A 3D UNSTEADY PANEL METHOD FOR VERTICAL AXIS WIND TURBINES

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Abstract
A 3D, unsteady, multi-body, free-wake panel method is developed to model a vertical axis wind turbine of arbitrary configuration. The panel code is intended as a design and research tool to capture the 3D nature of a VAWT and its wake. The model contains methods to realistically treat blade-wake interactions, vortex stretching/contraction and viscous diffusion. Validation is demonstrated by comparison with 3D-stereo Particle Image Velocimetry (PIV) and smoke trail studies for a straight-bladed VAWT. New insight into the near wake structure of a VAWT is developed in addition to how this structure changes as a function of tip speed ratio and height-diameter ratio. Tip vortex dynamics are also discussed.

Keywords: VAWT, panel method, near wake structure

1. Nomenclature

\begin{align*}
c & \quad \text{airfoil/blade chord, m} \\
D & \quad \text{rotor diameter, m} \\
k & \quad \text{reduced frequency, } k = \frac{\omega c}{U_\infty} \\
Re & \quad \text{Reynolds number } Re = \frac{c \lambda U_\infty}{\nu} \\
Re_v & \quad \text{Vortex Reynolds number } Re_v = \frac{\Gamma}{\nu} \\
U_\infty & \quad \text{Unperturbed velocity (wind speed) m/s} \\
\alpha & \quad \text{angle of attack} \\
\lambda & \quad \text{tip speed ratio } \frac{\Omega R}{U_\infty} \\
\psi & \quad \text{azimuth angle} \\
\Omega & \quad \text{rotation frequency} \\
\Gamma & \quad \text{circulation } m^2/s \\
\mu & \quad \text{doublet strength} \\
\sigma & \quad \text{source strength} \\
\sigma_s & \quad \text{solidity } \frac{N c}{R} \\
\Phi & \quad \text{velocity potential outside body} \\
\Phi_i & \quad \text{velocity potential on interior} \\
\nu & \quad \text{kinematic viscosity}
\end{align*}

2. Introduction
The notion of utilizing urban sites for wind power generation has renewed interest in the Vertical Axis Wind Turbine (VAWT). While appearing radically different from the conventional Horizontal Axis Wind Turbine (HAWT), VAWTs and HAWTs accomplish the same function and have a similar maximum theoretical performance. The key advantage of the VAWT concept is that it is insensitive to yaw and can handle a wind field that rapidly changes directions, making it ideally suited for application in the built environment.

VAWT aerodynamics are a complex three-dimensional unsteady problem that includes phenomena such as wake-blade interactions, substantial wake deformation, tip vortex effects, dynamic stall and separation.

As a consequence of its cycloidal geometry shown in Figure 1 a VAWT blade will experience a continuously oscillating angle of attack. For low tip speed ra-
tios λ, the α range will fall well outside the stall limits for a given Reynolds number and airfoil section. The variation of geometric angle of attack is illustrated in Figure 2. High α causes a dynamic stall cycle with each blade rotation and perhaps complete separation for portions of the azimuth depending upon the airfoil section, Re, λ and solidity σ. The current work focusses on the near wake structure within two rotor diameters from the upwind blade at tip speed ratios greater than 3 where dynamic stall/separation effects are less dominant. This permits application of relatively computationally inexpensive inviscid methods to capture the 3D problem.

Existing approaches for modelling the aerodynamics of a VAWT include double multiple streamtube models (equivalent of BEM), vortex methods and full Navier-Stokes simulation using a commercial CFD package.

The double multiple streamstube (DMST) models such as those implemented by Paraschivoiu [5] are very useful for overall performance calculations but are restricted to known airfoil sections, are limited in their ability to handle transient behavior and model performance at higher λ and also give no indication as to the near wake structure. The last point is particularly important when designing an urban turbine whose geometry often features exposed tips/blade supports for a more rectangular profile to facilitate integration into the built environment- In contrast opposed to the curved troposkien blades used by VAWTs of the 1970s.

