A NEW APPROACH FOR
LONG TERM MONITORING OF DEFORMATIONS
BY DIFFERENTIAL SAR INTERFEROMETRY
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PROEFSCHRIFT

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof. ir. K.F.Wakker,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen
op maandag 18 juni 2001 te 13:30 uur

door

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This research has been partially funded by TNO-FEL.

Published and distributed by: DUP Science
DUP Science is an imprint of
Delft University Press
P.O.Box 98
2600MG Delft
The Netherlands
Telephone: +31 15 2785121
Telefax: +31 15 1781661
E-mail: DUP@Library.TUDelft.NL
ISBN 90-407-2189-0

Keywords: SAR Interferometry, deformations, temporal decorrelation

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Printed in the Netherlands.
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Introduction

Synthetic Aperture Radar (SAR) Interferometry, notwithstanding its youth, has already been applied successfully in different research fields, from topographic mapping to earthquake studies, from the monitoring of deformations to volcanology. With this technique it is possible to measure both terrain height and terrain deformations by computing the phase difference between two SAR images of the same area. When the two images are taken simultaneously but under a slightly different viewing angle, this phase difference is directly related to the height of the imaged terrain. In this way digital elevation models can be produced. Alternatively, if the images are taken exactly from the same sensor position at two different times the same phase difference image is related no longer to the height, but to the height changes occurred in the meantime. This configuration permits therefore to map terrain deformations.

There are two main characteristics which make this technique very attractive to the scientific community, and in some sense even competitive with respect to other widely used deformation measurement techniques like levelling or Global Positioning System (GPS). The first one is that it uses differences of SAR images, and since the result is also an image, it gives a two-dimensional representation of the topography or deformation of the imaged terrain. This offers a completely new possibility for deformation studies, as techniques like GPS and levelling, although very precise, can give only point-wise information. Additionally, a SAR image has usually an extension of some tens of kilometres in both image directions and a spatial resolution of few- to few tens of metres per pixel, even for a spaceborne system. In other words, not only SAR Interferometry (INSAR) offers real ‘pictures’ of the to-
The second, but probably most important factor which makes the INSAR technique so attractive is the high accuracy of the phase differences which can be measured. The SAR systems use wavelength of the order of centimetres. The most commonly used bands are the X-band, which has a wavelength $\lambda$ of 3 cm, the C-band, with $\lambda = 5.6$ cm and the L-band, with $\lambda = 23.5$ cm. Since the measured quantities are phase differences, their accuracy is therefore in the order of one or few millimetres. When INSAR is used for the retrieval of heights, the accuracy of the determined topography is influenced more than by the accuracy of the phase difference, by the distance between the two imaging positions, and eventually this results in an accuracy of the final topographic model of the order of ten metres. The highest accuracies are however reached when INSAR is applied for the retrieval of deformations. In this case in fact the accuracy is directly related only to the used radar wavelength, and can reach thus the order of few millimetres. The result is then a two-dimensional high resolution image of the ongoing deformations, measured with accuracies of few millimetres and several kilometres wide. It is clear thus that, especially for deformations studies, this technique constitutes an important breakthrough.

### 1.1 The ERS satellites

Maybe the best examples of deformation measurements with SAR Interferometry are those obtained with SAR data from the ERS (ERS-1 and ERS-2) satellite systems. The first satellite, ERS-1, was launched in 1991, and has been decommissioned in 2000 after an operational life three times longer than the planned life of three years. Its twin satellite, ERS-2, launched in 1996, is still operative. The satellites are placed on the same 35-days repeating orbit, with ERS-2 following ERS-1 at a one-day distance. When both satellites were operational this configuration permitted to have the same area imaged at time intervals of one, 35 days and multiples of 35 ± 1 days. Combinations of images on short intervals, say of 1 or 35 days, have been used to measure deformations from sudden events such as earthquakes, volcanic eruptions and flooding, while larger intervals, covering more 35-days cycles, have been considered for the monitoring of slower events, such as deformations after major events and land subsidence. In now almost ten years of activity, the ERS satellites have provided a huge database of SAR images of a large portion of the Earth. Thanks also to its wide accessibility from the scientific community, this database is perhaps the most used for the development of the inter-
ferometric technique itself and for its application for both the topographic mapping and the study of a wide range of deformation phenomena.

1.2 The problem of temporal decorrelation

Every technique has its limitations, and for SAR Interferometry the major limitation is decorrelation. Decorrelation in interferometry can be defined as the loss of information due to differences in the backscattered signal between the two considered images. If the two images have different signatures, their interferometric combination is meaningless and gives as a result a pure-noise image of the area. A change in the backscattering and the consequent decorrelation can have different causes. It can be intrinsic, e.g. it can be caused by system noise or by processing errors, which can occur both in the elaboration of the signal to a SAR image and in the subsequent procedure of combining interferometrically the two SAR images. The greatest contribution to decorrelation comes however from the external sources, or in other words, from a real difference in the backscattering signal from the terrain. The backscattering changes for example when the images are taken under a different viewing geometry, as it is done on purpose in the topographic application of INSAR. It is intuitive that this kind of decorrelation increases as the viewing geometry becomes more and more different, and since this happens when the baseline, i.e. the distance between the two sensor position, increases, it is called baseline decorrelation. When two images are not taken simultaneously but at a certain time interval from each other, as it is the case when INSAR is applied for deformation studies, and the viewing geometry remains the same, baseline decorrelation does not occur. Still, another source of decorrelation can manifest itself, called temporal decorrelation.

When two SAR images taken at different times are combined, one has to assume that the area they represent has not changed in the meantime. The backscattering characteristics of a terrain are in fact dependent on several factors, such as for example the terrain composition, moisture, roughness, etc. and if any of these factors is different in the two images, then also the reflection from the terrain will be different. Depending on the amount of change of this backscattering, the interferometric combination will be more or less decorrelated. Temporal decorrelation affects different kind of land coverage on different time scales, depending on how fast they are likely to change (some of) their properties. For example, water decorrelates in fractions of a second, due to its fast surface movements, while vegetated areas decorrelate in few days, due to the vegetation growth, and fields usually in a few months or less, depending on the farming activities. The most slowly decorrelating terrains
are deserts in non windy conditions: for these areas, interferograms can be obtained also on time scales of one or more years.

In general, however, most areas are almost completely decorrelated after two or three months. The result is that, in spite of the availability of several years of SAR data (as already mentioned the ERS archive goes back to 1991, so in theory combinations of images over a 10 year time interval would be possible), for the majority of the areas slow deformations, like those caused by land subsidence, which would request long time spans in order to be detected, fall therefore outside the range of applicability of this technique. For the same reasons, long term monitoring of hazardous areas, subject for example to frequent seismic activity, is also only partially feasible.

Note that temporal decorrelation is the major problem when INSAR is applied for deformation measurements, but is nonetheless of importance also for topographic applications. Although the ideal configuration for these application uses two images taken simultaneously, in the real case this is not always possible, and two different imaging times have to be used. To this has also to be added that, in order to correct for different error sources and to achieve a better accuracy in the topographic mapping, often more than two SAR images of the studied area are combined interferometrically, and this also forces to the use of temporally spaced images.

1.3 Objective of this thesis

If the research objective of this thesis would have to be formulated in one single question, then the question could be: "Is it possible to extend the applicability of SAR Interferometry over time scales of years despite the presence of temporal decorrelation?". As this question and the title itself of this thesis suggest, the main research issue is temporal decorrelation. This is the most important source of error when INSAR is applied for deformation studies and, as it will be shown in this thesis, especially over time intervals of some years its effects can be so heavy, that the interferometric combination itself of the images is almost impossible. In such cases it becomes clear that in order to compensate for the loss of information in a single combination of images, one has to generate as much interferometric information as possible by combining several images. This led to the use of a database approach, developed for the particular case of long term deformation analysis. The database was generated for two test sites, in the North of the Netherlands and in the Neapolitan area in Italy. The two areas are very different from each other from the geomorphological and climatic point of view and for this reason they are also af-
fected in a different way by processing errors and temporal decorrelation. Moreover, their deformation sources, namely the extraction of natural gas in the Netherlands and volcanism in Italy, cause subsidence at different rates. As a consequence, the generated database was used differently in the two cases to retrieve the deformations.

1.4 Outline of this thesis

This first chapter concludes here with an outline of the structure of this thesis. After an explanation of the basic concepts of Radar and of the SAR interferometric technique in Chapter 2, the concept of decorrelation is introduced in Chapter 3 where particular emphasis is given to its statistical significance and properties. The last part of the same chapter focuses then on the effects of temporal decorrelation on long term interferograms. In Chapter 4 is explained the approach used for the generation of the interferometric database for both the sites under study. This general explanation is followed by an overview of the characteristics of each site and of its deformation history and by more detailed information on the specific interferometric database. Chapters 5 and 6 are dedicated to the long term deformation analysis of the two areas. Two different methods for the retrieval of deformations have been developed, and their explanation is followed by the results obtained with their application. The first method is intended for the case of almost total decorrelation, and it is based on the use of interferometric information from very small, highly coherent features mostly of anthropogenic nature, like houses and roads. The second method can be applied in less serious cases of decorrelation and consists in a least squares inversion of the database to describe the temporal evolution of the deformations. Since the analysis of anthropogenic features illustrated in Chapter 5 revealed remarkable interferometric properties of these objects, it seemed interesting to assess the repeatability of the results by performing the same analysis on the second test site. The results of this second analysis on anthropogenic objects are presented in Chapter 7. Finally, the last chapter of this thesis, Chapter 8, reports the general results and conclusions of the presented research, with some suggestions for future work.
In Section 2.1, the basic principles of Radar and Synthetic Aperture Radar (SAR) will be explained. Only the information which is strictly necessary for a good comprehension of the interferometric technique will be given. For a more extended explanation of Radar, and in particular of SAR, the reader can refer for example to [12, 16, 22, 47]. Section 2.2 is devoted to SAR Interferometry. After introducing the interferometric principle (Section 2.2.1) and the processing which leads to the interferometric products (Section 2.2.2), the possible applications of the interferometric technique are explained (Section 2.2.3). Section 2.3 is then focused on the particular application which is the topic of this thesis, i.e. *Differential INSAR* for the retrieval of terrain deformations.

### 2.1 Radar and Synthetic Aperture Radar

#### 2.1.1 Radar

A radar is an active system, e.g. provides its own illumination by transmitting radiation. It is therefore not dependent on external radiation sources, such as the sun, and can work also in the night. The emitted radiation is in the microwave region of the electro-magnetic spectrum, with wavelengths of the order of one to few tens of centimetres. With these short wavelengths imaging is possible also in the presence of clouds, fog or precipitation, thus the radar, unlike for example optical
sensors, can work under all weather conditions. Fig. 2.1 depicts the configuration of a side-looking real aperture radar. The antenna is mounted on a platform (aircraft or satellite) moving with a certain velocity $V$ with respect to the Earth at a constant altitude; the flight direction is also called azimuth direction. From a certain position along its flight path, or azimuth position, the radar sends a train of pulses in the direction, called range direction, perpendicular to the flight path and with an inclination, or look angle, $\theta$ with respect to the vertical. The transmitted pulses are partially absorbed by the ground, partially reflected in all directions, the scattering depending on the surface physical properties, such as roughness and slope, but also on its electrical properties, such as the dielectric constant, which varies with temperature, humidity, etc. A certain amount of the scattered radiation is reflected back to the antenna, which detects the echoes and registers the corresponding values of intensity and phase, and the round-trip time which, since the pulses travel at the speed of light, is straightforwardly related to the range distance of the antenna from...
the imaged area. These echoes can be separated from each other by means of their
different return time, and can be therefore ordered as an array along the range direc-
tion. The motion of the sensor along its flight path causes then the illuminated area
to shift in the azimuth direction. The result is then a two-dimensional matrix of the
returned echoes, called a raw data matrix.

A radar system is characterised by its resolution in the range and azimuth
directions. A difference in range distance $\Delta r$ corresponds to a time difference
$\Delta t = 2\Delta r/c$, where $c$ is the speed of light. Since the smallest detectable time
difference is the pulse length, $\tau$ in Fig.2.1, then the smallest range distance which
can be resolved is:

$$R_r = \frac{c \tau}{2} \quad (2.1)$$

This is the resolution in slant range, i.e. in the line-of-sight direction. Usually it is
preferred to refer to the ground range resolution, which is defined as the minimal
distance between two points on the ground which can be resolved. This resolution
is the projection on the ground of the slant range resolution:

$$R_{gr} = \frac{c \tau}{2 \sin \theta} \quad (2.2)$$

For the satellite ERS-1 the ground range resolution is about 25 m.

As for the azimuth resolution, it is determined by the fact that all the points
illuminated at the same instant and lying at the same range distance cannot be dis-
tinguished from each other. Therefore, two points can be discriminated along the
azimuth direction only if they are not illuminated simultaneously or, in other words,
if they are separated by a distance at least equal to the footprint length. The azimuth
resolution is thus equal to the footprint length:

$$R_a = \frac{\lambda}{L_a \cos \theta} \frac{h}{r} = \frac{\lambda}{L_a r} \quad (2.3)$$

where $\lambda/L_a$ is the antenna beam width in the azimuth direction, $\lambda$ is the wavelength,
$L_a$ the antenna length and $r$ the distance of the sensor from the surface. From
Eq.(2.3) it can be seen that the longer the antenna and the smaller the altitude, the
higher the resolution. By using this equation one obtains for example for ERS-1
an azimuth resolution of about 4.8 km, thus much poorer than the one along the
range direction. In order to achieve acceptable values very large antennas would be
required: for example, for ERS-1 an azimuth resolution of the same order of the
range one, i.e. 25 m, could be achieved only with an antenna having a length of
about 440 m, which is clearly technically not feasible. It is exactly the poorness of
the achievable azimuth resolutions in real aperture radar which leads to the concept
of synthetic aperture radar.
2.1.2 Synthetic aperture radar (SAR)

The concept of synthetic aperture radar is based on the observation that a given target is imaged in a certain number of echoes during the passage of the sensor [7]. This is illustrated in Fig.2.2, where $Q$ is the target, while $P_1$ and $P_2$ are the radar positions when $Q$ is imaged for the first and for the last time respectively. The echoes received from $Q$ as the sensor moves between $P_1$ and $P_2$ have undergone different Doppler shifts due to the sensor movement [20]. These Doppler shifts can be used to discriminate the different echoes from $Q$ and to combine them into one single image, with the same resolution which would be given by an antenna with length $L_a = (P_2 - P_1)$. The resulting maximal azimuth resolution achievable with such an antenna length is then, according to Eq.(2.3), equal to:

$$ R_a = \frac{\lambda}{L_a} t $$  \hspace{1cm} (2.4)
2.1 Radar and Synthetic Aperture Radar

Figure 2.3  ERS images of the Vesuvius (Italy): on the left side is shown the intensity image, on the right the interferometric phase.

and since the synthesised antenna length \( L_s \), equal to the footprint on the azimuth direction, can be expressed as [16, 36]:

\[
L_s = \frac{2\lambda r}{L_a}
\]  

(2.5)

with \( L_a \) the real antenna length, then the resolution in azimuth in Eq.(2.4) turns out to be

\[
R_a = \frac{L_a}{2}
\]  

(2.6)

The important result is that the azimuth resolution of a SAR system is independent from the antenna distance from the surface and that a smaller antenna gives a better resolution. The signal processing on the received echoes gives the image of the scene. Fig.2.3(a) is an example of intensity image obtained from actual measurements. The black zones are the high reflective ones: most of the radiation is reflected at an angle equal to the incidence angle and a very small portion of it is returned back. The white areas, on the contrary, are those where a great part of the radiation is scattered back, for example because of a strong surface roughness.

Since the SAR signal is an EM wave, besides the intensity image there is also a phase image. One single SAR phase image however does not give useful information. The phase of an image resolution element, or pixel, is in fact, in most cases, the sum of the values of a large number of single scatterers, none of them dom-
inatating on the others\textsuperscript{1}. Since these scatterers can be assumed to be uncorrelated, due to the central limit theorem their resultant phase can be modelled as a circular Gaussian white noise [30]: the phase has a uniform distribution in the interval \([0, 2\pi]\) and, as a result, a SAR phase image is a random noise image. The phase information becomes meaningful only when two phase images of the same resolution element are compared. The phase value at a certain pixel is in fact determined by two factors, namely by the backscattering properties of all the scatterers within the resolution element on the ground and by the slant range distance of the antenna from the ground. If the backscattering properties of the scatterers remain the same, then the comparison of two phase values relative to the same pixel permits to eliminate their effect. The only contribution to the phase value which is left is then the one related to the difference in the distance scatterer-antenna in the two images and this is the information which is used for SAR Interferometry.

### 2.2 SAR Interferometry

#### 2.2.1 The interferometric principle

A SAR intensity image, like a photographic image, is the projection of a three-dimensional surface, the Earth surface, on a two-dimensional plane. What is missing is the height information: two different points A and B having the same range distance and azimuth position will appear in the same pixel in the image and therefore cannot be distinguished from each other (see Fig.2.4). The third dimension, i.e. the height information, can however be retrieved by considering the phase information and applying the so-called SAR interferometric technique, or \textbf{INSAR}.

The key to the interferometric technique is the measurement of the phase of the radar signal. The radiation generated by a radar antenna is in fact \textbf{coherent}, which means that the emitted EM wave is an harmonic wave: consequently, the amplitude and the exact point in the oscillation, i.e. the phase, can be measured in the returned echo. The phase is directly related to the distance covered by the radiation, i.e. to the distance between sensor and terrain. Suppose, for simplicity, that the distance antenna-terrain-antenna (the signal reaches the ground and bounces back to the antenna) is a multiple of the wavelength: this means that the wave, when is received back from the antenna, has covered a large but integer number of oscillations, or \textbf{phase cycles}, and its phase will be thus exactly the same as when

\textsuperscript{1}This assumption is valid in the case of backscattering from extended objects and apply to almost all the different kind of natural land coverage. It is not valid if there are one or few dominating scatterers and, as it will be shown in the next chapters, this happens typically in urban areas.
the signal was emitted. Suppose now that the distance is not equal to a multiple of the wavelength, but, let us say, to a multiple plus one centimetre: then the signal will have to travel two centimetres more and, when detected from the antenna, its phase will be shifted by an amount corresponding to this additional distance. The shift, which can be determined as the difference $\phi$ between the phase values of the signal in the two cases, is directly related to the difference in distance $\Delta r$ according to the following formula:

$$\phi = \frac{4\pi}{\lambda} \Delta r$$  \hspace{1cm} (2.7)

By computing the difference in the phase value of each pixel in two images, one is in fact mapping the differences in distance. This is indeed the basic concept of SAR Interferometry and such a phase difference image, called interferogram, is the basic tool for this technique. An example of interferogram is given in Fig.2.3(b); since the phase differences can be measured only in the interval $[0, 2\pi]$, the interferogram is characterised by a pattern of fringes, each one representing the difference in distance corresponding to a phase cycle. Since the used wavelengths are of the order of some $10^{-2}\text{m}$, the phase differences allows us to estimate variations in the distance antenna-target with accuracies of $10^{-2}$ and even $10^{-3}\text{m}$.

### 2.2.2 The interferometric processing

We will consider the interferometric processing starting from two high resolution complex images, called Single Look Complex (SLC). These are the images obtained by synthesising the returned echoes from the same resolution element on the ground in an unique image pixel, as explained in Section 2.1.2. Actually, one can also
start from the raw data matrix and produce own complex images before combining them interferometrically; however since for this work the processing started from ESA SLC products and not from raw data, this pre-processing (it is called in this way to distinguish it from the processing, by which it is intended here the pure interferometric processing) will not be treated here.

Basically, interferometric processing consists of the following main steps:

- Matching of the two SLC images;
- Complex multiplication of the two matched images and generation of interferometric products;
- Correction of the interferometric products for the distortions due to the radar slant range geometry;
- Phase unwrapping.

The matching of the two images, or co-registration, consists essentially in determining the transformation which brings one of the two images, called slave, to be superimposed on the other, the master. This operation is usually performed in two steps. First, a "rough" matching, called coarse co-registration is performed: the shifts, in both range and azimuth direction, of the slave with respect to the master are estimated with an accuracy of a few pixels. Coarse co-registration can be performed by selecting pass points in both images. Due to the distortion of the image (for example ERS images have resolution 5 times higher in azimuth than in range and look therefore strongly stretched) and to the presence of speckle, it is however usually rather difficult to identify the same object in both images with an uncertainty of few pixels. An alternative method uses the satellite orbits: since the coordinates of the image centre and the start-, centre- and end-of-image acquisition times are given, with the help of the satellite position it is possible to determine the shift of the slave with respect to the master in the two directions [56].

After the coarse co-registration, a fine co-registration is performed, which refines the matching between the two images to a sub-pixel level. A matching accuracy of 1/8th of a pixel is usually the limit considered for a good co-registration [26]. Fine co-registration can be accomplished with different methods, either in the space or in the frequency domain. Most of these methods consist basically in comparing approximately corresponding areas in the two images and solving for a set of local transformation parameters. Several different transformations are applied at sub-pixel level to the considered area in the slave image, each time comparing the transformed slave area with the corresponding one on the master. The comparison is performed by computing a certain evaluation function and the best set of parameters
is chosen as the one minimising or maximising this function. So, for example the method outlined in [40] searches for the minimum of the average fluctuation function, the one used in [23] the maximum of the signal-to-noise ratio (SNR), while other algorithms ([26, 56]) search for the maximal correlation. Once its parameters have been determined, the transformation is applied to the slave. After the transformation, the pixel coordinates of the slave are no longer integer with respect to the new pixel coordinates, i.e. the master pixel coordinates. The pixel values have therefore to be re-sampled in order to determine the slave new pixel values in the master coordinate grid. The value of each pixel of the slave is interpolated in order to give the new value in the matched slave according to the applied transformation. Due to the high number of pixels contained in one SLC image, this is usually the most time- and memory-consuming processing step.

In order to improve the matching of the two images, co-registration is generally preceded by a filtering of the images in both azimuth and range directions [6, 26]. The image spectra are in fact shifted with respect to each other, and overlap only partially. This happens in both directions, but for two different reasons [22, 35]. In the range direction, the different look angles under which the same resolution element is imaged in the two scenes cause different frequency bands of the reflectivity spectrum to be detected. In the azimuth direction the spectral shift is caused by different squint angles in the two scenes, due to either to the non-parallelity of the orbits, or to a different alignment of the radar beam. The non-overlapping part of the spectra does not contribute to the interferometric information and constitutes only a source of noise. This noise can therefore be eliminated by bandpass filtering to select only the common part of the spectra, and this results in a general improvement of the co-registration [26].

After the slave is co-registered and re-sampled, the two images can be complex multiplied and the interferometric products generated. If $S_1$ and $S_2$ are the complex signals respectively of the master and of the co-registered slave at a certain pixel:

$$S_1 = |S_1|e^{i\phi_1} \quad S_2 = |S_2|e^{i\phi_2}$$

where $|S|$ is the amplitude and $\phi$ the phase of the signal, then the result of the multiplication is still a complex signal $I$ equal to:

$$I = S_1S_2^* = |S_1||S_2|e^{i(\phi_1 - \phi_2)} = |I_0|e^{i\phi}$$

where the asterisk "*" indicates the complex conjugate and $\phi = \phi_1 - \phi_2$ is the interferometric phase.

Besides the interferometric phase image also an interferometric coherence image is usually generated. Interferometric coherence is defined as the amplitude of
the complex correlation coefficient:

$$\gamma = \frac{E\{S_1 S_2^*\}}{\sqrt{E\{|S_1|^2\} E\{|S_2|^2\}}}$$

(2.10)

$E\{\cdot\}$ is the mean operator and it is approximated by the average on a finite number of elements. The estimated complex correlation coefficient is therefore:

$$\hat{\gamma} = \frac{\sum S_1 S_2^*}{\sqrt{\sum |S_1|^2 \sum |S_2|^2}} = |\hat{\gamma}| e^{i\phi}$$

(2.11)

where the summations are performed over a number of pixels within a shifting window. The interferometric coherence $|\hat{\gamma}|$ has a value comprised in the interval $[0, 1]$ and, as per definition of correlation, gives a measure of the similarity of the information in the two images. The significance of the interferometric coherence and the factors affecting its value will be discussed in detail in the next chapter. Notice finally that, after the multiplication, the resulting interferometric products, i.e. phase and coherence images, are usually multi-looked, i.e. the values of the phase (or coherence) at neighbouring pixels comprised within a window of fixed size are averaged. The result is an image with resolution lower than the original ones, but with a better signal-to-noise ratio. In the case of ERS SLC images, since the resolution is about 4 metres in range and 20 metres in azimuth, averaging is usually performed considering a factor 5 more pixels in azimuth than in range (multi-looking windows of $2 \times 10$ or $3 \times 15$ pixels are most often used) in order to obtain approximately square pixels.

