 2.3 of the single-phase transformer) can be associated with each limb of the transformer. Also in this case the total field will be described by considering the fields of two sections, the longitudinal (Fig. 2.11a) and the cross section (Fig. 2.11b). Notice, that for the sake of convenience only one of the three cross section fields (corresponding with the three limbs) have been displayed. Recognize also, that the field of a three-limb three-phase transformer can be viewed as a concatenation of single-phase like fields.

As in the case of the single-phase transformer, attention will particularly be paid to the air paths. The significance of these flux paths will be related to some specific (three-phase) transformer conditions. It will be shown that from a physical point of view these air paths are a logical and integral part of the three-phase transformer description.

Consider for this purpose the air paths between the inner and outer windings (henceforth also referred to interwinding paths) of every limb of the transformer. It can readily be appreciated that like the single-phase transformer model, these air paths are related to the (three-phase) short-circuit conditions of the transformer. Like the single-phase transformer, these interwinding paths are really addressed in short-circuit conditions.

In this consideration of the air paths between the inner and outer windings, it has been assumed that only three-phase short-circuit conditions are taken into account. However, it will be instructive to consider not only the well-known three-phase (symmetrical) conditions, but also the non-regular ones. For this

![Figure 2.11](image-url)  
*(a)* magnetic field, front view  
*(b)* magnetic field, side view
purpose, the three-phase way of thinking is given up, and the transformer is viewed as a magnetic system consisting of six (independent) windings with twelve terminals. Obviously, there are numerous ways of connecting these terminals, provided not only the usual transformer connections are considered.

For conceptual reasons the transformer will be subjected to a theoretical experiment. The aim of this theoretical experiment is to uncouple the understanding of the three-phase transformer flux behaviour from the known three-phase considerations, and to illustrate the application of the integral flux approach for the three-phase transformer. For this purpose, both windings of the centre limb are connected so as to constitute a single winding, whereas the remaining (four) windings of the two outer limbs are being short-circuited. When applying a voltage across the newly made winding, a short-circuit behaviour is obtained which is similar to the single-phase transformer. Actually, a short-circuit situation has been created which is similar to the single-phase transformer (of the type discussed in the previous section), of which the outer winding has been short-circuited. It is worthwhile to notice that for this short-circuit condition the flux is not forced into the interwinding paths. Since the flux is not allowed to enter the limbs nor the interwinding flux paths, the flux is forced to complete its way through the remaining air paths (henceforth referred to as interphase paths). It will be seen later that these interphase paths typically will be called upon in the zero-sequence behaviour, a behaviour which is directly related to three-phase systems. Consequently, these interphase paths are sometimes also referred to as zero-sequence paths. Notice at this point, that what essentially was intended with this theoretical experiment, is to show that idealizations of the magnetic flux of any magnetic device must be carried out so as to obtain a realistic flux description for all kinds of situations (winding connections).

Next, the generality of the magnetic field will be illustrated qualitatively by considering the two typical (symmetrical) three-phase conditions, the no-load and the short-circuit condition, of the three-phase transformer. For both conditions it is assumed that the excitation consists of a set of symmetrical, three-phase voltages. As already known, the transformer core principally determines the transformer behaviour in the no-load (steady-state) condition. Unnecessary to say that the no-load condition can easily be represented by the magnetic description given in Fig. 2.11. In this case the air paths may be neglected, since the reluctances of these paths are very high compared with that of the iron core. Conversely, it can be seen, that for three-phase short-circuit conditions, the air paths may not be neglected. As mentioned, in these cases the flux is being forced to complete its paths through the corresponding interwinding air paths. Later, when the model parameters are discussed quantitatively, these aspects will be reconsidered more closely.

It is important to recognize that for the three-phase transformer, the same distinction between the windings has been made as for the single-phase transformer. Magnetically seen, the condition of short-circuited inner windings differs from that of short-circuited outer windings. Recognize that (deliberately) the idealization of the magnetic field has been chosen so as to
distinguish only six windings (two for every limb) and consequently six independent magnetic sources.

The above described symmetrical behaviour of the three-phase transformer is quite similar to that of the single-phase transformer. Accordingly, the single-phase representation is often used in describing symmetrical three-phase conditions, and one can get a long way with it. It is evident that for asymmetrical conditions of the transformer this single-phase representation cannot be used anymore. This problem has been overcome by using the well-known symmetrical-component analysis, a method mainly developed for hand calculation purposes. The problem is then reduced to the solution of three single-phase systems with given conditions for connecting them together [38-40]. Notice here, that in that case it is tacitly assumed, that the elements of the three-phase system and consequently the transformer, possesses cyclic symmetry. Observe at this point, that released from any restrictions concerned with hand computation matters, it was not for any moment felt necessary to make similar assumptions with regard to the (computer) transformer model resulting from the integral flux approach.

An important aspect in the asymmetrical operation mode of the three-phase transformer, is the zero-sequence behaviour. A three-phase system exhibits zero-sequence behaviour if some of its (state) variables, but not necessary all, contain zero-sequence components. In the transformer case this behaviour can easily be studied by taking zero-sequence magnetic sources in the scheme given in Fig. 2.11, that is, the magnetic sources associated with the three limbs are in phase. Observe, that in this zero-sequence driven magnetic system, zero-sequence fluxes are produced. These fluxes are diverted from the core and caused to close their ways through the interphase air paths. With respect to the trajectory of the zero-sequence fluxes, it can be noted that these fluxes in their way through the air path, actually not only have to overcome the reluctance of the air, but also that of other things such as bottom, top and walls of the transformer tank. Since the character of this trajectory predominantly is determined by air [13,17], these paths are

---

Figure 2.12 Magnetic field with zero-sequence flux path
referred to as air paths. Recognize at this point, that the zero-
sequence flux behaviour (a mode resulting from the decomposition
of the three-phase description) can readily be described by the
transformer model, obtained with the integral flux approach.

In the conventional description of the three-phase
transformer, the zero-sequence behaviour is not treated as being
one of the various states of a magnetic system. The transformer
is seen as a component of a three-phase system, in which - in
contrast with single-phase considerations - allowance must be
made for one of the modes, the zero-sequence mode [14]. It should
be realized that, while the behaviour of the single-phase
transformer can satisfactorily be described with the divided flux
approach, the three-phase transformer cannot. When considering
the three-phase behaviour of the transformer, one is confronted
with a zero-sequence flux which cannot readily be related to a
separation of fluxes. One of the most plausible solutions that
has been found for coping with zero-sequence fluxes is
illustrated in Fig. 2.12. As can be seen, an additional flux path
is introduced at the joint of the centre limb and yokes. This
path is meant for interception of the flux when the sum of the
fluxes in the three limbs does not equal zero.

\[\text{Figure 2.13} \quad \text{Idealized magnetic field corresponding with (zero-
sequence) short-circuited conditions}\]

\text{a) outer windings short-circuited, front and side view}
\text{b) inner windings short-circuited, front and side view}
Above example shows how the three-phase, three-limb transformer has been modelled in the conventional way. Next, it will be considered more closely, how the zero-sequence behaviour is represented by a model resulting from an integral approach.

The idealized fields of two examples of zero-sequence short-circuit behaviour are given in Fig. 2.13. A zero-sequence short-circuit behaviour is obtained by impressing zero-sequence voltages on one set of windings, while the other set is being short circuited. Figure 2.13a shows the flux picture corresponding with short-circuited outer windings and excited inner windings. It is seen that in this case the fluxes are concentrated in the transformer limbs for building up the voltage across the inner windings. These fluxes are completing their ways through the interwinding space, since the outer windings are short-circuited. From Fig. 2.13b it is seen that when the interwinding fluxes are prohibited to close their paths through the limbs (owing to the short-circuited inner windings), these fluxes are closed through what are named the interphase air paths; these interphase air paths have been mentioned before (in the theoretical experiment). Recognize further, that these examples of zero-sequence behaviour clearly show that in describing this behaviour, the distinction between inner and outer winding is automatically introduced.

The symmetrical component approach has tackled the zero-sequence behaviour in a very typical way. For describing the proper behaviour of the various transformer connections - only the usual ones - several equivalents have been proposed [38,39]. This apparently tedious procedure has proven to be adequate as far as hand computation of linear cases is concerned. However, in pronounced nonlinear cases this method will come into trouble, since it lies heavily on the superposition principle. Unnecessary to say that a model resulting from a physically based integral approach will behave well in any of those cases. Also recognize, that in an early stage of the modelling process one can get a good idea of some specific transformer states, merely by anticipating the corresponding magnetic characteristics.

2.3.2 Five-limb Transformer

The main purpose of this section is to show how the concept of the integral flux approach can readily be applied to other magnetic configurations. For this purpose some typical characteristics of the five-limb transformer will be discussed, though this core configuration is not one of the most common used in practice. Like has been done for the three-limb transformer, the integral approach will be related to some characteristic three-phase modes.

Figure 2.14 shows the idealized field of the five-limb transformer. It can be easily verified that this idealization is similar to that of the three-limb transformer, except that in this case two outer unwound limbs are involved. Since the side view does not provide any new information - this is the same as for a three-limb transformer -, this has been left out here. From this idealization the magnetic network given in Fig. 2.15 can be derived using the previously discussed construction rules.
Recognize, that both, the three-limb transformer as well as the five-limb transformer, are represented by the same network. However, the main difference lies in the values of reluctances of the two outer network branches. The reluctances corresponding with the unwound limbs of the five-limb transformer are much smaller than the reluctances corresponding with those of the outer air paths of the three-limb transformer. The consequence of this difference can be understood by considering the zero-sequence behaviour of the five-limb transformer. For this purpose the inner windings of the limbs are short-circuited, while zero-sequence voltages are being impressed on the outer windings. As a result, the fluxes in the three limbs will vanish, and in order to build the voltage across the outer windings, the fluxes will be concentrated in the interwinding space. From the idealized field shown in Fig. 2.14 it can be seen, that these fluxes are not condemned to close their ways through the remaining interphase paths air paths, but also through the two outer
unwound limbs. Actually, the flux in these air paths may be neglected due to their high reluctance compared with that of the outer limbs. In particular, these air paths may be omitted in the model, since in any such cases, the outer limbs will always constitute a magnetic short-circuit for these air paths.

The field associated with above discussed zero-sequence short-circuit test is depicted in Fig. 2.16.a. Notice, that this field is a special case of the general field given in Fig. 2.14, where in this case field lines, corresponding with paths containing negligible fluxes, are omitted. It can be verified that the short-circuit test with short-circuited inner windings and a symmetrical set of voltages impressed on the outer windings, produces almost the same field as given in Fig. 2.16.a. The same more or less holds for the zero-sequence and normal short-circuit test with short-circuited outer windings and inner windings being excited. The field corresponding with this test is given in Fig. 2.16b. In the symmetrical component approach this feature of the five-limb transformer is converted into a zero-sequence short-circuit reactance with the same value as the normal short-circuit reactance [38].

Another salient characteristic of the five-limb transformer is its zero-sequence no-load behaviour. When (for example) the inner windings are excited with a set of zero-sequence voltages
and the outer windings are open, a picture of the magnetic field is obtained which is comparable with the normal no-load condition. Like the normal no-load condition, the flux is (almost) completely intercepted by the iron core in the zero-sequence no-load condition. Consequently, it can be verified that the zero-sequence no-load characteristic is amenable to saturation effects. It can also be shown that the magnitude of the corresponding equivalent zero-sequence inductance has the same order as its normal counterpart [40].

This is an appropriate point to make some final remarks with regard to the concept of the integral approach. It is seen that in a rather early stage of the modelling process one can subject the model to various characteristic tests. Actually, with every test a characteristic behaviour is considered and the significance of the flux paths (or branches of the magnetic network) is established. It is evident that flux paths which are not producing substantial contributions in any of the states being considered, can be omitted. For example, a close look at the idealized magnetic field of the five-limb transformer will show that the two interphase air paths between the windings of the outer limbs and the centre limb may be ignored. In this respect, it is emphasized that in an early stage of the modelling process, one should try to establish a complete physically based picture of the magnetic device. It must be assured that a model is obtained which contains functional fluxes paths, that is, flux paths with physical significance. It is also noted that in the integral flux approach, the essence of the flux paths is to split the driving force of the magnetic system into independent (distinctive) magnetomotive forces. It can be verified that the driving force of the single-phase transformer is divided into two independent magnetomotive forces, whereas that of the three-phase transformer is divided into six independent magnetomotive forces.

In modern three-phase transformer studies the configuration of the core (e.g. three-limb, five-limb, etc.) is also taken into account. However, the driving force of the model resulting from these studies is divided in only three magnetomotive forces. In fact, this division (in three magnetomotive forces) automatically follows from the three windings which are physically separated by three limbs. Recognize here, that as a result of only three magnetomotive forces, these models do not make a distinction between inner and outer windings of a limb. It is assumed that the flux through the limbs is fully linked with the two corresponding windings. Consequently, in these studies, the flux in the core is referred to as main flux. As can be verified, this is an extension of the divided flux approach, discussed in the previous section.

Finally, it may be noted, that the simulation study will be carried out for the three-limb transformer since this is the most used core configuration. In this study the general magnetic network given in Fig. 2.15 will be used. The fundamentals of the algorithm used for solving the magnetic circuit are explained in Appendix A. The choice of the model parameters of this network determines the type of the transformer. The next section deals with the determination of these parameters.
2.4 Model parameters

In this section the transformer model will be discussed quantitatively. In particular, the use of the magnetic circuit for determining the parameters of a three-limb transformer will be illustrated. The determination of model parameters of the three-limb transformer are discussed here, since in Chapter 5 transformer responses of this core type will be calculated and discussed.

Depending on the specific transformer states being considered, simplifications of the previously derived magnetic network are given. As a result simple and tractable networks are obtained for hand calculation.

As known, the transformer core characteristics are predominant in the no-load condition of the transformer. Hence, the no-load condition is very well suited for determining the transformer core parameters. In particular, no significant errors are introduced if the air paths are neglected for describing the no-load condition. What finally remains is the remarkable simple magnetic configuration given in Fig. 2.17.a, with its associated network given in Fig. 2.17.b. In the following it will be seen that this network can be hand-computed without exaggerated efforts. The reader is reminded that the assumption of the flux relationship being linear still is not abandoned.

For computing reasons all circuit parameters and quantities are referred to the three excitation windings. Needless to say, that the turns of the windings are equal. Also notice that by taking the excitation windings as a reference, the flux is considered to be generated by three windings consisting of only one turn. In terms of the integral flux approach, this means that a magnetic system is considered with the following essentials: three magnetomotive forces and consequently, three flux paths (see Fig. 2.17).

Suppose next, that the sum of the three exciting currents $i_U$, $i_V$ and $i_W$ equals zero. It is emphasized that this constraint

\[ \sum_{i=U,V,W} i_i = 0 \]

\[ \begin{align*}
\psi_U & = B S U \\
\psi_V & = B S V \\
\psi_W & = B S W \\
\end{align*} \]

\[ \begin{align*}
i_U & = i_0 \\
i_V & = i_0 \\
i_W & = -2i_0 \\
\end{align*} \]

Figure 2.17  
(a) Simplified magnetic field of the three-limb transformer  
(b) Magnetic network of the simplified magnetic field
is externally imposed, that is, it does not follow from the magnetic network given in Fig. 2.17b. As known, this occurs when the transformer is wye-connected and the neutral is not grounded. With the aid of the network (Fig. 2.17) it can easily be verified that the following three-phase (coupled) current-flux relationship holds:

\[
\begin{array}{ccc|c|c}
\text{i}_U & \alpha & (\alpha-1)/3 & 0 & \psi_u \\
\text{i}_V & 0 & (\alpha+2)/3 & 0 & \psi_v \\
\text{i}_W & 0 & (\alpha-1)/3 & \alpha & \psi_w \\
\end{array}
\]

(2.4.1)

where:

- \( R \): the reluctance of the centre limb
- \( \alpha R \): the reluctance of an outer limb plus upper and lower yoke, henceforth treated as a whole, and referred to as outer limb
- \( \alpha \): the ratio of the median length of an outer limb and the median length of the centre limb
- \( \psi_i \): flux related to the voltage \( v_i \) across winding \( i \) (i = U, V, W) through the equation

\[
v_{iN} = v_i - v_N = \frac{d}{dt} \psi_i
\]

where the voltages \( v_i \) (i = U, V, W) are related to three terminals of the windings U, V and W, whereas \( v_N \) is related to the common terminal N (the neutral) of these winding.

The factor \( \alpha \), being the ratio of the median length of outer limb and centre limb, depends on the transformer construction and its values ranges from 2.1 to 2.5 [14]. It will be shown that this factor can be determined by current measurements.

First it will be shown, that Eq. (2.4.1) can be seen as a good approximation of a wye-connected transformer whose neutral is not grounded. It can be verified that the transformer description given in Fig. 2.17 will yield a zero voltage at the neutral - henceforth also referred to as the neutral voltage - in the calculation. As known, the neutral voltage is related to zero-sequence fluxes, and these fluxes are not allowed for in this simplified transformer description (see Fig. 2.17a and 2.17b). With the neutral voltage \( v_N \) equal to zero, the voltage \( v_{iN} \) across the transformer winding \( N \) is equal to the terminal voltage \( v_i \). Hence, Eq. (2.4.1) can be seen as the characteristic equation of a wye-connected transformer.

Equation (2.4.1) reveals the relationship of the (fundamental component of the) excitation currents and the transformer core geometric. Further, it is seen that the currents through the outer windings \( iU \) and \( iW \) can be seen as the resultant of two components, one being in phase with respectively \( \psi_U \) and \( \psi_W \), and the other in phase with \( \psi_V \). This last component is a
measure for the inequality of the transformer limbs. Indeed, it can be verified that this component would vanish in the practically impossible case of \( \alpha \) being equal to unity. It is noted, that this insight was already obtained by Vidmar [14]. However, since he did not take the advantage of a magnetic network, this was achieved in a rather tedious way.

The factor \( \alpha \) can be determined from the ratio of above discussed no-load currents. For this purpose it is assumed that the impressed voltages \( v_i \) (\( i = U, V, W \)) constitute a set of normal symmetrical voltages. Since the neutral voltage is neglected, the fluxes \( \psi_i \) can directly be related to \( v_i \), that is, the fluxes \( \psi_i \) also constitute a set of symmetrical quantities. Hence, Eq. (2.4.1) can be used for the determination of the factor \( \alpha \). It can be verified that after some straightforward manipulation, the ratio \( r \) of the currents \( i_U \) and \( i_V \) can be expressed as

\[
r^2 = \left| \frac{i_U}{i_V} \right|^2 = \frac{(7 \alpha^2 + \alpha + 1)}{(\alpha + 2)^2}
\]

which yields

\[
\alpha = \frac{4r^2 - 1 + 3\sqrt{3}(4r^2 - 1)}{2(7 - r^2)}
\]

(2.4.2)

A quick check on Eq. (2.4.2) affirms that only in the theoretical case of a transformer having three limbs equal to each other, i.e. \( \alpha \) being equal to unity, the magnitude of the (fundamental component of the) currents would be equal to each other. Further, it is noticed that in practice, the factor \( \alpha \) can be determined by using values obtained with current measurements in the no-load condition of the transformer.

As mentioned above, the neutral voltage is directly related to the zero-sequence fluxes of the transformer. These zero-sequence fluxes will now be considered. Actually, the aim of the following is to gain some more analytical insights into this part of the transformer behaviour, since up to now it has been only

![Figure 2.18 Magnetic network with zero-sequence flux paths](image-url)
qualitatively discussed.

For this purpose, the network given in Fig. 2.17 can be used as starting point. A close examination of this network shows that the fluxes \( \psi_U, \psi_V, \) and \( \psi_W \) cannot be chosen independently. These fluxes are related to each other by the Kirchoff's constraint, imposing their sum, and thus the zero-sequence flux, being zero. If the fluxes \( \psi_U, \psi_V, \) and \( \psi_W \) are to be chosen independently, another equation must be introduced. From a physical point of view, this means that new flux paths must be introduced for carrying zero-sequence fluxes.

Figure 2.18 is an extension of the network given in Fig. 2.17b, and shows how this extension has been carried out; two paths have been introduced for the three-limb transformer. In fact, one additional flux path would have been enough to meet the requirement of having three independent fluxes. However, when deriving the three-phase transformer model (six independent magnetic sources and six independent fluxes) more flux (air) paths have been considered. In the general network of the three-limb transformer given in the previous section (Fig. 2.11), several air paths have been used for describing the zero-sequence behaviour. Particularly, the air paths not belonging to the interwinding flux paths are responsible for the zero-sequence behaviour. Notice however, that with the magnetic network given in Fig. 2.18 the flux is allowed to divert from the iron core at only two points of the yoke. It should be emphasized that this simplification of the general network has been done solely for (hand) computation reasons. The general network does not lend itself to study the transformer behaviour by means of simple formulas. Thus, the zero-sequence behaviour will be studied using the paths introduced in the network given in Fig. 2.18.

It would be virtually impossible to point out where exactly the flux will leave the yoke, in other words, where the magnetic node must be defined. So far, this aspect has not yet been discussed, and with this simplified model the effect of the positions of these nodes can more or less be analysed. In the network of Fig. 2.18 the looseness of the additional magnetic nodes is effectuated by the factor \( \beta \). If \( \beta \) is equal to zero, the flux is supposed to leave the core at the top of the centre limb. In most existing three-phase transformer models, the zero-sequence behaviour is represented in this way \([13, 63, 72]\). Recognize, that in these models only three magnetotive motive forces are considered, and that therefore the inner windings cannot be discriminated from the outer windings. Also notice, that this discrimination has not been allowed for in the network of Fig. 2.18. This has only been done for computational reasons; the analysis given here is tentative: the purpose is to have to a certain extent some quantitative grasp of this part of the flux behaviour, rather than a rigorous analytical treatment on this subject.

For this purpose it is assumed that the transformer is in no-load condition, and that the terminal voltages \( v_i \) can be related to fictitious fluxes as follows.

\[
v_i = \frac{d}{dt} \phi_i \quad i = U, V, W\]  

(2.4.3)
Notice that these fictitious fluxes $\Phi_i$ can be related to a set of voltages impressed at the transformer terminals. The reason for using fictitious fluxes instead of voltages is that by working so we are able to use algebraic equations (current-flux relations) instead of differential equations.

Suppose further, that the voltage $v_i$ constitute a set of normal symmetrical voltages, that is

$$v_U + v_V + v_W = \frac{d}{dt} (\Phi_U + \Phi_V + \Phi_W) = 0 \quad (2.4.4)$$

Then, it can be shown that the fictitious fluxes $\Phi_i$ can be related as follows to the fluxes $\Psi_i$ of a wye-connected transformer with an ungrounded neutral:

$$\begin{pmatrix}
\Phi_U \\
\Phi_V \\
\Phi_W
\end{pmatrix} = \begin{pmatrix}
2 & -1 & -1 \\
\frac{1}{3} & -1 & 2 \\
-1 & -1 & 2
\end{pmatrix} \begin{pmatrix}
\Psi_U \\
\Psi_V \\
\Psi_W
\end{pmatrix} \quad (2.4.5)$$

It can also be shown, that the voltage at the neutral, $v_N$, can be related to the zero-sequence flux $\Psi_o$ as

$$v_N = -\frac{d}{dt} \Psi_o \quad (2.4.6)$$

with

$$\Psi_o = \frac{1}{3} (\Psi_U + \Psi_V + \Psi_W) \quad (2.4.7)$$

The last equation which is imposed by the transformer connection is the current balance at the neutral point. This can be expressed as

$$i_U + i_V + i_W = 0 \quad (2.4.8)$$

Finally, it can be verified that the magnetic network of Fig. 2.18 can be described by the following current-flux relationship

$$\begin{pmatrix}
\alpha + \beta (\alpha - \beta) R/2R_o & -1 & 0 \\
1 + (\alpha - \beta) R/2R_o & 1 & 1 + (\alpha - \beta) R/2R_o \\
0 & -1 & \alpha + \beta (\alpha - \beta) R/2R_o
\end{pmatrix} \begin{pmatrix}
\Psi_U \\
\Psi_V \\
\Psi_W
\end{pmatrix} =$$
\[
\begin{vmatrix}
1 + \beta R/2R_o & -1 & 0 \\
R/2R_o & 0 & R/2R_o \\
0 & -1 & 1 + \beta R/2R_o \\
\end{vmatrix}
\begin{bmatrix}
i_U \\
i_V \\
i_W \\
\end{bmatrix}
= (2.4.9)
\]

Notice, that the form of above current-flux relationship enables a quick verification of these equations. Suppose for example that the reluctance of the two additional paths \( R_o \), that means that these paths are omitted, and that the original network of Fig. 2.17 is obtained. Then it is easily seen that the first and the last Eqs. of (2.4.9) - corresponding with the first and last row entry in the matrix equation (2.4.9) - are related to the two corresponding remaining meshes. The middle equation of Eq. (2.4.9) changes into the Kirchoff's constraint - the sum of the three fluxes being zero - at the joint of the centre limb and yokes.

After a few but straightforward manipulations, and neglecting higher order terms of \((R/R_o)\), the following approximation of the zero-sequence flux \( \psi_o \) is obtained.

\[\psi_o \approx \frac{1}{3} \left\{ (\alpha - 1)R/R_o - \frac{3}{2} \beta R/R_o \right\} \Phi_V \]  \hspace{1cm} (2.4.10)

Examination of Eq. (2.4.10) shows that the fundamental component of the neutral voltage is low, and that it is mainly determined by the reluctance \((\alpha - 1)R\) of the yoke length and the reluctance \(R_o\) of the zero-sequence flux paths. It is worth to be noted, that since in this model the flux is not forced to cover the whole yoke length, a term shows up in Eq. (2.4.10), which is taking the flux outlets of the yoke into account. As can be seen, the effect of taking only one point at the yoke as a flux outlet, i.e., \(\beta = 0\), results in a higher magnitude of the fundamental component of the neutral voltage.

Consider next, the case of a wye-connected transformer with an earthed neutral. This configuration can be expressed with the following equations

\[v_i = \frac{d}{dt} \Phi_i = \frac{d}{dt} \psi_i \]  \hspace{1cm} (2.4.11)

and

\[i_N = i_U + i_V + i_W \]  \hspace{1cm} (2.4.12)

where \( \Phi_i \) is a fictitious flux related to a set of voltages impressed at the transformer terminals, and \( i_N \) is the current through the neutral wire, henceforth also referred to as the neutral current. With the given current-flux relationship (2.4.9) and some mathematical manipulations, neglecting higher order terms of \(R/R_o\), the neutral current can be expressed as
\[ i_N = \left\{ (\alpha - 1)R - \frac{3}{2}\beta (R/R_o) \right\} \Phi_V \] (2.4.13)

Like the previous case of the neutral voltage, the neutral current is closely related to the symmetry of the transformer core via the reluctance of the yoke \((\alpha - 1)R\). Notice, that the provided flux outlets at the transformer yokes, has again been taken into account by a term containing the factor \(3/2\beta\). Later, in Section 5.5 of Chapter 5, we will have recurrence to this finding.

As a last example, the zero-sequence paths will be related to the open zero-sequence impedance-test. Notice, that this terminology, emanated from the symmetrical components approach, is based on a pure mathematical concept. Briefly considered, the method of symmetrical components introduces a transformation (diagonalization) for obtaining decoupled descriptions of the symmetrical three-phase components in a three-phase system, and the coupling conditions resulting from the asymmetrical components (usually faults in power systems). The three modes of the transformed system, the zero-, positive- and the negative-sequence, have shown to be physically interpretable. Physically seen, by subjecting a three-phase component to a zero-sequence excitation, its zero-sequence mode can be analysed. Thus, in the case of an open zero-sequence impedance test, it must be assumed that the impressed set of voltages \(v_i\), or the fictitious set of fluxes \(\Phi_i\) given in (2.4.11) now are a set of zero-sequence voltages or fluxes. In practice this corresponds with a star connected transformer with the same voltage impressed on the windings. Leaving out the mathematical details it can be proven that the following simple expression can be obtained (ignoring the higher order \(R/R_o\)-terms)

\[ i = i_U + i_V + i_W = \left\{ 9R_o + (1 + 2\alpha + 3\beta)R \right\} \Phi \] (2.4.14)

Notice, that in above equation \(R \gg R_o\), and thus, it is seen that for a three-limb transformer the open zero-sequence impedance is mainly determined by the yoke-to-yoke air paths, and therefore can be considered as almost constant. This has been experimentally confirmed in Ref. [40].

In the foregoing it has been shown that the factor \(\alpha\), eventually can be related to a magnetic node, and that \(x\) can be determined by considering the no-load excitation currents. Just like \(\alpha\), the factor \(\beta\) can also be related to magnetic nodes, but cannot be identified from alike considerations. The expressions of the neutral voltage and/or neutral current given in Eqs. (2.4.10) - (2.4.14) are not suited for this purpose. It can be appreciated that when using the fundamental components of the neutral voltage and/or neutral current, the accuracy of the method can become questionable. It will be seen (in Section 5.5 of Chapter 5) that the amplitude of (the fundamental component of) these quantities are relatively low, compared with that of the higher harmonics. Hence, the factor \(\beta\) must only be seen as an additional parameter suited for fine-tuning of the transformer zero-sequence behaviour. Since it is most likely that the flux will leave the core at the top and bottom of the limbs (in the
zero-sequence condition described above), the nodes of these paths have been attached at these positions (cf. Fig. 2.15, Section 2.3). Each of these paths is assigned with the same reluctance. Notice, that this idea has already been signalized in reference [60,73]. However, as already mentioned, these existing magnetic descriptions do not make a distinction between inner and outer windings, and hence some zero-sequence states cannot be described. This has already been discussed in the previous section.

Finally, our attention will be focussed on the air paths between the inner and outer windings of the limbs, the interwinding flux paths. It has been shown that these paths are responsible for the distinction between inner and outer windings. It has also been shown, that especially in the three-phase short-circuit conditions, these air paths will be occupied by the flux. Hence, for the determination of the reluctance of these paths, it suffices to consider the three-phase (symmetrical) short-circuit condition of the transformer.

Fig. 2.19a shows the mapping of the magnetic field corresponding with short-circuited outer windings and energized inner winding. From this figure it is seen that the flux is partially in the core and partially in the inner winding space. Further, it is seen that the field at the centre limb will differ in some degree from the field of the outer limb. As can be seen, this difference is inherent in the construction of the transformer core. However, in this case, the predominant effect of the air is so as to allow the effect of the core to be disregarded. In particular, no significant errors will be introduced when this three-phase short-circuit condition is represented by a magnetic network associated with one limb, as given in Fig. 2.19b. From this network it is seen that the reluctance of the air paths of the inner winding space simply follows from
\[ i = (R + R_{SC}) \Psi = R_{SC} \Psi \tag{2.4.15} \]

where \( i \) is the short circuit current in the windings and \( \Psi \) follows from the impressed flux. The reluctance \( R_{SC} \) which is dictating the short-circuit behaviour, can be determined provided \( \Psi \) and \( i \) are known.

Recognize, that in above determination of the model parameters some characteristic states, steady states, of the transformer have been used. Also notice, that it has been assumed that the magnetic circuit is linear. Since the transformer is a nonlinear device, only the fundamental component of these states (and other similar states) will satisfactorily be described. As will be seen when discussing transformer responses in Chapter 5, the harmonics in the voltage and current waveforms, a result of the nonlinearity, are not so well described. This fact can directly be related to the identification method, described here, which basically is for linear systems. However, for most of the cases to be considered, this identification will yield a reasonable accuracy. When the accuracy becomes more critical, one should use identification methods developed for nonlinear systems, which implies that the transformer can not be characterized anymore by the conventional data.

Before calculating the transformer responses, it will be shown in the next section how the flux equations from the magnetic circuit can be integrated into the voltage equations of the transformer.

2.5 Transformer Voltage Equations

In the previous sections a physically based concept has been discussed for the description of the flux behaviour of the transformer. In particular, a magnetic network has been constructed for representing this behaviour. With this magnetic network formulation the flux equations required for the transformer voltage equations can easily and systematically be derived.

This section deals with the transformer voltage equations. In fact, the coupling from the magnetic network to the electric network will be elaborated. An example of the single-phase transformer will be used to illustrate how this coupling between magnetic and electric network can be described.

As already known, the terminal behaviour of the transformer is expressed by the voltage equations. The most compelling reason for considering these equations, of course, stems from the fact that one wishes to analyze the individual transformer behaviour as well as the mutual interaction between transformer and the rest of the electric power systems. These voltage equations are derived by using the well-known Faraday's law.

Generally speaking, the transformer can be considered as a multi current-carrying winding system, interacting with one another (in the presence of magnetic materials). For the sake of simplicity - the problems concerned with skin effect are circumvented here -, the conductivity of the windings are assumed to be zero, that is, the current can only flow on the surface of
the conductor constituting the windings. From the quasi-static approach it is known [1,2], that for any winding consisting of closely spaced turns, the terminal voltage can be defined as the line integral of the electric field over a closed path, identical to the paths associated with the winding but with opposite reference direction, that is,

$$ \oint_{C_k} \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_{S_k} \mathbf{B} \cdot d\mathbf{s} $$

or

$$ v_k = \frac{d\psi_k}{dt} $$

(2.5.1)

where $S_k$ is a (two sided simply-connected) surface bounded by a closed contour $C_k$ related to the geometry of the $k$th-winding, $v_k$ is the terminal voltage of the $k$th-winding and $\psi_k$ is the flux linked by the $k$th-winding. It is noticed that the contour $C_k$ coincides with the turns of the winding, except for a short path joining the terminals of the winding. Further, it is worth knowing, that the reference direction of $C_k$ is the same as that of the winding current $i_k$.

At this point we would like to emphasize, that in applying the quasi-static approximation - according to Ref. [1,2], the field is approximated by a zero-order magnetic field and a first order electric field - the model is limited to slow time-varying phenomena. In general, when analyzing phenomena whose frequencies are extremely high, one often must fall back upon Maxwell's equations. Particularly, higher order approximations of the electric and magnetic field can be introduced for systems whose mode of operation lies on the line of demarcation between lumped and distributed elements. The study of the consequence - expressed in the voltage distribution along the winding - of a surge striking the transformer winding is a typical example. Under these circumstances the displacement currents cannot be disregarded, in other words, the capacitance of the turns of the winding must be taken into account. Obviously, a detailed picture of the winding must be used, by considering higher order approximations of the electromagnetic field. Actually, these higher order approximations will lead to the well-known L-C-circuits used for describing the response of the windings under above mentioned conditions. This intricate behaviour has been, and still is the concern of many investigators, as may be illustrated by the compilation of 1263 papers dealing with this subject which has been given by Abetti in 1964 [51].

Turning back to the derivation of the voltage equations, it is noticed that the flux which links the winding, logically appears in the branches of the magnetic network, corresponding with these windings, specifically, the branches containing the magnetic sources. When the resistance of the windings is taken into account, by assuming them lumped and in series with the above assumed windings of infinite conductivity, the transformer behaviour can be described by
\[ v_k - v_1 = R_{kl} i_{kl} + \frac{d}{dt} N_{kl} \psi_{kl} \]  \hspace{1cm} (2.5.2)

with the current-flux relation, which follows from the magnetic network, given by

\[ \psi_{kl} = P N_{kl} i_{kl} \]  \hspace{1cm} (2.5.3)

where

- \( v_k - v_1 \) : vector of terminal voltages of the windings
- \( R_{kl} \) : diagonal matrix of winding resistances
- \( i_{kl} \) : vector of currents through the windings
- \( N_{kl} \) : diagonal matrix of the numbers of turns of the windings
- \( \psi_{kl} \) : vector of fluxes linking the windings
- \( P \) : matrix of permeances resulting from the magnetic network.

