HYBRID MODELS OF RANDOM UTILITY MAXIMIZATION AND RANDOM REGRET MINIMIZATION: RESULTS FROM TWO EMPIRICAL STUDIES

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ABSTRACT

This paper shows how the co-existence of different decision rules can be accommodated in hybrid discrete choice-models. Specifically, the paper presents a generic hybrid model specification that allows for some attributes being processed using conventional linear-additive utility-maximization based rules, while others are being processed using regret-minimization based rules. We show that on two revealed and stated choice datasets hybrid models outperform – in terms of model fit – choice models where all attributes are assumed to be processed by means of one and the same decision rule. Implications, in terms of marginal Willingness-to-Pay measures (WtP), are derived for the different hybrid model-specifications and applied in the context of the two datasets. It is found that in the context of our data hybrid WtP-measures differ substantially from conventional utility-based WtP-measures, and that the hybrid WtP-specifications allow for a richer, because choice set-specific, interpretation of the trade-offs that people make.
1. INTRODUCTION

In transportation as well as in urban planning, the interest in discrete choice-models that provide behavioural alternatives to conventional Random Utility Maximization-theory (RUM) is increasing (e.g., [1], [2], [3], [4]). One recently proposed alternative is that of Random Regret Minimization [5]. The RRM-approach to discrete choice modelling provides a logit-based account of multi-attribute multinomial decision-making, which allows for capturing semi-compensatory choice-behaviour and choice set-composition effects. RRM’s distinguishing features are its formal tractability, parsimony and user-friendliness – the model can be estimated using conventional discrete choice-software packages.

Generally, the first and foremost question to be answered when introducing a behavioural alternative for RUM-theory, is how it performs empirically when compared to RUM’s workhorses, the MNL model2 ([6], [7], [8]). Usually this is done by estimating the two competing model paradigms on the same data, and discussing differences in model fit and estimation outcomes. In the context of the RRM-model a series of such comparisons have highlighted a promising empirical performance on a number of datasets, ranging from mode choices, car-type choices and parking lot choices to nature park visits and transport policy decisions (see [9] for a review of recent empirical evidence).

However, it is increasingly being recognized that it may be naïve to perform such generic comparisons: recent studies have highlighted the importance of allowing for heterogeneity in terms of decision-making strategies (e.g., [4], [10]). In the context of RRM, this has led to modelling approaches which allow for different segments of the population to choose in a way more in line with RUM-premises, while others may choose in a way more in line with RRM-premises (e.g., [11], [12]). However, notwithstanding the obvious value of these approaches to capture heterogeneity, they only capture one or two dimensions of heterogeneity (across decision-makers and/or decision-contexts).

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2 More specifically, RUM’s ‘workhorse’ is the linear-additive form of the MNL-model. With linear-additive we mean to say that the systematic (or ‘observed’) part of the utility is a linear-additive function of tastes (estimable parameters) and attributes. In the remainder of this paper we will omit, for brevity and ease of communication, the term ‘linear-additive’ when discussing RUM-models, although this is in fact the type of RUM-model we refer to.
This paper aims to contribute to the above-described literature by studying heterogeneity in decision-making along another dimension: that of the attributes. We propose that while some attributes may be processed using one type of decision-rule (e.g., utility-maximization), other attributes may be processed using another rule (e.g., regret-minimization). More specifically, we aim to present the following contributions to the literature: first, the conceptual idea, put forward in this paper, that different attributes may be processed in different ways (i.e., using different decision rules) and that this can be effectively modelled in a hybrid discrete choice framework is – to the best of our knowledge – new. Second, we present an empirical proof-of-concept by means of estimating hybrid choice model-specifications on two revealed and stated choice datasets (concerning shopping destination choices and route choices). Estimation results are discussed and used as input for out-of-sample validation exercises. Third, we derive Willingness-to-Pay (WtP) formulations for the different specifications of our hybrid choice model. Based on model estimation results we compute, in the context of our datasets, WtP-measures for the relevant hybrid model forms. Furthermore, we show how the interpretation of these hybrid-WtPs allows for new insights in the ways in which choice set-composition affects attribute trade-offs made by decision-makers.