The interferometric products generated with Eq.(2.9) contain still a strong contribution due to the slant range geometry of the system. The effect is particularly visible in the interferometric phase. As can be deduced from Fig.2.1, even for a perfectly flat terrain the side-looking configuration of the radar system causes the slant range distance of the sensor from the terrain to vary passing from the inner (near range A) to the outer (far range B) edge of the swath. The result is the presence, in the interferometric phase, of an "artificial" fringe pattern, which is superimposed to the topography and/or deformation fringe pattern of interest [42]. This effect is illustrated in Fig.2.5: the true topography, shown in Fig.2.5(a), is "seen" by the radar as represented in Fig. 2.5(b), i.e. with an inclined plane superimposed to it (Fig.2.5(c)). The resulting interferometric phase is shown in Fig.2.5(d), where a vertical pattern of fringes is clearly visible. These "flat earth fringes", which can be a considerable number across the image, are parallel to the sensor track, like in Fig.2.5(d), if the orbits remain parallel to each other along the whole image. In the real case the non-parallelism of the orbits however can cause more distorted fringes.
The evaluation and subtraction of these fringes, a procedure called flat earth correction, is usually performed by using the orbits [26]. By knowing the sensor positions at a certain instant one can estimate the fringe pattern along the corresponding range line in the image. The phase value, say $\phi_{flat}$, due to this geometrical effects is then subtracted when the complex multiplication is performed; in practice thus, the following formula is used instead of Eq.(2.9):

$$ I = |I_0|e^{j\phi} = |S_1||S_2|e^{j(\phi_1 - \phi_2 - \phi_{flat})} $$

(2.12)

SAR Interferometry is based on the interpretation of a phase difference as a measure of a difference in distance between sensor and terrain. The only measurable quantity, however, is the fractional part of this phase difference, i.e. the principal value, which lies in the interval $[-\pi, \pi]$ (or $[0, 2\pi]$) and causes the typical interferometric fringe pattern, as shown, for example, in Fig.2.5(d); a whole gray scale cycle, say from black to black, corresponds to a phase cycle of $2\pi$. Let us take as a starting value the phase value indicated as "A" in figure, which is more or less equal to $-\pi$; proceeding along the line perpendicular to the fringe, the represented value increases up to $\pi$ and when it reaches the edge of the fringe, say in "B", "jumps" back to $-\pi$. The true value of the phase difference B-A is however not zero, but $2\pi$, or one phase cycle. Running across the fringes one should therefore be able to add, for each phase jump, $2\pi$ to the measured value, and obtain in this way the true phase of all the points in the image with respect to a starting point taken as a reference ("A" in this example). Since the measured phase can be seen as a value "wrapped" around the interval $[-\pi, \pi]$, the procedure is called phase unwrapping.

In the ideal case of a perfectly delineated interferometric fringe pattern, phase unwrapping would be a quite straightforward procedure. In the real situation however the interferogram is always more or less affected by noise, which causes the fringe contours to be less sharply defined, distorted or even interrupted. The result is that the number of phase cycles to be added to a phase value can be influenced by the path followed in the unwrapping procedure, leading to their wrong estimate, and the error is propagated to the whole interferogram. For this reason phase unwrapping is maybe the most difficult processing step in interferometry.

Several phase unwrapping methods have been developed up to now, based on totally different approaches, and in the context of this introductory chapter only

\[2\] Notice that although the effect has been explained in terms of an additional plane, in the real case when estimating the flat earth correction the approximation, for the zero-topography earth surface, of a tangential plane applied in the centre of the image is not sufficient. In fact, at the typical scales of a SAR image, the earth curvature is not negligible, and in the computation of the correction one has to take as the reference surface the Earth ellipsoid.
Figure 2.5  The effect of the SAR slant range geometry on the interferometric phase: (a) "true" topography, (b) topography as seen from the side-looking radar, (c) difference between (a) and (b), (d) resulting phase image.
a short reference list of the most known can be given, e.g. [10, 13, 21, 28, 29].
What can be said in general about them is that they all work well in conditions of low noise level, but that in most cases their efficiency decreases rapidly as the noise increases. There is not a generally better algorithm, but they are differently affected by effects such as decorrelation noise and foreshortening or lay-over, so that algorithms based on a certain approach can be more suitable than others, depending on the particular interferogram.

It is finally worth noting that the increasing importance of long term application of interferometry, which uses interferograms spanning interval of more years, is leading to the development of special phase unwrapping methods for such interferograms which, as it will be shown, are often reduced to sparse patches of interferometric information in an otherwise totally noisy image, [11, 15].

2.2.3 Applications

Digital elevation model generation

As explained in Section 2.2.1 SAR Interferometry is based on the measurement of phase differences caused by a path difference, in the slant range direction, of the radar signal. A path difference, and consequently an interferometric pattern, can be caused by a slight change in the angle under which the same terrain is seen in the two images (see Fig.2.6). By considering images taken from the sensor at two slightly different positions one can deduce then the height of the terrain. This approach of INSAR is used to produce Digital Elevation Models (or DEM’s) of the surface. The phase fringes obtained in this way are directly related to the terrain height $z$, the (approximated) relationship being:

$$\phi = \frac{4\pi}{\lambda} \Delta r \simeq \frac{4\pi}{\lambda} \frac{B_\perp}{r \sin \theta} z \quad \text{with} \quad B < < r$$

(2.13)

where $B_\perp$ is the perpendicular component of the baseline $B$, as is called the distance between the sensor positions for the two images. The approximation $B < < r$ used in Eq.(2.13), or “far field approximation”, assumes that the baseline length is much smaller than the slant range distance from the sensor to the ground. This is a reasonable approximation, as $r$ is typically of the order of some kilometres to hundreds of kilometres (for airborne and spaceborne systems respectively), while the

---

3 Foreshortening and lay-over are geometrical distortions of the image due to the side-looking imaging geometry. The projection into the radar line-of-sight causes two surfaces of different inclination to be imaged with different lengths. As a consequence, the side of a mountain towards the radar will appear shorter than the side away from it. See [12] for more details.
considered baselines $B$ are usually not larger than a few hundreds of metres. An example of a "topography-interferogram" is the one shown in Fig.2.3(b). The interferometric fringes can be interpreted as a sort of contour lines: the distance between the edge of two subsequent fringes corresponds in fact to a certain fixed height $z_{2\pi}$, called **height ambiguity**. The height ambiguity is a function of the baseline, as can easily be deduced from Eq.(2.13) by posing $\phi = 2\pi$:

$$z_{2\pi} = \frac{\lambda r \sin \theta}{2} \frac{B}{B_\perp}$$  \hspace{1cm} (2.14)

From Eq.(2.14) is clear that the larger the baseline the smaller the height ambiguity. From the same equation and from Eq.(2.13) is also evident that the accuracy of the retrieved height is directly related to the accuracy of the phase measurement, which means that, assuming the latter fixed, larger baselines give higher height accuracies in the estimated DEM. Unfortunately one cannot take an arbitrarily large baseline in order to increase indefinitely the accuracy. As the baseline becomes larger, in fact, the difference in viewing geometry between the two images increases and consequently the backscattering from the same resolution cell becomes more and more different in the two cases [39]. Speaking in the same spectral terms used for the
2.2 SAR Interferometry

explanation of range filtering in Section 2.2.2, this means that the overlap between the image spectra in the range direction reduces with increasing baselines [25]. At a certain point the spectra become disjoint indicating that the backscattering is totally different in the two images, and their interferometric combination produces only noise. The baseline corresponding to this point is called critical baseline and the effect is called baseline decorrelation, [39, 53]. For ERS the critical baseline is of about 1100m [23], however already for values of the order of 300m the baseline decorrelation is well noticeable. The application of interferometry for topography estimation is thus dependent on the existence of pairs of images with suitable baseline, and the choice of the baseline length is a tradeoff between the request of the highest obtainable accuracies in the resulting DEM, and the necessity of limiting as much as possible the effect of baseline decorrelation.

**Deformation measurements**

Another application of INSAR is the detection of height changes, [42, 68]. This also is done by comparing two images of the same area, but this time the images are taken from exactly the same position of the sensor at two different times, as illustrated in Fig.2.7. In this case the viewing geometry in the images is identical, thus if the distance of the sensor from the terrain has not changed between the two acquisition times, then the returned echoes will have in principle exactly the same phase and the difference (given by Eq.(2.7)) between these two phase values will be zero. If on the contrary the terrain has shifted even of few millimetres in the time between the two images are taken, then the phase in the second image will be also shifted with respect to the first one and the shift will give an estimate of the terrain movement. The result is again the presence of an interferometric pattern, but this time it describes the terrain deformations. The amount of deformation corresponding to a phase cycle can be determined by posing $\phi = 2\pi$ in Eq.(2.7):

$$\Delta r = \frac{\lambda}{2}$$ (2.15)

Therefore, while in the topographic applications the height corresponding to a phase cycle is dependent on the used baseline, for a deformation interferogram a fringe corresponds always to the same amount of deformations, equal to half the used wavelength.

An example of a deformation interferogram is shown in Fig.2.8. The interferogram is obtained from images of the ERS satellites, whose radar has a wavelength of 5.6cm, thus a phase cycle (represented as a grayscale cycle), corresponds to a deformation of 2.8cm. In the case shown in figure the deformation has a semicircular pattern and amounts to about three phase cycles.
2.3 Differential INSAR for deformation measurements

The use of the interferometric technique for the retrieval of terrain deformations requires the SAR images to be taken from exactly the same position in space at two different times. In practice however it is highly improbable that the sensor will return exactly to the same position twice, and the baseline will always be different from zero. This causes the presence of a topographic component in the measured interferometric phase, which adds up to the deformation information. The topographic contribution can be separated and subtracted from the deformation by using the so-called differential technique [24, 33, 43, 52, 66]. A second interferogram is generated with two images taken over a time interval during which no significant deformation had occurred. This interferogram, which will thus contain only topographic information, is re-scaled and subtracted from the one of interest, leaving in this latter only the deformation signal.

If $\phi_f$ is the flat earth corrected, unwrapped phase of the "topography + deformation" interferogram with perpendicular component of the baseline $B$, and

![Figure 2.7](image) The principle of SAR interferometry for the retrieval of terrain deformations.
2.3 Differential INSAR for deformation measurements

\[ \Delta \phi = \phi_f - \phi_f' \frac{B_\perp}{B'} \]  

\( \Delta \phi \) is also called differential phase. Notice that the rescaling is performed according to the perpendicular component of the baseline (parallel component if the phase values are not flat earth corrected, [68]): as explained in the topographic application of INSAR, in fact, the sensitivity of the interferometric phase to the slant range height of the terrain is an inverse function of the baseline. It is also straightforward from Eq.(2.16) that it has to be \( B'_{\perp} > B_{\perp} \). The differentiation of Eq.(2.16) gives in fact the propagation of the phase errors as:

\[ \delta \Delta \phi = \delta \phi_f + \delta \phi_f' \frac{B_\perp}{B'} \]  

where \( \delta \phi_f \) and \( \delta \phi_f' \) are the errors associated with the phase measurements in each interferogram and \( \delta \Delta \phi \) is the resulting error in the differential phase. The operation of topography subtraction therefore adds an error source, represented by the term
\[ \delta \phi'_I \frac{B_1}{B_2} \], to the phase accuracy of the deformation interferogram \( \delta \phi_I \). Assuming 
\[ \delta \phi_I = \delta \phi'_I, \] in order to keep this additional error as low as possible the baseline of the topography-only interferogram has to be much larger than the one of the interferogram of interest.

An alternative method to the use of a second interferogram is to use a pre-existing DEM of the area. For this purpose the DEM has to be transformed in the azimuth-slant range geometry, scaled to the baseline of the interferogram, and finally the topography resulting in this geometry has to be converted from length-to-phase units.
This chapter introduces the concept of decorrelation in SAR Interferometry. Particular attention is paid to the temporal component, which is the main source of decorrelation for repeat pass INSAR. The definition and theoretical background of decorrelation are explained in Section 3.1. As it will be shown in Section 3.2, its influence on the determination of the interferometric phase and coherence measurements can be estimated quantitatively for the case of scattering from an extended target, an assumption which suits most cases. Under this hypothesis the backscattering can be modelled as a Gaussian process, and the statistics of the interferometric process is then its second order statistics. Additionally, for extended targets also the statistics for averaged (multi-looked) interferometric values can be determined. At a certain point of this thesis, however, man made features will be considered and, due to their reduced size, their backscattering is probably more similar to the scattering from a point target than from an extended one. The statistics is in this case quite different, and becomes more complicated already at the first order, which corresponds to the case of one single image. The second order statistics, which models the interferometric process, can be determined also for point targets, however this is definitely not straightforward [44]. As shown in Section 3.2.3, a simpler approximation of this statistics can be found for high coherence.

The chapter concludes with a description of the effects of temporal decorrelation, in Section 3.3, in particular on long term interferograms.
3.1 Definition

As explained in the previous chapter, the interferometric technique consists basically in the comparison of two SAR images. These images can be taken simultaneously or at different times, leading to different kind of implementations and applications. The first possible implementation uses two distinct antennas illuminating the ground simultaneously. This is the typical configuration for airborne INSAR, but there is also a spaceborne example in the Shuttle Radar Topography Mission (SRTM) launched in January 2000 [2]. It is the most favourable configuration when INSAR is applied for topographic mapping, because temporal decorrelation does not play a role at all, as it will be clear when the concept of temporal decorrelation will be explained below. The alternative implementation for interferometry is the one which uses the same antenna on a nearly-exactly repeating track. This is the typical configuration used for satellite systems, such as ERS-1, ERS-2 and JERS-1, but also widely used on airborne systems. This repeat pass implementation is obviously the one which permits the application of the INSAR technique for the detection of height changes by comparing, as explained in Section 2.2.3, images taken at different times to retrieve their differences. The fact that the images are not taken simultaneously, which is the key for this application, leads however also to a major drawback, namely temporal decorrelation.

In order to compare two SAR images taken at different epochs, we have to assume that the terrain is imaged exactly in the same way both times, i.e. that the backscattering properties of the ground have not changed in the meantime. This is unfortunately never the case, as vegetation growth, climatic changes and the human presence cause all kind of modifications in the terrain coverage, changing, as a consequence, also its backscattering response. So, for example, a cultivated field undergoes several modifications in the course of the year, as it will be most probably bare soil in winter while in summer it will be covered with grown crop. Two images of such a field taken one in winter and one in summer will therefore contain information from completely different backscattering sources which cannot be compared, and their interferometric combination will result in pure white noise. Analogously, the same terrain can have different backscattering properties depending on its moisture content. Even a ground coverage which does not change significantly in time will therefore still reflect the radar signal in a totally different way if, for example, it is covered with snow or if it has rained, with respect to when it is dry.

One way to measure the differences between the signals in the two images is

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1The dependence of the decorrelation times on these characteristics leads indeed to another application of SAR Interferometry, which is the classification of ground coverage. This application will not be discussed here, however the reader can find several examples of it in the literature, [65, 3].
3.1 Definition

To compute their correlation coefficient. The statistical definition of the correlation coefficient between two complex signals was already given in Section 2.2.2 by Eq.(2.10) (here repeated for convenience):

$$\gamma = \frac{E\{S_1^*S_2\}}{\sqrt{E\{|S_1|^2\}E\{|S_2|^2\}}} \tag{3.1}$$

with $E\{\}$ the operator of mean. The quantity $\gamma$ is also complex: its amplitude $|\gamma|$ varies in the interval $[0,1]$ and it is called interferometric coherence. An interferometric coherence equal to 1 indicates that the ground properties are identical in the two images, i.e. that the signal comes from the same backscattering sources in the same conditions. A zero value indicates on the contrary that the signal in the two images is completely different, and in this case their interferometric combination is said to be uncorrelated or, better, decorrelated, since we assume that correlation has been lost due to some effect. The number $|\gamma|$ gives thus the degree of decorrelation of the signal. As a statistical quantity the interferometric coherence can in practice only be estimated. The estimator for the complex correlation coefficient corresponding to Eq.(3.1) is (see Eq.(2.11) in Section 2.2.2):

$$\hat{\gamma} = \frac{\sum |S_1||S_2|e^{i\phi}}{\sqrt{\sum |S_1|^2\sum |S_2|^2}} \tag{3.2}$$

It is intuitive that, since it is estimated from a limited number of samples of two stochastic variables, in real life $|\hat{\gamma}|$ is never exactly 1, as it is practically impossible to have identical backscattering in two images; for the same reason the value of $|\hat{\gamma}|$ cannot either be equal to zero. What has to be defined in the computation is the number of elements to consider for the summation. Since we are dealing here with two-dimensional arrays of values, such as images can be considered (see Fig.3.1), the summation is computed by considering all the elements on a two-dimensional subset, or window. The size of the window must be neither too small, otherwise the computed value would be too unstable and therefore statistically meaningless, nor too large, which would cause a loss of details, a "flattening" of the coherence values in the interferogram. Often a window size of $2 \times 10$ of $3 \times 15$ pixels is used for ERS images, because it gives already a sufficient number of samples for the statistical estimation of the coherence, while keeping still a high resolution; moreover, the ratio $1:5$ in the window size results in multi-looked pixels which are approximately square, making the image more agreeable to the eye. Different (larger) window sizes have however also been used, depending on the particular application. Notice that the use of a shifting window causes a dependence of the coherence values in neighbouring pixels. This is due to the fact that the same pixel
in the original images is used for the computation of the coherence in more pixels (as much as the size of the window) in the interferometric image. The inevitable result is therefore the "spreading" of the correlation values of each resolution element of the ground in the resolution elements of the images, i.e. the pixels. Notice finally that in the definition of Eq.(3.2) also the phase values of the pixels are taken into account in the estimate. These phase values have to be already flat earth corrected, in order to eliminate what would be a spurious effect, on the coherence estimation, of the slant range geometry.

### 3.2 Statistical modelling of decorrelation

#### 3.2.1 Sources of decorrelation

Temporal decorrelation is the major cause for the loss of coherence in repeat pass interferometry. Decorrelation can however be caused also by other effects than real changes in the backscattering from the imaged terrain, namely by factors related to the imaging process. There are three possible sources for this kind of decorrelation:

- thermal noise from the radar receiver;
- data acquisition and interferometric processing errors;
• different imaging geometry in the considered images, the so-called baseline decorrelation

The total decorrelation $\gamma$ resulting from these three sources and from the temporal component can be written as [6, 69]:

$$
\gamma = \gamma_{\text{temporal}} \times \gamma_{\text{scene}} \times \gamma_{\text{thermal}}
$$

(3.3)

where $\gamma_{\text{temporal}}$ accounts for the temporal decorrelation, $\gamma_{\text{thermal}}$ for the decorrelation due to the receiver noise and $\gamma_{\text{scene}}$ for the component purely related to the imaging process, i.e. due to processing errors and to the imaging geometry. $\gamma_{\text{scene}}$ accounts in general for both the surface and the volume scattering from the scene, however in the two case studies considered here the volumetric component can be assumed to be negligible. The amount of the volumetric component is in fact a function of the perpendicular component of the baseline and of the height difference of targets imaged in the same resolution element, [25]. In the case of the first test site, located in the Netherlands, the extremely reduced topography makes the volumetric scattering negligible at the baseline lengths considered (up to 150 metres in the perpendicular component). Even shorter baselines (less than 50 metres in the perpendicular component) will be considered for the second test site, the Neapolitan area, which has conversely a more significant topography. According to [25], volumetric effects for a (perpendicular) baseline of 50 m would cause decorrelation only for height differences of 190 metres, a value which is not likely to occur with the used resolutions.

### 3.2.2 Statistical modelling for extended targets

The effects of the decorrelation on the interferometric phase, which is the information we are interested in, can be estimated quantitatively by expressing the dependence of the standard deviation associated to the measured phase at a certain pixel on its corresponding coherence value (Eq.(3.2)). The starting point is the statistical modelling first of the single image, and then of the interferometric process as the combination of two such images. The reflection from a ground resolution element is assumed to be the resultant of the reflections from a certain number of scattering centres, or phasors, with the following assumptions [30]:

1. the number of phasors contained in one resolution element is extremely large;

2. the phasors have phase and amplitude statistically independent from each other and are also independent from all the other phasors both in phase and in amplitude;
3. the phase values of the phasors are uniformly distributed, which means also that the surface is rough with respect to the considered wavelength.

Under these assumptions, we can model the resulting field, in its real and imaginary parts, as the sum of a large number of independent random variables and, from the central limit theorem [51], this yields for both parts an asymptotically Gaussian distribution. The joint probability density function of the real and imaginary components is thus also a circular \(^2\) Gaussian process:

\[
pdf(A_r, A_i) = \frac{1}{2\pi\sigma^2} \exp\left[ -\frac{A_r^2 + A_i^2}{2\sigma^2} \right] \tag{3.4}
\]

where \(\sigma^2 = \langle |A|^2 \rangle / 2\) (with \(\langle |A|^2 \rangle = |A_r^2 + A_i^2|\)) is the mean intensity of the field. Equation (3.4) can also be expressed in terms of amplitude and phase by using the following relations:

\[
A_r = E \cos \theta \quad A_i = E \sin \theta \tag{3.5}
\]

Since the Jacobian of this transformation is \(E\), the probability density function (pdf) takes then the form:

\[
pdf(E, \theta) = \frac{E}{2\pi \sigma^2} \exp\left[ -\frac{E^2}{2\sigma^2} \right] \quad E \geq 0 \quad -\pi \leq \theta \leq \pi \tag{3.6}
\]

The marginal pdf for the intensity and phase of the field can be determined by integrating respectively in the phase and in the intensity, obtaining:

\[
\begin{align*}
\pdf(E) &= \int_{-\pi}^{\pi} \pdf(E, \theta) d\theta = \frac{E}{\sigma^2} \exp\left[ -\frac{E^2}{2\sigma^2} \right] \quad E \geq 0 \tag{3.7} \\
\pdf(\theta) &= \int_{0}^{\infty} \pdf(E, \theta) dE = \frac{1}{2\pi} \quad -\pi \leq \theta \leq \pi \tag{3.8}
\end{align*}
\]

Notice that the phase and intensity of the resulting field are statistically independent:

\[
\pdf(E, \theta) = \pdf(E) \pdf(\theta) \tag{3.9}
\]

Note also that the phase is uniformly distributed; this explains why the phase from only one image does not bear any useful information.

The interferometric image can, at this point, be modelled statistically as the combination of two complex stationary circular Gaussian processes of the type of \(^2\) "Circular" means that the contours of constant probability are circles in the complex plane.
3.2 Statistical modelling of decorrelation

Eq. (3.4). Supposing that the measured field at a given pixel in image 1 has real and imaginary components \( A_1, A_2 \) and in image 2 components \( A_{i1}, A_{i2} \), then the joint probability density function of the two fields is \([6, 35, 30, 51]\):

\[
pdf(A_{r1}, A_{i1}, A_{r2}, A_{i2}) = \frac{1}{4\pi^2\sigma^4(1 - |\gamma|^2)} \times \exp\left[ -\frac{|A_1|^2 + |A_2|^2 - \gamma A_1 A_2^* - \gamma^* A_1^* A_2}{2\sigma^2(1 - |\gamma|^2)} \right] \tag{3.10}
\]

or, in terms of \( E_1, E_2, \theta_1 \) and \( \theta_2 \):

\[
pdf(E_1, E_2, \theta_1, \theta_2) = \frac{E_1 E_2}{4\pi^2\sigma^4(1 - |\gamma|^2)} \times \exp\left[ -\frac{E_1^2 + E_2^2 - 2E_1 E_2|\gamma|\cos(\theta_1 - \theta_2 + \phi_0)}{2\sigma^2(1 - |\gamma|^2)} \right] \tag{3.11}
\]

where it has been assumed that the two processes have equal mean intensity, that is \( <|A|^2> = <|A_1|^2> = <|A_2|^2> \), \( \sigma^2 = <|A|^2>/2 \), while \( \phi_0 = \arg(\gamma) \) (i.e. \( \gamma = |\gamma|\exp(i\phi_0) \)) is the expected value of the interferometric phase for the particular pixel considered. The marginal pdf for the phase can be determined by integrating Eq. (3.11) with respect to the intensities \([35, 30]\):

\[
pdf(\theta_1, \theta_2) = \int_0^\infty \int_0^\infty pdf(E_1, E_2, \theta_1, \theta_2) dE_1 dE_2
\]

\[
= \frac{1 - |\gamma|^2}{2\pi} \times \frac{1}{1 - |\gamma|^2 \cos^2(\phi - \phi_0)} \times \left\{ 1 + \frac{|\gamma|\cos(\phi - \phi_0)\arccos[-|\gamma|\cos(\phi - \phi_0)]}{\sqrt{1 - |\gamma|^2 \cos^2(\phi - \phi_0)}} \right\} \tag{3.12}
\]

where \( \phi = \theta_1 - \theta_2 \) is the interferometric phase and \( |\gamma| \), the correlation coefficient between the two Gaussian processes, corresponds to the interferometric coherence.

