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Testing of a method for spectral analysis of long wave data

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delft hydraulics
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Contents

List of figures

1 Background and objectives ........................................... 1

2 Algorithm .................................................................. 2
  2.1 Principle ............................................................... 2
  2.2 Fast implementation .............................................. 3
  2.3 Illustrative example .............................................. 4

3 Application to sea surface elevation data from Europlatform .......... 5
  3.1 Description of the data ......................................... 5
  3.2 Some results ....................................................... 5
  3.3 Smoothing of spectra ........................................... 6
  3.4 Selecting algorithm .............................................. 7
  3.5 Comparison with estimates from windowed discrete Fourier transforms . 8
  3.6 Practical considerations ...................................... 9

4 Conclusions and recommendations ........................................ 10

References

Figures
List of figures

1. Example of time-varying spectral estimates from test sequence
2. Sea level data from Europlatform as analyzed: 30 s averages
3. Periodograms (.) and smoothed estimates (-) from data in Figure 2
4. Surface plot of smoothed spectral estimates as a function of time
5. Periodograms (.) and estimates smoothed on a logarithmic frequency scale (-) from data in Figure 2
6. Smoothed estimates from Figure 3 and Figure 5 as a function of time, plotted on a linear frequency scale
7a. Time-averaged spectra from filter bank (-) and windowed Fourier transform (.)
7b. Reference spectra with (lower) and without (upper) data-windowing
8a. Time-averaged spectrum from filter bank (-) and reference spectra (--)
8b. Time-averaged spectrum from windowed Fourier transform (.) and reference spectra (--)


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1 Background and objectives

In port basins, long waves (seiches) in certain narrow frequency bands can be amplified considerably, depending on the dimensions and configuration of the basin. In order to control the effects of a new storm surge barrier in the port of Rotterdam on long waves, several studies were commissioned by Rijkswaterstaat to assess amplifications and statistics of long waves in the port of Rotterdam before and after the construction of the storm surge barrier and to study the effect of measures to reduce long waves; see e.g. [Vogel et al., 1991].

One of the outcomes was that the shape of the spectrum of incoming long waves is a rather important factor determining the ratios of long wave energy in different parts of the basin and for different basin configurations. In [Vogel et al., 1991], a white-noise spectrum (uniform variance density) of the long waves at the entrance of the port of Rotterdam was assumed, based on estimates derived from measurements in the port basin. Long wave spectra averaged over a three month period at three offshore locations appeared rather flat [De Valk, 1992]. However, to reach a definite conclusion about the shape of the offshore long wave spectrum, more than three months of data should be analyzed. Moreover, if the spectral shape varies in time, the spectral shape during a single storm can be less favourable than the average. Therefore, more information is needed in particular about the variability of the spectral shape of offshore long waves during periods of relatively high long wave energy. Besides that, our knowledge of the mechanisms of long wave generation on the North Sea should be improved in order to relate long waves to other phenomena such as the shorter gravity waves and atmospheric pressure variations. Understanding and quantifying such relationships is crucial for reliable statistics of long waves, of which no long term records are available. Second- and higher-order spectra of offshore waves over a wide frequency range can provide insight into the relationships between long waves and shorter gravity waves [De Valk, 1992]. Summarizing, information about the temporal variation of the offshore long wave spectrum is needed both for assessment of long wave statistics in port basins and to understand the relationships of long waves to better known phenomena such as surface gravity waves.

Currently, Rijkswaterstaat (The Netherlands Ministry of Transport and Public Works) is collecting and storing long wave data on the North Sea using the already available instruments and data-infrastructure [De Ronde en Dillingh, 1992]). These data need to be analyzed, based on which efforts in long wave measurement and data-processing can be planned for the next few years. In a study carried out previously for Rijkswaterstaat, Dienst Getijdewateren (Tidal Waters Division), methods and tools for collection and analysis of long wave data on the North Sea were examined; see [De Valk, 1992]. In that study, a method for estimation of time-dependent spectra was proposed for processing of long wave data.

The objective of the present study is to test this spectral estimation method and demonstrate its value for processing of long wave data by applying a prototype algorithm to a data record supplied by Rijkswaterstaat.

