Including secondary illumination in seismic acquisition design

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SUMMARY

A subsurface image obtained from seismic data is influenced by the acquisition geometry, as it contains an acquisition footprint which can obscure the true reflection response of the subsurface. Hence, the acquisition geometry should be designed in such a way that it allows high-quality images and fulfils the criteria for reservoir characterization. A comprehensive and quantitative assessment of 3-D acquisition geometries, taking into account the effects of the overburden, is provided by the so-called focal beam analysis method. Both the resolution and the amplitude accuracy can be estimated. So far, the primaries-only source wavefield is taken into account by this method. As using multiples in imaging and characterization is an emerging technology (Jiang et al., 2005; Verschuur and Berkhout, 2011), it is important to analyze their significance in acquisition geometry design as well. In this paper, the benefit of including surface-related multiples in acquisition geometry design through the focal beam method is analyzed. This analysis is important as the multiples may illuminate the subsurface from other angles than primaries, leading to a higher resolution at the desired target location. The concept of a secondary source beam (related to surface multiples) similar to a primary source beam (related to primaries) is formulated. The extra angles, by which a subsurface target point is illuminated, are displayed by the secondary source beam and illustrated here with the help of numerical examples.

INTRODUCTION

Conventionally, acquisition geometry analysis and design is based on common midpoint analysis. In this approach, the attributes that are used to judge an acquisition geometry are measures like CMP fold, offset distribution and azimuth distribution. In these methods, the influence of the subsurface on the data quality and image quality is not taken into account. However, in areas with a complex subsurface structure, survey design can no longer be based on midpoint analysis, since reflection points are not situated at the midpoint locations (Campbell et al., 2002). The assumption of a horizontally layered earth needs to be replaced by the introduction of a subsurface model. Awareness of this requirement has led to model based illumination analysis approaches (Chang et al., 2002; Lu et al., 2002).

Quantitative measures for image quality can be obtained by simulating migration results, as in such results the final image quality will be visible. A method providing quantitative and qualitative measures is focal beam analysis. The assessment of seismic acquisition survey geometry based on focal beam method was described by Berkhout et al. (2001); Volker et al. (2001); van Veldhuizen et al. (2008) and Wei et al. (2012). The assessment includes the computation of focal source beam and detector beam from which two further diagnostics are ob-

tained. The properties of these two focal beams are determined by respectively the source and detector geometries. The other two produced diagnostics are: resolution function and angle versus ray-parameter (AVP) imprint. These diagnostics are target point oriented i.e., they are determined per selected target point.

The separate analysis of source and detector geometry in the focal beam method provides the opportunity to obtain angle dependent information for illumination (source side) as well as sensing (detector side). In the case of primary illumination, the illumination part concerns the primary source. However, the subsurface is not only illuminated by primary source wavefields (generating the primaries), but also by secondary source wavefields (generating the surface-related multiples) as shown in Figure 1.

Figure 1: Extra illumination angles obtained at the target point from the secondary sources.

METHOD

The focal beam analysis method originates from the imaging by double focusing concept (Berkhout, 1997; Berkhout et al., 2001). This can be well explained by the **WRW** framework. In this framework, each monochromatic component $\mathbf{P}_0(z_0, z_0)$ of the primary wavefield (data without surface multiples) that is recorded at the surface z_0 , can be described in the space-frequency domain as:

$$\mathbf{P}_0(z_0, z_0) = \mathbf{D}^-(z_0) \sum_{m=1}^M [\mathbf{W}^-(z_0, z_m) \mathbf{R}^{\cup}(z_m, z_m) \mathbf{W}^+(z_m, z_0)] \mathbf{S}^+(z_0). \quad (1)$$

The subscript m in z_m indicates the different depth levels and it varies from 1 to M , if M is the total number of depth levels under consideration. The superscripts $+$ and $-$ denote downgoing and upgoing direction, respectively. $\mathbf{S}^+$ represents the source