The probability density function of Eq. (3.12) is a function only of the phase difference \( \theta_1 - \theta_2 \) and is centred on \( \phi_0 \), as it is shown in Fig. 3.2(a) for \( \phi_0 = 0 \). As can be seen from the figure and can be also deduced from Eq. (3.12), for \( |\gamma| \rightarrow 0 \), i.e. at the limit of total decorrelation, the pdf tends to a uniform distribution, which is the expected distribution for uncorrelated noise; for high coherence values, i.e. \( |\gamma| \rightarrow 1 \), on the contrary the distribution tends to a delta function.
The standard deviation as function of the correlation value is represented in Fig. 3.2(b).

\[ \sigma_{\phi}^2 = E\{(\phi - \phi_0)^2\} = \int_{-\pi}^{\pi} (\phi - \phi_0)^2 pdf(\phi)d\phi = \int_{-\pi}^{\pi} \phi^2 pdf(\phi + \phi_0)d\phi \] (3.13)

The corresponding standard deviation of the interferometric phase can be finally computed as the second moment of the pdf in Eq.(3.12):

\[ \sigma_{\phi}^2 = E\{(\phi - \phi_0)^2\} = \int_{-\pi}^{\pi} (\phi - \phi_0)^2 pdf(\phi)d\phi = \int_{-\pi}^{\pi} \phi^2 pdf(\phi + \phi_0)d\phi \] (3.13)

The standard deviation as function of the correlation value is represented in Fig. 3.2(b).

### 3.2.3 Statistical modelling for point targets

The probability density function computed in the previous section is valid in the assumption of a speckle-like reflection, which is the reflection from an extended target, where “extended” means of size significantly greater than the resolution cell size. This assumption is however not valid when reflection comes from one or few very strong scattering centres. If this is the case, then the starting hypothesis of a statistically large number of scatterers, which leads to the modelling as a Gaussian process, is no longer valid and the statistical model explained above cannot be applied. Although most of the reflection is from extended surfaces, there are some cases of point-wise scatterers. The most common sources of such scattering are anthropogenic objects, mostly buildings. Indeed, man made features are more likely to backscatter the radiation in a similar way to corner reflectors. Buildings have usually several edges and corners and their walls form also a right angle with the ground. Some of these right angles happen to be “properly” oriented with respect
3.2 Statistical modelling of decorrelation

Figure 3.3  Standard deviation of the phase as a function of the coherence $|\gamma|$ for point targets (continuous line). The corresponding standard deviation in the case of an extended target is plotted for comparison (dashed line).

to the incidence angle of the radiation, and reflect most of it back to the antenna, giving the same strong backscattering as corner reflectors [61].

In the case of strong scatterers, the imaging process can be modelled as a constant signal plus background noise [30, 44]. The probability density function of the resulting interferometric process takes then a rather complicated form [44], which cannot be integrated directly to give the standard deviation of the interferometric phase. An alternative much simpler expression for such standard deviation can however obtained for high coherence (thus unbiased, see next section) values [35]:

$$\sigma_\phi^2 = \frac{1 - |\gamma|^2}{2|\gamma|^2}$$  \hspace{1cm} (3.14)

The standard deviation is shown in Fig.3.3 where, since Eq. (3.14) is valid only for very high coherence values, $\gamma$ has been taken in the interval $[0 \ 1]$. As can be seen in the figure, for all the coherence values the standard deviation in this case is always considerably smaller than in the case of an extended target, represented as a dashed curve for the comparison.
Figure 3.4  The multi-looking windows, differently from the shifting window used for the estimation of the coherence, do not have pixels in common. The estimate in A and B are therefore independent.

3.2.4 Statistics for multi-looked data

The statistics presented in Section 3.2.2 is computed for a single resolution element in the interferogram. In other words, Eq.(3.12) and Eq.(3.13) refer to the interferometric combination of two image samples. As it has been already mentioned in the previous chapter, however, in general the coherence and phase values are computed on more samples, or multi-looked, i.e. they are averaged on a certain window of values (see Fig. 3.4). This averaging procedure, is applied in order to obtain a less noisy interferogram, though the price to pay is a lower resolution. If more interferometric samples are used to determine a single multi-looked interferometric sample, then the pdf’s of the resulting phase and coherence take another form. In the case of extended targets, assuming that the multi-looking is performed by considering L samples with constant phase, the probability density function for the estimated phase \( \hat{\phi} \) results to be [5, 59]

\[
pdf(\hat{\phi}; L) = \frac{\Gamma(L + 1/2)(1 - |\gamma|^2)^L |\gamma| \cos(\hat{\phi} - \phi_0)}{2\sqrt{\pi} \Gamma(L)(1 - |\gamma|^2 \cos^2(\hat{\phi} - \phi_0))^{L+1/2}} \\
\times \frac{(1 - |\gamma|^2)^L}{2\pi} _2F_1(L, 1; \frac{1}{2} |\gamma|^2 \cos^2(\hat{\phi} - \phi_0))
\]

(3.15)

where \( \Gamma \) is the Gamma function and \(_nF_m(a_1, \ldots, a_n; b_1, \ldots, b_m; z)\) is the generalised hyper-geometric function [31, 38]. The pdf of the phase for different number of
3.3 Effects of temporal decorrelation on long term interferograms

The reliability of the interferometric phase information depends strongly on the degree of correlation, as it has been shown quantitatively in the previous section. Although different factors contribute to the total decorrelation, in practice the temporal decorrelation constitutes the major source of noise in repeat pass interferometry.

Figure 3.5 Probability density function of the phase (left) and of the coherence (right) for different number of looks $L$ ($\phi_0$ is assumed to be zero).

samples (looks) is shown in Fig.3.5(a), where a coherence value of 0.5 has been assumed. As can be seen in the figure, with increasing number of looks the distribution becomes narrower. As for the coherence, the probability density function of its estimated value $\hat{\gamma}$, represented in Fig.3.5(b), takes the form [5, 59, 60]:

$$pdf(\hat{\gamma}; L) = 2(L - 1)(1 - |\gamma|^2)^L |\hat{\gamma}|(1 - |\hat{\gamma}|^2)^L^{-2} \times _2F_1(L, L; 1; |\gamma|^2 |\hat{\gamma}|^2)$$  \hspace{1cm} (3.16)

The mean value is given by [5, 59, 60]:

$$E\{|\gamma|\} = \frac{\Gamma(L)\Gamma(1 + 1/2)}{\Gamma(L + 1/2)} \times {_3F_2(3/2, L; L; L + 1/2; 1; |\gamma|^2)} \times (1 - |\gamma|^2)^L$$  \hspace{1cm} (3.17)

It has been demonstrated that such a mean coherence value is biased for low coherence, as they tend to take higher values [59, 60]. As shown in Fig.3.6, the bias affects the lower estimated coherence values, and decreases for increasing number of looks.

3.3 Effects of temporal decorrelation on long term interferograms

The reliability of the interferometric phase information depends strongly on the degree of correlation, as it has been shown quantitatively in the previous section. Although different factors contribute to the total decorrelation, in practice the temporal decorrelation constitutes the major source of noise in repeat pass interferometry.
Figure 3.6  Bias of the coherence estimate for different values of the number of looks \( L \).

As mentioned before, decorrelation can affect an interferogram on different time scales depending on the kind of terrain coverage. The slowest decorrelating areas are those having particularly favourable characteristics, such as poor vegetation and dry and no windy climate: in this conditions interferograms over more years can be rather easily obtained (see for example [43, 68]). Desert areas constitute however (fortunately) only a small part of the Earth surface, and excluding water, where decorrelation is practically instantaneous, the most part of the remaining imaged areas are vegetated and/or cultivated, mostly urbanised, and in rather unfavourable climatic conditions for INSAR. Speaking in terms of ERS repeat times, Tandem interferograms of these regions are usually well correlated, but after a few 35 days-cycles decorrelation is rather heavy, if not already total. If, with a good co-registration algorithm, an interferogram can still be generated, then its coherence image will most probably look like the one in Fig.3.7. The corresponding phase image will be almost totally pure noise. The interferometric fringes, when present, are corrupted by the general noise and sometimes even broken by patches of complete decorrelation. The result is, in the best cases, a deformation image which is incomplete and most probably affected by phase unwrapping errors. The situation becomes even worse if the deformation is not enough to generate fringes, or if it is so extreme that only sparse single pixels are still coherent. With such decorrelation, application of the interferometric technique in the "conventional way" becomes practically impossible.
3.3 Effects of temporal decorrelation on long term interferograms

Figure 3.7 Coherence image of an interferogram of Naples and surroundings (time interval: August 1995 - August 1996). Notice the high-coherence of urban areas, in particular of the city of Naples.
The database approach

In this chapter the procedure used for the construction of a database of interferograms is explained, from the choice of the SAR images to the generation of the interferometric products (Section 4.1). Both the selection of the images and their processing are determined by the special purposes of this research and they are guided by particular choices which are motivated in the different sections. These choices are of course not always strictly necessary when a database of interferograms is generated for more general purposes, however also in that case often they simplify the task. The two test sites considered for this research are then presented in Section 4.2, where also the specific characteristics of their corresponding databases are explained.

4.1 The processing method

The set of images to be used for the construction of the time series has been selected with the purpose to obtain:

- the highest possible number of interferometric pairs with short baselines;
- interferograms with very short baselines on long time intervals (of the order of a couple of years);
- at least one (well correlated) interferogram on 1 or 35 days.
Short baselines were preferred for two reasons. First of all, they are less affected by baseline decorrelation. Since this study focuses on the effects of temporal decorrelation, it is in fact desirable that all the contributions to decorrelation from the other possible sources are negligible. Decorrelation from receiver noise and, with the exclusion of phase unwrapping errors, from processing errors, should not be a problem, if a standard, good working software is used. As for the decorrelation due to the imaging geometry, its effect increases with the baseline length: the shortest possible baselines were thus preferable for the purpose of this study. The observation of the interferograms from ERS data suggested that baseline decorrelation starts to affect them heavily for baseline lengths of the order of 300m. An initial criterion was, therefore, to use only interferometric pairs with baseline below this limit. In practice, the idea was to consider baselines as small as possible, and eventually the used interferograms did not exceed a baseline length of about 150m.

The second reason for preferring short baselines is the reduced influence of the topographic component of the phase with respect to the deformation one. As explained in Chapter 2, the ideal interferometric configuration for the retrieval of deformations is a zero-baseline one, in which the interferometric phase is directly related to height changes. In the real situation, since the baseline is practically never zero, the smallest baselines are preferred, because they tend to the ideal case, as the phase becomes less and less sensitive to the topography. Eventually, if the baseline is small enough the topography can be neglected and the interferogram can be considered a pure-deformation one, without applying the differential technique. The possibility to obtain interferograms with baselines so short to avoid the application of the differential method is even more desirable for long term interferograms. The strong decorrelation which usually occurs in these interferograms makes it very difficult to identify and unwrap the fringes, operations which are required for the application of the differential method. It has to be said however that, in the context of this research, the topography has always been extracted from the interferograms by applying the differential method before any other procedure was applied. As it will be shown in the following chapters, the study of the interferometric information from highly coherent pixels involved considering quantities, like standard deviations of the differential phases, of the order of $10^{-1}$ radians. At this level, even very short baselines may produce a still significant topographic component. Moreover, the ultimate purpose is to determine the deformations occurring in the two specific test sites and, especially in the case of the first one, where the subsidence rate is very low, on the considered time interval deformations and topography are in the same order of magnitude. In order to apply the differential technique, therefore, at least one well correlated interferogram on 1 or 35 days with a sufficiently large baseline had to be available, which could be used as the reference interferogram. Notice
that, for the topography estimation, one could alternatively use a pre-existing Digital Elevation Model (DEM), and this has indeed been done for the second test site. At the time when the databases were generated, however, a DEM of the test sites was not directly available. Moreover, the subtraction of an elevation model from an interferogram requires the simulation of the interferometric information from the topography, which usually, due to the lower resolution of the DEM itself with respect to the interferometric image, results in an approximate model. An interferogram containing only topographic information, if available, has on the contrary the following advantages: a) it has the same resolution of the deformation interferogram, b) its subtraction from the latter is a simpler, more straightforward procedure and c) it gives usually more up-to-date information on the topography, as DEM’s can be sometimes several years old. For these reasons, the use of an interferogram for the estimation of the topography was the preferred choice.

The whole interferometric processing was performed using the ERS precise orbits produced at DEOS [54]. The interferograms were generated with a particular procedure, which consisted basically in co-registering first all the SLC images on a unique master and in combining then the slaves with each other to form the interferometric products. In this way all the database interferograms were computed on the same interpolation grid, and a given pixel came to represent, in each of them, exactly the same area. This is particularly important in the context of the research presented here. At a certain point, in fact, the information contained in anthropogenic features will be considered. Due to their limited extension of one or just a few pixels, these features have to be localised with great accuracy in an interferogram. Also when, as in the second test site under study, instead of single pixels whole interferograms will be considered, they will be merged with a least squares adjustment on a pixel by pixel basis. In this case too, one has to be sure that a pixel represents the same resolution cell on the ground in all the interferograms, therefore a precise matching among them all is required. Such matching is usually obtained by performing, after the interferometric processing, the so-called geocoding, i.e. the conversion from image coordinates to a cartographic reference system. This is a quite laborious processing step, moreover it must be preceded by a conversion of the phase values from slant range to height, which is also a rather complicated procedure because of the presence of lay-over and foreshortening effects. Alternatively, the co-registration on the same master permits to have the desired matching among interferograms without the need to perform these additional processing steps.

Notice that the use of images both already co-registered on a third one obviously limits the resulting interferogram to the common part among the three SLC’s. For the ERS products the loss with respect to the area covered by the ”direct” interferogram between the two images amounts usually to a couple of pixels in range.
and some tens to about a hundred lines in azimuth. This is however a rather small portion of the images, which are 5,000 pixels wide in range and about 25,000 lines long in azimuth (in the case of a full scene).

The master of the database can be in principle any image of the selected set of SLC’s, however it is convenient to choose the one which gives the largest number of short baselines when combined with the others. With “short” it is intended here that they are short enough to permit a good co-registration, not necessarily that they can be used for the retrieval of deformations. Indeed, the purpose is here to co-register as many images as possible on the master. So, if one image forms with the master a baseline of about 150m, co-registration should not be a problem, but an interferogram between them most probably will not be used for the retrieval of deformations. The co-registered slave however can still give a lot of interferograms with baselines of a few to few tens of metres when combined with other co-registered slaves.

The co-registration of each slave on the master has been performed by determining first the co-registration parameters on the azimuth-filtered, and then applying the determined parameters to the original (not filtered) slave. In this way co-registration is optimal but at the same time the spectrum of the images is preserved entirely. Once the set of co-registered slaves was obtained, they were combined with each other and with the master to form as many short-baseline interferograms as possible. The two chosen images of the database were first azimuth filtered with respect to each other. Since the slaves were already co-registered on the same master, complex multiplication could in principle be applied straightforwardly to obtain the interferometric products. Nonetheless, an additional fine co-registration step has been applied, as it turned out to be effective for the refinement of the matching between the two slaves.

The last step was then the generation of the interferometric coherence and phase images. The products used for the tests are coherence and phase images 2 × 10 muli-looked, the coherence was estimated on a 2 × 10 window. A coherence estimation window of 2 × 10 pixels seemed a good trade off between the necessities, on one side, of choosing a large enough window in order to give a reliable estimate of the coherence and, on the other side, of working on the highest possible resolution, which is desirable especially when anthropogenic features are considered, since these are usually very small compared to the spatial scale of the image. Moreover, the fact that the coherence estimation window and the multi-look size are the same allows us to refer directly the estimated coherence to the corresponding phase value

1Tests confirmed what was already highlighted in the literature ([27, 55]) slave and master, i.e. that azimuth filtering highly improves the coherence. This is particularly important for the long time scale interferograms, where the co-registration is more difficult because of the low coherence. In some cases co-registration did not succeed without first filtering in azimuth.
4.2 The database

The whole research presented in this thesis was performed on two test sites having completely different topographical and geophysical characteristics. This was done in order to assess the degree of dependence of the results from the particular selected test site and at the same time to arrive at conclusions which are independent from this choice.

4.2.1 Test Site 1: Groningen

The first site considered is located in the province of Groningen, in the North of the Netherlands. Here one of the largest gas fields of Western Europe was discovered in 1959. The main field stretches over an area of almost 900 km² and has an initial reserve estimated in about $2900 \times 10^9$ m³ [32]. The exploitation of the gas fields, started in 1964, causes subsidence in more than half the province of Groningen. The subsidence has a bowl shape and a mean rate of up to 1 cm per year. Prognosis indicate that when the fields will be totally exploited the maximum total subsidence will amount to about 40 cm. The subsidence of Groningen is carefully monitored. A portion of the area is in fact below sea level, and therefore monitoring of the sea level is extremely important. Moreover, due to its large extent the deformation affects also infra-structural works like roads, canals, bridges, etc. The database of interferograms of this site was generated by using 13 SLC images both from ERS-1 and ERS-2. The images, listed in Table 4.1, are quarter scenes (50 × 50 km wide) spanning a time interval of three and half years, from 1992 to 1996. Notice that in the table each image has been associated with a "day number" (the same for images forming Tandem pairs), which will be used in Chapter 5 to represent the distribution of the time intervals spanned by the used interferograms (Fig. 5.2).

From the interferometric point of view the northern Netherlands can be said to be a fast decorrelating zone. The main reason for this is that most of the land cover-

---

Notice that this does not imply that neighbouring multi-looked pixels are uncorrelated. In fact, multi-looking uses an averaging window, while the coherence estimation is performed with a shifting window: therefore, the single-look pixels at the border of two adjacent multi-looking windows, which will contribute to two different average values, share actually a portion of the estimation window. The choice of a multi-looking window having the same size of the estimation one reduces however the statistical correlation only to the adjacent multi-looked pixels.
The database approach

4.2.2 Test Site 2: Campi Flegrei

The Campi Flegrei (Phlegrean Fields) test site (Fig.4.2) can in some sense be considered complementary to the Groningen area: high rates of deformation, less atmospheric disturbances, smaller influence on temporal decorrelation due to meteo-
Table 4.1  
ERS images of Groningen considered for the generation of the database. All the images have frame no.2529. The 4th column shows the day number, i.e. the serial number assigned to the image (see also text).

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<th>day</th>
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</table>

rological variability and a significant topography.

Situated about 15km to the West of Naples, Campi Flegrei must be considered one of the most dangerous volcanic areas in Italy. Campi Flegrei is a nested caldera, formed mainly during two extremely violent explosive eruptions, about 12,000 and 37,000 years ago, which produced Campanian Ignimbrite and Neapolitan Yellow Tuff respectively [49]. The last eruption took place in 1538, and gave birth to the Monte Nuovo mountain. The caldera of Campi Flegrei is still active, as testified by the fumarolic and magmatic activity at the Solfatara Crater and by the phenomenon of Bradyseism, which is the alternating sinking and uplift of the ground within the caldera. The phenomenon is known since the Roman times. Levelling surveys made at the beginning of the century showed that the maximum sinking was occurring in the city of Pozzuoli and regularly decreased eastward and westward along the coast. This slow sinking of the ground continued until 1968. In the periods 1969-1972 and 1982-1984 two strong uplift phases occurred, with a maximum uplift, again in the city of Pozzuoli, of 170 and 180cm respectively; those uplift phases were accompanied by intense seismic activity. In between these two episodes a subsidence period has occurred (from 1972 to 1974), with a maximum subsidence of 22cm, followed by 8 years of no significant terrain deformations and no seismic activity. From 1985 subsidence is occurring again, interrupted only by two small uplift episodes. The uplift episodes are always associated with seismic events, while subsidence phases are not.

The higher current deformation rate, estimated from levelling data at about 4mm per month, on one hand makes the detection of deformations easier than in the case of Groningen. On the other hand this, together with the fact that the topography
Figure 4.2  The Campi Flegrei area.

Figure 4.3  Deformation measured at the maximum deformation point (benchmark no.25. in downtown Pozzuoli) from 1969 to 1997 (courtesy of Osservatorio Vesuviano, Naples).
4.2 The database

Figure 4.4  Height variations in the directions E-W (upper plot) and N-S (lower plot) in the period January 1982-December 1984 (courtesy of Osservatorio Vesuviano, Naples).

is relevant even for short baselines, causes the presence of both topographic and deformation fringes. Phase unwrapping is therefore necessary in the application of the differential technique, and unwrapping errors are an additional error source with respect to the Groningen test site. Due to its high volcanic risk, the area is monitored on a regular basis by means of different techniques, among which GPS and precision levelling [1]. The availability of such spatially and temporally dense measurements from levelling campaigns permits thus to check the INSAR results, in particular with respect to phase unwrapping errors.

Finally, it has to be remarked that, as they are full scenes (100 × 100km wide), the images used for the retrieval of deformations at Campi Flegrei, comprise ac-
The database approach

Table 4.2  
ERS images of Campi Flegrei considered for the generation of the database. The images have all frame no.2781. The 4th column shows the day number, i.e. the serial number assigned to the image (see also text).

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Eventually the whole Neapolitan area. Another imaged portion of the area, not subject to deformation, was then used to repeat the tests on highly coherent anthropogenic features performed in the Groningen test site. Due to the different characteristics of the two sites, in fact, this seemed a good method to assess whether, and which part, of the results from the study of the first one could be considered as site-independent and therefore lead to general conclusions.

The images of Campi Flegrei considered for this research are listed in Table 4.2. Here also, as for in Table 4.1 for Groningen, the images are given a day number, this time indicated as $b\#$; this day number will be used for representation purposes in Chapter 7, Fig. 7.2.
As already mentioned in the previous chapter, the Groningen area is subsiding at the rate of, at most, 1 cm per year, and therefore interferograms over a minimum time span of one year are necessary in order to have a significant amount of deformation to be detected. Interferograms of this site on such time scales, and actually already on intervals of a couple of months, are however heavily affected by temporal decorrelation and look like the one on the right side of Fig. 5.1: a completely decorrelated image dotted with what is still strong, almost point-wise information, mostly from anthropogenic features. Only "patches" of coherence can be seen, related to the presence of cities and groups of buildings. Due to the general decorrelation and also to the small rate of subsidence (the total deformation expected in the whole time interval considered here does not amount even to one fringe), no clear deformation pattern is visible in the corresponding interferometric phase image. The only possibility to measure deformations is therefore to try to use the information from these highly coherent features and pixels. In order to do this, it has first to be assessed whether the phase information from these objects is still reliable. For this purpose, an analysis of the interferometric characteristics of these features is performed (Section 5.2). Based on the conclusions from this analysis, part of which has been published in [64], a method for the retrieval of the deformations has been developed, which is presented in Section 5.4, and its results are compared with those from a pre-existing subsidence model.
Figure 5.1  Interferometric coherence images on intervals of 1-day (Tandem pair 16/17-3-1996, above) and three and half years (16-3-1996/ 10-9-1992, below).
Table 5.1 The dataset of interferograms used. The 4th column shows the time span in terms of days and the 5th the serial number of the interferogram.

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5.1 Interferometric database

In Table 5.1 are shown the characteristics of the 21 interferograms generated for this study as combinations of the ERS images of Table 4.1. Figure 5.2 additionally shows the time span covered by each one of them: on the y-axis is represented the interferogram serial number, while on the x-axis are given the day numbers, as defined in Table 4.1 of Chapter 4. In the first two columns are given the dates corresponding to the master and slave images, followed by the length, as computed at the image centre, of the perpendicular component of the baseline. The fourth column gives the time interval, in days, spanned by the interferogram, the fifth column its serial number. As can be seen in the Table, the interferograms are labelled with a number from 1 to 21 and are ordered in terms of the length of the time interval they
Figure 5.2 The set of interferograms generated from the SLC images listed in Table 5.1. The time spans are depicted as horizontal bars on a linear scale, with the days labelled according to Table 4.1 of Chapter 4.