Recognize, that the behaviour of the transformer is described by Eqs. (2.5.2) and (2.5.3), but that generally speaking, no solution can be obtained. For a solution, an additional set of equations is required which describe the other components of the circuit the transformer is connected with. In order to elucidate this process of mathematical bookkeeping, an example will be given.

For this purpose, the very simple network given in Fig. 2.20 is considered. This network represents a single-phase transformer having a resistive load, and being connected to a voltage source. This example is given, since the essence of the coupling of the transformer equations with the equations of the network (the transformer is connected with) is very clearly illustrated.

The voltage equations of the single-phase transformer can

![Network representation of a single-phase transformer with a resistive load](image)
be written as

\[
\begin{bmatrix}
\mathbf{v} - \mathbf{v}_1 \\
\mathbf{v} - \mathbf{v}_2
\end{bmatrix} =
\begin{bmatrix}
\mathbf{R} & 0 \\
0 & \mathbf{R}
\end{bmatrix}
\begin{bmatrix}
\mathbf{i}_1 \\
\mathbf{i}_2
\end{bmatrix} + \frac{d}{dt}
\begin{bmatrix}
\psi_{pl} \\
\psi_{s2}
\end{bmatrix}
\quad (2.5.4)
\]

whereas the flux equations (obtained from the magnetic network, as can be verified in Section 2.2, Eq. (2.2.8)) are given by

\[
\begin{bmatrix}
\psi_{pl} \\
\psi_{s2}
\end{bmatrix} =
\begin{bmatrix}
p_{11} & p_{12} \\
p_{21} & p_{22}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
n_2
\end{bmatrix}
\quad (2.5.5)
\]

with

\[n_{21} = \frac{n_{s2}}{n_{pl}}\]

and

\[n_{pl} : \text{number of turns of the primary winding } p_1\]

\[n_{s2} : \text{number of turns of the secondary winding } s_2\]

The fluxes \(\psi_k\) and the permeances \(p_{kl}\) (\(k = 1, 2\), \(l = 1, 2\)) are referred to the primary side. For solving this network problem, information of how the transformer is connected with the rest of the network must be given. This is confined in the following network equations:

\[
\begin{bmatrix}
\mathbf{v} - \mathbf{v}_1 \\
\mathbf{v} - \mathbf{v}_2
\end{bmatrix} =
\begin{bmatrix}
\mathbf{e}(t)
\end{bmatrix}
\quad (2.5.6)
\]

Having given these equations the network problem can readily be solved. For this purpose Eqs. (2.5.6) and (2.5.4) are substituted in (2.5.4), which yields the following expression:

\[
\begin{bmatrix}
\mathbf{e}(t)
\end{bmatrix} =
\begin{bmatrix}
\mathbf{R} & 0 \\
0 & \mathbf{R} + \mathbf{R}
\end{bmatrix}
\begin{bmatrix}
\mathbf{i}_1 \\
\mathbf{i}_2
\end{bmatrix} + \frac{d}{dt}
\begin{bmatrix}
\mathbf{i}_{pl} \\
\mathbf{i}_{s2}
\end{bmatrix}
\quad (2.5.7)
\]

with
\[ L_{12} = p_{11} \]
\[ L_{12} = n_{21} p_{12} \]
\[ L_{22} = (n_{21})^2 p_{22} \]  

(2.5.8)

As can be seen, the transformer circuit given in Fig. 2.20 is described by a set of ordinary differential equations given by (2.5.7). These equations can be solved, provided the initial values of the primary and secondary current are given.

A few comments about above formulation would not be misplaced here. The matrix of induction co-efficients introduced in (2.5.7), directly follows from the magnetic network of the single-phase transformer. However, this induction co-efficient formulation has its pros and cons. Working with induction co-efficients has the advantage of reducing the equations describing the problem. Realize however, that with an induction co-efficient formulation (e.g., [79]), the description of the flux behaviour of the transformer can become intransparant. Often, especially in transient studies, the information of the transformer flux behaviour can be of incalculable value for the explanation of the transformer terminal behaviour. Examples of how to draw on the flux behaviour will be given in Chapter 5. Finally, it is noted that an induction co-efficient formulation is mostly used when one is not interested in the flux behaviour of the transformer (in loadflow studies for example).

As explained above, Eqs. (2.5.4) through (2.5.7) are chosen so as to facilitate the understanding of a generalized attack of the problem. It is seen that the current-flux relation is kept separately. This has been done, since this relation in fact is nonlinear. In Chapter 4 a method will be discussed which solves this set of nonlinear equations simultaneously with the network equations. In particular, a structured and generic approach for a network containing nonlinear elements, especially transformers, will be considered.
For conceptual reasons, the constituent relationship between the flux, density $B$ and the magnetic field intensity $H$ has been taken linear in Chapter 2. This chapter is concerned with the nonlinear $B$-$H$ relationship, responsible for the pronounced transformer behaviour such as inrush, ferro-resonance and residual flux conditions of the transformer core. Unnecessary to say that for studying these phenomena, an adequate representation is indispensable. Actually, the major purpose of this chapter is to give a hysteresis description, that is, a description that is suited for calculating residual core conditions for a three-phase, three-limb transformer.

In most studies on transformers the $B$-$H$ relation is taken very simple, whereas in studies concerned with, for instance, magnetic registration the $B$-$H$ relation is often very complex. In this chapter a compromise between these two approaches is presented; a description of the $B$-$H$ relation is given which is suited for transformer studies.

The description of the $B$-$H$ relation is considered separately, because of its particular characteristics, but above all, because this part of the modelling process will be tackled fundamentally different from the physical approach. Unlike the physical approach followed in Chapter 2, the phenomenologic approach will be used for describing the $B$-$H$ relation in this chapter. This approach is used here, since it is doubtful whether the result of a physical attack of this modelling problem would warrant the effort expended. In most cases the physically based models are too complex and must be stripped in order to fit in larger system description.

In Section 3.1 of this chapter, a single-valued function, the saturation curve, is given for the $B$-$H$ relation. The saturation curve is considered, since it forms the base for explaining the typical nonlinear behaviour of the transformer. It is also shown, how this curve can be derived from (conventional) measurements.

Section 3.2. is concerned with the far from easy task of describing the hysteresis phenomenon. First, the dynamic hysteresis - a terminology used in the theory of magnetic to allow for frequency dependence - will be considered. It is shown that this description is strongly related to the eddy-current effect of the iron core. Then, the frequency-independent part of the hysteresis behaviour, also referred to as static hysteresis, is considered. This multi-valued $B$-$H$ relation is described by a family of functions which has a similar shape as the saturation curve. It will be discussed in detail how these functions are joined together as to describe the static hysteresis.
3.1 Saturation Curve

The empirically established nonlinear relationship between B and H, usually takes the form as illustrated in Fig. 3.1. As can be seen, the hysteresis characteristics of the ferromagnetic material have not (yet) been taken into account.

Over the years many investigators have addressed themselves to fabricate analytical expressions, in order to predict this curve, referred to as the saturation curve [1-7]. In most of these studies, analytical expressions describing the nonlinear B-H relationship as an assembly of functions have been avoided. This mainly has been done to circumvent mathematical difficulties, which would arise when attempts are being made to reach analytical solutions. Since it is not the intention to produce analytical solutions - numerical solutions will be considered -, we do not have to deal with any of these mathematical restrictions. Therefore, in this study, the saturation curve is approximated by an assembly of exponential functions, producing both a proper prediction of the curve and a well-behaved differential permeability dB/dH [3,4].

![Saturation curve](image)

**Figure 3.1 Saturation curve**

It is worth to note, that the choice of a particular function does not harm the generality of our consideration. In fact, any suitable function can be used. The general analytical expression of the saturation curve used in this thesis is given by

\[
B = \mu_0 H + \sum_{j=1}^{n} K_j \left[ 1 - \exp(-K_{j+1} H) \right]
\]  

(3.1.1)

where the constants \( K_j \) and \( K_{j+1} \) can be evaluated using an iterative procedure, described in Ref. [4].

Instead of instantaneous values required in Eq. (3.1.1), one often has to deal with measured data given in RMS values.
Data acquisition of the saturation curve of transformers usually is accomplished with a sinusoidal input (i.e. an applied sinusoidal voltage), the magnetic induction \( B \). Its peak value is \( \sqrt{2} \) times the RMS value of \( B \). For the corresponding RMS-value of \( H \), no simple expression exists from which the peak value of \( H \) can be obtained. A procedure of determining the peak value of \( H \) iteratively from RMS values will be discussed in the following. More specifically, the main lines of an algorithm which converts the "measured" RMS \( H \)-values - in fact, the "measured" RMS \( H \)-values are derived from measured RMS values of currents - into instantaneous values will be outlined. This algorithm is discussed in detail in Ref. [8].

Starting from the assumption that some of the peak values of \( H \) corresponding with the given peak values of \( B \) (\( \sqrt{2} \) times the measured RMS values of \( B \)), have been determined, we are at one of the steps of the algorithm. Given these data, it is not difficult to construct a piece-wise linear function which relates the thus far determined peak values of \( B \) and \( H \). The next step of this converting algorithm - assume the \( k \)th step - consists of an extension of the piece-wise linear description, which can be used to determine the next peak value of \( H \) associated with the \( k \)th peak value of \( B \) (greater than that of the previous step). From this point, the algorithm proceeds as follows.

The response \( H(t) \) and its associated peak value, resulting from a sinusoidal excitation \( B(t) \) with its maximum equal to the peak value of this step, is calculated. This can be carried out, since the \( B-H \) characteristic is supposed to be known, that is,

![Diagram](image)

**Figure 3.2** Determination of the instantaneous \( B-H \) relationship
- first quadrant: saturation curve and its estimation
- second and third quadrant: excitation \( \sqrt{2} B_{\text{rms}} \cos(\omega t + \phi) \)
- fourth and third quadrant: \( H(t) \) and its estimation
the piece-wise linear approximation with the extension made in this step (see Fig. 3.2). Since a (candidate) response \( H(t) \) is known, the corresponding RMS value can be calculated. With this calculated RMS value of \( H \) and the corresponding given (measured) RMS value of \( H \), a test for the correctness of the peak value of \( H \) in this step of the algorithm, can be carried out. The RMS value \( H_{\text{rms}} \) of this \( k \)th step is calculated by applying

\[
H_{\text{rms}} = \sqrt{\frac{1}{T} \int_{0}^{T} H^2(t) \, dt}
\]  

(3.1.2)

to the calculated instantaneous \( H(t) \), resulting from the excitation \( B(t) \), where \( T \) is the period of \( B(t) \) and consequently, of \( H(t) \). This RMS value related to this first guess is compared with the given RMS value of \( H \) of this step. The discrepancy between these RMS values is used to obtain the next guess for the extension of the function

\[
H_k^{(i+1)} = H_k^{(i)} + (H_{\text{rms},k} - H_k^{(i)}) \, C
\]  

(3.1.3)

where \( C \) is a suitable constant, whereas in this \( k \)-th step, \( H_k^{(i)} \) is the \( i \)-th guess, \( H_k^{(i)} \) is the \( i \)-th calculated RMS value and \( H_{\text{rms},k} \) is the given RMS value. In most cases the calculated RMS value will compare favourable with the given RMS value after some iterations. A well-behaved iterative process is obtained by using small adjacent intervals of the measured data.

In obtaining a proper peak value of \( H \), we have reached the very step at which the loop of the algorithm has been cut. Thus, this iterative scheme has to be maintained, starting from the first value of \( B \) (the minimum) to the last value of \( B \) (the maximum) inclusive.

The relationship established in this way relates the peak values of \( B \) and \( H \), and is at the same time the relationship required for the instantaneous values. With the aid of a fitting procedure a suitable curve (3.1.1) passing through these converted points can be found.

Recognize, that in above algorithm of finding peak values of \( H \) from a set of measured RMS \( B \) and \( H \) values, it is tacitly assumed that the \( B-H \) relation is single-valued. Since this is not true in practice, an inaccuracy is introduced in the calculation of the peak value of \( H \). In order to reduce this inaccuracy the algorithm can be refined by introducing a simple correction. This correction is based on the knowledge (see also Section 3.2.1) that \( H \) can be considered as being built up by two components. The first component of \( H \) is related via a non-linear single-valued relation to \( B \), whereas the second component of \( H \) is related to the total iron core losses. This can be easily conceived when the network given in Fig. 3.3 is considered. It can be seen that for this purpose the quantities \( c \) and \( i \) are used instead of the fundamental quantities \( B \) and \( H \). This change of quantities can be carried out as follows. Let the \( B-H \) relationship (e.g. Eq. (3.1.1)) be given as general as possible, that is,
Figure 3.3  Network representation of the two components related to the measured magnetizing current

\[ B = f(H) \]  \hspace{1cm} (3.1.4)  

where \( f(.) \) is a monotonically increasing function exhibiting the characteristic shown in Fig. 3.1. Ignoring the physical background, it is seen that, in fact we are dealing here with a simple mathematical scaling procedure. Thus, Eq. (3.1.1) can be easily converted into

\[ (\psi_n/B_n)B = f((i_n/H_n)H) \]

or

\[ \psi = f(i) \]  \hspace{1cm} (3.1.5) 

with

\[ \psi = (\psi_n/B_n)B \quad i = (i_n/H_n)H \quad \text{and} \quad \psi_n = u_n/\omega \]

where \( \psi, B, u, i \) and \( H \) are the nominal (peak) values of the corresponding quantities, and \( \omega \) is the radial frequency. Having introduced this change of quantities, it can now be easily verified that the current (instead of \( H \)) is the sum of two components, as given below.

\[ i = i_{Fe} + i_m \]

or

\[ i = G \frac{d\psi}{dt} - f^{-1}(\psi) \]  \hspace{1cm} (3.1.6) 

where

\[ i_{Fe} \quad : \text{component of the current related to the core losses} \]
\[ i_m \quad : \text{component of the current related to the restoring function} \ f(.) \]
conductance taking into account the core losses.

With $G \neq 0$, a guess of the current in the $k$-th step of the converting algorithm can be calculated as follows:

$$i_k(t) = G_k \hat{u}_k \cos(\omega t) + f_k^{-1}(\hat{u}_k/\omega_k \sin(\omega t)) \tag{3.1.7}$$

where

- $G_k$: conductance taking into account the core losses for the $k$-th step
- $\hat{u}_k \cos(\omega t)$: sinusoidal voltage excitation for the $k$-th step
- $\omega_k$: radial frequency
- $f_k(.)$: the piece-wise linear $\psi$-$i$ function with the extension of the $k$-th step (see Fig. 3.2)

Notice, that for $G = 0$, the previously discussed case is obtained. Also notice, that the rest of the algorithm remains unaltered; the difference between the calculated and measured RMS values is used to obtain a next guess for the extension of the last step.

### 3.2 Hysteresis

When the electromagnetic behaviour of electric power systems containing devices with cores of ferromagnetic materials is being studied, sometimes one cannot ignore the hysteresis phenomenon. A survey on this subject learns, that there is no question of any lack of papers dealing with hysteresis. On the contrary, the non-single valued relationship between the magnetic flux density $B$ and the magnetic field $H$, with its intricate character appeals to many scientists, and an abundance of papers has appeared on this subject. However, most of the descriptions given in these papers, are not suited for our purpose due to their mathematical complexity, or conversely, due to the oversimplifications that have been introduced. In fact, most of these models are only tractable for (simple) case studies and not for system studies.

On solving the hysteresis modelling problem, again two basic approaches can be distinguished. One approach, based on physical principles, is trying to get a macroscopic picture by elaborating the physical laws, governing the micro mechanisms of the material. Obviously, this approach is the most appealing, because if carried out properly, this phenomenon would be described in a quite realistic way. However, various idealizations would be necessary in order to arrive at appropriate transformer equations. Hence, the accuracy of this physically based model will be affected.

Compared with the still complex descriptions which result from the physical approach, the phenomenological approach is much more attractive and pragmatic. In this case, suitable mathematical (nonlinear) expressions, simulating the hysteresis phenomenon, are being postulated. In doing so, we readily encounter one of the greatest problem in mathematics.
As contrasted with linear systems, no theorem exists which is concerned with the validity of a nonlinear model (the postulated nonlinear expressions). For linear systems fulfilling fairly mild conditions, there exists a theorem which states, that if a linear model is found to give an exact response - for example, the impulse response - corresponding to a given test signal, then the model is valid in that it will give the correct response to all excitations [9]. In the absence of a similar theorem, practically applicable to nonlinear systems, a quantitative proof of the validity of the nonlinear model cannot be given. Strictly speaking, the nonlinear model can only be established by considering the responses of an infinite set of test-signals. Hence, the hysteresis model will be discussed within the confines of a finite set of excitations, in other words, hysteresis properties will be extracted from a finite set of observations (excitations). Considering our purpose - allowance for the hysteresis phenomenon in a transformer model - a hysteresis model will be constructed which is entitled to carry the designation of simplicity, but at the same time representing the hysteresis peculiarities in an acceptable manner. Recognize, that in this context (hysteresis) modelling often is more of an art than science.

3.2.1 Dynamic Hysteresis Model

The dynamic nature of hysteresis is realized by introducing a (nonlinear) differential equation. Using the mathematical notation of Chua et al. [10,11], the dynamic hysteresis [10-14] is defined by

\[
\frac{dB}{dt} = g( H(t) - f(B(t)) )
\]  

(3.2.1)

where \( g(\cdot) \) and \( f(\cdot) \) are strictly monotonically increasing functions, with nonzero slopes (throughout the entire real line) and satisfying the property \( g(0) = f(0) = 0 \).

It can be shown that above equation, being related to dynamic hysteresis, is the description of a phenomenon which in the electrical world is better known as the eddy current effect. To show this, we must fall back upon the physical approach. Consider for this purpose a lamination of the transformer core (see Fig. 3.4). Assume, for this purpose that the magnetic induction \( B \) possesses only one component, the \( z \) component. Assume further that the thickness of the lamination is such that the eddy-currents are practically parallel with the x-y plane. Accordingly, in the following discussion it is assumed that the eddy-currents have one component, the x-component. It is also assumed that the reaction of the eddy currents on the magnetic induction can be ignored. Then, the following fundamental Maxwell's equations will be considered as point of departure.

\[
\text{rot } E = - \frac{dB}{dt}
\]

\[
\text{rot } H = J
\]
With the next constituent relations, which for the sake of simplicity are taken linear,

\[ J = \sigma E \]

\[ B = \mu H \]

the following equation is produced

\[ \mu \frac{\partial^2 B}{\partial y^2} = -\mu \sigma \frac{\partial B}{\partial E} \]

Taking into account the early made assumptions the well-known diffusion equation is obtained

\[ \frac{\partial^2 B}{\partial y^2} = \mu \sigma \frac{\partial B}{\partial E} \] (3.2.2)

Above equation shows that the eddy-current effect is described with a partial differential equation. Generally speaking, partial differential equations take into account the distributed nature of a phenomenon. Since the transformer equation will be incorporated into a system describing only lumped models, (ordinary differential equations), Eq. (3.2.2) is not suited for our purpose. Obviously, this means that this distributed behaviour must be converted appropriately into a suitable lumped behaviour. This conversion process of obtaining a (satisfactory) lumped description from the distributed behaviour, can be discussed in terms of system theory.

For this purpose Eq. (3.2.2) is considered as a description of a linear distributed system from which the transfer characteristics is being sought. Since the system is linear, this can be done by calculating the system response to a given excitation. It can be shown [15] that if the lamination with zero
initial conditions is exposed to a step function of the magnetic induction $H$ - the surface of the lamination is being held on a constant value of $H$ - the response (solution) can be easily calculated.

This response contains the $x$-coordinate, expressing the distributed nature of the eddy-current effect. In order to obtain a lumped behaviour, the $x$-coordinate must be eliminated. This can be done for example by taking the average of the response ([16]), or considering the losses dissipated in the system ([17]).

In this way a response is obtained which does not depend on the $x$-coordinate, whereas it can also be shown that an approximation of this response can also be obtained from the linear system described by the following differential equation:

$$\frac{dB}{dt} = a(H - \mu^{-1}B) \quad (3.2.3)$$

where $a$ follows from the approximation method used for obtaining the lumped behaviour. As can be seen, above equation is the linear case of the general non-linear dynamic hysteresis description given in (3.2.1).

This dynamic hysteresis (or eddy-current effect) can be understood easily when the following obvious change in notation, and re-arrangement are introduced (cf. Eq. 3.2.1)).

$$i(t) = R^{-1} \frac{d\psi}{dt} + L^{-1} \psi \quad (3.2.4)$$

with

$H \rightarrow i$, $B \rightarrow \psi$, $f(.) \rightarrow L^{-1}$ and $g^{-1}(.) \rightarrow R^{-1}(.)$

or for the linear equation (3.2.3)

$a \rightarrow L^{-1}$ and $\mu^{-1} \rightarrow R^{-1}$

![Electric network representation for dynamic hysteresis](image_url)
Considering the particular case of $R(.)$ and $L(.)$ being linear functions, enables us to elucidate this model by means of an electric circuit. Figure 3.5 shows an electric circuit consisting of an inductance in parallel with a resistance. As can easily be verified, this circuit is a representation of the linear application of the general dynamic hysteresis description (3.2.1). Consequently, for this circuit the restoring function $f(.)$ and the dissipation function $g(.)$ are chosen to be linear.

Suppose now that this circuit is subjected to a sinusoidal excitation, then Lissajous figures of the network quantities can be constructed. The $\Psi$ - i-Lissajous figure resulting from the sinusoidal variation of $\Psi$ is elliptical with the properties diagrammed in Fig. 3.6a, and can be considered as the dynamic hysteresis loop of the linear model. The width of the loop is determined by the radial frequency, the resistance $R$, the inductance $L$ and the peak value of the flux. The area of the loop decreases with decreasing frequency and narrows down to a simple straight line as $\omega \to 0$ (dc excitation). The same effect can be achieved by increasing the value of the resistance; for $R \to \infty$ (no eddy-currents), the loop is approaching the straight line $\Psi = Li$.

Now that we have arrived at this point, the step to the nonlinear hysteresis representation can easily be taken. For this purpose the inductance $L$ of Fig. 3.5 is replaced by a nonlinear inductance described by a single-valued function - for example, the saturation curve discussed in Section 3.1 - and then again the hysteresis loop (Lissajous figure) for a sinusoidal flux excitation is considered. The result given in Fig. 3.6b can be viewed as a distorted ellipse, the distortions becoming more and more severe as the deviation of the linear function is increasing. Consequently, the characteristic form of the hysteresis loop is mainly determined by the nonlinear restoring function. All of above mentioned properties of the linear model, more or less holds for the nonlinear hysteresis model. The resistance $R$, being either a constant or a function of $i$, can be

![Figure 3.6](image-url)

**Figure 3.6** Hysteresis loops (Lissajous figures) obtained with the electric circuit given in Fig. 3.5

- a) function $L(.)$ is linear
- b) function $L(.)$ is nonlinear
seen as a model parameter which basically takes care of the width of the loop at a particular frequency.

For electrical engineers the term dynamic hysteresis is rather confusing, especially when the electric circuit shown in Fig. 3.5 is being introduced. In the electrical power world, this circuit is known as the model of the iron losses of the transformer core. Almost from the beginning of the transformer theory these losses have been translated into a resistance in parallel with a (nonlinear) inductor [18].

Swift has experimentally proved that the transient condition of the transformer core is described satisfactorily by this model [19]. In his model, the dynamic behaviour of the hysteresis, a result of the eddy-current in the core, is obtained by the parallel resistance in the circuit representation of the model, whereas the pure (and therefore termed static) hysteresis is described by the restoring function represented as the nonlinear inductance.

Since in the electric circuit given in Fig. 3.5, the restoring function is single-valued, the static property of hysteresis has not been considered here. As has been shown the c-i-trajectory narrows down to a simple curve as the operating frequency approaches zero. Consequently, the model discussed here is not suited for analysing residual flux conditions of the transformer core. Residual flux conditions can be studied by allowing for a multi-valued restoring function. The next section deals with this static hysteresis model.

3.2.2 Static Hysteresis Model

In order to predict the hysteresis phenomenon, particularly the behaviour of minor loops, some basic characteristics have been formulated. Actually, these characteristics provide us with information required to generate (algebraic) equations for describing hysteresis trajectories. The phenomenological model resulting from this approach are far much easier than models based on micro physical grounds [20-23] (for example used in analyses of the magnetic recording process). However, one should bear in mind, that inherent in this black box approach is, that by no means the model equations can be related to the mechanisms of the material.

The commonly accepted characteristic properties, describing the behaviour of the minor loops [24-27] are stated as follows:

1. in addition to the symmetric hysteresis excursions, the hysteresis developments exhibits asymmetric (minor) loops
2. these minor loops are closed - a minor loop is closed at the point where the original trajectory was left - and after closure have no longer influence on the subsequent evolution.

Above assumptions are underlying most of the recent static hysteresis models, models whose behaviour agrees with the Preisach hysteresis description [20]. Since in above characterizations only the behaviour of the loops is dealt with and less if nothing with their forms, or more precisely, nothing with the ambiguity of the equations predicting this phenomenon,
the end of the ever increasing number of hysteresis models has not yet come.

Next, a simple hysteresis model will be outlined which satisfies the postulated loop properties as closely as possible. First the model will be discussed qualitatively by following various M-H excursions (M = B - Ψ H), whereupon a mathematical implementation will be given. During the explanation of this new model it will become clear that a procedure is followed which in some respects is similar to the procedure followed in the discussion of the dynamic hysteresis. Actually, the basic idea for the static model stems from this linear approach. Consequently, the approach followed here may be called a pseudo linear approach.

Suppose the material is in the initial (virgin) state, point 0 in Fig. 3.7a, and is taken into state 1 via the straight

![Diagram of hysteresis trajectories](image)

**Figure 3.7**

Hysteresis trajectories representing the characteristics of the hysteresis model

a) symmetrical loop  
b) failure of loop closure  
c) loop closure  
d) innerloops
line 0-1. Let point 1 be a reversal point, that is, the
derivative (dm/dh) changes sign in this point. Then, the next
curve (trajectory) 1-1' is determined by this reversal point.
Point 1' simply follows from point 1, if the symmetrical
considerations are taken into account. In a similar manner the
trajectory 1'-1 can be constructed.

The loop 1-1'-1 constructed in this manner is symmetrical
and satisfies the symmetrical loop properties. Notice, that this
constructed symmetrical loop shows great conformity with the loop
(ellipse) obtained in the linear dynamic hysteresis model with
sinusoidal excitation, except that in this case the loop is not
elliptical. Keeping in mind that the typical hysteresis form can
be seen as a distortion caused by the restoring function, the
static hysteresis model will be further discussed in a similar
way as the dynamic hysteresis model.

Suppose next, that when traversing the trajectory 1-1', a
reversal point 2 has been encountered (see Fig. 3.7b). Then a
second curve 2-1 is defined. This trajectory 2-1 is defined by
the reversals 2 and 1, and is necessary for constructing the loop
1-2-1.

Many of the existing hysteresis models use the thus far
constructing process for describing the hysteresis phenomena [28-
35]. In particular, the reversals 1 and 1', defining a
symmetrical loop, are used as end points for all intermediate
trajectories. These intermediate trajectories are constructed by
the curve passing through an arbitrary reversal and point 1 or
1', and are respectively being defined as ascending or descending
trajectories. The consequence of this departure is depicted in
Fig. 3.7b. For example, if in the evolution of trajectory 2-1,
another reversal occurs, point 3 in Fig. 3.7b, the minor loop
originating from this reversal will not obey the loop closure
rule. Consequently, models considering only one end (point 1 or
1') for the construction of minor loops, will always exhibit this
shortage of loop closure.

In Fig. 3.7c a closure of a minor loop is illustrated. The
loop is closed in point 2, and after closure the subsequent
trajectory is curve 1-1', indicating that minor loop 2-3-2 is
completely forgotten.

The major point of the hysteresis curve is point 1 (see
Fig. 3.7a), since with this reversal point a symmetrical loop is
defined. Evolutions of inner loops take place within this
symmetrical loop. If reversal 2 is known (see Fig. 3.7b) curves
of the type 2-1 can be constructed; according to the loops
properties these curves must pass through the point 2 and 1. From
Fig. 3.7c it can be seen that for a proper closure of loop 2-3-2,
points 3 and 2 are required. After closure these two points are
forgotten, and in the subsequent trajectory only point 1 is
needed. Notice, that whenever trajectory 1-1' is considered,
point 1' follows directly from point 1, since loop 1-1'-1 is a
symmetrical loop. Therefore, point 1' is never considered in the
stack (and in the further discussion).

Consider next the closure of loop 4-5-4 whose character is
illustrated in Fig. 3.7d. Notice, that this state can be reached
by traversing the trajectory 1-2-3-4-5-4, and that 4-5-4 is an
innerloop of innerloop 2-3-2, which has not yet been closed. For
the closure of loop 4-5-4, the trajectory between points 5 and 4
must be traversed, and, according to the loop properties, after
closure, part 4-2 of trajectory 3-2, and finally, part 2-1' of
trajectory 1-1'. It is clear that for carrying out this
particular loops closing procedure, the points 5, 4, 3, 2 and 1
must be stacked, and that closing of \( n \) innerloops will require a
stacking of \( 2n + 1 \) points, and that closing of \( n \) innerloops will
require a stacking of \( 2n + 1 \) points. With the aid of a stack the
algorithm of this loop generating process will be elucidated.

For this purpose it is assumed again that five reversals
have occurred, that means, that data related to these five
reversals - more precisely, data necessary to construct a curve
between to successive reversals - , can be represented by the
following stack:

```
   0  1  2  3  4  5
```

where the points of the stack (the dots) and associated numbers
stand for the corresponding reversals and related data.
Consequently, the discussion on loops and corresponding curves
can be held in terms of the (ordered) points of the stack.
The stacking mechanism is strikingly simple. Whenever a reversal
occurs, the stack is extended with one point, point \( n+1 \), whereas
the stack is shrunken to point \( n-2 \) by dropping the two last (most
right) points from the stack, whenever a loop is closed. This
implies that the actual curve, that is, the curve currently being
traversed, is always determined by the last two points of the
stack. In above representation of the stack this has been
visualized by an arrow, indicating that the direction of
evolution is from right to left.

It is important to know whether the first curve, curve 1 -
2, is an ascending or descending curve, since an ascending curve
is always followed by a descending curve, and vice versa. If
curve 1 - 0 is a descending curve, all \( (2n+1) \)-curves (these are
curves beginning at point \( 2n+1 \)), are descending, whilst the \( (2n) \)-
curves are ascending. On the other hand, \( (2n+1) \)-curves are
ascending and \( (2n) \)-curves descending, if curve 1 - 0 is
ascending. In other words, the ascending or descending nature of
a curve can be read from its begin point and the nature of the
first curve.

Above description of the static hysteresis is generic in
two ways. Recognize, that above approach allows the ability of
working out the two basic hysteresis properties without filling
in the mathematics. The other generic description is the stack
formulation, which can be seen as the formulation of the former
description in a more formal plane, the algorithm plane. From
there the distance to a computer program is very small.

A complication accompanying this loop constructing
mechanism arises if the loops become smaller and smaller. In
order to close all these loops according to the loop closure
properties, all related reversals must be stored. We think that
the effort expended to apply an algorithm that covers the loop
constructing mechanism for all possible reversals, is not in
proportion to the requirements that are being met in modelling
power transformers. Hence, a new model will be presented, in
which the (unwanted) bookkeeping, resulting from a stringent
application of the loop closure rule, is circumvented. For this
purpose the loop properties as defined above, will be relaxed somewhat, knowing that these properties are idealizations of tendencies in the excursions of the experimental magnetization curves [25].

Our aim is the construction of a model with a minimum of stored reversals, but nevertheless, of a model which complies in a reasonable manner with the tendencies of the hysteresis phenomenon. A model which needs fewer reversals for the construction of hysteresis trajectories will be discussed in the following.

Observe that the extension of the stack to the right is unbounded, meaning that an infinite number of reversals can occur, whilst the shrinking to the left is bounded; it can be verified that the shortest stack is stack 1-0 or stack 1-2. For practical reasons a right bound is introduced - an option of keeping the necessary computer storage within reasonable limits - , and the algorithm is slightly adapted. Effectively, this adaption enables us to describe the evolution of hysteresis trajectories with fewer points in the stack.

For this purpose the stack is now reconsidered as given below.

```
  0 1 2 3 4 5 m-1 m
```

The stacking procedure is the same as before except that after a certain number of reversals - in this example, after \(7 = 5 + 2\) reversals - the stack stops growing. This is achieved by freezing all points of the stack minus the two most right points (m-1 and m) when a certain reversal (reversal 5) is reached. For these two most right points, the pushing mechanism is maintained. That means that also in this case the actual curve is determined by the last two points, since these points are related to the last and the last but one reversal. Also notice that, since the pushing mechanism is limited to only two points, some reversals are not stored. Hence, closing of innerloops will not always comply with the loop closure rule. In the following example this will be considered more closely.

Suppose that curve 9-8 is being traversed - Fig. 3.8a and 3.8b should be used to follow the discussion -, in other words, points m and m-1 stand for 9 and 8 respectively. Suppose further, that the loop associated with reversals 9 and 8 (loop 8-9-8) is now closed, then, according to the loop closure rule, point 9 and 8 must be dropped from the stack. The next curve to be traversed should be defined by point 7 and 6. However, it can be directly verified that in the absence of point 7 and 6, curve 7-6 cannot be constructed, and thus, loop 6-7-6 (of which the just closed loop was an innerloop) cannot be closed according to the loop closure rule.

At this point the algorithm needs an adjustment. On account of the "frozen" part of the stack, curve 5-4 is the most eligible curve to proceed with the hysteresis excursions. As explained above, curve 5-4 has the same nature as curve 9-8, since both have the same nature as curve 1-0, ascending or descending. However, the continuation of curve 5-4 cannot be carried out,
Figure 3.8  Construction of innerloops

a) unlimited stack of reversals
b) limited stack of reversal
since then a discontinuity will occur; reversal 8 does not lie on curve 5-4. To avoid this discontinuity, a new curve is defined which has reversal 8 (instead of 5) as beginning and reversal 4 as end.

It is important to know that before forming loop 8-9-8, or more precisely, before the occurrence of reversal 8, the hysteresis evolution was heading for reversal 6 via curve 7-6. At the occurrence of reversal 8, both reversal 8 and the tendency of trajectory 7-6 (which is defined here as the derivative of curve 7-6 in reversal 8), are stored. This tendency could later be passed on to curve 9-8 and curve 8-4. In doing so it is achieved that after closure of loop 8-9-8 a trajectory is traversed which is close to the original curve 7-6.

Notice, that the discrepancy between the newly constructed curve 8-4 and the original curve 7-6 depends on how far reversals 4, 6 and 8 are relative to each other; the closer to each other, the more curve 8-4 will coincide with curve 7-6. Generally speaking, the distance between these reversals are modest, since the occurrence of reversals are closely related to oscillatory transients. As far as the occurrence of successive reversals is concerned, most oscillatory transients behave well in practice.

It is also worth knowing that the modification of the algorithm can be generalized as follows. If in a "frozen" stack a loop of type \(2n-(2n+1)-2n\) is closed, a new curve \(2n-4\) is constructed, whereas if the loop is of type \((2n-1)-2n-(2n-1)\), a new curve \((2n-1)-5\) is constructed. Hereupon the stack becomes alive again, and the first algorithm can be followed.

