PRINCIPLES OF MIXING
IN TIDAL BASINS
IN THE NETHERLANDS
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Abstract

Some fundamental notions related to the flushing of tidal basins are reviewed and some important mixing mechanics are discussed. It is shown that the characteristics of mixing and flushing in tidal basins can be described in various but connected ways, introducing the concepts of time scales and dispersion coefficients. For some simple geometrical configurations formulas for the computation of time scales and dispersion coefficients are given. For complex-shaped tidal basins field data are necessary in order to obtain quantitative information on time scales, dispersion coefficients or on the contribution of different mixing processes. The theoretical topics dealt with in this paper are illustrated by field data collected in some tidal basins in the Netherlands.
1. INTRODUCTION

Due to the progress of numerical computational techniques of the past years a successful simulation of water quality processes is possible in many cases. However, a good understanding of the physics of the individual mechanisms is still lacking. Such an understanding is useful for qualitative predictions and is also of special importance for the choice of a numerical model as well as to the schematisation and to the interpretation of the results. The purpose of this paper is to review some fundamental notions related to the flushing of tidal lagoons and to set forth some important mixing mechanisms. The theoretical topics discussed in the text are illustrated by field data collected in some tidal basins in the Netherlands. Therefore, the reference list does not reflect most of the field work carried out in other tidal lagoons around the world. For a more extensive literature list the reader is referred to Fischer et al. (1979).

In many coastal regions the tide penetrates into embayments behind the shoreline. These embayments will be called tidal basins, or tidal lagoons, if the inlet allows free passage of the ebb and flood flows at each stage of the tide, the tidal range being in the order of a few meters, and if the tidal discharge through a cross-section is much larger than the total discharge of rivers flowing into the basin. This definition covers a certain type of "bar built estuaries", but also a class of well mixed "coastal plain estuaries" (Pritchard, 1967).

The presence of large tidal flats and curved branching channels are characteristics of tidal lagoons. The channelbeds are sandy, siltation takes place on marsh areas near the borders. The maximum tidal velocity in the channels typically ranges between 1 and 2 ms⁻¹. The large tidal velocities and the irregular bottom topography cause a strong vertical mixing. Density stratification of salt and fresh water will only occur if the fresh water inflow of rivers is large, say at least 10 percent of the tidal inflow. The tidal lagoons considered here, however, are supposed to experience a smaller fresh water inflow. Therefore, no important density stratification will occur and the lagoon waters, as a consequence, are well mixed vertically.
Tidal lagoons as described above are found in many places in the world, but mostly in the temperate regions where the semi-diurnal tide dominates, for instance along the European coast and along the eastern coast of the United States.

The strong vertical mixing in tidal lagoons does not necessarily imply a strong longitudinal mixing, or a high flushing rate of lagoon waters. Fluid particles which are introduced into the lagoon, either from the river or from the sea, will remain there for some time. This is one of the reasons why a specific ecosystem may develop in a tidal lagoon, different from ecosystems in the river or in the sea. Therefore, it may be very convenient for lagoon-ecosystem studies to express the longitudinal mixing processes in terms of time scales (Zimmerman, 1981a). From a hydrodynamic point of view (theoretically as well as experimentally) often another quantity for the description of the mixing processes is preferred: the so-called dispersion coefficient.

Both descriptions are presented in the next section, in which the relation between time scales and dispersion coefficients is discussed. The third section is devoted to the hydrodynamic processes which are responsible for the renewal of lagoon waters. It will be shown that the tidal motion is the main generator of the mixing mechanisms. There are two important categories of generating factors:
- tide-induced large scale residual circulations,
- spatial variations of the oscillatory tidal velocity distribution.

The magnitudes (the r.m.s. velocity, say) of both residual circulation and spatial variations of the tidal velocity distribution are determined to a large extent by the geomorphology of the lagoon. If the ratio of these magnitudes is fixed (the r.m.s. value of the deviation of the tidal velocity amplitude from its cross-sectional average is of the same order as the average itself and a typical value of a residual velocity is 5-10 % of the average tidal velocity amplitude) then the flushing in wide lagoons tends to be forced mainly by residual circulations, whereas in lagoons with rather narrow channels the tidal variations of the velocity distributions tend to be the dominant factor.
Here some important mixing mechanisms of each category are analyzed in more detail and the order of magnitude of their contribution to the mass transport is indicated. The numerical values refer to a hypothetical lagoon, with the following tidal and geometrical characteristics:

- tidal range = 3 m,
- maximum velocity (cross-sectionally averaged) = 1 m/s,
- channel width = 1500 m,
- channel depth = 15 m,
- tidal flats = 50 % of the surface.
2. TIME SCALE ANALYSIS

In order to illustrate that time-scales related to the flushing of a lagoon should be carefully defined and interpreted, imagine a watersample taken from the lagoon at a particular position and time. For simplicity we assume that the lagoon has only one outlet to the sea and that there is only one fresh water discharge, for instance a river. The sample of "lagoon water" now, will be mixture of water-parcels which either originate from the sea, called the subsample of "sea water" henceforth, or from the river (the subsample of "river water"). It will be evident that both subsamples have experienced a different history before becoming mixed in the particular sample taken. Let the time that has elapsed since any waterparcel present in the sample entered the lagoon be called its age.

We may now expect that:

1. due to the fact that the transport processes in a lagoon occur in a highly random way, the ages of all the parcels constituting a subsample of one and the same origin will follow some probability distribution. The distribution itself, apart from the character and strength of the transport processes in the lagoon, is a function of the position at which the (sub)sample is taken and of its origin, so that in particular:

2. the average age of all waterparcels from one and the same subsample will differ according to its origin.

Another way of defining a time-scale for the same sample is to use the time it takes for any waterparcel of the sample to leave the lagoon through its outlet to the sea, called the residence time. For the same reason as for the age, we may expect that also the residence time will follow a probability distribution. However, in contrast to the age we do not expect that, after being mixed this probability distribution differs for the different subsamples present. In particular, we expect the average residence time to be independent of the origin of the subsamples and of their relative contribution to the sample as a whole. I.e.:

The residence time will only depend on the position at which the sample is taken.
[Cautionary note: The fact that the residence time is independent of the origin of the subsample closely adheres to the assumption that the constituents of the sample as a whole will behave dynamically passive after having been mixed in the sample. An example, where this does not occur is given by the subdivision of the sample in the subsample of all water parcels and the subsample of all its suspended matter. Even after being mixed at a particular position and time, both subsamples will in general follow different pathways before leaving the lagoon, as the suspension is not dynamically passive.]

