two-dimensional breaking of waves on a beach

M.J.F. Stive

report on laboratory investigation

M 1585, part II

May 1983
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Laboratory report

part 2 - Pressure field in waves shoaling and breaking on a plane beach

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General introduction

This report is part of an experimental study of the two-dimensional breaking of waves on a beach, performed under laboratory code M 1585 in the framework of the TOW-Coastal Research Programme. The initiation of the study took place in Oktober/November 1978 when a programme was carried out which comprised the measurement of external and internal characteristics of periodic, progressive waves shoaling and breaking on gentle beach slopes. The external, i.e. surface elevation related, characteristics were determined for a variety of beach configurations and incoming wave conditions. This variation in physical conditions was feasible because of the relative simplicity of this type of measurement. This is also the reason that experimental surfzone studies have generally been restricted to the external characteristics. In order to analyse the surfzone wave motion more in "depth" rather than at the "surface" also internal, i.e. kinematic and dynamic characteristics of near-breaking and breaking waves were measured. In this case only one beach slope and two incoming wave conditions were considered because of the relative complexity of these measurements.

In the analysis of these measurements priority has so far been given to the analysis in depth in view of our limited fundamental knowledge of the surfzone. The results are presented in three papers (referred to as I, II and III) which describe three consecutive phases of the analysis, viz.

- instantaneous velocity and pressure fields are analyzed in
  Velocity and pressure field of spilling breakers

- time and depth averaged momentum fluxes are analyzed in
  A study of radiation stress and set-up in the nearshore region
  Coastal Eng., 6, pp 1-25

- time and depth averaged energy fluxes are analyzed in
  Energy dissipation in waves breaking on gentle slopes
  Submitted for publication to Coastal Eng.
The present part 2 of the laboratory report M 1585 is one of four simultaneously published parts which aim to describe some remaining aspects of the measurements not yet published.

The four parts are:

part 1 - Shoaling and breaking waves on a plane and on a composite beach
part 2 - Pressure field in waves shoaling and breaking on a plane beach
part 3 - Kinetic energy in waves shoaling and breaking on a plane beach
part 4 - Small and large scale waves breaking on a gently sloped beach.
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REFERENCES

FIGURES
PART 2 - PRESSURE FIELD IN WAVES SHOALING AND BREAKING ON A PLANE BEACH

1. Introduction

Some effort during the measurement programme of October/November 1978 was put in internal pressure measurements in waves shoaling and breaking on a plane beach. This effort was more directed towards investigating the feasibility of pressure measurements than towards providing pressure data as an integral part of the internal measurements. A preliminary result of measurements in one cross-section in the quasi-steady breaking region is described semi-quantitatively in I. A more complete account of the obtained results is given in the present Part 2.
2. Experiments

2.1 Arrangements

The experiments were conducted in a wave flume (the Schelde flume) of the Delft Hydraulics Laboratory. The flume is 55 m long, 1 m wide and 1 m high. A plane, concrete beach of a 1:40 slope was installed (see Figure 1a). To enable the installation of instruments in the bottom of the horizontal section a false, concrete floor was made, which reduced the water depth in the horizontal section to 0.70 m.

Periodic waves with minimal free second harmonic components were generated in a water depth of 0.85 m. To obtain a control signal for the wave generator which resulted in a maximum suppression of the free second harmonics a simple trial and error procedure was applied, using Fontanet's theory as a first approximation (see Part I, Section 2.2).

2.2 Instrumentation

Surface elevations were measured by means of conductivity type wave gauges. Three gauges were installed on a carriage to cover the horizontal section and most of the shoaling zone. Twelve fixed gauges were positioned near and in the surfzone. Calibrations confirmed that deviations from linearity were negligible, viz. 0.5% of the full scale deflection, which amounted to 10, 20 or 50 cm depending on the wave heights. It is noted that conductivity type wave gauges must be used with care in the cases of sharply peaked waves and of waves aerated by breaking. In both cases the gauges response may be affected (see Part I, Section 2.3).

The internal pressures were measured by means of relatively small differential pressure transducers (φ 8 mm). One set of measurements was conducted with the transducers mounted flush with the bottom: two in the horizontal flume section and seven in the shoaling and breaking region (see Fig. 1a for positions). The other set of measurements was conducted with the transducers mounted flush in a movable, transparent, plane board of 2.0 m length, positioned parallel to the flume walls. Instrumental inaccuracies are exceeded by inaccuracies introduced by the data-analysis procedure (see 3.1), with the exception of inaccuracies due to drift of the transducers which was found to amount to ± 0.2 cm water column generally and up to −0.4 cm in one case.
2.3 Measurements and procedures

Two types of pressure data runs were conducted, each for two wave conditions. One pair comprised the measurement of surface elevation and bottom pressure in a limited number of cross-sections (see Fig. 1b). The other pair comprised the measurement of pressures at different levels in 4 cross-sections in the breaking and near-breaking region (see Fig. 1b). In each of these cross-sections the velocities were measured consecutively at different levels, i.e. non-simultaneously. To establish a common reference phase for the non-simultaneous measurements in one cross-section the command signal of the wave generator and the surface elevation in the same cross-section were recorded simultaneously with the velocities.

