On factors affecting the quality of tomographic reconstruction

Adam Cheminet, Benjamin Leclaire, Frédéric Champagnat, Philippe Cornic and Guy Le Besnerais

1 Department of Fundamental and Experimental Aerodynamics, ONERA the French Aerospace Lab 
adam.cheminet@onera.fr
2 Department of Modeling and Information Processing, ONERA the French Aerospace Lab

ABSTRACT

In this paper we present experimental factors affecting the quality of tomographic PIV reconstruction. Our main contribution is to introduce a new factor built on the geometrical considerations applied to the cameras field of views and the laser volume. By performing numerical tests, we show the influence of this parameter the reconstruction using different quality measurements. We also analyze the reconstruction quality loss when illuminating larger volumes and when defocussing occurs. Finally, the effects of light scattering in the Mie regime on the reconstruction are studied. We show how Mie scattering effects ultimately lead to a signal to noise ratio loss in the reconstruction volume.

1. Introduction

In its most common form, the three-dimensional measurement of flow velocity by PIV relies on a two-step process, in which one first seeks to reconstruct a volumic intensity distribution from several camera images at two time instants, and then correlates the corresponding volumic distributions [1]. The first step, i.e. tomographic reconstruction, is entirely new compared to traditional plane PIV, and is known to have a strong impact on the final measurement quality. Its optimization is crucial to ensure the accuracy of the vector fields, and has received much attention since the introduction of the technique. In past studies, the influence of various experimental factors have been studied, in order to assess their effect, both on the reconstruction quality in itself, and on the displacements. The number and position of the cameras, their calibration, the particle image diameter in the images and the seeding density have been recognized as the most influential ones [1-2], and corresponding ranges of operation guaranteeing a good measurement quality have been derived.

While they have managed to provide useful guidelines and directions for designing more efficient reconstruction strategies, some of these studies have limitations, in the sense that they have partly oversimplified some experimental problems. This has recently led de Silva et al. [3] to propose a refined view, by considering 3D instead of 2D synthetic experiments, and introducing a simplified model for Mie scattering, among others. The present contribution proceeds from the same motivation, and aims at providing further landmarks and optimization guidelines for the experimentalists, by taking a step forward in the physical model complexity. As an expected by-product, our characterizations should also provide foundations for further refinement of the reconstruction algorithms, if possible. The influential parameters investigated in this paper are mostly of geometrical nature, in a direct or indirect way. We consider ranges of variations for these parameters which roughly match the case of high repetition rate measurements in air flows, raising specific constraints and challenges. We conduct synthetic reconstruction tests using MLOS-SMART [1][2], which we assess with the usual quality criterion initially proposed by Elsinga et al. [1], and also introduce new performance measures adapted to detection problems. We here focus on the reconstruction only, the subsequent impact on the displacement estimation being left to future work.

Four parameters are varied. The first parameter is seeding density, whose impact we will simply recall in order to provide a comparison with the following ones. Then we will show that when choosing the angular positions of the cameras relative to the illuminated volume, a new parameter should be considered, which is the ratio between the intersection and the union volumes of the cameras’ fields of view. As briefly mentioned in [1], the fact that some particles are not seen by all cameras acts as a source of noise and degrades the reconstruction. We show that this ratio is the adapted control parameter to quantify this phenomenon. In a third step, we investigate the impact of out-of-focus particles in the images. Here as well, we build a tuning parameter to control the degree of defocusing. Such a test is important in practice, as depth of field limitations are frequently encountered in tomo-PIV. In particular, these tests will allow to quantify the amount of degradation obtained if one tries to extend the size of the illuminated volume beyond the cameras depth of field, but when this defocussing is not taken into account. In other words, this may help define a boundary above which it is necessary to model it, such as in the approach of Schanz et al., or in our companion communication [7/8]. Finally, we will consider images synthesized using the full Mie theory, comparing with images built with different steps of approximation classically found in the literature. We will quantify the resulting differences.
in reconstruction quality, which will help to determine the degree of realism of these approximations depending on the setup geometry and nature of the seeding.