Vortex methods such as those pioneered by Strickland [11] are able to model the near wake structure. Such methods use the Biot-Savart equation to calculate the inflow induced by the wake at each blade segment, and from the apparent angle of attack use a look-up table of experimentally determined Cl and Cd data for a given Reynolds number and section. Like DMST, they are limited to known sections where experimental data exists.

Reynolds-averaged Navier-Stokes/ large eddy solution which can handle an arbitrary geometry is possible but for a useful level of fidelity particularly to prevent excessive numerical diffusion of the wake structures, a 3D solution is currently not practical as a design tool.

Consequently, a design and research tool is needed to evaluate aerodynamic performance while taking into account the 3D unsteady nature of the near wake whilst not being restricted to specific geometries. A level of computational economy is required so that design trends may be thoroughly investigated. The 3D unsteady panel method presented here is a first step towards that goal.

3. Model

A source-doublet formulation is implemented following the approach taken by Katz and Plotkin [4]. Beginning with a general body-wake problem shown in Figure 3, the Laplace equation in terms of the velocity potential is solved for a Dirichelet boundary condition on the body with the Kutta condition being enforced at the trailing edge. The steps to arrive at a general solution are not given here but may be found in Katz and Plotkin [4] or Hess [3]. Considering the total velocity potential on interior of the body Φ₁:

\[ \Phi_{t_{total}} (x) = \Phi_{t} (x) + \Phi_{∞} (x) \]  

and the general solution to the Laplace equation,

\[ \Phi_{t_{total}} (x) = \frac{1}{π} \int_{S_B} {\frac{1}{r} \left[ \gamma \left( \frac{1}{2} - \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right) \right] dS} \]

\[ + \frac{1}{π} \int_{S_{BW}} {\mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) dS (x) = \text{const.} \]  

Fig. 1: 2D VAWT schematic of azimuth definition as viewed along axis of rotation from bottom

Fig. 2: Geometric angle of attack at various tip speed ratios; this image courtesy C.Ferreira, note that azimuth definition differs from Figure 1
Where the source $\sigma$ and doublet $\mu$ have been introduced with $\Phi$ being the velocity outside the body, and $\Phi_1$ inside:

$$-\mu = \Phi - \Phi_1 \quad (3)$$
$$-\sigma = \frac{\partial \Phi}{\partial n} - \frac{\partial \Phi_1}{\partial n} \quad (4)$$

The zero normal flow boundary condition requires that the internal potential is constant; fluid is neither able to enter or leave any closed surface, so the velocity potential inside is constant. The constant is arbitrary, and can be set in a way such that it will simplify the equations. In an inertial frame that is fixed to the moving body (and non-rotating) it is convenient to choose $\Phi_{i,tot}=\Phi_{\infty}$, yielding Equation 5. This is the same as choosing $\Phi_{i,tot}=0$ in a fluid fixed inertial frame where $\Phi_{\infty}=0$ by definition. The final result is a boundary element integral equation shown in Equation 5:

$$\Phi_i(x) = \frac{1}{4\pi} \int \left[ \sigma \frac{1}{r} - \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right] dS$$
$$+ \frac{1}{4\pi} \int \left[ \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right] dS = 0 \quad (5)$$

Equation 5 has an unknown distribution of sources and doublets over the body, and finding a distribution of each that will satisfy the boundary conditions is the crux of the problem.

### 3.1. Discretization

The body is discretized by flat panels with a constant singularity distribution (doublet and source) over each panel. The wake is discretized as a sheet of constant doublet panels or a as a lattice of vortex segments, both representations are equivalent and needed for numerical reasons in order to handle wake intersections as part of normal VAWT operation. Higher order approximations of the geometry and singularity distributions are possible but add significant complexity for only slight gains in accuracy. Other improvements such as a higher-order time marching scheme (discussed later) have the potential to have a larger impact on the modelling performance.