The preceding qualitative discussion will now be given a quantitative formulation in the next paragraphs, following more or less Bolin and Rodhe (1973) and Zimmerman (1976a). Particularly, we are interested to know how the different time-scales are related to the strength of the diffusive transport processes in the lagoon. The age and residence time distributions can be determined experimentally. For the age distribution \( p_\alpha \), consider an instantaneous injection at time \( t-\tau \) at position \( \varrho \). The resulting concentration distribution at time \( t \) is called \( c(\varrho ; t-\tau; \zeta, t) \)

Then

\[
p_\alpha (\varrho ; \zeta, t) = \frac{\int c(\varrho , t-\tau; \zeta, t) \, dt}{\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty c(\varrho , t-\tau'; \zeta, t) \, dt' \, \delta^3 \zeta \, d\zeta} (1)
\]

where henceforth the integral \( \frac{1}{T} \int_0^T dt \) stands for averaging over the tidal period. Hence, the age and residence time distributions \( p_\alpha \) and \( p_\tau \) (to be defined next) are tidal averages.

For the residence time distribution, consider an instantaneous injection of a unit mass at position \( \varrho_\alpha \) at \( t-\tau \) and call \( c(\varrho_\alpha , t-\tau; \zeta, t) \) the resulting concentration distribution at time \( t \). Then

\[
p_\tau (\varrho_\alpha , t) = -\frac{\partial}{\partial t} \frac{1}{T} \int_0^T dt \int \int \int \int d^3 \zeta \, c(\varrho_\alpha , t-\tau; \zeta, t) (2)
\]

where \( V(t) \) is the contents of the lagoon varying with the tide.
The average age $\tau_\alpha$ and the average residence time $\tau_r$ are now given by the integrals:

$$\tau_\alpha(\zeta, x_0) = \int_0^\infty t \rho_\alpha(\zeta, x_0, t) \, dt \quad (3)$$

$$\tau_r(x_0) = \int_0^\infty t \rho_r(x_0, t) \, dt \quad (4)$$

The average residence time may also be determined from another type of experiment. Substituting equation (2) into (4) and integrating by parts it follows that

$$\tau_r = \lim_{Q_o \to 0} \frac{1}{Q_o c_o} \frac{1}{T} \int_0^T \int_0^\infty \int_0^{v(t)} c_\infty(x_0, \zeta, t) \, d^2 \zeta \quad (5)$$

In this expression $c_\infty(x_0, \zeta, t)$ is the stationary concentration distribution corresponding with a steady injection of mass $Q_o c_o$ at the position $x_0$. With this formula the residence time can easily be related to the dispersion coefficient, as will be shown later.

In most tidal lagoons water quality parameters mainly change in the longitudinal direction, between the river entrance and the sea boundary. If the lateral change in water quality (not the lateral gradient) is much smaller than the longitudinal change, it may be convenient to consider a partition of the lagoon in sections perpendicular to the longitudinal axis and to regard the average distributions of the water quality parameters over the cross-sections. This leads to a one-dimensional description of the lagoon. One may again define age and residence time distributions for water parcels in a section of the lagoon between the planes $x_o$ and $x_o + dx$. 
The one-dimensional age distribution is given by

\[ \tau_a(g;x,t) = \frac{\int_0^T C(g,t-t';x,t) \, dt}{\int_0^T dt \int_0^\infty C(g,t-t';x,t) \, dt'} \quad (6) \]

where \( C(g,t-t';x,t) \) represents the cross-sectionally averaged concentration corresponding with an instantaneous injection in \( g \) at \( t-t' \). The one-dimensional equivalent of the expression (5) for the residence time is

\[ \tau_r(x_o) = \lim_{Q_o \to 0} \frac{1}{Q_o c_o} \int_0^\ell \int_0^T A(x,t) C_\infty(x_o;x,t) \, dx \quad (7) \]

Here \( C_\infty(x_o;x,t) \) represents the cross-sectionally averaged stationary concentration distribution corresponding with a steady injection \( Q_o c_o \) at \( x_o \), homogeneously spread over the cross-section. The other symbols are: \( \ell \) = length of the lagoon along the longitudinal axis, \( A(x,t) \) = cross-sectional area.

In the case \( C_\infty \) is interpreted as the fresh water fraction of lagoon water due to a single fresh water source having a discharge \( Q_o \), (7) equals an often used expression for the residence time of fresh water:

\[ \tau_r = \frac{F}{Q_o} \quad (8) \]

where \( F \) stands for the stationary tidal average total fresh water content of the lagoon.

The average total age \( \tau_a(o,\ell) \) and the average total residence time \( \tau_r(o) \) for a lagoon with a single river inflow at \( x=o \), are equal. This time is also called the flushing or transit time \( \tau \) of the lagoon (Zimmerman, 1976a):

\[ \tau = \tau_a(o,\ell) = \tau_r(o) \quad (9) \]
It should be noted that the sum of the average age and the average residence time in the cross-section \( x_0 \) is not equal to the transit time:

\[
\tau_a (\sigma, x_0) + \tau_r (x_0) \neq \tau.
\]

There are two reasons for this:
- the water parcels passing at different parts of the cross-section \( x_0 \) have a different transit time through the lagoon; relatively more water parcels choose a path through the lagoon corresponding with a short transit time. This is not taken into account with the averaging procedure over the cross-section.
- the average residence time \( \tau_r (x_0) \) corresponds with the average time water parcels spend in the lagoon after the first time they have passed through the plane \( x_0 \). However the average age \( \tau_a (\sigma, x_0) \) does not refer to the time elapsed since the water parcels have passed for the first time through the plane \( x_0 \), but to the average time spent in the lagoon by all water parcels present in the plane \( x_0 \). For instance: \( \tau_a (\sigma, 0) \neq 0 \)