It should be reminded that in above discussion, the characteristics of the hysteresis phenomenon have been considered in a relatively simple way. Via a linear function the initial state is left and a particular state is reached; from there loop descriptions are obtained by an assembly of curves. As already mentioned, the typical hysteresis forms are obtained by distorting this description. For this purpose it is assumed, that the various loops can be assembled by a family of functions \(F(H)\). Then, the distorting i.e., the real hysteresis model, can be introduced by \(M = g(F(H))\), with the function \(g(.)\) possessing the characteristic nonlinear form. Incidentally, notice that every phenomenological model based on a single-valued function (the saturation curve) and its translation can be reduced to this principle.

As example of this principle, two responses of the hysteresis model (Fig. 3.9a and 3.9b) are compared with similar real responses (Fig. 3.9c and Fig. 3.9d) given in Ref. [36]. It can be verified that in both cases of the model, the loop-forgetting mechanism has worked, and that in this respect, the response of the model agrees with the real responses. Notice, that the experimentally obtained hysteresis excursions show that the loop forgetting mechanism as defined above - assumption II - is an idealization: it is seen that after closing of a loop certain trajectories do not exactly coincide as in the case of the model (the idealized behaviour). Further it is noted that the appliance of a frozen stack until 7 (=5+2) reversals (as given in above example) will yield an idealized forgetting mechanism. When more than 7 reversals has occurred, trajectories will also not always coincide after closing of a loop.

What now remains is the mathematical filling in of above skeleton description, that is, the mathematical expression for a
Figure 3.9  Hysteresis excursions, computed and measured (see ref. [36]) results

(a) and (b) computed inner loops
(c) and (d) measured inner loops
family of functions $F(.)$, assuming the restoring function $g(.)$ is already known — for example, the single-valued restoring function used in the dynamic hysteresis model. The function used for describing the loop properties is shown in Fig. 3.10 in a changed co-ordinate system. For convenience the origin has been shifted so as to obtain curve 1-1' in the first quadrant of the new coordinate system, while the values of $H$ and $f(H)$ have been normalized.

The family of $F(.)$ describing the hysteresis excursions is given by the following function

$$\Delta F = C_n \left[ 1 - \exp(-\alpha_n \Delta H) \right]$$  \hspace{1cm} (3.2.5a)

$$0 \leq \Delta H \leq \Delta H_n$$

where

$$C_n = \frac{\Delta F_n}{1 - \exp(-\alpha_n \Delta H_n)}$$ \hspace{1cm} (3.2.5b)

$$\Delta F = F - F_{n-1}$$

$$\Delta H = H = H_{n-1}$$

$$\Delta F_n = F_n - F_{n-1}$$

$$\Delta H_n = H_n - H_{n-1}$$

with

$$F_n = F(H_n)$$

for $n = 1, 2, \ldots$

and

![Figure 3.10](image)

**Figure 3.10** Members of the family of functions used for describing hysteresis loops
$F_0 = H_0 = 0$

Notice, that the above defined curves $F(.)$ will always pass through two successive reversals $(H_{n-1}, F_{n-1})$ and $(H_{n}, F_{n})$ for all possible values of the parameter $\alpha$. Since $\alpha$ has not been related to anything, one degree of freedom is left to construct a curve between two successive reversals.

This degree of freedom is used by prescribing the derivative at the end of curve $n-(n-1)$, point $(H_{n-1}, F_{n-1})$. However, one exception is made in case of a "frozen" stack. As already mentioned, when an innerloop is closed of which the begin and end point are related to the two most right points of the stack, the continuity requirement of the derivative in the end point (of the loop), guarantees a smooth continuation of the hysteresis evolution. Hence, for curve $2n-4$ (curve 8-4 in above example) or $(2n-1)-5$, the corresponding parameter $\alpha_{2n}$, $\alpha_{2n-1}$ respectively are determined by the specified derivative at the beginning of these curves.

It can be easily appreciated that when the hysteresis evolutions are described by $g(F(H))$, with the family $F(.)$ as defined in Eq. (3.2.5), the width of the symmetric hysteresis loop is strongly related to the first member of the family, specifically to $\alpha_1$. In the next section a method is given to relate $\alpha_1$ to the more regular transformer data.

3.2.3 Determination of the Parameters of the Hysteresis Model

In the previous section a family of functions has been defined with which a hysteresis description can be easily obtained. When using this description, the area enclosed by the symmetric hysteresis loop can be related to the parameter $\alpha_1$ of the first member of this family. It will be shown by means of a simple expression that this parameter $\alpha_1$ can be related to the more commonly used transformer data. For this purpose it is assumed that the (measured) iron core losses consist of eddy-current losses and static hysteresis losses [18], and that the eddy-current losses are a fraction $r$ of the static hysteresis losses. Thus, in terms of equivalent resistances we have

$$R_H = (1 + r)R_{Fe} \quad (3.2.6a)$$

$$R_e = \frac{(1 + r)}{r} R_{Fe} \quad (3.2.6b)$$

where $R_H$ and $R_e$ are equivalent resistances, which allow for static hysteresis and eddy-current losses, and $R_{Fe}$ is an equivalent resistance representing the iron-core losses. Above relations can easily be derived from a circuit representation of the single-phase transformer, in which the iron losses are represented by two parallel resistance $R_H$ and $R_e$.

Suppose the iron losses are introduced by $e$. 
\[ P_O = \frac{P_{Fe}}{S} \times 100 \% \]

where \( P_O \) stands for the iron core losses and \( S \) for the rated power of the transformer. It can be shown that above introduced resistances can be expressed in terms of these quantities, in other words

\[ R_{Fe} = \frac{U^2}{S} \times \frac{100}{P_O} \quad (3.2.7a) \]

\[ R_H = (1+r) \frac{U^2}{S} \times \frac{100}{P_O} \quad (3.2.7b) \]

and

\[ R_e = \frac{(1+r)}{r} \frac{U^2}{S} \times \frac{100}{P_O} \quad (3.2.7c) \]

The equivalent resistance \( R_{Fe} \) can also be related as follows to the no-load condition of the transformer.

\[ i_O = \frac{I_m}{I} \times 100 \% = \sqrt{\frac{R_{Fe}^2 + X_m^2}{R_{Fe}X_m}} \frac{U^2}{S} \times 100 \% \]

where \( I_m \) is the magnetizing current and \( X_m \) is the equivalent magnetizing reactance.

Notice, that by introducing the equivalent resistance \( R_H \), the static hysteresis losses can be related to an ellipse. This is easily recognized by considering the linear electric circuit given in Fig. 3.11.

![Electric network with \( R_H \) and \( R_e \) representing core losses](image)

The Lissajous figure \( \psi-\theta \), caused by a sinusoidal excitation with a particular (power) frequency, is elliptical. The area enclosed by this (symmetrical) ellipse is equal to the energy loss per cycle, and can be associated with \( R_H \). From the discussion of the static hysteresis it is known, that for a given
peak value of the flux, a symmetrical loop can be constructed of which the width depends on the model parameter $\alpha_1$. It is also known that the static description in principle arises from a pseudo-linear approach based on the linear dynamic description. The next step is to choose $\alpha_1$ so as to obtain a symmetrical loop (based on this pseudo-linear approach) with enclosed an area which approximates the area of the ellipse. This can be done by letting the intersection of the symmetrical loop with the $\Psi$-axis coincide with the intersection of the ellipse with the $\Psi$-axis. For this purpose the major curve of the static description is rewritten and given as

$$\frac{\Psi}{\alpha} = \frac{2}{1 - \exp(-\alpha_1)} \left(1 - \exp(-1/2\alpha_1(\frac{\theta}{\Theta} + 1))\right) - 1$$  \hspace{1cm} (3.2.8)$$

with

$$\left|\frac{\Theta}{\Theta_0}\right| \leq 1 \text{ and } \frac{d\psi}{dt} \leq 0$$

where $\psi$ is the peak value of the flux $\Psi$ and $\theta$ is the peak value of the current corresponding with the m.m.f. From Section 3.2 (Fig. 3.3) it is known that the intersection of the ellipse with the $\Psi$-axis can be expressed as

---

**Figure 3.12** Hysteresis characteristics

a) factor $\alpha_1$ as function of $p_o/i_o$

b) $\psi-\theta$ loops

1: $\psi/\psi - \theta/\theta$

2: $\psi/\psi - \theta/\theta_e$

3: $\psi/\psi - \theta/\theta_e$ (linear)

4: $\psi/\psi - \theta/\theta_n$ (nonlinear)
\[ \frac{\nu_R}{\psi} = \frac{X_m}{\sqrt{R_H^2 + X_m^2}} \]

which changes into

\[ \frac{\nu_R}{\psi} = \frac{(p_o/i_o)}{\sqrt{(1+r)^2 - ((1+r)^2 - 1)(p_o/i_o)^2}} \]  \hspace{1cm} (3.2.9)

if \( R_H \) and \( X_m \) are eliminated, using (3.16) and (3.2.17). It can be easily verified that the factor \( \alpha_1 \) follows from

\[ 2 \left( \frac{1 - \exp(-1/2\alpha_1)}{(1 - \exp(-\alpha_1))} - 1 \right) = \frac{(p_o/i_o)}{\sqrt{(1+r)^2 - ((1+r)^2 - 1)(p_o/i_o)^2}} \]  \hspace{1cm} (3.2.10)

Note, that the left hand side of Eq. (3.2.10) is obtained by letting \( \theta/\theta = 0 \) in Eq. (3.2.8).

Figure 3.12a shows factor \( \alpha_1 \) as a function of \( (p_o/i_o) \) for several values of \( r \) (the ratio of eddy-current loss \( v.8\) hysteresis loss). Further, an example of the complete hysteresis model is given in Fig. 3.12b. As can be seen, the area enclosed by the static hysteresis loop (loop 4) approximates the area enclosed by the loop associated with the resistance \( R_H \) and the reactance \( X_m \) (loop 3). With \( r = 3.0 \), the specified ratio of the eddy-current losses to the (static) hysteresis losses, a hysteresis model is obtained with a calculated ratio of \( r = 2.7 \).
4 TRANSIENT NETWORK ANALYSIS

The basic steps in modelling one of the most important components in electrical power systems, the transformer, have been discussed in Chapter 2 and 3. In particular, the magnetic flux behaviour of the transformer is represented by magnetic networks. Since the transformer is a component of a larger system, it is worth to study its behaviour in conjunction with the other components. Needless to say that this must be done systematically. It will be seen that this is precisely the scope if this chapter.

This chapter is concerned with all the material necessary for solving the transient network problem. In the first section, Section 4.1, the network equations used in a general algorithm are briefly considered. In fact, it is shown which method of organization will be used for solving the network equations. Section 4.2 deals with the Modified Nodal Approach (MNA). This method of organizing network equations is discussed separately, since this method is not so well known in the electrical power engineering. Moreover it will be seen that this method can easily handle transformer equations. Since the MNA has been explained more comprehensively in the literature, only a mathematical treatment of this method is given.

Basically the methods mentioned in Section 4.1 and 4.2 are developed for solving linear networks. How these methods, in particular the MNA, can be adapted for solving nonlinear networks is shown in Section 4.3. It is shown that the solution of the nonlinear network, eventually can be reduced to the solution of a set of nonlinear equations of a lesser order. For the sake of completeness, the method used for solving this set of nonlinear (algebraic) equations is given in Section 4.4. Actually, a modified Newton-like method suited for solving problems of this kind will be considered.

Further, the numerical integration method used for solving the transient network problem is given in Section 4.5. Numerical integration methods are of key importance for solving differential equations, since they greatly affect the organization of the resulting algebraic (discretized) equations. It will be seen that if an implicit method of numerical integration is used for solving a network described by ordinary differential equations, a resistive network is obtained. The stability of the method of numerical integration is also considered in terms of discretized networks. Finally, a few examples which form the base in solving transformer circuits, will be worked out. The examples given are for a single-phase case, however, they can readily be extended for the three-phase case.

4.1 Network equations

For the sake of simplicity, only networks made of arbitrary connected lumped elements are being considered. Hence, it is convenient to recognize the network equations in the ensuing
discussion as consisting of two constituent parts: a set of ordinary differential equations, describing the elements of the network, and a set of algebraic equations describing the topological structure of the network. The network elements usually are models of devices of the circuit being studied, whereas the topology of the circuit (network) is captured in the Kirchoff's laws.

The bookkeeping chore of interconnecting the network elements is systematized in terms of the well-known incidence matrices (a detailed treatment can be found in [1,2]). In the following, three of the major approaches of organizing network equations will be outlined, merely by considering their basic equations. These three methods have been used in a computer program which has been developed for solving transformer equations, or more general, network equations.

To begin with, it is assumed that the elements of a network are described by linear ordinary differential equations, and that these equations are algebraized. The introduction of algebraic equations as an element description is a result of the discretization of the time (Section 4.5 deals with this matter). The consequence of this algebraization is that from a network point of view only resistive elements constituting dc-networks, can be considered. Accordingly, only resistive networks with a dc-excitation will henceforth be discussed, that is, networks constructed with standard branches, being particularly arranged descriptions of these resistive elements. The standard branch, used in the derivation of the network equations is given in Fig. 4.1. Recognize, that the symbols used for the quantities in this branch, generally are used for representation in the complex plane. This may lead to confusion, since the complex method of analysis is not used for handling dc-networks, but for representing the steady state solution of sinusoidal driven linear networks. However, no confusion will be caused, since in this chapter only resistive branches (dc-networks) will be treated. The constituent relationship of this branch is given by

\[
\tilde{u}_b = Z_b \tilde{i}_b \quad \text{or} \quad \tilde{i}_b = Y_b \tilde{u}_b \quad (4.1.1)
\]
and
\[ \tilde{r}_b = i_b - j_b \]  \hspace{1cm} (4.1.2)
\[ \tilde{u}_b = u_b - e_b \]  \hspace{1cm} (4.1.3)

where
\[
\begin{align*}
\tilde{r}_b & : \text{resistive branch element} \\
\tilde{y}_b & : \text{conductive branch element} \\
e_b & : \text{(independent dc-)voltage source} \\
i_b & : \text{(independent dc-)current source} \\
u_{\tilde{b}} & : \text{voltage across } z_b \text{ or } y_b \\
j_{\tilde{b}} & : \text{current through } y_b \text{ or } z_b \\
u_b & : \text{voltage across the entire branch} \\
i_b & : \text{current through the entire branch.}
\end{align*}
\]

The vector expression of all branches of a network is given by
\[ U = I - (ZJ - E) \]  \hspace{1cm} (4.1.4)
or
\[ I = YU - (YE - J) \]  \hspace{1cm} (4.1.5)

where the variables used in (4.1.1) through (4.1.3) are stored in the corresponding column vectors \( U, I, E \) and \( J \), and matrices \( Z \) and \( Y \) of Eqs. (4.1.4) and (4.1.5). Recognize, that these equations represent an (ordered) collection of not (yet) interconnected branches. The interconnection of these branches is imposed by the network topography, embodied by the Kirchoff's laws.

The Kirchoff's laws can be formulated in various ways, each having their own merits. A brief review of three basic methods will be given, since these methods will be used in formulating the network problem.

The Loop Method

For this purpose, the Kirchoff's constraints on the branch voltages and the branch currents imposed by the topological structure of the network, are written in the following matrix equations.

\[ CU = 0 \]  \hspace{1cm} (4.1.6)

\[ I = C^T I_l \]  \hspace{1cm} (4.1.7)

An expression of the loop currents can be derived, using the branch equation (4.1.4). The loop currents are given by
\[ I_l = (CZC^T)^{-1} C(ZJ - E) \]  \hspace{1cm} (4.1.8)

where

- \( C \): loop-branch incidence matrix
- \( I_l \): vector of loop currents
- \( CZC^T \): loop impedance matrix

As can be seen, this method is used when the network branches are, or (some times) can only be, described with the current as the independent variable. Henceforth this description will also be referred to as the 'z-description'.

The Cutset Method

When using this method, a set of branch voltages is expressed as a linear combination of an independent set of branch voltages. This independent set of branch voltages are called tree-branch voltages. The linear combination of branch voltages can be written as

\[ U = Q^T U_t \]  \hspace{1cm} (4.1.9)

whereas the Kirchoff's current law is given by

\[ QI = 0 \]  \hspace{1cm} (4.1.10)

Substitution of (4.1.9) and (4.1.10) in (4.1.5) yields the equation of the tree-branch voltages

\[ U_t = (QYQ^T)^{-1} Q(YE - J) \]  \hspace{1cm} (4.1.11)

where

- \( Q \): cutset-branch incidence matrix
- \( U_t \): vector of tree-branch voltages
- \( QYQ^T \): cutset-admittance matrix

Unlike the loop method, an 'y-description' of the network branches has been used, implying that the branch voltage is the independent variable.

These two methods, the loop- and the cutset method, have been used for arranging the equations of the magnetic circuit of the transformer. In practice the elements of the magnetic circuit can have both forms, the z-description, or the y-description, depending on how the current-flux relationship of the transformer is given. The z-description is used when the current is the independent variable, whereas the y-description is used when the
flux is the independent variable. In Appendix A these two methods are used in formulating the network equations of the magnetic circuit of the transformer. In summary, both, the loop and the cutset method are used in order to include the transformer equations in the set of equations of the network in which the transformer occurs.

The Nodal Method

The Kirchhoff’s laws for the nodal voltage (versus a common datum node), and the branch currents, correspond to the following vector relations

\[ U = A^T U_n \]  \hspace{1cm} (4.1.12)

and

\[ AI = 0 \]  \hspace{1cm} (4.1.13)

The expression of the nodal voltages follows directly from Eq. (4.1.5), using (4.1.12) and (4.1.13)

\[ U_n = (AYA^T)^{-1} A (YE - J) \]  \hspace{1cm} (4.1.14)

with

\[ A \] : reduced node-branch incidence matrix
\[ U_n \] : vector of node to datum voltages
\[ AYA^T \] : reduced nodal admittance matrix

It is noted, that in addition to above mentioned methods, other network formulations have been developed. Detailed information can be found for example in [3].

Compared with the two other methods, the loop and the cutset method, the algorithm of the nodal approach is very simple. In contrast with these methods, no tree (set of independent variables) need to be constructed (see Appendix A and Ref. [2]): the nodal admittance matrix can be built readily from the input data of the network configuration, exhibiting a very efficient way of processing data. Another advantage of this approach is that nodal voltages are calculated, mostly being of greatest importance to the electrical engineer. Further, it has been proven, that with the nodal approach a numerical well-behaved diagonal is obtained, which warrants a numerical solution in most cases. Apparently, these promising aspect of the nodal approach have been recognized very early in the electrical power world. From the beginning this method has been successfully applied in load flow calculations and stability analysis of power systems [4,5]. As far as can be seen, this method has commonly been recognized as one of the most powerful methods for analysing the network problem.
Illustrative, especially for electrical engineers, is the international use of EMTP [6], a computer program based on the nodal approach. The nodal approach is also frequently used in modern computer aided circuit analysis and design techniques [7-10].

However, confronted with elements which can only be described with currents being independent variables (z-description), the nodal approach gets into trouble. For example, this method is incapable of including ideal voltage source branches. In order to cope with difficulties arising from implementing these elements, also referred to as current-dependent or current-controlled elements, laborious interventions are required [3,11,12].

Generally speaking, these problems can be overcome by using hybrid methods. These methods can handle current-dependent and voltage-dependent elements at the same time. Since the transformer can be described as a current-dependent as well as a voltage-dependent element, a hybrid method has been chosen for solving the network problem. For this purpose the nodal approach is extended. Particularly, the set of nodal equations is extended with the set of equations related to the current-dependent elements. Accordingly, the unknown vector resulting from this hybrid method, better known as the modified nodal approach (MNA) [13,14], consists of the nodal voltages and the currents of the elements whose constitutive relation is given in a current-dependent form. Next, only a mathematical justification of this method will be given. The more detailed treatment of the MNA-method can be found in [3,13,14].

4.2 The Modified Nodal Approach (MNA)

As illustrated in the preceding section, for the simple case of a network consisting of conductances and independent (equivalent) sources, the following matrix relation holds

\[(AYA^T)U_n = A(YE - J)\]

or

\[\tilde{Y}U_n = I_{eq}\]

\[
(4.2.1)
\]

with

\[\tilde{Y} = AYA^T: \text{nodal admittance matrix}\]

and

\[I_{eq} = A(YE - J): \text{vector of equivalent sources}\]

If the network contains ideal voltage source branches and/or other elements whose constitutive relations are given in a current-dependent form, the nodal approach is extended. For this purpose, the elements of a network are divided into:
voltage-dependent elements given by

\[ I_v = YU_v - (YE_v - J_v) \]  \hspace{1cm} (4.2.2)

and

current-dependent elements given by

\[ U_i = ZI_i - (ZJ_i - E_i) \]  \hspace{1cm} (4.2.3)

As a consequence of this distinction, the Kirchoff's constraints are written in the following partitioned form

\[ AI = \begin{bmatrix} A_v & A_i \end{bmatrix} \begin{bmatrix} I_v \\ I_i \end{bmatrix} = 0 \]  \hspace{1cm} (4.2.4)

and

\[ U = \begin{bmatrix} U_v \\ U_i \end{bmatrix} = T U_n = \begin{bmatrix} A_v \\ A_i \end{bmatrix} T U_n \]  \hspace{1cm} (4.2.5)

where

\[ U_v, I_v: \] vector respectively containing the voltages and currents related to the voltage-dependent branches.

\[ U_i, I_i: \] vector respectively containing the voltage and currents related to the current-dependent branches.

\[ A_v, A_i: \] submatrices of the nodal branch incidence matrices.

Notice, that the association with the voltage-dependent and current-dependent elements, is made via the indices v and i, respectively. Equations (4.2.1) through (4.2.5) can easily be rearranged in the matrix relation given below.

\[ \begin{bmatrix} A_v Y A_v^T & A_i \\ -A_i^T Z & I_i \end{bmatrix} \begin{bmatrix} U_v \\ I_i \end{bmatrix} = \begin{bmatrix} A_v (Y E_v - J_v) \\ Z J_i - E_i \end{bmatrix} \]

or
\[
\begin{bmatrix}
\tilde{Y}_v & A_i & U_n \\
-A_i & Z & I_i
\end{bmatrix}
= \begin{bmatrix}
I_{v,eq} \\
I_{i,eq}
\end{bmatrix}
\]

(4.2.6)

with

\[
\tilde{Y}_v = A_v Y A_v^T
\]

: the nodal admittance matrix of the network without the current-dependent branches.

\[
I_{v,eq} = A_v (Y E_v - J_v)
\]

: vector containing equivalent currents of the voltage-dependent branches.

\[
U_{i,eq} = Z J_i - E_i
\]

: vector containing equivalent voltages of the current-dependent branches.

From Eq. (4.2.6) it is readily seen which modifications are being introduced in the algorithm of the basic nodal approach. The current-dependent branches are deleted from the original network, and from the resulting network the nodal admittance matrix is constructed, using the well-known procedures [4,15]. In particular, the admittance matrix \( Y_v \) of Eq. (4.26) is built according to the well-known admittance construction procedures. The extension of these procedures for taking into account the additional rows and columns (see matrix in Eq. (4.26)) is straightforward.

Like the nodal approach, the MNA is very well suited in handling compound elements as entities for the network construction. Each compound element is characterized by its (indefinite) admittance matrix and a companion vector of equivalent voltage sources. Current-dependent compound elements are characterized by their constitutive relation, in other words, by the Z-matrix and a vector containing the equivalent voltages sources. Thus, the organization of the network equations is reduced to a simple processing of these element characteristics.

4.3 Nonlinear Network Formulation

Yet so far, the simple case of equations related to linear networks has been considered. It will be shown that if the network being analysed, contains nonlinear elements, transformers for example, a slight adaption is required to deal with these nonlinear elements. In Section 4.5 it will be shown that the basic idea of solving (nonlinear) networks, is to replace at every time step the characteristics of its elements by algebraic equations, and then solve the equivalent resistive (nonlinear) dc-network. Consequently, in terms of network analysis the problem is reduced to the evaluation of the dc-distribution in nonlinear networks. With this, we are confronted with the inevitable question of the existence of a solution, and if one exists, with the subsequent question of the uniqueness of the solution. This very complex problem of the existence and the uniqueness of the solution of a nonlinear resistive network is beyond the scope of this thesis; necessary and sufficient
conditions for the existence of a unique solution have been formulated for only a few networks [16,17].

From a practical point of view however, a method based on the assumption of a unique solution will be used for solving the nonlinear resistive network problems. Intuitively, one may conjecture that, considering our problem, this assumption is legitimate, since most physical phenomena will mathematically behave well as the time proceeds. Consequently, in the course of numerically evaluating the time response, it is most likely that at any instant a neighbourhood can be defined, in which the solution is unique, provided the time step is not taken too large.

The method used for solving the nonlinear resistive network problem is suited for networks containing only relatively few nonlinear branches. The non-linear network problem will be tackled by a special handling of non-linear branches. This will be illustrated in the following, where a method used for solving networks containing voltage-dependent and current-dependent

![Diagram](image-url)

**Figure 4.2** Standard branch for nonlinear networks

a) voltage-dependent branch (cf Eq. (4.3.1))

b) current-dependent branch (cf Eq. (4.3.2))
branches will be outlined. For this purpose, it is assumed that the voltage-dependent branches of a network are given by

\[ I_v = YU_v - (YJ_v - J_v) + \mathbf{\mathbf{J}_v} (U_v) \]  

(4.3.1)

and the current-dependent branches by

\[ U_i = ZI_i - (ZJ_i - E_i) + \mathbf{\mathbf{E}_i} (I_i) \]  

(4.3.2)

where

\[ \mathbf{\mathbf{J}_v}(.) \]: vector of branch currents (being nonlinear functions of the voltages) in the voltage-dependent branches.

\[ \mathbf{\mathbf{E}_i}(.) \]: vector of voltages (being nonlinear functions of the branch currents) in the current-dependent branches.

The standard branches corresponding to these nonlinear equations are given in Figs. 4.2a and Fig. 4.2b.

Since the same division of the network branches into voltage-dependent and current-dependent branches has been made, the Kirchoff's constraint (4.1.12) and (4.1.13), given in Section 4.1, holds for above equations. Consequently, using these constraints, equations (4.3.1) and (4.3.2) can be arranged into

\[
\begin{bmatrix}
\mathbf{\mathbf{J}_v} & \mathbf{A}_i & \mathbf{U}_n \\
\mathbf{-A}_i & \mathbf{Z} & \mathbf{I}_i
\end{bmatrix}
= \begin{bmatrix}
\mathbf{I}_{v,eq} \\
\mathbf{U}_{i,eq}
\end{bmatrix} + \begin{bmatrix}
\mathbf{\mathbf{J}_{v,eq}} \\
\mathbf{\mathbf{E}_i}
\end{bmatrix}
\]  

(4.3.3)

where

\[ \mathbf{\mathbf{J}_{v,eq}} = A_v \mathbf{\mathbf{J}_v}(.) \]: vector of equivalent currents being nonlinear functions of the voltages of the voltage-dependent branches.

It is important to recognize that above nonlinear description (Eq. (4.3.3)) is made up of a linear and a nonlinear part. The linear part of the network is the description of a linear subnetwork, which results from excluding the nonlinear elements from the original network. It is noticed here, that this procedure of excluding branches from a network is not new, but has already been used in the network theory when computation of large networks must be simplified [18,19,20]. This method of network analysis can be very well used in simplifying the solution of nonlinear network problems. With the aid of a simple example this approach will next be elucidated.

Suppose we have a network which contains only one nonlinear element, and suppose further, that this element is voltage-dependent. Then, it can be shown that the solution of this
nonlinear network is obtained after the solution of a nonlinear equation associated with the nonlinear element has been found. For this purpose, the nonlinear element is excluded from the network. In order to maintain the original currents and voltages of the given network, equivalent sources must be introduced at the two nodes the element has been connected with, assuming a two node incidence of the element. Actually, Eq. (4.3.3) can be viewed as the mathematical formulation of above. Suppose now, that the nonlinear element is incident with node k and l, and that its voltage-dependent equation is given by

\[ i_{kl} = y_{kl}u_{kl} - (y_{kl}e_{kl} - j_{kl}) + j_{kl}(u_{kl}) \]  

(4.3.4)

then it can be verified that this nonlinear element can be characterized by the matrix equation given below

\[
\begin{vmatrix}
  i_{kl} \\
  i_{lk}
\end{vmatrix} =
\begin{vmatrix}
  y_{kl} & -y_{kl} \\
  -y_{kl} & y_{kl}
\end{vmatrix}
\begin{vmatrix}
  u_k \\
  u_l
\end{vmatrix}
+ \begin{vmatrix}
  1 \\
  -1
\end{vmatrix}
\begin{vmatrix}
  j_{kl} \\
  j_{lk}
\end{vmatrix}
+ \begin{vmatrix}
  i_{kl,eq} \\
  -i_{lk,eq}
\end{vmatrix}
\]  

(4.3.5)

This matrix equation contains the elements from which the MNA-equation (Eq. (4.2.6)) can be built, namely, the indefinite nodal admittance matrix, the nodal-nonlinear-branch incidence vector accounting for the nonlinear portion of the current, and the (known) equivalent current sources.

It can be verified that, when a network contains a nonlinear element characterized by Eq. (4.3.5), the general form of the MNA-equation (Eq. (4.3.3)) will change by the appearance of a current \( j_{kl} \) in the row entries k and l. Notice in this connection that the contribution of each element - and thus, that of the nonlinear element -, to the nodal equations can be found in rows and/or columns corresponding to the nodes the element is incident with. Hence, the contribution of the nonlinear element can be expressed as:

\[
\begin{vmatrix}
  U_n \\
  I_i
\end{vmatrix} =
\begin{vmatrix}
  \tilde{Y} & T \\
  -A_i & Z
\end{vmatrix}
\begin{vmatrix}
  k \to 1 \\
  1 \to l
\end{vmatrix}
\begin{vmatrix}
  j_{kl} \\
  i_{kl,eq}
\end{vmatrix}
+ \begin{vmatrix}
  I_v,eq \\
  U_i,eq
\end{vmatrix}
\]  

(4.3.6)

Above set of equations can be solved, provided current \( \tilde{j}_{kl} \) is known. Consequently, our next concern is the determination of
\( j_{kl} \). From Eq. (4.3.6) it is seen that an arbitrary nodal voltage \( u_v \) can be written as

\[
u_v = -(\tilde{Z}_{vk} - \tilde{Z}_{vl}) j_{kl} + \hat{u}_v \tag{4.3.7}
\]

where

\[
u_v \quad : \text{the nodal voltage of the } v\text{-th node of the original network}
\]

\[
\hat{u}_v \quad : \text{the nodal voltage of the } v\text{-th node of the linear network}
\]

\[
\tilde{Z}_{vk}, \tilde{Z}_{vl} : \text{elements of the solution matrix of (4.3.6) on the entries } (v,k) \text{ and } (v,l).
\]

It is now easily seen that with the aid of (4.3.7) the branch voltage, \( u_{kl} = u_k - u_l \), can be expressed as

\[
u_{kl} = -[(\tilde{Z}_{kk} - \tilde{Z}_{kl}) - (\tilde{Z}_{lk} - \tilde{Z}_{ll})] j_{kl} + \hat{u}_k - \hat{u}_l \tag{4.3.8}
\]

with the nonlinear equation (see Eq. (4.3.4))

\[
j_{kl} = j_{kl} (u_{kl}) \tag{4.3.9}
\]

The current \( j_{kl} \) can be determined from above set of Eqs. (4.3.8) and (4.3.9). Subsequently, with the aid of (4.3.6), all nodal voltages and control currents of the original network can be determined.

Above exposed procedure of cutting and composing networks can be summarized as follows. First of all, the nonlinear part of the network has been excluded from the network. Then, currents have been injected into the two nodes \( k \) and \( l \), in order to maintain the original behaviour in the resulting network. These injection currents are equal, except for their sign. And finally, for the determination of the solution of the nonlinear network, a Thevenin equivalent is constructed, and the nonlinear element is connected with this equivalent network. Actually, in doing so, a solution scheme is obtained which confines the solution of the nonlinear equations to a much smaller equivalent nonlinear network. It is obvious that with this method an appreciable reduction of computation time can be obtained. In Section 4.5 (see Fig. 4.8) this method is further elucidated by working out an example of a single-phase.

The generalization of processing nonlinear elements within the MNA is straightforward. Equations (4.3.1) - (4.3.3) will be used as starting point. For this purpose Eq. (4.3.3) is worked out and re-arranged as

\[
\begin{bmatrix}
U_n \\
I_i
\end{bmatrix} =
\begin{bmatrix}
\hat{U} \\
\hat{I}
\end{bmatrix} +
\begin{bmatrix}
\tilde{Z} & \tilde{C} & \tilde{J} \\
\tilde{T} & \tilde{Y} & \tilde{E}
\end{bmatrix}
\begin{bmatrix}
C_i \\
\tilde{I} \\
\tilde{E}_i
\end{bmatrix} \tag{4.3.10}
\]
where

\[
\begin{pmatrix}
\hat{u}_n \\
\hat{i}_i
\end{pmatrix}
= \begin{pmatrix}
\tilde{z} \\
-\tilde{c}_i
\end{pmatrix}
\begin{pmatrix}
\alpha \\
\gamma
\end{pmatrix}
\begin{pmatrix}
i_v,eq \\
i_{i,eq}
\end{pmatrix}
\]  
(4.3.11)

and

\[
\begin{pmatrix}
\tilde{z} \\
-\tilde{c}_i
\end{pmatrix}
= \begin{pmatrix}
z \\
-\gamma
\end{pmatrix}
\begin{pmatrix}
c_i \\
\gamma_i
\end{pmatrix}
\begin{pmatrix}
A_i^{-1} \\
A_i^{-1}
\end{pmatrix}
\begin{pmatrix}
y_v \\
y_i
\end{pmatrix}
\begin{pmatrix}
A_i \\
A_i
\end{pmatrix}
\]  
(4.3.12)

with

\[
\tilde{z}_v = \tilde{y}_v^{-1} - (\tilde{y}_v^{-1} A_i) \tilde{z}_i^{-1} (\tilde{y}_v^{-1} A_i)^T 
\]  
(4.3.13a)

\[
c_i = -(\tilde{y}_v^{-1} A_i) \tilde{z}_i^{-1} 
\]  
(4.3.13b)

\[
\tilde{y}_i = \tilde{z}_i^{-1} 
\]  
(4.3.13c)

\[
\tilde{z} = z_i + A_i^T \tilde{y}_i^{-1} A_i 
\]  
(4.3.13d)

Notice, that in above derivation it is assumed that \(\tilde{y}_v\) and \(z_i\) are symmetrical, and that \(\tilde{y}_v\) is nonsingular.