In Chapter 2 of this report, a short description of the spectral analysis method is given and results of its application to test data are shown. The application of the method to offshore sea surface elevation data is described in Chapter 3. Conclusions and recommendations pertaining to implementation and application of the method are given in Chapter 4.
2 Algorithm

2.1 Principle

An algorithm for estimation of time-varying spectra from data records was proposed in [De Valk, 1992]. Here, the ideas underlying this algorithm will be sketched; the interested reader can find the technical details in [De Valk, 1992] and in the next Section.

The principles can be summarized as follows. Time-varying spectral densities are defined at every instant of time as the instantaneous variances of the signal components within frequency bands of fixed nonzero width; the temporal scale of variation in the spectral density is then equal to the inverse of the bandwidth. The spectral analysis algorithm essentially consists of a bank of filters, each passing the signal component within a certain band of positive frequencies and stopping all other components of the signal. The result for a particular frequency band \([\omega_q, \omega_{q+1}]\) is a complex band-limited signal \(y_q\) of slowly varying amplitude and phase, which can be written as

\[
y_q(j) = a(j)e^{2\pi i \omega j}
\]  

(1)

in which \(\omega\) is any frequency in the band considered, and \(a\) is a complex-valued signal which only contains components in the low-frequency band \([0, \omega_{q+1} - \omega_q]\); \(a\) can be regarded as the time-dependent analog of a Fourier transform coefficient. A crude estimate of the time-varying spectral density (the time-varying periodogram) is obtained by multiplying the bandlimited signal \(y_q\) with its complex conjugate and normalizing, which eliminates the factor \(e^{2\pi i \omega j}\) in (1). This estimate can be refined by smoothing in the frequency domain.

In an implementation of this method, the band-pass filters applied to filter the signal have finite support only for practical reasons; the ideal band-pass filter has infinite support. This implies that frequencies are strictly localized within bands of nonzero width, but there is no strict time-localization in the sense that spectra relate to signal sections of finite length: the Fourier coefficients just vary as much as needed within the frequency band between zero and the filter bandwidth. This lack of strict time-localization does not need to be regarded as a disadvantage; strict localization in both time and frequency is fundamentally impossible, and the idea of decomposing the signal by band-pass filtering and monitoring the signal components is straightforward and easy to interpret. The only difference with the more familiar spectral analysis of stationary signals is that instead of evaluating the periodogram at discrete frequencies, its averages over frequency bands of nonzero width are evaluated. When comparing the filter bank with the more commonly applied method of windowed Fourier transforms (taking discrete Fourier transforms over short subsequences on which a data-window of a suitable shape has been applied; see e.g. [Daubechies, 1992]), differences are:

1. estimates obtained from a properly implemented filter bank do not suffer from the effects of limited record length which are inevitable when using windowed Fourier transforms;
2. spectra can be computed at arbitrary instants of time without much additional work.
The first difference is by far the most important. In particular, time-averages of spectra from a filter bank will be close to accurate spectral estimates for the entire data sequence, whereas the time-averages of estimates from windowed Fourier transforms will be biased, because a properly chosen data-window can at best only partially compensate for the effects of a limited record length. In Section 3.5, a comparison will be made which shows that for measurement of offshore long waves, this bias can be significant.

In the current implementation of the algorithm (see Section 2.2), all frequency bands are of equal width. One motivation for this choice is computational efficiency. Also, it should be possible to compare changes in the spectral densities over different bands and to relate those to other information such as the surface gravity wave spectrum and (spectra of) atmospheric parameters. Such comparisons require a common framework for data-analysis. It is expected that changes in spectra of long waves (as well as waves of shorter frequencies) are ultimately caused by atmospheric phenomena such as surface wind and pressure fields. It makes sense, therefore, to choose a minimum time-scale of changes in the spectrum which is also suitable for describing these atmospheric phenomena. Another motivation for using a uniform bandwidth is that computation of higher-order spectral estimates from the periodograms is straightforward. Summarizing, the choice of a uniform bandwidth has merely operational advantages; it does not reflect a general viewpoint. In fact, better temporal resolution over the higher frequency range (for example above 0.005 Hz) can be obtained by applying the algorithm separately with different resolutions of time and frequency to the components of the signal above, and below this frequency. It just takes a little extra work.