In the hypothetical case that no mixing at all takes place in the lagoon and that density currents are absent, the flushing of the lagoon is caused by the river discharge \( Q_0 \). The seawater is pushed out of the lagoon and the salinity of the lagoon waters at low water slack is zero. The flushing, or transit time of the lagoon then equals:

\[
\tau = \tau_a \equiv \frac{V}{Q_0} \quad (10)
\]

The presence of mixing processes in the lagoon is related to random displacements of individual fluid parcels with respect to the cross-sectional mean displacement of the water body. These relative displacements follow a probability distribution. The average relative displacement in longitudinal direction of an individual water parcel in time interval \([t, t+\Delta t]\) is called \( \sigma (r, t; \Delta t) \)

If the flow properties in the lagoon are periodical and the same everywhere, then \( \sigma \) is approximately independent of the time \( t \) and
the place $\mathcal{E}$, if $\Delta t$ is an entire number of tidal periods. If the successive relative displacements in the time interval $\Delta t$ are uncorrelated [one-dimensional random walk, (Taylor, 1921)], then

$$\sigma(n\Delta t) = n^{1/2} \sigma(\Delta t).$$

From this it follows that the average transit time of water parcels through the lagoon is given by

$$\tau_D = \frac{L^2 \Delta t}{\sigma^2(\Delta t)} \quad (11)$$

This expression only yields the timescale for flushing of the lagoon by mixing processes. If turbulence is the only mixing process in the lagoon, then

$$\frac{\sigma^2(\Delta t)}{\Delta t} \approx 1 \text{ m}^2 \text{ s}^{-1}.$$ 

In that case $\tau_D \gg \tau_Q$, even for a very small river inflow. This means that discharge is the principal flushing process, implying a very low lagoon salinity. In practice, however, the salinity of tidal lagoons is comparable with the seawater salinity, even in absence of evaporation. This shows that the mixing processes which operate in a tidal lagoon are much stronger than what can be expected from turbulence proper. These mixing mechanisms, which are mainly generated by tidal motion will be investigated in the next section.

In real lagoons the flow properties are different in different parts of a cross-section. If the time interval $\Delta t = nT$ is taken long enough, i.e. at least equal to the time scale for cross-sectional mixing, then the average relative longitudinal displacement

$$\sigma(\tau; nT)$$

is approximately the same in every point of the cross-section (Dronkers, 1982). In that case one may define a dis-
The dispersion coefficient is related to the average mass transport caused by the mixing processes. The tidally averaged mass transport $\dot{M}_D$ generated by the mixing processes (often referred to as "dispersion") is normally given by

$$\dot{M}_D = -D(x)A_0(x) \frac{\partial C}{\partial x}$$

where the important assumption of gradient-type character of the dispersive mass transport is assumed. Conditions for this assumption are discussed by Dronkers (1982). Here $A_0(x)$ represents the tidally averaged cross-sectional area, and $C(x,t)$ is the concentration distribution averaged over the cross-section and the time interval $nT$:

$$C(x,t) = \frac{1}{A_0} \frac{1}{nT} \int_t^{t+nT} \int \int_{A(t')} c(x,y,z,t') dydz .$$

The concentration distribution $C(x,t)$ follows the advection-dispersion equation:

$$A_0 \frac{\partial C}{\partial t} + \frac{\partial}{\partial x} [Q_0 C] - \frac{\partial}{\partial x} [DA_0 \frac{\partial C}{\partial x}] = 0$$

By solving this equation and substituting the results into equations (6) and (7) an expression can be found for the age and residence time distributions $p_a$ and $p_r$ as functions of the dispersion coefficient $D$. If a lagoon is considered with a small river inflow ($Q_0 \ll \frac{DA_0}{T}$), the
average residence time $\tau_R(x_o)$ is given by

$$
\tau_R(x_o) = \int_{x_o}^{l} \frac{dx}{A_o(x)D(x)} \int_0^{x} dx' A_o(x') \quad .
$$

(16)

To obtain this expression the steady state solution of equation (15) has been substituted in equation (5). Instantaneous complete mixing has been assumed at the sea-boundary. It should be noted that the residence time only depends on the mixing processes between the section $x_o$ and the sea-boundary; the dispersion coefficients for this region only enter the expression for $\tau_R$.

In figure 1 the residence time distribution is shown for the Oosterschelde, a tidal basin in the Netherlands with characteristics close to the prototype lagoon described in section 1.

In the case of a constant dispersion coefficient and a constant cross-section, expression (16) reduces to

$$
\tau_R(x_o) = \frac{1}{2} \frac{l^2-x_o^2}{D} \quad .
$$

(17)

For this particular case a simple analytical expression is also found for the average age $\tau_a(o,x_o)$ of river parcels discharged at $x=0$:

$$
\tau_a(o,x_o) = \frac{1}{2} \frac{l^2}{D} - \frac{1}{6} \frac{(l-x_o)^2}{D} \quad .
$$

(18)

This expression is found by solving equation (15) for an instantaneous injection at $x=0$, and by substituting the result in the equations (6) and (3). In the same particular case an expression can be found for the age of seawater parcels, $\tau_a(l,x_o)$, by considering an instantaneous injection at the sea-boundary $x=l$. 
Figure 1: Isolines of equal residence time in the Oosterschelde.
Time unit is $T$ ($M_2$ tidal period $\approx 12$\:hr \:25$min).
The result is

$$\tau_q (l, x_o) = \frac{1}{2} \frac{l^2 - x_o^2}{D}$$  \hspace{1cm} (19)$$

Obviously the ages of river parcels and seawater parcels in a particular cross-section of the system are different. Note that, by definition, using the boundary condition $C(l, t) = 0$, the average age of seawater parcels in the cross-section at the sea-boundary, $x = l$, is always zero, though there will be parcels present in that cross-section that definitely have an age different from zero. Care should therefore be taken in interpreting ages near $x = l$.

