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Surface Wave Tomography at Exploration Scale

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Chapter 1

Introduction

In this work we present the application of a direct tomographic inversion method, similar to that historically used in global seismology, to exploration scale. We will start with an *excursus* on the physical background necessary to understand how surface waves propagate and the characteristics which make these waves suitable for inversion. In the following section the optimal acquisition methods will be described. These are not very different from those for the datasets used for industry-standard reflection seismology imaging; however a few remarks have to be made. For instance, the length of the line, the spacing between receivers and the dominant wavelength are essential parameters to take into account when acquiring a dataset for surface wave analysis. The following section, regarding the processing, will present the most used methods for inversion of surface waves at exploration scale, those at regional (seismological) scale and the method used to perform the inversion. These can be classified into *tomo- graphic* and *non tomographic* approaches. In both cases, the aim is to extract phase velocity dispersion curves from the data and invert them to get 1D models. The key difference lies in how the dispersion curves are extracted and the "meaning" we give to them. When performing a *non tomographic* inversion it is possible to extract the curves in many ways, e.g. by computing a Fourier transform in space and time (*f* − *k* domain) and pick the curve as energy maxima, as surface waves are the most energetic event in a seismogram. Most commonly, the *f* − *k* spectrum is an average one over a certain number of stations. Each of these curves will be inverted to get a 1D model and each of these models can be connected to the others via constraints (*regularisation*). On the other hand, when one wants to perform *tomographic* inversion, the dispersion curves are extracted by computing the average slowness of surface waves travelling along a path from one receiver to another, illuminated by the same source. Therefore, since more paths can cross the same model point, more dispersion curves have to fit the model parameters at each model point. Seismologists, at this point, generally compute phase velocity maps from which they extract further dispersion curves to be inverted. What’s new in the method used for this work is the fact that we invert dispersion curves, without computing phase velocity maps. This is why it’s called "direct inversion". This has several positive aspects, e.g. the computational efficiency. Another advantage, common to all tomographic approaches is the fact that it uses only two stations and therefore does not need a dataset with a large number of channels. On the other hand, it has some limitations both on the acquisition parameters (especially offset and distance between the two receivers) as well as the fact that it only inverts for the fundamental mode dispersion curve.

Two datasets have been analysed: one shot in New Zealand by ETH Zurich, one shot in the area of Torino by the Italian National Centre for Research (CNR). The first constitutes of five 2D seismic lines, one of which at ultra-high resolution (1m receiver spacing and 2 m source spacing). This dataset has already been inverted using the established *multichannel analysis of surface waves* method. Therefore, it was a good dataset to start with as results could be compared with those from this previous inversion. The CNR dataset is a small 3D dataset, above an artificially made tank filled with loose sand. The fact that the map of the subsurface was already known made it a good dataset to test the validity of the method also in 3D.

In the appendix, we show tests performed on a large 3D seismic survey, acquired in Oman for oil exploration purposes. This dataset is known in the geophysical community (e.g. Gouédard et al., 2012) for having a low signal to noise ratio. Furthermore, the subsurface to be mapped shows strong lateral variations. For this reason, preliminary tests were performed in order to find a method to increase the signal to noise ratio and therefore assess the feasibility of a direct tomographic inversion. A good solution could be to stack the traces from two different sources aligned with the same couple of receivers.
Chapter 2

Theoretical Background

2.1 Free Surface Waves

2.1.1 Introduction

Surface waves can be defined as those waves which propagate in a limited layer close to the boundary between two media; in particular one could say that they can only exist where there is a free surface, like, for instance, at the boundary between the Earth and the atmosphere. These waves are generally referred to as ground roll and are easily identifiable on a seismogram as they commonly show the most energetic event. An example of surface waves on a seismogram can be seen in fig. 2.1

Figure 2.1: Example of seismic survey, with the main events shown (after Drijkoningen, 2011)
In common processing for seismic reflection imaging, these waves are considered noise and therefore suppressed. However, they have several properties which make them suitable for inversion and imaging. In particular, the fact that the thickness of the layer they propagate in is roughly comparable to the wavelength of the wave. This implies that different waves at different frequencies will propagate at different depths and therefore, if the medium is vertically heterogeneous, waves at different frequencies will have different velocities of propagation. This phenomenon, known as geometric dispersion, can be described via the so-called dispersion curves. The inversion of the phase velocities dispersion curves allows an estimation of the dispersive characteristics of the subsoil and therefore build a velocity map of the shallow subsurface. There are several types of surface waves (Love, Scholte, Lamb, Stoneley, Rayleigh, etc.), each of these waves may be used for imaging purposes, but the most widely used, and those this thesis will focus on, are Rayleigh waves.

2.1.2 Rayleigh Waves

Rayleigh waves are surface waves, generated by the interaction of inhomogeneous plane p-waves and vertical s-waves under specific boundary conditions. The resulting propagation can be seen in fig. 2.2.

Propagating approximately 67% of the energy coming from a source located at the free surface, Rayleigh waves are by far the most energetic during a seismic experiment. Furthermore, they are subject to cylindrical spreading, i.e. their amplitude decreases with distance as $\frac{1}{\sqrt{r}}$. This makes them more energetic at far offset with respect to body waves, as these are subject to spherical spreading and therefore their amplitude decreases as $\frac{1}{r}$. Similar to all other surface waves, Rayleigh waves are subject to the phenomenon of geometrical dispersion; in fact its propagation velocity $V_R$ is dependent mainly on shear-wave velocity $V_S$, as well as on Poisson’s ratio and bulk density. In a layered medium, $V_R$ becomes therefore frequency-dependent. Furthermore, the depth at which the waves propagate depends on the frequency: high frequency (i.e. short wavelength) waves will propagate in the shallow layers, while low frequency (i.e. long wavelength) waves will propagate at larger depths. A schematic representation of this phenomenon is shown in fig. 2.3.

![Figure 2.2: Propagation of Rayleigh waves (after Bolt, 1976)](image)

An interesting and problematic aspect of propagation of Rayleigh waves in vertically heterogenous media is that propagation can occur at different modes; in other words, several modes with different propagation velocities for the same frequency can occur at the same time, making it difficult to interpret the dispersive properties of the investigated medium.