This paper is outlined as follows. Section 2 describes the principles of the synthetic tests, in terms of image generation, and choice for the quality criteria. The results of the parametric studies are then exposed in Section 3. Finally, concluding remarks are given in Section 4.

2. Principle of the tests

2.1. Image generation

We here describe the setup and parameters defining the synthetic test cases that will be used throughout the article.

![Figure 1: Typical camera and laser setup for synthetic experiment](image)

2.1.1 Optical Setup

All our simulations involve four cameras, which are positioned on a single side of the laser sheet at the vertices \((\pm 1/2, \pm 1/2, \pm 1/\sqrt{2})\) of a square of 1 m side. They are positioned at a distance of 1 m from the center of the reconstructed volume, the latter defining the origin \((0,0,0)\) and point at it. A pinhole model is assumed for the cameras, without Scheimpflug adapter for simplicity, and calibration is supposed to be perfectly known and to obey a pinhole model. The focal length is 100 mm, thus the magnification factor \(M\) is equal to 0.1, and the pixel size is 10 \(\mu\)m with a fill factor of 100%. The images' size, and hence the field of view, depends on the test cases and will be specified for each simulation.

The laser sheet is modeled as a 20 mm thick parallelepiped. Its intensity profile is assumed to be Gaussian in the \(z\) direction with a standard deviation \(\sigma_L\). However in this study the dependence in \(z\) will be very weak as we will consider the laser sheet as an almost perfect top hat with, \(\sigma_L = 0.05\) m. The direction of the light is taken as the \(\hat{x} = (1,0,0)\) axis. It’s wavelength is \(\lambda = 532\) nm.

The reconstructed volume, also 20 mm thick, is always the smallest parallelepiped including the illuminated volume seen by all the cameras. Thus it depends on the field of view and is given for each experiment. The voxel-to-pixel ratio is in this paper always set to one.

2.1.2 Tracer particles

Tracers particles are uniformly distributed in the light sheet volume. The density is controlled by the particle per voxel count (ppv). Horizontal and vertical extension of the laser sheet is larger than the field of view covered by all the cameras. Thus all illuminated particles cannot be seen by all cameras, a fact that always occurs in real dataset and is however often overlooked in synthetic experiments.

The particles are assumed to be spheres with diameters small enough (a few microns, i.e. we focus on experiments in air flows) to neglect the size of their geometric image \(M.d_p\). Given the size of particles typically used in a real experiment, emitted intensity is governed by Mie scattering, which applies to a sphere of diameter \(d_p \approx \lambda\). In this article, we will consider two choices for the intensities.

Unless otherwise specified, we will consider a traditional approximation, which considers the scattered light as proportional to the square of the particle physical diameter \(d_p\) and determines the intensity of a particle by its diameter and its depth only, via:
Here the particles' physical diameters are randomly drawn in $[\text{min}_{d_p} \text{ max}_{d_p}]$ according to a Gaussian distribution law with mean $m_{d_p}$ and standard deviation $\sigma_{d_p}$, with $m_{d_p}=0.5$ max$_{d_p}=2.5$ $m_{d_p}=1.5$. The distribution is controlled by $\sigma_{d_p}$, $\sigma_{d_p}=0.15$ yielding medium-low diameter scattering and $\sigma_{d_p}=0.5$ high scattering.

As already noted by de Silva et al. [3], and will be further confirmed in this paper, accounting more finely for Mie scattering may lead to significant changes in algorithms performance. In another series of synthetic experiments, we will thus introduce a complete model for this scattering in the generated images, contrary to de Silva et al. who rely on an approximation. Assuming the light to be non-polarized, Bohren and Huffman [6] show that the scattered intensity of a sphere particle is proportional to a scattering function $S_{11}(d_p, \theta)$ which depends on $d_p$ and $\theta$, the scattering angle being defined as the angle between the light source direction and the detector direction in the scattering plane of the particle. Figure 2 shows a typical scattering function for a particle of 2 $\mu$m in diameter. The refractive index of the particles in the air is taken as $n=1.47$.