Section 2 presents the RUM- and RRM-based MNL-models, introduces a hybrid RUM-RRM model formulation, and derives relevant WtP-measures. Section 3 presents the data and empirical analyses (including model estimation, out-of-sample validation, and computation and interpretation of hybrid WtP-measures). Section 4 provides conclusions and points at avenues for further research.

2. RANDOM UTILITY MAXIMIZATION, RANDOM REGRET MINIMIZATION AND A HYBRID ALTERNATIVE

2.1. Random Utility Maximization

Assume the following choice situation: a decision-maker faces a set of $J$ alternatives, each being described in terms of $M$ attributes that are comparable across alternatives. The focus is on predicting the choice probability for an alternative $i$ from this set. A conventional, linear-additive utilitarian specification would assign the following deterministic utility to alternative
\( V_i = \sum_{m=1}^{M} \beta_m x_{im} \). Adopting the classical RUM paradigm (that is: adding i.i.d. Extreme Value Type I-distributed errors to the deterministic utilities of all alternatives to represent heterogeneity in unobserved utility) implies the following MNL formulation of the resulting choice probability [6]:

\[
P_i = \frac{\exp(V_i)}{\sum_{j=1}^{J} \exp(V_j)}.
\]

### 2.2. Random Regret Minimization

The RRM-based model postulates that, when choosing between alternatives, decision-makers aim to minimize anticipated random regret, and that the level of anticipated random regret that is associated with the considered alternative \( i \) is composed of an i.i.d. random error \( \epsilon_i \), which represents unobserved heterogeneity in regret and whose negative is Extreme Value Type I-distributed, and a systematic regret \( R_i \). Systematic regret is in turn conceived to be the sum of all so-called binary regrets that are associated with bilaterally comparing the considered alternative with each of the other alternatives in the choice set. The level of binary regret associated with comparing the considered alternative with another alternative \( j \) is conceived to be the sum of the regrets that are associated with comparing the two alternatives in terms of each of their \( M \) attributes. This attribute level-regret in turn is formulated as follows:

\[
R_{ij}^m = \ln \left(1 + \exp \left[ \beta_m \cdot (x_{jm} - x_{im}) \right] \right).
\]

This formulation implies that regret is close to zero when alternative \( j \) performs (much) worse than \( i \) in terms of attribute \( m \), and that it grows as an approximately linear function of the difference in attribute-values in case \( i \) performs worse than \( j \) in terms of attribute \( m \). In that case, the estimable parameter \( \beta_m \) (for which also the sign is estimated) gives the approximation of the slope of the regret-function for attribute \( m \).

Systematic regret can then be written as:

\[
R_i = \sum_{j \neq i} \sum_{m=1}^{M} \ln \left(1 + \exp \left[ \beta_m \cdot (x_{jm} - x_{im}) \right] \right).
\]

Acknowledging that minimization of random regret is mathematically equivalent to maximizing the negative of random regret, choice probabilities may be derived using a variant of the well-known multinomial logit-formulation: the choice probability associated with alternative \( i \) equals

\[
P_i = \frac{\exp(-R_i)}{\sum_{j=1}^{J} \exp(-R_j)}.
\]
Although the two modelling approaches (RUM and RRM) have important similarities (they are equally parsimonious and both models result in closed form logit-type choice probabilities), there are important differences between them: as discussed in-depth in [9], an important difference between the two approaches is that the RRM-based MNL-model does not exhibit the IIA-property even while errors are i.i.d. That is: the attractiveness of a particular alternative is a function of its own performance, as well as the performance of competing alternatives relative to the considered alternative and one another.

Furthermore, RRM implies semi-compensatory behaviour and choice set-specific preferences: the extent to which a deterioration of one attribute can be compensated by improving another attribute depends on the position of the alternative relative to the other alternatives in the set in terms of these two attributes. Finally, note that the interpretation of parameters differs between RUM- and RRM-model specifications (although parameters should have the same sign in both models): whereas in a RUM-framework, parameters reflect the added utility of a unit increase in an attribute’s value, parameters in a RRM-setting parameters reflect the upper bound of the extent to which a unit in- or decrease in performance on an attribute influences the level of regret that is associated with a comparison with another alternative. Whether or not this upper bound is reached for a one unit in- or decrease in an attribute’s value depends on the relative performance of the two alternatives that are compared in terms of the attribute.