For computation, a coupled algebraic system of equations is formed by summing the influence of each body and wake panel at every other panel; each body panel corresponds to a single equation according to Equation 5. This results in the following:

$$\sum_{j=1}^{N_{\text{panels}}} - \frac{1}{4\pi} \int_{S_{\text{body},j}} \mu_j \frac{\partial}{\partial n} \left( \frac{1}{r_{ij}} \right) dS_i$$
$$+ \sum_{j=1}^{N_{\text{panels}}} \int_{S_{\text{body},j}} \sigma_j \frac{1}{r_{ij}} dS_i$$
$$+ \sum_{k=1}^{M_{\text{wakepanels}}} \int_{S_{\text{wakepanel}}} \mu_k \frac{\sigma}{\mu \pi} \left( \frac{1}{r_{ik}} \right) dS_k = 0 \quad (6)$$

Solutions for each of the integrals in can be found in Hess [3]. Collectively, the integrals evaluated over a panel to determine its influence at a point are called the aerodynamic influence coefficients.

$$A_{ij} = - \frac{1}{4\pi} \int_{S_j} \frac{\partial}{\partial n} \left( \frac{1}{r_{ij}} \right) dS_j$$
$$B_{ij} = \int_{S_j} \frac{1}{r_{ij}} dS_j$$
$$C_{ik} = \frac{1}{4\pi} \int_{S_k} \frac{\partial}{\partial n} \left( \frac{1}{r_{ik}} \right) dS_k \quad (7)$$

The final system as expressed in Einstein notation is:

$$A_{ij} \mu_j + B_{ij} \sigma_j + C_{ik} \mu_k = 0$$
$$i,j = 1 \ldots N_{\text{panel}}$$
$$k = 1 \ldots M_{\text{wakepanel}} \quad (8)$$

Both source $\sigma$ and doublet $\mu$ terms are unknown, and an assumption must be made in order to solve the system. While completely arbitrary, a reasoned assumption can applied to $\sigma$ when considering the Neumann boundary condition that requires zero normal velocity at the surface; thus:

$$\frac{\partial \Phi}{\partial n} = - \left( \vec{V}_{\text{origin}} + \vec{v}_{\text{rel}} + \vec{\Omega} \times \vec{r} \right) \cdot \vec{n} \quad (9)$$

Knowing that the internal potential due to the disturbance only (Equation 5) is zero, it follows that the derivative normal to the surface of disturbance potential should also vanish $d\Phi_i/dn = 0$. Hence Equation 4 is reduced to the following:

$$-\sigma = \frac{\partial \Phi}{\partial n} \quad (10)$$

Therefore the source distribution is required to be equal to the normal component of the local kinematic
velocity, where the vector \( \mathbf{n} \) points into the body:

\[
\sigma = \left( \mathbf{V}_{\text{origin}} + \mathbf{v}_{\text{rel}} + \mathbf{\Omega} \times \mathbf{r} \right) \cdot \mathbf{n}
\]  

(11)

With \( \sigma \) now known, Equation 8 represents the algebraic system for all \( i \) panels that is solved for the unknown body doublet strengths \( \mu_j \). \( A_{ij} \) and \( B_{ij} \) are known from the problem geometry, and \( C_{ik} \) is calculated at each time step given the current location of the wake. The wake strengths \( \mu_k \) are known since each wake panel strength remains constant in time. The most recent wake panel however is a function of the unknown top surface and bottom surface doublets at the point where the wake is shed as in Equation 12.

The effect of each near wake is included by adding or subtracting its influence from the appropriate columns in \( A_{ij} \) which represent the influence of the top or bottom surface where a particular near wake panel originates.

\[
\mu_w = \mu_{\text{top}} - \mu_{\text{bottom}}
\]  

(12)

3.2. Removal of Singularities

Because the VAWT intersects its own wake as part of normal operation, modifications must accommodate this. Body-wake interactions may be divided into a strong and weak category. Weak interactions are those resulting from the shedding of the wake form the trailing edge and are handled naturally by the source-doublet formulation. Strong interactions are those where a wake surface comes in close proximity to many panels (besides those at the trailing edge) or intersects a panel directly. This can occur when a VAWT crosses its own wake or the wake of another blade. The interactions are mutual, so singularities must be removed from the influence generated by the body on the wake, vice-versa, and in the wake’s self-influence.