**Figure 2:** Typical scattering function ($\log_{10}(S_{11}(d_p,0))$) of a 2 $\mu$m diameter particle of refractive index $n=1.47$

An algorithm developed by Bohren and Huffman [6] was implemented to compute the scattering function based on $d_p$, $\lambda$ and $n$. Within this exact model, the particles' diameter are still randomly drawn in $[\text{min}_{d_p} \text{ max}_{d_p}]$ according to a Gaussian distribution law with mean $m_{d_p}$ and standard deviation $\sigma_{d_p}$, this time with physical, dimensional values of $d_p$. In particular, one considers $\text{min}_{d_p}=0.2$ $\mu$m and $\text{max}_{d_p}=2$ $\mu$m.

In our camera setup, described in section 2.1.1, 2 cameras are in a forward scatter configuration (with a 60° scattering angle) and 2 cameras are placed in a backward scatter configuration (with a 120° scattering angle).

2.1.3 Final image intensities

Considering $P$ particles with intensity $E_p$ located at point $X_p$ in 3D-space, the intensity distribution in the image is given by:

$$I(x) = \sum_{p=1}^{P} E_p \cdot h(x - F(X_p)) \quad (2)$$

where $x=(x,y)$ denotes any location in the image plane, $F$ is the geometric projection function in the image, and $h$ the so-called Point Spread Function (PSF), which models the aperture limited diffraction and pixel integration.

For the tests presented in this paper, we assume a Gaussian Optical Transfer Function PSF with standard deviation $\sigma_{psf}$ set to 0.6 averaged on the pixel surface, with a 100% fill factor.

$$h(x, y) = \frac{1}{4} \left[ \text{erf} \left( \frac{x+0.5}{\sqrt{2} \sigma_{psf}} \right) - \text{erf} \left( \frac{x-0.5}{\sqrt{2} \sigma_{psf}} \right) \right] \left[ \text{erf} \left( \frac{y+0.5}{\sqrt{2} \sigma_{psf}} \right) - \text{erf} \left( \frac{y-0.5}{\sqrt{2} \sigma_{psf}} \right) \right] \quad (3)$$
Unless otherwise specified, we assume an image dynamic range of 8 bit, and a Gaussian noise with mean=5 and standard deviation 2 is added to the images. Its amplitude is set at about 10% relative to the maximum particle intensity. Note that in particular different settings of dynamic range and noise will be chosen when studying the influence of Mie scattering.

2.2. Reconstruction and quality measures

To reconstruct the volume, we build a first guess using an MLOS step [4], and then refine this reconstruction using SMART iterations [1]. In the context of our study, it is useful to recall some fundamentals of this algorithm, and what can be expected in some of the situations we consider.

SMART iteratively inverts the linear system

\[ I = WE \quad (4) \]

which links the image and volume intensities, I and E, through the sensing matrix W. Each iteration combines a projection from the volume, and then a back-projection from the images. Matrix W is here built in the exact same way as initially proposed by Elsinga et al. [1], as widely done in the literature. It is now well-known that since this system is highly underdetermined, reconstruction is affected by the presence of ghost particles, which act as a source of noise during the subsequent velocity estimation by cross-correlation. Aside from this lack of unicity, another, more fundamental issue is the existence problem, i.e. how ill-posed the problem is. Indeed, any linear system such as (4) is at least a little ill-posed, as approximations cannot be avoided; however, the approximation can be more or less relevant depending on the situations. In the present formalism, W is built on the basis of geometrical considerations, and does not include any angular variation due to Mie scattering, for instance. Thus, in the hypothesis of significant scattering differences, the projection step of SMART will consist in projecting a single voxel intensity with the same weight on all images, whereas the actual images will have different intensities. In such a situation, the problem will thus be more ill-posed than when scattering differences are negligible.