In a wider context, there are some fundamental advantages in choosing the bandwidth proportional to frequency. In that case, a spectral estimate applies effectively to a fixed number of waves. This is characteristic of a class of methods for temporal/spectral analysis known as wavelet transforms and subband-filtering [Daubechies, 1992]. A well-designed subband filtering scheme may be a suitable alternative for the filter bank proposed in this report, at least for the purpose of analyzing sea surface elevation records over a very wide range of frequencies and corresponding temporal scales.

### 2.2 Fast implementation

A fast implementation of this filter bank is described in [De Valk, 1992; Appendix A2]. Each of the band-pass filter in the filter bank is a convolution of the signal with a sequence of filter coefficients which can be implemented efficiently using the FFT (fast Fourier transform). Computing these FFT’s for all frequency bands would still require a considerable effort in practice. This can be further reduced if the bands are all of equal width and spectral estimates are only computed at instants of time separated by the inverse of the bandwidth. This is sufficient because the spectrum cannot vary at a time scale shorter than the inverse of the bandwidth. In this case, equation (39) in [De Valk, 1992; Appendix A2] can be used to compute the output of the filter bank, which can be written as

$$ y_q(jm) = \mathcal{F}^{-1} \sum_{k=0}^{n-1} \left[ \mathcal{F}^{-1} \sum_{r=0}^{m-1} \hat{x}(k+rp) \hat{h}_q(k+rp) \right] e^{2\pi i jk p} \tag{2} $$
with \( y_q \) the output sequence for band number \( q \), \( m \) the number of frequency bands, \( n \) the length of the filter (no. of coefficients) and \( p = n/m \) and \( \hat{h}_q \) and \( \hat{x} \) and \( \hat{h}_0 \) the discrete Fourier transforms of length \( n \) of a section of the signal and of the filter sequence, resp. Equation (2) represents the inverse discrete fourier transform of length \( p \) of the average between the square brackets. This average can be computed for all bands at the same time by means of discrete fourier transforms by realizing that for the fixed bandwidth of \( p \) bins, all bandpass filters can be obtained from one of them, say \( \hat{h}_0 \), by shifting along the frequency axis over multiples of \( p \) bins. Accordingly,

\[
\hat{h}_q(k) = \hat{h}_0(k-qp)
\]

and (2) can be written as

\[
y_q(jm) = p^{-1} \sum_{k=0}^{p-1} m^{-1} \sum_{r=0}^{m-1} \hat{x}^k(r) \hat{\bar{g}}_0^k(q-r) e^{2\pi i jkp}
\]

with \( \hat{x}^k(r) = \hat{x}(k+rp) \) and similarly, \( \hat{\bar{g}}_0^k(r) = \hat{h}_0(k-rp) \). The expression between the square brackets in (4) is a convolution, which can be computed in a single stroke for all \( q \) by a discrete Fourier transform, a multiplication and an inverse discrete Fourier transform.

The filter sequence represented in the discrete frequency domain by \( \hat{h}_0 \) is chosen such that in the time domain, its middle \( n/2 \) coefficients vanish, so by applying it repeatedly on half-overlapping subsequences of length \( n \) and saving the middle half of the result, it effectively represents a finite impulse response filter with a length of \( n/2 \) samples. The \( n/2 \) nonzero filter coefficients are determined analytically by the inverse Fourier transform of the ideal filter in the continuous frequency domain \([-\pi, \pi]\) to the discrete time-domain. Then a window is applied to reduce side-lobe leakage; in the tests carried out, a Hamming window [Rabiner and Gold, 1975] was used. This type of filter can be made arbitrarily accurate by taking \( n \) large enough while keeping the number of bands \( m \) fixed.