From a more mathematical point of view, Rayleigh waves can be modelled in terms of eigenvalues and eigenfunctions. In fact, assuming a plain strain field, imposing the boundary conditions of a free surface (lower halfspace with no stress and strain at infinity and higher halfspace with no stress) and the continuity of both stress and strain at the interface between the two halfspaces, one can write the...
equation of motion as follows:
\[
\frac{df}{dz} = A(z) \cdot f(z).
\]  
Vector \( f \) is formed by two displacement eigenfunctions and two stress eigenfunctions; the matrix \( A \) is a \( 4 \times 4 \) matrix, which depends on the vertical distribution of the soil properties. The eigenvalues to this equation are specific values of the wavenumber \( k \); therefore, the solution to this equation is non-trivial only for these specific values of \( k \). For a layered system, it can be found that this is verified for a special relation between frequency and wavenumber of the type \( k = k_j(\omega) \). To be noted, that the latter is a multivalued function of the frequency: for a given frequency, more values of \( k \) are possible. Therefore, this is a reasonable mathematical expression for the so-called modal curves. It is possible to write the equation of motion using this relation, getting the so-called Rayleigh secular equation, which can be expressed in implicit form as
\[
F(k, f) = 0,
\]
\[
F_R [\lambda(z), G(z), \rho(z), k_j, f] = 0.
\]  
\( k \) is the wavenumber, \( f \) is the frequency, \( \lambda \) is the Lamé parameter, as defined in Hooke’s law, \( G \) is the shear modulus and \( \rho \) is the mass density. The modal curves are therefore a possible solution to the problem. Each mode (except the fundamental one) has a minimum frequency at which it propagates. Each mode (except the fundamental one) has a minimum frequency at which it propagates. An example of modal curves can be seen in figure2.4 (Taken from Socco and Strobbia, 2004). The modal curves solution has several properties, which can help implement an inversion method. Firstly, the Rayleigh-wave velocity is mostly dependent on the shear wave velocities of the layers; therefore bulk density and Poisson ratio could be assumed \( a \ priori \), decreasing the total number of unknown parameters. Furthermore, because of the way Rayleigh waves propagate, information about deeper layers is carried only by the low frequency components of the waves, while those for the shallow layers are carried by all frequencies. For this reason, the inversion problem is a mix-determined one. The modal curves are theoretical solutions; on the data, practically, these are detected as energy maxima in the spectral domain. This may be a problem, since the different modes may be difficult to separate and recognise as they may intersect each other and the data itself may contain apparent dispersions.
2.2 Data Acquisition

A dataset acquired with the purpose of surface wave analysis should be designed so that the information contained in the data allows the further processing and inversion. In detail, the data should have a high signal to noise ratio over the widest frequency range as possible, the different modes of propagation should be detectable and it should be possible to estimate the uncertainties. It has to pointed out, that the receivers have to be aligned with the source. Furthermore, the acquisition technique depends on the purpose of the survey, the desired investigation depth as well as the scale of the survey.

The pioneers of surface wave analysis for seismological studies used surface wave data to detect the depth of the Mohorovic’s discontinuity or to create a map of the deep crust and upper mantle (Knopoff et al. 1966, Landisman et al. 1968). To do this, they used recordings from macro seismic events at very long periods (from 10 to 80s) with receivers at very large distances (from a few hundred to several thousand kilometres). On the other hand, at exploration scale, a controlled source is used and receivers are geophones with central frequencies of a few Hz. This, together with the limited amount of energy produced by a controlled source, the necessarily limited size of the recording array and the short spacing between receivers allows detection of only signals with short wavelength. In fact, the longer the distances involved, the larger the difference in traveltimes, i.e. phase velocities, between the different modes of propagation. Therefore, at this scale, only higher frequencies are detectable and the vicinity of the receivers together with the shorter array length can lead to difficulties in separating the modes of propagation. Moreover, the limited amount of energy released by active sources In fact, the array length influences the wavenumber resolution and therefore the ability to separate the different modes. Array length determines the maximum detectable wavelength and therefore the maximum penetration depth. On the other hand, long arrays have a lower signal to noise ratio and are less sensitive to lateral variations. In order to avoid spatial aliasing, receiver spacing should be short enough, while sampling rate should also be low enough to avoid temporal aliasing. Both of these parameters can be determined with the Nyquist theorem. Because we will use the so-called two-station method, the distance between the two receivers should be at least half the dominant wavelength to make reliable measurements even at lower frequencies, as pointed out by Yao et al. (2006). Therefore, too small datasets or datasets with too long receiver spacing may not be suitable for two-station tomography. In conclusion, the acquisition modes of a dataset for surface wave analysis are not very different from those of other seismic
exploration methods. For this reason, datasets shot for reflection seismic imaging or refraction seismic tomography are generally suitable for most surface wave analysis processing methods.

2.3 Data Processing

Generally a dataset shot for reflection/refraction seismology imaging purposes is of good enough quality for surface-wave tomography. In fact, these provide a sufficiently high signal to noise ratio as well as arrays of the correct dimensions. Instead, processing has the purpose of extracting all the information on the surface waves necessary for the inversion and their uncertainty. In the following sections the most commonly used processing procedures are explained. Aim of all of these methods is to extract phase velocities dispersion curves; two approaches are possible: tomographic and non-tomographic. The former has been historically used by seismologists, while the latter is mostly used at exploration scale. The aim of this work is to apply a tomographic approach at exploration scale.

2.3.1 Non-tomographic approach

The non tomographic approach is most commonly used at exploration scale and involves extraction of dispersion curves and their inversion to build 1D models. In order to identify the surface waves and their propagation modes, a well established method is to perform a transformation to the frequency-wavenumber (or \( \omega - p \)) domain. In fact, since we are dealing with an eigenvalue-eigenfunction problem, the discrete eigenfunctions corresponding to an eigenvalue should appear as lines in the f-k domain. Due to spectral leakage, on a real dataset - where also random and coherent noise are present - the surface waves are identifiable as energy density maxima or as the dominant energy events. Once these energy maxima have been identified, the velocities are simply computed as \( v = \frac{2\pi f}{k} \). From this, a curve representing the positions of energy maxima at each frequency can be extracted: the so-called dispersion curve. The extraction is not always easy: the dispersion curve may be discontinuous and it can be related to more than one mode. In order to gain more information on the dispersion curve related to the fundamental mode, it can be helpful to consider also relative maxima and not only absolute maxima and compare the obtained curve(s) with a reference curve. Sometimes, however, the modes may be superimposed and therefore the fundamental one not identifiable. If the recorded dataset is multichannel, a processing method named multichannel analysis of surface waves can be used. The aim of this analysis is to build dispersion curves from a multichannel seismic recording. Due to the fact that the recording is multichannel, a decomposition of the record in single-frequency components is possible. In fact, in multichannel records it is possible to identify the phase of the signal with good accuracy for each wavelength. The separation of single frequency components of the signal allows, via an analysis of the frequency patterns, arrival times and amplitude of each seismic event, the removal of noise. Several algorithms have been implemented to compute the \( f - k \) spectrum. One of the most commonly used is the MUSIC algorithm (Schmidt, 1986). This has the advantage of not needing evenly spaced receivers, which makes it useful for large 3D datasets where one might want to select a certain number of receivers not necessarily equally spaced. Therefore multichannel recording can deliver significantly higher signal to noise ratio, increasing the accuracy of the dispersion curves. These can be computed by identifying the phase velocity at each frequency. The computed dispersion curves are then inverted to build 1D models, one per each dispersion curve. The models are linked to each other via some constraints in the model parameters, i.e. regularisation. The main drawback of non-tomographic methods is the low sensitivity to lateral variations in the subsurface.