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matrix where each column corresponds to the source wavefield due to one source (array). $\mathbf{D}^-$ represents the detector matrix where each row represents one detector (array). The $\mathbf{W}^+$ and $\mathbf{W}^-$ matrices are the downgoing and upgoing propagation operator, respectively. $\mathbf{R}^\cup$ represents the reflection matrix at each level z_m . Seismic migration can be represented as the application of two focusing operators to the recorded seismic data:

$$P_{ij}(z_m, z_m) = \vec{F}_i^\dagger(z_m, z_0) \mathbf{P}_0(z_0, z_0) \vec{F}_j(z_0, z_m),$$

$$P_{ij}(z_m, z_m) = \vec{F}_i^\dagger(z_m, z_0) \mathbf{D}^-(z_0) \mathbf{W}^-(z_0, z_m) \mathbf{R}^\cup(z_m, z_m) \mathbf{W}^+(z_m, z_0) \mathbf{S}^+(z_0) \vec{F}_j(z_0, z_m). \quad (2)$$

In this expression, one reflection level (z_m) is considered. The subscript i indicates the i^{th} row of the matrix is selected. A row vector is marked by a dagger symbol ($\dagger$). The subscript j indicates that the j^{th} column of the data matrix is selected. Row vector $\vec{F}_i^\dagger(z_m, z_0)$ represents focusing the detectors from the acquisition level (z_0) to a subsurface grid point at depth (z_m) with lateral location $(x, y)_i$, and similarly column vector $\vec{F}_j(z_0, z_m)$ represents focusing the sources from the acquisition level (z_0) to a subsurface grid point at depth (z_m) with lateral location $(x, y)_j$. Applying some imaging principle to equation 2 gives the reflectivity of that particular level.

In equation 2, the terms on the right hand side of the reflection matrix $\mathbf{R}^\cup$ represents the downward propagating focal source wavefield. This part of the double focusing expression is called focal source beam $\vec{S}_j$ at the target point at depth (z_m) and lateral location $(x, y)_j$:

$$\vec{S}_j(z_m, z_m) = \mathbf{W}^+(z_m, z_0) \mathbf{S}^+(z_0) \vec{F}_j(z_0, z_m). \quad (3)$$

The terms on the left hand side of the reflection matrix $\mathbf{R}^\cup$ concern the detector side of the seismic experiment: the upward propagation to the focal detector array. This part of the double focusing expression is called focal detector beam $\vec{D}_i^\dagger$ at the target point at depth (z_m) and lateral location $(x, y)_i$:

$$\vec{D}_i^\dagger(z_m, z_m) = \vec{F}_i^\dagger(z_m, z_0) \mathbf{D}^-(z_0) \mathbf{W}^-(z_0, z_m). \quad (4)$$

The focal source and detector beams can be combined (multiplied) to a full migration result: an image of the target point (the resolution function). By multiplication of the Radon transformed beams, the AVP function is obtained. This function shows the angle-dependent amplitude effects at the target point that are caused by the acquisition geometry. For the mathematical formulation of the focal beams in the Radon domain, the reader is referred to van Veldhuizen et al. (2008).

Including the surface-related multiples in the focal source beam formulation:

In seismic acquisition, sources and detectors are often situated at or near the Earth's surface (z_0), where the surface acts as a strong reflector. The monochromatic expression of surface seismic data ($\mathbf{P}$) - including surface-related multiples, is given by the following feedback model (Berkhout, 1982):

$$\begin{aligned} \mathbf{P} &= \mathbf{D}^- \mathbf{W}^- \mathbf{R}^\cup \mathbf{W}^+ (\mathbf{S}^+ + \mathbf{R}^\cap \mathbf{P}^-), \\ &= \mathbf{D}^- \mathbf{W}^- \mathbf{R}^\cup \mathbf{W}^+ \mathbf{S}^+ + \mathbf{D}^- \mathbf{W}^- \mathbf{R}^\cup \mathbf{W}^+ \mathbf{R}^\cap \mathbf{P}^-, \\ &= \mathbf{P}_0 + \mathbf{M}_0. \end{aligned} \quad (5)$$

For simplification, we have dropped the terms related to depth levels between the brackets from all the matrices. Here $\mathbf{R}^\cap$ represents the surface reflectivity and $\mathbf{P}^-$ is the total upward traveling wavefield. The total upward traveling wavefields are measured discretely at the detector locations and indicated as $\mathbf{P}$. It is important to realize that in a practical implementation, secondary downgoing wavefield is known only at the position of detector locations ($\mathbf{R}^\cap \mathbf{P}^- \Rightarrow \mathbf{R}^\cap \mathbf{D}^- \mathbf{P}^- = \mathbf{R}^\cap \mathbf{P}$). This means that the benefits to be obtained from the secondary illumination depend on the detector distribution of the acquisition geometry (Berkhout et al., 2012).