In practice, we use 20 SMART iterations, with a relaxation parameter of 1.1. The quality of the reconstruction is assessed by several means, the first of which being the well-known Q criterion [1]:

\[ Q = \frac{\sum_{x,y,z} E_0(x,y,z) \cdot E_r(x,y,z)}{\sqrt{\sum_{x,y,z} E_0^2(x,y,z) \cdot \sum_{x,y,z} E_r^2(x,y,z)}} \]

where \( E_r(x,y,z) \) is the reconstructed intensity field and \( E_0(x,y,z) \) is an ideal reconstruction intensity field (considered as the ground truth), in which a particle is seen as a 3D isotropic volume of Gaussian intensity whose standard deviation is the same as the standard deviation of the PSF in the images \([1][2]\). This quality factor Q is very well suited for classical tomographic reconstruction analysis where a particle is seen with the same intensity in every camera, i.e. when the scattered light is considered to depend only on the particle diameter and depth. However, taking Mie scattering into account means that the intensity of a particle differs from one camera to the other. In that case, the question of building a volumic ground truth becomes less obvious, as one cannot associate a single intensity value to a given particle. Computing a Q criterion then necessitates a choice for this intensity (e.g., for each particle, the highest intensity among all images), which diminishes its meaning as an objective quality criterion. In our study of Mie scattering, we will restrict our use of this quantity to the most relevant cases, and indicate precisely how we approximate the ground truth.

Another performance diagnostic is thus required in addition, which should be an accurate quality measure and compatible with all the experimental nuisance factors we consider here. We propose to introduce two quantities, which are well known measures in pattern recognition and information retrieval, Precision and Recall. A “detection” (i.e. here, a local maximum voxel of the reconstruction) is a True Positive (TP) if it is in the neighborhood of a true particle. Unless otherwise specified, the neighborhood is here a 3x3x3 voxels cube centered on the voxel of the true particle. A detection is a False Positive (FP), i.e. a ghost, if it is not in the neighborhood of a true particle. A particle is recorded as False Negative (FN) if there is no detection in its neighborhood. Precision quantifies the fraction of true particles among all detected particles, and Recall is defined as the number of true positive divided by the total number of true particles, i.e:

\[ \text{Precision} = \frac{\#TP}{\#TP + \#FP} \quad \text{and} \quad \text{Recall} = \frac{\#TP}{\#TP + \#FN} \]

where # stands for “number of”. The best achievable performance is given by Recall=1 (#FN=0, every particle is detected) and Precision=1 (#FP=0, all the detected particles are true).
Note that, whatever the quality criterion considered, and in all the simulations below, the ground truth will consist of the particles that are seen by all the cameras exclusively, which is consistent with the fact that all reconstructions are initialized with MLOS.

3. Results

3.1. Seeding density

The effects of image seeding density (Nppp) on the quality of the reconstruction has been widely explored in the literature, as the concentration of particle tracers is directly linked to the spatial resolution of the Tomographic PIV technique. Elsinga et al [1] showed that an increased particle density produces a larger amount of ghost particles, consequently decreasing the reconstruction quality and so the velocity measurement. Ultimately, a compromise must be reached between high spatial resolution and accuracy of the technique.

As this dependence is a well-known landmark to tomo-PIV users, we will here simply recall its amplitude, as a reference enabling to assess the relative impact of the other experimental parameters which we will consider in the following. Figure 4 (left) shows the decrease in quality of the MLOS-SMART reconstruction with respect to the seeding density Nppp.

3.2. Ratio between intersection and union volumes of the camera's fields of view

In this section, we focus on geometry-related considerations. In one of the conclusions of their study, Elsinga et al [1] recommended that the reconstructed volume should include all illuminated particles, since particles which lie outside this volume but are still visible by the cameras act as an important source of noise. However, it turns out that this situation inevitably occurs in practice, partially due to the reconstruction algorithms used. Indeed, common algorithms such as MART or SMART are multiplicative in nature, so that they only reconstruct particles seen by all cameras and automatically eliminate the others. Geometrically, actually reconstructed particles lie in the Intersection volume between the cameras' fields of view and the laser sheet, as depicted in Figure 3. As seen in this figure, the image recorded by each camera also includes particles which are not seen by the remaining cameras. These "added" particles lie in the Union between the cameras' fields of view and the laser volume, but do not lie in the Intersection. Figure 4 (left) shows the influence of Nppp on two tests: in one case, all the particles lie in the Union volume, in the second case, the particles lie in the intersection only. The quality difference between the two cases can be observed.