In the next step those equations related to the nonlinear elements are extracted from Eq. (4.3.10). For this purpose, the sparse vector \(\tilde{\mathbf{j}}_v(.)\) and \(\tilde{\mathbf{e}}_i(.)\) introduced in Eqs. (4.3.1) and (4.3.2) are now given as:

\[
\tilde{\mathbf{j}}_v(.) = \mathbf{b}_v \tilde{\mathbf{j}}_{br}(.) 
\]  
(4.3.14)

\[
\tilde{\mathbf{e}}_i(.) = \mathbf{b}_i \tilde{\mathbf{e}}_{br}(.) 
\]  
(4.3.15)

where \(\tilde{\mathbf{j}}_{br}(.)\) and \(\tilde{\mathbf{e}}_{br}(.)\) are completely filled vectors of nonlinear functions of respectively branch voltages and branch currents. It can now be readily seen that Eq. (4.3.10) changes into

\[
\begin{pmatrix}
\hat{u}_n \\
\hat{i}_i
\end{pmatrix}
= \begin{pmatrix}
\tilde{z} \\
-\tilde{c}_i
\end{pmatrix}
\begin{pmatrix}
\mathbf{b}_v \tilde{\mathbf{j}}_{br} \\
\mathbf{b}_i \tilde{\mathbf{e}}_{br}
\end{pmatrix}
\]  
(4.3.16)
and that the following set of nonlinear equations can be extracted from Eq. (4.3.10)

\[
\begin{bmatrix}
U_{br} \\
I_{br}
\end{bmatrix}
= 
\begin{bmatrix}
\hat{U}_{br} \\
\hat{I}_{br}
\end{bmatrix} + 
\begin{bmatrix}
T T & T T & \sim (U) \\
B A Z B & B A C B & \sim (I)
\end{bmatrix}
\begin{bmatrix}
V V V V \\
V V i i
\end{bmatrix}
\begin{bmatrix}
T T \\
- B C B \\
B Y B \\
i i i
\end{bmatrix}
\begin{bmatrix}
E (U) \\
E (I)
\end{bmatrix}
\]

with

\[
U_{br} = B_v T U_v, \quad U_{br} = B_v T A_v U_n
\]

and

\[
I_{br} = B_i T I_i
\]

The set of nonlinear equations given in (4.3.17) is solved first and thereupon, using Eq. (4.3.16) or (4.3.10), the set of the original nonlinear network equations. In the next section the (modified) Newton method which has been used to solve Eq. (4.3.17) will be discussed. For convenience, the form \( F(x) = 0 \) will be used in the discussion. As can be seen from Eq. (4.3.17), the vector \( x = [U_{br} I_{br}]^T \), while the function \( F(.) \) is implicitly given.

4.4 Nonlinear Equations

In the preceding section a method is given which reduces the nonlinear network problem to the solution of nonlinear elements connected with an equivalent network. In this way, a considerable reduction of the dimension of the nonlinear problem is obtained. Moreover, this construction - a nonlinear element connected with an equivalent network - enables us to use any solution method tailored to the characteristics of the nonlinear element involved. This section is concerned with a modification of the Newton method used for solving this nonlinear problem.

Suppose for this purpose that the formerly derived set of nonlinear equations (Eq. (4.3.17)), or more generally, of equation

\[
F(x) = 0
\]

with

\[
x \in \mathbb{R}^n
\]

and
F : n-dimensional vector function

has a unique solution. Then, for solving this equation, the very popular Newton iterative scheme can be used, providing well-behaved partial derivatives of the functions [21]. The reason of the popularity of this method is its quadratic convergence in the vicinity of the solution. Most known applications of this method are, for example, the calculation of the loadflow of power systems and the dc-analysis of nonlinear networks [22-28]). In most of these applications Newtonlike methods are used. The main concern of these modified Newton methods, is the reduction of the number of evaluations and inversions of the Jacobian matrix, which basically has to be done at every stage of the iteration. More specifically, in the Newtonlike methods the inverse Jacobian matrix is replaced by an approximation which is modified in a simple manner at each iteration, while allowance can be made for preventing divergence [22,23,24].

The general Newton iterative procedure applied to equation (4.4.1) is given by

\[ x_{i+1} = x_i - \lambda_i d_i \quad (4.4.2) \]

where

\[ d_i = J_i^{-1} F(x_i) \quad (4.4.3) \]

with

\[ \lambda_i : \text{scaling factor} \]
\[ J_i : \text{Jacobian matrix} \]
\[ F(x_i) : \text{residual vector} \]
\[ i : \text{the ith-iteration step} \]

In every iteration step, the value of \( \lambda_i \) is chosen so as to minimize the function \( h(\gamma) = F(x_i - \gamma d_i) \) in the direction of the correction vector \( d_i \) [21]. Hence, divergence of the iterative scheme is avoided. Further, Broyden [22] has given algorithms for calculating approximations of the inverse Jacobian matrix. However, in some (of our) cases these methods yield bad approximations, resulting in a slow movement towards the solution, or in the worst case a non-convergent movement. A very practical procedure, lying on above ideas, but with some more efforts, apparently yields favourable results in most cases. This procedure, will be outlined next.

Starting with a suitable choice of the solution vector \( x_0 \), the Jacobian matrix \( J \) and its inverse

\[ J^{-1} = \left( \frac{\partial F(x_0)}{\partial x} \right)^{-1} \]

and the residual vector

\[ F_0 = F(x_0) \]
can be evaluated. Generally speaking, the residual vector $F_0$ will not be equal to zero, in other words, $||F_0||$ will be greater than a given accuracy $\epsilon$. With the aid of Eqs. (4.4.2) and (4.4.3), the subsequent solution vector $x_1$, can now be computed, provided the scaling factor $\lambda_0$ is known. To begin with, we set $\lambda_0 = 1$. Subsequently, $x_1$, respectively $F_1$, are computed. Suppose we now have, $||F_1|| > ||F_0||$, where $||\cdot||$ is a suitable norm, then $\lambda_0$ is bisected and with this new $\lambda$, a new $x$, is calculated. This process is maintained as long as it is not successful in finding a magnitude of the residual vector which is less than that of the previous iteration step. Thus, at the end of every iteration step it is assured that the magnitude of the residual vector is at least less than the magnitude of the previous step, assuring non-divergence of the process.

The next step, that is, if $||F_1|| < ||F_0||$, $d_1$ is calculated, provided the inverse Jacobian matrix is known. The number of evaluations and inversions can be limited when it is decided to update the Jacobian matrix, whenever the elements of the matrix strongly differ from the corresponding elements calculated in the last Jacobian evaluation. Accordingly, the Jacobian matrix and its inverse are determined in this step, if at least one of the derivatives of the nonlinear functions which represent the nonlinear elements appreciably differs from the corresponding derivatives of the functions in point $x$. With this criterion a region round an operating point of the nonlinear element is created in a simple manner. Within this region the derivatives may be held constant in the iteration process, that means, that no evaluation nor inversion of the Jacobian takes place. This region is determined by those points of which the derivatives of the nonlinear functions are lying within given constraints, that is, if

$$c_{1,m}^{(k)} \leq \frac{\partial f(x)}{\partial x}^{(k)} \leq c_{r,m}^{(k)}$$

$$x = 1, \ldots, n$$

$$m = 1, \ldots, n$$

where

$c_{1,m}$ and $c_{r,m}$: left and right hand constants respectively

$f^{(k)}(x)$ : the $k$-th element of the vector function $F$

$x^{(m)}$ : the $m$-th element of the vector $x$

If above introduced constraints (by means of the left and right hand constants) are relaxed, this may result in a reduction of the computation time due to fewer updating and inversion at each iteration stage. Obviously, the extent of this region greatly influences the iteration scheme. A great number of iterations is obtained with too liberal constraints, resulting in an increase of the computation time. On the other hand, we may arrive at this very result with too narrow constraints, for in this case the computation time is affected by the increase of matrix manipulations. The region in which the partial derivatives may be kept constant in the iterative scheme, is determined by
the rule of thumb, and can be best determined in practical situations.

With the determination of $d_*$, the same steps must be carried out for computing $x_*$, in general $x_{i+1}$.

It is noticed, that above procedure greatly agrees with the methods used for solving piece-wise nonlinear resistive networks [29–32]. The derivatives of the functions describing piece-wise linear elements, of course are constant in a given region. Consequently, in these methods the inverse of the Jacobian matrix is continuously updated, whenever a jump is made to another part of the piece-wise linear function. In most of these studies the new inverse Jacobian matrix is evaluated from the old inverse matrix by applying a few matrix manipulations.

Since the procedure discussed in this section has a more general character, and moreover, allowance is made for the rate of change of the nonlinear functions, this approach is also very well suited for handling piece-wise resistive networks. Except that in the approach used here, the inverse is calculated all over again, because of inaccuracies introduced by a repeated modification of the inverse matrix [31, 33].

Finally, it is noticed that the evaluation of the scaling factor has been kept very simple. This has been put to $\lambda_i(m) = 1/K^m$ with $m = 0, 1, 2 \ldots$ and $K = 2$, or more general $K > 1$. In most cases this choice leads to satisfactory results.

4.5 Numerical integration

The behaviour of most electrical circuits can be described by a set of ordinary differential equations and algebraic equations. These equations are first discretized, whereupon the resulting algebraic equations are solved. In the previous section it has been shown that these algebraized equations can systematically be solved by means of resistive dc-networks. This sections deals with the discretizing of the differential equations. It is well-known that as far as the differential equations are concerned, the applied numerical integration methods are of key importance to the solution of the network problem, since they affect the modelling details to a great extent. Consequently, a brief review of the commonly used integration method will be given.

Numerical integration methods fall into the main categories explicit or implicit, and single-step and multi-step methods [34]. In explicit methods the integration formulas are applied directly to the previously solved variables, that is, the state variables can readily be evaluated using information obtained from the previous steps. In implicit methods, on the other hand, an additional calculation is required. At every time step $t_n$ a set of algebraized differential equations has to be solved, because additional information confined in the derivatives at $t_n$ is considered. If in these methods we fall back upon information from the last time interval $[t_k, t_{k+1}]$, the method is called a single-step method. A multi-step method requires more information than obtained from the last time interval and goes further back in the time.

Suppose now, that a system is described by $n$-ordinary differential equations, corresponding to
\[ \frac{dx}{dt} = f(x,t) \]  
(4.5.1)

where

\[ t_0 \leq t \leq t_e, \quad x(t_0) = x_0 \]

and

\[ x(t), f(x,t) \] are \( n \)-element column vectors. Further it is assumed that \( f(.) \) is Lipschitz-continuous, that is, for \( 0 < L < \infty \), the following inequality

\[ ||f(x,t) - f(y,t)|| \leq L||x - y|| \]

guarantees the existence and uniqueness of the solution of Eq. (4.5.1). Then, a numerical solution of (4.5.1) can be obtained with the linear multi-step methods (see e.g. [46]), given by

\[ \sum_{j=0}^{k} \alpha_j,n \ x_{n-j} = h_n \sum_{j=0}^{k} \beta_j,n \ \dot{x}_{n-j} \]

(4.5.2)

with

\[ \alpha_0,n = 1 \]

and where

\[ h_n = t_n - t_{n-1}, \]

\[ n = 1,2, \ldots \]

and

\[ \dot{x} = \frac{dx}{dt} \]

The method is called explicit if the coefficient \( \beta \_0,n \) is equal to zero, whereas in the other case the method is implicit. A great deal of the attractiveness of the explicit methods disappears if we are dealing with the transient analysis of systems of which the ratio between the largest and smallest time constant is high. Application of the explicit methods for solving these what are called stiff differential equations, leads to excessive computation time. In order for the computer solution to be numerically stable (bounded), the step size must be smaller than the smallest time constant of the system [34]. In this respect, the additional effort of the implicit method is repaid with significant computation time reduction and a greater stability.

When using the term stability, we more or less have made an appeal for our intuition of stability. Intuitively it is assumed, that numerical stability is concerned with the propagation of errors over many successive steps; a numerical unstable method is one in which the error tends to an inadmissible accumulation of errors in the computation. For a detailed discussion on stability of numerical integrated methods, the reader is referred to e.g. [34,36,37]. Dalquist has proved that the trapezoidal rule is the
best single step method of order two - the order two is a guarantee for absolute stability - in that it produces the smallest truncation error [35]. However, the trapezoidal rule has not been chosen for solving our network problems. In the following it will be shown how this method can come into severe trouble, producing unwanted results in a network formulation.

For this purpose, a one to one Z-operator is related to the differential operator d/dt. Notice that this can be done only when it has been decided which numerical integration will be used to solve the differential equations [38-41]. The general form of this Z-operator is

$$
\frac{d}{dt} \leftrightarrow \frac{1}{h} \sum_{j=0}^{k} \alpha_j z^{k-j} \sum_{j=0}^{k} \beta_j z^{k-j}
$$

(4.5.3)

which yields the trapezoidal rule Z-operator

$$
\frac{d}{dt} \leftrightarrow \frac{z - 1}{z + 1}
$$

(4.5.4)

Application of the trapezoidal rule operator, or more general, application of the operator of any linear multi-step method, can be best explained by means of an electric circuit, which of course appeals to most electrical engineers. For this purpose a circuit consisting of an inductance L and a resistance R (Fig. 4.3) is considered. With the aid of this circuit some typical characteristics of trapezoidal rule will be discussed, while at the same time the application of the MNA will be illustrated once more.

This network is described by the following differential equation

![diagram]

Figure 4.3 L-R-circuit used in the stability considerations of the numerical integration methods
\[ e = L \frac{d}{dt} i + R i, \quad i(0) = 0 \]  
(4.5.5)

which can be written in the following MNA form

\[
\begin{bmatrix}
\frac{1}{R} & -1 \\
1 & L \frac{d}{dt}
\end{bmatrix}
\begin{bmatrix}
u_1 \\
i
\end{bmatrix}
= \begin{bmatrix} 0 \\ e \end{bmatrix}
\]  
(4.5.6)

of which the solution vector is

\[
\begin{bmatrix}
u_1 \\
i
\end{bmatrix}
= \frac{1}{\tau \frac{d}{dt} + 1}
\begin{bmatrix} 1 \\ e \end{bmatrix}
\]  
(4.5.7)

with \( \tau = L/R \), and its Z-transform

\[
\begin{bmatrix}
U_1 \\
I
\end{bmatrix}
= \frac{ah z + 1}{2 + ah z - \frac{z}{\tau}}
\begin{bmatrix} 1 \\ E \end{bmatrix}
\]  
(4.5.8)

where

\[
z_t = \frac{2 - h/\tau}{2 + h/\tau} = \frac{2 - ah}{2 + ah} < 1 \quad \text{and} \quad a = 1/\tau
\]  
(4.5.9)

with \( U_1, I \) and \( E \) are the Z-transforms of respectively \( u_1, i \) and \( e \). The impulse response (\( E = 1 \)) of this discrete system is

\[
\begin{bmatrix}
u_{1(nh)} \\
i_{(nh)}
\end{bmatrix}
= \frac{ah}{2 + ah}
\begin{bmatrix} (z)^n \\ t \end{bmatrix}
\begin{bmatrix} 1 \\ L/R \end{bmatrix}
\]  
(4.5.10)

and for the condition, \( 0 < ah < 2 \), (4.5.10) can be rewritten as

\[
\begin{bmatrix}
u_{1(nh)} \\
i_{(nh)}
\end{bmatrix}
= \frac{ah}{2 + ah}
\begin{bmatrix} \exp(-\alpha nh) \\ 1 \end{bmatrix}
\begin{bmatrix} 1 \\ L/R \end{bmatrix}
\]  
(4.5.11)

\[ n = 1, 2, 3, \ldots \]
with

$$z_c = \exp(-\alpha h) \quad \text{or} \quad \alpha = -1/h \ln(z_c)$$

From a study of (4.5.9) it is seen that for every $h$, the inequality $|z| < 1$ holds, that is, the discrete system described by (4.5.8) is stable. A comparison of this discrete response with the following response of the continuous system (described by Eq. (4.5.7))

$$
\begin{bmatrix}
1 \\
i(t)
\end{bmatrix}
= a \exp(-at)
\begin{bmatrix}
1 \\
1/R
\end{bmatrix}, \quad t > 0
$$

(4.5.12)

shows that the discrete response will loose its fidelity in representing the continuous response for $ah > 2$: instead of an approximation of a decaying exponential function the response is a damped oscillation. Figure 4.4 shows the responses for $ah=2$, 4, 10 and 40. Although we are dealing with a stable discrete solution (bounded solution), for simulation purposes this is inadmissible. The fidelity of the response has vanished.
Figure 4.5  L–R–circuit with switch, used in the stability consideration of the trapezoidal rule

completely. This fidelity problem prohibits the use of a large step size. Actually, we should have, \( h < 2\tau \), where \( \tau \) is the smallest time constant of the system.

The following example, which is an extension of the first example, clearly illustrates some misunderstanding that can be introduced in this respect. For this purpose, the network given in Fig. 4.3 is extended with an ideal switch in series with the resistance (Fig. 4.5). It is quite easy to verify that two MNA representations are needed for the description of the two states of the (ideal) switch. For a closed switch the MNA form in the Z-plane of the circuit illustrated in Fig. 4.5 is

\[
\begin{bmatrix}
1/pL & 0 & 1 \\
0 & 1/R & -1 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
I_{sw}
\end{bmatrix}
= \begin{bmatrix}
(l/pL)E \\
0 \\
0
\end{bmatrix}
\] (4.5.13)

with \( p = \frac{2}{h} \frac{z-1}{z+1} \) and with solution vector

\[
\begin{bmatrix}
U_1 \\
U_2 \\
I_{sw}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{pL/R + 1} \\
1 \\
1/R
\end{bmatrix}
\] (4.5.14)

and for an open switch
\[
\begin{bmatrix}
1/pL & 0 & 1 \\
0 & 1/R & -1 \\
1 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
U_1 \\
U_2 \\
I_{SW}
\end{bmatrix}
= \begin{bmatrix}
(1/pL)E \\
0 \\
0
\end{bmatrix}
\quad (4.5.15)
\]

with
\[
p = \frac{2}{h} \frac{z-1}{z+1}
\]
and solution vector
\[
\begin{bmatrix}
U_1 \\
U_2 \\
I_{SW}
\end{bmatrix}
= \begin{bmatrix} 1 \\ 0 \\ E \end{bmatrix}
\quad (4.5.16)
\]

So far, apparently nothing peculiar has happened: the solution for a closed switch is the same as in the preceding example, whereas the solution for an open - the opening of the switch occurs at zero current - switch is evident. If however, the response of this circuit is computed, we surprisingly are confronted with an unexpected undamped oscillation of the kind described above. Instead of the response depicted in Fig. 4.6b, spurious oscillation as shown in Fig. 4.6a is obtained. At first

\hspace{1cm}

Figure 4.6  Voltage response after opening of the switch, calculated with

a) trapezoidal rule
b) backward Euler method
sight this oscillation seems to be inexplicable, especially if
Eq. (4.5.16) is considered. Consequently, some users of the
trapezoidal rule, being faced with these phenomenon, try to seek
physical explanations for this pure numerical problem [42].

Let us however, try to look at this problem more closely.
In this respect, it is important for clarification, to point out
that in fact we have defined discrete systems with the discrete
MNA-description given in (4.5.13) and (4.5.15). As is known,
discrete systems are amenable to stability analysis [38], and for
this purpose, the characteristic equations of these discrete
systems are evaluated, leading to the equation of a closed
switch:

$$z + \frac{1 - (L/R)2}{1 + (L/R)2} = 0$$

(4.5.17)

and the equation of an open switch

$$z + 1 = 0$$

(4.5.18)

Obviously, the characteristic equation (4.5.17) yields a value
for z which is given in Eq. (4.5.9). Since this value for z is
smaller than unity, the system is stable. From the characteristic
equation (4.5.18) it is readily seen that the discrete system
described by (4.5.15) is a borderline case of instability, since
z=1. Consequently, every disturbance as a result of, for
example, round-off errors in the computation, will propagate
without attenuation in the subsequent calculations. In this
specific case, the switch was opened at a zero-crossing of the
current. As already noticed, this has lead to the phenomenon
illustrated in Fig. 4.6a, a voltage oscillating with frequency
1/(2h). A complete understanding of this phenomenon can be
obtained by considering the following equation

e(nh) + e((n-1)h) =

$$u_1(nh) + u_1((n-1)h) + \frac{2}{h} L[i_{sw}(nh) - i_{sw}((n-1)h)]$$

n=1, 2, ...

which is the discretized equation of voltage $u_1$, the voltage at
the terminal of the inductor (see Fig. 4.5).

Suppose now, that $i_{sw}(0) = i_{sw0} \neq 0$, and that $i_{sw}(nh) = 0$
for n=1, 2, ..., or in other words, the opening of the switch at a
zero-crossing is being considered. Then, it can be easily
verified that voltage $u_1$ can be expressed as:

$$u_1(nh) = e(nh) - (-1)^n \frac{2}{h} L_i_{sw0} \Leftrightarrow U_1 = E - \frac{z}{z + 1} \frac{2}{h} L_i_{sw0}$$

Above expressions clearly explains the response as observed in
Fig. 4.6a, namely, oscillation with frequency 1/(2h) superposed
on the voltage waveform e(t) of the source. It is also seen that
for \( i_{\text{sw}} = 0 \) – this represents the opening of a switch at zero current, and not at zero-crossing – no oscillation will occur. When confronted with this fact, numerical oscillations when the current is interrupted at a nonzero value of the current, and none when this happens at zero current, one is tempted to have the following thoughts. Chopping of the current, which is a violation of physics, has caused numerical oscillation here. When numerical oscillations (of this kind) occur in other cases, this will have alike cause, namely, representation of phenomena which do not exist physically. However, it can be shown that this kind of numerical oscillations will also occur when physically well-behaved phenomena are being represented.

This can be appreciated by letting \( R \) (in Eq. (4.5.10)) obtain large values. Notice, that this can be related to the response given in Fig. 4.4 for various values of \( \alpha h \). An extreme consequence of this is a time constant \( \tau = L/R = 0 \), or \( \alpha h = h/\tau = \infty \), and thus, an oscillatory response (cf. Eq. (4.5.10)): \( u_1(nh) = (-1)^n \). It can easily be verified that in this case the condition for obtaining responses which show reasonable fidelity (0<\( \alpha h < 2 \), or \( 0<h<2\tau \)) is not satisfied.

It should also be indicated that the condition \( \tau = L/R = 0 \), or \( \alpha h = h/\tau = \infty \), can also be read here as the inductance \( L \) being much smaller than the resistance \( R \). In practice, this mostly is the case for saturable inductors. It can be easily shown, that the voltage \( u_1 \) (see Fig. 4.5) will contain alike numerical oscillations when the inductor is taken nonlinear. Assume for this purpose, that the saturation characteristic can be represented by two values of the inductance \( L \), namely, \( L \neq 0 \) for the unsaturated state, and \( L = 0 \) for the saturated state. Then, it can be shown, that in the saturated state the voltage \( u_1 \) can be described as follows

\[
u_1(nh) + u_1((n-1)h) = e(nh) + e((n-1)h) + \frac{2}{h} \frac{\mathrm{Li}}{s}
\]

for \( i(nh) > i_s \) or \( i(nh) < -i_s \) 

\[n=1,2, \ldots \]

and

\[i(0) = \pm i_s \]

where

\( i(nh) \): current through the inductor

\( i_s \): value of the current for which the following inequality holds: \( L \neq 0 \) for \( -i_s \leq i \leq i_s \), and \( L = 0 \) for \( i > i_s \), or \( i < -i_s \)

The closed form of above recurrent equation can be expressed as follows

\[
u_1(nh) = e(nh) + (-1)^n \frac{2}{h} \frac{\mathrm{Li}}{s} \quad U_1 = E \pm \frac{z}{z + 1} \frac{2}{h} \frac{\mathrm{Li}}{s}
\]
Notice that above equations are mathematically identical to those related to current chopping.

It has been verified that, when the saturation characteristic is not represented by a simple two slope description (as done here), but by a more complex function, as for instance given in Chapter 3, these very numerical oscillations occur. Clearly, this must be related to the fact that due to saturation $ah=\frac{h}{\gamma}$ becomes greater and greater (seen from a linear standpoint of view), and thus, that the condition $0<ah<2$, or $0<h<2\gamma$ is violated and not physics.

The conclusion that can be drawn from above discussion should be obvious. Application of the trapezoidal rule for solving transient network problems will get us undoubtedly into trouble.

The problems encountered with the trapezoidal rule do not show up in the simple backward Euler method. Incidentally, notice that the voltage waveform given in Fig. 4.5b has been calculated with the backward Euler method. The backward Euler method can be characterized in the Z-plane as follows:

$$\frac{d}{dt} I = \frac{2}{h} \frac{z+1}{z}$$  \hspace{1cm} (4.5.19)

Application to the test equation given in (4.5.7) leads to

$$\begin{bmatrix} u \\ i \end{bmatrix}_1 = \begin{bmatrix} ah & z \\ 1 + ah & z - z_e \end{bmatrix} \begin{bmatrix} 1 \\ 1/R \end{bmatrix}$$  \hspace{1cm} (4.5.20)

with

$$z_e = \frac{1}{1 + ah} < 1 \text{ and } a = 1/\gamma$$  \hspace{1cm} (4.5.21)

The impulse-response of this discrete system is given below,

$$\begin{bmatrix} u_2(nh) \\ i(nh) \end{bmatrix} = \begin{bmatrix} ah \\ 1 + ah \end{bmatrix} \exp(-\alpha nh) \begin{bmatrix} 1 \\ 1/R \end{bmatrix}$$  \hspace{1cm} (4.5.22)

with

$$z_e = \exp(-\alpha h) \text{ or } \alpha = -1/h \ln(z_e)$$

and $u_2(nh)$ is plotted in Fig. 4.7. From inequality (4.5.21) $(ah>0)$ it is seen that in this case no oscillations will appear.
Figure 4.7 illustrates the impulse responses for the same values of $ah$ used in the trapezoidal rule case. As can be seen, for every value of $ah$ the impulse response has a decaying shape, indicating that round-off noise will not be amplified.

Some users of transient network-analysis programs who have suffered great inconvenience with the trapezoidal rule, did have notified this [43-48], and have proposed the application of linear multistep methods described by the following form:

$$\frac{d}{dt} \leftrightarrow \frac{1}{h} \sum_{j=0}^{k} \alpha_j z^{k-j}$$

(4.5.23)

The coefficients $\alpha_j$ and $\beta_j$ are determined by approximating $x(t)$ (of Eq. (4.5.1)) by a polynomial $p(t)$ in the interval $[t_{n-k}, t_n]$, and by letting $p(t_{n-j}) = x(t_{n-j})$ for $i = 0, 1, \ldots, k$ and $p_k(T) = x(T)$. An efficient algorithm for evaluating these coefficients can be found in Ref. [47], where it is shown that the following differentiation formula can be used for carrying out numerical integration:

$$x_n = \sum_{j=1}^{k} \left( x_n - y_n^{(j)} \right) / h_j$$

(4.5.24)
with
\[ h_j = t_n - t_{n-j} \]
\[ \hat{h}_j = t_{n-1} - t_{n-1-j} \quad j = 1, 2, \ldots, k-1 \]
\[ y_n^{(j)} = y_n^{(j-1)} + d_{j-1} \left( x_{n-1} - y_n^{(j-1)} \right) \]
\[ d_j = d_{j-1} \left( h_j / \hat{h}_j \right) \quad j = 2, 3, \ldots, k \]
\[ y_n^{(1)} = x_{n-1} \]
\[ d_1 = h_1 / \hat{h}_1 \]

Notice that in this case no use is made of an operator in the Z-domain, since the non-uniform division of the time interval would hamstring application of the Z-transform.

It is easily verified that for a constant step size (4.5.24) changes into the following equivalent form

\[ \dot{x}_n = \frac{1}{h} \sum_{j=1}^{k} \frac{x_n - y_n^{(j)}}{j} \quad (4.5.25) \]

with
\[ h = t_n - t_{n-1} \]
\[ y_n^{(j)} = y_n^{(j-1)} + \left( x_{n-1} - y_n^{(j-1)} \right) \]
\[ j = 2, 3, \ldots, k \]
\[ y_n^{(1)} = x_{n-1} \]

In this case there is no need to store the specific coefficients related to the various time step number \( k \).

With the aid of an example, while attention is being paid to the above-mentioned handling of nonlinear elements, it will be demonstrated how the time response can be numerically calculated, using integration formula (4.5.24). The example given here explicitly shows the basic steps which must be taken (in a computer program) to calculate the behaviour of transformers in electrical networks. Also notice, that since allowance is made for a variable step, but above all, since the circuit is nonlinear, no use is made of the Z-transform.

In Fig. 4.8 a circuit is depicted with a nonlinear inductor described by the following nonlinear equation

\[ i_{12} = \sum_{j=1}^{q} c_{2j-1} \psi^{2j-1} \quad (4.5.26) \]
where \( i_{12} \) is the current through the inductor.

It can be easily verified that after discretizing the voltage equation of the inductor

\[
    u_1 - u_2 = \frac{d\psi}{dt}
\]

using the \( d/dt \)-operator expressed in (4.5.23) - actually, expression (4.5.23) can be seen as a shorthand notation of a linear implicit differentiation formula of order \( k \), the following equation is obtained

\[
    u_1(n) - u_2(n) = \frac{1}{h\beta_0} \psi(n) + U_L \quad (4.5.27)
\]

where

\[
    U_L = - \frac{1}{h\beta_0} \psi
\]

will serve as a dc-voltage source in the resistive network, and with

\[
    \psi = \sum_{j=1}^{k} \alpha_j \psi(n-j)
\]

Substitution of Eq. (4.5.27) in (4.5.26) yields

\[
    i_{12}(n) = \sum_{j=1}^{q} c_{2j}\left(h\beta_0(u_{12}(n) - U_L)\right)^{2j-1} \quad (4.5.28)
\]

with

\[
    u_{12}(n) = u_1(n) - u_2(n)
\]

It is important to recognize that Eq. (4.5.28) is characteristic for the nonlinear inductor and that the nodal voltages are the
independent variables (also, compare this equation with the more general equation (4.3.4) of Section 4.3). In Section 4.3 a procedure has been discussed to handle voltage controlled nonlinear elements. This procedure will now be followed for this specific case.

For this purpose the capacitance is also discretized, and it can be shown that the resistive nonlinear dc-network given in Fig. 4.9 can be derived. The MNA description of this network can be expressed as

\[
\begin{bmatrix}
0 & 0 & -1 & u_1(n) & 1 & 0 \\
0 & \frac{C}{h\beta_0} & 0 & u_2(n) & -1 & i_{12}(n) \\
1 & 0 & R & i(n) & 0 & E
\end{bmatrix}
\begin{bmatrix}
i_1(n) \\
i_2(n) \\
i_{12}(n)
\end{bmatrix} = \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]

\[i_{12} = \frac{C}{h\beta_0} \sum_{j=1}^{k} \alpha_j u_2(n-j)\]

is the dc-current source as given in Fig. 4.8, and

\[E = e(t_o + nh)\]

is the dc-voltage source

Notice that Eq. (4.5.29) describes the linear resistive network shown in Fig. 4.9b, that is, the original (nonlinear) network, except that the nonlinear branch (the inductor) is
replaced by a current source \(i_{12}(n)\). This linear network cannot be solved directly, since the current source \(i_{12}(n)\) is an unknown source. In first instance the quantities of the linear network must be expressed in terms of \(i_{12}(n)\), whereupon this current can be solved from the Thevenin equivalent network the nonlinear inductor is connected with.

For this purpose the solution vector of Eq. (4.5.29) is written as follows

\[
\begin{bmatrix}
  u_1(n) \\
  u_2(n) \\
  i(n)
\end{bmatrix} + \begin{bmatrix} R \\ -h\beta/C \\ -1 \end{bmatrix} \begin{bmatrix} i_{12}(n) \end{bmatrix} = \begin{bmatrix} R & 0 & 1 \\ 0 & h\beta/C & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \hat{u}_1(n) \\ \Delta \hat{u}_2(n) \\ \Delta \hat{i}(n) \end{bmatrix}
\]  

(4.5.30)

From above equation the nodal voltages \(u_1(n)\) and \(u_2(n)\) are used in the Thevenin equivalent, that is, the branch voltage \(u_{12}(n) = u_1(n) - u_2(n)\) is expressed as

\[
u_{12} = -(R + h\beta_0/C)i_{12}(n) + \Delta \hat{u}_{12}
\]  

(4.5.31)

with

\[
\Delta \hat{u}_{12} = \Delta \hat{u}_1(n) - \Delta \hat{u}_2(n)
\]

and substituted in Eq. (4.5.28) of the nonlinear inductor for solving \(i_{12}(n)\). Once the current \(i_{12}(n)\) has been solved, the solution of the whole nonlinear network follows from Eq. (4.5.30).

Suppose now, that instead of the flux, the current is the independent variable. In other words, let for example, the current-flux relationship be given by

\[
\psi(n) = Li(n) + \sum_{j=1}^{q} c_{2j-1} \left[1 - \exp(-c_{2j}i(n))\right]
\]  

(4.5.32)

which, substituted in Eq. (4.5.27), yields the following algebraic equation for the inductor

\[
u_1(n) - u_2(n) = \frac{1}{h\beta_0} \left[Li(n) + \sum_{j=1}^{q} c_{2j-1} \left[1 - \exp(-c_{2j}i(n))\right]\right] + U_L
\]  

(4.5.33)

where

\[
U_L = -\frac{1}{h\beta_0} \sum_{j=1}^{k} \alpha_j \psi(n-j)
\]
Equation (4.5.33) shows that when the flux equation is given in a current controlled form, e.g. Eq. (4.5.32), the constitutive equation of the nonlinear inductor (transformer) logically is current controlled. In the following, a similar procedure as followed above for the voltage controlled inductor, will be given. However, the mathematical details will now be left out.

It can be verified that, after similar mathematical manipulations as above, the solution vector of the linear resistive network can be expressed now as

\[
\begin{pmatrix}
  u_1(n) \\
  u_2(n) \\
  i(n)
\end{pmatrix} + \begin{pmatrix}
  \frac{1}{RC/h_0} + \frac{1}{1} \\
  0 \\
  -C/h_0
\end{pmatrix} \begin{pmatrix}
  u_1(n) \\
  u_2(n) \\
  i(n)
\end{pmatrix} = \begin{pmatrix}
  E/R \\
  0 \\
  0
\end{pmatrix}
\]

where

\[
\begin{pmatrix}
  \hat{u}_1(n) \\
  \hat{u}_2(n) \\
  \hat{i}(n)
\end{pmatrix} = \begin{pmatrix}
  1/R & 0 & 1^{-1} \\
  0 & C/h_0 & -1 \\
  1 & -1 & 0
\end{pmatrix} \begin{pmatrix}
  E/R \\
  1 \\
  0
\end{pmatrix}
\]

\[E = e(t_o + nh)\]

and

\[I_C = \frac{C}{h_0} \sum_{j=1}^{k} \alpha_j u_2(n-j)\]

The branch current for the Thevenin network follows from the last row of Eq. (4.5.34) and is given by

\[i(n) = \frac{-RC/h_0}{RC/h_0 + 1} u_{12}(n) + \hat{i}(n)\]  \hspace{1cm} (4.5.35)

Then, the voltage \(u_1(n)\) can be solved by substituting Eq. (4.5.35) in (4.5.33), the equation of the inductor, whereupon the solution of the whole network is obtained from (4.5.34).

Recognize at this point, that above examples very clearly demonstrate how easily the MNA-approach can cope with both forms of the current-flux relation. The examples given here for
(nonlinear) inductors are characteristic for single-phase elements, specifically for two-node-incidence elements. However, the extension to single-phase and three-phase transformers, or in general, to multi-phase nonlinear elements is straightforward; Section 4.3 deals with the formulation of this matter.

Since the inductor considered in the examples is a two-node incidence element, the current-flux relationship is simple and easy to handle. Notice, that the current-flux relationship for transformers has similar forms, except that for transformers this relationship is given by a set of (nonlinear) equations. This means that essentially the same approach can be used for transformers. In fact, this requires the processing of nonlinear vector equations of the same form.