Figure 5.3 Amplitude images of the road (on the left) and of the factory (on the right) being examined.
Interferogram no.20 has been used for the estimation of the topography, as it was both generally well correlated and with a relatively large baseline (about 145 metres).

5.2 Analysis of highly coherent features

5.2.1 Selection of the features

The features considered for the tests are two single objects, which can be identified as a road and a very large building, and seven larger areas, of different size, delimiting either a group of buildings or an urban settlement. Their amplitude images are shown in Fig.5.3 and Fig.5.4 respectively, where the areas are labelled with letters from A to G. The features selected for the tests were identified by observing the longest term interferograms. They are formed by a certain number of highly coherent pixels, more or less the same in all the long term interferograms, which define their shape and make them therefore recognisable as candidates for the stability analysis.

In both cases of single features and of urban areas, defining the contours of the area to be considered is not immediate. The problem is evident when an urban area is considered, as the definition of the limits of such an area is arbitrary. Also in the case of a single feature, defining its contours may be not trivial: different interferograms show in fact variations in the coherence of the pixels at the borders of the feature itself. Visual analysis of the coherence images covering the longest time spans usually gave a clue on the extension of the cluster of highly coherent pixels to be considered, especially in the case of a single feature. A window was therefore taken, centred on the feature and small enough to minimise the risk of including highly coherent pixels from the surroundings. As it will be shown in the analysis, such pixels are highly coherent only in one or few interferograms and they are detached from the usually rather compact image of the feature, therefore they are often clearly distinguishable from those of the feature itself. As for the urban areas, the higher density of buildings resulted also in an higher density of highly coherent pixels and, as it will be shown further in this chapter (see Fig.5.22), for this reason the cities are clearly distinguishable in the long term interferograms. The decrease in the density of highly coherent pixels was assumed to coincide to the borders of the urban area, and the selected section of image was then the smallest possible rectangle enclosing that area.
Figure 5.4  Amplitude image showing the 7 areas selected for the study.
5.2 Analysis of highly coherent features

5.2.2 Tests

The interferometric properties of the selected anthropogenic features were analysed by applying the differential technique as explained in Chapter 2 on a pixel by pixel basis and performing different tests to the resulting differential phases. The procedure and the meaning of each test, in terms of this study, are explained in the next subsections. As for the computation of the differential phases, the following procedure was applied.

First of all, after the same feature or area of interest was extracted from each interferogram, the set of all its pixels having coherence above a certain fixed threshold $\Gamma_{\min}$ was considered. Coherence thresholds from 0.5 up to 0.8 were taken: pixels with coherence lower than 0.5 were not used because the phase at that level resulted to be very noisy and also because the effects of the bias in the coherence estimation are not negligible [59]. As for the upper limit of 0.8, it was chosen because usually (although not always) the number of pixels having coherence above this value is still large enough to enable a statistical approach to be applied, while higher coherence thresholds result often in a number of selected pixels too poor for such an approach.

For a certain $\Gamma_{\min}$, a set of pixels $S_i$, with their coherence and phase values, was associated with each interferogram $I_{F_i}$. Subsequently, one of the interferograms, $I_{F_r}$, was taken as the reference interferogram for the topography extraction, i.e. its corresponding set of selected pixels $S_r$ was taken as the reference for the comparison with the sets of pixels of the other interferograms (in our case $r = 20$). Each set $S_i$ (with $i \neq r$) was in turn compared with $S_r$, and, for each pixel belonging to both the sets, the differential phase value was computed by rescaling and subtracting from the phase value of the pixel itself in $S_i$ the corresponding value in $S_r$ [63].

Notice that all the used interferograms had baselines short enough to assume that the interferometric signal does not contain a significant topographic component (the maximum height difference in the area is less than 11 metres). Since however subsidence occurs in the area at such a slow rate, that the expected deformation on the considered time intervals would result in less than one fringe, the differential method has still been applied in order to be sure that even the smallest contribution from topography was subtracted. Moreover, as the absence of fringes causes the interferometric phase to be already unwrapped, in this particular case phase unwrapping was not necessary. Of course, phase unwrapping must always be performed before applying the differential method whenever the topographic and/or deformation component of the signal is large enough to cause the presence of interferometric fringes. In these cases it can be a rather challenging issue if the interferogram is decorrelated, however attempts to solve this problem are already being made, [11, 15].
Once the differential method had been applied, for every interferogram, except for the reference one, a set of differential phases \(DF_i\) \((i = 1, \ldots, 21, i \neq 20)\) was obtained, determined on a pixel-by-pixel basis. To these phases different tests were applied, based mainly on the evaluation of three statistical quantities. The first aim of these tests was the estimate of the degree of spatial homogeneity of the set of pixels, i.e. how similar is, in a certain interferogram, the phase information coming from different pixels. Secondly, the stability in time of the phase of single pixels was studied by comparing the phase values of the same pixel in the different interferograms of the series. Finally a correlation analysis of the phase values of the set in different interferograms was performed in the cases when the presence of a deformation signal within the set of pixels was suspected.

It is worth observing here that the results from the presented tests are unlikely to be affected by atmospheric (e.g. clouds) or meteorological (e.g. rain) effects. On one hand, in fact, most of such effects are expected to occur on a larger spatial scale than the considered features: in this case the effect would be a bias of all the phase values of the feature. However since each feature is examined separately and intrinsically such a bias can neither be detectable, because the phase values are not referred to a reference phase value external to the feature, nor have any effect on the results, which are based on measurements of standard deviations. On the other hand, supposing the presence of atmospheric/meteorological effects of smaller scale than the scale of the feature does not make the results less valid. The purpose is in fact to estimate the phase stability of a feature, e.g. the degree of phase noise present at the features scale. Not only atmospheric and meteorological effects, but also incidental ones such as temporary modifications in the structure or backscattering centres of the feature, acting at the features spatial scale reveal themselves as noise in this analysis (since they are not likely to repeat themselves in two different interferograms, they cannot be considered signal). The aim is, however, not to discriminate the sources of this noise, but to estimate it in its totality, estimating in this way the stability of the feature itself.

"Homogeneity" of a feature

The first quantity considered is the standard deviation of the differential phases for each interferogram:

\[
\sigma_i = \sqrt{\sum_{j=1}^{N_i} (d\phi_{ij} - d\phi_i)^2 / (N_i - 1)} \quad \forall d\phi_{ij} \in DF_i \quad j = 1, \ldots, N_i \quad (5.1)
\]

where \(DF_i\) \((i = 1, \ldots, 21\) in this case, \(i \neq r\)) is the set of differential phases \(d\phi_{ij}\) of interferogram no.\(i\) having coherence above the chosen threshold, \(N_i\) is their number.
and \( \overline{\phi_i} = \frac{\sum_{j=1}^{N_i} d\phi_{ji}}{N_i} \) is their mean value.

This standard deviation gives a measure of the \textit{spatial homogeneity} of the feature or area. In fact, if either the area is not displaced in the vertical direction or, in presence of terrain deformations, is subject to a rigid movement, then all the pixels in which it is imaged will show more or less the same phase shift. The differential phases will therefore be very similar and their standard deviation will be small. Conversely, if the standard deviation of the differential phases is high, this means that pixels imaging different parts of the area, are (at least partially) subject to different effects, i.e. the area carries spatially inhomogeneous information. A loss of homogeneity can be caused by incidental effects, such as structural modifications (in the case of a single feature), a partial change of the backscattering characteristics due for example to the presence of snow or water, or changes in the scattering centres. These effects could add noise to the differential phases causing the standard deviation to increase. However, a high standard deviation could also be caused by a true deformation affecting different parts of the area to a different degree, which means that this latter has undergone an intrinsic deformation. As it will be shown in the results, it is possible to discriminate between the two cases by comparing the differential phases obtained in different interferograms, using the fact that, while incidental effects are unlikely to be the same in different interferograms, a true deformation signal would show up the same in interferograms ranging over the same period of time.

**Phase stability in terms of time**

Instead of computing the standard deviation of the differential phases with the \textit{spatial} approach just explained above, i.e. examining all the pixels in each interferogram, one can use a \textit{temporal} approach, by considering the same pixel in a set of interferograms. The comparison of the differential phase values assumed by a certain pixel in different interferograms, can give an estimate of the \textit{stability in time} of the pixel itself.

Given a set of interferometric images of a feature or area covering a certain time interval, if they carry the same (deformation) information, then this implies that in each of these interferograms the phase of a certain pixel is shifted by the same quantity with respect to a "reference" phase value (in the sense of the zero phase of the considered area, which is not to be confused neither with the reference interferogram, nor with the reference phase value of the whole interferogram). In such a case, the phase values of the pixel in all the interferograms would be the same and their standard deviation would be zero. Analogously to the first test, a
standard deviation gives here also a measure of stability. The difference is that while in the first case a sample of different pixels in the same interferogram was considered, here the sample consists of the phase values from the same pixel in the different interferograms, thus giving an estimate of the intrinsic phase stability over the course of time of that particular pixel.

In order to compute the standard deviation per pixel, a certain number $N_i (N_i \leq 21)$ of sets $DF_i$ was considered, for which one can assume that they either are spatially homogeneous, i.e. no internal deformation has taken place, or have undergone the same intrinsic deformation. In each of the sets $DF_i$ the mean value was subtracted from all the differential phases:

\[ d\phi_{ij} = d\phi_{ij} - \overline{d\phi_i} \quad \forall j = 1, \ldots, N_i \quad i = 1, \ldots, N_l \quad (5.2) \]

Each pixel was then searched for in all the sets $DF_i$ of new values $d\phi_{ij}$ and, when the pixel was found in at least two sets, the standard deviation of its phase values was computed. If $M$ is the total number of pixels occurring at least twice in the series, then to each pixel the following standard deviation $\sigma_k$ can be associated:

\[ \tau_k = \sqrt{\frac{\sum_{p=1}^{L_k}(d\phi_{pk} - \overline{d\phi_k})^2}{L_k - 1}} \quad p = 1, \ldots, L_k \quad k = 1, \ldots, M \quad (5.3) \]

where $L_k$ is the number of occurrences of the $k$-th pixel, and $\overline{d\phi_k}$ is the mean of the $L_k$ values taken by the pixel.

Notice that the meaning of $\tau$ is completely different from the meaning of the standard deviation $\sigma$ computed in the previous section and the results of the two tests are indeed independent from each other. So a high $\sigma$ can be the result of either incidental effects or a true internal vertical deformation of the feature or area. In this latter case, however, each single pixel could still result to be very stable over the course of time – and have therefore a small $\tau$ – maintaining more or less the same shift with respect to the mean value in each interferogram. The fact that this shift is different for different pixels will cause a deformation pattern, very similar in all the considered interferograms, which can be detected by means of a correlation analysis.

### Correlation analysis

The "spatial" standard deviation, i.e. the standard deviation related to the spatial homogeneity of a feature, is always different from zero. This is caused by what can

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1The author is aware of the poor significance of a standard deviation estimation based on only two measurements. However, at this stage she was only interested in excluding pixels with only one value, for which the standard deviation is not defined.
be called in the context of this research "noise" and which comprises all effects other than real deformation such as, for example, meteorological effects and occasional changes in the feature itself. However, a high standard deviation could also indicate the presence of a signal due to a deformation that occurred intrinsically in the feature or area. The suspect of an intrinsic deformation can be enforced by a high stability in time of the majority of the pixels. A way to check the presence of a deformation signal, is therefore to compare the different sets with each other by computing their correlation coefficient. The correlation coefficient between two random variables $x$ and $y$, defined as [51]:

$$
\text{Corr}(x, y) = \frac{E\{xy\} - E\{x\}E\{y\}}{\sigma_x \sigma_y} \tag{5.4}
$$

where $\sigma_x$ and $\sigma_y$ are the standard deviations of $x$ and $y$ respectively, gives in fact an estimate of the "similarity" of the two random variables. As in the case of the computation of the "temporal standard deviation" $\tau$ in the previous section, we want to consider only the $N_I$ sets from those interferograms which are supposed to contain the same deformation information. For each possible pair of these $DF$’s then the correlation coefficient $C_{rij}$ is computed by considering their common pixels:

$$
C_{rij} = \text{corr}(DF_i \cap DF_j) \quad i, j = 1, \ldots, N_I \quad j \neq i, \quad i, j \neq 20 \tag{5.5}
$$

It is clear that a generally high correlation coefficient between all the interferograms will indicate the presence of a common deformation signal. Conversely, all the effects other than deformation are not likely to occur in the same way in all the interferograms, and for this reason one can expect that their presence will cause a decrease in the value of the correlation coefficient.

### 5.2.3 Results on selected features

#### Case I: a "road"

The first object to be examined shows up in coherence images as an highly coherent linear feature the size of 1 pixel in the range direction and about 60 pixels in the azimuth direction. Its coherence image in one of the long term interferograms is shown in Fig.5.5. It is localised in correspondence of a road, and it will be indicated as a "road", although it cannot be stated where exactly the signal is reflected from. Likely the backscattering comes not from the road itself, but from its edge or from a structure located along its edge such as a guard rail. It has to be stressed, however, that it is not much relevant in this context which kind of feature is giving such an highly coherent signal, but only whether to such a high coherence corresponds also
phase stability in time. First of all, the spatial homogeneity of the road was checked
by computing the spatial standard deviations $\sigma$ in each interferogram according to
Eq. (5.1). The $\sigma$'s obtained for different values of the coherence threshold $\Gamma_{\text{min}}$ are
shown in Fig.5.6, together with the corresponding number of pixels considered for
the computation. It can be observed that, with the only exception of interferograms
$IF$ 3 and 13, where passing from a $\Gamma_{\text{min}}$ of 0.7 to 0.8 causes an increment in the
standard deviation, in all interferograms $\sigma$ becomes smaller as $\Gamma_{\text{min}}$ increases. This
is to be expected if one assumes that having an higher coherence implies having a
less noisy – and therefore more reliable – information. However, the improvement
in the standard deviation with increasing coherence thresholds varies greatly from
one interferogram to another. In some cases, in fact, a higher $\Gamma_{\text{min}}$ will cause a
considerable reduction in $\sigma$ (see for example $IF$ 5). In other cases, such as for
example in $IF$ 7, the improvement is definitely less. A difference in standard devi-
ation values between long and short term interferograms is also visible in Fig.5.6:
interferograms ranging from 1 day (no.21) up to 70 days (no.15) have a $\sigma$ at the
level of 0.4 radians even for $\Gamma_{\text{min}} = 0.5$, while in long term interferograms (from
no.1, over 1319 days, up to no.14, over 829 days), the standard deviation is in most
cases higher and varies more strongly with different thresholds. As we shall see
later on, there is a possible explanation for this difference between short and long
term interferograms. There are anyway still long term interferograms showing $\sigma$'s
at the level of 0.4 to 0.6 radians for $\Gamma_{\text{min}}$ of 0.7 and 0.8: see for example interfer-
grams no.1,2,4 and 6. Notice also that these same interferograms show smaller

Figure 5.5  Detail of a coherence image showing the road being examined.
standard deviations than other (long term) interferograms spanning shorter time intervals, such as those from no.9 to no.14. This could suggest that $\sigma$ does not depend on the considered time interval, at least on time scales longer than one year.

Finally, let us consider the groups of interferograms $I_1=\{1, 2, 4, 6\}$ and $I_2=\{3, 5, 7, 8\}$. The interferograms in each of these two groups are the combinations of the same four SLC images with a unique other SLC, say $SLC_1$ in $I_1$ and $SLC_2$ in $I_2$. As may again be seen in Fig.5.6, the standard deviations of the differential phases are all of the same order of magnitude for interferograms of the same group. On the other hand, the $\Gamma$'s in $I_1$ are remarkably lower than those in $I_2$: this seems to indicate a lower quality of $SLC_2$ than of $SLC_1$, at least with respect to the four SLC’s with which they have both been combined.

Fig.5.6 shows also the number of pixels used for the computation of the standard deviations. The long term interferograms have about the same number of pixels for each considered $\Gamma_{min}$, while the number of highly coherent pixels changes clearly as one progresses from long to short term interferograms (note that the scale in the plots is logarithmic). The only exception is no.17, which has a lower number of highly coherent pixels. An analysis of the coherence image revealed that this interferogram indeed suffers from a heavier decorrelation than the other short term ones and it is strongly suspected that this was because of the presence of snow at

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**Figure 5.6** Standard deviation $\sigma$ of the differential phases for the road (above) and number of pixels used (below) for different values of the coherence limit $\Gamma_{min}$.
Figure 5.7  Differential phases along the road in four cases for $\Gamma_{\text{min}} = 0.5$. On the x-axis is represented the line number, on the y-axis the differential phases computed with respect to their mean value.

the time when one of the images was taken.

Let us now take a closer look at the differential phases from which the $\sigma$’s have been computed. Fig.5.7 shows the differential phases along the road in four of the considered interferograms. The x-axis represents the line coordinate of the pixels (as already mentioned, in the multi-looked products the road is a vertical line with constant range coordinate) and the y-axis is the differential phase value in radians. Note that, since we do not have a reference phase, the differential phase values in themselves do not have a real significance. The purpose here is only to visualise the spread of the values along the road. On the upper part of Fig.5.7 are shown the differential phases for two short term interferograms, namely a 1-day (Tandem) and a 35-days interferogram: the phase along the road is indeed rather constant, as already indicated by the low values of $\sigma$ in Fig.5.6. In the lower part of the same figure the differential phases of two interferograms spanning about three and half years are represented. Here the phase values are more dispersed than in the short term case, which corresponds again with the higher standard deviations observed in Fig.5.6. However, they show a similar pattern: what is remarkable is that indeed this
5.2 Analysis of highly coherent features

Figure 5.8  Comparison of differential phase values in different pairs of interferograms. In order to compare the values, the mean was subtracted in each interferogram. The vertical lines in the plot delimit the three sections in which the road can be divided.

pattern is very much the same in all the first ten interferograms of the series. This might suggest that a deterministic signal is present in the differential phase. In other words, this could mean that the road has changed in the course of time, and the pattern could actually represent the deformation that has occurred, which amounts to about 0.5cm. The similarities can easily be seen if the differential phases of two interferograms are placed in the same plot, as it is done in Fig.5.8 for four different pairs. The values of correlation coefficient $\rho_{ij}$, computed according to Eq.(5.5), for all the possible pairs of the fourteen long term interferograms ($N = 14$) are shown in Fig.5.9. They confirm the presence of similarities in the pattern of the differential phases within the interferograms, indicating a particularly strong agreement between the first 8 differential interferograms. The correlation values for the 4 cases shown in Fig.5.8 were respectively equal to 0.97, 0.88, 0.89 and 0.98. Notice that, although in the cases represented in Fig.5.8 (a) and (d) the two interferograms had one image in common, in the cases shown in (b) and (d) they are obtained from different pairs of images, and still they are highly correlated. Since the road has undergone an intrinsic deformation, the standard deviation $\sigma$ cannot be considered
as a measure of its phase stability in space with respect to all the possible incidental effects. However, by looking at the differential phases, like for example those in Figs.5.7(c) and 5.7(d), the road appears to be divided into three sections, as indicated in Fig.5.8, and while the third section consists of few pixels and varies rather greatly from one interferogram to the other, the first two parts remain quite similar in all the ten longest term interferograms. In particular, along the first section the phase values seem to be rather uniform, suggesting that within this part of the road no intrinsic deformation occurred. The test on the spatial homogeneity was therefore repeated by considering only this section. In Fig.5.10 the standard deviations $\sigma$ of the first section are plotted together with the number of considered pixels. As one can see, for this section the values are, in most cases, lower than when we consider the whole road. Except for those cases where too few pixels where selected (i.e. $IFs$ 14 and 17 for $\Gamma_{min} = 0.8$ and $IF$ 14 also for $\Gamma_{min} = 0.7$), and which therefore should not be considered, for the other interferograms $\sigma$ drops well below 0.4 radians already at $\Gamma_{min} = 0.6$, and in some cases this happens even with $\Gamma_{min} = 0.5$. Finally, the phase stability in terms of time of single pixels was estimated, as described in Section 5.2.2. For this purpose, the long term interferograms from no.1 to no.8 were considered, which, as also indicated by the correlation values in Fig.5.9, showed the most similarities and thus can be assumed to contain the same deformation information. The coherence threshold for the pixels selection was set
at $\Gamma_{min} = 0.5$, and in each interferogram the phase values were referred to their mean value, which was taken as a reference (zero) value. The standard deviations $\tau$ were computed for every pixel occurring at least twice with coherence $> 0.5$ in the eight interferograms: they are shown in Fig.5.11, together with the corresponding number of occurrences, for each considered pixel. As one can see, there are several pixels occurring in all the eight long term interferograms and showing a remarkable temporal stability, with $\tau$ ranging from 0.2 to 0.4 radians.

**Case II: a building**

After the analysis of a linear feature like a road, a two-dimensional object, namely a building housing a factory, was considered. The building, of which a coherence image in a long term interferogram is shown in Fig.5.12, is remarkably long (about 1km) and therefore covers an area of some tens of pixels. Since different decorrelation conditions cause slight variations in each coherence image, it is difficult to define with certainty the building’s contour. A window was therefore considered, centred on the feature and small enough ($10 \times 30$ pixels) to include as few pixels as possible reflecting from the surroundings rather than from the building. Most
Figure 5.11 Standard deviations per pixel $\tau$ computed on the set of the first 8 interferograms and no. of times that each pixel occurs in the set. $\Gamma_{\text{min}}$ is here 0.5.

Figure 5.12 Detail of a long term coherence image showing the building being examined.
5.2 Analysis of highly coherent features

Figure 5.13  Standard deviation $\sigma$ for the factory and no. of used pixels for different values of $\Gamma_{\min}$.

such pixels were discarded anyway through the masking of $\Gamma_{\min} = 0.5$. Figure 5.13 represents the standard deviations $\sigma$ of the phase differences for the building, computed here also, as for the road, with the assumption that the structure is homogeneous with respect to deformations and that therefore the standard deviation is a measure of the phase stability in the course of time. Notice that 5 out of the 21 interferograms are not represented in these figures or in the next figures concerning the building: the feature is in fact located rather at the edge of the imaged scene and in some interferograms it falls outside the overlapping area between the two SLC images. Note also that, while the road is known to be in an area of no subsidence, subsidence is indeed taking place where this factory is located.

The results show similarities with those of the road. With the exception of a few cases, the $\sigma$ decreases here also with increasing $\Gamma_{\min}$. The remaining interferograms of the groups indicated as $I_i$ in the analysis of the road, namely $IF$’s 2,4 and 6, show here also similar values and the same can be said for the remaining interferograms of the group $E_j$, i.e. $IF$’s 5,7 and 8. Contrary to the road case, the latter group shows standard deviation values that are much smaller than those of group $I_i$. What is noticeable is that here too interferograms with one image in common seem to produce similar results.

A difference with respect to the case of the road is that there is no visible worsening of the standard deviation as one passes from short to long term interferograms. Since in that particular case the worsening was probably due to the presence of a
Figure 5.14 Images of the differential phases $DF$ of the factory in the 3 interferograms of group $I_1$ (left) and in those of group $I_2$ (right) for $\Gamma_{\text{min}} = 0.5$ (the colour scale represents a $2\pi$ cycle).

Figure 5.15 Correlation coefficient $Cr$ in the case of the factory for all the possible couples of long term interferograms (left) and no. of used pixels (right), for $\Gamma_{\text{min}} = 0.8$. 
5.2 Analysis of highly coherent features

Figure 5.16  Standard deviation per pixel $\tau$ for the factory in the first 14 interferograms (left) and corresponding no. of occurrences (right), for $\Gamma_{\text{min}} = 0.6$.

Figure 5.17  Standard deviation per interferogram $\sigma$ for the factory and no. of pixels used for only pixels with $\tau < 0.4$. 
intrinsic deformation signal, its absence here could indicate that there is no such signal in the case of the building.

Finally here also, as with the road, the number of selected pixels is much the same in all interferograms with the exception, of course, of the tandem interferogram. Due to the larger size of the feature, the number of pixels considered is generally almost double with respect to the corresponding value for the road.