2.2. A hybrid model form and Willingness-To-Pay-formulations

The combination of the RUM- and RRM-approaches is relatively straightforward. Assume, without loss of general applicability, that the first \( q \) attributes are being processed by means of a linear-additive utility-maximization rule, while the remaining \( M - q \) attributes are being processed in terms of a regret-minimization rule. This results in a Random Modified Utility form for alternative \( i \) (where the random error is i.i.d. Extreme Value Type I-distributed):

\[
RMU_i = MU_i + \epsilon_i = \sum_{m=1}^{q} \beta_m \cdot x_{im} - \sum_{m=q+1}^{M} \sum_{j \neq i} \ln \left(1 + \exp \left[\beta_m \cdot (x_{jm} - x_{im})\right]\right) + \epsilon_i.
\]

Maximization of this modified utility function across alternatives implies that choice probabilities are given by

\[
P_i = \exp(MU_i) / \sum_{j=1}^{J} \exp(MU_j).
\]

Note that we have at this point no expectations concerning which attributes are likely to be processed by means of an RRM-rule, and which are likely to be processed by means of a RUM-rule.
It may be noted that there is no a priori reason to assume that the units of the regret-part and the utility-part of the modified utility function are directly comparable. By estimating a weight for either the regret-part or the utility-part this can be tested empirically in the context of particular datasets. On the two datasets we use for empirical analyses (see section 3), the estimated weight did not in a statistical sense differ from 1 at any reasonable level of significance. As such, at least in the context of the datasets used in this paper, it appears that the units of the regret- and utility-parts of the modified utility function are comparable.

Note that the above hybrid formulation has non-trivial implications for the derivation of measures of marginal Willingness-To-Pay (WtP). Basically, four different situations are possible, each with an associated formulation of WtP. Assume that the $r$-th attribute is the cost-attribute, which has of course a negative parameter (in both the RUM- and RRM-formulations), and that the $t$-th attribute represents the attribute for which the marginal WtP for a one-unit increase is being studied. A first scenario is that where both attributes are being processed in a utility-based fashion. In that case, conventional WtP-measures apply:

$$WTP_{RUM} = -\frac{\partial MU_i}{\partial x_t} \left/ \frac{\partial MU_i}{\partial x_r} \right. = \frac{\beta_i}{\beta_r}$$

(1.a)

Another possible scenario is that where the cost-attribute is being processed in a utility-based fashion, while the $t$-th attribute is being processed in a regret-based fashion. In that case, the following WtP-measure can be derived:

$$WTP_{RUM} = -\frac{\partial MU_i}{\partial x_t} \left/ \frac{\partial MU_i}{\partial x_r} \right. = \frac{\sum_{j \neq i} \left( -\frac{\beta_j}{1 + \exp \left[ \beta_j \cdot (x_j - x_t) \right]} \right)}{\beta_i}$$

(1.b)

A third scenario arises when the $t$-th attribute is being processed in a utility-based fashion, while the cost attribute is being processed in a regret-based fashion. In that case, the following WtP-measure applies:
Finally, the situation may occur where both attributes are being processed in a regret-based fashion, which results in the following formulation for marginal WtP (see also [9]):

\[
WTP_{RUM}^{RRM} = -\frac{\partial MU_i}{\partial x_{it}} \cdot \frac{\partial MU_i}{\partial x_{jt}} = \frac{\beta_i}{\sum_{j\neq i} \left( \frac{-\beta_j}{1 + \exp[\beta_j \cdot (x_{jt} - x_{it})]} \right)}
\]  

(1.c)

A crucial difference between on the one hand the situation where both attributes are evaluated in a utility-based manner (Eq. 1.a) and on the other hand the situations where one or both are evaluated in a regret-based manner (Eqs. 1.b-d), is that the latter situations result in choice set-specific measures of WtP. That is, the attribute-values of all available alternatives enter the RRM-based WtP-equation, which implies that RRM-based WtP-measures will generally change when the attributes of alternatives that compete with a considered alternative change. This is of course fully in line with the notion, highlighted above, that the RRM-model implies choice set-specific preferences. Note that inspection of the derivatives of these WtP-measures (not presented here for reasons of space limitations) shows that they are monotonic in terms of the values of relevant attributes.