Finally, the flushing time of the lagoon is equal to

$$\tau = \tau_q (0, l) = \tau_r (0) = \frac{1}{2} \frac{l^2}{D}$$  \hspace{1cm} (20)$$

The principal reason for relating the time scales to the dispersion coefficient is the experimental determination of these quantities. After a sufficiently long period of constant river inflow $Q_o$ into the lagoon, the tidally and cross-sectionally averaged salinity distribution $S(x)$ follows the equation:

$$Q_o S = -DA_o \frac{dS}{dx}$$  \hspace{1cm} (21)$$

Thus, the dispersion coefficient $D(x)$ follows from a single measurement of the salinity distribution. If the river inflow does not remain constant for a sufficiently long time, then the dispersion coefficient can be found by solving equation (15) and by adjusting $D(x)$ in such a way that the solution $S(x, t)$ matches the measured salinity distributions. To illustrate the connection between the longitudinal dispersion coefficient derived from the salinity distribution and the flushing time of the lagoon [defined in (20) as the residence time of water introduced into the lagoon at its landward end] we give in table 1 orders of magnitude of $\tau$ and $D$ for some tidal lagoons along the Dutch coast. Note that since $\tau$ is pro-
portional to the squared length of the lagoon, it is primarily the length which determines the flushing time, the more so since for three out of four of the examples (the Eems, Oosterschelde and Westerschelde) the diffusion coefficient is of the same order of magnitude. Only for the Waddensea, which has a larger width than the other three areas, is the diffusion coefficient appreciably larger. Therefore, together with its shorter length, this area experiences a very rapid flushing. The values given for the flushing times are really only an order of magnitude, as in all cases the dispersion coefficient varies according to the position in the area.

<table>
<thead>
<tr>
<th>Area</th>
<th>Length</th>
<th>Mean dispersion coefficient, (derived from salinity distributions)</th>
<th>Flushing time</th>
<th>References</th>
</tr>
</thead>
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<tr>
<td>Oosterschelde</td>
<td>50</td>
<td>250</td>
<td>50</td>
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<tr>
<td>Westerschelde</td>
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<td>Western Dutch Waddensea</td>
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<tr>
<td>Eems</td>
<td>45</td>
<td>250</td>
<td>40</td>
<td>Dorrestein and Otto, 1960</td>
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3. MIXING MECHANISMS

Consider a lagoon with a periodical tidal motion and no fresh water discharge. In that case, renewal of lagoon waters only occurs if water parcels experience a net displacement with respect to one another during each tidal period. Such net displacements are either produced by turbulence or by residual circulations. Residual circulations are generated by:
- density differences, due to fresh water inflow,
- wind,
- interaction of the tidal flow with the bottom topography and shoreline geometry.

As discussed in section 2, turbulent fluctuations of the velocity field will not cause large residual displacements if acting separately. Interacting with the large scale gradients in the tidal velocity field, however, the dispersive action of turbulence is greatly amplified. This is explained in figure 2.

The mixing processes related to residual currents and to the non-uniformity of the tidal velocity distribution will be illustrated by the way they act upon a patch of dye. Therefore consider a longitudinally uniform system (cross-sectional area and velocity distribution independent of $x$) with a patch of dye located initially in a plane $x = 0$ and spread homogeneously in the cross-section. The velocity distribution is supposed to be non-uniform in the cross-section. Thus, in a frame moving with the average cross-sectional velocity, there will be currents in opposite directions in different parts of the cross-section, causing a longitudinal stretching of the patch of dye.

In the case of a residual circulation one finds opposite currents in different parts of the cross-section even after averaging over a tidal period. At intervals of one tidal period, the patch of dye will be stretched with the speed of these residual currents, as long as no cross-sectional mixing occurs. Thus if the cross-sectional mixing is only weak, a strong longitudinal dispersion results, which implies a fast renewal of lagoon waters. In the case of strong cross-
Figure 2 : Horizontal view of a part of a lagoon and the corresponding cross-section.

In the horizontal plane the tidal pathways of two water-parcels are indicated, corresponding to a non-uniform velocity distribution over the cross-section. Water parcels in the deeper (shallower) part of the channel follow a longer (shorter) tidal path. If it is supposed that water parcels are interchanged by turbulent mixing at a certain phase of the tide - here at some time before high water slack (HWS) - they travel to their mutual starting positions at low water slack (LWS). Thus the water parcels experience a net longitudinal displacement over a tidal cycle.
sectional mixing the patch of dye will be stretched only slightly, resulting in a small longitudinal dispersion and a slow renewal of lagoon waters.

Call $\tau_\perp$ the timescale for cross-sectional mixing, and $\bar{u}_S$ a measure for the tidally averaged velocities in the moving cross-section. Then a water parcel will on the average travel a distance $l_S \approx \bar{u}_S \tau_\perp$ before, under the influence of lateral mixing, it reaches another part of the cross-section where the residual current has the opposite direction. Over a time interval much longer than $\tau_\perp$, the motion of a water parcel under the influence of residual currents and cross-sectional mixing resembles turbulent motion with eddies of a length scale $l_S$ and a timescale $\tau_\perp$ (see figure 3). The corresponding longitudinal dispersion is given by the expression (cf. Okubo, 1967)

$$D_S = \frac{l_S^2}{\tau_\perp} = f_S \bar{u}_S^2 \tau_\perp$$

(22)

with $f_S \approx 1$. This formula displays the $\tau_\perp$ -dependence described before.

We now consider the case that residual circulations are absent, but in which lateral variations in the tidal velocity amplitude are present. The direction of the currents in the center-of-mass frame is inverted during the tidal period. Thus, if there is no cross-sectional mixing, the patch of dye which has been stretched in a first stage of the tidal period, will be contracted during the second stage and return to its initial size! So there is no longitudinal dispersion. Now consider an increase of cross-sectional mixing such that $\tau_\perp$ is in the order of a tidal period. In that case the patch of dye is to a certain extent stretched, but it remains well spread in the cross-section. After the turning of the currents, the patch cannot be contracted any more into its initial size and longitudinal dispersion results (see figure 4). If cross-sectional mixing increases further, so that its time scale becomes much smaller than a tidal period, then the patch of dye will hardly be stretched by the non-uniform velocity distribution and the longitudinal
Figure 3: In the presence of a residual velocity distribution as shown in the lefthand side of the picture (assumed to be uniform in the longitudinal direction), fluid parcels experience an eddy-like displacement due to the interaction of residual currents and turbulence, if viewed in a frame of reference moving with the cross-sectional average of the tidal flow.
Figure 4: The combined action of a lateral variation of the tidal velocity field and the turbulence produces enhanced longitudinal dispersion by stretching a streak-patch of dye that was originally located in a narrow section around a specific position in the channel. Stretching increases the concentration gradient in the direction perpendicular to the streak, together with increasing its effective length. This causes an increase in the effectivity of turbulent mixing, so that after one tidal cycle the patch is dispersed strongly in the longitudinal direction.
dispersion decreases again. If \( \langle u'^2 \rangle \) is a measure for the mean square deviation from the average velocity in the cross-section during a tidal period, then the longitudinal dispersion coefficient is given by (Okubo, 1967; Holley et al., 1970):