3.2.1. Strong Body-Wake Influence

The influence equation for a 3D doublet panel [3][4] has a singularity at the panel edges. The singularity is of the form is equivalent to an ideal vortex whose circulation is equal to the difference in strength between adjacent doublet panels. From the governing equations, the velocity tangential to the body surface can be determined from derivative of the doublet distribution over the surface. The velocity is also determined a short distance away from the body at ‘outer collocation points’ which are usually placed 5-10% above the surface depending on grid fineness, well outside of the singular regions. A local tessellation scheme is used to interpolate between the velocity on the surface at that at the outer collocation points which removes the non-physical velocities at the panel edges and gives realistic values.

3.2.2. Strong Wake-Body Influence

To handle the strong wake-body influence the wake is split into ‘near’ and ‘far’ sections. The near section is those wake panels which have been shed recently and are still within 5-10 chordlengths away. The near wake is represented as a doublet sheet and the far wake is represented as a velocity component in the source term. The wake can be represented entirely as a velocity but in the velocity potential Dirichelet formulation used, it results in a higher error level owing to the propagation of discretization error (as an error analysis of the discretized equations will reveal). The entire wake cannot be represented purely as a potential either owing to the fact that there is a discrete jump in the potential calculated by a vortex point (and equivalently a doublet sheet) which is a result of the angular nature of the influence equations when a wake panel is intersected by the body. A solution is to represent those panels which have recently been shed and
have little chance of intersecting the body as a potential, and older panels as a velocity, resulting in a hybrid scheme which alters the system of equations somewhat as shown in Equation 13. A velocity representation for older panels also allows a viscous vortex model to be integrated naturally.

\[
A_{ij}\mu_j = -B_{ik}(\sigma_j + \sigma_{wake,j}) - C_{ij}\mu_k
\]  

(13)

3.2.3. Wake-Wake Influence

The wake will exert a self influence. Because it is represented as a vortex lattice, each vortex lattice segment must have a core model to remove the singularity as \( r^- \to 0 \). The Rankine model is commonly used to represent a viscous vortex. It linearly interpolates the swirl velocity inside of a fixed vortex core which does not grow with time. This representation is at odds with experimental results that show a different core structure where for vortices of the same strength, the Rankine model over predicts the peak swirl velocity, and the core grows in time as a function of vortex Reynolds number. Lamb-Oseen (LO)[9] have derived a solution for a single isolated vortex in 2D from the Navier-Stokes equations assuming laminar and incompressible flow. The Lamb-Oseen model produces a more realistic swirl velocity trend, but does not include Reynolds number effects and thus has a core growth rate which is far slower than that seen in experiment [7][8].

The Ramasamy-Leishman model (RL) is implemented to capture Reynolds numbers effects on vortex core growth and give a realistic swirl velocity profile. The RL model is an improvement on previous attempts in that it takes into account the layered structure of real vortices, whose interacting layers influence the core growth rate and swirl velocity profile. The flow inside the vortex core is approximately mostly laminar at low \( Re_v = 10^3 \) and fully turbulent at higher \( Re_v = 10^5 \). While the RL model is developed for helicopters, whose tip vortices are within the transitional regime, it is also applicable to the VAWT case where the Rev is approximately in the same regime.

The RL model is based upon an analytical solution derived from Navier Stokes for an isolated 2D viscous vortex where the eddy viscosity inside the vortex is allowed to vary continuously between the different layers via an intermittency function. Several parameters in the model are determined through comparison with experiment over a range of Rev. Further information about the RL derivation can be found in [7] and [8].

At low \( Re_v \), the RL model reduces to the LO model and at high Rev it gives the fully turbulent behavior seen for example in the strong tip vortices in an aircraft wake. The model has been validated against experimental data from many sources over a wide range of Rev. In its original form, the RL model cannot give the swirl velocity explicitly, but rather must be solved for iteratively. In [8] Ramasamy et al., give an explicit approximation as a sum of three exponential terms whose coefficients are a function of Rev. Core growth as well cannot be given in a convenient form, but in comparison with experimental data, a simple approximation can be made.