2.3.2 Tomographic approach

The second approach, called tomographic, has historically been used in seismology and consists in extracting dispersion curves by computing the average phase slowness between two aligned receivers illuminated by the same source. This is the so-called two station method and it consists in reconstructing the dispersion curve by cross-correlating the signal of two receivers. Time lags of the maxima in the cross-correlation matrix correspond to the travel time of a certain mode of propagation from the first to the second receiver. This method was originally implemented by seismologists with the purpose of mapping the deep crust of the Earth. In 1966, Knopoff et al. (1966) built a map of the lower crust and upper mantle in the western Alpine region via a primitive inversion of very low frequency dispersion curves. The authors took the records from four seismological stations in the area; they considered four
different paths, of different lengths, crossing the Alps and built dispersion curves (from average slowness) along these paths. They then compared the models obtained from these dispersion curves with previous images obtained from geological observation as well as refraction seismic experiments. Similar studies were performed by Landisman et al. (1969), Dziewonsky et al. (1968 and 1969). Bloch and Hales (1968) found phase velocities of Rayleigh waves between seismological stations in southern Africa. The method included time windowing, in order to isolate the Rayleigh wave fundamental mode, via computation of the group velocity envelope. In fact, it is very rare that only the Rayleigh wave fundamental mode propagates and it is therefore advisable to isolate it before any further processing. After this, the phase velocity dispersion curves were evaluated in two different ways. The first consisted in determining the time lags at which the traces from two stations are in-phase by summing (subtracting) the two traces. Wherever a maximum (minimum) occurred, at that time lag the two signals were in-phase and therefore the time lag is the travel time of the Rayleigh waves from the first to the second receiver. The second method consisted in computing the cross-multiplication matrix of the two traces and finding the maxima in such matrix. Again, the time lags corresponding to the maxima give the travel-time of the waves from one receiver to the other. The conclusion of the authors is that the latter method is more accurate. Based on these, in more recent years, Boiero (Ph.D thesis, 2009) used a similar method to perform an inversion of seismological data to map the depth of the Mohorovic’s discontinuity in south-east asia. The network of the stations used and the paths along which he computed the dispersion curves are visible in 2.5. Each seismological station also corresponds to a point where a 1D model is computed. It is evident how multiple paths cross each model point, i.e. multiple dispersion curves are inverted to

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{network.png}
\caption{Network of seismological stations available (after Boiero, 2009)}
\end{figure}

compute each model point. This is a characteristic of the tomographic approach, which makes, when compared to non-tomographic methods, less sensitive to outliers and increases the reliability of the geological features shown by the computed 1D models. However, it can happen that this also leads to an increase in the misfit between the experimental and computed dispersion curves. In fact, one model may fit some curves passing through that point, while it may not fit curves through other points very well. This especially happens if the picked dispersion curves refer to more than one mode, which, especially in controlled-source seismic, can easily be the case. To avoid this, the fundamental mode has to be
isolated as efficiently as possible.

2.4 Methodologies

The study presented in this thesis was made using a code based on the above mentioned code implemented by Daniele Boiero and illustrated in his Ph.D. thesis. It is a fundamental-mode tomographic inversion of dispersion curves. Our aim is to adapt this code to make it suitable also for exploration scale. This is not trivial. In fact, the first difference to be taken into account is the fact when using a controlled source, higher frequencies are generated and more modes of propagation are excited. It is therefore much harder to isolate the fundamental mode dispersion curve required by the inversion algorithm. To do this, a suitable time-window from the group velocity envelope had to be computed. Bergamo (personal communication, 2013) implemented a procedure to extract phase velocity dispersion curves based on the already mentioned work from Bloch and Hales (1968). A flowchart of the procedure is visible in figure 2.6.

2.4.1 Time Windowing

First of all, the group velocity for the seismograms of both the receivers involved has to be computed. To do this, following Dziewonski et al. (1969) the following algorithm was implemented:

1. The input seismogram $s(t)$ is filtered by means of a series of gaussian narrow band-pass filters, centered around the frequencies of interest: the output is a series of filtered signals $a_f(t)$ where $f$ is the central frequency of the band pass filter.

2. For each filtered signal $a_f(t)$ the relevant quadrature signal is computed $q_f(t)$.

3. The envelope of $s(t)$ for frequency $f$ is therefore $A_f(t) = (a_f(t)^2 + q_f(t)^2)^{1/2}$.

4. The envelope $A_f(t)$ which is a function of time is eventually translated into a function of group velocity $E_f(v_g)$, being $A_f(t') = E_f\left(\frac{\Delta x}{t'}\right)$, where $\Delta x$ is the distance between the source and the receiver.

The following step consists in identifying the event of interest, which in our case is the Rayleigh wave fundamental mode and isolate it by means of a time window. It was decided to follow Yao et al. (2005) and use a time window with cosine shoulders centred at the group arrival time at frequency of interest $w(t, f_c)$ (see equation 2.3). The main reason for this is the fact that, since after the windowing a Fourier transformation is performed, a sinusoidal window will not lead to computation of numerical artefacts such as inexistent frequency components.

\[
w(t, f_c) = \begin{cases} 
1 & \text{if } t_g(f_c) - \frac{n}{f_c} < t < t_g(f_c) + \frac{n}{f_c} \\
\cos \left(\pi \left|\frac{t - t_g(f_c)}{\frac{n}{f_c}}\right| + \frac{\pi}{2}\right) & \text{if } -\frac{2}{f_c} < |t - t_g(f_c)| < \frac{2}{f_c} \\
0 & \text{elsewhere}
\end{cases}
\]  

(2.3)

where $f_c$ is the frequency of interest $t_g(f_c)$ is the arrival time of the group velocity envelope (computed with the system described above) for frequency $f_c$ and $n$ is a window constant with value 2.5. The optimal window size (determined by $n$) and width of its shoulders were retrieved through a series of tests. An example of group velocity envelope with picking can be seen in chapter 3. The two traces are then cross-correlated and the maxima in the matrix should indicate where the two traces are in phase, therefore the time lag should correspond to the travel time of the wave from one receiver to the other. Another possibility, following Bloch and Hales (1968), could be to sum or subtract the two traces, but the cross-multiplication method performs better at low frequencies and therefore allows a greater investigation depth.