\[
D_o = f_1 \frac{\langle u'^2 \rangle T^2}{\tau_L} \quad \text{for } \tau_L \geq T, \tag{23}
\]

\[
D_o = f_2 \frac{\langle u'^2 \rangle \tau_L}{T} \quad \text{for } \tau_L \ll T. \tag{24}
\]

The coefficients \( f_1 \) and \( f_2 \) still depend on the cross-sectional velocity distribution \( u'(y,z,t) \). Their order of magnitude is \( f_1 \approx 0.03 \) and \( f_2 \approx 1 \).

It should be noted that the derivation of (23) and (24) presupposes that the tidal currents change direction at the same time at all positions in a cross-section. In reality, however, due to friction, significant phase differences may occur, which can lead to an effective increase of \( \tau_L \) in (22) or (24), as has been discussed by Taylor (1974) for vertical tidal shear flow.

Because of the small fresh water inflow with respect to tidal discharges, the vertical mixing of the water body is not inhibited by density stratification. In that case the time scale for vertical mixing is much smaller than the tidal period. From the formulas (22) and (24) it follows that neither the steady nor the unsteady component of the vertical velocity distribution gives rise to a strong longitudinal dispersion.

On the other hand, the time scale for lateral mixing is generally larger than a tidal period, owing to the large width-to-depth ratio of lagoons. Fixing the ratio \( \langle u'^2 \rangle / \tilde{u}_s^2 \), the largest contribution to the longitudinal dispersion is supplied by horizontal residual circulation if the time scale for lateral mixing is very large, i.e. in wide lagoons. In narrow lagoons where the time scale for lateral mixing does not exceed a few tidal periods, lateral gradients of the tidal velocity distribution tend to be the dominant
factor for longitudinal dispersion and for renewal of lagoon waters. This is illustrated in figure 5 where the ratio $\frac{\overline{u'^2}}{D_s/\overline{u_s^2} D_0}$ is given as a function of the dimensionless width of the lagoon $\beta = b/(\varepsilon_y T)^{1/2}$, $b$ denoting the dimensional width, $\varepsilon_y$ a (width independent) turbulent diffusion coefficient to be discussed in the next chapter and $T$ the tidal period. If $\beta \ll 0$ (narrow lagoons) the ratio becomes a constant of order 1. As now $\overline{u'^2}$ is of the order of the average squared tidal velocity $\overline{u^2}$, and in general $\overline{u_s^2} \ll \overline{u^2}$, this means a dominance of oscillatory shear diffusion over residual shear diffusion. On the other hand if $\beta \gg 1$ (wide lagoons), the ratio is proportional to $\beta^4$ and rapidly the residual shear effect dominates the longitudinal dispersion process.

The physical processes behind the parameters which appear in the above analyses, are reviewed more in detail and illustrated with experimental evidence in the following sections.
The dimensionless width $\beta$ of the lagoon as defined in the text.

Figure 5: The ratio $\frac{\langle u^2 \rangle_D S}{\langle \tilde{u}_s^2 \rangle D_0}$ shown as a function of $\beta = b / (e_y)^{1/2}$.
4. LATERAL MIXING

From the analysis of the preceding section it is clear that lateral dispersion plays an important role in the longitudinal dispersion in a tidal system.

Lateral mixing is usually expressed in terms of a dispersion coefficient $\varepsilon_y$. If it is supposed that the longitudinal variation of $\varepsilon_y$ and the width $b$ is not too strong, then an order of magnitude of the lateral mixing time $\tau_1$ is given by

$$\tau_1 = \frac{(\frac{1}{2}b)^2}{2\varepsilon_y} \quad \ldots \quad (25)$$

Lateral mixing, just like vertical mixing, is caused by turbulence generated at the bottom. However, other processes may also contribute: from experiments it follows that the dispersion coefficient $\varepsilon_y$ not only depends on the depth, but also on the width (Lau and Krishnappan, 1977). Other geometrical characteristics also seem to play a role. For example, behind contractions in the channel section, or other shoreline irregularities, the tidal flow may create horizontal eddies, which contribute to the lateral mixing.

From field measurements in rivers it follows that (Okoye, 1970)

$$\varepsilon_y = b u_* f\left(\frac{h}{b}\right) \quad \ldots \quad (26)$$

In this expression $u_*$ represents the shear velocity, and $f$ an increasing function of the depth-to-width ratio:

$$f \approx 10^{-2} \quad \text{for} \quad \frac{h}{b} = 4.10^{-2}, \quad f \approx 5.10^{-3} \quad \text{for} \quad \frac{h}{b} = 10^{-2}.$$ 

For strongly curved channels the values of $f$ are a few times larger (Yotsukura and Sayre, 1976).

As a result of the interaction between the tidal flow and the channel geometry, the transverse mixing coefficient may become several times larger than the values indicated by (26). The lateral dispersion coefficients for tidal flow reported by Fischer et al. (1979)
amount to

\[ f \left( \frac{h}{b} \right) = (0.5 - 1.5) \cdot 10^{-2} \quad \text{for} \quad \frac{h}{b} \approx 10^{-2}. \]

From this the time scale for lateral mixing in the prototype lagoon described in section 1 can be estimated at about ten tidal periods.