### 2.3 Illustrative example

To validate the algorithm described in 2.1, tests were carried out with simulated signals. One of these tests involved a sequence consisting of two sinusoids of distinct frequencies in the middle and white noise at the beginning and the end of the sequence. Figure 1 shows crude spectral estimates (periodograms) computed from this sequence. Clearly, the two sinusoids in the middle of the record are correctly identified; at the beginning and end of the sequence, messy periodograms are obtained without any clear trend, as should be expected from a white noise signal.
3 Application to sea surface elevation data from Europlatform

3.1 Description of the data

The data used for testing the spectral estimation algorithm were sea level data collected at Europlatform over the period Feb 16 to March 20, 1993 with a sampling frequency of 4 Hz. The data have been obtained from a capacitive sea level sensor which gives quantized values with a quantization interval of 0.05 m. The data file received contained raw data obtained from the sensor without correction. Upon a first inspection of the data, the first part was skipped from the record because too many wide gaps were present. Then to reduce the data volume, averages over 7.5 s, 15 s and 30 s intervals were computed and stored. It was decided to use the 30 s averages in the tests because the record could easily be manipulated on PC without running into storage problems. A sample spacing of 30 s is small enough to obtain accurate spectral estimates at frequencies up to at least .007 Hz (or 2.5 minutes). The record of 30 s averages was then plotted and inspected for gross errors, which were corrected manually by interpolation. However, many apparent errors of relatively small magnitude were found in the data which could not be corrected. The main reason for not running tests with smaller sampling intervals was that (manual) correction of these data would have taken too much time.

A plot of the data is shown in Figure 2; plotted is the data segment over about 14.2 days which corresponds with the time interval over which spectra have been computed (see Section 3.2). The beginning of this record is at Feb 26, 1993. In the first data segment plotted, waves of various frequencies are clearly visible; the waves attenuate in consecutive data segments. Fluctuations on the tidal signal in the last segment are apparently just errors on the data.

3.2 Some results

The first test of the method was carried out using a filter array length $n$ (see 2.2) equal to 16384 samples (about 137 hours) and with $m = 1024$ frequency bands (so 512 nonnegative frequency bands). With the time step of 30 s, this gives estimates of spectral density with a frequency bandwidth of $3.3 \times 10^{-5}$ Hz. Output spectra (periodograms) were produced every 30,720 s (every 8:32 h). Some periodograms are plotted as small dots on a logarithmic scale in Figure 3; they correspond to instants separated 42:40 h, printed in the upper right corner of each figure. In order to facilitate comparison of the spectral estimates with the data, the instants printed in Figure 3 refer to the time-coordinate along the horizontal axis in Figure 2. In addition, spectral estimates obtained by smoothing the periodogram are plotted in the same figure (continuous curve); the smoothing will be discussed later.

In all spectra, the astronomical components are clearly dominating in the low frequency range. In the beginning, we see a rather high level of the long wave energy, in particular in the higher frequencies, with the spectral density increasing with frequency from about .002 Hz (8 minutes). Later, the spectral densities decrease and the spectra become rather flat over the higher frequency range, settling at a level of approximately .001 m²/Hz. In the
plotted periodograms starting from that of 6.4 days, narrow regularly spaced peaks are visible with a density of a little under 0.1 m²/Hz, showing up as about ten dots. This is very regular and is probably related to the occurrence of small disturbances in the data. A remarkable feature are the occasional blobs of energy in the low frequency range next to the astronomical peak; one is visible in the spectra at 2.88 days and 13.51 days in Figure 3 but they are more clearly visible in a surface plot of the smoothed spectral estimates as a function of frequency and time in Figure 4. Inspection of the data learned that similar features corresponded to relatively large errors in the data; they disappear when the errors were corrected by replacing bad samples by interpolation. Clearly, such errors can spoil spectral estimates over the entire range of periods of interest from several minutes to a few hours. Therefore, good data validation and correction procedures are absolutely necessary for measurement of spectra of offshore long waves. Whether all remaining blobs of energy in the low frequencies in Figure 4 are due to errors or some represent features of the signal is not clear yet; this can only be discovered by computing spectral estimates from a thoroughly validated record.