Because it is not possible in practice to overcome this situation due to the extension of the laser volume, this source of noise should be quantified. A natural choice for the associated control parameter is then the ratio between intersection and union volumes,

$$R_{IU} = \frac{\text{Vol(Intersection)}}{\text{Vol(Union)}}$$

Figure 3: Typical setup, with two cameras for simplicity, showing the difference between reconstructed volume (dashed black rectangle), union (green) and intersection (red) of the camera field of views.

$R_{IU}$ is varied without changing other experimental parameters by varying the sensor sizes (i.e., varying the number of pixels while keeping the size of each pixel constant), and thereby, the solid angles of the cameras' fields of view. In practice, this ratio will more probably vary with the cameras' angular positions. The reconstructed volume is systematically chosen as the smallest parallelepiped containing the Intersection volume. Figure 4 right shows that this parameter is indeed of utmost importance, as both $R_{IU}$ and Q follow the same evolution with image size, with a...
decrease in quality with decreasing $R_{I/U}$. Besides, the amplitude of this decrease within the present range of variation is comparable to the typical variation due to $N_{ppp}$ (Figure 4, left).

Figure 4: Influence of the seeding density $N_{ppp}$ (left), and of the $R_{I/U}$ ratio (right) on the reconstruction quality.

These “added” particles, lying in the Union but not in the intersection, act as a strong source of noise in the reconstruction: indeed, the inversion algorithm will try to explain the particle detected in the image as a particle in the intersection volume, where it is not. This will lead in an increase in the number of ghost particles. This phenomenon is clearly seen in Figure 5 where it is showed that the Precision increases with the $R_{I/U}$ ratio.

Figure 5: Influence of $R_{I/U}$ ratio on the Precision and Recall performance measures

3.3. Out-of-focus regions

When considering larger volumes than currently obtained when restricting to the camera depth of fields, defocusing of particle images will be observed. Some aspects of this issue have already been tackled in a recent study by Schanz et al. [5]; in particular, these authors have considered one instance of defocusing and varying seeding densities. Our companion paper [7/8] also proposes a complete "particle" approach, where this varying Point Spread Function can be accounted for. As a first quantifying step when these methods should be preferred, we propose here a systematic point of view in which the degree of defocusing is varied. This is equivalent to the progressive increase of the laser volume thickness ($z$ direction in Figure 1). In order to account for the limited depth of field, each camera is given a defocusing function $\sigma_{psf}(z)$ where $z$ is the space variable along the camera optical axis. The corresponding control parameter is built as the difference between the standard deviation $\sigma_{psf}$ of the particle with the largest image among all cameras and that of in-focus particles, ie $\delta \sigma = \sigma_{max} - \sigma_{focus}$. Simulations are run at $N_{ppp} = 0.055$ and $N_{ppp} = 0.098$.

Results in Figure 6 show that the main effect of increasing $\delta \sigma$ is not to increase the number of ghost particles neither after MLOS nor after SMART. Indeed, as seen the precision is only weakly dependent on $\delta \sigma$. However, as this parameter increases, the recall, i.e. the missed detections, decreases. At given $\delta \sigma$, the drop increases in magnitude with the seeding density. This behavior can be explained by unavoidable threshold applied to the image during the MLOS step. Increasing the defocussing parameter leads to a decrease in the intensity of the particle image which is subject to a
threshold, thus increasing the number of missed detection. Higher seeding densities can also lead to particle overlapping in the image, well-known for degrading the reconstruction quality. Therefore, conducting an experiment with a degree of defocusing, but without accounting for it in the reconstruction, will be at the cost of signal to noise ratio in the volumes used in the correlation. Contrary to the precedent conclusion referring to $R_{xy,ij}$, this will stem from a decrease in the number of true particles detected, the number of ghost remaining comparable.