Direct experimental evidence concerning lateral mixing in tidal lagoons is very scarce (Fischer et al., 1979). It can be expected that the values for the lateral mixing time scale given above are sometimes considerably decreased by lateral velocity components other than those induced by turbulence proper. For instance geometrically induced flow patterns normally also show lateral velocity components. In the case of residual circulations, it has been shown by Zimmerman (1976b) that such circulations tend to decrease the lateral mixing time scale.
5. DISPERSION BY RESIDUAL CURRENTS

In section 3 several factors responsible for residual flow have been indicated. The corresponding processes will be reviewed in this section. Most attention is paid to the generation of residual circulations by tidal flow over or along an irregular topography, which is thought to be the most important factor in many tidal lagoons.

Wind-driven residuals

In the presence of variations in water depth, wind stress on the water surface induces horizontal circulations, particularly when it acts in the direction of the longitudinal axis of the lagoon (see a simple explanation in Groen, 1969). The strength of the circulation depends on the variation of the depth in the cross-section. Wind-induced circulations are most outstanding if the channels in the lagoon are deep and bounded by wide shallow regions. In that case, there is a wind driven current in the direction of the wind stress on the shoals and a return current in the deep part of the cross-section. The residual wind driven currents are dominated by the tidal currents in the type of lagoon considered here: at a wind speed of 8-10 m/s (10 m above sea level) the wind driven current $u_o$ on the shallows is typically in the order of a few centimeters per second.

If $\gamma$ is the fraction of the cross-section corresponding to the shallow region bounding the channel, then $u_S \approx 2\gamma u_o$ is the cross-sectionally averaged modulus of the residual velocity. Following equation (20) the longitudinal dispersion coefficient is given by

$$D_s \approx u_S^2 \tau_\perp \approx 4\gamma^2 u_o^2 \tau_\perp . \quad (27)$$

For the prototype lagoon described in section 1, $\tau_\perp = 5.10^5 s$ and $\gamma \approx 0.2$, which yields $D_s = 50 m^2 s^{-1}$ for residual wind circulations. The different values entering this estimate are very
crude approximations; the uncertainty in the dispersion coefficient is of the order of magnitude of the computed value.

In lagoons where tidal currents are weak, wind driven currents may develop more strongly, and become the principal agent of mixing and renewal of lagoon waters.

**Residuals driven by horizontal density gradients**

Even if river inflow is small and if the salinities at the bottom and the surface are approximately the same, density effects may influence the longitudinal dispersion. When the width of the channels exceeds about 1000 m the most important effect of a longitudinal density gradient, is the generation of a horizontal residual circulation (Smith, 1980). The origin of this circulation lies in the fact that water with the higher density tends to concentrate in the deeper part of the channel. Because of strong vertical mixing the surface density is on the average higher at the center of the channel than near the boundaries. This explains the transverse circulation shown in figure 6b, the water with lower density flowing over the water with higher density. The pressure gradient due to the horizontal density gradient increases proportionally with distance from the water surface. Thus the depth averaged pressure gradient is proportional to the local depth. The first statement explains the residual velocity distribution of figure 6a: highest velocities in landward direction occur near the bottom of the deepest part of the channel. The second statement explains the residual circulation pattern shown in figure 6c: depth averaged residual transport in landward direction only occurs in the deepest part of the channel.

An order of magnitude for the cross-sectionally averaged modules of the residual longitudinal velocity $u_s$ can be derived from Smith (1980)

$$
\bar{u}_s \approx u_s = 4\pi \frac{5^{7/2}}{7^{9/2}} \frac{g}{g} \frac{d\sigma}{dx} \frac{h^2}{u_i C_B}.
$$

(28)
Figure 6: Residual circulations due to density differences.

a. Distribution of longitudinal residual velocities in a cross-section,

b. Transverse circulation pattern in a vertical cross-section, generated by the lateral density gradient,

c. Horizontal depth-averaged residual circulation pattern generated by a longitudinal density gradient together with lateral differences in depth.
For the derivation of this expression it is assumed that the cross-section may be approximated by an equilateral triangle.

The symbols used in equation 28 are:
\( g = \) density, \( C_B = \) coefficient for bottom friction, 
\( u_1 = \) amplitude of tidal velocity.

Substitution of characteristic values for the Oosterschelde, which resembles the prototype lagoon described in section 1:
\[
\frac{1}{g} \frac{dg}{dx} \approx 10^{-7} \text{ m}^{-1}, \quad h^2 = 100 \text{ m}^2, \quad C_B = 0.005, \quad u_1 = 1 \text{ m s}^{-1},
\]
yields the result \( u_s = 0.01 \text{ m s}^{-1} \). Using equation (22) with \( T_L = 5 \times 10^5 \text{ s} \), the longitudinal dispersion in the Oosterschelde generated by longitudinal density differences amounts to an order of magnitude of \( D = 50 \text{ m}^2/\text{s} \).

It should also be noted that the transverse circulation, due to lateral density differences, reduces the lateral mixing time. So even in cases where the longitudinal dispersion is not dominated by density driven horizontal circulation, the presence of density differences may affect the longitudinal dispersion by decreasing the lateral mixing time scale.

Topographically generated residuals

Tidal lagoons generally show irregular shorelines and bottom topographies. These irregularities produce inhomogeneities in the tidal current velocity field, mainly by frictional and Coriolis forces. Nonlinear interactions, in turn, then give rise to rectification, i.e. the generation of ebb or flood surpluses (the so-called tidal residual currents). Their dynamics and relationship to the geomorphology of seabed or coastline has recently been reviewed by Zimmerman (1981b) in terms of vorticity transfer. According to the morphological agent producing the residual current, three different types of residuals were distinguished in Zimmerman (1981b). Here, as an example, we shall only show in some more detail one of these types: the headland eddy.