Importantly, we at this point do not wish to argue that any of these measures of WtP should be *a priori* preferred over other measures. Instead, it is our viewpoint that this is largely an empirical question to be answered in the context of the data at hand, in the sense that WtP-measures belonging to model-specifications that perform relatively well on the data (in terms of model fit and/or predictive ability) may have an edge over other measures *in the context of*
those data. However, as we will highlight using empirical examples below, we do find that the hybrid WtP-measures allow for additional insights into the ways in which choice set-composition affects the trade-offs people make. See further below for empirical applications of the WtP-measures presented in this section.

3. TWO EMPIRICAL APPLICATIONS

The RUM-, RRM- and hybrid RUM-RRM modelling approaches are tested on two datasets as follows: first, models are estimated that assume full RUM- and full RRM-behaviour. Then, the one with the best model fit is taken, and for each of the attributes it is analysed and recorded whether a change in decision strategy for that particular attribute from RUM to RRM or vice versa leads to an improvement in model fit. If for only one attribute the decision strategy results in an increase in fit, the search is ended and the resulting model is kept for further analyses. This situation occurred for both the datasets discussed below. Note that this strategy of analysis implies the underlying assumption that attribute processing strategies are selected by decision-makers for each attribute in isolation, and independently of the process selected for other attributes. In other words, it is assumed that there are no interaction-effects between different attributes in terms of the adopted evaluation rule. After having compared models in terms of fit and predictive ability, relevant measures of Willingness to Pay are computed and compared as well in the context of fictive examples drawn in the context of the collected data, as well as in the context of the data themselves.

3.1. Revealed shopping destination choices: Estimation and validation

A dataset was collected concerning shopping centre-choices [13], containing 1503 revealed choices from consumers that went shopping with the aim of buying groceries. Variables included were FLOOR_GROCERIES (m\(^2\) floorspace concerning groceries in a particular centre), FLOOR_OTHER (m\(^2\) floorspace concerning other items) and TRAVEL_TIME (seconds, taking into account the used travel mode). Given the very large number of available

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3 For reasons of space limitations, and in line with the scope of this paper, details about the data collection efforts are left out to the extent possible. The reader is referred to [13] and [14] for details. Models are estimated using BIOGEME ([15], [16]); robust t-values are reported throughout this paper.
stores in the study-area (Noord-Brabant province, The Netherlands), choice sets were imputed: next to the chosen alternative, four other stores were selected based on shortest travel time calculations\(^4\). Table 1 shows estimation results for RUM- and RRM-models, and the best-fitting hybrid model (\(U\) stands for a utility-based evaluation of the attribute, and \(R\) for a regret-based evaluation).

Table 1: A comparison between RUM, RRM and hybrid RUM-RRM (shopping destination choices)

<table>
<thead>
<tr>
<th></th>
<th>RUM</th>
<th>RRM</th>
<th>RUM-RRM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>beta</td>
<td>t-val.</td>
<td>beta</td>
</tr>
<tr>
<td>FLOOR_GROCERIES</td>
<td>0.000106</td>
<td>5.71</td>
<td>0.000068</td>
</tr>
<tr>
<td>FLOOR_OTHER</td>
<td>0.000011</td>
<td>4.05</td>
<td>0.0000029</td>
</tr>
<tr>
<td>TRAVEL_TIME</td>
<td>-0.000448</td>
<td>-6.47</td>
<td>-0.000155</td>
</tr>
<tr>
<td>Nr. of observations</td>
<td>1503</td>
<td></td>
<td>1503</td>
</tr>
<tr>
<td>Null-LL</td>
<td>-2419</td>
<td></td>
<td>-2419</td>
</tr>
<tr>
<td>Final-LL</td>
<td>-2305</td>
<td></td>
<td>-2301</td>
</tr>
</tbody>
</table>

A few observations can be made directly: parameters are significant and of the expected sign. The full RRM-model provides a small improvement in fit when compared to the full RUM-model. This difference, when put to the Ben-Akiva & Swait test for nonnested models [17], is significant at a 1%-level. However, a further improvement in fit can be achieved by taking the full RRM-model and postulating that travel times are being evaluated by means of a utility-based, rather than a regret-based evaluation rule. Also this difference in fit is small, yet again significant at a 1%-level. Note that in line with expectations the utility-based estimate in the

\(^4\) It may be noted at this point that a priori, RRM’s postulate that preferences are choice-set specific implies that it may be less suitable for the analysis of choices from imputed choice sets like those considered in this subsection. However, it is our viewpoint that this in the end remains an empirical question: to the extent that the RRM-model performs well in terms of model fit and/or predictive ability, it may even be argued that the imputation process has in fact resulted in credible, behaviorally realistic choice sets.
A hybrid model is almost equal to its counterpart in the full RUM-model, and that the same holds for regret-based attributes when compared with the full RRM-model.