2.4.2 The Inversion Algorithm

The inversion algorithm was implemented by Daniele Boiero and presented in his Ph.D. thesis (2009). The aim is to invert average phase slowness dispersion curves computed via the two station method.
with a 2-step only inversion algorithm. In fact, inversion procedures commonly used by seismologists involve 3 steps:

1. The average-phase slowness dispersion curves are extracted.

2. From the tomographic inversion of the extracted dispersion curves, one can build a volume (for the spatial coordinates $x$, $y$ and frequency $f$), mapping the average slownesses. This volume is discretised as in a regular grid. At this point, one average slowness dispersion curve along a path crossing the point where the 1D model should be computed is extracted.

3. These curves are then inverted to obtain the 1D models.

The algorithm used for this work skips step 2, as it already discretises the area by means of regular grid of 1D models, inverting directly the experimental average phase slowness dispersion curves. In the following sections, the used algorithm is described in detail.

**Forward Model**

A phase slowness map is built, using equation 2.4

$$t_{AB}(\omega) = \int_A^B p(l, \omega) \, dl = \sum_{i=1}^I p_i(t_{AB}, \omega) \, dl_i,$$

(2.4)

where $p_i$ is the phase slowness for each path segment $dl_i$ along $AB$. Using the bilinear interpolation coefficient $f_{ik}$, which is non-zero only for the four surrounding points, we can then write the phase slowness from point A to point B as

$$p_{AB}(\omega) = \sum_{k=1}^K \left[ \frac{1}{I} \sum_{i=1}^I f_{ik} \right] p_k(\omega) = \sum_{k=1}^K (f_k)_{AB} p_k(\omega),$$

(2.5)

where $(f_k)_{AB} = \frac{1}{I} \sum_{i=1}^I f_{ik}$. The relative average velocity dispersion curve $c(\omega)$ between $A$ and $B$ is then

$$c_{AB}(\omega) = \frac{t_{AB}(\omega)}{t_{AB}(\omega)} = \frac{1}{p_{AB}(\omega)}.$$

(2.6)

**Inversion Algorithm**

The algorithm inverts averaged slowness dispersion data $p_{obs}$ for the $K$ parameters $m = m_k (k = 1\ldots K)$. The algorithm uses a damped least squared method and the misfit function $Q$ to be minimized is

$$Q = \left[(p_{obs} - p(m))^T C_{obs}^{-1} (p_{obs} - p(m))\right] + \left[(-R_p m)^T C^{-1}_{R_p} (-R_p m)\right],$$

(2.7)

This equation matrix could be interpreted as: the forward response $p(m)$ evaluates for model $m$ the phase slowness along a path according to equation 2.5. $C_{obs}$ is the covariance matrix inferred from the measurements and it links the model parameters $m$ to the observed slownesses $p_{obs}$. Since the measurements are not repeated, a statistical error cannot be computed. Lai et al. (2005) show how uncertainty in seismic signals follows a gaussian distribution and its value should be assigned following experimental estimations. These, in the group of the Politecnico di Torino, have shown 5% to be a
reasonable value. The other covariance present in the equation, \( C_{RP} \), contains the weights for the spatial regularisation defined by \( R_p \) and is computed from the residuals (see further on in this paragraph). The matrix \( R_p \) has the aim of relating the models \( m_k \) at nearby points and contains 1 or \(-1\) for the constrained parameters and zeros elsewhere:

\[
R_p = \begin{pmatrix}
1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & -1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & -1
\end{pmatrix}
\] (2.8)

Minimising 2.7 constitutes an ill-posed and ill-conditioned inverse problem. Regularisation does reduce the illness of the problem and keeps the error propagation under control, but, if excessive or only chosen by mathematical arguments, can lead to results not connected to reality; see for example Tarantola (2005). The regularisation \( R_p \) was designed to minimize the differences between model \( m_k \) and the models at surrounding points; the effectiveness of this regularisation depends on the covariance matrix \( C_{RP}^{-1} \), even though the covariance matrix may not, generally, be known \textit{a priori}, its effect on the inversion can be estimated by computing the residual

\[
R = p_{\text{obs}} - p(m_{\text{end}}),
\] (2.9)

where \( m_{\text{end}} \) is the final model from the inversion. Excessive regularization and smoothing of the model parameters will produce an unnatural increase in the residuals. At the \( n_{th} \) iteration, the solution of the minimisation of 2.7 can be expressed as

\[
m_{n+1} = m_n + \left[ G^T C_{\text{obs}}^{-1} G + R_p^T C_{RP}^{-1} R_p + \lambda I \right]^{-1} \times \left[ G^T C_{\text{obs}}^{-1} (p_{\text{obs}} - p(m_n)) + R_p^T C_{RP}^{-1} (-R_p m_n) \right]
\] (2.10)

where \( \lambda \) is the damping parameter (as defined by Marquart (1963)) and \( G \) is the sensitivity matrix, describing the sensitivity of dispersion slownesses \( p_k \) to model parameters \( m_k \).

\[
G = \begin{pmatrix}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\partial p_{AB}}{\partial m_1} & \frac{\partial p_{AB}}{\partial m_2} & \cdots & \frac{\partial p_{AB}}{\partial m_n} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\partial p_{CD}}{\partial m_1} & \frac{\partial p_{CD}}{\partial m_2} & \cdots & \frac{\partial p_{CD}}{\partial m_n} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{\partial p_{EF}}{\partial m_1} & \frac{\partial p_{EF}}{\partial m_2} & \cdots & \frac{\partial p_{EF}}{\partial m_n} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \
\end{pmatrix}
\] (2.11)

Each element of the matrix can be expressed via the bilinear interpolation coefficient \( f_k \) as

\[
\frac{\partial p_{AB}(\omega)}{\partial m_k} = (f_k)_{AB} \frac{\partial p_k(\omega)}{\partial m_k}
\] (2.12)

Since for our purposes a higher sensitivity on the slownesses was desired, we decided to normalise each component of \( G \) by the model parameters.