In figure 7 the observed horizontal distribution of residual currents in the Oosterschelde is shown. Around the headland at the southern
shore clearly two residual circulation cells are visible, the western one rotating anticlockwise, the eastern one clockwise. Such a pattern can be easily explained from a vorticity argument as in Zimmerman (1981b), but to avoid the introduction of vorticity here we shall, with the aid of figure 8, treat the dynamics of the residual eddies in terms of the residual momentum budget. The explanation is that the tidal current in flowing around the bend in the coastline experiences a centrifugal force towards the outer channel wall, which tends to be balanced by a pressure gradient inwards. The latter means that the sealevel is raised in the outer part of the channel and lowered at the headland. Away from the headland no cross channel sea surface slopes are supposed to exist. A crucial element is, that the disturbed sealevel pattern is the same for the ebb and flood stages of the tide, so that it persists even after averaging over the tidal period. It can then easily be seen that along the coast the residual pressure distribution drives the water towards the headland at both sides, whereas in the outer channel the currents flow in the opposite direction. Of course, the residual pressure gradients, driving the currents, are balanced by bottom friction. The residual momentum balance thus is given approximately by:

\[ \frac{\langle u_\theta^2 \rangle}{R} + g \frac{\partial \zeta}{\partial r} = 0 \]  \hspace{1cm} (29)

centrifugal force pressure gradient

for the radial direction in a polar coordinate system with its center coinciding with the center of the circle described by the radius of curvature at the headland.

For the azimuthal direction, we then have the residual momentum balance:

\[ \frac{g}{R} \frac{\partial \zeta}{\partial \theta} + \frac{C_B}{h} \langle u_\theta | u_\theta | \rangle = 0 \]  \hspace{1cm} (30)

pressure gradient bottom friction

Here \( \zeta \) is the deviation of sealevel from its equilibrium position. Other symbols have their previously given meanings.
Figure 7: Horizontal tide-induced residual circulation measured in a hydraulic scale model of the Oosterschelde. The circulation cells, or "residual eddies" are generated by centrifugal forces acting on the tidal currents flowing around headlands. The center of the curvature of the flow and the radius are shown.
The residual sealevel distribution around a headland is shown by +, - or 0 for respectively raised, lowered and unaffected sealevel. The arrows denoted by P show the direction of the pressure gradient forces. In the radial direction this force is balanced by the tidal average centrifugal force C. In the azimuthal direction the pressure gradient forces are balanced by bottom frictional forces F. The direction of the latter sets the resulting residual circulation pattern shown by the solid curves.
In equation (30) a term \( \frac{1}{2R} \frac{\partial}{\partial \theta} \langle u_\theta^2 \rangle \) has been neglected. For bottom shear flows, which are considered here, this neglect is often justified, as the current velocity along the shallow headland is smaller than the current velocity in the outer channel. For potential flow however, this term is important: it balances precisely the longitudinal pressure gradient \( g \langle \xi \rangle / R \delta \), so that no residual circulation is generated.

Equations (29) and (30) allow an estimate of the residual current velocities near the headland.

Substituting

\[
    u_\theta = \langle u_\theta \rangle + u_{1\theta} \cos \left( \frac{2\pi}{T} t \right),
\]

then \( \langle u_\theta \rangle \) is found to be:

\[
    \langle u_\theta \rangle = \frac{\pi}{16} \frac{h}{C_B} \frac{b}{RL_s} u_{1\theta},
\]

where \( b \) is the width of the channel, \( R \) the radius of curvature at the headland and \( L_s \) the lengthscale of the residual eddy.

Characteristic values for the example of the Oosterschelde, as shown in figure 7, are:

\[
    L_s \approx R = 6000 \text{ m}, \quad h = 20 \text{ m}, \quad b = 3000 \text{ m}, \quad u_{1\theta} = 1.5 \text{ ms}^{-1}
\]

and \( C_B = 0.005 \), yielding the crude estimate \( \langle u_\theta \rangle \approx 0.1 \text{ ms}^{-1} \).

Should the above given value of \( \langle u_\theta \rangle \) be used as an order of magnitude for \( \tilde{u}_s \) in (22), a value of \( 3000 \text{ m}^2\text{s}^{-1} \) for \( D_s \) is obtained, if \( \tau_L = 5 \times 10^5 \text{ s} \).

This certainly is too large an order of magnitude. The problem we face is twofold. First the very existence of residual eddies may decrease the lateral mixing time scale as has already been discussed before. This will decrease the dispersion coefficient. Second, the derivation of (22) assumes the lateral structure of the residual velocity in the moving frame to be more or less homogeneous over a distance much larger than the tidal excursion, which obviously is not satisfied in the present example where the tidal excursion spans more than the length scale of the residual vortex pair. In that case an alternative expression for the residual longitudinal dispersion coefficient may do:
This expression was proposed independently by Sugimoto (1975) and Zimmerman (1976b, 1978). The former author estimated \( c \) to be of order 0.4, but Zimmerman (1976b) expressed \( c \) as a function of the ratio \( \lambda \) of tidal excursion amplitude and eddy length scale and of the ratio \( \delta \) of the kinetic energy density of residual current velocity field and tidal current velocity field. A value of 0.4 for \( c \) in (32) gives \( D_s = 320 \text{ m}^2\text{s}^{-1} \) for an eddy diameter of 8 km. For \( \lambda \approx 2.7 \) and \( \delta \approx 4.4 \times 10^{-3} \), \( c(\delta, \lambda) \) following Zimmerman's (1976b) tidal random walk method, is of the order 0.5, thus \( D_s = 400 \text{ m}^2\text{s}^{-1} \). Though still somewhat too large, these estimates have the right order of magnitude. Thus, the example shows that great care be taken in applying (22) in cases where the residual currents are strongly non-uniform in the longitudinal direction. As the dynamics of residual circulation is such as to favour residual eddies of a size comparable to the tidal excursion amplitude (Zimmerman, 1981b) this means that for the dispersive effect of these eddies application of (22) may lead to wrong results, especially when fixed-frame residual velocities are substituted.
6. DISPERSION IN AN OSCILLATING (TIDAL) VELOCITY FIELD

In section 3 it has been shown that lateral mixing generates longitudinal dispersion if the oscillating velocity field is non-uniform in the lateral direction. For $\tau_\perp \geq T$, the corresponding dispersion coefficient is given by (23). This expression shows that the dispersion is enhanced by an irregular channel geometry, for two reasons:
- a decrease of the lateral mixing time,
- an increase of the lateral variation of the velocity field.