All electric signals were recorded on analog tapes. At the same time they were digitised at 25 Hz by a mini-computer for limited processing, allowing a check on the data-acquisition immediately after the data runs. The analog tapes were digitised at 100 Hz for extensive processing.

2.4 Wave conditions

The experiments were restricted to two conditions (see Table below), which are referred to as test 1 and test 2. In the data runs conducted with the pressure transducer installed board slight differences occur in the local wave motion compared to the situation without the wave board. In Figure 2 some wave profiles are given to indicate the differences. It appears that the wave profiles are somewhat disturbed by the presence of the pressure board (the wave heights differ up to 12%). These quantitative differences however do not question the relevancy of the pressure results to the 'undisturbed' situations of test 1 and 2.

The breaking motion for these test conditions corresponds to that found on gently sloping beaches. The initial process of breaking of these conditions is mostly described as 'spilling' breaking. After a horizontal distance of several times the breaker depth, i.e. if the depth continues to decrease, the rapid transitions of initial breaking develop soon into a relatively well-organized, depth-similar breaking motion, where the white-cap and the mean flow in a reference frame moving with the wave are nearly steady. At that stage of their breaking motion, breakers may be described as bores, which again are commonly named 'spilling' breakers. After Svendsen et al. (1978) this region is called the inner region, while the region in which the process of initial breaking takes place is called the outer region.
Table of wave conditions (wave height $H$, period $T$, wave length $L$, where the subscripts $o$, $h$ and $b$ denote deep water, horizontal section and breakpoint)

<table>
<thead>
<tr>
<th>Test</th>
<th>$H_o$ (m)</th>
<th>$H_h$ (m)</th>
<th>$H_b$ (m)</th>
<th>$T$ (s)</th>
<th>$H_o/L_o$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.159</td>
<td>0.145</td>
<td>0.178</td>
<td>1.79</td>
<td>0.032</td>
</tr>
<tr>
<td>2</td>
<td>0.142</td>
<td>0.145</td>
<td>0.226</td>
<td>3.00</td>
<td>0.010</td>
</tr>
</tbody>
</table>
3. Pressure results

3.1 Data-analysis

With the differential pressure transducers dynamic pressures, \( P \), are measured which describe the pressure fluctuation around the still water hydrostatic pressure, thus

\[
P = p + \rho g z
\]

where \( p \) is the total pressure and \( z = 0 \) is located at the still water level.

The dynamic pressure data were processed with the aid of an ensemble averaging technique in which each wave cycle in the time series was considered to be one realization. The technique results in a description of the dynamic pressure as the sum of an 'organized', periodic component (denoted by a tilde) and an 'unorganized', residual component (denoted by a dash), i.e. \( P = \tilde{P} + P' \). With the aid of the command signal of the wave generator as a precise periodic reference signal and applying a high density sampling rate (100 Hz) it was possible to select 20 data points in each wave cycle and assign each point to 1 of 20 'bins'. Each bin corresponds to a phase between 0 and \( 2\pi \) relative to the primary wave period. Thus each bin consists of \( N \) data points at a fixed phase \( Q_i \), where \( i = 1,20 \) and \( N = \) the number of wave cycles measured, being 20 minimally.

The ensemble mean dynamic pressure as a function of phase is

\[
\bar{P}_i = \frac{1}{N_i} \sum_{k=1}^{N_i} P_{ik} \quad i = 1,20
\]

where \( P_{ik} \) are the values in each bin. \( \bar{P}_i \) is defined as the periodic or organized fluctuation and is a measure of that part of the fluctuation that is repeatable from wave cycle to wave cycle. The rms deviation from the ensemble mean in each bin is

\[
P_{i_{rms}}' = \left[ \frac{1}{N_i} \sum_{k=1}^{N_i} (P_{ik} - \bar{P}_i)^2 \right]^{0.5} \quad i = 1,20
\]

\( P_{i_{rms}}' \) is defined as the nonperiodic or unorganized fluctuation and represents the part of the fluctuation not repeatable from cycle to cycle. The average value of \( P_{i_{rms}}' \) over a cycle gives a measure of the average unorganized fluctuation:
\[ \overline{P_i^{\text{rms}}} = \frac{1}{20} \sum_{i=1}^{20} P_i^{\text{rms}} \]

Similarly, the rms value of \( \tilde{P}_i \) gives a measure of the organized flow averaged over a wave cycle:

\[ \tilde{P}_i^{\text{rms}} = \left[ \frac{1}{20} \sum_{i=1}^{20} (\tilde{P}_i)^2 \right]^{0.5} \]

The ratio (expressed as a percentage)

\[ P_{\text{var}} = \left( \frac{\overline{P_i^{\text{rms}}}}{\tilde{P}_i^{\text{rms}}} \right) \times 100 \]

is a measure of the relative size of the unorganized and organized fluctuations averaged over a wave cycle.