The model parameters to be chosen are: number of layers, thickness of the layers, bulk density, poisson ratio. The number of layers should be chosen considering the desired investigation depth and the resolution necessary to map all important geological features. Of course, this decision should also be taken considering the amount of information present in the data; an over- or under-parametrisation of the model could lead to artefacts or non-significant results. The maximum penetration depth depends on the wavelengths at which the phase velocity is reliably detectable; the longer the wavelength, the higher the penetration depth. Further \textit{a priori} information which can be input regard the position of known faults and of the water table. The latter has to be taken into account as it strongly influences the p-wave velocity and therefore Poisson’s ratio.
2.4.3 Checkerboard Test

Seismologists assess the ability of the algorithm to reconstruct the features of a given area by testing it on a synthetic dataset: the so called checkerboard test. This consists in initialising an arbitrary 1D model with certain model parameters (number of layers, bulk density, Poisson ratio, shear wave velocity and, for seismologists, Moho depth) and creating a canvas with alternating positively and negatively perturbed areas. The resulting bird-eye view looks like a checkerboard. Looking at a vertical section, one can see that to positively perturbed areas in an upper layer correspond negatively perturbed areas in the lower layer. Perturbation is generally chosen to be between 5% and 20%. In this canvas, the latitude and longitude correspond to those where the real data was acquired, the points corresponding to seismological stations are positioned. A forward model is run, simulating the 3D propagation of waves inside the checkerboard, along all possible paths from one receiver to another and extracting the corresponding phase velocity dispersion curves. These are then inverted, and the model obtained from the inversion is compared with the original one. Looking at the results (figure 2.7 and 2.8), one can see that, while at shallow depth the model is quite well reconstructed, at higher depths resolution decreases significantly. Furthermore, there is also horizontal variability in resolution: the longitudinal cross-section at Lon 102.5° is better reconstructed than the longitudinal one at 27.5°, especially the layers at medium depth (between 50 and 150 km). This could be explained by differences in density of the paths along the section and their average length (at longer path lengths, one gets more information at low frequencies, i.e. larger depth). The conclusion is that, with the number of station available, the checkerboard model can be reconstructed and, therefore, the resolution is adequate.
Figure 2.7: Horizontal resolution tests: a) first input $1.5^\circ \times 1.5^\circ$ model; b) recovery of $1.5^\circ \times 1.5^\circ$ model at 10 km depth; c) recovery of $1.5^\circ \times 1.5^\circ$ model at 100 km depth; d) second $1.5^\circ \times 1.5^\circ$ model; e) recovery of $1.5^\circ \times 1.5^\circ$ model at 25 km depth; f) recovery of $1.5^\circ \times 1.5^\circ$ model at 250 km depth. After Boiero (paper submitted).

Figure 2.8: Vertical resolution tests: a) input model at latitude 27.5°; b) recovery of the model at latitude 27.5°; c) input model at longitude 102.5°; d) recovery of the model at longitude 102.5°. After Boiero (paper submitted).
Chapter 3

New Zealand

The dataset processed in this work is part of a wider project from ETH Zurich. In fact, along the seismic lines also GPR lines were acquired with the purpose of mapping the fault and groundwater in the area of Inchbonnie, Souther Island, New Zealand. See fig. 3.1 for a map showing the location of the site.

3.1 The Location

The dataset was recorded to map the Alpine Fault, a transpressional strike slip fault, which signs the border between two major plates of our planet: the Australian and Pacific. Two subduction zones are identifiable along the fault: at the Tonga-Kermadec Trench the Pacific plate is subducted under the Australian plate, while at the Puysegur trench the opposite is observed (see fig. 3.1). The oblique motion of the plates along the fault has given place to major seismic events in the past 150 years; the most significant events occurred in 1848 (magnitude 7.8), 1929 (magnitude 7.8), 1968 (7.1), 2003 (7.2) and 2009 (7.8). Another indication of the activity of the fault is the fact that uplift is still occurring in the south-east side; in fact the mountain range called "Southern Alps" is still rising at a rate of approximately 7 mm per year. Better understanding of the fault is vital for a more accurate evaluation of seismic hazard and risk; a better mapping of the main and secondary faults can help plan new constructions as well as enhance the earthquake hazard assessment. Furthermore, along ground-mechanical parameters, also a study of the groundwater is essential to identify areas where possible side-effects of earthquakes (e.g. liquefaction) can occur.

From a geologic point of view, the area of Inchbonnie was subject to several glaciations, the last of which ended approximately 15000 years ago. During these glaciations glaciers carved valleys and deposited glacial and lacustrine sediments. In more recent years, rivers flow in the valley depositing fluvial sediments, mainly gravel and other unconsolidated material. Previous studies (e.g Langridge et al. 2010) estimated a simplified geologic model of the area; the first meter is made of sand and silt, from depth of 1 to 15 meters uncompacted braided-river gravel can be found, while below the water table, at 15 m depth, the gravels become more consolidated.
3.2 The Dataset

The New Zealand dataset is a 2D high-resolution dataset acquired by ETH with the purpose of mapping the Alpine Fault. 5 lines were shot, one in ultra high resolution (1m receiver spacing, 2m source spacing) and 4 in high resolution (2m receiver spacing, 4m source spacing). The receivers were vertical 30 hz geophones, while the sources were 40g power gel capsules at approximately 1.2 metres depth. Since some of the lines passed close or through inhabited areas and railway lines, for some of the shots a 5 kg sledge hammer was used as source. The system used for recording included 240 channels with 10 Geometric Geodes and a field laptop computer. The sampling rate was chosen to be 0.25 ms, in order
to avoid temporal aliasing. A typical shot can be seen in 3.2; the data is of very good quality, with high signal to noise ratio, visible reflections, refractions and the ground roll can be easily identified. The location of the lines and their lengths are visible in fig. 3.3.