![Figure 6: Recall and precision obtained at varying intensity of defocusing in the images, for $N_{ppp} = 0.055$ and $N_{ppp} = 0.098$, 512x512 images. MLOS (left), MLOS-SMART (right).](image)

3.4. Polydisperse seeding and Mie scattering

3.4.1 General remarks

In this section, we analyze the influence of both the dispersion in particle physical diameter $d_p$, which is common in practical seeding, for instance due to coalescence or when seeding is injected at a given upstream of the illuminated zone, and of Mie scattering. Both factors are indeed closely related to one another: taking Mie scattering into account means that the intensity of a particle image depends on two parameters: $\theta$ the scattering angle, and $d_p$ the particle diameter (the refractive index $n$ and laser wavelength $\lambda$ are constant throughout the whole paper). Figure 7 (left) shows the scattering function $S_{11}$, as a function of both variables, while Figure 7 (right) focuses on the dependence with the particle diameter at chosen scattering angles. Both plots show $\log_{10}(S_{11})$. The scattering function has a complex behavior with respect to both $d_p$ and $\theta$, which raises several issues that will be addressed below. An efficient way to distinguish between the various effects at play and unfold this complexity is to consider first a strictly monodisperse particle distribution (fixed $d_p$), and then the polydisperse case.

![Figure 7: Logarithm of the scattering function $S_{11}$ as a function of $d_p$ and $\theta$ (left) and as a function of $d_p$ for given $\theta$ (right).](image)

Assuming at first a fixed diameter $d_p$, the immediate consequence of Mie scattering regime is that, as expected, the intensity difference between backward and forward scatter may be of several orders of magnitude, i.e. one may observe
important differences in the average intensity levels of the images. A simple way to compensate them could be to determine the relative values of \( S_{11} \), and to rescale the darkest images in order to reach the level of the brightest images. This could be done as a pre-processing, or even be included in the construction of matrix \( W \), and would simply lead to images with different signal to noise ratio, as one would amplify the noise in the same way as the particles. However, this operation is only made possible if, within a given image, the variation in scattering angle and diameter remains moderate enough to avoid strong variations of \( S_{11} \) between different particles, or at different locations in the camera sensor for a fixed value of \( d_p \).

Our second interest will thus be to determine, still for a given particle diameter \( d_p \), under which conditions significant intensity variations should be expected within an image, due to the variation of \( S_{11} \) with \( \theta \), i.e. due to the variations in viewing angle from one end of the camera sensor to the other. To the best of our knowledge, this point has not been discussed or accounted for in past studies.

In a third step, we will then consider the full complexity of the problem, by assuming a dispersion in \( d_p \), i.e. a polydisperse seeding, which may also prevent the simple intensity compensation mentioned above. This corresponds to a very frequent situation, since even in well controlled experiments, a finite degree of dispersion is inevitably present. In that respect, we will particularly compare the usual approximation, where the scattered intensity is assumed to vary as \( d_p^2 \) with no dependence with \( \theta \), to the exact physical situation. Given the behavior observed in Figure 7 (right), this approximation may be a strong one, especially in air flows where small particles are frequently used. For instance, the \( d_p^2 \) behavior corresponds to the largest particles, while for the smallest a variation as \( d_p^6 \) is observed. The transition occurs roughly at 0.3 microns, which is among the typical values for seeding in the air. In this realistic situation, simple methods for compensating the illumination variations, or models accounting for them, should be very difficult to derive. Thus our goal will be to quantify the error which one does when synthetic tests only consider a partial modelling of Mie scattering and physical diameter dispersion.

### 3.4.2. Intensity difference between cameras

As mentioned above, the intensity difference between two cameras in backward and forward scatter configuration is the most obvious setback for tomo-PIV technique due to Mie scattering behavior. Indeed, it may lead to a practical situation where a particle seen in the forward scatter camera is missed on the backward scatter camera because its intensity is too low or comparable to that of the CCD noise and thus cannot be reconstructed, ultimately leading to a decrease in reconstruction and velocity field quality. Understanding the mechanism of such a phenomenon is therefore crucial.

For the sake of simplicity, and to clarify ideas, we introduce a first experiment in which a pure mono-disperse seeding is considered, with a simplified account of Mie scattering and not the full model. The particle intensities are computed through equation (1), without any angular dependence at first. Two cameras (\( x>0 \)) are considered in forward scattering configuration and the remaining two (\( x<0 \)) in backward scatter configuration; To account for this difference in scatter and corresponding intensity loss, the images of the cameras (\( x<0 \)) are multiplied by an attenuation coefficient \( \delta_{\Delta I} \). Note that in practice, such a coefficient can be determined quite easily, by reading for instance the values of \( S_{11} \) (Figure 2) for both sets of cameras. The seeding density was such that \( N_{ppp}=0.024 \). In this experiment, the reconstruction was done using 16 bit images for a better dynamics and the signal to noise ratio was increased to have a better understanding of the issue and avoiding too many particle detection losses in the backward scatter cameras due to thresholds in the images during the MLOS step.
**Figure 8**: Influence of the attenuation parameter $\delta_{\Delta I}$ between backward and forward scatter cameras on the Q reconstruction quality and on the Recall parameter.