3.3 Smoothing of spectra

Periodograms (the dots shown in Figure 3) are not suitable for routine presentation. For that purpose, some experiments with smoothing of spectra were carried out. The smoothed spectral estimates displayed in Figures 3 and 4 were obtained with cubic spline smoothing of the logarithm of the periodogram, using the method implemented in the MATLAB Spline Toolbox [Der Boor, 1990]. The rationale behind smoothing of the logarithm of the periodogram is that errors in the periodogram are approximately proportional to the spectral density, so the error statistics of the logarithm of the periodogram are uniform over the entire frequency range. The degree of smoothing can be controlled by a real parameter which should be chosen between zero and one. A disadvantage of this ways of smoothing is that the degree of smoothing is everywhere the same, whereas it is clear that the magnitudes of derivatives of the spectral density are much higher near the astronomical peak than in the high frequency range. This became particularly obvious when plotting the smoothed spectra with a double logarithmic axes scaling, showing a very poor fit in the lower frequencies where more detail is required. Therefore, the same method of smoothing was applied to the logarithm of the periodogram as a function of the logarithm of frequency. The resulting estimates as well as the periodogram are shown in Figure 5; periodogram values are shown as dots in the centres of the bands, and the smoothed estimates are displayed by a staircase plot which is constant over a band. In Figure 6, the results of both methods of smoothing are displayed as surfaces similar to Figure 4, using a linear frequency scale. Smoothing of the logarithm of the periodogram as a function of the logarithm of frequency clearly gives an estimate which is easier to interpret. This way of smoothing gives estimates which have lower variance in the high frequency range than in the low frequency range. Intuitively, one would expect this when using a uniform bandwidth, because a fixed record contains more waves of high frequency than of low frequency.
3.4 Selecting algorithm parameters

There is some freedom in choosing filter length $n$ and number of frequency bands $m$ (related to bandwidth $1/(m\Delta t)$ with $\Delta t$ the time step) which can be used to optimize the results. A very small bandwidth is desirable, but this also limits the temporal resolution because the time-interval between spectral estimates is equal to $m\Delta t$. The choice of 8:32 h for $m\Delta t$ (30,720 s) is not bad. In the low frequency range, upper limits of bands correspond to periods of 8:32 h, 4:16 h, 2:57 h, 2:08 h, 1:42 h, 1:25 h, 1:13 h, 1:04 h, 0:57 h, 0:51 h, etc. so the resolution is not high but probably sufficient in practice. Higher resolution in frequency would result in estimates varying on a longer time-scale than every 8 hours. Lower resolution is not desirable in particular because to compute accurate spectral estimates, still some smoothing is required. It appears from Figure 4 or 7 that variations in the spectrum on shorter time-scales are probably not very significant in this case; if this holds in general, then the temporal resolution is sufficient. However, if from other tests the temporal resolution would appear insufficient, then the bands should be chosen wider.

Improvement may be expected from increasing the filter length $n$ because this will improve the separation of components in different frequency bands. This will not reduce the variance of the estimates, however! The length of the record used in the test was not sufficient to be able to try much larger values of $n$; it is recommended to determine final value of $n$ by increasing it with $m$ fixed until the estimates do not change much any more, or otherwise the largest value which is feasible.

The time step of 30 s over which the data were averaged before sampling seems to produce reasonable results; however, the results of simulating averaging of sea level data given in [De Valk, 1992; Figure 19], show that the spectral estimates are reliable up to a frequency of about 0.006 Hz (2.5 minutes) only. The larger part of a plot as in Figure 3 is therefore not reliable and may give a wrong impression. Therefore, it is recommended to plot spectral estimates only up to about one fifth of the sampling frequency, or to plot them on a logarithmic frequency scale as in Figure 5.