As known both considered forms of the current-flux relationship are used in transformer studies. The formal framework discussed in Section 4.3 takes this into account. Actually, the examples worked out in this section can be fit in this framework. Both representation forms have been implemented in a computer program. In Appendix A it is shown how these two forms of current-flux relations of transformers can be organized (using the loop method and the cutset method) in order to fit in the adapted version of the MNA-method that has just been explained.

Finally, a few relevant remarks concerned with the implemented integration formula are made. In the computer program the integration is carried out as prescribed by the algorithm given by (4.5.24), though in all calculations the time step is held constant. In doing so - using algorithm (4.5.24) instead of (4.5.25) -, allowance is made for automatic adaption of the step size as suggested in Ref [47]. This mechanism has not (yet) been implemented, since in all the considered cases there was no need to change the step size; the transformer responses, which are discussed in the next chapter, have been calculated with a fixed number k=2.
5 TRANSFORMER RESPONSE

In the preceding chapter much attention has been paid to represent the transformer behaviour as realistic as possible. This has primarily been done to have a physically based description to fall back upon when analyzing the (external) behaviour of the transformer. In addition, a generic approach has been given for the study of circuits containing transformers and/or various transformer connections.

In this chapter some simulation results obtained with a computer program (discussed in Chapter 4) will be considered. The results discussed in this chapter are not so much meant to stress the disparities in the different modelling concepts of the transformer as to show how clarifying a physically based transformer model works for a variety of transformer topics. More specific, this approach enables us to elicit new aspects of the transformer behaviour. Moreover, an example of exquisite application of the concept is shown in the explanation of a newly observed voltage jump phenomenon.

In Section 5.1 modelling of core losses is investigated. Particularly, the effect of the representation of these losses - Chapter 3 has been expended in describing this magnetic characteristic of the core - on a transient are considered.

Residual core conditions are the subject of Section 5.2. It is shown how in a three-phase, three-limb transformer the residual condition of the core depends on the transformer load.

Section 5.3 is concerned with one of the most continual topics in transformer studies, namely ferroresonance. More, and deeper insight in this matter will be gained by working out a case of two-phase switching (which results in ferroresonance) of a three-limb transformer.

A voltage jump phenomenon occurring at the transformer terminals is explained in Section 5.4. This recently observed phenomenon occurs when the transformer is being energized with a circuit breaker whose poles are not closing simultaneously.

Finally, the no-load conditions of the three-phase, three-limb transformer will be considered closely in Section 5.5. Waveforms emanating from two transformer models are compared with measured waveforms. It will be seen that the models fail in predicting the magnitude of third harmonics (and that of higher harmonics). Responsible for this discrepancy between models and reality is the identification method used (see Section 2.4), which has been developed for a linear and not a nonlinear model.

5.1 Hysteresis and Eddy-current Effect

In this section the influence of the core characteristics, the eddy-current effect and hysteresis, on the transient behaviour will be discussed. The effect of several representations of core losses will be considered in relation with transients.

It is noticed that some authors, among whom Swift [53], subdivide the total core loss in a frequency-dependent part and a
frequency-independent part. Swift has experimentally proved that for power transformers the frequency-dependent part can be closely simulated by a resistance in parallel with a nonlinear inductor branch. This nonlinear inductor branch, in this thesis being described by a static hysteresis model, is the representation of the frequency-independent part of the total core loss. In Section 3.3 and 3.4 of Chapter 3 this idea has been worked out with much emphasis on the improvement of the frequency-independent part (the static hysteresis model). In order to avoid confusion, it is important to note that in practice (of the electrical engineer) the frequency-dependent part of the core loss is referred to as the eddy-current part and the frequency-independent part as the (static) hysteresis part. Henceforward this terminology will be used.

It is well known that the eddy-current losses, and thus the resistances representing these losses, can be easily determined from measured values of the total core losses at two different frequencies. Once the eddy-current losses are known, the hysteresis part of the model can be described; Section 3.2 of Chapter 3 explains the link between eddy-current losses and dynamic hysteresis.

In order to investigate the transformer model under the most simple transient conditions, three capacitances of $C=10 \, \mu F$ are applied to the secondary side of the transformer as shown in Fig. 5.1. The transformer model is a model of a 9.5 kVA (laboratory) test-transformer. Further it can be seen, that the neutral of the wye-connected secondary windings is grounded, whilst the primary winding, being delta-connected, are directly connected to a symmetrical voltage system. In this numerical experiment, the steady state of transformer with its pure capacitive load is first calculated. Then, when the transformer is disconnected from the voltage sources - the disconnection of a phase takes place at the zero-crossing of the phase current $-$, a transient is set in. This transient can be followed by observing the secondary voltages. It will be seen that the secondary voltage will decrease rapidly, oscillating with the frequency determined by the capacitance and the inductance associated with the transformer core. The damping in this oscillating voltage

![Figure 5.1 Delta-wye connected transformer with capacitive load](image-url)
response is mainly introduced by the dissipation in the transformer core. This experiment has been numerically carried out for the following cases:

(a) eddy-current effect and hysteresis effect excluded
(b) eddy-current effect included and hysteresis effect excluded
(c) eddy-current effect excluded and hysteresis effect included
(d) eddy-current effect and hysteresis effect included.

It is noted that the simulation has been carried out with the integrated flux model (if-model). The simulation has been restricted to this model, since the transient being discussed is strongly related to the iron-core. Further, it is noted that in cases (b) through (d), the dissipative effect in question has been chosen so as to cover the total core losses. Recognize, that the aim of this experiment is to show by means of simulation rather than tedious mathematical expressions, the damping effect introduced by the four types of core modelling.

Computers waveforms of above mentioned cases and the experimentally obtained waveforms are shown in Fig. 5.2. Since the other phases do not show essential differences, only the voltage of the centre phase are given. Figure 5.2a clearly illustrates the poorly damped voltage waveforms obtained when the core losses are neglected. The damping seen in the calculated transient is the result of the copper losses (ie, resistance of the winding) which have been allowed for in the model.

If however, the core losses are taken into account by an (parallel) equivalent resistance, case (b), the response will be damped, as can be seen from Fig. 5.2b. Case (c), in which the core losses are represented by a pure static hysteresis model, merely is of academic interest. With this simulation the hysteresis model is tested in the sense that minor loops are described. Due to traversing of minor loops, a dissipative effect is introduced, which is responsible for the voltage decay. The voltage decays as in case (b), however, in this case the response is less damped. It is important to note, that hysteresis models incapable of describing minor loops (the reader is referred to Section 3.3 of Chapter 3), will not yield satisfactory results. It can be verified that the voltage response then produced, will be more of the character given in Fig. 5.2a.

Finally, Fig. 5.2d shows the voltage response as calculated with both, eddy-current and hysteresis effect. A comparison with the experimentally obtained and the calculated voltage waveforms shows that as far as the damping is concerned, satisfactory results are obtained in cases (b), (c) and (d), i.e. when the eddy-current and/or hysteresis effects are taken into account. From a modelling point of view, the model provided with an equivalent resistance for representing the core losses, is preferable to the hysteresis model owing to its simplicity. As far as introduction of damping in transient phenomena (in which transformers are involved) is concerned, the effort expended in modelling hysteresis is disproportional. However, it will be seen later that a hysteresis model is necessary when considering residual core conditions.

To consider the behaviour of the hysteresis models in above discussed simulation more closely, the traversed hysteresis trajectories of the centre limb are plotted in Fig. 5.3. As can be seen, the main difference is the width of the major loop
Figure 5.2  Calculated and measured secondary voltage waveforms after the disconnection of the transformer

a) eddy-current and hysteresis effect excluded
b) eddy-current effect included and hysteresis effect excluded
c) eddy-current effect excluded and hysteresis effect included
d) eddy-current effect and hysteresis effect included
e) measured voltage waveform
Figure 5.3  Calculated hysteresis excursions related to

(a) the voltage waveform of Fig. 5.2c

(b) the voltage waveform of Fig. 5.2d

(being the loop traversed in the steady state condition), which is a measure of the core losses per cycle. Since the minor loops are related to this major loop, the minor loops related to case (c) will, generally speaking, be greater than those related to (d), and thus, introducing more damping in the transients. Notice, that a pure capacitive load seems to have a de-magnetizing effect when the transformer is de-energized. This effect and other effects of the load on residual conditions will discussed in the following section. But before considering residual core conditions, the transformer model will be subjected to a case of trapped charge.

For this purpose, the same circuit given in Fig. 5.1 can be used, except that the secondary side contains a circuit breaker for connecting and disconnecting the capacitances to and from the transformer. The object of this numerical experiment is not so much a thorough study of trapped charges as an illustration of how the transformer model (and computer program) can handle these kinds of problems. Actually, in doing so, the hysteresis option of the transformer model is being subjected to a severe test (benchmark).

To start with, the steady state condition of the circuit is computed. This is done by calculating an initial state of the network (Fig. 5.1), using the complex method of analysis. In other words, a linear description is being used for the nonlinear element (the transformer). Subsequently, the calculation of the steady state condition is completed by integrating the nonlinear circuit equations, starting from the obtained initial condition. The steady (periodical) state is reached when the transients become negligible. The idea behind this procedure is that according as the discrepancy between initial condition and final state decreases, the transient to the final state loses its severity. In this case, the initial state does not differ very
Figure 5.4 Calculated current and voltage waveforms of the trapped charge study

a) currents through the transformer breaker  
b) currents through the capacitor breaker  
c) secondary voltages of the transformer  
d) capacitor voltages  
e) flux linkages  
f) hysteresis loop
much from the steady state condition, and hence, the integration is only extended over a few periods. In most cases, the steady state was reached within two or three periods. It is worth knowing, that generally speaking, this so called 'brute force' approach could extend over hundred of cycles, making the computational cost prohibitively expensive. Recent studies on this subject propose more sophisticated algorithms for solving the (periodic or almost periodic) steady state response of nonlinear systems [54-58].

The numerical experiment is then continued with the disconnection of the capacitances from the transformer. This is done (see Fig. 5.4b) at the current zero (or nearly the zero) of the secondary current by the three (ideal) switches, which connect the capacitances with the transformer. Hereafter, the transformer is de-energized, applying the same switching procedure — disconnection at zero-crossing of the currents (see Fig. 5.4a) — with the transformer switches. What now is left, is a transformer with remanent fluxes (see Fig. 5.4e) and three capacitances with trapped charges (see Fig. 5.4d). Hereupon, the three capacitances are discharged over the transformer by re-closing the three capacitance switches simultaneously (see Figs. 5.4b and 5.4d).

The complete simulation of isolating (charging) the capacitances, disconnecting the transformer and discharging the capacitances, is reflected by the responses of current, voltage and flux waveforms given in Fig. 5.4. The switching instants can be perceived by considering the primary and secondary currents (Figs. 5.4a and 5.4b).

It is also seen (see Fig. 5.4d) that after removal of the capacitances from the transformer, the voltages of these capacitances remain constant. Further it can be verified that the transient related to the discharging of the capacitances over the not excited transformer is moderate. The waveforms related to the primary voltage (Fig. 5.4c) reveal that the discharging is transferred to the primary side of the transformer; after diminishing of the voltages the transformer suddenly becomes alive. As far as the fluxes are concerned, the de-magnetizing tendency of capacitances is moderate. In fact, it is seen that (in this case) discharging of trapped charges effectively caused a small change relative to the previous residual fluxes.

5.2 Residual Core Fluxes

In the previous section it has already been suggested that one of the major consequence of hysteresis is the residual core condition. This condition is established when the transformer is disconnected from its excitation. It is well appreciated that switching transients in power systems containing transformers will be affected by the residual flux. As far as can be traced, only the effect of the residual flux on the inrush currents of the transformer is comparatively well understood. The inrush phenomenon has been analytically set down in some well written, classical papers [20-23]. More recently, there exists a tendency to use (computer) simulation techniques for describing this phenomenon [61,70,71,81,83].
This section deals with the generation of various residual core conditions. As known, the solution of nonlinear differential equations is greatly affected by the initial conditions. Ferro nonlinear oscillations, for instance, are greatly affected by the initial conditions, and thus, the residual core conditions [29]. Therefore, the calculation of residual core conditions, being a subset of the possible initial conditions, is very important. The calculation of the residual core conditions must be seen as the pre-calculation of the initial conditions for a subsequent problem.

As mentioned already, when a transformer is reconnected to its supply, the residual core flux influences the transient following hereupon to a great extent. Therefore, it is important to have some more insight in this 'permanent magnet' state of the core relative to different circuit parameters. Recognize however, that this state of the transformer is related in a very complicated way to various parameters, such as circuit breaker characteristics (incoming of the poles), transformer core characteristics, transformer connections, load characteristics, etc. What will be treated here is a simple description of residual core conditions of a three-limb transformer relative to its condition before de-energizing. Actually, the description is stripped from most of above mentioned interrelations, in order to have a more clear understanding of the fundamentals in three-phase cases.

For this purpose, the transformer is connected with a simple load, namely, a capacitance, an inductance or a resistance. The transformer with its load is taken into the steady state condition (as described in the previous section), whereupon the transformer is disconnected from the supply, which is considered to consist of a set of symmetrical three-phase voltages. The primary side of the transformer is delta-connected,

![Diagram](image)

*Figure 5.5  Delta-wye connected transformer with three simple loads (L, C, or R)*
whereas the secondary side, with its neutral earthed, is wye-connected (see Fig. 5.5). Further, it is noted that the simulations are carried out with the 9.5 KVA test-transformer discussed in the previous sections.

As far as the load is concerned, this consists of three wye-connected simple elements (L, C or R) with the neutral likewise earthed. It is emphasized that the operation of the circuit breaker (de-energizing of the transformer) has been reduced to the opening of three ideal switches (poles) at the zero-crossing of the primary currents.

The computed flux waveforms related to these numerical experiments are given in Fig. 5.6. These waveforms are characteristic for the three simple loads. The residual flux condition can be understood by realizing that de-energizing of the transformer is the result of two successive switch operations. Since it is assumed that the opening of a pole is a clear cut matter, this instant can be related to a certain point at the flux waveforms. Thus, by close inspection of the flux waveforms, these switching instants can be detected. The opening of the first pole can be conjectured when a change in the course of two flux waveforms is detected. This conjecture becomes true if at the same time the course of the third flux waveform remains unchanged. To understand this, one must realize that when the

Figure 5.6  Calculated residual flux conditions

a) flux waveforms related to a capacitive load
b) flux waveforms related to an inductive load
c) flux waveforms related to a resistive load
first pole opens at a current zero, the two windings connected at that particular pole - remember, the transformer is delta-connected - must carry the same current at that instant, and also will in the sequel. Effectively, this means that these two windings, now in series, have been placed in parallel with the third winding on which the phase to phase voltage is being impressed. For the two windings this implies that after switching this voltage will be distributed (almost) equally across these windings, and according to this, the corresponding fluxes. This explains why after opening of the first pole, two fluxes will change course and one will not.

With the opening of the second and the third pole, the transformer is completely disconnected from the voltage source. It can be verified that under ideal conditions this will happen 1/4 period after opening of the first pole. This is directly related with the fact that in the energized state of the transformer, there exists a phase difference of π/4 radians between the current through the poles and the current through the transformer windings.

A simple mathematical description of above discussion will give extra and some quantitative insight into this matter. For this purpose, it is assumed that the excitation consists of an ideal three-phase voltage system. Further, the transformer is reduced to a lossless electromagnetic device of which it is assumed that all fluxes are intercepted by the iron core. Then, it can be shown that the flux waveforms related to the condition with one pole open can be expressed as

\[
\begin{align*}
\psi_U^\wedge (\tau) / \psi &= \cos(\omega \tau - \varphi) \quad (5.2.1a) \\
\psi_V^\wedge (\tau) / \psi &= \frac{-1}{1+\alpha} \cos(\omega \tau - \varphi) + \frac{1-\alpha}{1+\alpha} \frac{1}{2} \cos(\varphi) - \frac{1}{2} \sqrt{3} \sin(\varphi) \quad (5.2.1b) \\
\psi_W^\wedge (\tau) / \psi &= \frac{-\alpha}{1+\alpha} \cos(\omega \tau - \varphi) - \frac{1-\alpha}{1+\alpha} \frac{1}{2} \cos(\varphi) + \frac{1}{2} \sqrt{3} \sin(\varphi) \quad (5.2.1c) \\
\end{align*}
\]

with

\[
\tau = t - t_0, \quad t \geq t_0
\]

\( \varphi \) angle between winding current and the corresponding flux linkage

\( t_0 \): opening instant of the first pole.

As discussed before, the transformer is completely disconnected at \( \omega t = \pi / 2 \). Hence, the remaining flux can be expressed as

\[
\begin{align*}
\psi_{U_I} / \psi &= \sin(\varphi) \quad (5.2.2a) \\
\psi_{V_I} / \psi &= - (\sqrt{3} + \frac{2}{1+\alpha}) \frac{1}{2} \sin(\varphi) + \frac{1-\alpha}{1+\alpha} \frac{1}{2} \cos(\varphi) \quad (5.2.2b) \\
\psi_{W_I} / \psi &= (\sqrt{3} - \frac{2\alpha}{1+\alpha}) \frac{1}{2} \sin(\varphi) - \frac{1-\alpha}{1+\alpha} \frac{1}{2} \cos(\varphi) \quad (5.2.2c) \\
\end{align*}
\]
In above equations the factor \(\alpha\) accounts for the inequality between the inner limb and the outer limb. Recognize, that by changing the subscript, these equations can be used for the two possible switching conditions (sequence of opening of the poles). Only two switching conditions need to be considered, since the two outer limbs are equal. The factor \(\alpha\) must be set to unity (due to symmetry), in the case were after opening of the first pole the flux of the centre phase is directly being impressed.

Equation (5.2.2) clearly shows that the residual core condition is strongly related to the transformer load. Generally speaking the residual fluxed will be very small for \(\phi = 0\), i.e. the current through the winding is in phase with the corresponding flux linkage, whereas maximum residual fluxes can be expected for \(\phi = \pi/2\). In fact Eq. (5.2.2) shows that basically, the magnitude of the residual flux can take all values in the range from zero to the peak value of the flux, and even higher as can be seen in Eq. (5.2.2b). In the following these equations will be used for explaining the calculated waveforms. Figure 5.6a shows the residual core condition as obtained with a load consisting of three capacitances (\(C = 10 \mu F\)). A study from the flux waveforms reveals that the pole incident to winding \(U\) and \(V\) is opened first. From the continuation hereupon of flux \(\psi\), it can be seen that angle \(\phi\) must be next to zero. According to Eq. (5.2.2) the remanent flux level, which is defined here as the maximum magnitude of the three fluxes, should be accordingly low. However, Fig. 5.6a shows a discrepancy between calculation and what has been expected in virtue of Eq. (5.2.2). As can be seen, when the first pole is opened, an oscillatory transient is initiated of which the frequency is predominantly determined by the capacitance and the inductance associated with the air paths. Obviously, this transient cannot be expressed in Eq. (5.2.1), since it is assumed that, when transformed, the capacitance is directly fixed to an ideal voltage source. Observe further, that after opening of the second pole, the residual flux level is increased relative to what may be called the ideal level. In consequence, when a transformer with a pure capacitance load is de-energized, the oscillating transient involved tends to increase the residual level above its ideal level. Figure 5.6b displays the de-energizing of the transformer in case of an inductive load (\(L = 16\) mH). Like the previous case, the first pole opens at the positive top of the flux in limb \(W\). From the calculated waveforms it can be seen, that after opening of the first pole no (noticeable) transient is initiated. As a consequence, the flux is re-distributed, and it can be seen that the fluxes in the limb \(U\) and \(V\) stay close to each other until the transformer is completely disconnected. Observe, that unlike the previous case, this flux behaviour is satisfactorily described by Eqs. (5.2.1) and (5.2.2) with \(\phi = 0\). It is found that when a transformer with a pure inductance load is de-energized, the residual level is very low.

Next, the de-energizing of the transformer with a resistive load (\(R = 5\) \(\Omega\)) will be considered. From the flux waveforms given in Fig. 5.6c it can be seen that the first pole opens at or nearby the zero-crossing of the flux through the outer limb \(U\). Observe also that the magnitude of the fluxes in limb \(U\) and \(V\) have reached a maximum value. The maximum value of the magnitude of \(\psi\), is greater than the peak value (in the normal steady state condition). This is due to the unfavourable initial value of the
flux as related to the transient following the opening of the first pole. It is well-known that the transient of the flux (in a circuit being composed of only resistances and inductances) will contain a decaying dc-component and a sinusoidal part. The former is directly related to the initial conditions, and the latter to the steady state condition. Notice that basically, this descriptive discussion of the waveforms is confined in Eqs. (5.2.1) and (5.2.2) with $\phi = \pi/2$.

From the foregoing it can be concluded that maximum residual core conditions can be expected as $b$ (the phase angle between the current through the winding and the corresponding flux linkage) approaches $\pi/2$ radians. It can been shown that for relative large values of the resistance $R$ (i.e., relative to the magnetizing reactance) the angle $\phi$ will take small values. This is also true for values of $R$ smaller than the short-circuit impedance of the transformer. Between these extremes of the resistance $R$ there exists a value of $R$ and thus for $\phi$, for which the residual level reach its maximum. The flux waveforms given in Fig. 5.7 are meant to express these considerations. Figure 5.7a shows the de-energizing of a short-circuited transformer. It can be seen, from the instant of opening of the first pole, that the

![Figure 5.7](image_url)

**Figure 5.7** Calculated residual flux conditions for resistive loads

- **a)** $R_{\text{oad}} = 0$ (short-circuit)
- **b)** $R_{\text{oad}} \to$ (no-load)
- **c)** $R_{\text{oad}} = 0.1$
- **c)** $R_{\text{oad}} = 5.0$
Figure 5.8  Residual flux conditions for a short-circuited transformer computed with

a) the 1f-model
b) the 1f-model

angle $\phi$, although small, does not equal zero. Therefore, a low residual flux level is obtained. For similar reasons, the residual flux level related to the de-energizing of a transformer in no-load condition (see Fig. 5.7b) is relatively low. On the other hand, it can be seen from Fig. 5.7c and 5.7d that if the transformer has a resistance load, the residual value can reach maximum values, when de-energized.

Careful observations of the flux waveforms related to these cases shows that this will occur on the condition that the angle $\phi$ is nearby $\pi/2$ radians. Observe further, that due to the relative smaller time constant (compared with that related with a short-circuited transformer), the flux level decreases after opening of the second (and third) pole.

Above simulations have been carried out with both, the 1f-model and the 1f-model. Though a physical interpretation of the flux behaviour of the 1f-model is not always easy to give, it can be compared with that of the 1f-model. Recall, that instead of giving an integral description of the flux behaviour, the 1f-model basically gives a divided flux description, consisting of two set of equations. The first set consists of a set of nonlinear equations which is related to the iron core. The second set consists of linear equations which are somehow related to the air fluxes. If the 1f-model is subjected to the same (numerical) test, the same residual conditions are obtained. However, a slightly different (lower) residual level is obtained in the short-circuit condition case. From Fig. 5.8 it can be seen that this is a result of the smaller magnitude of the fluxes in the steady state. Recognize, that if the leakage inductions had not been equally divided, they could have been chosen so that the models match for this short-circuit condition.
5.3 Ferroresonance

This section is concerned with a phenomenon, associated with unloaded or lightly-loaded transformers, and known as ferroresonance. Ferroresonance has mainly been analyzed by means of a single-phase transformer. Specifically, the oscillatory consequences caused by a capacitance and a nonlinear inductance (mostly being connected in series) have been studied [24-29]. These studies have shown that even with a simplified single-phase approach this phenomenon hardly lends itself to an exhaustive and revealing mathematical analysis, let alone with a three-phase approach. In this section a study is made of ferroresonance in a three-phase circuit, and as such, contributes to a better understanding of this phenomenon.

Since the term ferroresonance is not used unambiguously, the term ferroresonance will be defined here for the sake of clarity. In general, many modes and forms exist for the nonlinear oscillations in a nonlinear circuit. The term ferro nonlinear oscillations (f.n.o.) is used to refer to all types of oscillations if the nonlinear element of the circuit is an iron-core inductance [29,64]. Ferroresonance is the special case of a jump resonance. More specific, ferroresonance is a jump phenomenon which occurs if in a sinusoidally excited system one of the system parameters is changed. Consequently, in electrical power systems, ferroresonance is the jump resonance occurring at power frequency (50 Hz), and is a steady state phenomenon [29].

Electrical engineers have been confronted with ferroresonance since the early 1900's [24,25]. Since then, numerous of papers on this subject have been presented, and as this interesting but highly undesirable phenomenon is not fully understood, this subject remains a topic in electrical engineering. The problem of ferroresonance, or more generally, of f.n.o., is usually tackled in two ways. One method is spending much effort in producing analytical solutions for this problem. Once an analytical solution has been found, ferroresonance or any other mode of the nonlinear oscillations can relatively easily be predicted [31,32,33,36,37]. In most cases, the circuits being studied are simplified as to make mathematics reasonable. The work done in this field by Hayashi [29] is beyond praise. More recently, an attempt has also been made to predict f.n.o. for the more complex three-phase power systems [72]. The second method of studying f.n.o. uses the "brute force" approach. Actually, with this method the occurrence of f.n.o. is shown by means of simulation. Analog simulation [28,30,31], or digital simulation [34,35,69] are being used for studying this phenomenon.

Today's topic in this field is the investigation of the influence of hysteresis on the ferroresonant conditions. Most studies are dealing with single-phase circuits [35], or in case of a three-phase system, are using three single-phase transformers, not allowing for the magnetic coupling existing in three-limb transformers [69]. In addition, most of the hysteresis models being employed are poor in the sense that minor loops are not described properly [34].

In the following, ferroresonance in a three-phase system will be considered. First it will be shown that for a specific transformer connection - wye-connected primary windings and isolated neutral -, ferroresonance can be expected when the transformer is energized and one pole of the circuit breaker is
retarded. To prove this, it suffices to derive the basics of the ferroresonance phenomenon by means of the simplified circuit given in Fig. 5.9. Hereafter, a few cases will be computed with a more realistic model of the transformer.

![Diagram of a circuit](image)

**Figure 5.9** Three-phase network for the analysis of ferroresonance

The network shown in Fig. 5.9 is used to prove analytically that configurations like this are amenable to ferroresonance. Notice, that in this network use has been made of ideal transformers. It is also seen that the primary winding of the transformer is wye-connected, whereas the secondary winding is delta-connected. Further, it is important to notice that the neutral of the transformer is not earthed, since this is one of the necessary conditions for ferroresonance to occur. In advance it may be noted that the delta-connected windings do not carry current. Yet these windings are considered, although it is tempting to ignore them. Figure 5.9 shows three capacitances at the three terminals of the transformer. These capacitances can be best seen as a simplified representation of a cable. It will be shown that this network can be reduced to a basic ferroresonant circuit, a series LC-circuit, being liable to ferroresonance.

For this purpose, it is assumed that the transformer consists of three single-phase transformers, that is, the flux equations consist of three decoupled equations. Recognize, that this assumption is only introduced for the sake of simplicity, and does not fundamentally affect the explanation. This can also be verified in the following section where a similar circuit is used for explaining a voltage jump phenomenon. From the network given in Fig. 5.9 it can be seen that the transformer consists of three nonlinear inductors and three ideal transformers. This network also reveals why the primary windings may carry currents, although the delta-connected secondary windings do not carry current. This does not imply that the effect of the secondary side can be disregarded. As will be seen, the effect of the delta-connected windings is taken into account by forcing the sum of the time derivatives of the fluxes to zero.

It can be verified that for an open phase T, the following equations hold.
\[ \frac{d\psi_V}{dt} - \frac{d\psi_W}{dt} = u_R - u_S = u_{RS} \]
\[ \frac{d\psi_U}{dt} - \frac{d\psi_W}{dt} + u_W = u_R \]
\[ \frac{d\psi_U}{dt} + \frac{d\psi_V}{dt} + \frac{d\psi_W}{dt} = 0 \]

\[ \frac{du_W}{dt} + i_W = 0 \quad \leftrightarrow \quad L(i_W)C \frac{d^2 u_W}{dt^2} + \frac{d\psi_W}{dt} = 0 \]

with
\[ \psi_W = L(i_w) i_W \]

After a few manipulations, above equations can be reduced to

\[ \frac{3}{2} L(i_w) C \frac{d^2 u_W}{dt^2} + u_W = u_R - \frac{1}{2} u_{RS} \]

Recognize, that above equation can be related to the single-phase ferroresonant circuit given in Fig. 5.10. As can be verified, the amplitude of the excitation in this circuit is half the amplitude of the phase voltage, whereas the capacitance \( C_{eq} \) is 3/2 the phase capacitance. Consequently, the conditions for ferroresonance due to two-phase switching can be reduced to ferroresonance conditions in a single-phase ferroresonance circuit. At this point, it is important to note, that for a three-limb transformer a similar equivalent single-phase circuit can be derived. (As already mentioned, for more details, the
reader is referred to the next section.) Unnecessary to say that this also holds for a five-limb transformer. Clearly, two-phase energizing of a star-connected transformer whose neutral is not earthed, can cause ferroresonance. A necessary condition for this to occur is that the value of the capacitance is small enough and that the flux related to the disconnected phase increases beyond its saturation level. The single-phase circuit given in Fig. 5.10 can be used for the determination of a combination of capacitance and nonlinear inductance (the transformer) for which ferroresonance is liable to occur.

In the following, some simulation results of this two-phase switching will be considered. The simulations are carried out with a more realistic description than the one used in above discussion. It is assumed that the transformer is not directly connected to the (ideal) three-phase voltage source, but through an impedance which allows for the short-circuit MVA at the point of the circuit breaker. The remaining of the circuit is described as before, except for the transformer. For this, a three-limb transformer model is used which has been discussed in the preceding chapters. In particular, a flux description of the transformer is used, in which the air fluxes (leakage fluxes) have been integrated, and hence is referred to as the integrated flux model. As far as the core losses are concerned, three cases will be considered as to show how the calculated waveforms are affected by the description of these losses.

---

The circuit shown in Fig. 5.11 is used to examine the two-phase switching transients of a 450 MVA transformer. The value of the capacitance (C = 10 uF) is chosen so as to obtain a circuit which is most likely to support ferroresonance (or more generally, f.n.o). This circuit must be viewed as a crude but not unrealistic representation of a transformer and cable which are being energized at the same time. With this circuit the following numerical tests have been conducted.

---

Figure 5.11 Circuit for the calculation of ferroresonance following upon a two-phase switching
A two-phase energizing of cable (capacitance) and transformer. The phases R and S are closed non-simultaneously, while phase T remains open, for instance, due to a failure of the circuit breaker. The core losses of the transformer are neglected in the calculations.

The same as (a), except that the core losses are represented by an equivalent resistance, thus neglecting hysteresis.

The same as (a), except that the core losses are represented by an equivalent resistance and the (static) hysteresis description of the core, discussed in Chapter 3. It is important to know, that for this numerical experiment it is assumed that the transformer core has been demagnetized.

The same as in (c), but now with residual fluxes \( \psi_U = 0.23 \) pu, \( \psi_V = -0.39 \) pu, and \( \psi_W = 0.16 \) pu. These residual fluxes, related to the three transformer limbs U, V, and W, have been established by disconnecting the unloaded transformer in the steady state condition.

The calculations are carried out for a 450 MVA - 380 kV transformer with a short-circuit impedance of 38 % and an excitation current of 0.1 %. The copper losses and the core losses has been set to 0.6 % and 0.04 % respectively. Further, it is assumed that the three-phase short-circuit power at the circuit breaker amounts to 130 GVA, and that the voltage source has an amplitude of 220√2 kV. Finally, it is worth knowing that the closing of the poles can be related to the positive voltage peak of the first phase R. The first acting pole, the pole in phase S, closes 235 degrees away (in the positive direction) from the positive voltage peak of the phase voltage \( u_R \); 2.6 ms hereafter the pole in phase R closes. Throughout the whole calculation the pole in the last phase, phase T, remains open, and thus, the transformer terminal W remains disconnected. It will be seen that this voltage \( u_W \) can reach considerable high values in the ferroresonant state.

The calculated waveforms related to the cases (a), (b), (c) and (d) are displayed in the Figs. 5.12 and 5.13. The waveforms shown in Fig. 5.12 are meant to consider the transients of the two-phase switching in more detail, whilst Fig. 5.13 shows the ferroresonant state of the two-phase excited transformer. From the waveforms given in Figs. 5.12a, 5.12b and 5.12c it is seen that up to 8 periods (= 160 ms) after switching, the shapes of the transients are alike, but with the lapse of time, the differences between the waveforms are becoming more pronounced. The waveforms (5.12a), which are related to a lossless core (case (a)), are predicting the ferroresonant state of the transformer relatively early. This can be seen from the voltage waveform which tends to a steady state, characterized by a high voltage. It is important to recognize, that the fluxes plotted here are also carrying this information. Actually, when considering the fluxes, one could be more affirmative in answering the question of ferroresonance. In the preparatory discussion of this ferroresonance study, it has been stated that a necessary condition for ferroresonance to occur is that the magnitude of the flux related to the disconnected phase should increase beyond its saturation level. Notice by inspection from the flux waveforms in Fig. 5.12a, that this condition is fulfilled after
Figure 5.12  Voltage and flux waveforms following upon a two-phase switching

- core losses of the transformer have been neglected
Figure 5.12  Voltage and flux waveforms following upon a two-phase switching (continued)

- resistance representation of the transformer core losses
Figure 5.12 Voltage and flux waveforms following upon a two-phase switching (continued)

- hysteresis has been taken into account
Figure 5.12 Voltage and flux waveforms following upon a two-phase switching
(continued)

- transformer core with residual fluxes
Figure 5.13 Voltage and flux waveforms reaching the ferroresonant state

--- core losses of the transformer have been neglected
Figure 5.13  
Voltage and flux waveforms reaching the ferroresonant state  
(continued)  
- resistance representation of the transformer core losses
Figure 5.13  Voltage and flux waveforms reaching the ferroresonant state (continued)

- hysteresis has been taken into account
Figure 5.13 Voltage and flux waveforms reaching the ferroresonant state (continued)
- transformer core with residual fluxes
about 10 periods (= 200 ms). Without any noticeable announcement, the flux related to the disconnected phase, flux $\psi_w$, reaches its saturation level, while on the other hand, the flux related to the centre phase, $\psi_{cw}$, is decreasing.

From the erratic character of the voltage waveforms related to the cases (b) and (c) it cannot be told which steady state condition will be reached (and in case of sustained f.n.o., which mode will be established). Observe, that the flux waveforms too cannot give a decisive answer on this question. Compared with case (a), no clear change in the evolution of the flux waveforms can be noticed. For obtaining more clarity in this matter, the simulation must be continued. But before continuing this discussion, case (d) will first be treated.

As can be seen from the waveforms shown in Fig. 5.12d, the two-phase energizing yields a different picture when the transformer core contains residual fluxes. Particularly, the voltage waveform $u_w$ and the flux waveform $\psi_w$ show great differences. Examination of these waveforms will show that within two cycles after switching the transient pattern entirely differs from the above-mentioned cases (a), (b) and (c). Notice, that in this case the mode of a possible sustained f.n.o. also cannot be predicted.

The waveforms plotted in Figs. 5.13a-5.13d prove that this two-phase energizing leads to the most well known mode of sustained f.n.o., namely ferroresonance. This stage is reached by enlarging the observation time of the simulation. In the case shown here the ferroresonant state was reached after 30 periods (= 600 ms).