Figure 3.2: Shot n.24 from line HR

In their 2013 paper Konstantaki et al. describe the results of the inversion they performed using the Multichannel Analysis of Surface Waves (MASW). Being a multi-channel dataset, this method was very suitable for the dataset and results are remarkable. Our purpose is to perform a direct tomographic inversion using the "Two-Station" method. To do this, we needed first of all an indication about the ideal offsets and distances between the couples of receivers. We therefore computed, using the MUSIC algorithm, an f-k spectrum of some traces (an example is shown in 3.4), in order to get information about the dominant wavelength and the maximum significant wavelength. From the latter, we can get the optimal offset from the first receiver, which should be equal to approximately 1 wavelength, while from the former it is possible to estimate the optimal distance between the two receivers. Looking at fig. 3.4 we see how the most energetic event is located at approximately $k=0.6$ rad/m and $f=23$ Hz; this implies that, at 23 Hz, we expect velocities around 240 m/s. Dominant wavelength should therefore be at approximately 10 m. Similarly, the maximum available wavelength can be estimated as approximately 35 m ($k=0.2$ rad/m at $f=20$ Hz). Further tests were performed and it was assessed that it was possible to retrieve better information at lower frequencies and that it was easier to separate the different modes when picking the dispersion curves if the offset was set to 20 m
Using the same MUSIC algorithm, reference curves were computed. This was done in order to pick the dispersions curve more accurately. The decision was to compute curves averaged over 48 receivers, i.e.
48 metres, which is a reasonable length for which we can assume that the subsurface does not present sharp discontinuities.

### 3.3 The Zebra Test

As exposed in Chapter 1, the inversion algorithm was originally intended to invert surface waves at seismological scale with the purpose of mapping Mohorovic’s discontinuity. Therefore, the code has been modified so that it could trace the raypaths for a small-scale 2D dataset. First of all, modifications to the function which set the geometry had to be implemented, since our dataset was 2D, with 1m receiver spacing and because the position in the southern hemisphere required opposite direction for increasing latitude. Furthermore, the raytracing function had to be adapted: the integration step was set to 0.5m; the time step was set so that 24 timesteps were used per each raypath; the tolerance was set to 0.5m, so that the traced ray could not end on another receiver. Since the dataset we are processing is 2D, the checkerboard test usually performed for 3D datasets would not have been significant. On the other hand, the propagation is anyway considered in three dimensions, therefore testing the code on a 3D canvas was necessary. A good solution was found in an arbitrary model in which, from a bird's-eye view, low and high s-wave velocities areas are alternated as in the fur of a zebra (see 3.6). For this reason we gave it the name Zebra Test. The aim of this test was the same of that of the checkerboard test; test the validity of the forward model and assess whether it is able to resolve all the features we wanted to map. The initial 1D model used consists of a simplification of one of the 1D models used later in the inversion of experimental data. Looking at a vertical section, 4 layers (plus halfspace), each 7.5 metres thick were considered. Values for all other parameters can be read in table 3.1. This reference model has been perturbed by ±20%. following the sequence indicated by the stripes: black stripes indicate negative perturbation, while white stripes indicate positive perturbation. Looking at a vertical profile, as in fig.3.9 (top), the layers alternate the distribution of high and low velocity zones. The canvas considered had 16 “stripes”. As a first step, the synthetic dispersion curves relative to this model were computed. These can be seen in 3.7 and look consistent, since, at high frequencies, phase velocities range from 200 m/s to 300 m/s, which are the velocities of the first layer of the model perturbed by 20%. These same curves have then been inverted, in order to check whether our algorithm was actually capable of reconstructing this striped phase velocity map using 24 1D models. Results are encouraging, as the global RMS error is of only 40 m/s and the dispersion curves obtained from the inversion fit very well those from the synthetic model. An example of synthetic and inverted dispersion curves can be seen in fig.3.8.

Overall, the zebra-like canvas has been quite well reconstructed, as can be seen in 3.9. A plot of the misfit per each dispersion curve, computed as

\[ E = \sum_{i=1}^{N} \sqrt{\frac{(v_{\text{obs},i} - v_{\text{comp},i})^2}{\sigma_{\text{obs},i}^2}}, \]  

(3.1)

where \( v_{\text{obs},i} \) is the experimental value of phase velocity at a certain frequency, \( v_{\text{comp},i} \) are the values of the computed phase velocity at the same frequency and \( \sigma_{\text{obs},i} \) is the variance of the observed phase velocity. As one can see in fig. 3.10 in some model points, the error is very high. In some cases, like in the one with the highest error, this is due to the fact that not many paths cross that model point, while other points are located at the boundary between two zebras, where it is difficult to resolve the sharp velocity difference. As expected and shown by Boiero (Ph.D thesis, 2009), with increasing depth, the model is not reconstructed as well as in the shallower layers. However, until a depth of 20 m results look quite reliable.
Figure 3.5: Group velocity picking for windowing the traces and picking of the phase velocity dispersion curve from the cross-multiplication matrix
Figure 3.6: Bird-eye view of the canvas for the "Zebra" test. Green points indicate where the 1D models were computed.
Figure 3.7: Synthetic dispersion curves of the zebra model.
Figure 3.8: Two examples of dispersion curves obtained from the “zebra test” and the inversion.
Figure 3.9: Comparison between true model and model obtained from the inversion of the synthetic curves
3.4 The Inversion

As a first attempt, no constraints were set and the initial model was a 4 layer (plus halfspace) model; parameters were taken from the results presented by Konstantaki et al. (2013). The line was divided in two zones: one from 0 to 150m (left of the fault) and from 150 to 360m (right of the fault). The chosen values can be seen in table 3.4 and also graphically visualized in fig. 3.11.