It is recommendable to produce reliable spectral estimates up to a frequency of at least 0.02 Hz (50 s) in order to be able to relate the long wave spectrum to that of short waves, which usually begins at about 0.03 Hz. When using simple averaging before resampling as in this study, this requires a time step of at most 10 s. When using a more sophisticated lowpass filter, for example of the same type as used in spectral estimation method, a slightly longer time step may be used or spectral estimates are reliable up to a somewhat higher frequency, but this is probably not worth the extra effort.
3.5 Comparison with estimates from windowed discrete Fourier transforms

The common method for estimating time-varying spectra is to apply a window on data subsequences of fixed length and computing periodograms from the FFT’s of the windowed subsequences. To compare this method with the results of the filter bank described in Chapter 2, estimates by both methods were made using a band-width (or frequency spacing) of 1.3 \(10^4\) Hz (\(m = 256\)) and for the filter bank, a filter length \(n\) of 49152 samples (about 410 h). Estimates were computed at instances separated by 7680 s (2.08 h). Averages of these estimates are plotted in Figure 7a; for the filter bank, the values have been plotted in the central frequency of each band. The estimates agree in the high-frequency range, but estimates from the windowed FFT’s are clearly higher in the low frequency range, differing a factor of about two over a rather wide range of frequencies.

As a standard for comparison of these results, two high-resolution spectral estimates were made based on an FFT of the entire data sequence, which were subsequently averaged over the same bands as used in the filter bank. One of these estimates was made using a rectangular data-window; for the other estimate, a Hamming window was used. These two estimates are shown in Figure 7b. As expected, the estimate based on the Hamming window falls off much more rapidly above the astronomical peak than does the estimate based on the rectangular window. Because of the lower bias, the estimates based on the Hamming window are the most reliable in the lower frequencies. However, the rectangular window also leads to higher spectral densities in the high frequency range, were both estimates should be identical. The difference can be explained by the fact that the highest variance in the higher frequency range is found at the beginning of the sequence, which is largely reduced by the Hamming window. Therefore, in the higher frequency range, the estimates from the rectangular window are more representative.

Comparing the averages of spectra from filter bank and windowed FFT with the two reference spectra obtained from an FFT over the entire sequence, we see that in the high frequency range, the result of filter bank and windowed FFT both agree with the reference spectrum based on a rectangular window; see Figure 8. In the lower frequency range, the average spectral estimate from the filter bank agrees closely with the reference spectrum based on the Hamming window; the result of the windowed FFT is somewhere between both reference spectra over most of the lower frequencies (except the lowest, where the peak is wider than for all other estimates). Another difference which can be observed is that the estimate from the filter bank follows the smaller narrow peaks much better than the estimate from the windowed FFT does. Summarizing, the comparison shows that the filter bank performs considerably better than the windowed FFT using a Hamming window, according to the standard that time-averaged spectral estimates should agree with accurate spectral estimates from the entire data-sequence. Using certain other types windows in the windowed FFT, it may be possible to obtain somewhat better results than with a Hamming window, but the effects of a short record length can never be fully compensated. Moreover, when errors in the data can be suppressed more effectively, the spectral estimates in the lower frequency range will become more affected by bias because the slope of the astronomical peak will be even steeper. If one chooses to use the windowed FFT, then it is recommendable to present the averages over a long time-interval together with overall spectral estimates of the same interval as above, in order to enable assessment of the effect of bias.
3.6 Practical considerations

An important issue in practice is which data to store. As the spectral estimation hardly reduces the data volume, it is recommended to store the original (validated and if necessary, corrected) 10 s averages, rather than the estimates. The estimation can always be rerun at a later stage.

Most errors present in the data will already have been removed or considerably suppressed after the 10 s averaging when using appropriate data-correction procedures. Rejected 10-s averages can be corrected by interpolation (crude approach), or probably much better, by using a variation on the Lomb method [Press et al., 1992; pp 569-577] to obtain a reliable approximation of the Fourier transform of the signal. In the presentation of a spectral estimate at a given instant, the quality can be indicated by printing the percentage of missing or bad samples in a time-interval of length of twice the reciprocal of the bandwidth, centered at that instant.