Caution must be exercised though when applying such models to all vortex segments because the inboard sections represent a decaying vorticity layer and not a tip vortex. The vorticity layer is composed of two regions of opposing vorticity shed from bottom and top surfaces, that average out to give the total the circulation change over the wing in both temporal and spanwise directions. In ideal flow this region is compressed to an infinitely thin doublet surface (or equivalently as two vortex sheets whose directions are perpendicular) over which there is a step change in tangential velocity. The tangential velocity is equal but opposite on each side, and pressure being independent of reference frame, is the same on either side (a wake surface can not support a pressure differential). The physics of the vorticity layer are different from those of a tip vortex and a model derived for the tip vortex should not be applied to capture the vorticity layer. This detail is only relevant for wake-wake interactions in close vicinity of a vortex segment, since all models converge to \( \frac{1}{2} \) behavior at even modest distances from the core. Data from recent Particle Image Velocimetry (PIV) experiments on a straight-bladed VAWT indicate that the region of vorticity making up the shear layer (the shed and trailing vortex sheet) grows rapidly in size and diffuses quickly even within a quarter blade rotation. Future work should develop an analytical model of this process, and investigate how to best represent such behavior using a vortex lattice (if possible).

3.3. Time Marching Scheme

The current method evaluates the problem at each time step and updates with wake successively until it is required to stop. As currently implemented, the time-marching process uses a simple Eulerian update, whereby the induced velocity caused by the wake and body at every wake marker is calculated, and the distance a marker moves is simply the velocity multiplied by the time step. This relationship is expressed in Equation 14 where \( x_i \) is the position of a wake marker at time step \( t \):

\[
\bar{x}_{i+1} = \bar{x}_i + \bar{v}(\bar{x}_i, t) \Delta t + O(\Delta t^2)
\]  

(14)

With each step the Eulerian method has an error of order \( O(\Delta t^2) \) and while computationally cheap, it does accumulate errors over time that may or may not be acceptable. Higher order methods exist but require evaluation of the induced velocities more than once. The bulk of the computational time for each time step is spent calculating the wake-update, so every additional velocity evaluation increases the computational cost significantly. Implementation of an improved update method could make a higher order time-stepping scheme practical, but this is left as a future topic.

4. Validation
The method is validated against experimental data and theory results for simple cases such as an oscillating flat plate (Theodorsen theory), elliptical finite wing and unit sphere. Correspondence is shown with inviscid theoretical results; comparison with the oscillating wing data of Piziali [6] is also acceptable considering the current method is inviscid in its prediction of body forces. For reasons of brevity, these validation cases are not presented here; only those cases pertaining directly to the VAWT case will be shown. The full validation and verification may be found in [2].

The model is compared against 3D stereo Particle Image Velocimetry (PIV) data for a straight bladed VAWT configuration experiments at a $\lambda$ of 4. The experimental setup consists of a two bladed VAWT with one blade having a NACA 0015 section and another with NACA 0018. Tip geometry is also varied: the 0015 blade having quarter chord sweep and straight tip, and the 0018 blade with a leading edge and trailing edge swept tip. Each blade has a chord of 6cm and span of 70cm and rotates about a diameter of 56.8cm. The model is shown to predict the tip vortex trajectory and subsequent deformation downstream as illustrated in Figure 6 where the PIV results for vorticity are shown superimposed by the wake vortex lattice produced by the model. Further information about this specific experiment may be found in [1].

The PIV data is restricted in its field of view, and only allows a validation in the neighbourhood of the blade tip. Smoke trail studies however, carried out as part of Ferreira et al. [10] are able to provide validation over the entire diameter. These are conducted again for a straight bladed VAWT with two NACA 0018 of chordlength 8cm blades at $\lambda = 3$, $\beta = 0.67$, $\sigma = 0.43$. Over the blade diameter the model is shown to predict the inward motion of the tip vortex and its convection downstream as seen in Figure 7 which shows the tip vortex segments superimposed on an image of smoke trail data.

5. General Structure of the Near Wake for a Straight Bladed VAWT

Neglecting for the time being wake deformation, the VAWT wake when viewed along its axis of rotation is epicycloidal; i.e. it can be described by a combination of a point rotating about a circle and translating at the same time. The VAWT will usually cross through its own wake during normal operation, with a frequency that is dependent on $\lambda$. Additional blades further complicate the wake pattern as each blade will pass through its own wake and that of every other blade at least once every rotation. Figure 8 shows a typical undeformed wake locus for 1, 2 and 3 blades.