3.2. Revealed shopping destination choices: Willingness to Pay-measures

When computing marginal WtP (in terms of travel time) for increases in floorspace (e.g. for groceries), equation 1.b can be applied since the ‘cost’-parameter is utility-based while the floorspace-parameter is regret-based. This implies that marginal WtP varies with the floorspaces of the considered alternative and its competitors (but it does not vary with travel times to the considered shop and its competitors). Before computing sample-averages of WtPs in the context of the RUM-, RRM-, and hybrid specifications, we present a numerical example to see how the hybrid WtP-measure\(^5\) can be interpreted. Consider the following (fictive but realistic) situation: someone faces a choice set containing five shops. Shops 1 to 4 have floorspaces (grocery-related) of 1,000, 2,000, 3,000 and 4,000 square meters respectively (red line in Figure 2), or of 2,000, 3,000, 4,000 and 5,000 square meters respectively (dashed green line in Figure 2). WtP for one extra square meter of floorspace is computed for shop 5, whose current floorspace is varied between 1,000 and 30,000 square meters (which is in line with the range observed in the data). Note that a classical full RUM-specification would imply a marginal WtP for floorspace equalling 0.24 seconds of extra travel time per square meter, irrespective of the relative current size of shop 5. Figure 1 depicts marginal WtP for the hybrid RUM-RRM model.

Clearly, WtP is a function of the current shop size, relative to the other shops: when the shop is currently relatively small because the shop itself is small and/or the competing stores are bigger, there is a large WtP for extra floorspace (larger than that computed with a full RUM-model). As the current shop size increases relative to its competitors, WtP for extra increases decreases up to a point where the hybrid model predicts a lower WtP than the one predicted by the full RUM-model.

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\(^5\) See [9] for discussion of a numerical example of the RRM-based WtP-measure (Equation 1.d).
Figure 1: Marginal WtP for extra floorspace (Hybrid model)

Note that the interpretation of the above WtP-example is in line with the RRM-premises presented in section 2.1: when the shop is currently small, there is a sizeable regret associated with the comparison with the four competing shops when it comes to floorspace. The regret-function depicted in Figure 1 shows that in such a situation, an improvement of the attribute comes with a considerable reduction in regret. Hence the large WtP when the current shop size is small. When the shop is big when compared to its competitors, the regret-function in Figure 1 shows that there is little regret associated with the comparison with other shops in terms of shop size, and that further improvements only lead to very small reductions in regret. Hence the small WtP for extra floorspace when the shop is already large compared to other shops.

It is at this point important to note that any non-linear specification of a RUM-model would lead to WtP-measures that are specific for the current values of relevant attributes of the considered alternative (such as in this case the shop’s floorspace). What is new here, is that – on top of this attribute-value specificity of WtP-measures – hybrid WtP-measures are also sensitive to changes in relevant attribute-values of competing alternatives (i.e., floorspaces of other shops). That is, hybrid WtP-measures depend on the composition of the choice set in terms of relevant attributes. This allows for a richer interpretation of the trade-offs people make as a function of the composition of the choice set they face (as exemplified by the
differences between the solid and dashed lines in Figure 2). This in turn allows for the development of new perspectives for policy-implementation. For example, policy-makers or planners may distinguish between segments of decision-makers not only based on their sociodemographics or tastes for different attributes (as is currently being done), but also based on the choice sets different segments of the population face. Taking into account the specific composition of the choice set, optimal policy- or planning-measures can be developed by capitalizing on the choice-set specific WtP exhibited by decision-makers and captured by hybrid (RUM-RRM) choice models.

To conclude the analysis of WtP-measures relating to shopping destination choices, we compute decision-makers’ average WtP in terms of travel time for extra grocery-related floorspace for the three estimated models (RUM, RRM, and hybrid RUM-RRM). Since RRM- and hybrid measures of WtP are conditional on the considered alternative, these are computed in the context of each observation and subsequently averaged. Table 2 presents results.