Examination of the waveforms in the ferroresonant state reveals, that in fact the transient has not diminished completely. This can be quickly verified by considering the flux $\psi_w$ which still contains a considerable dc-component. Yet, it can be said that a ferroresonant state (a steady state) will settle down in which the voltages will reach very high values (up to and above 5 pu for the voltage at the disconnected phase) and the fluxes related to the two outer limbs will grow beyond their saturation level.

As already mentioned, above calculations have been carried out with the integrated flux transformer model (if-model). For comparison a case will be calculated with the classical transformer model, that means, a model which contains leakage inductions (arising form the division of the flux in a main flux and leakage fluxes), and is referred to as leakage flux model (lf-model). Only one case, case (a) (two phase switching and core losses of both models neglected), will be considered, since the other cases will not shed any new light on the physics of the transformer nor on ferroresonance.

It is very important to note that the aim of this numerical experiment is to show how delicate "ferro-transients" - i.e., transients in a ferroresonance circuit -, are with respect to the nonlinear magnetic characteristics, using these two concepts of transformer modelling. Particularly, it will be shown that two models (the if-model and the lf-model), which are displaying (nearly) the same specification behaviour - i.e., steady state behaviour corresponding with the transformer specifications -, can have considerable differences in responses in certain cases, especially in cases of "ferro-transients".
Figure 5.14  Voltage waveforms (of the if-model and if-model respectively) following upon a two-phase switching
As can be seen from Fig. 5.14, within 40 ms after switching the transient waveforms of both models deviate from each other. Also notice that the voltage and flux waveforms related to the if-model are reaching a higher crest value, which is also maintained in the steady state condition. No clear explanation could be found for this discrepancy in nonlinear transient behaviour. To this end, (computer) simulation has a too phenomenologic nature, i.e., a solution is given which is less explanatory then an analytical solution.

The scope of ferroresonance, or more generally f.n.o., is so wide that it would be virtually impossible to cover the entire area in this section. Instead some simulation results have been presented to show that under special circumstances, ferroresonance can occur in power transformers. It has been proven analytically and by means of simulation that two-phase energizing of transformer (wye-connected and ungrounded neutral) and cable can lead to ferroresonance. The simulation study has been carried out with several models of the transformer core losses. From the calculated waveforms it has been seen that especially the transient part of the waveforms are affected by the various type of core losses modelling. The ferreosouter state however, is less affected by the various type of core losses modelling.

The importance and necessity of predicting ferroresonance can be read from the very high voltages involved in this state. Obviously, simulation programs have come an indispensable tool for studying ferroresonance, or more generally, system responses. The power of this tool has been underlined once more in this section. However, the results of this section has exposed one of the major deficiencies of simulation programs. Basically, ferroresonance is a steady state condition, and to study this phenomenon with the transient analysis program, the brute force method has been used, that is, each calculation has passed through all transient events leading to that particular steady state solution, called ferroresonance. In fact, the study of this section has pointed out that, apart from the fact that the economic attractiveness of this method is questionable, the brute force approach is a cumbersome and distressing method for studying ferroresonance more thoroughly. For example, it would be virtually impossible to determine domains of attraction using the brute force method. Other methods are being developed to solve the f.n.o. problems, and thus ferroresonance, more satisfactorily.

One method proceeds with the adopted course of simulation, and is trying to shorten the passage through the transient events. The other method, the classical one, is trying hard to make full advantage of the computer in giving analytical solutions (not only for a single-phase circuit). Unnecessary to say, that these two methods are complementary, and that both these methods need more attention than has been given up to now. The following section illustrates the power of the synthesis of an analytical and numerical approach.
5.4 Voltage jump at transformer terminals due to non-simultaneous switching

In this section an analytical explanation of a voltage jump phenomenon at the transformer terminals is given. Actually, the explanation of this behaviour has been set down in a publication, which is inserted in this section.

It is shown in this paper that the voltage of the last disconnected transformer terminal may jump to considerable values. From this study it follows that this jump phenomenon can be expected when the poles of a circuit breaker close non-simultaneously, and the neutral of a wye-connected, three limb transformer is ungrounded.

At this point, of few additional remarks will be made. A wye-connected transformer (at the source side) has been considered, since this is commonly used in Europe. However, it can be shown — following the same explanation steps used in the paper — that this jump phenomenon can also occur at the terminals of a transformer whose delta-connected (at the source side).

The explanation is given for a three-limb transformer. However, it can be shown that at the terminals of a five-limb transformer this behaviour can also occur.

Finally, it should be noted that it can be shown that the jump behaviour will not occur when the transformer consists of three single-phase transformers, and/or the neutral of transformer is grounded.
VOLTAGE JUMP AT TRANSFORMER TERMINALS
DUE TO NON-SIMULTANEOUS SWITCHING

H.E. Dijk
N.V. KEMA, Dept. of Electrical Research
47-48, Holtschotweg 210
6800 ET ARNHEM
the Netherlands

Abstract - This paper gives an analytical explanation of a voltage jump phenomenon noticed at the transformer terminal. In particular, it is proven that as a consequence of certain non-synchronous switching conditions, the voltage of the last disconnected transformer terminal will suddenly increase. Overvoltages as a consequence of non-synchronous switching are calculated and compared with measured voltage waveforms.

INTRODUCTION

It has been noticed that under certain conditions switching operations can cause voltage oscillations with a considerable crest value at the transformer terminal [1,2]. More precisely, the energizing of a three-limb power transformer (star-delta connected and non-earthed neutral) has been recorded. These measurements revealed that in the period the last transformer terminal was still disconnected - the closure of the circuitbreaker poles is non-synchronous - sometimes a sudden rise of the voltage followed by oscillations occurred at this terminal. In few cases the arrester on this terminal sparked over.

Most of the papers on the response of transformers due to switching operations deal with the investigation of resonances within the windings. However, the registration of above mentioned transformer behaviour, pointed out that this phenomenon essentially is nonlinear, and that therefore the oscillatory behaviour of the windings is of minor importance. Actually, the observed switching transient is caused by the saturation of the transformer core.

In reference 1 the first steps were made to explain this non-linear transformer behaviour. However, since hardly any attention at all has been paid to the transformer core characteristics, the effects of relevant system parameters have not satisfactorily been considered.

In this paper the pertinent mechanism of the transformer behaviour will be outlined. A three-phase description of the three-limb transformer core will be presented which allows for saturation effects in a simple manner. With this (coupled) magnetic description of the transformer the jump character of the voltage at the disconnected terminal will be explained. Furthermore, it will be shown that its oscillatory nature is caused by the capacity (transformer connections etc.) at the terminal. Finally, an example of the response of the transformer to the non-synchronous switching will be calculated and compared with the measured waveforms.

THE IDEAL VOLTAGE JUMP BEHAVIOUR

Evaluations of energizing tests on transformers [2] have pointed out that the voltage of the last disconnected transformer terminal may jump to considerable values. This behaviour is closely related to the switching sequence of the circuit breaker poles and the nonlinear characteristics of the transformer. Hence, in seeking a satisfactory description of these peculiar switching overvoltages, it is conjectured that particularly the interaction between circuit breaker and transformer must be considered more closely.

For this purpose, the circuit representation shown in Fig. 1 will be used. It is assumed here, that the supply consists of a symmetrical three-phase voltage system, whereas the circuit breaker is represented by three ideal switches, i.e., switches characterized by an infinitely small time of transition from the opened state (zero conductance) to the closed state (zero resistance), and vice versa.

As far as the transformer is concerned, the resistance of the winding is neglected, and as can be seen from Fig. 1, the primary windings are wye-connected, while the secondary (and tertiary, if any) are not taken into account. Further, it should be emphasized, that the neutral of the transformer is isolated, and that this is one of the necessary conditions for the jump phenomenon to occur.


© 1986 IEEE
Further, it can be shown, using the above equation and the current flux relationship related to the case \( i_W = 0 \) (Eq. A.7) that, generally speaking, the applied voltage \( u_W \) is not divided into two equal voltages across the connected windings \( U \) and \( V \), but that the following voltage distribution is obtained

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} u_V \\ u_W \end{bmatrix} &= \begin{bmatrix} 1 - C_{RS} \\ 1 - C_{RT} \end{bmatrix} u_{RS} \\
\frac{d}{dt} u_W &= (1 - C_{RS}) u_{RS}
\end{align*}
\]  

(2)

where the factor \( C_{RS} \) depends on the magnetic state of the transformer.

It can now be rapidly verified, that the voltage at the disconnected terminal \( W \) can be expressed as

\[
\begin{align*}
u_W &= u_{RT} - (d_{RT} u - d_{RT} 0) = u_{RT} - (3C_{RS} - 1)u_{RS} \\
&= u_{RS} - d_{RT} u_{RT}
\end{align*}
\]  

(3)

with \( d_{RS} = 3C_{RS} - 1 \)

In other words, the voltage \( u_W \) can be obtained merely by considering the factor \( d_{RS} \) for all magnetic states of the transformer. Recognize at this point, that a jump in the voltage \( u_W \) is a result of a jump (discontinuity) in the difference of the time derivation of the two fluxes \( \phi_U \) and \( \phi_W \), being the voltages across the windings \( U \) and \( W \) respectively, and that this has been reduced to a discontinuity in the factor \( d_{RS} \).

Before continuation of the discussion the equations related to the case of a retarded pole \( S \) (pole \( R \) and \( T \) closed) are given. In this case, the mesh-equation is

\[
u_{RS} = u_R - u_T - \frac{d}{dt} u_U - \frac{d}{dt} u_W
\]  

(4)

which, substituted in the current-flux Eq. A.9 (Appendix A), yields the following voltage distribution

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} u_V \\ u_W \end{bmatrix} &= \begin{bmatrix} 1 - C_{RT} \\ 1 - C_{RT} \end{bmatrix} u_{RT}
\end{align*}
\]  

(5)

and with above equation, the voltage of the disconnected terminal \( V \) can be written as

\[
\begin{align*}
u_V &= u_R - (d_{RT} u - d_{RT} 0) = u_R - (3C_{RT} - 1)u_{RT} \\
&= u_{RS} - d_{RT} u_{RT}
\end{align*}
\]  

(6)

with \( d_{RT} = 3C_{RT} - 1 \)

Eqs (3) and (6) are the basic expressions of the voltage at the disconnected terminal for two-phase switching conditions. It is noted, that for symmetrical reasons, the S-T two-pole switching is not considered.

From Eqs (3) and (6) it is seen, that the voltage at the disconnected terminal can be considered as being built up by two compo-
ments, of which the component related with the phase to phase voltage $u_{gg}$ or $u_{pg}$ varies with the magnetic state of the transformer via the factor $d_{gg}$ or $d_{pg}$. Also note, that the theoretical maximum of this voltage can be easily determined, provided the variations $d_{gg}$ or $d_{pg}$ with the magnetic state is known.

A tractable and for this purpose suitable description of the transformer magnetic state is obtained by assuming that to each of the transformer limbs, two magnetic states can be associated, characterized by two slopes of the saturation curve (see Fig. A.2, Appendix A), that is

$$L_V^{(m)} = \frac{dL_V}{d\theta} = B_L \quad 0 < \theta < 1$$

and

$$L_U^{(m)} = \frac{dL_U}{d\theta} = \alpha L_V^{(m)} \quad 0 < \alpha < \frac{1}{\alpha}$$

where $L_U^{(m)}$, $L_V^{(m)}$ and $L_W^{(m)}$ are the slopes of the saturation curves of respectively limbs $U$, $V$ and $W$ in the $m$-th ($m=0,1$) magnetic state. Also notice, that the magnetic properties of the outer limb $U$ and $W$ have been taken equal to each other, and that the difference in magnetic permeability of the inner and outer limbs is expressed by the factor $\alpha$.

![Figure 2](image-url)

**Figure 2** Schematic magnetic representation of a three-limb transformer

It can be easily recognized that with above assumption of two states per limb, the three-limb transformer has eight ($=2^3$) magnetic states, and that it needs only straight forward calculations, (using Eqs (7), (8), (A.8) and (A.10)), to express the factors $c_{gg}$ and $c_{pg}$ in terms of $\alpha$ and $B_L$. Table 1 shows the factors $c_{gg}$ and $c_{pg}$ and the corresponding factors $d_{gg}$ and $d_{pg}$ for the eight magnetic states of the transformer, defined as state 0 to 7. The usefulness of this table will be illustrated with two examples of two-pole switching which drives the transformer into saturation.

To start with, it is assumed that the poles $R$ and $S$ are closed at the zero-crossing of voltage $u_{gg}$. The outlines of the transient related to this B3-two-pole switching are given in Fig. 3. From Fig. 3a it is seen, that directly after switching, both fluxes $\psi_U$ and $\psi_W$ start to increase (in magnitude), flux $\psi_V$ being faster. Apparently, the latter stems from the difference between outer- and inner limb of the transformer, and can be easily verified with $c_{gg} < 1/2$ (state 0 in table 1) in Eq. (2). Consequently, limb $V$ will be the first to enter into state 2. Table I shows that this change of magnetic state (from 0 to 2) is attended by a considerable change in the factors $c_{gg}$ and $d_{gg}$. The consequence of this abrupt change of magnetic state is a voltage jump at the disconnected transformer terminal $W$ (see Fig. 3c). This voltage jump can be best explained by considering (in the complex plane) the phasors related to $u_W$ (according to Eq. 6) for the states 0 and 2. Fig. 3d shows the phasor-diagram with in it, the construction of this phasor. In this phasor-diagram, the phasors $R$, $S$ and $T$ are related to the voltages of

![Figure 3](image-url)

**Figure 3** Explanation of the ideal voltage jump

a flux linkages $\psi_U$, $\psi_V$ and $\psi_W$

b $\psi$-9 magnetic characteristics of the transformer limbs
c voltage at the disconnected terminal
d phasor diagram
the corresponding phases, whereas \( W_0 \), \( W_2 \), and \( W_6 \) represent the phasor of \( U_0 \) in state 0, 2, and 6 respectively.

The instantaneous value of \( U_0 \) can be found by projecting the phasor \( W_0 \) \((x=0.2, 6)\) on the time axis (line revolving with angular velocity of \( \omega = 2\pi \)), of which the relevant lines \( t_0 \), \( t_2 \) and \( t_6 \) are displayed, respectively representing the two-pole switching instant \( t_0 \), and the two instants \( t_2 \) and \( t_6 \), of change in the magnetic state of 1/1m V and U (see Fig. 3a and 3b).

<table>
<thead>
<tr>
<th>U</th>
<th>V</th>
<th>W</th>
<th>c_0</th>
<th>c_1</th>
<th>c_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3/2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2/4</td>
<td>2/4</td>
<td>3/4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3/4</td>
<td>3/4</td>
<td>3/2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
<td>3/2</td>
<td>3/2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>4</td>
<td>2/3</td>
<td>2/3</td>
<td>3/4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5</td>
<td>3/4</td>
<td>3/4</td>
<td>2/3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>2/3</td>
<td>2/3</td>
<td>3/4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
<td>3/4</td>
<td>3/4</td>
<td>2/3</td>
</tr>
</tbody>
</table>

Table 1 Factors describing the magnetic state of a three-limb transformer for two-pole switching conditions.

Table 1 also reveals that for \( S=0 \) (zero slope in the saturation characteristics) the theoretical maximum of \( d_0 = 2 \) is obtained, namely in state 2 of the transformer. The length of the corresponding maximum phasor \( W_0 \), then, is \( \sqrt{(3/2)^2 + (1/2)^2} = 2.65 \) times the length of phase phasor \( R, S \) or \( T \), and will be defined as the theoretical maximum value of the voltage at the disconnected terminal.

While in the ideal case the RS-two-pole switching can produce a jump in the voltage, the RT-two-pole switching cannot. A similar process of reasoning as above will lead to this conclusion. From Eq. 5 and table 1 it is seen, that with the RT-two-pole switching only state 0 and 5 of the transformer can be obtained, and that accordingly, no jump will occur at the transitions.

Above consideration very clearly explains why in practice a voltage jump at the disconnected terminal is most likely to occur in case of a RS-two-pole switching, but does not explain the discrepancy between the theoretical maximum and the measured maximum value of above three times the rated voltage.

### VOLTAGE OVERSHOOT

Now that the principles of the voltage jump phenomenon have been given, the step to a more thorough analysis can be easily taken. In particular, capacitive effects are taken into account for obtaining closer agreements (of shape and magnitude) with practice. Fig. 4 shows the circuit that will be used for this purpose.

As can be seen, the basic circuit (Fig. 1) has been extended with three capacitances \( C \) to earth. With these capacitance allowance is made for the capacitive effects (of transformer connections and transformer) at the transformer terminals.

Since it has been shown that a considerable voltage jump can occur for a RS-two-pole switching, the discussion will be concentrated on those cases in which terminal \( W \) will not be connected, or at least, stays sufficiently long disconnected.

The voltage response of this terminal will be explained, making use of the insights obtained with the basic circuit.

Let pole \( S \) of the circuit breaker be closed on the instant \( t=t_0 \). Then, from this moment on, the magnitude of the transformer quantities (fluxes, voltages and currents) will gradually increase. The rate of change of these quantities is determined by the magnetic characteristics of the transformer and the capacitance at the terminals.

If at \( t=t_0 \), pole \( R \) is closed, then a situation is obtained which is similar to the one discussed in the previous section. In fact, the transient will not show fundamental differences, except that the fluxes will not start to increase (in the absolute sense) from zero initial values, and that the voltage at the disconnected terminal \( W \) will not show any discontinuities. The two discontinuities, which still occur in the time derivative of the fluxes (the two-slope representation of the saturation characteristic has not been abandoned) at the instances \( t_2 \) and \( t_6 \), will be passed to the capacitance of the terminal \( W \), resulting in an oscillatory response of the voltage across this capacitance.

A more fundamental apprehension of this voltage behaviour can be obtained by considering the equivalent circuit shown in...
Fig. 5. In this circuit, C is the capacitance at terminal W. \( L_{e,k} \) and \( e_{W,k} \) respectively are the equivalent inductance and excitation seen from terminal W. It is also important to recognize that \( e_{W,k} \) being the voltage that would appear at terminal W if nothing had been connected to it, will show the basic voltage jump behavior as discussed before. Due to the oscillatory nature of \( u_W \), the maximum value of \( u_W \) can exceed that of \( e_{W,k} \).

Figure 5 Equivalent circuit describing the oscillatory nature of the voltage at terminal W

A condensed mathematical justification of above will now be given. The main equations and derivation can be found in Appendix B. Particularly, it can be proven, that with one pole closed, the transient will contain, generally speaking, two oscillation frequencies. In practice these frequencies will be low and can be observed by the relatively slow variations of the transformer quantities.

Mathematically seen, the condition with one pole closed produces the initial values (at \( t=t_0 \)) for the differential equation which applies to the condition with two poles closed. It can be shown that, if the second pole is closed, i.e., terminal U and V are connected with the voltage source, the configuration can be described by

\[
L_{e,k} \frac{d^2 u}{dt^2} + u - u_R - k_R u = e_{W,k}
\]

where \( L_{e,k} \) and \( e_{W,k} \) are the previously introduced equivalent inductance and excitation. It is easily verified that for all states of the transformer, \( u_W \) can be expressed as

\[
\frac{d}{dt} \left[ \frac{v_{e,k}}{v_{e,k}} \right] = e_{W,k} \cos(\omega t + \delta_k) - \delta_k \sin(\delta_k) - \frac{v_{e,k}}{L_{e,k}} \left( \frac{v_{e,k}}{v_{e,k}} \right) \cos(\omega_{e,k} t) + \delta_k \sin(\delta_k) \sin(\omega_{e,k} t)
\]

\[
\frac{1}{L_{e,k}} \left[ \frac{v_{e,k}}{v_{e,k}} \right] = e_{W,k} \sin(\delta_k) + \frac{v_{e,k}}{L_{e,k}} \cos(\omega_{e,k} t)
\]

and for \( v_{e,k} = 1 \)

\[
u_W = \frac{e_{W,k}}{2} \cos(\omega t) + \left( \frac{u_{W,k}}{L_{e,k}} \right) \sin(\omega t) + \delta_k \sin(\delta_k) \sin(\omega_{e,k} t)
\]

(9b)

where

\[
\tau = t - t_k, \quad t_k < t < t_{k+1}, \quad k = 0, 2, 6
\]

\( t_0 \): switching instant of the second pole

\( t_k \): Instant of change of magnetic state of the transformer (k=0)

\[
u_{W,k} = u_W(t_k)
\]

\[
u_{e,k} = u_{e,k}/\omega;
\]

\[
\omega_{e,k} = 1/(L_{e,k} C)
\]

\[
\delta_k = \omega t_k + \delta_k
\]

\[
\tau(\delta_k) = \frac{-R_{RS,k}}{L_{e,k}} 1/2 \sqrt{3}
\]

\[
\delta_{W,k} = \delta_k \sqrt{3} RS_{RS,k} - 3RS_{RS,k} - 1
\]

For comparison with the basic case of the RS-two-pole switching, another RS-two-pole switching with SR-sequence is given in Fig. 6. This figure reveals that the basic mechanisms of the jump phenomenon are still the same, except that changes in the magnetic state of the transformer will now cause oscillatory voltage variations at the disconnected terminal. It is now easily conceived that, due to these oscillatory effects, the overvoltages at the transformer terminal will reach higher values than obtained with the basic circuit. Compared with reality, these oscillatory responses show good agreement, albeit that, due to the two-slope representation of the saturation curve, the magnetic transitions are still too abrupt.

Fig. 7 shows the calculated and measured voltage waveforms obtained when energizing a 150 kV-transformer. In the calculation, the magnetic representation of the transformer has been taken more realistic, i.e., allowance has been made for leakage fluxes, and eddy-current effects, while the magnetic characteristics of the core have been represented by a (single-valued) function and the resistance of the windings has not been ignored.

The transformer connection is star-delta with a non-earthed neutral. As far as the measurement is concerned, the measured waveforms have been obtained via capacitive voltage dividers and transient recorder. The sequence of pole closure of measurement and calculation has taken equal to each other, whereas (in the calculation) the excitation has been taken so as to obtain a peak voltage of 125 kV in the steady state. Recognize from the waveforms that pole 5 has been closed first and (2.1 ms) hereafter pole 8. Further, it can be seen from both calculation and measurement at
the disconnected transformer terminal (W). Careful observation of the calculated waveforms also learns, that 6.5 ms after connection of terminal V (closure of pole S) a minimum of -3.34 pu (1 pu=125 kV) occurred, and that 0.9 ms later a maximum of 4.47 pu. For the measurement these values are 6.1 ms and -3.07 pu for the minimum, and 0.3 ms and 2.33 pu for the maximum. Notice, that the spark-over of the arrester (on the trans-

Figure 6 Explanation of the oscillatory nature of the voltage jump
a flux linkages \( \psi \), \( \psi_0 \) and \( \psi_w \)
b \( \psi - \theta \) magnetic characteristic of the transformer core
b equivalent voltage \( e_{w, k}(t) \)
c and voltage \( u_w \) at terminal W
d phasor diagram

Figure 7 Voltage waveforms at the transformer terminals obtained after a non-synchronous closure of the circuit: a calculated b measured

calculation

200 kV

2 ms

\( u_u \)

\( u_v \)

\( u_w \)

\( t \)

operation of arrester

200 kV

2 ms

\( u_u \)

\( u_v \)

\( u_w \)

\( t \)

Finally, it should be noticed that the conditions for calculating the maximum overvoltage have not been considered in this paper. Since an expression for the voltage at the (last) disconnected transformer terminal has been derived (see Eq. 9) it is possible to formulate such conditions, and thus, to avoid (cumbersome) statistical procedures for obtaining the maximum overvoltage. Work has already been done in this direction and future publication is in hand.

CONCLUSIONS

In the work presented here, it has been analytically shown that when a transformer of which the neutral is not earthed, is being energized and the closure of the circuit breaker poles is non-synchronous, the voltage at the last still disconnected terminal may jump to considerable high values. The basic mechanism of this behaviour has been reduced to transitions in magnetic states of a three limb transformer. Using a two-slope representation of the magnetisation curve (discontinuous derivative at the knee) and coupled flux equations for a three-limb transformer, these transitions could very elegantly be expressed in discontinuities in the voltages across the
transformer windings. Then, it has been shown that a voltage jump will appear at the disconnected terminal if one of the transformer limbs gets saturated. It has also been shown that this jump will be aggravated by the capacitance at the transformer terminal: the voltage jump is followed by an overshoot and oscillations.

Since it has been proven that this jump phenomenon occurs when one of the transformer limb enters into the saturation state, the time interval between two successive pole closures should be less than 5 ms. Within this context this analytical approach must be appreciated and used as a tool for formulating requirements of the closure of the circuitbreaker poles.

Though the calculations have shown good agreement with the measurements, further improvement can be expected if hysteresis is taken into account. It should be noticed that allowance for this feature does not alter the essence of the jump phenomenon. Since this voltage jump phenomenon has analytically been made accessible, it must be easier to formulate switching conditions as to obtain the maximum overvoltage. Further work has to be done in this direction. It will also be worthwhile to study the effect of residual core conditions on the overvoltage.

ACKNOWLEDGEMENT

The author is grateful for the support of this research project by the N.V. SEP, and also wishes to thank dr. J.H. Blom of N.V. KEMA for his suggestions and encouragement in this work. The author also acknowledges the contributions of Messrs. H.C. Tempelaar and C.G.A. Koreman of N.V. KEMA, who have carried out the measurements. The N.V. PHNM has afforded the facilities for carrying out these measurements.

REFERENCES


APPENDIX A

TRANSFORMER FLUX EQUATIONS RELATED TO TWO-PHASE SWITCHING

The magnetic circuit given in Fig. A.1 describes the magnetic behaviour of a three-limb transformer. In this description the fluxes in the air paths have not been taken into account.

\[ \begin{bmatrix} \Delta \psi_U \\ \Delta \psi_V \\ \Delta \psi_W \end{bmatrix} = \begin{bmatrix} L_{UU} & L_{UV} & L_{UW} \\ -L_{UU} & L_{VV} & L_{VW} \\ -L_{UU} & -L_{VV} & L_{WW} \end{bmatrix} \begin{bmatrix} \Delta i_U \\ \Delta i_V \\ \Delta i_W \end{bmatrix} \]

(A.1)

Figure A.1 Magnetic circuit of a three-limb transformer

The magnetic resistances given in this circuit characterize the magnetic properties of the three transformer limbs. These magnetic characteristics have been idealized and are described by a piece-wise linear function consisting of two straight lines, the so-called two-slope representation. From Fig. A.2 it can be seen that it is assumed that the outer limbs (U and W), having equal magnetic properties, differ from the center limb V.

Figure A.2 Two-slope representation of the magnetic characteristics of the transformer limbs U, V and W

With some straight-forward manipulations, using the magnetic circuit given in Fig. A1 and the incremental induction coefficients depicted in Fig. A.2, the following coupled current-flux equation can be derived
where
\[
L_{XX} = \Sigma L_{XY} \quad \text{and} \quad Y \neq X \quad \text{(A.2)}
\]

\[
Y = U, V, W \quad \text{and} \quad X = U, V, W
\]

and
\[
L_{XY} = L(p) L(q) / (L(k) + L(l) + L(m)) \quad \text{(A.3)}
\]

\[
Y = U, V, W \quad \text{and} \quad Y \neq X \quad \text{and} \quad X = U, V, W
\]

\[
l = 0.1; \quad i = 0.1; \quad m = 0.1
\]

\[
p = k, l, m; \quad q = k, l, m
\]

It is noticed, that with each transformer limb, two magnetic states are associated, characterized by two induction coefficients \(L_p^{(0)}\) and \(L_p^{(1)}\) \((X = U, V, W)\), implying eight \((-2^3)\) magnetic states for the three-limb transformer. Thus, with Eqs. (A.1)-(A.3) a piece-wise linear description of the flux behaviour is given.

It is now assumed that the following additional equation is given
\[
i_U + i_V + i_W = 0 \quad \text{(A.4)}
\]

Obviously, the above equation results from the non-earthed neutral of the star-connected transformer windings.

With the aid of Eq. (A.4) and eliminating \(\Delta i_Y\), the current flux equation can be re-arranged into
\[
\begin{align*}
\Delta i_U &= \left[ \begin{array}{c}
L_{UU} + L_{UV} \\
L_{UV} + L_{VV} \\
L_{WV} + L_{WU}
\end{array} \right] \Delta i_U + \left[ \begin{array}{c}
L_{UV} - L_{UU} \\
L_{VV} - L_{UV} \\
L_{WV} - L_{WU}
\end{array} \right] \Delta i_W \\
\Delta i_V &= \left[ \begin{array}{c}
L_{UU} + L_{UV} \\
L_{UV} + L_{VV} \\
L_{WV} + L_{WU}
\end{array} \right] \Delta i_U + \left[ \begin{array}{c}
L_{UV} - L_{UU} \\
L_{VV} - L_{UV} \\
L_{WV} - L_{WU}
\end{array} \right] \Delta i_W
\end{align*}
\]

and eliminating \(\Delta i_W\) into
\[
\begin{align*}
\Delta i_U &= \left[ \begin{array}{c}
L_{UU} + L_{UV} \\
L_{VV} + L_{WV} \\
L_{WU} + L_{WV}
\end{array} \right] \Delta i_U + \left[ \begin{array}{c}
L_{UV} - L_{UU} \\
L_{VV} - L_{UV} \\
L_{WV} - L_{WU}
\end{array} \right] \Delta i_W \\
\Delta i_V &= \left[ \begin{array}{c}
L_{UU} + L_{UV} \\
L_{VV} + L_{WV} \\
L_{WU} + L_{WV}
\end{array} \right] \Delta i_U + \left[ \begin{array}{c}
L_{UV} - L_{UU} \\
L_{VV} - L_{UV} \\
L_{WV} - L_{WU}
\end{array} \right] \Delta i_W
\end{align*}
\]

It is now very easy to verify that with \(\Delta i_W = 0\), Eq. (A.5) can be written as
\[
\begin{align*}
\Delta i_U &= \left[ \begin{array}{c}
c_{RS} \\
-(1 - c_{RS}) \Delta i_U - \Delta i_V \\
1 - 2c_{RS}
\end{array} \right] \\
\Delta i_V &= \left[ \begin{array}{c}
-(L - c_{RS}) \Delta i_U - \Delta i_V \\
1 - 2c_{RS}
\end{array} \right]
\end{align*}
\]

with
\[
c_{RS} = (L_{UU} + L_{UV}) / (L_{UU} + L_{UV} + L_{VV} + L_{WV}) \quad \text{(A.8)}
\]

and with \(\Delta i_Y = 0\), Eq. (A.6) as
\[
\begin{align*}
\Delta i_U &= 1 - 2c_{RS} \left[ \begin{array}{c}
\Delta i_U - \Delta i_V \\
-1 - 2c_{RS}
\end{array} \right] \\
\Delta i_V &= \left[ \begin{array}{c}
\Delta i_U - \Delta i_V \\
-1 - 2c_{RS}
\end{array} \right]
\end{align*}
\]

\[
\begin{align*}
\Delta i_U &= 1 - 2c_{RS} \left[ \begin{array}{c}
\Delta i_U - \Delta i_V \\
-1 - 2c_{RS}
\end{array} \right] \\
\Delta i_V &= \left[ \begin{array}{c}
\Delta i_U - \Delta i_V \\
-1 - 2c_{RS}
\end{array} \right]
\end{align*}
\]

with
\[
c_{RT} = (L_{UU} + L_{UV}) / (L_{UU} + L_{UV} + L_{VV} + L_{WV}) \quad \text{(A.10)}
\]

Recognize that Eqs. (A.1)-(A.10) are the base equations for a two-pole switching, since in each case one of the currents is assumed to be identical to zero. Also note that the factors \(c_{RS}\) and \(c_{RT}\) are related to the magnetic state of the transformer.

**APPENDIX B**

**DIFFERENTIAL EQUATION RELATED TO TWO-POLE SWITCHING**

For deriving the differential equation, related to two-pole switching, Fig. 4 of this paper will be used. Appendix A describes the magnetic behaviour of the transformer related to two-pole switching. Equations related to situations with one pole closed are not given here, since it is assumed that the magnetic state of the transformer will not alter (short duration of these situations), and thus, from a mathematically point of view, these situations merely yield the initial conditions for the subsequent situation: a circuit breaker with two poles closed. The equations related to pole R and S closed will now be derived. For this purpose it is assumed that the second pole (R or S, depending on the sequence) is closed at \(t = t_0\). Then, with the aid of Fig. 4 is very quickly seen that the phase to phase voltage is being impressed on the phase windings \(U\) and \(V\), i.e.

\[
u_{RS} = \frac{d}{dt} \psi_U - \frac{d}{dt} \psi_V
\]

and that the voltage of the disconnected terminal \(W\) is given by

\[
u_W = \frac{d}{dt} \psi_U - \frac{d}{dt} \psi_W \quad \text{(B.2)}
\]

The current equations are now given by

\[
i_u + i_v + i_w = 0 \quad \text{(B.3)}
\]

and

\[
i_w = -c \frac{d}{dt} i_w + \frac{d}{dt} i_w = -c \frac{d^2}{dt^2} i_w \quad \text{(B.4)}
\]

whereas for this purpose flux equation (A5) is re-written as

\[
u_{RS} = \frac{d}{dt} \psi_U - \frac{d}{dt} \psi_V
\]
\[
\begin{bmatrix}
\Delta \psi_U \\
\Delta \psi_V \\
\Delta \psi_W
\end{bmatrix} = 
\begin{bmatrix}
c_{RS} \\
-(1 - c_{RS}) \\
1 - 2c_{RS}
\end{bmatrix} \left( \Delta \psi_U - \Delta \psi_V \right) - 
\begin{bmatrix}
L_{UV} - L_{UV}^* \\
L_{VV}^* - L_{VV} \\
L_{WW}^* - L_{WW}
\end{bmatrix} \Delta \psi_W
\]

which yields

\[
\Delta \psi_U - \Delta \psi_W = (3c_{RS} - 1)(\Delta \psi_U - \Delta \psi_V) \\
- \left[ (3c_{RS} - 1)(L_{UV} - L_{UV}^* L_{VV}^* - L_{WW}^*) \\
- (L_{UV} - L_{UV}^* L_{WW}^* - L_{VV}) \right] \Delta \psi_W
\]

\[
= d_{RS} (\Delta \psi_U - \Delta \psi_V) - L_e \Delta \psi_W \tag{B.5}
\]

with

\[
d_{RS} = 3c_{RS} - 1 \text{ and } \quad c_{RS} = (L_{UU} + L_{UV}^*)/(L_{UU} + L_{VV}^* L_{VV})
\]

and

\[
L_e = (3c_{RS} - 1)(L_{UV} - L_{UV}^* L_{VV}^* - L_{WW}^*) \\
- (L_{UV} - L_{UV}^* L_{WW}^* - L_{VV}) \\
= 9(L_{UU} L_{WW}^*)/(4L_{UU} L_{VV} L_{WW}^*) \tag{B.7}
\]

Finally, it is seen that Eqs. (B1), (B4) and (B5) yield the following (nonlinear) second order differential equation

\[
L_e C \frac{d^2 u_W}{dt^2} + u_W = u_R - d_{RS} u_{RS}
\]

with the initial conditions \( u_W(t_0) \) and \( \dot{u}_W(t_0) \) following from the situation prior to this situation.
5.5 No-Load Transformer Conditions

It is well known that the no-load condition of the transformer is principally determined by the transformer core. Hence, a good measure for correct modelling of the core is the accuracy that can be obtained in simulating the no-load condition, in particular, the no-load currents of the transformer.