### 3.2 Internal dynamic pressure variations

The dynamic pressure variations measured at different fluid levels in the four selected cross-sections are given in Figure 3...10. The figures relate to the tests, cross-sections and type of wave motion as follows:

<table>
<thead>
<tr>
<th>test 1</th>
<th>test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 35.5 \text{ m} )</td>
<td>Fig. 3 (near-breaking)</td>
</tr>
<tr>
<td>( x = 36.5 \text{ m} )</td>
<td>Fig. 4 (initial breaking)</td>
</tr>
<tr>
<td>( x = 37.5 \text{ m} )</td>
<td>Fig. 5 (quasi-steady breaking)</td>
</tr>
<tr>
<td>( x = 39.5 \text{ m} )</td>
<td>Fig. 6 (quasi-steady breaking)</td>
</tr>
</tbody>
</table>

The measurement results (in cm water column) are presented by vertical bars, where the centre of each bar is the ensemble mean value \( \overline{P_i/\rho g} \) and the length is twice \( P_i^{\text{rms}} \), respectively. The percent values give \( P_{\text{var}} \). The upper part of the figures present the surface elevation in an equivalent way. All dynamic pressure results are compared to the ensemble mean of the surface variations.

As indicated in the figures two sources of measurement errors are easily recognized by inconsistent behaviour of the results, i.e. transducer drift and signal disturbance. From a further inspection of the results the following general conclusions regarding the dynamic pressure fluctuations in near-breaking and breaking waves may be drawn:
1. Under the wave troughs the ensemble mean pressure fluctuations are hydrostatic.

2. Under the wave crests and also at the wave fronts the ensemble mean pressure fluctuations are non-hydrostatic, which effect is most pronounced near the bottom. In the near-breaking wave the deviation from the hydrostatic variation amounts up to 40%, in the initial breaking waves the deviation amounts up to 25% and in the quasi-steady breaking waves the deviation amounts up to 15%.

3. The relative size of the unorganized and organized fluctuations averaged over a wave cycle, below the level of the wave troughs, amounts to 5% in the near-breaking wave, 7-9% in the initial breaking waves and 10-20% in the quasi-steady breaking waves.

The degree of non-hydrostaticity of the dynamic pressure fluctuations may obviously be connected with the degree of streamline curvature as indicated by the wave surface curvature, \( r \). A rough numerical approximation to this effect for the quasi-steady breaking waves may be found as follows. Consider a control volume of length \( dx \) extending between a level \( z \) and the wave surface \( \zeta \), and moving with the wave at propagation speed \( c \), so that the mean motion is steady. The balance equation of vertical momentum may be written to yield the following solution for the deviation of the dynamic from the static pressure fluctuation:

\[
P/\rho g - \zeta = -\omega^2/g + 1/g \frac{\partial}{\partial x} \int_0^\zeta (u-c) w dz
\]

At the wave crest \( w = 0 \) and below the level of the wave troughs \( u = c \). The effect of the curved streamlines may then be expressed as

\[
P/\rho g - \zeta = -1/g \int_0^\zeta c^2/r dz
\]

The term on the right hand side may be further simplified by the realistic assumptions \( c = (gh)^{0.5} \) and \( 1/r \) varies linearly between zero at the bottom to \( 1/r_s \) at the surface (\( r_s > 0 \) for positive curvature). The maximum deviation is found at the bottom and may be approximated by

\[
P/\rho g - \zeta \approx -\frac{1}{2}(h + \zeta_c)h/r_s
\]
In the quasi-steady breaking waves a typical value of \( h/r_s \) is 0.15 which results in a deviation of \(-7.5\) to \(-15\) mm for the cross-sections in the quasi-steady breaking region. This analysis also shows the effect of the streamline curvature to decrease continuously to zero towards the wave surface. These results are well in accordance with the measurements.

re 3 As found for the velocity data (see Part 3) the nonperiodic pressure fluctuations are found to increase towards the inner region and towards the wave surface (note: the decreasing values of \( P_{var} \) above the level of the wave troughs are due to the averaging procedure). If it may be assumed that these fluctuations are turbulence induced this behaviour is consistent with that of the turbulent velocity fluctuations (see also I). Here a few remarks are however in order:

(a) turbulent pressure fluctuations would be of the same order of magnitude as \( \rho u' / \rho g \) which yields \( P'_{rms}/\rho g \) of the order of 0.5 to 1 mm below the level of the wave troughs in the inner breaking region. The measured variations amount to 2 to 10 mm, i.e. an order of magnitude larger;

(b) due to the size of the transducers the spatial resolution may be too small;

(c) the rather high value of \( P_{var} \) in the near-breaking wave (compared to that of the kinetic energy) indicates that a substantial part of the nonperiodic fluctuations may well be induced by deviations from the periodic pressure variations due to other causes than turbulence.