As explained in equation 3, the expression of the focal source beam for the primary data $\mathbf{P}_0$ contains one forward propagation term ($\mathbf{W}$) and a focusing term ($\mathbf{F}$). In terms of a matrix, it can be written as:

$$\mathbf{S}^p = \mathbf{W}^+ \mathbf{S}^+ \mathbf{F}^p. \quad (6)$$

Here superscript p represents the source beam for the primaries-only source wavefield ($\mathbf{S}^+$). In the ideal case, a focal beam that is computed at target depth will show one narrow peak at the location of the target point. This means equation 6 is an identity matrix in the ideal case:

$$\mathbf{W}^+ \mathbf{S}^+ \mathbf{F}^p = \mathbf{I}, \quad (7)$$

which leads to the following expression for primary focusing operator $\mathbf{F}_p$:

$$\mathbf{F}^p = \mathbf{I} (\mathbf{S}^+)^{-1} (\mathbf{W}^+)^{-1} \approx (\mathbf{S}^+)^H (\mathbf{W}^-)^*. \quad (8)$$

Here superscript H denotes the Hermitian operator (conjugate transpose in frequency domain). In a similar way, a secondary source beam can be written as:

$$\mathbf{S}^m = \mathbf{W}^+ \mathbf{R}^\cap \mathbf{P} \mathbf{F}^m. \quad (9)$$

Here superscript m represents the source beam for the secondary source wavefield ($\mathbf{R}^\cap \mathbf{P}$). Similarly, secondary focusing operator ($\mathbf{F}^m$) can be written as:

$$\mathbf{F}^m = (\mathbf{R}^\cap \mathbf{P})^{-1} (\mathbf{W}^+)^{-1} \approx \mathbf{P}^{-1} (\mathbf{R}^\cap)^{-1} (\mathbf{W}^-)^*. \quad (10)$$

Note that to compute secondary focusing operator, an inverse of data matrix ($\mathbf{P}$) should be taken. Inversion of the full data matrix in practice may produce cross talk noise, especially in the case of a complex model. In order to overcome this issue, we propose to select only strong surface-related multiples (e.g., water bottom multiples) and perform the inversion event by event via the conjugate transpose of data matrix. In this way, the conjugate transpose of a matrix is a good approximation of an inverse of a matrix.

EXAMPLES

To illustrate the concept of the primary and the secondary source beam, the following 3-D acquisition geometry is considered. It comprises of a densely sampled detector spread with a sampling interval of 50 m in both x and y -direction and only one source line of 3000 m aperture with a sampling interval of 100 m along the x -direction located at $y = 3500$ m as shown in Figure 2. Focal beams and focal functions are computed for

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Figure 2: The 3-D acquisition geometry template taken for the focal beam analysis.

this acquisition geometry for a target point located at $(x, y, z = 2000, 2000, 1000)$ m. For the computation of the forward and inverse extrapolation operators, a recursive wave field extrapolation in the (x, y, ω) domain with a weighted least-squares operator optimization method is used (Thorbecke et al., 2004). The velocity in the subsurface is assumed constant at 2500 m/s. The frequency range for which the analysis is carried out is 5-30 Hz.