The cycloidal nature of the VAWT and the fact that the wind direction is always perpendicular to the axis of rotation, the angle of attack seen by the blade will oscillate between two extremes as a function of tip speed ratio. This relationship is summarized in Fig-
The vectorial sum of the speed due to rotation \(\omega r\) and the wind \(V_\infty\) produces a peak \(\alpha\) just after \(\psi = 180\), and before \(\psi = 360\), using the angle definition from Figure 1. Applicability of the current method requires that the peak \(\alpha\) not rise much above the static stall limit for any given airfoil because dynamic-stall/stall phenomena will appear and if strong enough, will dominate wake behavior. The wake has a strong influence on the effective angle of attack; this will be discussed in the next section.

In the case of a VAWT in viscous flow, the blade will release vorticity layers from the top and bottom surfaces which act both parallel and perpendicular to the trailing edge- henceforth termed ‘shed’ and ‘trailing’ respectively. The shed vorticity layers have opposite sign on either surface, and in steady flight generally cancel each other out. When there is a change in lift either due to a gust or otherwise unsteady motion, one of the layers will generally be stronger, resulting in a net vorticity, that when integrated, cancels the change in circulation over the blade as per Kelvin’s theorem. The trailing vorticity has the same sign on both surfaces but opposite on either (spanwise) half of the blade. In both steady and unsteady flight, there is a spanwise pressure gradient inboard on the top surface and outboard on the bottom surface assuming a positive angle of attack. These gradients are usually only strong very near the blade tips, and form a strong vortex as the higher pressure air mass on the bottom surface seeks to roll over the tip into a low pressure area- this produces the well known tip vortex. Like aircraft, VAWT tip vortices at either blade tip will rotate in opposite directions.

The shed and trailing vortex sheet strengths are coupled to one another in that one is dependent on the lift distribution along the span, and the other is dependent on the rate of change of lift, so they are roughly 90° out of phase. The trailing vortex sheet strength will peak in strength approximately with the geometrical angle of attack as in Figure 2, with shed strength approximately 90° behind. The strength of the shed vortex sheet is generally larger at its second peak \((\psi = 270°)\). This is a result of the epicycloidal motion of the blades, as they undergo the same change in lift over a much smaller distance between 180° and 360° when the blade is receding relative to the oncoming wind than they do between 0° and 180° when it is advancing. Because the total change in lift is the same, the integrated strength of vorticity on each half cycle is the same, but is more concentrated in the receding phase.

Superimposing the shed and trailing vortex sheets, the major wake structures as viewed from the top are presented in Figure 10 for a two bladed VAWT. Even though wake deformation has been neglected thus far, the wake structure is already complex. The epicycloidal paths for each blade are shown as red and blue thin lines accompanied by the strong vortices that form at approximately \(\psi = 90\) and 270 due to the shed vortex sheet roll-up. The strongest segments of the tip vortex shown as a thicker long-dashed and a short-dashed line for the upwind and downwind blade pass between \(\psi = 120 – 260\) upwind and \(\psi = 340 – 60\) downwind. Based on simulation and experimental results the wake may be split into two regions: A ‘regular’ region where the structure is well defined and displays a periodic nature upon reaching steady state, and an ‘irregular’ region where the structure is generally more convoluted, and does not appear to follow any periodic pattern.

6. The Wake Influence
The wake is influenced by the presence of each blade and itself. This influence is mutual as the wake and its shape impact performance significantly. The best way to illustrate this is by plotting the effective angle of attack as calculated at the quarter chord for a 2D VAWT when the wake is frozen vs. free. From Figure 9 it can clearly be seen that the effective angle of attack is markedly reduced on the downwind blade pass, becoming nearly constant at \(\frac{1}{4}\) of its expected value. On the upwind pass, it also does not achieve the geometric angle by a significant margin. This
reduction is a strong function of solidity for a given tip speed ratio, increasing solidity further reduces the angles as induction velocities increase. Comparing the effective angles of attack for a VAWT with and without a free wake shows the impact wake deformation has on flow experienced by the blade. A frozen wake essentially reduces the overall induction of the VAWT because vorticity that would otherwise induce such velocities has already been forcefully convected downstream. As a result the reduction in induced angle of attack is not as marked, and as such will affect performance calculations, reinforcing the need to allow the wake to develop freely in order to achieve accurate results.