A visual inspection of the differential phase images does not give further clues on the interferometric characteristics of the feature. Although in some cases a pattern seems to be present in the differential phases, there is no evidence of striking similarities between the interferograms such as in the case of the road. Only the interferograms of group \( I_1 \) show a clearly similar differential phase pattern, shown in Fig. 5.14(a) for the case of \( \Gamma_{\text{min}} = 0.5 \), which is the cause of the increased standard deviation. However, since as already mentioned, the interferograms of group \( I_1 \) have an image in common and since those of group \( I_2 \) are on the contrary particularly homogeneous (see Fig. 5.14(b)), it is suspected that the pattern is caused by the presence of an effect, maybe an atmospheric one, in the common image.

The computation of the correlation coefficient \( C_r \) is shown in Fig. 5.15 for the case \( \Gamma_{\text{min}} = 0.8 \), together with the number of pixels used. Even with such a high coherence threshold, it was not possible to identify a group of differential interferograms that clearly correspond with each other: instead, each combination of two sets of differential phases gives a different correlation value. For example, the interferograms \( IIF' \)'s 2, 4 and 6, when combined with each other, give very high correlations, again most probably because of their common image. There are also combinations, like that of \( IIF' \) 9 with \( IIF' \) 14 and 16, which result in high correlations, although they do not have images in common: on the other hand, each of these three sets, when combined with others, gives very low and sometimes even high-negative correlations.

The last step in the analysis of the feature was the computation of \( \tau \). In the case of the road, the first 8 interferograms were considered for this purpose which, as the high correlation coefficients also showed, were in strong agreement with each other. For the building however there was not such a similar agreement within a set of interferograms, so all the long term ones, from no.1 to no.14, were considered. An example of the computed \( \tau \)'s is shown in Fig. 5.16 for \( \Gamma_{\text{min}} = 0.6 \), where each value is represented in the location of the pixel it refers to. As one can see, there are several pixels occurring at least 10 times, and most pixels at the centre of the building are present in all the 14 interferograms. Most pixels turn out to be quite stable, with \( \tau \leq 0.5 \) radians.

Since the pixels actually showing a high \( \tau \) are quite a few (see Fig. 5.16), it is likely that, by eliminating them, the standard deviation per interferogram \( \sigma \) will
become lower: in other words, eliminating the pixels which are less stable in time could result in an improved stability of the feature in space. For each value of $\Gamma_{\text{min}}$ only those pixels with $\tau < 0.4$ were therefore considered and the $\sigma$’s were recomputed. As can be seen in Fig.5.17 the resulting $\sigma$’s are, in general, much lower than when all the pixels above the coherence threshold were used (Fig.5.13).

**Case III: a city**

Unlike the first two cases, where single features were considered, the third case study is a city, i.e. a collection of anthropogenic targets. These are mostly buildings of size smaller than the road and the factory, which therefore cover only one or a few pixels in the images; moreover, they are spread over a much larger area, of about $4.8 \times 6.8$km. The standard deviations per interferogram $\sigma$ are shown in Fig. 5.18. The values are definitely larger than in the two previous cases, ranging from approximately 0.55 up to 1.25 radians in the first 17 interferograms of the series, and only in the shortest time intervals, from 35 days, their values become smaller. Since interferograms $IF^1$ to $IF^7$ relate to short intervals, this seems also to indicate that in this case short term interferograms do not necessarily lead to lower $\sigma$’s than long term ones. Moreover, the value of the standard deviations does not seem to be related to the value of $\Gamma_{\text{min}}$, as in the other two cases: for example, for interferograms $IF^7$ to $IF^{17}$ a threshold $\Gamma_{\text{min}} = 0.8$ gives worse results than
for $\Gamma_{\text{min}} = 0.5$. Notice finally the number of selected pixels, which is much larger than in the other two cases and drops to about half of the pixels, in other words more heavily, when $\Gamma_{\text{min}}$ is increased by 0.1.

The fact that the $\sigma$’s are generally higher than in the two previous cases is probably due to the fact that we do not have here one large, unique feature, but a collection of smaller ones. Of course these features are all different and each one can be subject in a different way to all the possible effects which could cause a bias in its phase value without affecting the coherence. After all, the selected pixels do not belong physically to the same feature, so they are expected to behave less homogeneously.

Figure 5.19 shows the standard deviations per pixels $\tau$ in the series of the first fourteen interferograms, together with the corresponding number of occurrences, for $\Gamma_{\text{min}} = 0.5$. Differently from the case of the building and the road, most of these pixels only occur in a limited number of interferograms (up to 6) with coherence above $\Gamma_{\text{min}}$. The information thus originates in general from different scatterers in different interferograms or, in an equivalent way, the coherence of most scatterers oscillates around the value 0.5. This is also confirmed by the fact that, using a coherence threshold $\Gamma_{\text{min}} = 0.6$, the number of pixels occurring at least in two out of the fourteen long term interferograms, and thus for which $\tau$ could be computed,
5.3 A general analysis of urban areas

The analysis of the three case studies presented above gave some insight in the interferometric characteristics of highly coherent features at pixel level. Based on the results, some hypothesis have been formulated which have been tested on a sample of seven urban areas and features, shown in Fig.5.22. Notice that the area was reduced to less than half.

Analogously to the case of the building, here also the $\sigma$’s were re-determined using only pixels with $\tau < 0.4$, in order to see whether selecting the most time-stable pixels could result in an improvement of the spatial homogeneity of the area. The $\sigma$’s so determined are shown in Fig.5.20. The improvement with respect to the values computed by using all the pixels in Fig.5.18 is clearly visible and ranges from 0.1 up to even 0.8 radians depending on the interferogram. The selection has considerably reduced the number of pixels which, however, with the exception maybe of a couple of cases for $\Gamma_{\min} = 0.8$, is always large enough to justify the statistical approach. Finally, the correlation coefficient has been computed for all the possible combinations of the first fourteen interferograms and for $\Gamma_{\min} = 0.5$ (Fig.5.21): no particularly high correlation values are noticeable, which confirms the random character of the differential phases and thus the absence of a signal.

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Figure 5.21  Correlation coefficient $C_r$ in the case of the city for all the possible couples of long term interferograms (left) and number of pixels used (right), for $\Gamma_{\text{min}} = 0.5$.

Figure 5.22  Long term coherence image of the 7 selected areas.
labelled with $A$ is the city of Assen, which is also the one analysed for the case study III presented in the previous Section.

### 5.3.1 Spatial stability of urban areas

The values of the spatial standard deviation for each interferogram and for the different coherence limits are shown in the Fig.5.23 for all the considered areas (indicated with the letters $A$ to $G$). In the lower part of each figure is also shown the number of pixels considered for the computation of the corresponding standard deviation value.

Except for the case of the Tandem interferogram ($IF$ 21), which has always considerably lower standard deviations than the other ones, there is no visible dependence of the $\sigma$ on the considered time interval. The standard deviations of the 35 and 36 days interferograms ($IF$s 18 and 19) are also in most cases rather low, however there are also some long term interferograms which reach about the same $\sigma$ values, especially for higher $\Gamma_{min}$ (see for example $IF$ 3,5,7 and 8 in Fig.5.23(d). For interferograms on intervals longer than one ERS cycle the standard deviations are anyway in the same (more or less wide) range of values. Indeed also a visual inspection of the whole interferograms reveals that temporal decorrelation occurs so fast, that often 70-days interferograms are already as decorrelated as 3-years interferograms. The fact that the standard deviations have similar values indicates that the spatial phase stability of highly coherent areas is not a function of time, but that it decreases abruptly after a few months, at least for this site. The standard deviation usually varies strongly in different interferograms. Since these variations are not time-dependent, it can be supposed that they are caused by the same image in all interferograms. In other words, one can suppose that combining one image where the backscattering is much different from the backscattering in the other images will cause variations in the differential phases and subsequently a worsening of the standard deviations. The change in backscattering properties could have different causes, for example it can be due to different meteorological conditions or to accidental effects on the examined pixels; due to its local, or even feature-related nature, such an effect can be different or even not visible on different areas. The hypothesis was indeed suggested from the analysis of the spatial homogeneity of the road and of the building, where two groups of interferograms having one image in common turned out to have also similar values of the spatial standard deviation $\sigma$. The standard deviations obtained among interferograms having one image in common were thus compared and, although in some cases they showed indeed similar values, in most cases however the values resulted to be significantly different and no general trend was noticeable. For example, $IF$s 3,5,7 and 8 have one image in common:
Figure 5.23  Spatial standard deviation $\sigma$ of the 7 considered areas for different values of $\Gamma_{min}$ (see legend on the next page). The letter in each plot indicates the area it refers to.
5.3 A general analysis of urban areas

Figure 5.23  Continued.
the $\sigma$ of IF's 3 and 5 are similar, but they are very different from those of IF's 7 and 8 (see again Fig.5.23(d)). In conclusion, it was not possible to identify, in each case, a single image as responsible for different high standard deviation values. This is, after all, to expect, since there are not "bad" and "good" images, but more or less similar images, from the point of view of the backscattering properties.

One could also suppose that the interferograms having lower standard deviations have a generally higher coherence, i.e. that they have an higher amount of pixels with coherence $> 0.8$, while those with higher values have most pixels with coherence on the low side of the considered interval, thus for example in the interval [0.5 0.6]. The plots showing the number of data however do not seem to suggest a relationship between the standard deviation value and the relative number of pixels within a certain coherence interval. Notice that this is equivalent to suppose that the coherence value is a good indicator of the reliability of the phase information (as reflected by the "spatial stability" which is being investigated) or, in other words, that higher coherence pixels are more stable. This assumption is investigated in the next section.

**Dependence of $\sigma$ on the coherence limit**

An hypothesis which has been considered is that with increasing coherence limit the standard deviation decreases. The standard deviation plots of the three case studies showed in fact that in many cases, though not always, this indeed happens. This would mean that the phases of pixels with higher coherence lie in a smaller range of values and therefore that these values are more likely to be a good estimate of a "mean differential phase" of the area. The observation of the standard deviations of all the areas considered (again in Fig.5.23), indicates however that this is not necessarily the case. Although here also, in probably more than the half of the cases, the standard deviation is lower for an higher coherence limit, there are however still many cases in which the contrary is true. In order to give an estimate of the general improvement given by increasing the coherence limit, the mean value, on the set of interferograms, of the ratio of the standard deviations for $\Gamma_{\text{min}} = 0.5$ and $\Gamma_{\text{min}} = 0.8$ was computed:

$$M_i = \frac{\sigma_i(\Gamma_{\text{min}} = 0.8)}{\sigma_i(\Gamma_{\text{min}} = 0.5)}$$

$i = 1, \ldots, 21 \quad i \neq 20$.

$^2$Since the pixels selected for $\Gamma_{\text{min}} = 0.8$ are a subset of those selected for $\Gamma_{\text{min}} = 0.5$, if this hypothesis is true, then given two interferograms with remarkably different standard deviations and more or less the same number of selected pixels for $\Gamma_{\text{min}} = 0.5$, the one with lower $\sigma$ should have an higher number of selected pixels for $\Gamma_{\text{min}} = 0.8$. 
where the mean was computed on all the interferograms for each area. The resulting values are represented in Fig. 5.24. For all the features the value of $M_r$ results to be about $0.8 \div 0.9$, i.e. the increase of the coherence limit from 0.5 to 0.8 causes a drop of the standard deviation of 20% at maximum. The improvement is thus rather small, especially with respect to the corresponding strong reduction in the number of selected pixels (the ratio between the amount of pixels selected for $\Gamma_{min} = 0.8$ and those for $\Gamma_{min} = 0.5$ ranges from about 0.03 till about 0.3!). This means that coherence is not the best criterion for the selection of the most stable pixels, at least in the used image resolution and for single pixels.

**Dependence of $\sigma$ on the temporal standard deviation limit $\tau$**

Since the spatial standard deviation does not seem always to reduce by increasing the coherence limit, another criterion has been considered in order to improve the selection of the most spatially stable pixels. One can suppose in fact that the pixels which are more stable in time are also those giving the most reliable information. If this is the case, then the phase value of such pixels should be very similar in the same interferogram, giving a low $\sigma$. The existence of a link between temporal and spatial stability was indeed suggested by the results of the temporal stability analysis for the first three case studies, where a considerable improvement of the spatial standard deviation was obtained by considering only those pixels having $\tau < 0.4 \text{rad}$. In order to check this hypothesis the $\sigma$'s of the selected pixels were recomputed by considering only pixels with temporal standard deviation $\tau$, estimated...
Figure 5.25  Spatial standard deviation $\sigma$ for different values of the temporal standard deviation per pixel $\Delta_{\text{temp}}$ (see legend on the next page). The letter in each plot indicates the area it refers to.
5.3 A general analysis of urban areas

Figure 5.25 Continued.
on all the 21 interferograms, below a certain limit $\Lambda_{\text{max}}$. The coherence threshold was posed at $\Gamma_{\text{min}} = 0.5$, and four different cases were considered. In the first case no temporal standard deviation limit was imposed, while in the other three cases the limits of, respectively, $\Lambda_{\text{max}} = 0.4$, 0.6 and 0.8 radians were posed. The results are shown in Fig. 5.25, for each area and for the different values of $\Lambda_{\text{max}}$. In all the cases there is an evident improvement of the $\sigma$ when only the most temporally stable pixels are considered: for $\Lambda_{\text{max}} = 0.4$ the standard deviations are of the order of 0.4 rad, and in most cases the number of selected pixels is still significantly high. The improvement was estimated by computing the mean ratio between the spatial standard deviation obtained with $\Lambda_{\text{max}} = 0.6$ rad and by using all points. With "mean" it is here also intended the mean value computed on all the interferograms for a certain area:

$$R_{\sigma} = \frac{\sigma_i(\Lambda_{\text{max}} = 0.6)}{\sigma_i(\text{all})} \quad i = 1, \ldots, 21 \quad i \neq 20. \quad (5.6)$$

As can be seen in Fig. 5.26, $R_{\sigma}$ values of about 0.5 were obtained for the majority of the features, indicating a reduction of the standard deviation to half its original value. Not only the improvement is in this case better than in the case when the coherence limit is used, but it is also obtained with a less severe reduction of the number of pixels. The amount of selected pixels varies in fact considerably depending on the area, but remains in most cases very high and anyway always sufficient for the statistical treatment. For $\Lambda_{\text{max}} = 0.4$ rad the improvement is even more significant, however since in some cases the number of pixels becomes critically low, the results for $\Lambda_{\text{max}} = 0.6$ rad have instead been shown here.
5.3 A general analysis of urban areas

5.3.2 Temporal standard deviation

The results shown in the previous section seem to indicate that the temporal standard deviation $\tau$ provides a good criterion for the selection of the interferometrically most phase-stable pixels. The following step was the search of a possible link between the value of $\tau$ and other properties of the pixels. The main purpose was to try to find a method for a selection of these time-stable pixels which could be more "automatic", i.e. not requesting the computation of all the $\tau$'s in order to discriminate the pixels of interest. Quantities which could give a criterion for this purpose are again the coherence limit and the number of occurrences of a pixel along the series, i.e. the number of samples on which the $\tau$ of that pixel is computed. In Fig.5.27 the temporal standard deviations per pixel of area $G$ are shown in correspondence of each pixel location. As in the majority of the cases, the urban area (or feature) of interest can be identified quite clearly as a patch of relatively lower temporal standard deviations within a "noisier" background of values varying on a wider interval (thus the highest values of the temporal standard deviation are usually in this "background" field).

Figure 5.27 Temporal standard deviation (in rad) $\tau$ for each pixel of the area $G$ (Groningen) and for a coherence limit $\Gamma_{\text{min}} = 0.5$. 
Dependence of $\tau$ on the coherence limit

In Fig. 5.28 is represented, for each section, the distribution of the $\tau$’s for the usual four values of $\Gamma_{\text{min}}$ considered. The plots are in practice histograms, as the asterisks ”*” represent the distribution of $\tau$ values within bins of width 0.1 rad. They are represented as curves in order to plot them together for different $\Gamma_{\text{min}}$ and to compare them.

For all the considered areas the distributions show a clear maximum, which is located usually at the low side with respect to the whole range of covered values. Notice that especially for $\Gamma_{\text{min}} = 0.5$ and for the areas A, C, E, F, G, which contain an higher number of selected pixels, the distributions resemble a Chi-square distribution. This could indeed be the case, if one assumes that $\tau$ has a normal distribution: the standard deviations of a variable distributed normally have in fact a Chi-square distribution.

As can be seen in particular for the areas A, C, E, F, G, increasing $\Gamma_{\text{min}}$ causes a flattening of the distribution, indicating a drastic reduction of the number of pixels with low temporal standard deviation, thus ”good” pixels. When instead of the absolute number of pixels their percentage with respect to the total is plotted, it can be seen that the percentage corresponding to the maximum increases with the coherence, indicating that we drop less ”good” pixels than ”bad” ones, while the tails of the distribution are reduced. This behaviour could mean that dropping samples of lower coherence makes pixels more stable: their standard deviation decreases and therefore their position shifts toward the left, causing the lowering of the tails and the increase around the maximum. Other pixels having only low coherence samples will disappear, and this also will lower the tails. Moreover, pixels with already low standard deviation will also improve their stability, and this could explain the small shift of the maximum towards left which can be observed in most plots. These results suggest therefore that dropping lower coherence samples at least does not worsen the temporal standard deviation.

If on one side posing an higher coherence limit, i.e. using only the samples with higher coherence, may result in an improvement of the $\tau$ for a certain number of pixels, on the other side however a considerable number of lower-coherence, temporally stable pixels are lost in the procedure. In Fig. 5.29(a) is represented, for each area, the mean temporal standard deviation on all the considered pixels for different values of $\Gamma_{\text{min}}$. The mean is weighted with the number of occurrences in the series of interferograms, based on the consideration that the standard deviation becomes more significant with increasing number of samples. The values represented are actually those of the ratio:

$$T_x = \frac{\tau(\Gamma_{\text{min}} = 0.5)}{\tau(\Gamma_{\text{min}} = 0.5)}$$

$x = 5, 6, 7, 8$
Figure 5.28  Distribution of \( \tau \) corresponding to different \( \Gamma_{\text{min}} \) (see legend on the next page) and for the different areas. On the x-axis is represented the value of \( \tau \), on the y-axis the corresponding number of pixels. The letter in each plot indicates the area it refers to.
Figure 5.28  Continued.
and constitute therefore a measure of the improvement of the mean $\tau$ when $\Gamma_{\text{min}}$ is increased. Although there is a clear improvement for all the features when the coherence limit passes from 0.5 to 0.6, the behaviour of $T_x$ becomes very different for higher coherence limits.

The same plot has been done by considering, instead of the mean $T_x$, the value corresponding to its maximum, i.e. the distribution mode. It is represented in Fig.5.29(b) and indicates clearly that, in terms of the mode, the improvement (or worsening) of the mean temporal standard deviation is not related to the chosen coherence limit.

**Dependence of $\tau$ on the number of occurrences**

The previous tests seem to indicate that a selection based on the coherence value is not the optimal criterion for the discrimination of the most stable pixels. The fact that the number of occurrences is almost always higher for pixels belonging to a feature or urban area (see example in Fig.5.30) than for those outside, suggests the hypothesis that the number of occurrences could be related to the temporal stability of pixels, and lead therefore to another possible criterion for the discrimination of the most stable ones. In other words, one can also suppose that pixels occurring with relatively high ($>0.5$) coherence in most of the interferograms are more likely to be temporally stable than those occurring a few times. Pixels having few high-coherence samples could in fact be suspected of being too much sensitive in their

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3The mode of a distribution is defined as its most frequent value ([37]).
backscattering characteristics and have thus strong oscillations in coherence. Moreover, if certain pixels are going to be used as targets of opportunity for the retrieval of deformations, it is desired that these pixels are present (in the sense that they result to be selected at least for $\Gamma_{min} = 0.5$) in all the interferograms. A plot of the temporal standard deviation values versus the corresponding number of occurrences (samples) is shown in Fig.5.31. It refers to section A, but for the other sections the plot looks the same, also independently from $\Gamma_{min}$.

As for the results which can be deduced from this and the other similar plots, they suggest that there is no relationship between the value of $\tau$ and the number of samples, only the range of values seems to be wider for a low number of occurrences. In order to confirm this the mean and the standard deviation of $\tau$ were computed separately for each set of pixels with a certain number of occurrences. The results for $\Gamma_{min} = 0.5$ are shown in Fig.5.32, where each line represents one of the areas under study. The mean $\tau$ remains indeed constant, for a certain area, independently from the number of occurrences (Fig.5.32(a)), while the standard deviation, which represents the spread of the values in Fig.5.31, decreases slightly as the number of samples increases, as it is particularly visible for all features except $B$ and $D$ which however have a very reduced number of selected pixels for higher occurrences.

The same plot of Fig.5.31 was done for each $\Gamma_{min}$ for the same feature, in order to see if the results deduced for $\Gamma_{min} = 0.5$ are also verified for higher coherence limits. The mean values of $\tau$ seem to be different only for low occurrences, but in that case also the problem of the significance of the standard deviation plays a role. For high occurrences the results are on the contrary similar.
5.3 A general analysis of urban areas

Figure 5.31 Temporal standard deviation $\tau$ vs. number of samples for all the pixels of section A and for each $\Gamma_{\text{min}}$ considered.

Figure 5.32 Mean (on the left) and standard deviation (on the right) of $\tau$ computed separately for pixels with different number of occurrences and for each area ($\Gamma_{\text{min}} = 0.5$).
5.4 Retrieval of deformations

5.4.1 Method

The urban areas whose interferometric characteristics have been studied in the previous section were used as targets of opportunity for the measurement of the subsidence. Four long term interferograms were used for this purpose, namely $IF_1, 3, 4$ and $6$, whose characteristics are shown in Table 5.2. In each of the seven areas the pixels with coherence value greater than 0.5 were selected (i.e. $\Gamma_{\min} = 0.5$) and the set of the corresponding differential phase values $DF$ was considered. Assuming that the areas are small enough in size, with respect to the spatial scale of the deformations, to suppose that in each of them the subsidence signal is spatially uniform, a "representative" phase value can be determined and assigned to the centre of the area itself. The best choice for such a representative value turned out to be the most occurring phase value among the selected pixels, i.e. the mode of the distribution of the phases. This value was determined as the maximum of the histogram representing the distribution of the phase values (an example of such an histogram for area $A$ is shown in Fig.5.34). The location of the maximum is however influenced by the position of the histogram bins. For this reason the determination was performed by shifting the bin edges of a small, constant step, and computing each time the distri-
5.4 Retrieval of deformations

Table 5.2 The interferograms used for the retrieval of deformations in the Groningen test site.

| 21-04-96  | 10-09-92 | -46   | 1319 | 1  |
| 21-04-96  | 15-10-92 | -57   | 1284 | 3  |
| 17-03-96  | 10-09-92 | 63    | 1284 | 4  |
| 16-03-96  | 10-09-92 | 52    | 1283 | 6  |

Figure 5.34 Histogram of the phase values of area A in one of the interferograms.

The deformations estimated with the method explained above were validated by comparison with a pre-existing subsidence model of the area⁴ obtained with other kind of measurements. The procedure consisted in comparing the set of deformations with the pre-existing model. This 'fine tuning' permitted to identify the bins position giving the highest value of the maximum, and this was taken as the best estimate of the distribution mode. The result is, for each of the four interferograms, a set of seven phase values, which were then converted to deformations in mm.

5.4.2 Results

The deformations estimated with the method explained above were validated by comparison with a pre-existing subsidence model of the area⁴ obtained with other kind of measurements. The procedure consisted in comparing the set of deformations with the pre-existing model. This 'fine tuning' permitted to identify the bins position giving the highest value of the maximum, and this was taken as the best estimate of the distribution mode. The result is, for each of the four interferograms, a set of seven phase values, which were then converted to deformations in mm.

⁴Courtesy of NAM.
A first comparison between the two sets of deformations obtained respectively from INSAR and from the model consisted in the computation of their correlation coefficient. Notice that as the values estimated from the interferograms are not referred to a zero phase, the deformation are not absolute, but only relative among areas. Of course this has no influence on the correlation coefficient, which is only a measure of the similarity, in terms of trends, between two sets of data.

For each interferogram, the solution giving the maximum correlation coefficient has been searched by alternatively adding a $2\pi$ to the phase at one or more of the seven considered point. Although this could seem an arbitrary operation, it is not. The interferograms showed in fact a phase jump, causing the presence of a $2\pi$ ambiguity. This ambiguity is only of $2\pi$ (and not for example of $4\pi$) because the amount of deformation corresponds to less than one fringe, but nonetheless is present and introduces an ambiguity in the solution. Because of the strong decorrelation it was not possible to define with certainty the position(s) of the phase transition, and determine therefore which areas were on either side of it. This left in principle the freedom to apply $2\pi$ to all the features: the only constraint was that obviously the correction had to be the same for all the areas where it was applied, otherwise differences of more than $2\pi$ in the relative deformations would have occurred.