Figure 3 shows the results of focal beams and focal functions for the considered 3-D geometry. Note that the detector beam shows that all angles are sensed and is ideal in the spatial as well as the Radon domain (Figure 3a and 3d), as expected from the densely sampled detector spread. On the other hand, the source beam suffers from poor illumination and does show only one direction of illumination in the Radon domain (Figure 3e). The focal functions are obtained by multiplication of the focal beams in the spatial and the Radon domain respectively. Because of multiplicative effects, this acquisition geometry is able to resolve the point diffractor properly but the AVP function shows a very limited range of angles in the ray-parameter domain (Figure 3c and 3f). It means that angle-

Figure 3: a) focal detector beam in the spatial domain, b) focal source beam in the spatial domain, c) the resolution function for a given target point, d) focal detector beam in the Radon domain showing sensing angles, e) focal source beam in the Radon domain showing illumination angles, and f) the AVP function showing illumination-and-sensing angles for the primaries-only wavefield.

dependent reflection information in the y -direction can not be retrieved properly from this acquisition geometry.

So far, we have seen the results of the focal beams and focal functions due to the primaries-only wavefield. Next, the computations are carried out for surface related multiples as a secondary source wavefield. In case of a homogeneous medium the target point can be illuminated by an additional angle by every additional order of surface multiple. The new angles of illumination are complementary to the angles of illumination due to primaries-only wavefield.

The secondary source beam is computed for the same target point using the modelled surface multiples. Figure 4, 5 and

Figure 4: Same as Figure 3 but for the 1st order secondary source wavefield.

Figure 5: Same as Figure 3 but for the 2nd order secondary source wavefield.

Figure 6: Same as Figure 3 but for the 3rd order secondary source wavefield.

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6 show the results of the focal beams and functions for each order of multiples separately. Note that the detector beam is unchanged as it should be, and the secondary source beam in the Radon domain provides different illumination angles than the primaries-only wavefield. For this simple, homogenous subsurface, as we increase the order of multiples, the apertures of the secondary sources get reduced which affects the focusing of secondary source beams in the spatial domain as can be seen in Figure 4b, 5b and 6b. But, at the end, we add all these secondary beams together with the primary beam. As a consequence, the resolution also gets improved because of the extra angle information as shown in Figure 7. As a final remark, we conclude that the primary and secondary source beam together will lead to a better focal source beam and AVP function.

Figure 7: a) Focal source beam (spatial domain) for the primaries-only wavefield, b) Focal source beam (spatial domain) for the primaries-plus-multiples wavefield, c) Focal source beam (Radon domain) for the primaries-only wavefield, and d) Focal source beam (Radon domain) for the primaries-plus-multiples wavefield.

In the previous example, the detector spread was densely sampled within the aperture range. As we mentioned, the benefits to be obtained from the secondary illumination depend on the detector distribution of the acquisition geometry. Next, we show the results of secondary illumination for the case that some of the detector lines are removed from the above acquisition geometry. In this example, detectors are now sampled sparsely along the x -direction with the sampling interval of 400 m (shown in Figure 8).

Focal beams and functions are computed for the same target point as in the previous example for the first order multiples only. The results are shown in Figure 9. Here as expected, the detector beam in the spatial domain shows some side lobes along the x -direction due to the sparse sampling. The larger line spacing along the x -direction is directly apparent in the Radon domain. The individual detector lines can be easily identified.

As the detector geometry is not perfect, its effects are clearly

Figure 8: A second example of a 3-D acquisition geometry.

visible on the secondary source beam both in the spatial as well as the Radon domain (Figure 9b and 9e). The poor focusing in the spatial domain and less extra angle information in the Radon domain compared to Figure 4e can be seen clearly. This explains the importance of a proper detector sampling for the use of secondary illuminations.

Figure 9: Same as Figure 4 but the effect of removing some of the detector lines on the focal beams and focal functions for the 1st order secondary source wavefield.

CONCLUDING REMARKS

The concept of a secondary source beam was introduced and demonstrated via numerical examples. It was shown that more angle-dependent information can be retrieved with the help of the secondary sources. It should be noted that the secondary wavefield ($\mathbf{R}^{\top}\mathbf{P}$) is measured at the detector locations, therefore, the benefits to be obtained from the secondary illumination depend on the detector distribution of the acquisition geometry. So the concept of secondary illumination favors an improved detector sampling, rather than the conventional rule of symmetric sampling.

It is expected that the information from multiples can be used for effective survey design which is our future focus, assuming that multiples will be used in imaging and inversion on a routine basis.

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EDITED REFERENCES

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