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Thickness [m]</th>
<th>S-Wave velocity [m/s]</th>
<th>Poisson</th>
<th>Bulk density [g/cm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>160</td>
<td>0.25</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>340</td>
<td>0.37</td>
<td>1.75</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>500</td>
<td>0.25</td>
<td>1.9</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>630</td>
<td>0.25</td>
<td>1.9</td>
</tr>
<tr>
<td>halfspace</td>
<td>∞</td>
<td>800</td>
<td>0.45</td>
<td>2.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Layer #</th>
<th>Thickness [m]</th>
<th>S-Wave velocity [m/s]</th>
<th>Poisson</th>
<th>Bulk density [g/cm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>200</td>
<td>0.18</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>310</td>
<td>0.45</td>
<td>1.72</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>400</td>
<td>0.45</td>
<td>1.84</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>530</td>
<td>0.45</td>
<td>1.89</td>
</tr>
<tr>
<td>halfspace</td>
<td>∞</td>
<td>810</td>
<td>0.45</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3.2: Table for parameters for zone 1
Figure 3.11: Initial model: 24 model points, divided into zones, as described in table 3.2.
After this, the inversion code was run on the 92 curves picked on the ultra-high resolution line for 40 iterations. The inversion ran without errors and the misfit looks reasonably low for most of the inverted curves. A plot of the misfit per each path can be seen in fig. 3.13. However, a significant number of curves show a large misfit. This may be due to two reasons: some of the picked curves refer to more than one mode, while the code inverts only for the fundamental one. A second reason could be that, as already stated in chapter 2, the method used implies that more than one curve is inverted per each model point. This means, that when computing the 1D models, if it converges rapidly for one of the curves, the algorithm will not fit those from the other paths crossing that same model point. Furthermore, a path might as well cross more than one model point, making it even more difficult for the algorithm to fit the curve. A solution to the first reason might be to pick the curves again and possibly use a more accurate time-windowing. The second issue is instead a weak point of the method itself and should be improved. A very significant improvement would be to make the inversion code work also with higher modes. Comparing the results with those obtained with the MASW method by Konstantaki et al. (2013), (see fig. 3.14), we see how the trends of the geological layers are similar, especially in the area around the fault. In fact, our inversion confirms the position and entity of the main fault. However, since no spatial constraints were imposed, the results of the tomography are less regular, with strong irregularities both in the area to the right and to the left of the fault; the trend is similar. As one can see, using the tomographic approach larger depths were mapped and a larger number of 1D models were computed.

Figure 3.12: An example of observed and computed dispersion curves for HR line.
Figure 3.13: RMS error (in m/s) for each path used during the inversion
Figure 3.14: Comparison of results from MASW and tomographic inversion of the data from the HR line.
Chapter 4

CNR Dataset

4.1 The Dataset

The dataset was acquired by the Centro Nazionale Ricerche (National Centre for Research) at a test site in Torino. The site is divided into two parts: one in which a tank was dug and filled with loose sand and another one which was instead left unchanged. Some pictures of the site can be seen in fig. 4.1 The dataset consists of 96 receivers with 1m spacing, disposed to form a $12 \times 8$ rectangle. 21 sources were shot, all of them outside the rectangle, with 2m spacing. As only 48 channels were available, 2 shots were performed at each source position, the first for the 48 receivers above the tank, while the second for the geophones above the unmodified ground. The traces of the two shots were then combined, so that the response of the full array was obtained. The geometry of the dataset can be seen in fig.4.2.

Figure 4.1: Location of the CNR test site (top) and the tank filled with the loose sand (bottom).
4.2 The Processing

The aim of the processing of this dataset is to identify the tank. To do this we need a fair number of couples of receivers aligned with a source both above and away from the tank. Firstly, in order to determine the velocities and frequencies available, a MUSIC f-k spectrum was computed taking the 48 traces from the geophones above the tank illuminated by one source (see fig. 4.3. The frequencies at which the data looks reliable range from approximately 15 to 27Hz, while the wavenumbers from 0.3 to $0.7\text{rad/m}^{-1}$. Therefore, velocities range from about 250 to 1000m/s. Furthermore, from such spectrum a reference dispersion curve was extracted and used for further processing.
As a second step, a script to select the couples of aligned (a tolerance of 5° was set) receivers for each source was implemented; the size of the dataset allowed couples with first receivers at offsets ranging from 5 to 8m, while the second receiver was at 6 to 9m from the first one. This is not ideal, as the offset should approximately be half of the maximum wavelength (20 m), while distance between receivers should be set at approximately one dominant wavelength (15 m). The issue coming from this is that it was hard to distinguish between the modes, as the traveltimes were all very similar and the modes many times “crossed” each other. Furthermore, the most energetic event in the group velocity envelope most of times did not correspond to the fundamental mode, but to a higher one. For this reason, in many cases it was not possible to isolate the fundamental mode in the group velocity matrix. An example of this can be seen in fig. 4.4. Despite this, after computing the cross correlation matrix, the picked phase velocity dispersion curve, when compared to the reference curve, looks reliable, therefore the processing was completed despite this issue (see fig. 4.5). It has to be noted, that the dispersion curves have been picked for relatively high frequencies (lower available frequency was generally 20 Hz). This is not a problem, as we are only interested in mapping the first few meters of the subsurface. The curves were picked so that both the tank and the area around the tank could have a good coverage. Also, some curves relative to paths crossing the border between the two areas were picked, in order to map the sharp border. The coverage can be seen in fig. 4.6.
Figure 4.4: Group velocity envelope of trace from the CNR dataset.
Figure 4.5: Example of picked curve from the CNR dataset
As can be seen in fig. 4.6, it was chosen to compute 12 1D models, of which 4 inside the tank. The starting model was chosen to be the same for all model points; 2 layers, each 1.4 m thick, plus a halfspace. The bulk density was taken to be 1.8 and the Poisson ratio set to 0.3, which are reasonable values for the sandy sediments in the area. The initial s-wave velocities was set to 100 m/s in the first two layers (corresponding to the 2.5 m where the tank should be mapped), while in the halfspace was set to 130. The parameters initialized in the starting model are summarized in table 4.1. The inversion code ran for 29 iterations, after which the ratio between the misfit of one iteration and the following was below 1.0001 and therefore the code exited the loop. The misfit is rather high, as can be seen in fig. 4.8. However, the tank was correctly identified, with four 1D models showing on the top-right corner lower velocities than those on the left-hand side. This was expected, since s-waves propagate at lower velocities in loose sands than in consolidated sediments. Furthermore, also the depth of the tank (2.5 m) was successfully mapped. In the end, due to the simplicity of the field mapped, this experiment could be considered like a checkerboard test, with successful results.
Figure 4.7: 3D view of the inversion of the CNR dataset, with the computed 1D models
Figure 4.8: Misfit per each path of the inversion of the CNR dataset
Chapter 5

Conclusions

In this thesis we presented a direct tomographic inversion method for surface waves, typically used for regional-scale seismological data, adapted to exploration scale. After a description of the physical phenomena such as the geometric dispersion which make surface waves suitable for inversion, the main aspects related to acquisition and processing of surface wave data were presented. In particular, the difference between a tomographic and non-tomographic approach was illustrated. The latter implies extraction of dispersion curves via transformation (e.g. to the $f-k$ domain) and picking of the most energetic event in the spectrum. Most commonly, such spectrum is the result of an average over several receivers. On the other hand, extraction of curves for a tomographic inversion is made via computing the average phase slowness at the different frequencies for a couple of receivers illuminated by the same source. In both approaches, these curves are then inverted using a forward 1D forward velocity model. In a non-tomographic inversion, only one curve is inverted per each model point. In the tomographic approach, instead, the number of models which can be obtained (i.e. the resolution with which the area is map) depends on the number of paths along which the curves have been picked. Furthermore, more paths can cross the same model point. It is common practice to first invert the experimental curves to obtain phase velocity map, from which other dispersion curves are computed and then inverted. Since we perform a direct inversion, no velocity maps were computed and the data was directly inverted to obtain 1D depth models.