For single-phase transformers this matter is well-understood. Not so equally well-understood are the no-load currents in three-phase, three-limb transformers. It will be seen that these currents are not single peaked like the excitation (no-load) current of a single-phase transformer. It can be verified that, in the ideal case, the current of the single-phase transformer will have only one peak. In this case the peak value of the current lies on top of the peak value of the fundamental component (e.g. see Eqs. B2 and B3 in Appendix B).

When excitation currents are seen as composite waves, the composition related to the three-phase transformer differs from the composition related to three single-phase transformers. In Section 2.5 of Chapter 2 it has been shown already that these currents are not only coupled with one phase, but also with the other phases. As a consequence, the phase angle difference between the fundamental component of the no-load current is not always exactly $2\pi/3$. The harmonic contents of these excitation currents will also differ as a consequence of this mutual coupling.

In a three-phase system containing a transformer, not only the no-load (phase) currents are important from a modelling standpoint of view, but also the other quantities such as the voltage at the neutral point, the current in delta-connected windings and the current through the connection from neutral point to earth. These quantities will be considered more closely, since they are related to the characteristics of the iron core as well as the flux paths outside the iron core. It will be seen that the third harmonic in these quantities is dominant. These third harmonics arise from the nonlinearity, and since a linear magnetic network has been used for model parameter identification, the prediction will be poor.

The attention given in this section to the no-load condition may seem somewhat overdone. However, one must realize that, once all quantities can be predicted within reasonable accuracy, an identification algorithm for the three-phase, three-limb transformer will automatically be at hand. For then it will be possible to determine the model parameters from measured waveforms.

In the following two models will be considered in their representation of the no-load condition. The models are equal in their description of the transformer core. One model, the leakage flux model (lf-model), uses separated leakage fluxes for discriminating between primary and secondary windings, and contains one interphase flux path. The other model, the integrated flux model (if-model) uses interwinding flux paths which form part of an integral flux description. This integral flux description contains three interphase flux paths. The transformer models have been tuned to a 9.5 kVA (laboratory) test-transformer, using the parameter identification method,
described in Section 2.4 of Chapter 2. In effect, the models have been chosen close to each other in that the states which have been adopted for identification will be described close to each other by both models. In other words, a good prediction of the magnitude of the no-load and short-circuit current will be obtained.

Since both models have an alike representation of the iron core - the (measured) saturation curve of the test transformer has been used for this representation -, the no-load currents of the models will show only small differences. Hence, only one diagram of these currents will be shown in the ensuing discussion. Discernible differences have been detected in the other quantities, and thus, these quantities will be considered more closely and will also be compared with measured waveforms. To begin with, the case of wye-connected primary windings and ungrounded neutral will be considered; the primary windings are being excited, while the secondary windings are not carrying current.

Since the calculated no-load currents of both the if- and the 1f-model are quite near to one another, the waveforms related to these currents are not shown here. What is worth to be discussed however, is the occurrence of a third harmonic component in these distorted current, which on account of simple three-phase considerations would not have been expected. Due to the current constraint at the neutral point (a common junction) - the sum of the three currents must be zero - the three currents of wye-connected windings with an ungrounded neutral cannot contain components being in phase with one another. Hence, no third-harmonic component is expected in a wye-connected three-phase system, where in each phase the third-harmonic would be $3 \times \frac{2\pi}{3} = 2\pi$ apart, or, in other words in phase with each other. However, an analysis of the harmonic content of the calculated no-load current reveals that these currents do contain a third harmonic. The phase of the third harmonic component of the current in the centre phase (winding) is opposite to that of the current in the two other phases (windings). In Appendix B it has been shown analytically that the occurrence of a third harmonic component in the no-load currents (in case of an ungrounded neutral) is directly related to the fact that the magnetic configuration consists of a three-limb core and not of three single-phase cores; the origin of the third harmonic component can be reduced to an inequality in the three-phase description, introduced by the three-limb core.

Figure 5.15 shows the waveforms of the voltage at the ungrounded neutral point, henceforth also referred to as neutral voltage. The waveforms in Fig. 5.15a and 5.15b are calculated with the if-model and the 1f-model respectively, whereas in Fig. 5.15c an oscillogram from the test-transformer is given. As can be seen, the calculated waveforms show satisfactory resemblance with the measured waveforms as far as the form (prominent third harmonic) is concerned. Notice however, that the peak values of the waveforms obtained with the if-model are smaller. A detailed explanation of why this is so, is given in the discussion of the parameter identification in Section 2.4 of Chapter 2, and the more extended analysis in Appendix B. Particularly, it has been shown that the voltage at the neutral point is strongly related to the interphase flux paths, also referred to as the zero-sequence flux paths. With the aid of a linear magnetic network it
Figure 5.15 Calculated and measured neutral voltage waveforms

a) calculated with the if-model
b) calculated with the lf-model
c) measured neutral voltage

has been shown that the magnitude of the calculated fundamental component of the neutral voltage can be affected by the modelling of the zero-sequence flux paths. More specific, it has been shown that this magnitude will increase when the place where the zero-sequence flux leaves the core is taken closer to the junction of centre limb and yokes (see Section 2.4). Since it has been assumed that in the if-model the zero-sequence flux will leave the core at three places, namely the three junctions of limbs and yokes, the fundamental component of the neutral voltage will be smaller than that of the lf-model, in which the other option has been used, namely the concentration of zero-sequence flux paths at one point, the junction of centre limb and yokes. It is noted at this point, that this assertion has not only been confirmed by (simplified) analytical considerations, but has also been verified - this has been done for all cases - by a (numerical) harmonic analysis. Accordingly, in the further discussion it will be seen that the peak values of the waveforms calculated with the lf-model will be greater than those of the if-model.

It is clear that if the neutral is earthed, the form of the excitation current will change. This change of form is mainly due
Figure 5.16  Calculated and measured current waveforms

a) excitation currents calculated with the $i_f$-model
b) excitation currents calculated with the $I_f$-model
c) measured excitation currents
d) neutral current calculated with the $i_f$-model
e) neutral current calculated with the $I_f$-model
f) measured neutral current
to the third harmonic component. The fourth lead, the connection between neutral point and earth, provides a return circuit for the third harmonic components. In Appendix B it has been shown that in this case the third harmonic components of the three no-load currents have the same phase. In this case, a three-limb transformer acts more like a three-phase bank of single-phase transformers.

The excitation currents and the current through the fourth lead (henceforth referred to as neutral current) of models and test-transformer have been plotted in Fig. 5.16. A comparison of the computed excitation currents with the real currents shows that the result obtained with both models is satisfactory, except for the representation of \( i_v \), the current in the centre phase. The form of the real current \( i_v \) (Fig. 5.16c) is more rectangular, whereas the calculated current \( i_v \) (of both models) has a more pointed form (Fig. 5.16a and 5.16b). From a harmonic point of view, this can be directly related to the phase of the higher harmonic (especially the predominant third harmonic) relative to the fundamental component. It can also be seen that the agreement of the calculated neutral current (Fig. 5.16d and 5.16e) with the real current (Fig. 5.16f) mainly lies in the third harmonic component being predominant. For reasons mentioned before - modelling of the zero-sequence flux paths -, the if-model generates a neutral current with lower peak values. Later it will be shown that a better tuning of the neutral current can be obtained.

In the two previously discussed no-load conditions of the transformer the secondary windings did not carry current. Consequently, the interwinding flux paths, introduced in the if-model to express the (subtle) distinction between the primary and secondary windings (as being two independent magnetic sources), have played a minor role up to now. In the following, two special cases will be considered in which the secondary windings are carrying current. This is simply achieved by connecting the secondary windings in delta. With delta-connected windings (and the primary windings still wye-connected) a situation is created in which both sides of the transformer are carrying current. Owing to this, a portion of the flux, albeit relatively small, is forced to occupy the earlier mentioned interwinding flux paths. In Appendix B it is shown that if in a transformer model the air paths are not taken into account, i.e. if it is assumed that the fluxes occupy solely the iron core, the neutral voltage will be zero, while the current through the delta connected winding, henceforth referred to as delta current, will attain a basic form which is solely determined by the characteristics of the iron core.

Both transformer connections will be considered again: the wye-delta with an ungrounded neutral and the wye-delta with a solidly grounded neutral. The calculated waveforms and the oscillograms of the neutral voltage and delta-current are displayed in Fig. 5.17. Compared with the previous case (Fig. 5.15a and 5.15b) the calculated waveforms of the neutral voltage (Fig. 5.17a and 5.17b) show that in this case this voltage has decreased. This can directly be related to the secondary windings being delta connected. In Appendix B it is shown that under these circumstances - the delta-connected windings are imposing a constraint on the sum of the corresponding flux linkages -, the zero-sequence flux will be small (zero in the ideal case). Owing
Figure 5.17  Calculated and measured voltage and current waveforms

a) neutral voltage calculated with the if-model
b) neutral voltage calculated with the lf-model
c) measured neutral voltage
d) delta current calculated with the if-model
e) delta current calculated with the lf-model
f) measured delta current
Figure 5.18  Calculated and measurement current waveforms

a) neutral current calculated with the if-model
b) neutral current calculated with the ifOmodel
c) measured neutral current
d) delta current calculated with the if-model
e) delta current calculated with the if-model
f) measured delta current
to this constraint, part of the flux is forced to occupy the interwinding flux paths, which eventually leads to a neutral voltage of lower magnitude.

Different from what is expected on account of these considerations, the voltage waveform displayed in the oscillogram (Fig. 5.17c) has not decreased in the same degree (cf. Fig. 5.15c); compared with the earlier obtained voltage waveforms, the fundamental component is now relatively larger. An explanation of this discrepancy between models and real transformer is not straightforward, and does not simply follow from the analysis given in Appendix B. Another marked difference between calculations and measurements is found in the delta-current. As can be seen, the delta-current displayed in the oscillogram (Fig. 5.17f) does not contain a fundamental component, whilst the existence of a fundamental component in the calculated delta-current of both models (Fig. 5.17d and 5.17e) is obvious. This difference too cannot successfully be explained by the analytical methods. At this point, it should be clear that both models are incapable of predicting the amplitudes of the third harmonics. The main reason of this deficiency is the fact that identification of the models has been carried out with linear networks. Therefore, only a good prediction of the fundamental component is obtained.

The waveforms of currents shown in Fig. 5.18 are the (calculated and measured) neutral- and delta-current of the wye-delta connected transformer whose neutral is grounded. As in the previous case, a similar discrepancy between models and test-transformer is detected. Compared with the measured neutral current given in Fig. 5.16f (for which the secondary side is currentless), the neutral current (Fig. 5.18f) has increased, while this is just the other way round with the calculated currents: the neutral current from both models has decreased (compare Figs. 5.16d and 5.16e with Figs. 5.18d and 5.18e for this purpose). As far as the delta current is considered, the following features are noteworthy. First, notice that the delta current obtained with the if-model (Fig. 5.18d) hardly shows any difference with the one calculated in the previous case of an ungrounded neutral (Fig. 5.17d). In contrast with the delta current obtained with the if-model, the delta current obtained with the lf-model changed by a factor 2 (cf. Fig. 5.17e and 5.18e). Presumably, this difference in reaction (on grounding of the neutral) of the models is mainly due to how the (magnetic) distinction between primary and secondary windings has been worked out, by introducing interwinding fluxes (if-model), or separated leakage fluxes (lf-model). Finally, it can be seen that the delta current of the test transformer, Fig. 5.18f, has changed substantially. Compared with the delta current given in Fig. 5.17f (ungrounded neutral), the delta current contains a considerable fundamental component now.

In seeking for a better match of transformer model with a real transformer, the if-model will be used for a sensitivity test. From the previously considered waveforms it is known, that the current and voltage waveforms obtained with the if-model were smaller than the waveforms obtained with the lf-model. As already has been recalled, this is a direct consequence of modelling the zero-sequence flux paths (of the if-model), being paths for fluxes leaving the core at the top (and bottom) of the three limbs. Next it will be shown how the level of the waveforms
Figure 5.19 Neutral and delta current waveforms calculated with the following ratios of the three zero-sequence flux paths

a) 1: 1: 1  
b) 1: 2: 1  \quad \text{neutral current}  
c) 1: 3: 1  
d) 1: 1: 1  
e) 1: 2: 1  \quad \text{delta current}  
f) 1: 3: 1
obtained with the if-model can be raised by changing the permeance of zero-sequence paths. In order to keep the same zero-sequence permeance in all cases, the sum of the three corresponding permeances - recall that the if-model has three zero-sequence paths - is taken constant and put equal to the zero-sequence permeance of the 1f-model which has only one zero-sequence flux path. For not getting entangled in numerous possibilities, only the last transformer connection, wye-delta with the neutral grounded, will be considered.

Figure 5.19 shows the waveforms of neutral current and delta current of the if-model for three cases. In the first case the three permeances of the three corresponding zero-sequence flux paths have been set equal to each other, and thus having a ratio of 1:1:1. Notice that this results is exact the if-model used in above discussion; the ratio 1:1:1 is used here as a departure. The ratios 1:2:1 and 1:3:1 respectively are used in the next two cases. From the waveforms shown in Fig. 5.19a, 5.19b and 5.19c the effect of increasing the permeance of the centre zero-sequence flux path can be read very clearly; as expected, this change of permeance increases the level of the neutral current. Observe further, that for the last ratio, the neutral current reaches a level comparable with the real neutral current (compare Fig. 5.19c with 5.18c for this purpose). Hence, there is no need to further increase the ratio of the permeances. Also note from the waveforms of the delta current shown in Figs. 5.19d, 5.19e and 5.19f that this tuning of the neutral current hardly affects the delta current. The three waveforms seem almost identical, although the attentive observer will not miss the small differences. For conceptual reasons it is worthwhile to note that it has been verified that when (in the if-model) the flux, which is leaving the core, is forced to occupy only the centre zero-sequence flux path, that means, when the permeance of the two outer zero-sequence flux paths is put to zero, the neutral current has increased to a (higher) level comparable with that of the 1f-model given in Fig. 5.18b, while the delta current again was hardly affected. This once more shows that the difference in modelling is not so much expressed in the delta current as in the neutral current.

From the foregoing it can be concluded that as long as the secondary windings are not carrying current (e.g., open delta connection), the neutral voltage or the neutral current can be satisfactorily represented by both models. However, as soon as both sides of the transformer are carrying current (closed delta connection, and no-load condition), the prediction of these quantities becomes poorer. Since the if-model has been equipped with some fine tuning possibilities, the waveforms related to these models, are amenable to improvements; the neutral current, for example, has been straightened up. Further, it has been seen, that the delta current is poorly predicted by both models. The delta current of both models reacts different to the grounding of the neutral point, both reactions being different from the reaction of the delta current of the test-transformer.

Having considered above confrontation of theory and practice, one cannot help thinking that there still are some opaque and inscrutable spots in the transformer description. Recognize however, that in above matching of models and real transformer not the usual quantities (phase currents and voltages) have been accentuated, but quantities, such as neutral
voltage, neutral current and delta current, which normally spoken no attention is paid to. Characteristic for these quantities is the presence of a prominent third harmonic. And indeed, harmonics generated by a nonlinear device will not satisfactorily be predicted when a linear network is underlying the identification of its model. It has been seen that the discrepancy between model(s) and real transformer is mainly expressed in the waveforms related to these quantities. However, the importance, which is attached to this discrepancy, depends upon the scope of study. Obviously, the models are not yet adequate enough to be inserted in a study concerned with transformer identification based on measured waveforms. Hence, a study will be started with the subject: (on line) identification of three-phase three-limb transformers. On the other hand, the accuracy attained with these models are sufficient enough for studies in which these harmonics play a less dominant role.
6 CONCLUSIONS

The work presented in this thesis ought to be seen within the framework of aligning (earlier) models of power system components with the now existing insights and techniques. In particular, a nonlinear network description of the transformer has been given, using the best approach now available, a combination of both the physical and the phenomenological approach.

Starting from the elementary Maxwell's equations, it has been shown which assumptions and simplifications are required to arrive at what can be seen as a physically based network description of the transformer. As a result of this analysis, the link between the T-network and the H-network, both describing the single-phase transformer, could be made visible. From a modelling point of view, the T-network results from the assumption that the total flux can be divided in main flux and leakage fluxes, while the H-network results from an integral flux approach. It has been shown that these two approaches are mathematically related via the star-delta transformation. As a consequence, these two transformer descriptions, of which the T-network is most popular, will yield the same behaviour in the hypothetical case of the transformer being a linear device.

Usually, in three-phase transformer descriptions leakage inductions are being used, as a result of a similar concept underlying the T-network, namely the flux division in a main flux - the flux in the iron core - and leakage fluxes. However, it has been shown that the three-phase transformer can perfectly and also more realistically be described, when the integral flux approach is used. Since in the transformer description given in this thesis magnetic networks and not electric networks have been used, fluxes could directly be related to reality, thus, exploiting the physical approach to a full extent. For instance, the difference in the no-load currents has been related directly to the difference in the transformer limbs (of a three-limb transformer), whereas quantities in the normal no-load condition such as, the voltage at the neutral point, or the current through the neutral lead of a wye-connected transformer could have been more closely studied than up till now.

The description of the nonlinear elements in the branches of the magnetic network constitutes the phenomenological approach applied in the modelling process. A single-valued function has been used for describing the saturation characteristics of the transformer. As far as hysteresis is concerned, the term dynamic hysteresis has been related to what the electrical engineers are conversant with, viz, the eddy current losses. A Preisach hysteresis model has been used for describing the frequency-independent part of hysteresis, the static hysteresis. It has been shown that when a stack mechanism is used for storing reversals occurring in the hysteresis evolution, its description, especially the forgetting property of minor loops, is straightforward. It has also been shown that a satisfactory hysteresis description can be obtained when a limited number of reversals
are stored. The strength of the algorithm described in this thesis is its generality; not until a late stage mathematical equations are introduced for describing a behaviour, only qualitatively discussed. With the formulation used in this thesis the relatively high degree of freedom in choosing mathematical equations becomes visible and explains why there are so much variations on this theme.

For carrying out system studies in which transformers are involved, a network transient analysis program has been written. The Modified Nodal Approach has been used for the network synthesis. Since the transformer is a nonlinear device, the Jacobian associated with the solution vector must be updated every time step. However, a significant reduction in the dimension of this nonlinear problem has been obtained by constraining the algorithm to solve a linear part and a nonlinear part, the latter being an equivalent network, the nonlinear device - the transformer in this particular case - is connected with.

An implicit integration formula has been used for the numerical integration of the differential equations. Actually, a backward differentiation formula (which reduces for a constant time step to the well-known Gear formulas ) and not the trapezoidal rule has been chosen for solving the differential equations. It has been shown that the spurious oscillations, often occurring unannounced when using the trapezoidal rule, will keep being an affliction to its users, since this is an inherent (mathematical) property of the trapezoidal rule and not always a matter of violating physics. The backward differentiation formulas are as easy to implement, have an option to change the time step, and lead to stable integration of (stiff) differential equations.

The strength of formulating the transformer equations as presented in this thesis, namely the possibility of having direct access to the transformer fluxes, has been shown in five case studies. Moreover, where necessary two transformer models - the leakage flux model (1f-model) and the integrated flux model (if-model) - have been compared with each other. The main conclusions from these five case studies will now be given.

With a simple but efficacious numerical experiment it has been shown, how necessary it is to take static hysteresis into account in transient studies. What actually has been considered, is the proportion of the effect of hysteresis on transients and the effort expended of taking it into account. It has been concluded that in nearly all transient studies, the main effect of hysteresis is one of damping. Hence, it will suffice to take a resistance representation for these cases.

In studies where the initial conditions of the fluxes play an important role however, such as inrush phenomena and ferroresonance, hysteresis modelling is important, since this is the most natural way to find these initial conditions. Residual flux conditions have been considered for two types of cases, namely, in a series of cases where the load of a wye-delta connected transformer was formed by a capacitance, an inductance, or a resistance, and in a case of trapped charges it has been
shown that when the load consists of a capacitance, the residual state is reached via an oscillatory response. Due to this oscillatory response the residual flux level will be greater than the ideal flux level, a level obtained by considering the ideal case in which no oscillations (nor transients) occur. Further it has been shown that the residual flux level is the smallest for an inductance load and the greatest for an resistance load. The residual flux conditions related to the case of trapped charge were similar to that of the transformer having a capacitance load. Actually, this case has been meant to demonstrate the capabilities of the computer program and transformer model in one run.

It is worth knowing that these residual condition studies have been carried out with both, the lf-model and the if-model. No discernable differences have been obtained, except for the short-circuited transformer. This difference - for the short-circuit - is directly related to the fact that in the lf-model the leakage inductions are (usually) equal to each other (except for the square of the turn ratio). Therefore, this model does not make a distinction between short-circuited inner windings and short-circuited outer windings, a distinction which is inherent in the if-model. The negligible differences (in the fluxes) of the other cases can be appreciated when one is realizing that in fact only those states have been considered, that are near to the states used for identification of the model parameters.

To simulate the behaviour of transformers having a lengthened stay in the saturation state, a three-phase ferroresonant circuit has been studied: the energizing of a cable - represented by capacitances - and a wye-delta connected transformer. First, it has analytically been shown, that two-phase energizing - i.e., failure of the circuit breaker - of the cable and transformer can be reduced to the well-known single-phase ferroresonant circuit. Then, it has been shown by means of simulation that in this case, two phase switching can lead to ferroresonance. Various descriptions of the transformer core (in the if-model) have been considered in this study in order to show the effect of core modelling on the ferroresonant waveforms. Considerable differences have been obtained for the following considered types of core modelling: no eddy current nor hysteresis losses, only eddy current losses, both eddy current and hysteresis losses starting from the virgin state of the hysteresis curve, and both eddy current and hysteretic losses starting from a residual flux condition. The differences are mainly occurring in the transient state towards the ferroresonant state; none of these transients were alike. However, in the ferroresonant state - a steady state condition - the differences are less pronounced; the magnitude of the calculated voltages were in the same order.

Similar differences have been obtained in a comparative study of the if-model and the lf-model. The transients of the lf-model differ considerable from those of the if-model, while in the ferroresonant state the magnitude of the voltage calculated with the lf-model is smaller. In fact, two things have been made clear. First, it has been shown that these two models, though behaving similar in the normal state, display different behaviour in the saturated state. Since no experiments have been carried out for this behaviour in the saturated state, one should be
careful in advocating either one of the models at this point. However, it may be expected that the if-model will be a better match because of its physical base. The second finding is directly related to the property of a ferroresonant circuit; the ferroresonant behaviour - the transient towards and the ferroresonant state - is affected by the nonlinear description of the element in the circuit to a great extent.

The importance and necessity of predicting ferroresonance can be read from the very high voltages involved in this state. Obviously, simulation programs, like the one described and used in this thesis, have become an indispensable tool for studying ferroresonance, or more generally, system responses. However, the study of ferroresonance in this thesis has also exposed one of the major deficiencies of simulation programs. Basically, ferroresonance is a steady state condition, and to study this phenomenon with a transient analysis program, all transient events leading to the ferroresonant state must be passed through. For studying ferroresonance more thoroughly, the use of this approach, better known as the brute force approach, will be cumbersome and distressing. For example, it would be virtually impossible to determine domains of attractions using the brute force method.

Other methods are being developed to solve ferroresonance or more generally ferro-nonlinear oscillation problems more satisfactorily. One method proceeds with the adopted course of (computer) simulation, and is trying to shorten the passage through the transients events. The other method, the classical one, is trying hard to make full advantage of the computer in giving analytical solutions - not only for a single-phase circuit. In particular, computer algebra systems will be of great help to understand this nonlinear problem. Unnecessary to say, that these two approaches are complementary, and that both these two methods deserve more attention.

The last but one of this series of case studies was concerned with a nonlinear switching transient. In a thorough study a voltage jump occurring at the transformer terminals has been explained. In a combination of analytical evaluation and simulation this phenomenon has been explained, and thus, showing cogently that the use of a simulation program need not be restricted to a "black box" use.

It has been shown (analytically), that at least the following conditions are necessary for the voltage jump to occur. The side of the transformer being energized must be wye-connected and also must have its neutral not grounded. Further, the transformer must not be loaded, and as far as the circuit breaker is concerned, the poles must not close simultaneously, whereas the time between the incoming of two successive poles must be greater than 4 ms. No need to say that the transformer must be connected to a normal symmetrical voltage system. Calculations (simulations) have shown that for these conditions the voltage at the last still disconnected transformer terminal can rise to values above 4 pu.

Special attention has been paid to the no-load condition of the widely used wye-delta connected transformer. In particular, the responses of the two models, have been compared with the no-load behaviour of a test-transformer. The model parameters have
been determined from the rated constants of this test-transformer. Generally speaking, the specific transformer (steady) states for which these constants stand, are satisfactorily represented by both models. However, when second order effects are brought into focus, the predictive ability of these models becomes lesser. That is, it has been found that the transformer models have some difficulties in predicting the voltage at the neutral point, the current through the neutral lead, and the current through the delta connected windings, quantities which usually are ignored. From the calculated waveforms it has been seen that as long as the secondary windings do not carry current (e.g. open delta connection), these quantities can be satisfactorily represented by both models. However, as soon as both sides of the transformer are carrying current (closed delta connection), the representation of these quantities becomes lesser. With some fine tuning possibilities of the 1f-model some improvements could have been obtained. Effectively, this means that a better match between model and test-transformer has been obtained by changing the permeance values of the interphase (zero-sequence) flux paths, while the sum of these permeances is being kept constant. It is noticed that mutatis mutandis a similar effect can be obtained with the 1f-model.

At first sight these second order effects do not seem to play an important role in model validation. If however, one wishes to use no-load measurements - a much better alternative than using rated constants - to identify some model parameters, the effects of these quantities can not be regarded as second order any more. For instance, if the saturation characteristic of the transformer must be determined from measured waveforms, the third harmonic, or more general, the higher harmonics, in the excitation currents must be taken into account. Since the waveforms related to the above mentioned quantities are of the same order as the higher harmonic components in the excitation currents, the information captured in these waveforms cannot be left unused in a parameter identification algorithm.

The parameter identification problem as discussed just now, was beyond the scope of this thesis. The physical approach used and the insights obtained in this thesis can be seen as the necessary conceptual steps for a future parameter identification study. New methods of measurements - measured rms-values are not suited for identification of nonlinear devices - must be set up, and when doing this one should take advantage of the possibilities of modern information and data processing. For instance, it will be a powerful improvement when the transformer properties such as characteristic waveforms and frequency characteristics are digitally handled and stored.

To underline the more general conclusions related to the modelling aspects from this treatise on transformer modelling, they are given below.

While the divided flux approach gives an equivalent flux behaviour, the integral flux approach describes a physically based flux behaviour. Therefore some transformer behaviour (for instance, the voltage jump behaviour and the harmonic contents in the no-load condition) could be thoroughly explained with the integral flux approach.
The rated constants of the transformer have been used for the identification of the model parameters. As a consequence, it is seen that models resulting from both approaches are equally good in predicting the almost linear behaviour of the transformer and the fundamental components of the waveforms.

Both models show considerable differences when predicting ferroresonance (a strong nonlinear behaviour of the transformer) and the harmonic contents of the waveforms in the no-load condition. Only the calculated no-load conditions have been compared with practice. It has been shown that in this case both models have some difficulties in predicting the harmonic contents of the voltage and current waveforms. This outcome is directly related to the method used for parameter identification (of a linear network). This also explains the discrepancy between the two models in their prediction of ferroresonance.

Thus, in this thesis a physically based approach for transformer modelling has presented, and it has been shown that the main advantage of models emanating from this approach lies in the ability of producing perspicuous explanations of unknown phenomena. Moreover, it is believed that this approach can be of great help in developing a suitable parameter identification method for the transformer.
APPENDIX A

Magnetic Network Equations of the Single-Phase Transformer

Figure A shows the network representation of the magnetic circuit of a single-phase two winding transformer. With each branch element (magnetic reluctance or permeance), two branch quantities are associated, namely, $\theta$, a portion of the m.m.f. (accounting for the contribution of that particular branch to the total m.m.f.), and $\psi$, the flux through that branch. For purpose of the ensuing analysis the branch quantities have been given numbers as index. In doing so, it will be always possible to associate unambiguously one of these numbers to a particular branch.

This network will be solved first by the cutset method, of which the formulas are given in Section 4.1 of Chapter 4. As known, the cutset method begins with the selection of a tree, being a set of independent branch m.m.f.'s. Then, the remaining
branch m.m.f.'s can be written as a linear combination of m.m.f.'s from this tree. If in this case, the branches 1, 2, 3, 4 and 5 are selected to construct a tree, the network equations can be expressed as follows:

\[
\begin{align*}
\psi_1 & \quad 1 & 1 & 0 & 0 & | & \psi_6 \\
\psi_2 & \quad 1 & 1 & 1 & 1 & | & \psi_7 \\
\psi_3 & \quad 1 & 1 & 1 & 1 & | & \psi_8 = 0 \\
\psi_4 & \quad 1 & 0 & 1 & 0 & | & \psi_9 \\
\psi_5 & \quad 0 & 1 & 0 & 1 & | & \\
\end{align*}
\]

\[
C_f \quad \psi_f \quad Q_f \quad \psi_1 = 0 \tag{A.1}
\]

\[
\begin{align*}
\theta_6 & \quad -1 & -1 & 1 & 1 & 0 & | & i_1 \\
\theta_7 & \quad -1 & -1 & 1 & 0 & 1 & | & i_2 \\
\theta_8 & \quad 0 & -1 & 1 & 1 & 0 & | & \theta_3 \\
\theta_9 & \quad 0 & -1 & 1 & 0 & 1 & | & \theta_4 \\
\theta & \quad \theta_5 & | & \\
\end{align*}
\]

\[
\theta_1 = -Q_f^T \quad Q_t^T \quad i_f \quad i_t \tag{A.2}
\]

where

\[
[\psi_f \ \psi_t]^T = [\psi_1 \ \psi_2 \ \psi_3 \ \psi_4 \ \psi_5]^T
\]

\[
[\psi_1] = [\psi_6 \ \psi_7 \ \psi_8 \ \psi_9]^T
\]
\[ |i_f \theta_t^T = |i_1', i_2', \theta_3, \theta_4, \theta_5^T] \]

\[ |\theta_1^T = |\theta_6, \theta_7, \theta_8, \theta_9^T \]

and

\[
Q_f^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad Q_t^T = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}
\]

The remaining equations of this network are the nonlinear relations of the branches 3 through 9 which for this purpose are written as:

\[ \psi = \psi(\theta_i) \quad i = 3, 4 \ldots 9 \]

Since the Newton method will be used for solving the nonlinear equations of networks containing nonlinear elements such as transformers (see Chapter 4), partial derivatives of the nonlinear functions are required to build the Jacobian matrix, which basically is necessary in every iteration step. For this purpose, above nonlinear equations are linearized as follows for the \((k+1)\)-th iteration step:

\[
\Delta \psi_t^{(k+1)} = \begin{bmatrix} p_t^{(k)} \\ p_1^{(k)} \end{bmatrix}, \quad \Delta \theta_t^{(k+1)}
\]

\[
\Delta \psi_1^{(k+1)} = \begin{bmatrix} p_t^{(k)} \\ p_1^{(k)} \end{bmatrix}, \quad \Delta \theta_1^{(k+1)}
\]

with

\[
\Delta \psi_i^{(k)} = \psi_i^{(k+1)} - \psi_i^{(k)}
\]

\[
\Delta \theta_i^{(k)} = \theta_i^{(k+1)} - \theta_i^{(k)}
\]

\[
\text{diag} \begin{bmatrix} p_t^{(k)} \\ p_1^{(k)} \end{bmatrix} = \text{diag} \begin{bmatrix} p_3^{(k)} \ p_4^{(k)} \ p_5^{(k)} \ p_6^{(k)} \ p_7^{(k)} \ p_8^{(k)} \ p_9^{(k)} \end{bmatrix}
\]

and
\[ p_i^{(k)} = \frac{\partial \psi_i^{(k)}}{\partial \theta_1} \quad i = 3, 4, \ldots 9 \]
\[ k = 0, 1, \ldots \]

It is easily verified that with some straightforward mathematical manipulations, (A.1) through (A.3) can be reduced to

\[
\begin{pmatrix}
\Delta \psi_f^{(k+1)} \\
\Delta i_f^{(k+1)}
\end{pmatrix} =
\begin{pmatrix}
-P_{ff}^{(k)} & P_{ft}^{(k)} \\
0 & P_{tt}^{(k)}
\end{pmatrix}
\begin{pmatrix}
\psi_f^{(k+1)} \\
\theta_t^{(k+1)}
\end{pmatrix}
+ \begin{pmatrix}
P_{ff}^{(k)} & P_{ft}^{(k)} \\
-P_{tt}^{(k)} & P_{tt}^{(k)}
\end{pmatrix}
\begin{pmatrix}
C_f^{(k)} \\
C_t^{(k)}
\end{pmatrix}
\]

(A.4)

where the submatrices \( P_{ff}^{(k)}, P_{ft}^{(k)}, \) and \( P_{tt}^{(k)} \) follow from

\[
P_{ff}^{(k)} = Q_f P_f^{(k)} Q_f^T
\]
\[
P_{ft}^{(k)} = Q_f P_f^{(k)} Q_t^T
\]
\[
P_{tt}^{(k)} = Q_t P_t^{(k)} Q_t^T + P_t^{(k)}
\]

Equation (A.4) is the final (linearized) form of the transformer flux equations, which can be solved, provided more information is available. Recognize, that from this equation only the vector 
\([C_f, C_t]^T\) is known, and that \( \Delta \psi_f \) must be eliminated. Hence, an additional equation, not imposed by the magnetic network, must by introduced. For this purpose, the algebraized equation obtained from the network equations - the reader is referred to Section 4.4 of Chapter 4 - related to the transformer, is written as follows:

\[
i_f = \hat{i}_f + R_f \psi_f
\]

or linearized

\[
\Delta i_f^{(k+1)} - R_f \Delta \psi_f^{(k+1)} = -N_f^{(k)}
\]

(A.5)

with

\[
N_f^{(k)} = i_f^{(k)} - R_f \psi_f^{(k)} - \hat{i}_f
\]

Then, by substituting (A.5) in (A.4), the following equation finally holds:
\[ \begin{bmatrix} E - R_f P_{\text{ff}}^k & R_f P_{\text{ft}}^k & \Delta i_f^{(k+1)} \\ -P_{\text{ft}}^T & P_{\text{tt}}^k & \Delta \theta_t^{(k+1)} \end{bmatrix} \begin{bmatrix} R_f C_f^k + N_f^k \\ \Delta \theta_t^{(k+1)} \end{bmatrix} = - C_t \]

(A.6)

where \( E \) is the identity matrix.

At the end of the iteration process (cf. examples in Section 4.4 of Chapter 4), all fluxes of the magnetic circuit will be known, and thus \( \psi_f \), the quantity necessary for obtaining the solution of the entire network the transformer is connected with.