The solutions giving the maximum correlation coefficient for each interferogram are shown in Fig.5.35. The correlation coefficients between the two sets of deformations are listed in Table 5.3 on the left. From these high values, and also

<table>
<thead>
<tr>
<th>IF</th>
<th>Correlation coeff.</th>
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<tr>
<td>1</td>
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<td>3</td>
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<tr>
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<table>
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<th>IF</th>
<th>IF</th>
<th>Correlation coeff.</th>
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<tr>
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<td>4</td>
<td>6</td>
<td>0.91</td>
</tr>
</tbody>
</table>
5.4 Retrieval of deformations

from Fig.5.35, one can deduce that indeed the deformation has the same trend. Moreover the agreement, again in terms of correlation, among the interferograms themselves is very good, as can be seen in Table 5.3 on the right side. Notice that IFs 1, 4 and 6 have an image in common, and therefore one could suspect that this indeed leads to high correlations between them. The fact that IF 3 uses a different image and still it is highly correlated with the others, confirms however that, although the image in common could maybe cause a some higher correlation between them, for the great part their similarity is genuine, and not due to such a spurious effect.

If the trend of the deformation as represented in the plots is pretty much the same in the model and in the interferometric results, for the latter the deformations in areas C and D are however unrealistic. Apparently, searching the maximal correlation is not the optimal criterion to apply $2\pi$ corrections. The maximum correlation criterion has indeed subtracted $2\pi$ in these two areas (notice by the way that area D is comprised in area C therefore it is reasonable to assume that both undergo the same correction), however if this correction is not applied the result, although it causes a lowering in the correlation coefficient, is in better agreement with the expected deformations. The mean deformation computed on the four interferograms

---

5Notice that the one represented is not the actual shape of the deformation, as the areas are not represented in any spatial order (see also the location of the areas in Fig.5.4 or Fig.5.22). The values in the plot are connected with lines only for a better visualisation.
for each area in this case is the one represented in Fig.5.36. Since, as already mentioned, no phase reference has been taken, a shift has been applied which seemed visually to fit best the model, keeping in mind that the city of Assen (area A) is in an area of no subsidence and therefore should have zero deformation. As can be seen in the figure, a good matching with the model could be found for four out of the seven areas, while for the other three the differences range approximately from 4 to 10 mm. Since the correlation coefficient criterion does not apply the $2\pi$ corrections which lead to a reliable result, in a second attempt the corrections were applied manually, here also adding or subtracting each time $2\pi$ to a certain number of areas, still maintaining the constraints explained previously on how to apply the corrections. For each interferogram, the solution giving the deformation values nearest to those of the model was considered the best one. The result of such procedure, represented again as mean over the four interferograms, is shown in Fig.5.37 and is definitely in better agreement with the deformations from the model. Also for area $E$ there is in fact a good matching with the model, while the difference between model and interferometric results in area $G$ is reduced. The two significant differences, in areas $C$ and $G$, amount now to about 4 mm. The correlation coefficient between the mean interferometric deformations and the corresponding values from the model is in this case equal to 0.8.

Finally, the same procedure was repeated by applying a more restrictive selection of pixels. The results from the previous tests on the phase stability of the same areas suggested in fact that the most reliable phases are those from the most time
5.4 Retrieval of deformations

**Figure 5.37** Mean interferometric deformations obtained after visual matching between model and interferograms.

**Figure 5.38** Interferometric deformations obtained with highest-correlation matching for $\Lambda_{\text{max}}=0.6$. 
Table 5.4  Correlation coefficients between deformations retrieved from interferogram and from subsidence model (on the left) and between pairs of interferograms (on the right) in the case of $\Lambda_{\text{max}} = 0.6$.

<table>
<thead>
<tr>
<th>IF</th>
<th>Correlation coeff.</th>
<th>IF</th>
<th>Correlation coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.96</td>
<td>1</td>
<td>0.70</td>
</tr>
<tr>
<td>3</td>
<td>0.83</td>
<td>1</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>1</td>
<td>0.71</td>
</tr>
<tr>
<td>6</td>
<td>0.93</td>
<td>3</td>
<td>0.72</td>
</tr>
</tbody>
</table>

stable pixels. In order to see whether this gives a better solution for the deformations, the procedure was repeated by keeping the coherence limit $\Gamma_{\text{min}}$ at 0.5, but this time taking only the pixels with $\tau < 0.6$ (thus $\Lambda_{\text{max}} = 0.6$ rad). The results are shown in Fig.5.38, and it is difficult to say whether they are better of worse than those obtained by using all the pixels and represented in Fig.5.35. Also in this case the solution depicted is the one giving the highest correlations, and the corresponding correlation values, given in Table 5.4, are slightly lower than in the previous case. The mean deformations equivalent to those of Fig.5.36 are shown in Fig.5.39, where also in this case a $2\pi$ correction has been applied to the phase values of areas $C$ and $D$. The search for the best matching without considering the correlation coefficient brings then to the results of Fig.5.40. The agreement with the model is definitely better than when the solution is found with the criterion of the highest correlation with the model, the maximum difference, in area $C$, amounts to 5-6 mm. A correlation of 0.95 has been found between this averaged solution and the model. The results are indeed very similar to those obtained when all the pixels were considered (Fig.5.37): it has to be kept in mind, in fact, that no zero-phase has been taken, therefore an arbitrary shift along the y-axis can be applied to the curve referring to the interferometric deformations.

Some remarks have to be made at this point in order to give a better judgement on the results obtained in this deformation analysis. First of all, in order to define, at least approximately, absolute deformation values, it has been assumed that in Assen, an area where it is known that no subsidence is taking place, the absolute deformation is about zero. With "about" it is intended is has not been posed strictly to zero, but that in searching the best fit for all the values, when possible the value for Assen has been kept near zero within a range of 1 or 2 mm., a range which seemed
5.4 Retrieval of deformations

Figure 5.39  Mean interferometric deformation obtained after highest-correlation matching for $\Lambda_{max}=0.6$.

Figure 5.40  Mean interferometric deformation obtained after visual matching for $\Lambda_{max}=0.6$. 
to be reasonable as an estimate of phase noise. The phase value of Assen, however, could be biased, therefore causing a shift of the whole curve. This is most probably the case in IF 3, (Fig.5.41), which has indeed been shifted to match the other areas when determining the solution of Fig.5.40. The approach presented here is indeed not sensitive to phase biases affecting a whole area. Since only patches of coherence are present, and they are considered each one separately for the determination of the representative phase value, biases at spatial scales larger than the considered area itself cannot be revealed.

Another factor which has to be taken into account is the accuracy of the model used for the comparison. The model is in fact a polynomial interpolation in time and space of different kind of measurements, principally levelling, and therefore it gives only an estimate of the deformations. Moreover, it is a model of the deformations occurring in the Pleistocene layer, and not on the surface, where the interferometric signal relates to. Since the deformation of the surface is both spatially and temporally less homogeneous, it cannot be expected even in the ideal case, that the measurements from INSAR match exactly the estimated quantities from the model.

5.5 Summary and preliminary conclusions

The deformations occurred in the area of Groningen in the period 1992-1996 have been mapped by means of SAR Interferometry. The particularly strong decorrelation of the interferograms on such time intervals led to the development of an alter-
native approach for the measurement of height changes. The method is based on the use of the interferometric information from anthropogenic features which, notwithstanding their limited size with respect to the considered resolution, in many cases maintained high interferometric coherence on long term. Of course the use of the information from these features requests some knowledge of their interferometric properties, in particular of the reliability of their differential phase. A series of tests was performed for this purpose on a selection of both single features and groups of them. The single objects considered were a road (or an small and long object running parallel to the road) and a remarkably large building. As for the groups of features, seven areas of high density of coherent objects were identified, corresponding to cities or villages. The properties examined for each feature or area were essentially the homogeneity, in each interferogram, of the phase information from all the highly coherent pixels, and the stability in time again of the phase, but this time of a single pixel, analysed across the database.

Most of the examined areas and features turned out to remain spatially homogeneous at the level of 0.2 to 1 rad even on intervals of some years (1 rad correspond to approximately 0.5 cm deformation). The urban areas resulted to be in general less homogeneous than the single features: probably this is due to the fact that they are collection of independent features, therefore disturbances in their phase is more likely to occur.

The homogeneity of the phase values did not seem to be related to the coherence value of the pixels, i.e. considering only those pixels having an higher coherence in general did not reduce the range of values of their differential phases. The dependence of the spatial homogeneity of the feature with respect to the time interval covered by the interferogram considered was checked. No indication has however been found that increasing the time span causes the phase values to become more spread.

As for the temporal standard deviation of single pixels along the database, a significant number of them turned out to be stable in time at the level of 0.4 rad. The most stable pixels were not necessarily the ones with the highest coherence values, nor those having high coherence in the highest number of interferograms. A significant link was however found between stability in space and in time: the high coherence pixels which were very stable in time resulted also to be more spatially homogeneous.

A particularly interesting fact was that in the case of the object identified as a road the same, clear deformation signature was found on the ten longest term interferograms. Apparently the feature has deformed during the last years, and despite the small amount of the deformation (estimated in 5 mm or even less) and the small size of the object, this information could be detected with INSAR.
After the analysis of its properties, the interferometric information from these highly coherent pixels was used to measure the deformation. To each of the seven urban areas under study a "mean" phase, and thus deformation value was assigned. The relative deformation among the areas obtained in this way in each interferogram were then compared with the expected values obtained at the same positions by interpolating a pre-existing model. The presence of a phase jump in the interferometric information introduced however a $2\pi$ ambiguity in the corresponding deformations. Solving automatically for this ambiguity with a maximum-correlation search led however in some cases to unrealistic deformation values. The application of corrections led to better results when performed manually, using as a criterion the minimal distance from the model. Even in this case a good agreement between measured and expected values was found, with correlations at the level of 0.7 to 0.9. However, a larger set of deformation values within each interferogram and also the possibility to apply this approach on an higher number of long term would be necessary to confirm these results.
Long term deformation analysis at the Phlegrean Fields

With a mean deformation rate of 4 mm/month, Campi Flegrei can be said to be a rapidly subsiding area [4, 50]. Conversely to Groningen, where a time span of at least one year is necessary to have deformations measurable by INSAR, here subsidence should be detectable already after one or two ERS cycles and on intervals of a few months it should generate fringes.

Due probably to the more favourable atmospheric conditions with respect to the Groningen area and to the denser urbanisation, decorrelation is here less severe. Nevertheless on a long term interferogram, such as the one shown on the left side of Fig.6.2, only less than 20 percent of the surface has coherence higher than 0.5. Despite this high decorrelation it is still possible to see clearly the deformation fringes (see also Fig. 6.2 on the right). As the time interval increases however the fringes are more and more disturbed by decorrelation noise and even patches of almost white noise appear. Phase unwrapping, where it can still be performed, is very difficult and leads often to biased solutions. Although Campi Flegrei has completely different characteristics with respect to Groningen thus, here also temporal decorrelation is the greatest problem when time scales longer than a few months are considered. A new approach has been developed to overcome, also in this case, the temporal decorrelation problem. Different from the one presented for the Groningen test site, this approach is still based on the use of a database of interferograms, processed as explained in Section 4.1. As it will be explained in more detail in Section 6.2, the
method integrates the information from several interferograms, "cleaning" in this way the fringes from their decorrelation noise. The approach has been applied on a set of interferograms spanning the years 1993-1999 (Section 6.1) and the results, presented in Section 6.3, have been used also for the geophysical interpretation of the ongoing processes in the caldera (Section 6.4).

6.1 Interferometric Database

The interferograms used for the retrieval of deformations were generated as different combinations of a set of 15 ERS images, listed in Table 6.1. Notice that this Table is not identical to Table 4.2. Some of the images listed there, in fact, have not been used for the analysis of the subsidence, but only for the study on urban areas and anthropogenic features analogous to the one carried out on the Groningen test site and presented in the next chapter. On the other hand, two of the images listed in Table 6.1 do not appear in Table 4.2 (and were not used for the stability study) because originally they were not present in the database. These images, indicated with orbit numbers 9700 and 8197, are actually from IRECE-CNR, which provided their interferometric combinations with some of the other images of Table 6.1. This
6.1 Interferometric Database

Figure 6.2 Interferometric coherence (left) and phase (right) images of Campi Flegrei on the time interval 8-10-1997 / 22-4-1999.

Figure 6.3 The set of interferograms generated from descending passes. The time spans are depicted as horizontal bars on a linear scale.
Table 6.1  *ERS images of Campi Flegrei considered for the generation of the interferograms (descending passes).*

<table>
<thead>
<tr>
<th>ERS1/2</th>
<th>orbit</th>
<th>frame</th>
<th>date</th>
<th>day no</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERS 1</td>
<td>9700</td>
<td>2781</td>
<td>8-2-1993</td>
<td>d1</td>
</tr>
<tr>
<td>ERS 1</td>
<td>8197</td>
<td>2781</td>
<td>24-5-1993</td>
<td>d2</td>
</tr>
<tr>
<td>ERS 1</td>
<td>19563</td>
<td>2781</td>
<td>12-4-1995</td>
<td>d3</td>
</tr>
<tr>
<td>ERS 1</td>
<td>21066</td>
<td>2781</td>
<td>26-7-1995</td>
<td>d4</td>
</tr>
<tr>
<td>ERS 2</td>
<td>1894</td>
<td>2781</td>
<td>31-8-1995</td>
<td>d4a</td>
</tr>
<tr>
<td>ERS 2</td>
<td>2896</td>
<td>2781</td>
<td>9-11-1995</td>
<td>d5</td>
</tr>
<tr>
<td>ERS 2</td>
<td>6904</td>
<td>2781</td>
<td>15-8-1996</td>
<td>d6</td>
</tr>
<tr>
<td>ERS 2</td>
<td>9409</td>
<td>2781</td>
<td>6-2-1997</td>
<td>d7</td>
</tr>
<tr>
<td>ERS 1</td>
<td>30585</td>
<td>2781</td>
<td>21-5-1997</td>
<td>d8</td>
</tr>
<tr>
<td>ERS 2</td>
<td>10912</td>
<td>2781</td>
<td>22-5-1997</td>
<td>d8</td>
</tr>
<tr>
<td>ERS 1</td>
<td>31587</td>
<td>2781</td>
<td>30-7-1997</td>
<td>d9</td>
</tr>
<tr>
<td>ERS 2</td>
<td>13417</td>
<td>2781</td>
<td>12-11-1997</td>
<td>d10</td>
</tr>
<tr>
<td>ERS 2</td>
<td>17926</td>
<td>2781</td>
<td>24-9-1998</td>
<td>d11</td>
</tr>
<tr>
<td>ERS 1</td>
<td>40605</td>
<td>2781</td>
<td>21-4-1999</td>
<td>d12</td>
</tr>
<tr>
<td>ERS 2</td>
<td>20932</td>
<td>2781</td>
<td>22-4-1999</td>
<td>d12</td>
</tr>
</tbody>
</table>

was done in the frame of a cooperation for the application, on the largest possible temporal scale, of the time series analysis of deformations developed by the author of this thesis and presented here [41].

The acquisition day of each image is labelled in chronological order with a "day number" (d#). Notice that, here also as in the previous cases, Tandem pairs have been given the same day number: at the deformation rates considered here it is reasonable to assume that the deformation does not increase significantly after only one day. Notice also that 26-7-1995 is defined as day "d4a" and, as explained in more detail at the end of this section, in the least squares computation this day is assumed to coincide with 31-8-1995. With these images 26 interferograms were generated, labelled as $I_1, \ldots, I_{26}$ in Fig.6.3. The time spans they cover, represented in the same figure on a linear scale, varies from two ERS orbital cycles (70 days, interferogram $I_5$), up to almost five years (interferogram $I_{26}$).
6.2 Method

6.2.1 Interferogram generation

As a first step, the resulting deformations along the slant range direction have been determined in each interferogram by applying the differential technique. The topographic component to be subtracted was estimated by using a DEM from the Italian Military Geographic Institute (IGM)\(^1\), with a spatial resolution of 1” in both latitude and longitude and an accuracy of about 5 to 10 m in the height. In order to retrieve the absolute deformations a reference, i.e. a zero-deformation point, had to be defined. An area not subject to subsidence has been considered for this purpose. In each interferogram, the mean value of the phases comprised within a square of size 0.004 degrees around a point of fixed coordinates was taken as the zero-deformation phase value. Subtraction of this value from the whole interferogram should give as a result, at least in principle, the absolute (“true”) deformations. Of course, the correctness of the absolute deformations determined in this way is dependent on the estimated mean reference value. The standard deviation of the phase values used for its determination gives a measure of how noisy the considered area is. It is clear that the noisier are the used values, the less reliable, as zero-deformation value, is the computed mean. In this case the standard deviations associated with the estimate were ranging from 0.02 to 0.17 rad, with an average of 0.08 rad.

Not only the presence of noise, but also a bias could affect the mean value. In this case, the whole deformation pattern would be shifted, causing therefore an over- or under-estimation of the deformations. Two kind of biases can be identified: the image-related ones, like for example those caused by atmospheric disturbances, which will be addressed in Section 6.3; and the interferogram-related ones, i.e. those which have been produced in the interferometric combination of two images. In the considered interferograms, interferogram-related biases were caused most probably by phase unwrapping errors. The highly urbanised, subsiding area is delimited by hilly, vegetated zones which, from the interferometric point of view, constitute an almost continuous decorrelated belt around it. As a consequence, also between the deforming area and the reference one there is a certain amount of noise, and phase unwrapping can be rather easily subject to errors when passing from one to the other. Indeed, though all the interferograms showed a similar deformation pattern, extending radially from the Pozzuoli area (see for example Fig. 6.2 and Fig. 2.8 in Chapter 2) and the relative deformations were in agreement with the expected values, some of them showed unrealistic values of absolute deformations, suggesting the presence of a interferogram-related bias in the chosen reference area. These

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\(^1\)Courtesy of IRECE-CNR.
Figure 6.4  Left: histogram of a linear combination of 3 interferograms forming a closed loop. On x-axis are represented the deformations in cm. The maximum is clearly shifted with respect to zero, indicating the presence of a bias. Right: the same histogram after the correction for a bias, which in this case was estimated at 0.5 cm.

Biases were estimated with a procedure based on the comparison of the interferograms. An interferogram suspected to be biased was compared with other two ones forming with it what can be called a "closed loop" in time. Let us suppose that three images are taken at subsequent days \(A\), \(B\) and \(C\), and that the interferograms \(AB\), \(AC\) and \(BC\) are formed. In principle, the linear combination \(AB + BC + CA\) of the absolute deformation values for each pixel in the three interferograms should give as a result zero:

\[
D_{AB}(i, j) + D_{BC}(i, j) + D_{CA}(i, j) = 0 \quad (6.1)
\]

where \(D\) is the absolute deformation expressed in length units (cm) and \((i, j)\) are the coordinates of a certain pixel. So, for example, the difference between \(I_1\) and \(I_2\) should be equal to the interferogram labelled as \(I_3\), giving the deformations occurred between days \(d_3\) and \(d_4\) (see scheme in Fig. 6.3). Due to the presence of noise in the three interferograms, a noise caused by all different effects and which can be assumed to be random, the linear combination of the deformation values will not be overall zero. An histogram of its values however should show anyway a clear maximum around the zero value, as for example in Fig.6.4(b). It is intuitive that if one of the interferograms is biased, for example if:

\[
D'_{AB}(i, j) = D_{AB}(i, j) + K \quad K = \text{const.} \quad (6.2)
\]
then Eq.(6.1) will take the form:

\[ D'_{AB}(i, j) + D_{BC}(i, j) + D_{CA}(i, j) = K \]  

(6.3)

which means also that the histogram of the values of the linear combination will be centred in \( K \) (see Fig.6.4(a)). One can therefore check for the presence of a bias by considering an histogram of a linear combination of interferograms, and take the value at its maximum as an estimate of this bias. Notice that in the evaluation of this maximum the same reasoning of Section 5.4.1 applies, i.e. to determine the exact value of the maximum it is necessary to compute the distribution a certain number of times shifting each time the bin edges.

Of course, assessing which one among \( AB, AC \) and \( BC \) has to be corrected is not always straightforward. Most of the times this can be determined by comparing each of the three interferograms with more other interferograms, forming other "closed loops": it can be expected that a biased interferogram will cause biases in all the loops where it is considered and can therefore be identified quite easily. Moreover, knowing the expected deformation in the area, for example from other survey methodologies, can help to identify the interferograms giving plausible deformation values. These can thus be taken as the "good" ones and considered as a starting set of reliable interferograms to which all the others can be compared with the histogram method. Eventually, if the biases are estimated and subtracted correctly, the interferograms should be consistent as a whole set. This can be assessed again by checking that all the possible closed loop combinations of the interferograms give an histogram centred on zero.

In the case of the considered set, the majority of the interferograms resulted to be already consistent with respect to the closed loop test. Only a couple of them turned out to be slightly biased: in one case a correction of about 1cm was applied, in all the other cases the biases were always of few millimetres. Even for these small biases, however, correction resulted in a better consistence of all the interferograms with each other, as it was assessed at the end of the correction procedure by checking once again the histograms of all the possible closed loop combinations.

### 6.2.2 A least squares approach for the retrieval of deformations

After the detection and, when necessary, the removal of biases, the deformations resulting from the interferograms were used as input information for a least squares adjustment. The day corresponding to the first image (8 February 1993) was taken as reference and the deformations at each of the other eleven days relative to this day have been found as solutions of the problem [58]:

\[ a = \mathcal{A}b \]  

(6.4)
with \( a = [I_1, \ldots, I_{26}] \) and \( b = [X_{d21}, \ldots, X_{d12}] \) respectively the set of interferograms and the (unknown) deformations at the eleven remaining days. The system matrix \( \mathcal{A} \) is the mathematical representation of Fig.6.3:

\[
\mathcal{A} = \begin{bmatrix}
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] (6.5)

In this matrix, each row corresponds to an interferogram, while the columns correspond to the days. For interferogram \( I_k = I_{ik} - I_{ij} \) the values (on row \( k \)) are all zero except at columns \( i \) and \( j \), where they are respectively +1 and −1, the sign being determined by the interferometric difference.

The solution \( \hat{b} \) of the least squares problem of Eq.(6.4) has subsequently been used to re-estimate the original interferometric deformations, i.e.

\[
\hat{b} = (\mathcal{A}^t \mathcal{A})^{-1} \mathcal{A}^t a
\] (6.6)

\[
\hat{a} = \mathcal{A} \hat{b} = [\mathcal{A} (\mathcal{A}^t \mathcal{A})^{-1} \mathcal{A}^t] a
\] (6.7)
6.3 Results

The \( \hat{a} \) values have been compared with the set of interferometric deformations \( a = [I_1, \ldots, I_{26}] \) in terms of the quantity \( Er_i \), defined as:

\[
Er_i = \sqrt{\frac{\sum_{N_i} (a - \hat{a})^2}{N_i}} \quad i = 1, \ldots, 26
\]  

(6.8)

where the sum is performed on the set of \( N_i \) deformation values measured in the original interferograms (notice that for obvious reasons all those values, generated in the least squares solution, which fill the decorrelated areas are not included in the summation).

6.3 Results

The result of the least squares adjustment explained in the previous section is a time series of deformations showing the evolution of the subsidence. It is represented in Fig.6.5, where each image represents the deformations at one of the eleven considered days with respect to the reference day. The images have been masked for the sea in the lower part, while the sparse masked areas on the land are those where, due to the presence of decorrelation noise, the deformations could not be resolved. The deformations are consistent with the results from the GPS and levelling surveys performed periodically in the area [48]. The subsidence has a radial pattern, centred on the east part of the city of Pozzuoli. Here the ground deformation is maximal, with a subsidence from February 1993 to April 1999, as measured from the interferometric data, of about \( 23 \) cm in the slant range direction, with a standard deviation estimated at the level of \( 0.4 \) cm. The mean subsidence rate is of the order of about \( 0.3 \) cm/month, quite in agreement with the expected value from levelling data of \( 0.4 \) cm/month, the yearly rate has been estimated in \( 38 \pm 2 \) mm/yr [41]. Variations in the deformation rate are however noticeable. In particular, the subsidence seems to almost stop in the period August 1996 - July 1997 (of course the dates are only indicative, as it can be referred only to those of the images used). From the second part of 1997 until the end of the considered time span, i.e. April 1999, the subsidence increases considerably, with a rate higher than the one measured in the period from the beginning of the series, i.e. May 1993, to August 1996. Additional interferograms would however be necessary in order to confirm these observations and to locate more precisely in time the variations in the deformation rate.