**Figure 8** (left) shows the effect of this coefficient on the Q criterion. To build the ground truth corresponding to this case, we attributed to each particle the intensity corresponding to that observed on the brightest camera. As discussed in section 2.2, this is a first case in which the ill-posedness of the problem is increased. The observed drop in quality evidenced in **Figure 8** (left) is firstly due to an associated reconstruction intensity decrease; indeed, given the different intensities on the camera images for a same particle, the solution with minimal error consists in averaging these intensities, which is what SMART will converge to. **Figure 9** (right and left) illustrates this effect: the probability density function of the real particles shifts to the left and becomes more peaked (**Figure 9** right). By contrast, the intensities of the ghost particles are almost not altered. Note that these Probability Density Functions (PDF) are built from the local intensity maxima taken on a 3x3x3 voxels neighborhood. A second effect explaining this quality drop is the slight increase in detection loss: **Figure 8** (right) of the Recall parameter indeed shows that this inevitably occurs when $\delta_{\Delta I}$ increases. This phenomenon will amplify when considering wider ranges of intensity differences between the cameras.

**Figure 9**: Intensity PDFs of real and ghost maxima in the reconstructed volume for $\delta_{\Delta I} = 1$ (left) and for $\delta_{\Delta I} = 0.01$ (right).

To sum up, the intensity difference between forward and backward scatter cameras will lead to two combined effects which decrease the reconstruction quality and may be detrimental to the cross-correlation step: a smaller number of real particles is detected, and their intensity is weaker, leading to an decrease in the signal to noise ratio.

### 3.4.3. Intensity differences within an image

We still consider a mono-disperse seeding, and seek now to determine if some angular settings may lead to significant variations of the particle image intensities within a given camera image. To do so, we simply need to consider a one-dimensional image, whose size allows to span the scattering angle variation of a real image. For each pixel along the obtained line, we compute the corresponding value of $S_{11}$, which yields an intensity distribution along the image. We then compute the average $\mu_i$ and root mean square $\sigma_i$ of this distribution. This finally yields a convenient way to quantify the "intra-image" variation, by considering the relative fluctuation $q_i = \sigma_i / \mu_i$.

Results are presented in **Figure 10**. As could be expected, the dependence in $d_i$ and $\theta$ is rather complex. Logically, for the smallest diameters, $q_i$ remains very low, whatever the angle, due to the high regularity of $S_{11}$ in that case. For larger diameters, three zones can be distinguished. A common point to all these zones, is the high-frequency and regular alternation of peaks and valleys. Close to forward and backward scattering (i.e., respectively, for $120^\circ < \theta < 180^\circ$ and $0^\circ < \theta < 20^\circ$, approximately), high values of $q_i$ can be reached, with an order of magnitude of 1. For intermediate values $20^\circ < \theta < 120^\circ$, the maximum values are smaller, with an order of magnitude of 0.2.

To model the impact of this phenomenon on the reconstruction, we again consider a simplified framework, which allows faster processing but contains the entire problem complexity. As in section 3.4.2, we consider a test case with pure mono-disperse seeding whose intensities are simply computed with (I), i.e. still with no exact account of Mie theory angular. For each camera and for each particle, dispersion coefficients are randomly drawn from a Gaussian law,
(mean \( I_0 \) and mean root square \( \sigma_{I_0} \)) leading to an equivalent intensity dispersion in the image. Results in term of True detection and False detection are plotted in Figure 11 as a function of the control parameter of the intensity image dispersion \( \sigma_{I_0}/I_0 \). The seeding density was such that \( N_{ppp}=0.024 \). One does not observe a significant decrease in missed detections, but the number of ghost increases significantly. However, this has little impact on Recall and Precision quantities. Only a wider range of energy dispersion would lead to an increase in the number of missed detection due to threshold method for the MLOS step. As a conclusion, the energy dispersion in the image leads to a decrease in the signal to noise ratio in the volume.