Sometimes channel irregularities are so pronounced that "trapping" occurs: water masses are retained inside topographic structures and communicate with the water masses in the channel by turbulent diffusion, tidal eddies or transverse circulations. If it is assumed that these "traps" are more or less regularly distributed along the channel, a longitudinal dispersion results which is given by (Okubo, 1973):

$$D_\parallel = \frac{\gamma \tau_\perp \langle u^2 \rangle}{1 + 4\pi^2 \frac{\tau_\perp^2}{T^2}}$$

(33)

Here $\gamma$ is the fraction of the cross-section where the water is trapped and $\tau_\perp$ is the time scale for renewal of water in the trap. In the limits $\tau_\perp \ll T$ and $\tau_\perp \geq T$ this equation reduces to the general expressions (24, 23), which shows that mixing by trapping is in fact an extreme form of lateral shear dispersion.

Tidal flats in a lagoon may also be considered as a kind of "trap". The water exchange between the tidal flats and the channel is a direct result of the tidal variation of water levels. Though the flow to and from the tidal flats does not produce longitudinal dispersion by itself, longitudinal dispersion results in one of the following cases (Dronkers, 1978):
- the water level variation is out of phase with the longitudinal transport in the channel, as a result of energy dissipation in the lagoon (Dronkers, 1978; Schijf and Schönfeld, 1953; Postma, 1954),
Figure 9: Characteristic circulation pattern on a tidal flat.
Figure 10: Experimental evidence for the occurrence of mixing on tidal flats in the Oosterschelde is shown by comparing the time history of current velocity and salinity for positions A, C on the tidal flat and B, D in the main channel. Note the leveling of the salinity curves during outflow at the tidal flats. This indicates strong mixing of water at the flats during inflow, producing a homogeneous outflow at the ebb stage of the tide.
the water masses on the tidal flat are mixed, as a result of
topographic residual circulations (see figure 9). If it is assumed that complete mixing occurs on the tidal flat, the following expression for the longitudinal dispersion coefficient results:

\[ D_o = \frac{1}{3\pi^2} \gamma <u^2> T \quad (34) \]

Here \( \gamma \) is the ratio of the cross-sectional areas of the tidal flat and the main channel. Experimental evidence for mixing on tidal flats is shown in figure 10. For the prototype lagoon the ratio \( \gamma \) is order 0.2, yielding for the longitudinal dispersion coefficient a value of approximately 250 m\(^2\)/s. As, in general, the mixing on tidal flats is not complete, the theoretical value of \( D \) should be reduced accordingly.

Finally, as a particular case of "trapping", one may consider the water motion which occurs at channel junctions when phase differences exist between the discharge in the different channel branches (see figure 11). This effect is similar to trapping on tidal flats when the water level variation and the longitudinal velocity are out of phase. From the analysis in Dronkers (1978) it follows that the longitudinal dispersion in the main channel (1), caused by a small phase-shift \( \Delta t \) of the discharge in a branching channel (2), depends on the distance \( x \) from the channel junction. If the channel cross-sections and the velocities are independent of \( x \), the longitudinal dispersion may be computed from the following approximate expression:

\[ D_o = 2 \frac{A_o^{(2)}}{A_o^{(1)}} u_1^{(1)} u_1^{(2)} \frac{(\Delta t)^2}{T} \left[ 1 - \left( \frac{2x}{L} \right)^2 \right] \quad (35) \]

for \( |x| \leq \frac{1}{2} L \),

in which \( L = u_1^{(1)} T \) is the tidal excursion in channel (1). In figure 12 an example of a phase difference at a channel junction in the Oosterschelde is shown. According to equation (35), this phase difference produces a longitudinal dispersion of approximately 100 m\(^2\)/s in a region \( |x| \leq \frac{1}{4} L \) around the channel junction.
Figure 11: Dispersion resulting from a phase shift between the velocities at a channel junction.
Figure 12: An illustration of the phase shift between discharges at a channel junction in the Oosterschelde.
Such a localized contribution to the dispersion near a channel junction is, for instance, clearly visible in the results of Dorrestein and Otto (1960). For the Eems-estuary these authors found a pronounced increase of the longitudinal dispersion coefficient near the confluence of the Dollard channel and the river Eems.
CONCLUSION

It has been shown that the characteristics of mixing and flushing in tidal lagoons can be described in various different but connected ways. The most simple parameter is the overall flushing time scale of a lagoon, which, being an integral measure, characterizes the rapidity of water renewal in the lagoon as a whole. More detailed information can be obtained by looking at local time scales such as ages and residence times. Apart from being a function of the longitudinal dimension of the lagoon, all time scales bear an intimate relationship with the physical transport-processes. If the latter are purely dispersive, it is the distribution of the longitudinal dispersion coefficient that determines all of the mixing time scales. The dispersion coefficients, in turn, are related to and can be derived from the characteristics of the complete velocity field set-up by tide, wind and density differences.

In order to arrive at values for the mixing time scales it would therefore be a natural procedure to derive first the distribution of the longitudinal dispersion coefficient from the velocity field and then to calculate local and finally integral time scales from the dispersion coefficients. Unfortunately, however, even if sufficient information of the velocity field is at hand, it is nearly always impossible to say at first sight which of the various possible mechanisms dominates the dispersion process. This is because it is principally not allowed to say that the effective dispersion coefficient is the sum of the coefficients produced by the various dispersion mechanisms separately. Different mechanisms may influence each other; for instance, in the example discussed before, where the presence of residual circulation decreases the lateral mixing time scale and hence may increase or decrease the oscillatory shear dispersion coefficient depending on whether (23) or (24) applies. One is therefore nearly always obliged to use the distribution of a natural tracer, such as salinity, to calculate the flushing time scale and the longitudinal dispersion coefficients. The order of magnitude of the latter may then indicate the most plausible dispersion mechanisms, particularly if a good knowledge of the velocity field is at hand. Here numerical modelling can be
of help, but often the inverse is even more instructive: the dispersion coefficients derived from a tracer distribution in the field can be used as a guide showing whether or not the main dispersion factors in the velocity field are reproduced successfully in the model.

In summary, the analysis of mixing time scales and dispersion coefficients from field measurements in a lagoon can provide:
- an indication which mixing process dominates in a particular lagoon;
- insight in the mixing processes necessary for the interpretation of water quality parameters obtained from field data or for the interpretation of water quality simulations in mathematical or hydraulic models;
- qualitative estimates of the influence of minor changes in the hydraulic regime of a lagoon by engineering constructions.
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