In the following chapters, we presented the results of the inversion of two datasets, the first, acquired in New Zealand, is 2D, while the second, shot at a test site in Torino, is 3D. The target of the former was mapping the alpine fault, while the that of the latter was mapping an artificially made tank, filled with loose sand. Furthermore, for the 2D dataset a synthetic model was also inverted in order to assess to validity of the method and the resolution available. Overall, we can say all of the experiments have been successful, as the inversion was able to map the main geological features with sufficient detail. In particular, for the dataset shot in New Zealand, the position of the main fault was confirmed, as well as the overall trend of the geological layers. However, since no regularisation was imposed, variations in the depth of the layers look too strong. An improvement would be to link the 1D models with some regularisation. As for the CNR dataset, the filled tank was correctly mapped, both in depth and velocity with lower velocity in the tank filled with loose sand. Despite these encouraging results, however, a few remarks have to be done and the whole method can still be improved.

In particular, one can see how in both inversions of the “real” datasets the misfit for a few of the paths is quite large. This can be due to the fact that the picked curve refers not only to the fundamental mode, but also to higher ones. Since seismologists only consider the fundamental mode, when inverting, also the algorithm we used was a single-mode one. The implementation of a more refined windowing algorithm to better isolate the fundamental mode would be helpful. In conclusion, the method presented in this thesis is a computationally efficient, robust way to perform a direct tomographic inversion of surface waves. Despite some limitations, which should be investigated and solved in further studies, it outputs reliable velocity models of the shallow subsurface. Therefore, it constitutes a viable method for building shear-wave velocity maps of the shallow layers for exploration-scale seismic surveys.
5.1 Acknowledgements

We would like to thank ETH Zurich and prof. Alan Green for letting us use the data acquired in New Zealand for this project. The Oman dataset was provided by the EAPS department of MIT, in the person of Robert van der Hilst. Special thanks also to Affan Anwar, for its contribution to the picking of the dispersion curves. We would also like to thank Flora Garofalo for the support provided on the inversion code and display of the results.
Appendices
Appendix A

Oman

A.1 The Dataset from Oman

The dataset from Oman is a 3D high-resolution seismic survey acquired for exploration purposes by Petroleum development Oman. Its geometry consists of 1600 receivers set to form a square. Spacing is 25m, therefore the area covered is of 1 km². The 1600 sources were shot in a similar grid, but shifted by 12.5m in both directions. The sources used were vibroseis with frequencies ranging from 5 to 120Hz. The recorded traces count therefore a number of 1600x1600, with a recording length of 4s and a sampling rate of 8ms. In figure A.1 the geometry is illustrated.

![Geometry of the Oman Dataset](image)

**Figure A.1:** Geometry of the dataset acquired in Oman.

A.1.1 MUSIC with one source and 88 receivers

As a first step for processing, a reference dispersion curve for the phase velocities needs to be computed. This has been done by selecting a subset of 88 receivers and a source located outside such selection, see figure A.2. The number of traces couldn’t be higher, as the algorithm used later is very memory expensive and the computers provided could not handle more data. The plotted traces, in offset domain, can be seen in A.3.

The output of the Music algorithm is a normalised f-k spectrum. The main advantage of this algorithm is that it does not need evenly spaced traces to run. From this spectrum (shown in A.4) a dispersion...
Figure A.2: Geometry of the selected receivers

Figure A.3: Traces of the selected receivers.
curve was extrapolated. This curve was then used as reference curve for all further processing steps. The result may be misinterpreted: even though one clear event is visible, this is not necessarily the fundamental mode. In fact, the normalisation performed by the algorithm may show relative maxima, which may be actually quite weak but displayed with the same colour as the stronger ones. Furthermore, the dispersion curve obtained with this method shows an anomalous trend: it looks like three modes of oscillation are covered, but with low sampling for all of them. In addition to that, the curve visible on the top right-hand side of A.4 may come from guided p-waves. Finally the trend shown at low frequencies, with increasing velocity at increasing frequency, does not look very convincing. However, this dispersion curve has been taken as reference for the further performed tests. It turned out to be a good decision, as dispersion curves obtained with the two-station method are consistent with this reference one, as shown, for example, in figure A.5.

![Fk spectrum](image1)

**Figure A.4:** f-k spectrum obtained with MUSIC algorithm and dispersion curve

### A.2 Two Station Method

The idea of the first test was to verify whether it made sense to stack the traces from two different sources and then use the output as input for the two-station method. The fact that this dataset is suitable for the so-called "Two-Station Method" has already been assessed by Paolo Bergamo (personal communication). Two tests were performed: the first with two sources, located at opposite sides of the couple of receivers, the second with two sources on the same side at different offsets. In both cases the sources were in line (with tolerance of 3°) with the couple of receivers. It has been chosen to use offsets and distances between receivers of approximately 100m, i.e. comparable to the dominant wavelength of the recorded waves. The results show how for sources located on the same side, the correlation matrices of the responses to the two sources are basically the same. On the other hand, the correlation matrices for the response to sources located on different sides differ from each other. In figure A.6 we can see the selected couple of receivers and the two sources (indicated by an arrow). The cross multiplication of the
Figure A.5: Reference (light blue +) and picked (dark blue 0) dispersion curves
responses, which can be seen in fig. A.7 are visibly similar to each other and the trend of the maxima is also similar to that in the f-k spectrum. The absolute and relative difference between the two cross multiplication matrices were computed (fig. A.8) and we see how, in the interval between 7 and 25 Hz, i.e. where the energy of the source is concentrated, differences are minimal as they don’t exceed 2%. The conclusion was that stacking the traces could be the way increase signal to noise ratio, but further investigation is necessary.

Figure A.6: Geometry of the selected receivers (full yellow circles) and sources (black arrows). Red dots indicate the other sources in line with the couple of receivers.
Figure A.7: Cross-multiplication matrices for the two sources, closest and furthest away from the couple of receivers respectively.
Figure A.8: Difference between the two matrices (top) and relative % difference between the two (bottom).
Bibliography


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