As already mentioned above, in the least squares computation 31-8-1995 and 26-7-1995 were assumed to be the same day, namely 31-8-1995 (this is why 26-7-1995 is indicated as \( d4a \) in Tab.6.1). The reason for this is that the sets of interferograms having one image acquired at 26-7-1995 \( (I_{15, \ldots, 21}) \) and of those having one
Figure 6.5  Resulting deformations (in cm.) at the 11 considered days with respect to 8-2-1993
Figure 6.5
Continued.

21–5–1997

30–7–1997

12–11–1997

24–9–1998

22–4–1999

cm: -24 -22 -20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2
Figure 6.6  Left: difference, in terms of the quantity $E_r$, between estimated and original interferometric deformations computed considering the whole region under study as depicted in the plot on the right (diamonds) and the $121 \times 61$ pixel subsection comprising the area of maximum subsidence delimited by the red square on the right (circles). Right: difference $(\hat{a} - a)$ for interferogram $I_{10}$

image acquired at 31-8-1995 (all the others) were disjoint, i.e. it was not possible to obtain two interferograms by combining a same (third) image with both $dI$ and $dA$ because of the presence of too large baselines. In order to compute an unique least square solution for the whole set of interferograms, therefore, it was necessary to bring the two days to coincide. Of course, this introduces in the interferograms using $dA$ an error in the estimate of the deformation. This error is equal to the subsidence occurred between the two days and is estimated to be less than 0.5cm (see also [62]).

Figure 6.6(a) shows the results, expressed in terms of the quantity $E_r$ defined by Eq.(6.8), obtained in the comparison between the original interferograms and the set generated from the least squares solution. The values of $E_r$ depicted as diamonds are computed on the whole region under study, as shown in Fig. 6.6(b), while the circles represent the values obtained by considering only an area, indicated by the red rectangle in the same figure, with size of $121 \times 61$ pixel and located in the zone of maximum subsidence. As can be seen in Fig. 6.6(a), most values are comprised between 2 and 6mm, and they are often slightly lower when only the maximum subsidence region is considered for the computation of $E_r$ rather than the whole area. This is probably due to the fact that the most decorrelating areas, like for example the vegetated regions in the upper part of the interferogram, fall outside the
red rectangle. As can be seen in Fig.6.2(b), the interferometric phase at the border of the considered region can be quite noisy and, as a consequence, errors in the determination of the deformations are more likely to occur. This assumption seems to be confirmed by the presence, in these regions, of larger discrepancies, as can be seen in the differences \((\hat{\alpha} - \alpha)\) computed for \(I_{10}\), shown as example in Fig.6.6(b).

The greatest discrepancies between the original interferogram and the one generated from the least squares solution occur for interferograms \(I_{25}\) and \(I_{26}\). The discrepancies are even larger in the smaller region case. Analysis of the difference \((\hat{\alpha} - \alpha)\) reveals a significant difference in the deformation pattern, whereas in the other cases the differences in the maximum deformation area seemed to have more the character of noise. The fact that the pattern of the difference is similar and that the two interferograms have an image in common suggests that these discrepancies could be caused by a disturbance in this image.

The presented method has several advantages with respect to retrieving the deformations by analysing each interferogram singularly. The use of a database of interferograms permits a cross-check on the consistence of their deformations results with respect to biases, as shown with the so-called “closed-loop” test. Solving all the deformations as a unique least squares problem provides then a chronologically ordered sequence, i.e. a picture of the development of the deformation pattern in time, even if no direct interferograms between subsequent dates are available, either because the area has not been imaged or because the baseline is too large. Moreover, by applying the least squares it is possible to reduce the effects of interferometric processing errors, which can be considered different (thus independent) in each interferogram. Notice that the same is not true for image-related errors, like for example atmospheric disturbances. If one image carries such an error, in fact, this will show up in two of the interferograms in Eq.(6.1) with different sign and will therefore be cancelled out. The closed-loop test is thus not sensitive to such an error. Finally, the least squares procedure is particularly useful when, as in this case, long term interferograms are involved. Such interferograms are in general very noisy due to temporal decorrelation, and often the interferometric information is patchy. The least squares permits however to insert information from other interferograms which span shorter time intervals, but sum up to the longer ones, helping to “fill” these patches.

### 6.4 Geophysical interpretation of the results

The least squares solution from the interferometric series was used, together with the comparison of pairs of interferograms from ascending and descending passes,
Mean deformations at the area of maximum subsidence obtained from the least squares solution (from [41]).

The source of the deformation at Campi Flegrei has been since years a matter of debate. One possible explanation is that the deformation originates from the magma chamber, located at a depth of 4 to 6km [9]: in this case it would be caused by a pressure increase in the chamber, due to either the input of new magma, or the differentiation of the existing one, for example because of thermal variations. The second hypothesis relates the ongoing deformation to pressure and temperature changes in a shallower aquifer, located above the magma chamber and influenced by it. According to this hypothesis ([9, 14]) the rapid uplift crisis are caused by the expansion of the magma chamber and, if the pressure increases enough, it causes fractures in the surrounding rock. This fractures augment the permeability of the rock, causing then an increase in the ground water flow from the aquifer.

The least squares solution of the set of 26 interferograms gave the time series analysis represented in Fig.6.5. Additionally, for each solution of the series the mean value of the subsidence was computed on a section of 10 × 101 pixels in the region of maximum deformation, in the East-West direction. These values are represented in Fig.6.7, where their corresponding standard deviation is estimated from the mean standard deviation of the 91, 10 × 10 pixels boxes centred on the

\(^2\)The work is reported in [41], of which this section is a summary.
6.4 Geophysical interpretation of the results

Figure 6.8 Ascending-descending pairs of interferograms used for the modelling of the deformation source (from [41]).
The plot shows quite clearly the non-linearity of the deformation rate. In particular two periods can be identified when the subsidence rate diminished considerably, namely in 1995 and from late 1996 to the first half of 1997. From 1997 to 1999, on the contrary, an increase in the deformation rate seems to have occurred. The modelling of the deformation source was performed by inverting four pairs of interferograms (represented in Fig.6.8), where each pair was formed by one interferogram from ascending and one from descending passes, covering approximately the same time interval. The use of measurements along two different (slant range) directions provided in fact a geometrical constraint for the determination of the three-dimensional deformation field [8].

The pairs of interferograms were inverted for three different types of sources, namely a point pressure source, or Mogi source [45], a finite dipping prolate spheroidal cavity (Yang solution, [67]) and a dipping tensile dislocation (Okada solution, [46]).

In all the three cases a unique best fitting solution could be found for the four pairs of interferograms. The Mogi solution is a point source located at 2.5 km depth, however it generates a deformation pattern which is more circular than the one observed with interferometry (see Fig.6.9). The Yang best fitting solution is a spheroid reaching to within 1 km of the surface, as shown in Fig.6.10. The best agreement with the INSAR deformations is however obtained, in the assumption of an elastic half-space, by an Okada deformation source, i.e. a contracting 1 \times 4\text{km} long rectangular dislocation at a depth of 2.8\text{km} in the East-South East direction beneath the city of Pozzuoli (Fig.6.11). This solution is similar, although slightly shallower, to another solution proposed in [14], obtained by fitting both levelling and trilateration data of the site.

The source of the deformation found as the best solution is significantly shallower than the magma chamber, which has been located deeper than 4\text{km} [19]. The pressure changes which should occur in such a deep magma chamber to justify the measured deformations at the surface would be unrealistic, although including the effects of caldera ring faults could reduce the amount of pressure required. Nevertheless, the found solution seems to be in better agreement with the hypothesis of a hydrothermal nature of the deformation mechanism. Moreover, although the magma chamber pressurization could still be accounted for the rapid and strong uplift of the two major crisis, it is expected that in this case the following subsidence phase would be characterized by eruptions. The slow subsidence which takes place seems on the contrary to support the hydrothermal hypothesis, and can be explained as a consequence of the lateral diffusion of the pressurized fluid.
Figure 6.9 Mogi [45] source inversion results. (a) Model solution for descending (top) and ascending (bottom) range displacement data. (b) Profiles corresponding to the E-W (red) and N-S (blue) observed and modelled range displacements. (c) Residual (observed - modelled) range displacements. (d) 3D plot of the DEM and of the shallowly dipping tensile fault solution represented as a Sphere with 1 km radius viewed from an azimuth of N120E and a elevation of 15 degrees from the horizontal (from [41]).
Figure 6.10  Yang [67] source inversion results. (a) Model solution for descending (top) and ascending (bottom) range displacement data. (b) Profiles corresponding to the E-W (red) and N-S (blue) observed and modelled range displacements. (c) Residual (observed - modelled) range displacements. (d) 3D plot of the DEM and of the shallowly dipping tensile fault solution viewed from an azimuth of N120E and an elevation of 15 degrees from the horizontal (from [41]).
Figure 6.11 Okada [46] source inversion results. (a) Model solution for descending (top) and ascending (bottom) range displacement data. (b) Profiles corresponding to the E-W (red) and N-S (blue) observed and modelled range displacements. (c) Residual (observed - modelled) range displacements. (d) 3D plot of the DEM and of the shallowly dipping tensile fault solution viewed from an azimuth of N120E and a elevation of 15 degrees from the horizontal (from [41]).
Additional tests on highly coherent features

The strong decorrelation of the Groningen area, which left only urban areas and man made features as possible source of interferometric information, led to an unconventional approach for the retrieval of deformations. Due also to the reduced subsidence and the consequent absence of whole deformation fringes, no deformation pattern was recoverable. The only possibility was therefore to try to retrieve the height differences between man made features themselves, either single ones or clusters of them (i.e. urban areas). Before this was done, the reliability of the phase information from these features had to be assessed in some way. This brought to the use of simple statistical tools on the interferometric phase of single, highly coherent pixels. The results indicated that even for single pixels the information can remain very stable (even at the mm level) for years.

Of course there is always the risk that the results are somehow influenced by the particular characteristics of the test site considered. In order to be able to draw some more general conclusions on the interferometric properties of man made features therefore it was necessary to repeat the same tests on the same sort of objects on a test site with different characteristics.

In this chapter are presented the results obtained by performing, on interferograms of the Neapolitan area, the same study done on the Groningen test site and presented in Chapter 5. The selected features and areas are shown in Fig.7.1. The interferometric products used for this tests are the SLC full scenes processed for the retrieval of deformations at Campi Flegrei. As can be seen in the same figure the scenes, which have an extension of 100 × 100 km., comprehend also the whole
Gulf of Naples, with a great portion of the inner land, and the Vesuvius. The chosen areas and features lie all outside Campi Flegrei, mostly around the city of Naples, where no subsidence is taking place. The reason for this choice is that in presence of ground deformations the phase change due to real movements would superimpose to the phase oscillations due to other effects, which we can define as “instability” of the feature as opposed to the true deformation signal. Since the purpose is exactly to give an estimate of such effects, this can be done better in a zone free from deformations, otherwise it would be necessary to estimate and subtract the true signal.

The structure of this chapter is similar to the one of Chapter 5, except for the general explanation about the meaning of the tests and of the statistical quantities considered, which is for obvious reasons not repeated here. After a short description of the interferometric database in Section 7.1, the results of the tests are shown in Section 7.2, followed by the conclusions specific to the test site considered (Section
7.3). The general conclusions on the interferometric properties of man made features which can be drawn from the results in both test sites will be given, among others, in the next chapter.

Finally, some remarks are reported in Section 7.4. Notice that, although they are placed in this chapter, they are actually valid also for the case of the Groningen test site. Indeed they can be assumed to have the same general character of the final conclusions drawn after the analysis in both sites (at least, as far as the conclusion after tests on two different test sites can be assumed to be general).

### 7.1 Interferometric database

In Table 7.1 is given the list of 35 interferograms used for the tests. The interferograms are ordered from the longest to the shortest time span. The series contains also three Tandem pairs and one of them, IF 35, has been used for the estimation of the topography. No DEM was in fact available for the locations corresponding to the selected regions (the available DEM covered only the Campi Flegrei area).

In order to minimise the effects of errors in the topography as estimated from the

**Figure 7.2** The set of interferograms generated from the SLC images listed in Table 7.1. The time spans are depicted as horizontal bars on a linear scale, with the days labelled according to Table 4.2 of Chapter 4.
interferogram therefore only baselines smaller than 50 metres were considered. The temporal extension of the interferograms of Table 7.1 is depicted in Fig.7.2, which is the analogous of Fig.5.2 in Chapter 5. Here also the y-axis represents the interferogram serial number while the x-axis the day number according to Table 4.2 of Chapter 4.

7.2 Analysis of urban areas

7.2.1 Spatial stability of urban areas

The procedure for the analysis of each area is the same one applied to the selected areas of the Groningen test site. The first step is the computation of the spatial standard deviation $\sigma$ for each interferogram and for the four coherence limits $\Gamma_{min}$ from 0.5 to 0.8. The results are plotted in Fig.7.3. Analogously to the case of Groningen, here also the $\sigma$ are in the same range of values both for long and short term interferograms. This seems to confirm the hypothesis that, at least for the resolution and estimation window used here, the spatial homogeneity of highly coherent pixels is not a function of time. The value of the spatial standard deviation does not seem to be dependent on the used images or, in other words, there is no evidence that one single image causes more or less the same values in all the interferograms where it has been used. Interferograms having one image in common showed no particular similarities in terms of the value of their spatial standard deviation, on the contrary the $\sigma$ are often quite different. Consider for example IFs 1, 2 and 15, which have in common the image marked as orbit no.20932 (date: 22-4-1999). While the $\sigma$ is of the same order of magnitude for IFs 1 and 2, however, it is often much lower for IF 15 (see for example plots (a) and (c) in Fig.7.3).

Dependence of the $\sigma$ on the coherence limit

The analysis of the urban areas in the Groningen test site suggested that choosing an higher coherence limit does not necessarily result in a lower spatial standard deviation. The plots of Fig.7.3, seem to confirm this result. Although in probably more than the half of the cases the standard deviation is lower for an higher coherence limit, in fact, there are still many cases in which the contrary is true. An estimate of the general improvement of $\sigma$ passing from $\Gamma_{min} = 0.5$ to $\Gamma_{min} = 0.8$ is again obtainable by computing the quantity $M_i$ of Eq.(5.3) (this time of course for $i = 1, 2, ..., 34$). The results, plotted in Fig.7.4(a), indicate that, with the exception of feature no.7, the value of the ratio lies around 0.65. The mean ratio
7.2 Analysis of urban areas

Table 7.1 The dataset of interferograms of Campi Flegrei used for the analysis of urban areas. As in the case of Groningen, here also the interferograms are given a serial number, in the order from the longest to the shortest time span.

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Figure 7.3  Spatial standard deviation $\sigma$ of the 7 considered areas for different values of $\Gamma_{min}$ (see legend on the next page). The number in each plot indicates the area it refers to.
Figure 7.3  Continued.
is 0.6, corresponding to an improvement of 40%. This is a better improvement than in the case of Groningen, however the selection affects heavily the number of pixels, which is reduced by a factor ranging from 10 to 50 or more. Actually the number of remaining pixels for $\Gamma_{\text{min}} = 0.8$ is in many cases dangerously low (below 10) with respect to the significance of the standard deviation. For this reason also the mean ratio between the standard deviations for $\Gamma_{\text{min}} = 0.5$ and $\Gamma_{\text{min}} = 0.7$ has been computed, and is shown in Fig.7.4(b). Again with the exception of area 7, the values fluctuate this time around 0.7, i.e. an improvement of about 30% is reached with this criterion.

**Dependence of $\sigma$ on the temporal std $\tau$**

The values of $\sigma$ for different temporal standard deviation limits $\Lambda_{\text{max}}$ are shown in the plots of Fig.7.5. The coherence limit for the selected pixels has been kept at 0.5. In many cases a $\Lambda_{\text{max}}$ of 0.4rad is too restrictive, as the number of selected pixels results to be very poor. Nonetheless, the improvement of the $\sigma$ values resulting from the constraint on the time stability is clearly visible, as in the case of the Groningen test site. Values of $\gamma$ of the order of 0.4 to 0.6rad are reached already for $\Lambda_{\text{max}} = 0.6$rad, while a limit of 0.4rad brings in some cases the spatial standard deviation even at the order of 0.2rad, see for example the plots (c) and (e) in Fig.7.5, corresponding respectively to areas no.3 and 5.

The ratio $R_\sigma$ between the spatial standard deviations obtained with $\Lambda_{\text{max}} = 0.6$rad and by using all points is shown in Fig.7.6. Again area 7 shows a different behaviour.
7.2 Analysis of urban areas

Figure 7.5  Spatial standard deviation $\sigma$ for $\tau_{\text{min}} = 0.5$ and for different values of the temporal standard deviation per pixel $\Delta_{\text{max}}$ (see legend on the next page). The number in each plot indicates the area it refers to.
Additional tests on highly coherent features

Figure 7.5  Continued.
7.2 Analysis of urban areas

with respect to the other sections. While for this area the improvement of $\sigma$ is more
or less the same than when the criterion of the coherence limit is used, in fact, for
the other six areas the results are better than in that case, although the improvement
remains small, as can be seen by comparing Fig.7.6 with Fig.7.4 (plot on the left).
The values of $R_{\sigma}$ are anyway similar to those obtained in the Groningen case, the
reduced improvement being caused by the better results, with respect to that site,
obtained by using the coherence as criterion. It has to be noticed finally that even if
the two criteria give in this case similar results, by using the temporal stability the
number of selected pixels is reduced less considerably. This suggests therefore also
in this case that $A_{\text{max}}$ is a better selector of reliable pixels than $\Gamma_{\text{min}}$.

7.2.2 Temporal standard deviation

Dependence of the temporal std on the coherence limit

The histograms of the distribution of $\tau$ for different $\Gamma_{\text{min}}$ values are shown in
Fig.7.7. Here also the distributions show a maximum which, as the coherence
limit increases, maintains more or less the same positions or shifts towards the left,
becoming less distinguishable. This indicates that, analogously to the results for
Groningen, increasing $\Gamma_{\text{min}}$ causes sometimes a decrease of the $\tau$, but has also as a
consequence the rejection of a large amount of samples which contribute positively
to the small $\tau$. The plots of the mean and mode for the distribution of $\tau$ are more
straightforward to interpret than the corresponding ones for Groningen. The mean
Figure 7.7 Distribution of the phase values corresponding to different $\Gamma_{\text{min}}$ and for the different areas. The number in each plot indicates the area it refers to. See legend in the next page.
Figure 7.7  Continued.
value of the ratio $T_x$ (see Eq.(5.3.2)) in fact diminishes clearly as the coherence limit increases. In terms of the mode, while in the Groningen case (Fig.5.29(a)) the results did not support the hypothesis of its dependence from the $A_0$, here, with the exclusion of feature 2, the other areas show a constant or a slightly improved mean $\tau$.

**Dependence of the temporal std on the number of occurrences**

The last hypothesis to be checked is that the most stable pixels are those occurring more frequently. The plot of the mean $\tau$ as a function of the number of occurrences is very similar to the one obtained in the case of Groningen (Fig.5.32). The mean value of the temporal standard deviation remains, for each feature, rather constant independently from the number of occurrences. Only the standard deviation of the values decreases, indicating that the range of values becomes more limited for pixels having high number of samples.

### 7.3 Summary and preliminary conclusions

The tests on highly coherent features and urban areas performed on the interferometric database of the Groningen test site were repeated also on the database of the Neapolitan area. This was done in order to check the character of generality of the results obtained in that case. Seven small areas were considered, all enclosing single buildings or a small group of them. Probably because of the presence of only one or
Figure 7.9  Mean (left) and standard deviation (right) of $\tau$ computed separately for pixels with different number of occurrences and for each area ($T_{\text{min}} = 0.5$).

Few objects, these areas turned out to be in general slightly more spatially homogeneous than the urban areas considered in Groningen, with $\sigma$s in many cases of the order of 0.2 to 0.6rad. The spatial homogeneity of the analysed areas did not seem to vary depending on the temporal extension of the interferograms, nor seemed to be influenced by the choice of the pairs of images. Differently from the case of Groningen, increasing the coherence limit results in an improvement of the $\sigma$, however the number of pixels discarded with this criterion is still extremely high and often puts at risk the statistical significance of the standard deviation. A criterion based on the upper limit $A_{\text{max}}$ for the temporal standard deviation $\tau$ gives slightly better results, and with a less severe reduction of pixels. As for the temporal stability of the single pixels, also in this case it does not seem to be related neither to the coherence value of a certain pixel in each of the interferograms, nor with its number of occurrences with high coherence in the series.

7.4 Some remarks

7.4.1 Dependence of the standard deviation on the number of samples

The most part of the analysis performed on the highly coherent features is based on the estimation of a standard deviation. In each case a different number of samples was used for this estimation, depending on the criterion adopted for the selection of
the pixels and/or on the feature of urban area itself. For some urban areas, thousands to a few hundred of pixels were selected, and it is intuitive that this is a large enough amount of sample to give a statistically meaningful standard deviation. When only few tens of pixels or less are selected, however, the statistical approach loses it significance. The question is therefore which is the minimal number of samples necessary to have an estimate of the standard deviation that has statistically some sense.

The relation between the correctness of the estimator of the standard deviation and the number of samples can be quantified in the case of a set of independent random variables with normal distribution $N(\mu, 1)$ [34]. In this case it can be demonstrated that the quantity $(n - 1)S^2$, where $S^2$ is the estimated variance, i.e.

$$S^2 = \frac{1}{n - 1} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

has a Chi-square distribution with $\nu = n - 1$ degrees of freedom. In the case of random variables $X_i$ with distribution $N(\mu, \sigma^2)$ then, the same result is valid for the random variable $\bar{X}$. The variance of this random variable, therefore

$$(n - 1)\frac{S^2}{\sigma^2} = \nu \frac{S^2}{\sigma^2} = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \bar{X})^2$$

has again a $\chi^2$ probability distribution with $n - 1$ degrees of freedom.

It is possible at this point, by consulting the tables of the $\chi^2$ distribution [57], to define the interval corresponding to a certain confidence limit. So for example the 95% confidence interval is given by:

$$\chi^2_{0.025} < \nu \frac{S^2}{\sigma^2} < \chi^2_{0.975}$$

which can also be written in the form [57]:

$$\frac{\sigma^2 \chi^2_{0.025}}{\nu} < S^2 < \frac{\sigma^2 \chi^2_{0.975}}{\nu}$$

The 95% confidence interval for the standard deviation $\sigma$ in terms of the number of samples and of the estimated standard deviation is therefore:

$$\frac{S \sqrt{\nu}}{\chi^2_{0.025}} < \sigma < \frac{S \sqrt{\nu}}{\chi^2_{0.975}}$$
Table 7.2  Examples of confidence intervals for $S^2/\sigma^2$

<table>
<thead>
<tr>
<th>P (%)</th>
<th>n</th>
<th>$\nu (= n - 1)$</th>
<th>confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>20</td>
<td>19</td>
<td>[0.68 1.31]</td>
</tr>
<tr>
<td>90</td>
<td>20</td>
<td>19</td>
<td>[0.73 1.26]</td>
</tr>
<tr>
<td>80</td>
<td>20</td>
<td>19</td>
<td>[0.78 1.20]</td>
</tr>
<tr>
<td>95</td>
<td>10</td>
<td>9</td>
<td>[0.55 1.45]</td>
</tr>
<tr>
<td>90</td>
<td>10</td>
<td>9</td>
<td>[0.61 1.37]</td>
</tr>
<tr>
<td>80</td>
<td>10</td>
<td>9</td>
<td>[0.68 1.28]</td>
</tr>
<tr>
<td>95</td>
<td>100</td>
<td>99</td>
<td>[0.86 1.14]</td>
</tr>
<tr>
<td>90</td>
<td>100</td>
<td>99</td>
<td>[0.88 1.12]</td>
</tr>
<tr>
<td>80</td>
<td>100</td>
<td>99</td>
<td>[0.90 1.09]</td>
</tr>
</tbody>
</table>

In Table 7.2 are given the 80%, 90% and 95% confidence intervals of the ratio $S^2/\sigma^2$ for different values of the number of samples $n$. The same three confidence intervals are depicted as functions of the number of samples in Fig.7.10, where each one of them is represented as the area between the corresponding plots of the upper and lower limit of the interval. As can be seen in this plot and from the Table above, with ten points the estimated standard deviation $S$ can deviate rather considerably from the true value $\sigma$ and only with about 20 samples $S$ has, in 80% of the cases, variations within 20% of the true value.