Table 2: A comparison between WtP for RUM, RRM and hybrid RUM-RRM (shopping destination choices)

<table>
<thead>
<tr>
<th>Willingness to Pay for extra floorspace (seconds per m²)</th>
<th>RUM</th>
<th>RRM</th>
<th>RUM-RRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.24</td>
<td>0.44</td>
<td>0.30</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.24</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.24</td>
<td>0.55</td>
<td>0.37</td>
</tr>
</tbody>
</table>

When comparing the hybrid WtP with conventional RUM-WtP⁶, a first thing that is noted is that, in line with the example presented in Figure 2, there is heterogeneity across cases (choice situations) in terms of the hybrid WtP-measure: there is a positive standard deviation, and the difference between the minimum (0.13 s/m²) and maximum (0.37 s/m²) WtPs is substantial. As expected, inspection of the sample shows that low values of hybrid WtP are found in cases

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where the floorspace of the chosen shopping mall is (much) larger than that of competing alternatives (the opposite holds of course for higher values of hybrid WtP).

A second element that catches the eye is that on average, the hybrid model’s WtP is higher than its RUM-counterpart. This can be explained by comparing the average floorspace of chosen alternatives with that of non-chosen alternatives in our data: it turns out that only in about one third of cases (more specifically, in 503 out of 1503 cases) did people choose the alternative with the largest grocery-related floorspace in the choice set. Apparently, in many cases these shopping malls performed particularly well in terms of other attributes (non-grocery floorspace and/or travel time). This implies that in many situations, the chosen alternative performs poorly compared to the competition in terms of grocery-floorspace, leading to a relatively large implied WtP for extra floorspace as computed in the context of our hybrid RUM-RRM model.

3.3. Stated route-choices: Estimation and validation

A three alternative, five attribute stated route choice dataset was collected among Australian car-drivers ([14]). The dataset consists of 300 car-drivers that made 16 hypothetical choices each, between their current or reference route and two alternative routes with varying trip attributes: free flow travel time (TRAV_TIME_FF), slowed down travel time (TRAV_TIME_SLOW), travel time variability (TRAV_TIME_VAR), running costs (RUNNING_COSTS) and toll costs (TOLL_COSTS). Time-related attributes are in minutes, costs-related attributes in Australian dollars. Variability in travel time was conceptualized as the difference between the longest and shortest trip time. Besides parameters for these attributes, a constant was estimated for the reference alternative (CONSTANT). Table 2 shows estimation results for RUM- and RRM-models, and the best-fitting hybrid model. Again, parameters are significant and of the expected sign. In line with expectations, congested travel time is valued more negatively than free flow travel time, and toll costs are valued more negatively than running costs. The full RUM-model provides a rather substantial improvement in fit when compared to the full RRM-model. This difference, when put to the Ben-Akiva & Swait test for nonnested models [17], is significant at a 1%-level. However, a further improvement in fit can be achieved by taking the full RUM-model and postulating that toll costs are being evaluated by means of a regret-based, rather than a utility-based evaluation rule. The resulting difference in fit is relatively small, yet significant at a 5%-level. Note again
that in line with expectations utility-based estimates in the hybrid model are almost equal to their counterparts in the full RUM-model, and that the same holds for the regret-based attribute when compared with the full RRM-model.

Table 3: A comparison between RUM, RRM and hybrid RUM-RRM
(route choices)

<table>
<thead>
<tr>
<th>Attribute</th>
<th>RUM</th>
<th>RUM-RRM</th>
<th>U/R</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta t-val.</td>
<td>beta t-val.</td>
<td>beta t-val.</td>
<td></td>
</tr>
<tr>
<td>TRAV_TIME_FF</td>
<td>-0.0574 -17.48</td>
<td>-0.0571 -17.37</td>
<td>U</td>
</tr>
<tr>
<td>TRAV_TIME_SLOW</td>
<td>-0.0742 -25.64</td>
<td>-0.0742 -25.53</td>
<td>U</td>
</tr>
<tr>
<td>TRAV_TIME_VAR</td>
<td>-0.00511 -2.41</td>
<td>-0.00536 -2.55</td>
<td>U</td>
</tr>
<tr>
<td>RUNNING_COSTS</td>
<td>-0.235 -12.68</td>
<td>-0.237 -12.64</td>
<td>U</td>
</tr>
<tr>
<td>TOLL_COSTS</td>
<td>-0.283 -26.53</td>
<td>-0.196 -25.41</td>
<td>R</td>
</tr>
<tr>
<td>CONSTANT</td>
<td>0.0513 0.80</td>
<td>0.0506 0.80</td>
<td>U</td>
</tr>
</tbody>
</table>

