Time scales and uncertainty

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Facets of uncertainty – Kos Island, 17-19 October 2013
1 Introduction
- Examples

2 Integral form + interval analysis
- Applied to models
- Background
- Application
What this talk is not about

- Epistemic versus Aleatory uncertainty
- Model uncertainty
- Uncertainty in measurement values
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So what is it about?

- Relation between system time scales (model parameters) and uncertainty
- Quantifying uncertainty due to time step size
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Assumptions

- Model is correct
- Measurements are exact
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Example

Rain in mm

\[ t_0 \int_0^s p(s)ds \]

Figure: Uncertainty about cumulative total.
Precipitation measurements

Example

Rain in mm

Figure: Uncertainty about cumulative total with upper bound on intensity.
Stream flow measurements

Example

Stream flow in cubic meters per second

Figure: Uncertainty.
Example

Stream flow in cubic meters per second

Figure: Uncertainty with bounds on rate of change.
Outline

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Time step and uncertainty
Time scales

- measurement time step
- model time step
- physics (of system)
- meteorology (duration of dry spells vs. wet spells)
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Examples

- multi reservoir system
- reservoir with threshold
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Multi reservoir system

\[ q_{out} = c_2 h_2 + c_3 h_3 \]

**Figure**: Schematic
Effect of system response time scales

**Figure:** Physical system time scale effects (q)
Effect of precipitation structure

Figure: Input time scale effects (q)

(a) precipitation sums

(b) precipitation sums

(c) convolution q lambda=0.25, c=(1,0.25,0.75), h = (1.1,1.1,1.1)

(d) convolution q lambda=0.25, c=(1,0.25,0.75), h = (1.1,1.1,1.1)
Reservoir with threshold

**Figure:** Level in reservoir with threshold (first attempt)
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[1, 2] * [3, 4] = [3, 8]

\[ \sin ([3/2, 4/2]) = \min (\sin (3/2), \sin (4/2)), 1 \]
Advantages

- Hard bounds on location of ODE solution
- Can represent additional uncertainty
ODE in integral form

ODE

\[
\frac{dh}{dt}(t) = p(t) - ch(t)
\]

\[
h(t_0) = h_0
\]

Integral equation

\[
h(t) = h_0 + \int_{t=t_t}^{t} p(\tau) d\tau - \int_{t=t_t}^{t} ch(\tau) d\tau
\]

Works for ODE’s and systems of ODE’s
Advantages

- Precipitation: often \( p \) unknown, but integral known at \( t_k \).
- Integral form more general
- Specialization possible for one specific output
Interval system state $q$ $\lambda=0.25$, $c=(1,0.25,0.75)$, $h_0=(1.1,1.1,1.1)$

**Figure:** Different approaches to $q_{out}$ calculation
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Time step and uncertainty
Example

1. Select parameter vector (or parameter interval vector)
2. Match model output (intervals) to system output measurements
3. Accept/reject vector or interval vector
4. Use this to restrict search space/generate better prior
Summary

- Time step and system time scales ⇒ bounds on uncertainty due to measurement time step
- Derivable for linear systems using interval analysis
- First results for systems with thresholds
Thank you for your attention