For the record, some characteristic parameters of the spectrum over the frequency range of interest can be stored. One option is to store the total variances in the five frequency intervals [10^{-2} Hz, 10^{-1.5} Hz], [10^{-2.5} Hz, 10^{-2} Hz], [10^{-3} Hz, 10^{-2.5} Hz], [10^{-3.5} Hz, 10^{-2} Hz] and [10^{-4} Hz, 10^{-3.5} Hz], which is just five numbers a few times a day. Based on these five numbers, a procedure determining whether to store or discard data can be made. When discarding data, it is important to make sure that the remaining records are long enough to estimate spectra reliably. In practice, storing all data over the winter season Oct 1 to March 30 will probably be most practical; it can be expected that significant long wave energy is present frequently during this season, and storing data records covering less than one month is not a good idea anyway. Given these facts, the easiest and most straightforward procedure might just be to store all data over a winter season. These will not take more than about 1.6 Mwords (say about 6 Mbytes) of memory for a 10 s sampling interval. Storing all data over a winter season will also simplify rerunning the spectral estimation considerably, because the algorithm can be applied routinely with standard values for filter length, bandwidth etc. without having to adjust these to the record length.
4 Conclusions and recommendations

1. A prototype of a filter bank algorithm for estimation of time-varying spectra has been implemented and applied to a data-set of sea level data from Europlatform collected in February/March 1993. A relatively undisturbed section of the data was first converted to 30 s averages and resampled and gross errors were corrected by interpolation. In a qualitative sense, the spectral estimates agree well with the temporal variation of the long wave amplitude observed in the data.

2. A quantitative comparison of the filter bank method with the commonly applied method of windowed Fourier transforms learned that time-averages of spectral estimates by the filter bank method are significantly less biased over a rather wide range of frequencies of interest. Based on this comparison, the filter bank method is recommended for spectral analysis of long wave data.

3. Thorough validation and correction of the data prior to analysis appears absolute necessary. The spectral estimates appear to be quite sensitive to errors in the data, in particular to outliers. Gaps can be corrected by interpolation (crude approach), or probably much better, by using a variation on the Lomb method [Press et al., 1992; pp 569-577] to obtain a reliable approximation of the Fourier transform of the signal. The quality of a spectral estimate at a given instant can be indicated by the percentage of missing or bad samples in a time-interval of length of twice the reciprocal of the bandwidth, centered at that instant.

4. The chosen bandwidth of 3.3 \times 10^{-5} \text{Hz} (30,720 s), giving output spectra every 8:32 h, appears satisfactory. If at a later stage the temporal resolution would appear to be insufficient, then the bandwidth can still be increased somewhat. Alternatively, this could be done only for the higher frequencies. The filter should be long enough that increasing it will not change the spectral estimates. This could not be tested on the available data but during the implementation, it can be easily tested on a longer record.

5. Averaging of the sea level data over 10 s intervals prior to analysis is recommended in order to produce spectral estimates which are reliable (in terms of bias) for frequencies up to 0.02 Hz (50 s); this will facilitate relating the results to the spectral estimates of short waves. Roughly speaking, spectral analyses based on averaged data are reliable for frequencies up to five times the sampling frequency.

7. Both for presentation and analysis, smoothing of crude spectral estimates is needed. Cubic spline smoothing of the logarithm of the spectral density on a logarithmic frequency scale is recommended. Presentation of spectral estimates on a double logarithmic scale is recommended. Surface plots (and other methods such as image plots) are recommended for presenting the evolution of the spectrum in time.
8. It is recommended to store data (e.g. the 10 s averages mentioned above) rather than spectral estimates. Storing spectral estimates instead of data would not reduce the data volume, and if data are stored, reanalysis or analysis of different aspects of the data is still possible. For practical reasons, continuous storage of data over at least winter seasons from October 1 to March 30 is recommended. The variances over five wide frequency bands can be stored for identification of periods of high long wave energy.
References


EXAMPLE OF TIME-VARYING SPECTRAL ESTIMATES
FROM TEST SEQUENCE

DELFt HYDRAULICS
Sea Level Data from Europlatform as Analyzed: 30 S Averages

Delft Hydraulics

H 1772

FIG. 2a
Data Europlatform.

sea level relative to mean [m]

0 0.5 1

-0.5 1 1.5 2

-1 2 2.5

time [days]

Data Europlatform.

sea level relative to mean [m]

0 0.5 1

-0.5 3 3.5 4

-1 4.5 5 5.5

time [days]

SEA LEVEL DATA FROM EUROPLATFORM AS ANALYZED: 30 S AVERAGES

DELFt HYDRAULICS
Data Europlatform.