| Nr. of observations | 4800       | 4800       | 4800       |
| Null-LL             | -5273      | -5273      | -5273      |
| Final-LL            | -4024      | -4068      | -4022      |

3.4. Stated route-choices: Willingness to Pay-measures

WtP (in terms of toll costs) for travel time reduction (e.g. congested travel time) can now be computed based on equation 1.c, since the cost attribute is evaluated by means of a regret-based decision-rule and the travel time attribute by means of a utility-based decision-rule. A minus-sign is added to reflect that WtP for a reduction in travel time is computed. Note that the full RUM-model would predict a marginal Value of Travel Time Savings of around 12 Australian dollars per hour. As highlighted in section 2, the estimated hybrid model implies that WtP varies with the current level of toll (but not with the current congestion level). Take the following fictive situation to see how this plays out: route A is a toll-free road, while route B comes with a toll of 5 Australian dollars (red line in Figure 2) – the dashed green line gives
the situation where route A and B come with a toll of 1 and 6 Australian dollars, respectively. WtP is computed for various levels of current toll for route C (Figure 2).

As expected, WtP appears to be a function of the current toll level, relative to that of the other routes: when route C is currently relatively cheap because its own toll-level is low and/or that of the competition is high, there is a large WtP (larger than that computed with a full RUM-model). As the route’s toll level increases relative to that of its competitors, WtP decreases up to a point where the hybrid model predicts a lower WtP than that predicted by the full RUM-model. This finding is again in line with the RRM-premises presented in section 2.1: when the current toll level is low, there is only a very small regret associated with the comparison with the two competing routes when it comes to toll levels. The regret-function presented in section 2 shows that in such a situation, an increase in toll comes with a modest increase in regret – hence the large WtP when the current route is cheap. When the route is already expensive when compared to its competitors, the regret-function postulates that a further increase in the toll level will lead to a large increase in regret. Hence the small WtP when the route’s current toll level is already high compared to that of the other two routes.

![Figure 2: Marginal WtP for Travel Time Savings (Hybrid model)](image-url)
To conclude the analysis of WtP -measures, we compute decision-makers’ average WtP for the three estimated models (RUM, RRM, and hybrid RUM-RRM). Again, since RRM- and hybrid measures of WtP are conditional on the considered alternative, these are computed in the context of the alternative that was chosen by each specific traveller. Table 4 presents results.

Again, the hybrid model-based WtP vary across cases (choice situations): there is a positive standard deviation, and the difference between the minimum and maximum WtPs is very substantial. As expected, inspection of the sample shows that low values of hybrid WtP are found in cases where the current toll level is (much) higher than that of competing routes (the opposite holds for high values of hybrid WtP). For example, the highest WtP is found in cases where the chosen route had a toll level of zero dollars, while competing routes had tolls of 10 and 16 AUS. dollars, respectively. The lowest WtP is found in cases where the chosen route had a toll level of 11 dollars, while competing routes had tolls of zero and 7 AUS. dollars, respectively.

| Table 4: A comparison between WtP for RUM, RRM and hybrid RUM-RRM (route choices) |
|---------------------------------|-----|-----|-----|
| **WtP (Aus. dollars / minute)** | RUM | RRM | RUM-RRM |
| Mean                           | 0.20 | 0.23 | 0.36 |
| Standard deviation             | 0    | 0.10 | 0.14 |
| Minimum                        | 0.20 | 0.03 | 0.18 |
| Maximum                        | 0.20 | 1.34 | 1.76 |

Also here, it appears that on average, hybrid WtP is higher than its RUM-counterpart. This too can be explained by comparing the average toll level of chosen alternatives with that of non-chosen alternatives: it turns out that in well over one half of cases (more specifically, in 53% of cases) did people choose the alternative with the lowest toll level in the choice set. Apparently, in these cases competing routes did not perform well enough in terms of other attributes (such as travel time and travel time variability) to offset their relatively poor performance on the toll attribute. This implies that in many situations, the chosen alternative
performs better than the competition in terms of toll levels, leading to a relatively large implied WtP in the context of our hybrid RUM-RRM model.

4. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This paper studied the co-existence of different decision rules for different attributes in discrete choice-models. Specifically, the paper presented a hybrid model framework that allows for some attributes being processed using conventional utility-maximization based rules, while others are processed using regret-minimization based rules. An empirical proof-of-concept was provided for particular specifications of hybrid RUM-RRM models. These empirical analyses based on revealed shopping choice data and stated route choice data suggest that on these data indeed, allowing for heterogeneity in terms of processing rules across different attributes leads to statistically significant improvements in model fit. More precisely, when a full regret-based model achieved a higher fit and better predictive ability than a full utility-based model it paid off in terms of model fit to allow for one attribute to be processed in a utilitarian way, and vice versa. It should be noted once more that these differences between models in terms of fit and predictive ability are very small on our data.

Furthermore, willingness to pay-measures for the resulting hybrid choice models are derived. When one or more of the attributes is processed in a regret-based fashion, this implies that associated WtP-measures become choice set specific for that attribute, in a way that is in line with the behavioural premises underlying RRM-theory. More precisely, presented hybrid WtP-measures are sensitive to changes in relevant attribute-values of competing alternatives. As such, hybrid WtP-measures depend on the composition of the choice set in terms of relevant attributes. We argued (and illustrated using numerical examples in the context of our data) that this allows for a richer interpretation of the trade-offs people make as a function of the composition of the choice set they face. As stated earlier, this choice set-specificity of WtP-measures allows policy-makers or planners to distinguish not only between segments of decision-makers based on their sociodemographics or preferences for different, but also based on the choice sets different segments face. Taking into account the specific composition of the choice set, optimal policy- or planning-measures can be developed by capitalizing on the choice-set specific WtP exhibited by decision-makers and captured by hybrid (RUM-RRM) choice models.
Another potential avenue for putting to use RRM and hybrid RRM/RUM models with the aim of arriving at more informed policy-making refers to forecasting studies. The idea here would be to simultaneously employ multiple choice model types (RUM and one or more RRM or hybrid models) for the analysis and prediction of travel behavior. The outcomes of the different models (in terms of, for example, willingness to pay measures, elasticities, and/or market share forecasts) may then be used to obtain what may be called ‘behavioral confidence intervals’ and/or to perform what may be called ‘behavioral sensitivity analyses’. That is, to the extent that different model types generate different outcomes, it makes sense to consider each model type (and associated outcome) a possible scenario, in a roughly similar way as one would work with different scenarios concerning, for example, demographic developments. Confronted with these different behavioral scenarios, policy-makers and planners may then apply conventional techniques for dealing with multiple scenarios with the aim of developing ‘behaviorally robust’ policies – i.e., policies that are likely to turn out effective, irrespective of which behavioral scenario (or: which underlying travel choice model) in the end turns out to be the most correct one. This approach seems to be particularly appealing when, as is the case in the context of datasets reported in this paper, difference between models in terms of goodness of fit are (very) small.

A number of potentially fruitful avenues for further research can be identified: a first and very obvious one is that it seems worthwhile to test whether obtained results hold on other datasets. More specifically, it would be interesting to study whether regularities can be discovered in terms of which attributes are most likely to be processed using a particular evaluation rule. Furthermore, it seems natural to broaden the analysis towards other RUM- and non-RUM-choice strategies, for example including reference-dependent processing strategies ([18]) or Elimination-by-Aspects heuristics ([19]). In addition, an integration of the approach presented in this paper with models of attribute (non-)attendance seems promising. At least theoretically, it should be possible to identify simultaneously whether a particular attribute is being processed and if so, using what kind of evaluation rule. [20] provides an example of how this combined heterogeneity may be captured in the context of reference-dependent choice-behaviour. Finally, it seems worthwhile to combine heterogeneity in evaluation rules across attributes with (latent class) models that aim to identify heterogeneity across decision-makers.
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