3.3 Dynamic bottom pressure variations

Mean values of the crest height, the trough height and the crest to trough height of the dynamic bottom pressure variations as measured in the nine selected cross-sections are given in Figures 11 and 12. The measurement results are presented in cm water column and compared to linear theory in the shoaling region (with the wave heights predicted by linear shoaling) and to the hydrostatic fluctuation in the breaking region. The results confirm the conclusions already drawn in the foregoing paragraph.
4. **Summary and conclusions**

The present Part 2 describes the experimental results of the internal dynamic pressure measurements in breaking and near-breaking waves on a gently sloping beach. In the description of the results a distinction is made between 'organized' or periodic and 'unorganized' or nonperiodic, residual wave motion. The periodic dynamic pressure fluctuations are shown to deviate from the hydrostatic pressure fluctuation at the wave crests where the streamline curvature is strong. The deviations amount up to 15% in quasi-steady breaking waves and up to 40% in near-breaking waves. The measured unorganized pressure fluctuations are stronger than may be explained from the breaking induced turbulent flow. The ratio of rms values of unorganized over organized pressure fluctuations amounts to 10 to 20% in quasi-steady breaking waves and 5% in near-breaking waves.

Finally it is noted that the distinction between unorganized and organized motion is somewhat arbitrarily based on an ensemble averaging technique. An alternative technique applying routine spectral analysis may yield a more objective distinction between unorganized, turbulent and organized, periodic pressure fluctuation. This approach is being undertaken at the moment and the results will be described in a subsequent part of this report.
REFERENCES

STIVE, M.J.F. (1980) referred to as (I)
Velocity and pressure field of spilling breakers

Wave characteristics in the surfzone
FIG. 1a Experimental set-up

<table>
<thead>
<tr>
<th>test 1</th>
<th>breakpoint</th>
<th>internal pressure measurements</th>
<th>(surfzone)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0/L_0=0.032$</td>
<td>[\text{breakpoint} ]</td>
<td>[\text{internal pressure measurements} ]</td>
<td>[\text{(surfzone)} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>test 2</th>
<th>breakpoint</th>
<th>internal pressure measurements</th>
<th>(surfzone)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0/L_0=0.010$</td>
<td>[\text{breakpoint} ]</td>
<td>[\text{internal pressure measurements} ]</td>
<td>[\text{(surfzone)} ]</td>
</tr>
</tbody>
</table>

FIG. 1b Detail experimental set-up
COMPARISON OF UNDISTURBED AND PRESSURE BROAD DURING WAVE PROFILES

WATERLOO KUNDIG LABORATORIUM
COMPARISON OF MEASUREMENTS OF PRESSURE FLUCTUATION IN A NEAR BREAKING WAVE WITH THE HYDROSTATIC FLUCTUATION

DELFT HYDRAULICS LABORATORY

FIG. 3
COMPARISON OF MEASURED PRESSURE FLUCTUATIONS IN A WAVE TRANSITIONING FROM INITIAL TO QUASI-STATIONARY BREAKING WITH THE HYDROSTATIC FLUCTUATION

DELFt HYDRAULICS LABORATORY
Comparison of measured pressure fluctuation in a quasi-steady breaker with the hydrostatic fluctuation.
COMPARISON OF MEASURED PRESSURE FLUCTUATION IN A WAVE TRANSITIONING FROM INITIAL TO QUASISTATIONARY BREAKING WITH THE HYDROSTATIC FLUCTUATION

DELF'T HYDRAULICS LABORATORY
Comparison of measured pressure fluctuations in a quasi-steady breaking wave with the hydrostatic fluctuation.

DELFt HYDRAULICS LABORATORY

FIG. 8
COMPARISON OF MEASURED PRESSURE FLUCTUATION IN A QUASI-STABLE BREAKING WAVE WITH THE HYDROSTATIC FLUCTUATION

WATERLOOKUNDIG LABORATORIUM
Comparison of measured pressure fluctuation in a quasi-steady breaking wave with the hydrostatic fluctuation.
Mean values of crest, trough, and crest to trough height of periodic dynamic pressure fluctuation for

Test 1

A = 300 cm, T = 1.79 s
RC 403012.11.78
M. S. & S. Fig. 11