The effective angle of attack will also contain a slight phase lag between the lift generated and its geometric angle of attack is introduced by the presence of the wake. This is largely a function of the reduced frequency $k$. For VAWTs, $k$ is equivalent to $\frac{\lambda}{c}$, and given Theordoen’s theory for an oscillating plate, the phase lag is maximum for $k$ values near 0.2 and amounts to approximately 10 degrees. The phase lag behavior will tend to push the peak upstream lift peak further past 180 and will counter-act the geometrical angle effect downstream by causing the downstream lift peak to lie closer to $360^\circ$.

7. Wake Deformation and Tip Vortex Dynamics

Beyond the basic structure shown in Figure 10, a number of deformation effects can be observed. Isolating the tip vortex provides some interesting results as seen in Figures 11 and 12. Contrary to what might be expected from HAWT experience, the tip vortices move quickly inwards after being released and eventually are intersected by the inboard sections of the downwind blade pass. This inward motion can be explained by two factors: firstly, the trailing vortex sheet is strongest near the tips, so will exhibit the strongest roll-up behavior there. Analytical solutions for the roll-up of wing-tip vortices on aircraft do show a general inboard motion of the vortex core as a function of time [9]; Secondly, with each rotation the shed tip vortex follows a roughly half-circle path, and just like a full vortex ring, a half-ring will convect under its own self-influence downwards. Further downstream the upwind and downwind vortices interact with each other, essentially inducing a rotation about each other, but beyond a diameter behind the VAWT, the wake structure rapidly breaks down, and cohesive trends are difficult to discern. From the front view in Figure 12, an asymmetric downwards motion is visible, this is owing predominantly to the fact that the roll-up mechanisms are stronger when the tip vortex is strongest and this happens after $\psi = 180^\circ$. Along with the strong tip vortex segments, the vortex sheet in general also undergoes significant deformation as shown in Figures 13, 14, 15, ?? From the side view it is clear that the edges of the vortex sheet which are strongest due to the large gradients in lift distribution towards the blade tips, cause the vortex sheet to quickly roll-up about with the tip vor-
Fig. 13: Straight bladed VAWT wake - top view. note the differing roll-up shapes of the shed vortex sheet on the advancing as compared to the receding side. \(\lambda = 3, \beta = 0.67, \sigma = 0.43\).

Horizontal wake expansion as seen in Figure 13 is slightly slower on the advancing side where the shed vortex sheet is weaker, and more rapid on the receding side where the vortex sheet strength is more concentrated. Because the blades oscillate between the same two extremes of lift generation, it is expected that while the advancing sheet is weaker than the receding sheet, if both are integrated over the distance they cover, the total circulation should be equal. Several simulations have shown this to be the case, whereby the wake on the advancing side eventually achieves the same expansion as the receding side, albeit slightly slower since it takes more time for the advancing sheet to roll-up into a well defined vortex.

An interesting result can be seen in the front view in Figure 15 where the wake is highly non-symmetric. The wake shows a greater vertical expansion on the side where the shed vortex is weaker, and a horizontal expansion where the vortex is stronger. The vertical expansion can be attributed to an interaction with the upwind tip vortex shed on the previous upwind blade pass. This strong tip vortex is convected downstream, and induces an upward flow on the wake outside its arc, so the new wake segments being released near \(\psi = 90^\circ\) are convected upwards.

8. Conclusions
A tool has been created that can provide new physical insight into the near wake dynamics of a verti-
cal axis wind turbine. The method has been validated against PIV data and smoke trail studies. A general structure of the near wake has been presented. The wake deformation is shown to have a noticeable effect the effective angles of attack. Initial analysis shows that tip vortices from a straight bladed VAWT are shown to move inwards due to wake roll-up behavior in addition to self induction. Wake expansion is shown to be initially asymmetric in the XY and YZ planes owing wake self-influence and as a consequence of the cycloidal motion of the VAWT blades.

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