![Figure 10](image1.png)

**Figure 10**: Level of intensity fluctuation \( q_I \) observed in the camera images as a function of the particle diameter and scattering angle. See the text for the definition of \( q_I \).

![Figure 11](image2.png)

**Figure 11**: True (left) and False (right) detected particles in the volume as a function of the image intensity dispersion \( \sigma_{I_0}/I_0 \)

### 3.4.3 Polydisperse seeding

We finally turn to the most difficult case of a polydisperse seeding, which we address in two steps, themselves of increasing complexity. The first one (Case 1) is the classically adopted approximation where particle intensities are independent of the scattering angle and vary as \( d_p^2 \), following equation (1). The second one (Case 2) is the exact computation of Mie scattering function \( S_{11}(d_p, \theta) \).
In order to discriminate between the effects, we consider another camera setup than that of Figure 1, in which cameras are placed on both sides of the laser volume, all of them in forward scatter with a scattering angle of 60°. As a consequence, there is no difference in the average intensities of the cameras and the observed effects will be due to the polydisperse character of the seeding and a particle’s intensity is function of its diameter with the dependency shown in Figure 7 (right). For both models, we consider a Gaussian distribution of \( d_p \) with variable standard deviation \( \sigma_{dp} \). In both cases, the control parameter is defined as the ratio between the standard deviation and the mean diameter called \( \gamma_{dp} \). The seeding density was such that \( N_{ppp}=0.024 \) in both cases.

Similar results between the two models are obtained in terms of True detected particles and false detections, and therefore in terms of Recall and Precision, as shown by Figure 12, right and left. One observes that as \( \gamma_{dp} \) increases, the precision does not vary much, and the recall tends to decrease. Both quality measures progressively drop. Case 2’s recall parameter drops faster than Case 1’s recall parameter. Indeed, taking into account the real diameter dependency due to Mie scattering increases the intensity dispersion in the volume compared to Case 1, thus increasing the number of missed detection due to a very low intensity level on certain particles.

**Figure 12**: Influence of normalized standard deviation of diameter distribution for simple Case 1 (left) and Case 2 (right)

This detection loss can be seen in the PDFs in Figure 13 and 14. The PDFs show that for a same \( \gamma_{dp} = 0.3 \), the reconstructed intensities in Case 2 are widely scattered compared to Case 1. Distinguishing between a real particle and a ghost particle becomes more difficult. The signal to noise ratio in the reconstruction volume is thus lowered in Case 2.

**Figure 13**: Case 1, for \( \gamma_{dp} = 0.01 \) (left) and \( \gamma_{dp} = 0.3 \) (right)
This shows that a proper modeling of the Mie function is crucial for an accurate prediction of accuracy loss in the reconstruction and ultimately in the velocity field.

4. Conclusion

In this paper, our aim was to further assess the influence of experimental factors on the quality of tomographic reconstruction for PIV. We identified a new factor built on geometric considerations. This parameter, $R_{IU}$, is the ratio between the intersection and union volume determined by the camera fields of view and the laser volume. The "added" particles, lying in the union but not in the intersection, act as a strong source of noise, degrading the reconstruction quality from a resulting increase in the number of ghost particles. By analyzing the influence of limited depth of field of the cameras, we showed that conducting an experiment with a degree of defocusing, but without accounting for it in the reconstruction, will be at the cost of signal to noise ratio in the volumes used in the correlation. Contrary to the precedent conclusion referring to $R_{IU}$, this loss comes from a decrease in the number of true particles detected, the number of ghost remaining comparable. Finally, we studied the effects of Mie scattering on the reconstruction. We classified the effects of light scattering in three physical phenomena: an intensity difference between images, intensity dispersion within an image and the dependency of intensity on the particle diameter. We showed that all three phenomena tend to decrease the signal-to-noise ratio in the reconstruction volume.

REFERENCES