Sea level relative to mean [m]

-1 -0.5 0 0.5 1

11.5 12 12.5 13 13.5 14

time [days]

SEA LEVEL DATA FROM EUROPLATFORM AS ANALYZED: 30 S AVERAGES

DELFH HYDRAULICS

H 1772 FIG. 2c
PERIODOGRAMS AND SMOOTHED ESTIMATES
FROM DATA IN FIGURE 2

DELFT HYDRAULICS
Periodogram (.), smoothed estimate (-)

Periodogram (.), smoothed estimate (-)

Periodograms and smoothed estimates from data in Figure 2

Delft Hydraulics
PERIODOGRAMS AND SMOOTHED ESTIMATES
FROM DATA IN FIGURE 2

DELFt HYDRAULICS
periodogram (・), smoothed estimate (—)

spectral density [m^2/Hz]

frequency [Hz]

11.73 days

periodogram (・), smoothed estimate (—)

spectral density [m^2/Hz]

frequency [Hz]

13.51 days

PERIODOGRAMS AND SMOOTHED ESTIMATES
FROM DATA IN FIGURE 2

DELFt HYDRAULICS
SURFACE PLOT OF SMOOTHED SPECTRAL ESTIMATES AS A FUNCTION OF TIME

DELFt HYDRAULICS
periodogram (•) and smoothed spectral estimate (−)

spectral density [m²/Hz]

frequency [Hz]

1.067 days

periodogram (•) and smoothed spectral estimate (−)

spectral density [m²/Hz]

frequency [Hz]

2.844 days

PERIODOGRAMS AND ESTIMATED SMOOTHED ON A LOGARITHMIC FREQUENCY SCALE FROM DATA IN FIG. 2

DELFt HYDRAULICS

H 1772 FIG. 5a
periodogram (.) and smoothed spectral estimate (-)

spectral density [m^2/Hz]

frequency [Hz]

4.622 days

periodogram (.) and smoothed spectral estimate (-)

spectral density [m^2/Hz]

frequency [Hz]

6.4 days

PERIODOGRAMS AND ESTIMATED SMOOTHED
ON A LOGARITHMIC FREQUENCY SCALE
FROM DATA IN FIG.2

DELFt HYDRAULICS

H 1772   FIG. 5b
periodogram (.) and smoothed spectral estimate (-)

spectral density [m²/Hz]

frequency [Hz]

8.178 days

periodogram (.) and smoothed spectral estimate (-)

spectral density [m²/Hz]

frequency [Hz]

9.956 days

PERIODOGRAMS AND ESTIMATED SMOOTHED
ON A LOGARITHMIC FREQUENCY SCALE
FROM DATA IN FIG.2

DELT HYDRAULICS
PERIODOGRAMS AND ESTIMATED SMOOTHED ON A LOGARITHMIC FREQUENCY SCALE FROM DATA IN FIG. 2

DELFt HYDRAULICS  H 1772  FIG. 5d
SMOOTHED ESTIMATES FROM FIG. 3 AND FIG. 5
AS A FUNCTION OF TIME, PLOTTED ON
A LINEAR FREQUENCY SCALE

DELFIT HYDRAULICS
average spectra: filter bank (-) and windowed fft (.).

spectral density [m²/Hz]

frequency [Hz]

TIME-AVERAGED SPECTRA FROM FILTER BANK AND WINDOWED FOURIER TRANSFORM

DELFt HYDRAULICS
average spectra with (lower) and without (upper) data-windowing.

**REFERENCE SPECTRA WITH AND WITHOUT DATA-WINDOWING**

DELFt HYDRAULICS

H 1772 FIG. 7b
average spectrum from filter bank (-).

average spectrum from windowed fft (-).

TIME-AVERAGED SPECTRA FROM FIG.7a (-) AND REFERENCE SPECTRA FROM FIG.7b (--)
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