7.4.2 Dependence of the results on the considered time span

In order to evaluate the significance of the obtained results, also their dependence on the particular circumstances in which they are generated has to be considered. For example peculiar characteristics of the test site, or of the deformations occurring in it, may have an influence on the results. This consideration has indeed led to the repetition of the same study on the two in many aspect different sites of Groningen and Campi Flegrei. Another factor which can influence the results is the temporal extension of the time series of interferograms. One could suspect in fact that, while the quantities estimated in each interferogram separately are independent on the extension of the time series, the order of magnitude of the quantities estimated along the series can on the contrary be different on different time intervals. In practice, it is possible that on shorter total time intervals the greatest part of the pixels under exam are more stable than when temporally more extended series are considered. Especially for the temporal standard deviations this could be the case. It is easy in
fact to imagine that in the course of time the stability of a single pixel degrades, because more accidents can occur which can affect the backscattering properties of the corresponding scattering centre on the ground. As a consequence, it seems logical to expect that on longer total time spans the $\tau$ of single pixels will generally increase.

In order to investigate this hypothesis the temporal standard deviations were re-determined by considering a shorter time interval from April 1995 to December 1997. This was done for each feature selected for the Campi Flegrei test site, and the resulting standard deviations were compared with those obtained by considering the whole series (time extension April 1995 to April 1999).

The differences between the temporal standard deviations obtained in the two cases resulted to be very small: about 85% of the points showed a difference within 0.1 rad between the two cases. Moreover, the differences did not have a preferential sign, which means in particular that considering a longer time interval does not bring worse results. It is therefore more likely that these differences are due to statistical oscillations in the standard deviation value.
Conclusions

8.1 Summary and conclusions

The INSAR technique has been applied for the monitoring of deformations over temporal scales of several years on two different test sites. Interferograms on such long time spans are often affected by temporal decorrelation to such degree that a database approach was preferred. The databases used in the context of this research have been generated with a particular procedure, which has revealed several advantages. The co-registration of all the images to a common master permitted to compare the interferograms at pixel level without the need of geocoding them first to a cartographic system, a rather complicated, certainly not error-free processing step, which also requests first a conversion of the phase values from slant range to height. Moreover, in this way each image had to be co-registered only once, and the subsequent interferometric combination of all the possible pairs of images could be performed more rapidly.

The database generated for each site was then used for the retrieval of deformations. The temporal decorrelation on the Groningen test site affected the interferograms so heavily, that only point-wise information was present, mainly from single buildings and urban areas. Single features, despite their small size, turned out to have very stable phase information in time. In absence of internal deformations the pixels containing the reflection from a single feature (usually few tens of pixels for a large building) had similar phase values in the same interferogram. The standard deviation of these values ranges in most cases from 0.2 to 0.8 rad. Due most probably to their being a set of different, independent man made objects, urban areas were
Conclusions

slightly less spatially homogeneous than the single features, with standard deviations of the phase reaching in many cases even 1 rad. Considering the fact that 1 rad corresponds to 0.5 cm deformation in the slant range direction and that these values refer to time intervals of years, the stability of anthropogenic features remains still remarkable also when entire urban areas are considered. Another rather surprising result was the high stability in time of the differential phase of single pixels. The standard deviation of the phase values of one single pixel in all the interferograms reached in many cases the level of 0.2 to 0.4 rad.

An attempt has been also made to establish a link between the temporal stability of the phase information and other properties, such as the coherence value of the pixels, their spatial stability, and their number of occurrences with high coherence value in the interferograms of the database. The only significant relationship which was found between these properties was that the most spatially homogeneous pixels revealed to be also the most temporally stable ones. No relationship between the number of occurrences or the coherence value of the pixels and their spatial and/or temporal stability has been evidenced, at least in the considered cases of pixels with coherence higher than 0.5 (i.e. $\Gamma_{\min} = 0.5$).

A fact which is worth mentioning, and can be considered as an additional proof of the validity of the information from single pixels, is the identification of a deformation signal within one of the features being examined, located in correspondence of a road.

Finally, the repetition of the analysis of highly coherent features on interferograms of the Neapolitan test site, performed to check the generality of the results found in the case of Groningen, gave results in general agreement with these latter. One of the few differences between the two cases was a slightly better spatial stability of the phase values for the analysed features in the Neapolitan site with respect to those in Groningen, with $\sigma$’s in many cases of the order of 0.2 to 0.6 rad. This is probably due to the fact that in that site the selected features were buildings or small complexes of buildings, while in Groningen also extended urban areas were considered. Another difference with respect to the case of Groningen was that, for the features in the Naples test site, increasing the coherence limit seemed to cause an improvement of the $\sigma$: since however the number of selected pixels with high coherence was extremely reduced, this result cannot be assessed definitely.

Once the relatively high phase stability of the small, highly coherent anthropogenic features was assessed, their phase information could be used for the retrieval of deformations in the Groningen area in the period 1992–1996. For this purpose, the relative deformation among seven urban areas was determined by taking, for each area, the most representative value of the differential phase of its highly coherent pixels. The resulting deformations turned out to be in good agreement with
those from a pre–existing model, with correlations at the level of 0.7 to 0.9. However, a larger set of deformation values within each interferogram and also the possibility to apply this approach on an higher number of long term would be necessary to confirm these results.

As for the deformation monitoring in the Campi Flegrei test site, the higher deformation rate and the less severe decorrelation permitted to use whole interferograms and not only point-wise information. Still, the fact that these interferograms showed patches of almost totally decorrelated areas made it quite difficult to extract reliable information from each one of them singularly. Their interferometric information was therefore merged in a least squares sense, to give as solution a time series of deformations computed with respect to the day corresponding to the oldest SAR image used in the database. The found solution, besides showing for the first time the temporal evolution of the twodimensional deformation pattern, was in good agreement with the expected deformations from other kind of measurement techniques, in particular from precise levelling. The total deformation, measured in the slant range direction, from February 1993 to April 1999 is estimated in about 23cm, with a standard deviation estimated at the level of 0.4cm. The found mean subsidence rate, of the order of about 0.3cm/month, is quite in agreement with the expected value from levelling data of 0.4cm/month. In addition, the interferometric results revealed variations in the deformation rate which would be hardly detectable at the temporal samplings realized with levelling.

Comparison of the original data with the simulated ones from the least squares solution revealed that in general the corrections applied by the adjustment were of the order of 2 to 8mm, and had a sparse, noise-like character. Only in two cases larger discrepancies between simulated and real interferogram were found, with heavy corrections also in the highest deformation area, where they took even a signal-like connotation. The reason for this could be the presence of a image-related bias in the image common to these two interferograms.

Finally geophysical modeling of the deformation source was attempted by using the least squares solution and the inversion of four pairs of interferograms, each pair consisting of one interferogram from ascending and one from descending passes spanning about the same time interval [41]. The best fitting solution found in the assumption of an elastic half-space is a contracting $1 \times 4$km long rectangular dislocation at a depth of 2.8km in the East-South East direction beneath the city of Pozzuoli. The solution is consistent with other studies suggesting that the primary deformation mechanism is of hydrothermal nature.
8.2 Discussion and recommendations

The analysis performed on the anthropogenic features demonstrates that, despite their small size, such features can maintain stable interferometric phase information for years. They can therefore be used as targets of opportunity for the retrieval of deformations in highly decorrelated interferograms. This fact has also been confirmed by the results obtained recently with the "Permanent Scatterers" method developed by the Politecnico of Milan [17, 18]. Of course not all features are phase stable, therefore it is always necessary to determine a procedure to identify the suitable objects. The selection procedure presented in this thesis is in first instance based on the coherence value: this is a kind of "natural" selection criterion in interferometry, as usually interferometric phase products are always coupled with their corresponding coherence products, which are considered as a measure of the quality of the phase information. The coherence has to be estimated on a certain shifting window, and the fact that the coherence at a certain pixel is estimated by using the coherence at its neighbouring pixels causes a "spreading" of the correlation values of each resolution element of the ground in the resolution elements of the images. It has to be said that multi-looking with the same window size used for the coherence estimation reduces this effect, reintroducing a direct correspondence between the coherence and the phase images: in other words, this assures that the phase information causing high coherence at a given pixel is certainly contained in the same pixel of the corresponding phase image. The drawback is that in this way the phase resolution is also reduced. Working in full resolution, i.e. without multi-looking, on one side allows us to pinpoint with greater precision the time-stable phase information we are searching for. On the other side this requests the definition of a new statistical model for this information, in particular for the purpose of determining a degree of quality of this information which could take the place of the interferometric coherence.

The results of the presented work suggest that the analysis of the temporal stability of a single pixel could be a better way to identify the stable scatterers than a selection based only on the coherence. This of course implies the availability of a large database of interferograms, in order to have for each pixel, a significant number of measurements for a statistical treatment. Such a requirement might limit considerably the number of regions where this kind of unconventional interferometry can be applied, however the feeling is that a method relying on sparse phase information in condition of extreme decorrelation can be scientifically strong enough only when used in a database approach. In the end, using several interferogram is the only way to compensate the poorness of the information in a single one.

The need for a larger dataset was even more evident when an attempt has been made to retrieve the relative deformations between seven urban areas in Groningen.
The ambiguity given by the presence of a $2\pi$ phase jump could maybe have been solved if more interferograms would have been available. In this sense, it would be certainly desirable to try the proposed approach with a larger number of interferograms and with more than the only seven representative phase values which have been retrieved in the presented case study.

It has to be stressed also that the proposed approach to retrieve deformations by defining a 'representative' value for a cluster of highly coherent pixel can be applied only in regions of very slow deformation. The underlying assumption was in fact that each of these clusters, which had at maximum the size of a city, would be subject to the same deformation. This is a reasonable assumption in an area like the one of Groningen where subsidence is slow and on a large spatial scale. The same approach would not be feasible in an area like Campi Flegrei, where the deformations are stronger and the signal varies on much smaller spatial scales. It would be in fact extremely difficult to isolate clusters of pixels for which we can assume that they are undergoing the same deformation.

Fortunately the fact that Campi Flegrei is less heavily affected by temporal decorrelation permits to develop a less risky approach than using the phase information of single pixel. The presented least squares approach has several advantages. First of all, it permits a constant check of the consistency of the information contained in the interferograms, allowing the detection and elimination of processing errors, especially phase unwrapping errors. Secondly, the combination in a least squares sense of redundant information from (partially) overlapping interferograms permits to add information where it is missing due to decorrelation. Decorrelation patches in the long term interferograms can in this way if not totally, at least partially, be filled with information from shorter term interferograms. Finally, the least squares solution generates a time series which describes the evolution of the deformations as a chronologically ordered sequence, without the need of having the "direct" interferogram for each pair of subsequent dates. The fact that such a time series was successfully used for the geophysical interpretation of the ongoing deformation process, demonstrates once again the great potentialities of the interferometric technique as an operational tool for deformation monitoring and modelling. Also in this case, the addition to the database of more interferograms would increase the temporal sampling allowing to describe both qualitatively and quantitatively more precisely the temporal evolution of the deformations. Weighting of the interferograms for the least squares adjustment would also be desirable, however this is strictly related to the interesting issue of the definition of a "quality measure" of an interferogram. Finally, the integration of interferograms from different tracks and from ascending and descending passes in the same database is also an important issue. This would not only extend the spatial coverage of the area under study but
also provide an ulterior way to increase the temporal sampling of the data reducing the time interval between (non-Tandem) scenes to less than 35 days. The use of both ascending and descending passes moreover would allow the necessary geometrical constraint for the determination of the three-dimensional deformation vector.
Bibliography


List of symbols

$A, B, C$ generic SAR images
$AB, BC, AC$ generic interferograms
$A_i$ imaginary component of circular gaussian field
$A_r$ real component of circular gaussian field
$\mathbf{A}$ model matrix of the least squares problem $a = \mathbf{A}b$
$a$ set of observations in the least squares problem $a = \mathbf{A}b$
$B$ interferometric baseline
$B_\perp$ perpendicular component of the interferometric baseline
$b$ set of unknowns in the least squares problem $a = \mathbf{A}b$
$c$ speed of light
$Corr$ correlation coefficient
$Corr_{ij}$ correlation coefficient between the sets $DF_i$ and $DF_j$
$D_{AB}(i, j)$ absolute deformation at pixel coordinates $(i,j)$ measured from interferogram $AB$
$DF_i$ set of differential phases
$d\#$ day number
$E$ amplitude of circular gaussian field
$E\{ \}$ mean operator
$Err$ difference between true and estimated observations in a least squares sense
$aF_b$ generalised hypergeometric function
$\phi$ interferometric phase
$\phi_0$ expected value of interferometric phase
$\phi_1, \phi_2$ phase signal of a SAR image
$\phi_{flat}$ flat earth phase
$\phi_{f_t}, \phi_{f_t}'$ flat earth-corrected interferometric phase
\( \delta \phi_f, \delta \phi'_f \) interferometric phase error

\( \Delta \phi \) differential phase

\( \delta \Delta \phi \) differential phase error

\( d\phi_{ij} \) differential phase of pixel \( j \) in interferogram \( i \)

\( \overline{d\phi_i} \) mean value of differential phases in interferogram \( i \)

\( d\phi_{ij}' \) same as \( d\phi_{ij} \) but with the mean value \( \overline{d\phi_i} \) subtracted

\( d\phi_k \) mean value of the \( L_k \) phase values of one pixel

\( \Gamma \) Gamma function

\( \Gamma_{min} \) coherence threshold

\( \gamma \) complex correlation coefficient

\( \hat{\gamma} \) estimated complex correlation coefficient

\( h \) antenna height from the ground

\( I \) complex interferometric signal

\( I# \) interferogram serial number

\( I_1, J_2 \) groups of interferograms

\( I_{di} \) SAR image corresponding to day \( d_i \)

\( IF_i \) generic interferogram

\( IF_r \) reference interferogram

\( K \) generic constant

\( L \) radar antenna length

\( L_1 \) number of multi-looking samples

\( L_k \) number of occurrences of one pixel in the selected sets of \( DF_i \)

\( L_s \) synthetic antenna length

\( \Lambda_{max} \) upper limit of \( \tau \)

\( \lambda \) wavelength

\( M \) no of pixels occurring at least twice in the series of selected sets of \( DF_i \)

\( M_{\sigma} \) mean ratio between standard deviation \( \sigma \)'s for \( \Gamma_{min} = 0.5 \) and 0.8

\( N_i \) number of selected pixels in interferogram \( i \)

\( N_1 \) number of sets \( DF_i \) spatially homogenous

\( N(\mu, \sigma^2) \) normal distribution

\( \nu \) degrees of freedom

\( pdf \) probability density function

\( R_a \) azimuth resolution

\( R_{gr} \) ground range resolution

\( R_r \) slant range resolution

\( R_{\sigma} \) mean ratio between \( \sigma \)'s for \( \Lambda_{max} = 0.6 \) and for all pixels

\( r \) slant range distance sensor-surface

\( \Delta r \) difference in range distance
List of symbols

\( S_{LC_{1,2}} \)  generic SLC image
\( S^2 \)  estimated variance of the r.v. \( X \)
\( S_1, S_2 \)  complex SAR signal
\( S_r \)  set of selected pixels
\( \sigma \)  mean intensity of circular gaussian field
\( \sigma_\phi \)  standard deviation of the interferometric phase
\( \sigma_i \)  standard deviation of differential phases in interferogram \( i \)
\( T_x \)  ratio of mean \( \tau \) for different coherence limits
\( \Delta t \)  time difference
\( \theta \)  look angle
\( \theta_1, \theta_2 \)  phase of circular gaussian field
\( \tau \)  radar pulse length
\( \tau_k \)  standard deviation per pixel
\( V \)  radar system velocity
\( X \)  generic random variable
\( X_{d_0} \)  deformation at day \( d_0 \) with respect to day \( d_1 \)
\( x, y \)  generic random variables
\( \chi \)  chi square distribution
\( z \)  terrain height
\( z_{2\pi} \)  height ambiguity
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>DEM</td>
<td>Digital Elevation Model</td>
</tr>
<tr>
<td>DEOS</td>
<td>Delft Institute for Earth-Oriented Space Research</td>
</tr>
<tr>
<td>EM</td>
<td>Electro-magnetic</td>
</tr>
<tr>
<td>ERS-1,ERS-2</td>
<td>European Remote Sensing Satellites</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
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<tr>
<td>GPS</td>
<td>Global Positioning System</td>
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<tr>
<td>IGM</td>
<td>Italian Military Geographic Institute</td>
</tr>
<tr>
<td>INSAR</td>
<td>SAR Interferometry</td>
</tr>
<tr>
<td>JERS-1</td>
<td>Japanese Remote Sensing Satellite</td>
</tr>
<tr>
<td>SLC</td>
<td>Single Look Complex Data</td>
</tr>
<tr>
<td>SAR</td>
<td>Synthetic Aperture Radar</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SRTM</td>
<td>Shuttle Radar Topographic Mission</td>
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This thesis addresses the problem of the temporal decorrelation in SAR Interferometry, with particular emphasis on its effects on long time scales. On intervals of more years the presence of heavy decorrelation makes it extremely difficult, if not impossible, to apply SAR Interferometry in the conventional way. Alternative methods have therefore been searched for the retrieval of deformation information in such extreme conditions.

Two test sites were considered, having completely different characteristics and different behaviour with respect to temporal decorrelation. For each of these sites a different, new approach for the measurement of deformations with interferometry has been developed. Both methods use a database of interferograms: the interferograms covering long time scales are in fact often so corrupted by decorrelation, that it is practically impossible to retrieve unambiguous information by considering them separately. The logical way to compensate for this reduction of information seemed therefore to be some combination of as many as possible interferograms. A particular procedure was followed to generate the database, based essentially on the co-registration of all the images on the same master prior to their interferometric combination.

The databases generated for each of the two considered test sites were then used to retrieve the deformations. The first test site under study was the province of Groningen, in the Netherlands, where subsidence due to extraction of natural gas occurs at a very low rate (less than 1cm per year). Long term interferograms of the area resulted to be almost totally decorrelated: only highly coherent, mostly pointwise phase information from anthropogenic features could be detected. Despite their small spatial scale with respect to the image resolution, however such features resulted to be particularly stable in time, in many cases even at the level of few millimetres. An unconventional approach has then been used to measure the deformations using the phase information from these objects, and led to results...
which were in good agreement with the subsidence as computed from a pre-existing model.

As for the second test site, the bradyseismic area of Campi Flegrei, the higher rate of the ongoing subsidence (about 4mm per month) with respect to the Groningen area, and the more favourable meteorological conditions resulted in general in a less severe decorrelation. Nevertheless, integration of information from more interferograms was necessary to give a reliable estimate of the deformations. This was accomplished by inverting, in a least squares sense, the set of deformations obtained from the interferograms of the database. The result, a time series of deformation figures describing, for the first time in a two-dimensional picture, the evolution in time of the subsidence from 1993 to 1999, was in particularly good agreement with the expected values from existing levelling data, and put in evidence in particular the non-linearity of the deformation rate in this highly hazardous area.
Samenvatting

Dit proefschrift behandelt het probleem van temporele decorrelatie in SAR Interferometrie, met name over zijn effecten over grote tijdschalen. De aanwezigheid van sterke temporele decorrelatie bij tijdsintervallen van jaren maakt het zeer moeilijk, zo niet onmogelijk, SAR interferometrie op de gebruikelijk manier toe te passen. Alternatieve methoden zijn daarom gezocht om deformatie-informatie onder zulke extreme omstandigheden te verkrijgen.

Twee proeflocaties werden onderzocht, die allebei compleet verschillend zijn in zowel aard als gedrag met betrekking tot temporele decorrelatie. Voor beide locaties werd een verschillende, nieuwe benadering voor het meten van deformaties met interferometrie ontwikkeld. Beide methoden gebruiken een database van interferogrammen. Doordat de interferogrammen over lange tijdschalen erg verstoord zijn door de decorrelatie, is het praktisch onmogelijk eenduidige informatie uit elk van hen afzonderlijk te krijgen. De logische manier om voor dit verlies van informatie te compenseren was de combinatie van zoveel mogelijk interferogrammen. Een speciale procedure werd voor het samenstellen van de database gebruikt: het co-registreren van alle beelden op dezelfde master voordat de interferometrische combinaties werden gemaakt.

De gemaakte databases voor elk van de twee gebruikte testgebieden werden gebruikt voor het bepalen van de deformaties. De eerste proeflocatie was in de provincie Groningen, waar de bodemdaling ten gevolge van aardgaswinning vrij langzaam is (minder dan een centimeter per jaar). Lange termijn interferogrammen van dit gebied bleken bijna geheel gede correleerd te zijn; slechts de hoog coherente, puntsgewijze faseinformatie van gebouwen en objecten kon worden gevonden. Ondanks hun kleine schaal ten opzichte van de beeldresolutie, bleken deze objecten zeer stabiel in de tijd te zijn; in veel gevallen op het niveau van een paar millimeter. Een nieuwe benadering is toen gebruikt om de deformaties met behulp van de faseinformatie van deze objecten te meten. Deze resultaten waren in geode overeen-
stemming met die uit een bestaand deformatiemodel.

Voor het tweede testgebied, het bradiseismische gebied van de Campi Fregrei (Italië), was de decorrelatie, door de grote zakkingssnelheid (ongeveer 4mm per maand) ten opzichte van Groningen en door de gunstigere weersomstandigheden, minder ernstig. Maar de integratie van informatie van meerdere interferogrammen was toch nodig om een betrouwbare schatting te geven van de deformaties. Dit werd bereikt door het inverteren, op kleinste kwadraten wijze, van de set van de- 
formaties verkregen uit de interferogrammen van de database. Het resultaat, een


tijdreeks van deformatieplaatjes die, voor het eerst in een twee-dimensionaal figuur, de ontwikkeling in de tijd van 1993 tot 1999 van de zakking van weergeeft, is in zeer goede overeenstemming met de verwachte waarden uit bestaande waterpasgegevens en laat in het bijzonder zien dat de deformatie snelheid in dit gevaarlijke gebied niet lineair is.
Acknowledgements

I owe a great debt to many persons who helped me during my PhD research. First of all I would like to thank my promotor, Prof. J.T.Fokkema, who welcomed me in his group about one year ago and gave me the support and trust I needed to finish my PhD thesis. I am grateful also to the people of the "AIO-zolder", in particular to Ranajit Ghose, for the nice atmosphere I experienced during my stay at the Faculty of Applied Earth Sciences.

Thanks go also to TNO-FEL, for providing the financial support for this PhD research.

Part of this research was performed in cooperation with Eugenio Sansosti, Riccardo Lanari, Paolo Berardino and Gianfranco Fornaro, of IRECE-CNR, and with Paul Lundgren of JPL. To them I want to offer very special thanks, for their enthusiastic participation in the Campi Flegrei research, and for the stimulating discussions and exchange of ideas (and data). I owe thanks also to the Vesuvius Observatory of Naples, for their kindly providing information on the deformation at Campi Flegrei as measured from precise levelling.

A particularly pleasant experience has been my staying at the German Remote Sensing Data Centre of DLR, and I wish to thank all the people of DFD for their kindness and hospitality. I am indebted in particular to Richard Bamler and Michael Eineder for the inspiring conversations during my stay in Oberpfaffenhofen and in different other occasions.

There are other persons who I like to thank for their interest in my research and for the interesting exchange of impressions and suggestions. First among them is Prof. Hartl, who suggested me five years ago this research topic, and who helped me to understand the principles of SAR Interferometry. Very stimulating were also the conversation with Profs. Rocca and Prati and with Alessandro Ferretti of the Politecnico of Milan: with them I share the interest for extremely decorrelated interferograms.
I am also very grateful to the (ex)-members of the FMR group at the department of Geodesy here in Delft. Many thanks go to Bert Kampes and Ramon Hanssen: with them I shared the room and the interest in INSAR. There are two persons which I want to thank in particular: Radboud Koop and Wil Luijten. Their moral support and friendship has been decisive for this PhD research and for the well-being of its author.

I want to thank also the DEOS members, and in particular Boudewijn Ambrosius and Prof. Wakker, for their interest in my research and the encouragement.

Finally, my deepest love and gratitude goes to my husband Martin and to my son Giacomo, for being simply wonderful as they are.
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