Next the magnetic network of Fig. A will be solved by the loop method. Essential for this method is that branch fluxes are written as linear combinations of independent loop fluxes. It will be seen that this method can easily cope with magnetic networks in which the nonlinear elements are flux dependent. Thus, suppose for this purpose, that the nonlinear relations of the branches are now given in the following flux-dependent form:

\[ \theta_i = \theta_i(\psi_i) \quad i = 3, 4, \ldots 9 \]

or linearized

\[ \begin{bmatrix} \Delta \theta_t^{(k+1)} \\ \Delta \theta_1^{(k+1)} \end{bmatrix} = \begin{bmatrix} R_t^{(k)} & \Delta \psi_t^{(k+1)} \\ R_1^{(k)} & \Delta \psi_1^{(k+1)} \end{bmatrix} \]

(A.7)

where

\[ \text{diag}[R_t^{(k)} \quad R_1^{(k)}] = \text{diag}[r_3^{(k)} \quad r_4^{(k)} \quad r_5^{(k)} \quad r_6^{(k)} \quad r_7^{(k)} \quad r_8^{(k)} \quad r_9^{(k)}] \]

and

\[ r_i^{(k)} = \frac{\partial \psi_i^{(k)}}{\partial \psi_i} \quad i = 3, 4, \ldots 9 \]

\[ k = 0, 1, \ldots \]

The equations of the loop method will be related to those used in the cutset method, that is, in this case the two sets of
equations associated with this method will be written as given below.

\[ M \Delta \begin{pmatrix} \theta_1 \\ i_f^T \end{pmatrix} + \begin{pmatrix} Q_f^T \\ Q_t^T \end{pmatrix} \begin{pmatrix} \theta_t \\ i_f^T \end{pmatrix} = 0 \] (A.8)

and

\[ \begin{pmatrix} \psi_f \\ \psi_t \end{pmatrix} = \begin{pmatrix} Q_f \\ Q_t \end{pmatrix} \] (A.9)

It can easily be proven, using (A.8) and (A.9), that the following linearized flux equation can be obtained:

\[ -Q_f^T \Delta i_f^{(k+1)} + R_{11}^{(k)} \Delta \psi_1^{(k+1)} = -M^{(k)} \] (A.10)

with

\[ R_{11}^{(k)} = Q_f^T R_f Q_f + R_1^{(k)} \]

Like before, an additional equation is required to solve Eq. (A.10); notice, that \( M \) is the only known vector, and that in this case \( \Delta i_f \) must be eliminated. This can be done by substituting the external equation (A.5) in Eq. (A.10), and then (A.9) in the resulting equation, which yields the following and final linearized form:

\[ (R_{11}^{(k)} - Q_f^T R_f Q_f) \Delta \psi_1^{(k+1)} = -(M^{(k)} + Q_f^T N_f^{(k)}) \] (A.12)

It is obvious that \( \psi_1 \) is solved first, then all fluxes of the magnetic network, and finally the network of which the transformer is a constituent part.
APPENDIX B

Analysis of Currents and Voltages of the Three-Limb Transformer in the No-Load Condition

To understand the harmonic contents of current and voltage waveforms of the three-limb transformer, the current-flux relation will be given here as a single-valued function. It should be noted that the aim of the exertion here is to gain some insight in the generation of these harmonics contents, rather than to give a stringent method for their determination; to this end the current-flux relation used, is too simple. Also notice, that for the sake of simplicity, the eddy-currents effect is not taken into account in the ensuing discussion.

First, the basic idea of the analysis will be given for a single-phase case, whereupon this idea will be extended for the three-phase three-limb transformer. Let, for this purpose, the nonlinear current-flux relation be described by the following polynomial function:

\[ \theta = a_1 \psi + a_3 \psi^3 + a_5 \psi^5 \]  \hspace{1cm} (B.1)

where

\[ \theta \]: the m.m.f.
\[ \psi \]: flux linkage
\[ a_v \]: the coefficients of the polynomial function (\( v = 1, 2, 3 \))

Assume further, that the flux is sinusoidal, that is

\[ \psi(t) = \hat{\psi} \cos(\omega t + \phi) \]  \hspace{1cm} (B.2)

where

\[ \hat{\psi} \]: the amplitude value of the flux
\[ \omega \]: the radial frequency
\[ \phi \]: the phase angle

Then, it can be easily verified that substitution of (B.2) in (B.1) yields

\[ \theta(t) = R_1 \hat{\psi} \cos(\omega t + \phi) + R_3 \hat{\psi} \cos(3\omega t + 3\phi) + R_5 \hat{\psi} \cos(5\omega t + 5\phi) \]  \hspace{1cm} (B.3)

where

\[ R_1 = \frac{5}{8} a_5 \hat{\psi}^4 + \frac{3}{4} a_3 \hat{\psi}^2 + a_1 \] \hspace{1cm} (B.4.a)
\[ R_3 = \frac{5}{15} a_5 \hat{\psi}^4 + \frac{1}{4} a_3 \hat{\psi}^2 \] \hspace{1cm} (B.4.b)
Equation (B.3) shows that when the flux of a nonlinear magnetic circuit (represented by Eq. (B.1)) is sinusoidal, the corresponding m.m.f. will contain odd harmonics. In above example, only two odd harmonics have been considered, the third - and the fifth harmonic. Recognize that the number of harmonics depends on the degree of the polynomial.

Next, this harmonic consideration will be extended for the three-limb transformer. In particular, this approach will be used to explain the harmonic contents of the current and voltage of the transformer. However, the analysis will be kept simple by only considering the first and the third harmonic.

For this purpose, the magnetic circuit of the three-limb transformer is re-considered here. Figure B.1 shows a simplified version of the magnetic circuit of the three-limb transformer. As can be seen, the zero-sequence path is taken in the middle; more precisely, it is assumed that the zero-sequence fluxes will leave the core at the junction of centre limb and yokes. It is noted that this has been done to facilitate the analysis.

A straightforward evaluation of the network given in Fig. (B.1) learns that the magnetic behaviour of the transformer can be described by the following equations

\[
R_5 = \frac{1}{16} a_5 \psi^4
\]
\( i_a + i_U = R(\psi_U) + (\alpha-1)R(\psi_a) + R_o (\psi_a + \psi_b + \psi_c) \)  
(B.5)

\( i_b + i_V = R(\psi_V) \) \( R_o (\psi_a + \psi_b + \psi_c) \)  
(B.6)

\( i_c + i_W = R(\psi_W) + (\alpha-1)R(\psi_c) + R_o (\psi_a + \psi_b + \psi_c) \)  
(B.7)

\( i_U = R(\psi_U) + R_1 \psi_U - R_1 \psi_a \)  
(B.8)

\( i_V = R(\psi_V) + R_1 \psi_V - R_1 \psi_b \)  
(B.9)

\( i_W = R(\psi_W) + R_1 \psi_W - R_1 \psi_c \)  
(B.10)

where

\( \alpha \) : factor accounting for the reluctance of the yokes

\( R(.) \) : the polynomial function given in Eq. (B.1), except that for sake of simplicity its degree is set to three, i.e., \( a_3 = 0 \)

\( R_1 \) : linear magnetic reluctance related with the short-circuit behaviour of the transformer

\( R_o \) : linear magnetic reluctance related with the zero-sequence behaviour of the transformer

Above set of equations can be solved provided the remaining (six) independent equations, following from the transformer connection, are given. Consider for this purpose the wye-connected primary windings with unearthed neutral point, as shown in Fig. B.2.

Notice, that the ideal case will be treated here, that is, the system of three-phase voltages sources is taken ideal, whereas the resistance of the windings is not taken into account.

![Figure B.2](image)

*Figure B.2*  
Wye connected primary windings of a transformer with unearthed neutral point
When fictitious fluxes are introduced for the excitation and the voltage at the neutral point, the three branches of the network (see Fig. B.2) produce the following equations:

$$\psi_U = \hat{\psi}\cos(\omega t) - \psi_N \tag{B.11.a}$$

$$\psi_V = \hat{\psi}\cos(\omega t - 2\pi/3) - \psi_N \tag{B.11.b}$$

$$\psi_W = \hat{\psi}\cos(\omega t + 2\pi/3) - \psi_N \tag{B.11.c}$$

In addition, the neutral point imposes the following constraint on the currents:

$$i_U + i_V + i_W = 0 \tag{B.12}$$

Suppose finally, that the secondary windings do not carry currents, that is,

$$i_a = i_b = i_c = 0 \tag{B.13}$$

then, Eqs. (B.11)-(B.14) constitute the external imposed constraints, necessary to calculate the specific flux flow in the magnetic network (of Fig. B.1). However, before the magnetic network equations will be worked out, a characteristic nonlinear function related to the three-limb transformer will be considered.

Since the nonlinear function, \(xR(\psi_U) + R(\psi_V) + xR(\psi_W)\), plays an important role in the further discussion, this function will be first worked out. It can be verified that, after tedious but straightforward manipulations, using Eqs. (B.1) and (B.11), the following polynomial in \(\psi_N\) with \(x\) as parameter can be found:

$$\theta(\psi_N; \alpha) = \alpha R(\psi_U) + R(\psi_V) + \alpha R(\psi_W)$$

$$= \alpha a_1(\hat{\psi}\cos(\omega t) - \psi_N) + \alpha a_3(\hat{\psi}\cos(\omega t) - \psi_N)^3 +$$

$$a_1(\hat{\psi}\cos(\omega t - 2\pi/3) - \psi_N) + a_3(\hat{\psi}\cos(\omega t - 2\pi/3) - \psi_N)^3 +$$

$$\alpha a_1(\hat{\psi}\cos(\omega t + 2\pi/3) - \psi_N) + \alpha a_3(\hat{\psi}\cos(\omega t + 2\pi/3) - \psi_N)^3$$

or

$$\theta(\psi_N; \alpha) = -(A_3\psi_N^3 + A_2\psi_N^2 + A_1\psi_N + A_0) \tag{B.14}$$

with

$$A_3 = (2\alpha + 1)a_3 \tag{B.15.a}$$
\[ A_\ell = 3(\alpha-1)a_3^{\hat{\psi}}\cos(\omega t - 2\pi/3) \]  
\hspace{1cm} (B.15.b)

\[ A_1 = (2\alpha+1)(a_1 + \frac{3}{2}a_3^{\hat{\psi}^2}) - \frac{3}{2}(\alpha-1)a_3^{\hat{\psi}^2}\cos(2\omega t + 2\pi/3) \]  
\hspace{1cm} (B.15.c)

\[ A_0 = (\alpha-1)(a_1 + \frac{3}{4}a_3^{\hat{\psi}^2})\psi\cos(\omega t - 2\pi/3) + (2\alpha+1)\frac{a_3^{\hat{\psi}^2}}{4}\psi\cos(3\omega t) \]  
\hspace{1cm} (B.15.d)

Recognize at this point, that the degree of the characteristic function \( \theta(\psi, \alpha) \) is the same as that of \( R(\cdot, \cdot) \), and that extension to a higher degree, thus higher harmonics, is straightforward.

For understanding the basics of wye-connected windings as expressed in Eqs. (B.11)-(B.13), the branch with reluctance \( R_0 \) in the network of Fig. B.1 is not essential, and will therefore be ignored in first instance. Having done this, it is easily verified that, when Eqs. (B.5)-(B.7) are added together, the following equation is obtained:

\[ i_a + i_b + i_c + i_U + i_V + i_W = \alpha R(\psi_U) + R(\psi_V) + \alpha R(\psi_W) \]  
\[ 3R_0(\psi_U + \psi_V + \psi_W) \]  
\hspace{1cm} (B.16)

Substitution of Eqs. (B.11)-(B.14) in this equation yields:

\[ A_3\psi_N^3 + A_2\psi_N^2 + A_1\psi_N + A_0 + 9R_0\psi_N = 0 \]  
\hspace{1cm} (B.17)

Basically, the solution of \( \psi_N \) follows from Eq. (B.17). However, due to nested roots (square root in a cube root), the general solution is not easy to see through. Determination of the harmonic content will be tedious, since the general solution is cube root of a complex time function. Important for the further discussion however, is not so much the exact form of the closed solution, and the exact content of the harmonics of \( \psi_N \) but the insight that the third harmonic is related via \( a_3 \) (\( a_3 \neq 0 \)) to the nonlinearity. This can be understood quickly when a nonlinear function of which \( a_3 \) is relatively small - notice that for \( a_3 = 0 \) the function is linear - is considered. In this case, the coefficients \( A_3, A_2, \) and the time-dependent part of \( A_1 \) can be ignored (see Eq. (B.15)), which leads to the following approximation of the fictitious flux:

\[ \psi_N = \frac{(\alpha-1)(a_1 + \frac{3}{4}a_3^{\hat{\psi}^2})}{9R_0 + (2\alpha+1)(a_1 + \frac{3}{2}a_3^{\hat{\psi}^2})} \hat{\psi}\cos(\omega t - 2\pi/3) \]  
\[ - \frac{1}{4}(2\alpha+1)a_3^{\hat{\psi}^2} \hat{\psi}\cos(3\omega t) \]  
\hspace{1cm} (B.18)
Once $\hat{\psi}_N$ is known, the current $i_U$, $i_V$, and $i_W$ can be easily obtained. For this purpose Eqs. (B.5)-(B.7) are rewritten as

\begin{align}
  i_U &= \alpha R (\hat{\psi} \cos(\omega t) - \hat{\psi}_N) - 3R_0 \hat{\omega}_N \\
  i_V &= R (\hat{\psi} \cos(\omega t - 2\pi/3) - \hat{\psi}_N) - 3R_0 \hat{\omega}_N \\
  i_W &= \alpha R (\hat{\psi} \cos(\omega t + 2\pi/3) - \hat{\psi}_N) - 3R_0 \hat{\omega}_N
\end{align}

(A.19.a)\hspace{1cm} (A.19.b)\hspace{1cm} (A.19.c)

A close inspection of the expression of the currents (Eqs. (B.19)) will learn that these currents must contain a third harmonic. This can be appreciated by noticing that the expression of the current is built up by two terms. The first term, $R(\cdot)$, is a polynomial, which, generally speaking, will produce harmonics. The second term, $3R_0 \hat{\omega}_N$ is subtracted from the first term to obtain the expression of the current. Since the second term contains a third harmonic (cf. Eq. 18), the same third harmonic must be generated by the first term of all three currents if this third harmonic should be canceled. However, as can be seen from Eqs. (B.19), the third harmonic generated by $R(\cdot)$ of $i_V$ will differ from the third harmonic generated by $\alpha R(\cdot)$ of $i_W$ and $i_W$. Hence no simultaneous cancelation of a third harmonic can be obtained, and thus, the currents $i_U$, $i_V$ and $i_W$ will contain a third harmonic.

Next, the case of a solidly grounded wye-connected transformer will be worked out. Like the previous case, the branch with reactance $R_1$ (see Fig. B.1) is not taken into account. It is easily shown that the external imposed constraints change into

\begin{align}
  \psi_U &= \hat{\psi} \cos(\omega t) \\
  \psi_V &= \hat{\psi} \cos(\omega t - 2\pi/3) \\
  \psi_W &= \hat{\psi} \cos(\omega t + 2\pi/3)
\end{align}

(B.20.a)\hspace{1cm} (B.20.b)\hspace{1cm} (B.20.c)

in other words, $\psi_N = 0$, whereas the neutral current is given by

\begin{equation}
  \begin{aligned}
  i_N &= i_U + i_V + i_W \\
  \text{(B.21)}
  \end{aligned}
\end{equation}

Substitution of these constraints (Eqs. (B.20) and (B.21)) into Eq. (B.16) and (B.19) respectively yields

\begin{equation}
  \begin{aligned}
  i_N &= -A_0 = -((\alpha-1)(a_1 + \frac{3}{4}a_3\hat{\psi}^2)\hat{\psi} \cos(\omega t - 2\pi/3) + \\
  &\quad - \frac{1}{4}(2\alpha+1)a_3\hat{\psi}^3 \cos(3\omega t)
  \end{aligned}
\end{equation}

(B.22)

and
\[ i_U = \alpha R(\dot{\psi}_0 \cos(\omega t)) \quad \text{(B.23.a)} \]
\[ i_V = R(\dot{\psi}_0 \cos(\omega t - 2\pi/3)) \quad \text{(B.23.b)} \]
\[ i_W = \alpha R(\dot{\psi}_0 \cos(\omega t + 2\pi/3)) \quad \text{(B.23.c)} \]

Recognize at this point that the three-phase three-limb transformer, whose windings are wye-connected and neutral solidly grounded, behaves in this case like three single-phase transformers. The expressions of the currents given in (B.23) are similar to the expression of a single-phase magnetic circuit as given in (B.3), except for the factor (scalar) \( \alpha \). Also notice, that in contrast with the neutral current of wye-connected single-phase transformers, the neutral current of the three-limb transformer contains a fundamental component. From Eq. (B.22) it is seen that this is due to the inequality of the limbs (\( \alpha \neq 1 \)).

The effect of the current in delta-connected secondary windings will now be considered more closely. The constraints imposed by the delta-connected windings are as follows

\[ i_a = i_b = i_c = i_D \quad \text{(B.24)} \]

and

\[ \frac{d\psi_a}{dt} + \frac{d\psi_b}{dt} + \frac{d\psi_c}{dt} = 0 \rightarrow \psi_a + \psi_b + \psi_c = \text{constant} = 0 \quad \text{(B.25)} \]

where the resistance of the secondary has been neglected.

In the case of an ungrounded primary neutral, the sum of Eqs. (B.8)–(B.10) yields the following equation

\[ i_U + i_V + i_W = R(\psi_U) + R(\psi_V) + R(\psi_W) + R_L(\psi_U + \psi_V + \psi_W) \]
\[ - R_L(\psi_a + \psi_b + \psi_c) \]

while substitution of the constraints related to the wye-connected primary windings ((B.11) and (B.12)), the characteristic function \( \Theta(\psi_N, \dot{\psi}) \) expressed in (B.14) and (B.15), and the flux constraint given in (B.25), in above equation yields

\[ a_3^3 \psi_N^3 + (a_1 + \frac{3}{2}a_3^2 \dot{\psi}^2 + R_L) \psi_N + \frac{3}{4}a_3^2 \dot{\psi}^2 \cos(\omega t) = 0 \quad \text{(B.26)} \]

The fictitious flux \( \psi_N \), related to the voltage at the neutral point, follows from Eq. (B.26). It is seen that this flux will contain a third harmonic which not only largely depends on the core characteristics, but also on the reluctance \( R_L \).

For the derivation of an expression of the delta-current \( i_D \), the sum of Eqs. (B.5)–(B.7) is considered. After substitution of \( (B.12), (B.24) \) and \( (B.25) \) the delta-current can be expressed as
\[ 3i_D = R(\psi_U) + R(\psi_V) + R(\psi_W) + (\alpha-1)(R(\psi_A) + R(\psi_C)) \]

or

\[ 3i_D = 3R_1\psi_N + (\alpha-1)(R(\psi_A) + R(\psi_C)) \quad (B.27) \]

Since \( \psi_n \) follows from Eq. \((B.26)\), \( i_d \) can be found provided the second part of Eq. \((B.27)\) is known. For this purpose Eqs. \((B.5)-(B.11)\) are used. Straightforward manipulations will lead to the following set of nonlinear equations

\[ (\alpha-1)R(\psi_A) + 2R_1\psi_A + R_1\psi_C = R_1\dot{\psi}(\cos(\omega t) - \cos(\omega t - 2\pi/3)) \quad (B.28.a) \]

\[ (\alpha-1)R(\psi_C) + 2R_1\psi_C + R_1\psi_A = R_1\dot{\psi}(\cos(\omega t + 2\pi/3) - \cos(\omega t - 2\pi/3)) \quad (B.28.b) \]

The fluxes \( \psi \) and \( \psi_n \) follow from the set of nonlinear equations given in \((B.28)\), and in the most general form, these fluxes can be expressed as a sum of harmonics. However, for a fundamental understanding, it suffices to consider the following approximation (of the first harmonic)

\[ ((\alpha-1)R_1 + 2R_1)\psi_A + R_1\psi_C = R_1\dot{\psi}(\cos(\omega t) - \cos(\omega t - 2\pi/3)) \]

\[ ((\alpha-1)R_1 + 2R_1)\psi_C + R_1\psi_A = R_1\dot{\psi}(\cos(\omega t + 2\pi/3) - \cos(\omega t - 2\pi/3)) \]

from which the following equation can be derived

\[ (\alpha-1)R_1(\psi_A + \psi_B) = -(\alpha-1)aR_1\dot{\psi}(\omega t - 2\pi/3) \quad (B.29.a) \]

where

\[ a = \frac{3R_1}{3R_1 + (\alpha-1)R_1} \quad (B.29.b) \]

and

\( R_1 \) an equivalent magnetic reluctance.

From Eq. \((B.27)\) and \((B.29)\) it is seen that the delta-current contains a fundamental component, and that this component is directly related to the reluctance of the yokes \((\alpha \neq 1)\) of the three-limb transformer. Observe further, that the magnitude is also affected by the reluctance \( R_1 \). From the derivation of \( \psi_N \) (see Eq. \((B.26)\) it is seen that this reluctance cannot be ignored; ignorance of \( R_1 \) \((R_1 \rightarrow \infty)\) would have resulted in a zero \( \psi_N \). Notice however, that an alternative approximation of the delta-current is obtained when \( R_1 \) is not taken into account. It can be verified that when \( R_1 \rightarrow \infty \), i.e., \( \psi_N = 0 \), the delta-current can be approximated as follows
\[3i_D = \alpha R(\psi_U) + R(\psi_V) + \alpha R(\psi_W) = -A_o\]
\[= - (\alpha - 1) (a_1 + \frac{3}{4} a_3 \hat{\psi}^2) \hat{\psi} \cos(\omega t - 2\pi/3) - (2\alpha + 1) \frac{a_2}{4} \hat{\psi}^3 \cos(3\omega t) \]  \hspace{1cm} (B.30)

Note, that for the derivation of this approximation the Eqs. (B.14)-(B.16) and (B.24), (B.25) have been used. This last approximation once more shows that the delta-current \(i_D\) is mainly determined by the core characteristics.

The currents in the wye-connected primary windings follow from Eqs. (B.5)-(B.7) and can be expressed as

\[i_U = R(\psi_U) + (\alpha - 1) R(\psi_a) - i_D \]  \hspace{1cm} (B.31.a)
\[i_V = R(\psi_V) - i_D \]  \hspace{1cm} (B.31.b)
\[i_W = R(\psi_W) + (\alpha - 1) R(\psi_c) - i_D \]  \hspace{1cm} (B.31.c)

since the quantities at the right hand side follow from (B.11), (B.26), (B.27) and (B.28). For reasons as discussed before, these currents will contain a third harmonic.

Finally, the wye-delta connection with the neutral grounded will be considered. The constraints imposed by this transformer connection have already been given, namely, the Eqs. (B.20) and (B.21) for the primary side, and (B.24) and (B.25) for the secondary side. Like the previous case, the sum of Eqs. (B.8)-(B.10) is considered, which yields

\[i_N = R(\psi_U) + R(\psi_V) + R(\psi_W) = \theta(\psi_N;1) = -A_o(1)\]
\[= - \frac{3}{4} a_3 \hat{\psi}^3 \cos(3\omega t) \]  \hspace{1cm} (B.32)

where \(A_o\) follows from Eq. (B.15.a). On the other hand, the current \(i_D\) follows from the sum of Eqs. (B.5)-(B.7), and is given by

\[3i_D + i_N = R(\psi_U) + R(\psi_V) + R(\psi_W) + (\alpha - 1) \left( R(\psi_a) + R(\psi_c) \right) \]  \hspace{1cm} (B.33)

which, after substitution of (B.32), changes into

\[3i_D = (\alpha - 1) \left( R(\psi_a) + R(\psi_c) \right) \]  \hspace{1cm} (B.34)

of which the right hand side follows from (B.28) and can be approximated by (B.29). Also notice, that the delta-current can be approximated by ignoring \(R_1\); Eq. (B.33) changes into

\[3i_D + i_N = \alpha R(\psi_U) + R(\psi_V) + \alpha R(\psi_W)\]
and after substitution of Eqs. (B.14) and (B.32) into

\[ 3i_D = -(\alpha - 1) \left\{ \left( a_1 + \frac{3}{4} a_3 \dot{\psi}^2 \right) \psi \cos(\omega t - 2\pi/3) - \frac{1}{2} a_3 \psi^3 \cos(3\omega t) \right\} \]  

(B.35)
REFERENCES

Chapter 4


Chapter 3


Chapter 1, 2 and 5


ACKNOWLEDGEMENTS

The research described in this thesis has been carried out in the Power Systems Laboratory of the Department of Electrical Engineering of the Delft University of Technology and in the Department of Electrical Engineering of the N.V. KEMA.

I am grateful to all my former colleagues and the students of the Power System group for creating the stimulating environment necessary for my work on transformer modelling. However, a special word of thanks is due to my roommate in that time, Ir. drs. R.A.M. Van Amerongen. I would also like to mention the Mr. H.G. Lems who has carried out the measurements.

I wish to express my gratitude to the management of N.V. KEMA for the permission to publish some of my research results in this thesis and for giving me the opportunity to produce this document. In particular, I am indebted to dr. J.H. Blom for his moral support and Ir. A.J Degens who has given me all the facilities I needed.

Furthermore, I should like to thank Miss C.T.M. Derks for typing the manuscript and Mr. Q. Hoogenboom for his assistance with the typing of the mathematical expressions.

Finally I should like to thank all the people who through their continuing interest encouraged me to fulfil this thesis.
SUMMARY

The work presented in this thesis must be seen within the framework of reviewing models of power system components and aligning them in accordance with the now existing insights and techniques. Despite the developments activated by the computer, there still is a need to review existing models, especially models based on concepts developed in the computerless era. In particular, there is a need to confront these classical models with the possibilities and facilities offered by the computer, and if necessary, give new descriptions, exploiting the advantages of the computer. In fact, there should be a regular updating of computer models considering the developments in the computer science.

In this thesis a nonlinear network description of the transformer will be given, using the best approach now available, a combination of both the physical and the phenomenological approach. Starting from the elementary Maxwell's equations, it will be shown which assumptions and simplifications are required to arrive at what can be seen as a physically based network description of the transformer. As a result of this consideration, the link between the T-network and the m-network, both describing the single-phase transformer, could be made clear. It will be seen that from a modelling point of view, the T-network results from the assumption that the total flux can be divided in main flux and leakage fluxes, while the m-network results from an integral flux approach. It will also be shown that these two networks are mathematically related via the star-delta transformation.

Leakage inductions are frequently used in three-phase transformer descriptions. This is a result of a similar concept underlying the T-network, namely, that the flux is divisible in a main flux - the flux in the iron core - and leakage fluxes. It will be shown however, that the three-phase transformer can perfectly - and also more realistically - be described, when the integral flux approach is used. In this thesis the transformer is described in terms of magnetic networks and not their dual form, electric networks. In doing so, the fluxes are readily accessible and can be more easily related to reality. The advantage of this description will be illustrated, for example, when the difference in the no-load currents of the three phases is explained and related to the difference in the transformer limbs (of a three-limb transformer). It will be also seen that second order effects (in the normal no-load condition) such as in the voltage at the neutral point, or the current through the neutral lead of a wye-connected transformer can be studied and understood more closely.

The phenomenological approach is used for modelling the nonlinear elements in the branches of the magnetic network. A single-valued function will be used for describing the saturation characteristics of the transformer. As far as hysteresis is concerned, a Preisach hysteresis model will be used for
describing the frequency-independent part of hysteresis, the static hysteresis. Reversals occurring in the hysteresis evolution are stored in a stack and by means of a stack mechanism the algorithm for simulating static hysteresis is explained. It will be seen that especially the forgetting property of minor loops is straightforward.

A network transient analysis program necessary for carrying out system studies in which transformers are involved, is discussed in this thesis. The Modified Nodal Approach will be used for the network synthesis. Extra attention will be paid to the incorporation of the transformer equations in the network equations. A method is presented with which significant reduction in the dimension of this nonlinear problem can be obtained.

An implicit integration formula will be used for the numerical solution of the differential equations. The numerical integration will be carried out using a backward differentiation formula. It is also shown that the spurious oscillations sometimes occurring when using the trapezoidal rule, is an inherent (mathematical) property of the trapezoidal rule and not a matter of violating physics.

The strength of formulating the transformer equations as presented in this thesis, namely the possibility of having direct access to the transformer fluxes, will be illustrated in five case studies. Where necessary two transformer models - a leakage flux model and an integrated flux model - will be compared with each other.

It is also worth knowing that the transient analysis program has also been used to carry out several transformer transient studies for the N.V. KEMA.
SAMENVATTING

In dit proefschrift wordt behandeld het modelleren van driefasen transformatoren ten behoeve van het bestuderen van laag frequente verschillen in elektriciteitsvoorzieningssystemen. De meeste driefasen transformatormodellen zijn voortzettingen van een eenvoudige transformatorbeschrijving waaraan geen fysische maar voornamelijk rekentechnische overwegingen ten grondslag hebben gelegen. De fysische aanpak wordt in dit proefschrift besproken.

In het klassieke eenvoudige transformatormodel, het bekende T-netwerk, wordt de flux gesplitst in een hoofdflux en lekfluxen. Vooral beperkingen van rekentechnische aard zijn bepalend geweest voor dit, rond 1900, geïntroduceerde concept. Thans, in deze computer-era, zijn er nauwelijks rekentechnische beperkingen, of beter gezegd, zijn deze van een geheel andere orde. Dit houdt in dat de noodzaak voor de invoering van deze niet op fysische gronden gebaseerde fluxsplittings is komen te vervallen.

Besporen wordt een transformatormodel waarbij, uitgaande van de fysische realiteit, het fluxgedrag van de transformator als één geheel wordt beschreven. Aangetoond wordt dat het aan de eenvoud transformatoren gerelateerde n-netwerk een representant is van deze geïntegreerde fluxbeschouwing.

Interessant is te weten dat de twee netwerken, voortkomende uit de twee genoemde benaderingswijzen, via een ster-driehoektransformatie aan elkaar gerelateerd zijn. Beide netwerken zijn lineair en equivalent indien verondersteld mag worden dat de transformator lineair is. Wanneer deze veronderstelling niet meer te rechtvaardigen is, dat wil zeggen, wanneer de transformator zich sterk niet-lineair gedraagt, moeten beide netwerken uiteraard niet-lineair zijn. In dit geval zijn beide netwerken niet equivalent meer.

Bij het modelleren van driefasen transformatoren is de fysische benadering gevolgd. Via een consistente voortzetting van de eenvoud aanpak is een driefasen transformatormodel ontwikkeld. Bovendien is ervoor gezorgd dat de niet-lineaire elementen van het model (magnetisch netwerk) de hysteresis eigenschap bezitten – in dit proefschrift wordt een voor transformatorstudies geschikte hysteresisrepresentatie behandeld –, waardoor het mogelijk is remanentie bij driefasen transformatoren, dat wil zeggen, bij gekoppeld fluxgedrag, te bestuderen.

Een computerprogramma is ontwikkeld om het transiënte gedrag van transformatoren in netten te berekenen. Het programma is gebaseerd op de MNA (Modified Nodal Approach), een methode die netwerkvergelijkingen op een systematische en elegante manier aanpakt. Besproken wordt een aanpassing van de MNA die het mogelijk maakt efficiënt te rekenen aan netwerken waarin niet-lineaire elementen – en dus transformatoren – voorkomen.

Berekend is het gedrag van driefasen transformatoren voor verschillende transformatorschakelingen in verschillende netwerkconfiguraties. Voor een aantal gevallen zijn berekeningen en metingen naast elkaar gelegd. Ook is aangegeven tot welke verschillen genoemde fluxbeschouwingen – het splitsen of geïntegreerd behandelen van de flux – kunnen leiden door
responsies van de twee hiermee corresponderende transformator-modellen met elkaar te vergelijken.
Het computerprogramma is ook nog gebruikt om enige studies uit te voeren ten behoeve van de N.V. KEMA.
CURRICULUM VITAE

Harold E. Dijk werd op 29 december 1950 geboren op Aruba, Nederlandse Antillen.

In 1969 behaalde hij het HBS-B diploma aan de Algemene Middelbare School in Suriname.

In augustus 1977 studeerde hij af aan de Technische Universiteit Delft (TUD) in de richting der Elektrotechniek.

Van 1978 tot 1982 was hij studentassistent bij de vakgroep Elektriciteitsvoorziening. In deze vakgroep heeft hij, naast het begeleiden van afstudeerders, het belangrijkste deel van het in dit proefschrift beschreven onderzoek verricht.

Sinds oktober 1982 is hij in dienst van de N.V. KEMA, waar hij als wetenschappelijk medewerker werkzaam is in groep Netberekeningen van de afdeling Elektrotechnisch Onderzoek. Het aan de TUD ontwikkelde transformatormodel en de bijbehorende programmatuur hebben hier vooral hun nut bewezen bij het beantwoorden van vraagstukken uit de praktijk.
STELLINGEN

behorende bij het proefschrift van
H.E. Dijk

Delft, 19 september 1988
1. Bij de afleiding van zowel het T-netwerk als het π-netwerk voor een eenfase transformator worden idealiseringen ingevoerd met het doel het rekenwerk te vereenvoudigen. Het T-netwerk staat echter verder af van de fysische realiteit dan het π-netwerk; de herkenbaarheid van fysische grootheden is groter bij het π-netwerk.

_Dit proefschrift, hoofdstuk 2._

2. Bij een drievoet transformator verschilt de middenpoet in effectieve lengte van de buitenste poten. Dit verschil verklaart niet alleen de grondharmonische in de stropuntsspanning aan de voedende zijde van de transformator in nullast, maar ook de derde harmonische in de nullaststromen van een transformator waarvan het stropunt niet is geaard.

_Dit proefschrift, hoofdstuk 2, 5._

3. Bij het inschakelen van een transformator kunnen de klemspanningen van de transformator onder bepaalde omstandigheden plotseling heel hoge waarden aannemen. Dit fenomeen kan worden voorkomen door schakelaars te fabriceren met een nog kleiner verschil in de sluittijden van de polen, of transformatoren waarvan het ijzer minder snel verzadigt.

_Dit proefschrift, hoofdstuk 5._

4. Gebruikers van EMTP (Electro Magnetic Transient Program) worden regelmatig geconfronteerd met oscillaties in de resultaten van de berekeningen. Deze oscillaties, die een periodeduur van tweemaal de staptweekte van de numerieke integratie hebben, zijn niet het gevolg van een verkeerd model van een fysisch fenomeen, maar hangen direct samen met het feit dat de trapeziumregel is gebruikt bij de discretisering.

_Dit proefschrift, hoofdstuk 4._

6. De huidige transformatormodellen kunnen in twee klassen worden ondergebracht, te weten, modellen die alleen geschikt zijn voor het bestuderen van laagfrequente verschijnselen en modellen voor hoogfrequente verschijnselen. Verschijnselen waarvan de frequentie ligt in het middengebied, worden door beide type modellen onvoldoende beschreven.

7. In dit door computers beheerste tijdperk moet het gebruik van wijzerdiagrammen worden gezien als een anachronisme. Slechts daar waar hun gebruik visueel ondersteunend is, zijn wijzerdiagrammen zinvol.

8. Expertsystemen zijn geschikt om specialistische kennis en dogma's in de computer op te slaan.

9. Kwalitatieve simulatie is een van de meest interessante ontwikkelingen op het gebied van Kunstmatige Intelligentie.

10. Sport zou een geïntegreerd onderdeel moeten zijn van het werken, omdat in sport een aantal eigenschappen worden ontwikkeld die van voordeel zijn bij andere aspecten van het leven, met name beroepsuitoefening.

11. In een wereld waarin tekstverwerkers gemeengoed zijn geworden, behoren mensen met een fatsoenlijk handschrift tot een uitstervend ras.